BULK PROPERTIES OF STRONGLY INTERACTING MATTER: RECENT RESULTS FROM LATTICE QCD

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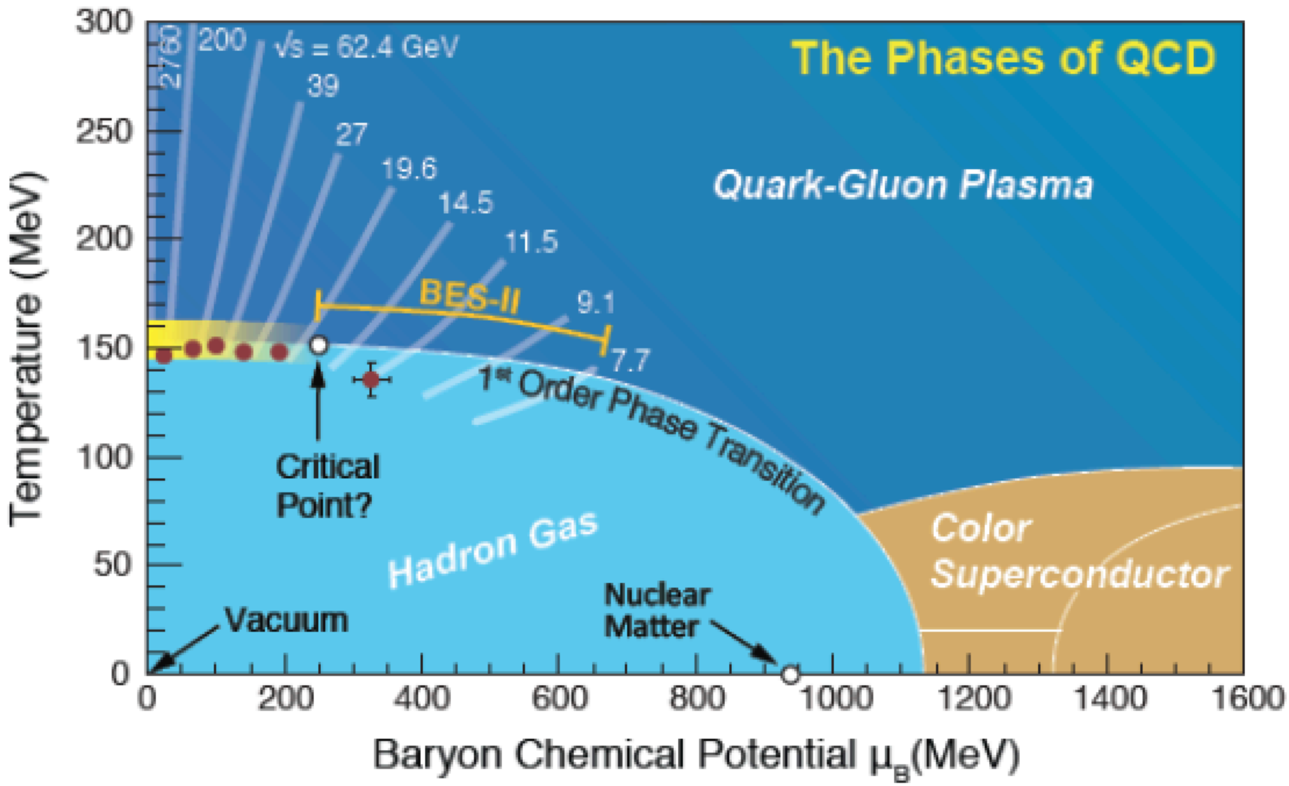
QCD phase diagram



- Lattice QCD: analytic crossover at $\mu_B = 0$
- Effective models suggest the presence of a critical point
- Experiments at lower collision energies explore higher μ_B region (BES @ RHIC)

Lattice QCD

- Best first principle-tool to extract predictions for the theory of strong interactions in the non-perturbative regime

- Uncertainties:
  - Statistical: finite sample, error $\sim 1/\sqrt{\text{sample size}}$
  - Systematic: finite box size, unphysical quark masses

- Given enough computer power, uncertainties can be kept under control

- Results from different groups, adopting different discretizations, converge to consistent results

- Unprecedented level of accuracy in lattice data

Low temperature phase: HRG model

Dashen, Ma, Bernstein; Prakash, Venugopalan, Karsch, Tawfik, Redlich

- Interacting hadronic matter in the ground state can be well approximated by a non-interacting resonance gas
- The pressure can be written as:

$$p^{HRG}/T^4 = \frac{1}{VT^3} \sum_{i \in \, mesons} \ln \mathcal{Z}^M_{m_i}(T, V, \mu_{X^a}) + \frac{1}{VT^3} \sum_{i \in \, baryons} \ln \mathcal{Z}^B_{m_i}(T, V, \mu_{X^a})$$

where

$$\ln \mathcal{Z}^{M/B}_{m_i} = \mp \frac{V d_i}{2\pi^2} \int_0^\infty dk k^2 \ln(1 \mp z_i e^{-\varepsilon_i/T}) \quad ,$$

with energies $\varepsilon_i = \sqrt{k^2 + m_i^2}$, degeneracy factors d_i and fugacities

$$z_i = \exp\left((\sum_a X^a_i \mu_{X^a})/T \right) \quad .$$

X^a: all possible conserved charges, including the baryon number B, electric charge Q, strangeness S.

- Needs knowledge of the hadronic spectrum

High temperature limit

- QCD thermodynamics approaches that of a non-interacting, massless quark-gluon gas:

$$\left(\frac{P}{T^4}\right)_{\text{ideal}} = \frac{8\pi^2}{45} + \sum_{f=u,d,s}\left[\frac{7\pi^2}{60} + \frac{1}{2}\left(\frac{\mu_f}{T}\right)^2 + \frac{1}{4\pi^2}\left(\frac{\mu_f}{T}\right)^4\right]$$

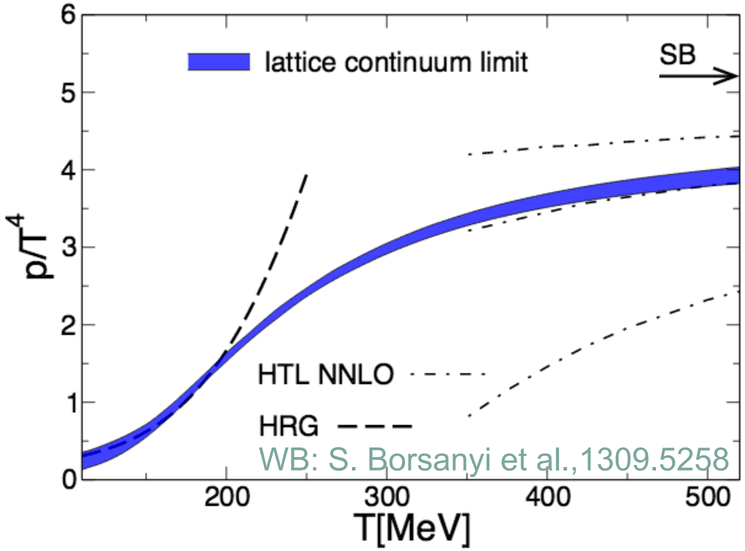
- We can switch on the interaction and systematically expand the observables in series of the coupling *g*

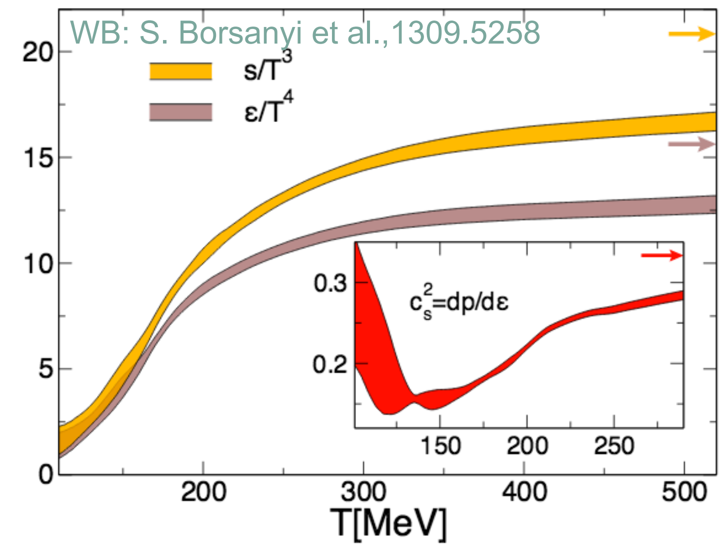
- Resummation of diagrams (HTL) or dimensional reduction are needed, to improve convergence

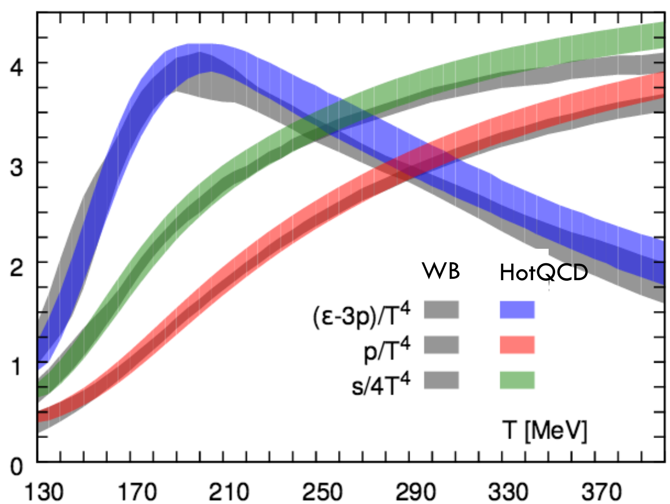
Braaten, Pisarski (1990); Haque et al. (2014); Hietanen et al (2009)

- At what temperature does perturbation theory break down?

QCD Equation of state at μ_B=0







- EoS available in the continuum limit, with realistic quark masses
- Agreement between stout and HISQ action for all quantities

WB: S. Borsanyi et al., 1309.5258, PLB (2014)
HotQCD: A. Bazavov et al., 1407.6387, PRD (2014)

Sign problem

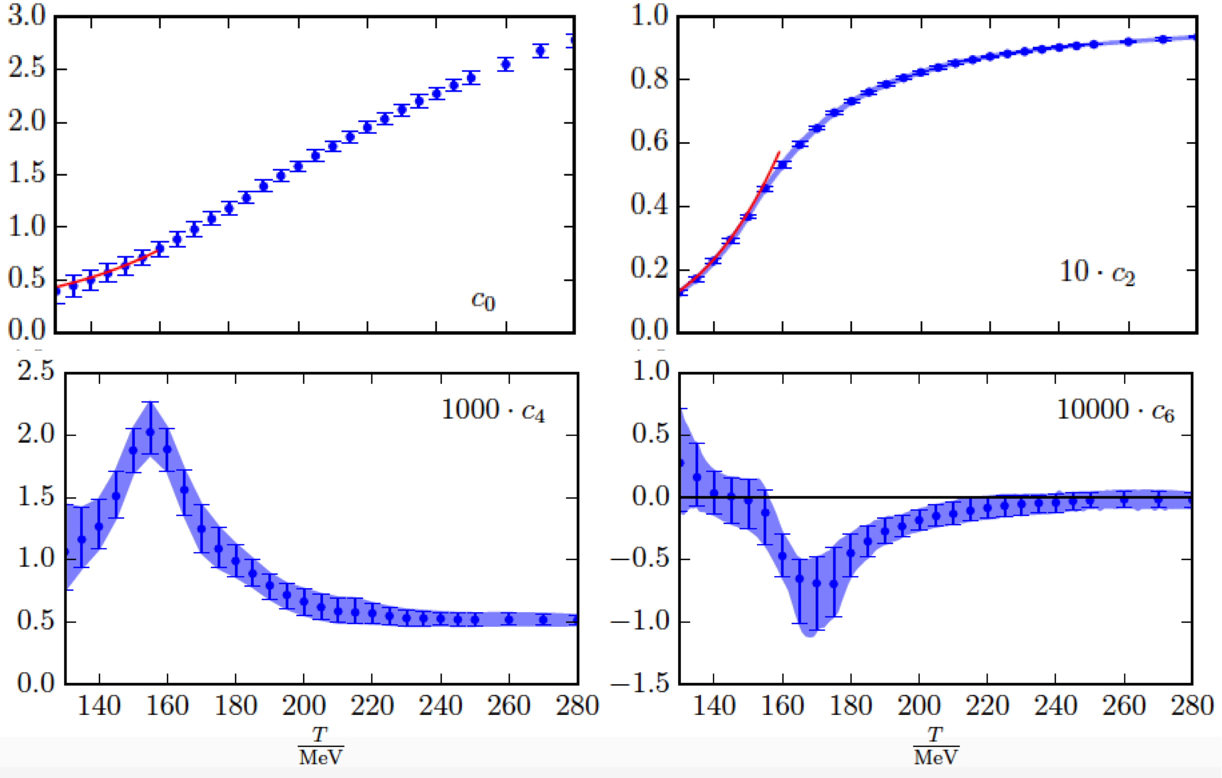
- The QCD path integral is computed by Monte Carlo algorithms which samples field configurations with a weight proportional to the exponential of the action

$$Z(\mu_B, T) = \mathrm{Tr}\left(e^{-\frac{H_{\mathrm{QCD}} - \mu_B N_B}{T}}\right) = \int \mathcal{D}U e^{-S_G[U]} \det M[U, \mu_B]$$

- $\det M[\mu_B]$ complex → Monte Carlo simulations are not feasible

- We can rely on a few approximate methods, viable for small μ_B/T:
  - Taylor expansion of physical quantities around $\mu_B=0$ (Bielefeld-Swansea collaboration 2002; R. Gavai, S. Gupta 2003)
  - Reweighting (complex phase moved from the measure to observables) (Barbour et al. 1998; Z. Fodor and S, Katz, 2002)
  - Simulations at imaginary chemical potentials (plus analytic continuation) (Alford, Kapustin, Wilczek, 1999; de Forcrand, Philipsen, 2002; D'Elia, Lombardo 2003)

Equation of state at $\mu_B>0$



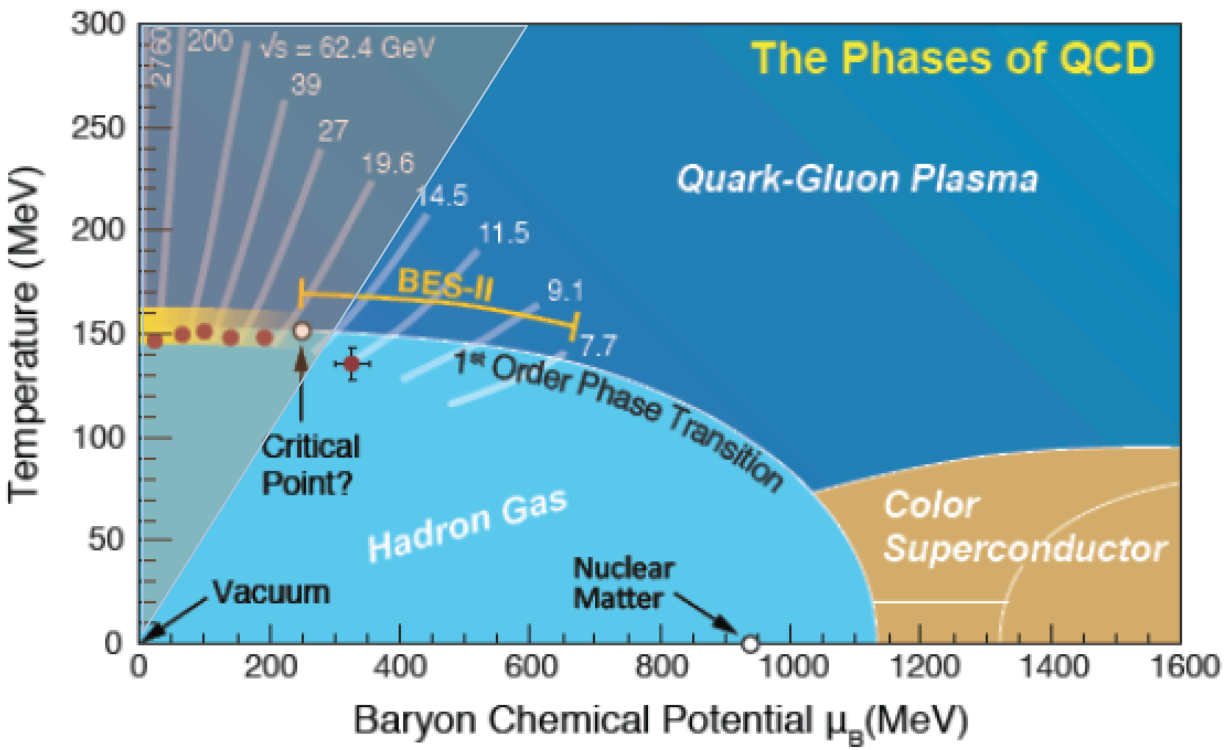
- Expand the pressure in powers of μ_B

$$\frac{p(\mu_B)}{T^4} = c_0 + c_2\left(\frac{\mu_B}{T}\right)^2 + c_4\left(\frac{\mu_B}{T}\right)^4 + c_6\left(\frac{\mu_B}{T}\right)^6 + \mathcal{O}(\mu_B^8)$$

- Continuum extrapolated results for c_2, c_4, c_6 at the physical mass

WB: S. Borsanyi et al. 1607.02493 (2016)

Equation of state at $\mu_B>0$



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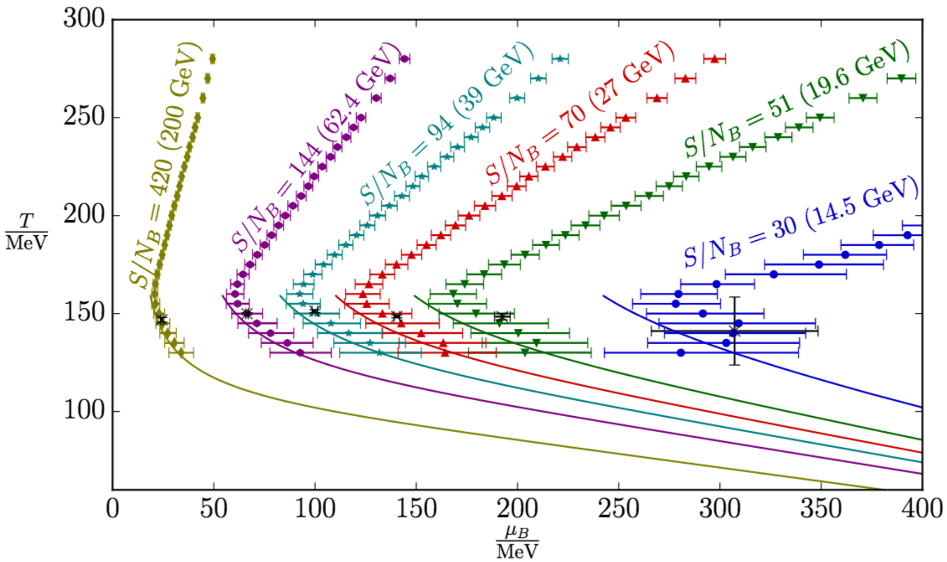
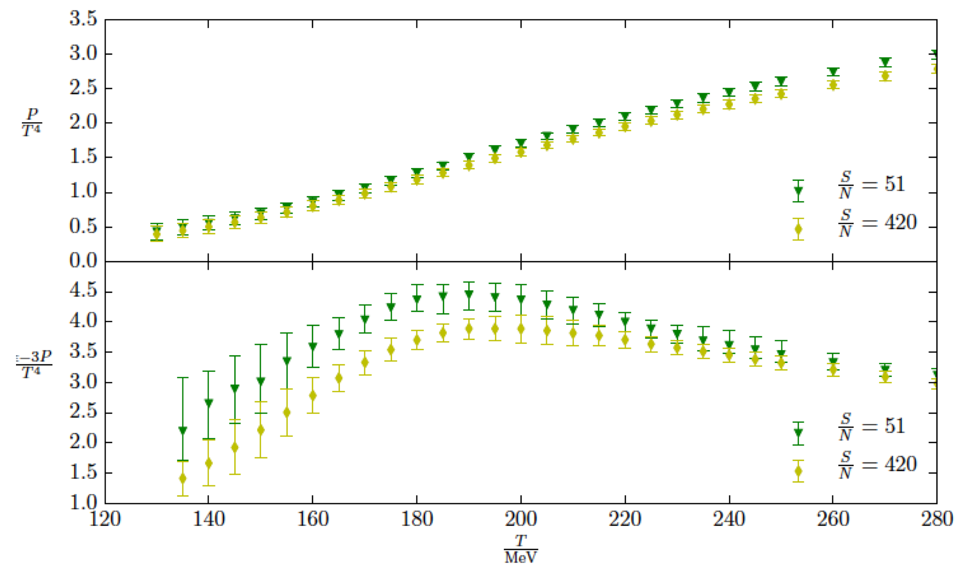
- Continuum extrapolated results for c_2, c_4, c_6 at the physical mass
- Enables us to reach $\mu_B/T \sim 2$

WB: S. Borsanyi et al. 1607.02493 (2016)

Equation of state at $\mu_B > 0$

- Extract the isentropic trajectory that the system follows in the absence of dissipation
- Calculate the EoS along these constant S/N trajectories





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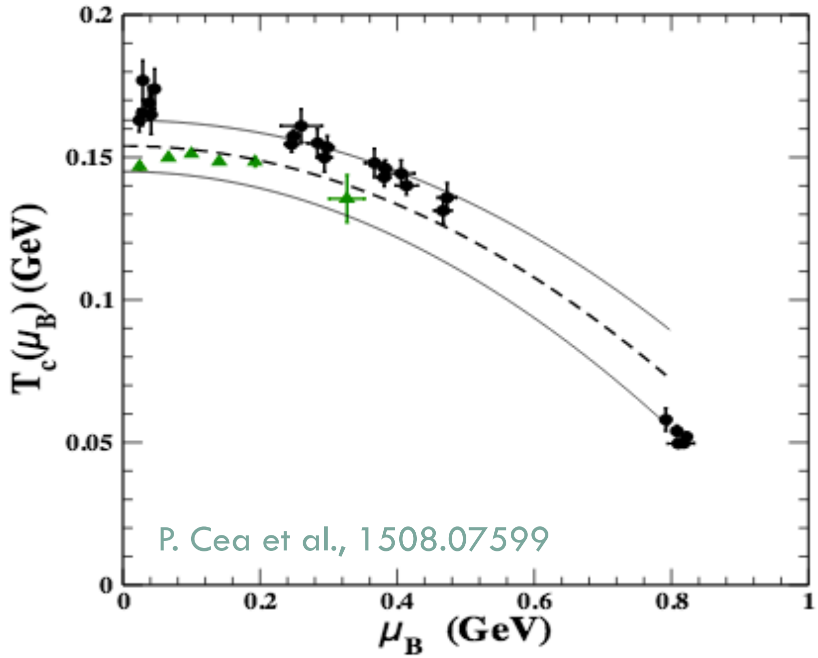
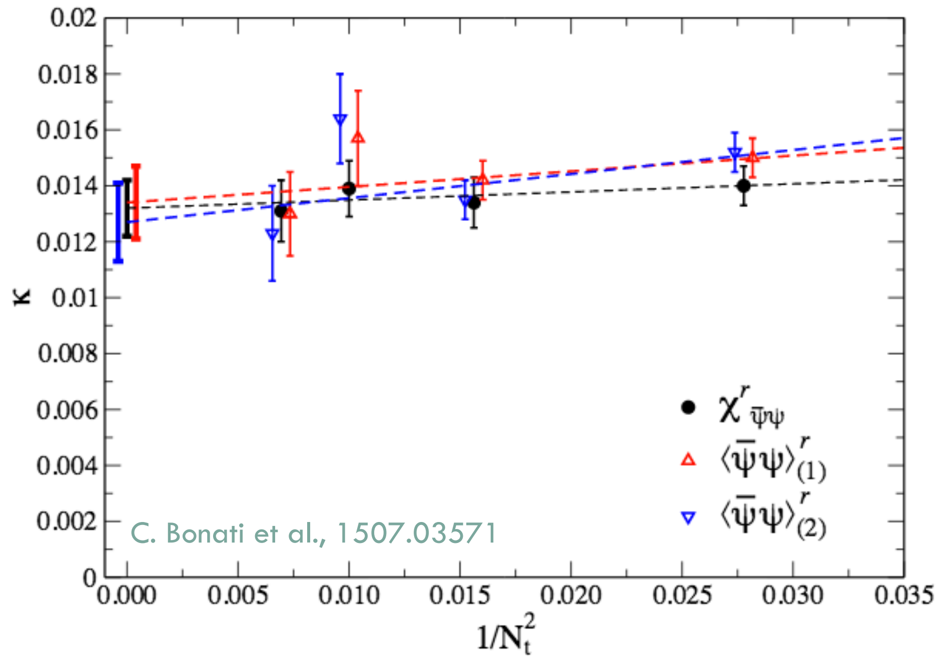
QCD phase diagram





Curvature κ defined as:

$$\frac{T_c(\mu_B)}{T_c(\mu=0)} = 1 - \kappa\left(\frac{\mu_B}{T_c(\mu_B)}\right)^2 + \lambda\left(\frac{\mu_B}{T_c(\mu_B)}\right)^4 \ldots$$

Recent results:

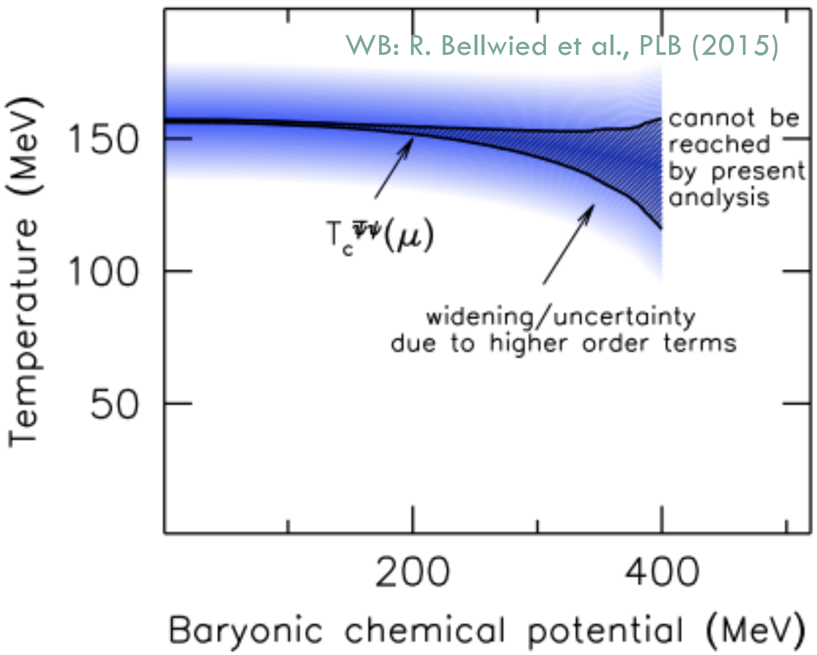
$$\kappa = 0.020(4)$$

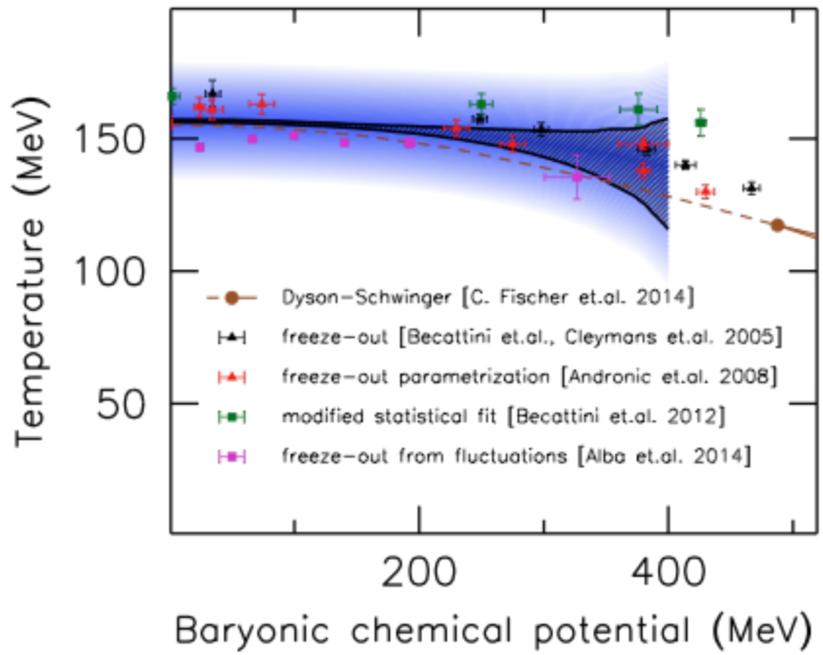
P. Cea et al., 1508.07599

$$\kappa = 0.0135(20)$$

C. Bonati et al., 1507.03571

QCD phase diagram





- Transition at μ_B=0 is analytic crossover

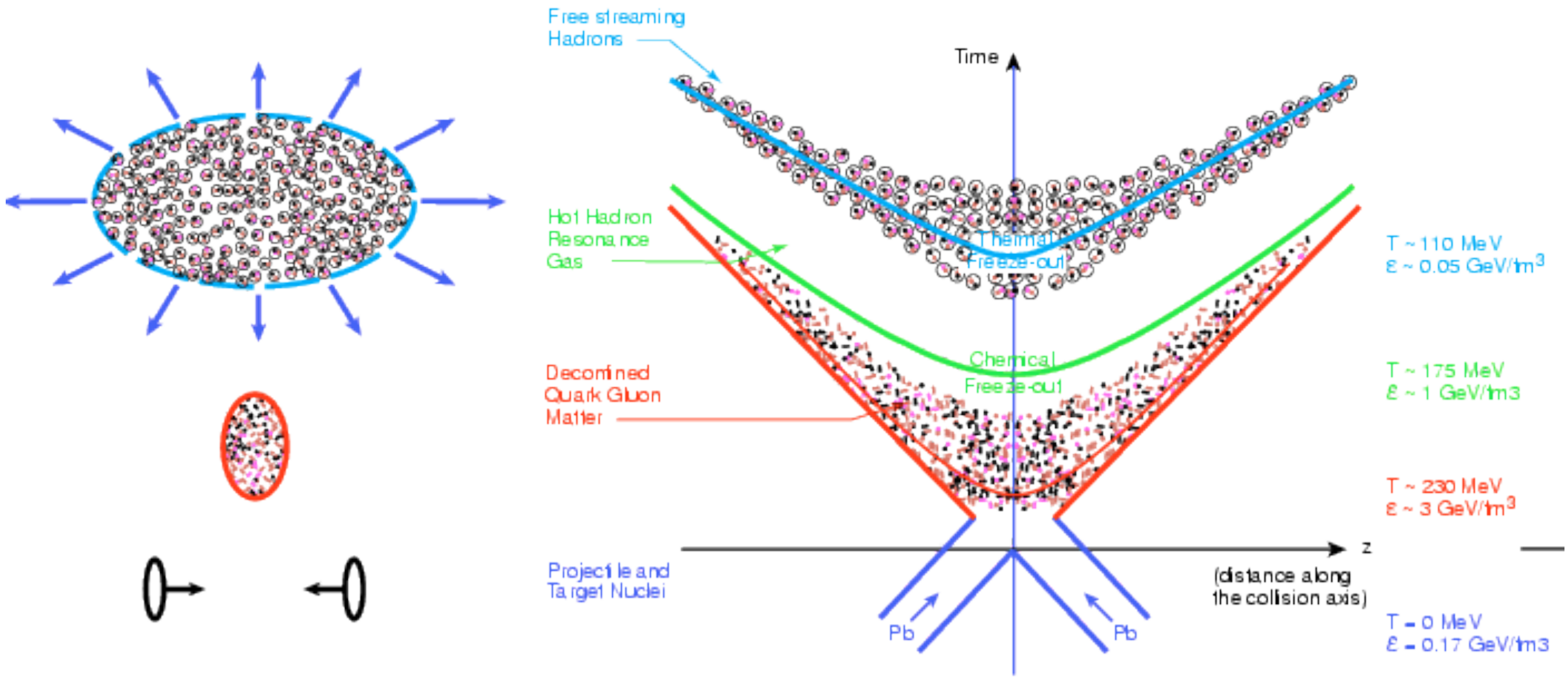
WB: Aoki et al., Nature (2006)

- Transition temperature at μ_B=0: T_c~155 MeV
- Curvature κ defined as: $$\frac{T_c(\mu_B)}{T_c(\mu=0)} = 1 - \kappa \left(\frac{\mu_B}{T_c(\mu_B)}\right)^2 + \lambda \left(\frac{\mu_B}{T_c(\mu_B)}\right)^4 \dots$$

Recent results: $$\kappa = 0.0149 \pm 0.0021$$

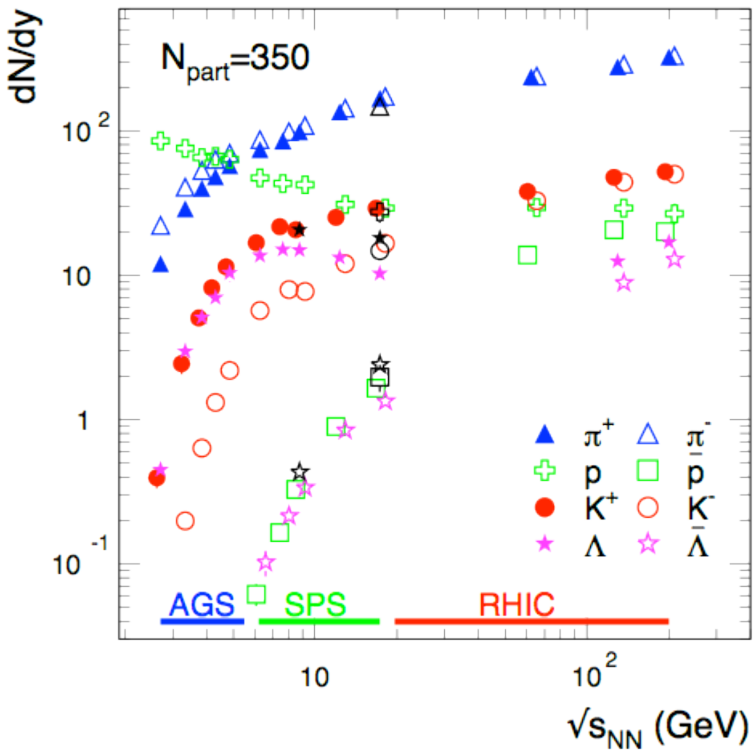
WB: R. Bellwied et al., PLB (2015)

Evolution of a Heavy Ion Collision



- Chemical freeze-out: inelastic reactions cease: the chemical composition of the system is fixed (particle yields and fluctuations)
- Kinetic freeze-out: elastic reactions cease: spectra and correlations are frozen (free streaming of hadrons)
- Hadrons reach the detector

Hadron yields

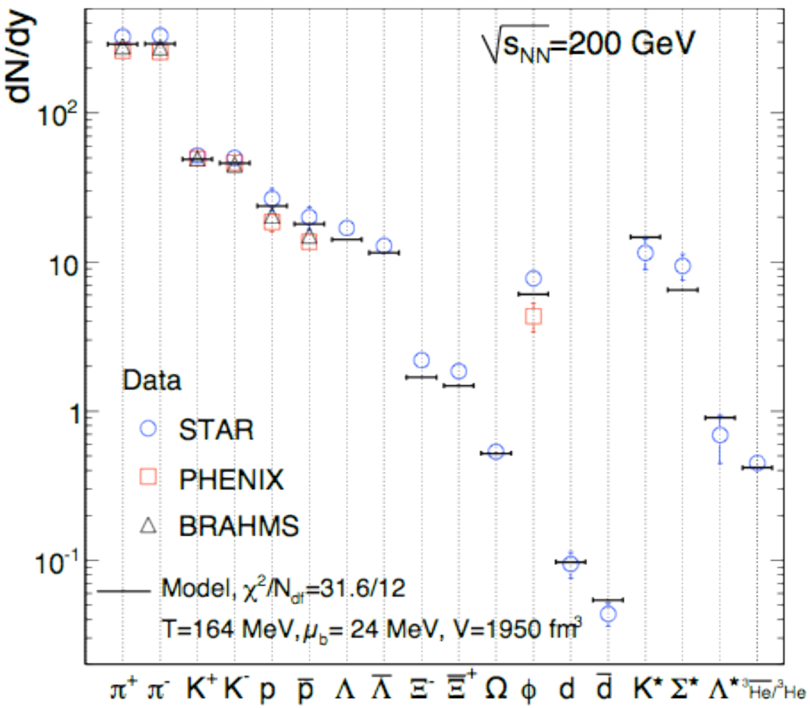


- $E=mc^2$: lots of particles are created
- Particle counting (average over many events)
- Take into account:
  - detector inefficiency
  - missing particles at low p_T
  - decays

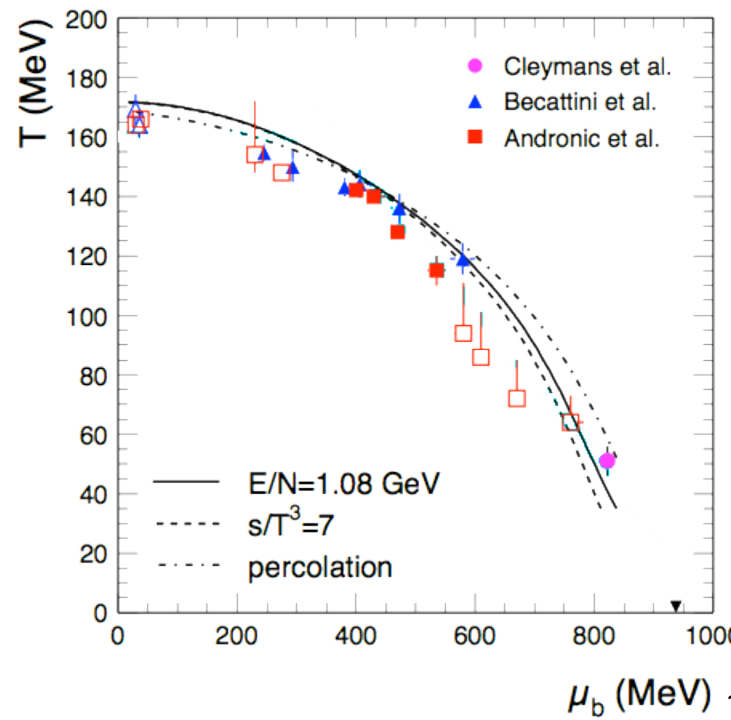
- HRG model: test hypothesis of hadron abundancies in equilibrium

$$N_i = -T\frac{\partial \ln Z_i}{\partial \mu} = \frac{g_i V}{2\pi^2}\int_0^\infty \frac{p^2 \mathrm{d}p}{\exp[(E_i-\mu_i)/T]\pm 1}$$

The thermal fits



- Fit is performed minimizing the χ^2
- Fit to yields: parameters T, μB, V
- Fit to ratios: the volume V cancels out



- Changing the collision energy, it is possible to draw the freeze-out line in the T, μB plane

Fluctuations of conserved charges

- Definition:

$$\chi_{lmn}^{BSQ} = \frac{\partial^{l+m+n} p/T^4}{\partial(\mu_B/T)^l \partial(\mu_S/T)^m \partial(\mu_Q/T)^n}.$$

- Relationship between chemical potentials:

$$\begin{aligned} \mu_u &= \frac{1}{3}\mu_B + \frac{2}{3}\mu_Q; \\ \mu_d &= \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q; \\ \mu_s &= \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q - \mu_S. \end{aligned}$$

- They can be calculated on the lattice and compared to experiment

Connection to experiment

- Fluctuations of conserved charges are the cumulants of their event-by-event distribution

$$\text{mean}: M = \chi_1 \qquad \text{variance}: \sigma^2 = \chi_2$$

$$\text{skewness}: S = \chi_3/\chi_2^{3/2} \qquad \text{kurtosis}: \kappa = \chi_4/\chi_2^2$$

$$S\sigma = \chi_3/\chi_2 \qquad \kappa\sigma^2 = \chi_4/\chi_2$$

$$M/\sigma^2 = \chi_1/\chi_2 \qquad S\sigma^3/M = \chi_3/\chi_1$$

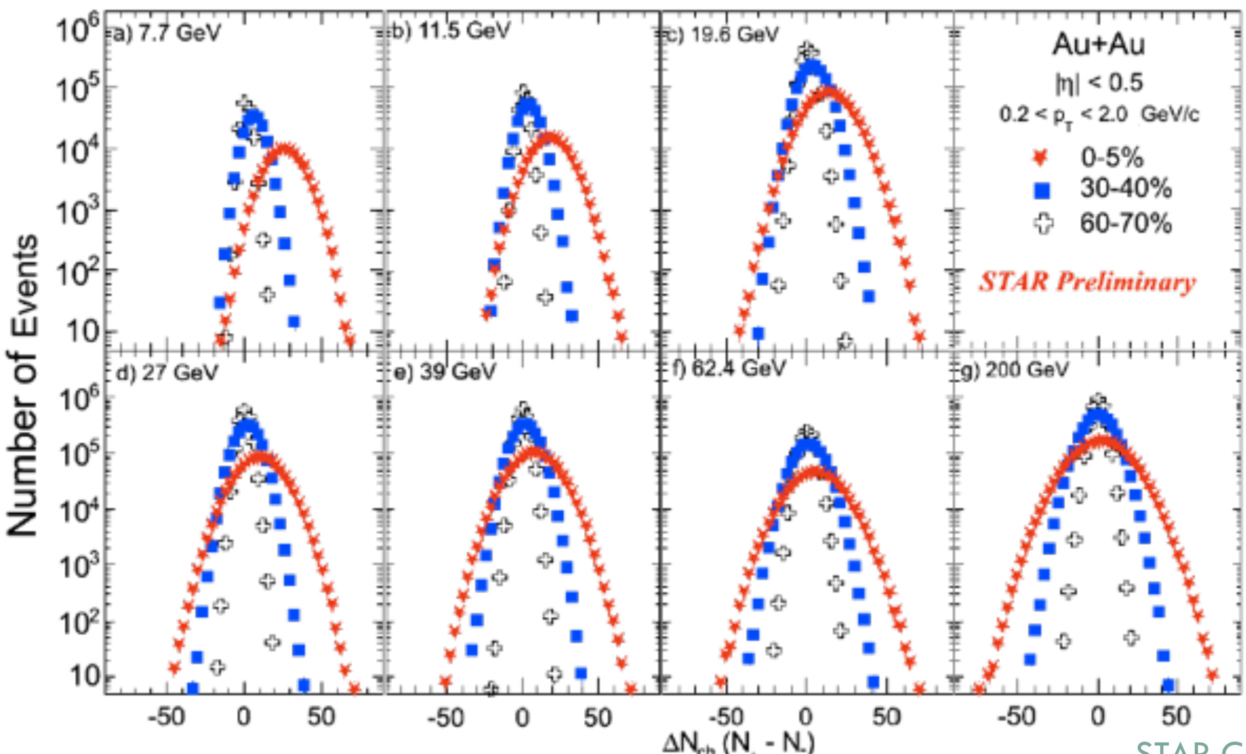
F. Karsch: Centr. Eur. J. Phys. (2012)

- The chemical potentials are not independent: fixed to match the experimental conditions:

$$\langle n_S \rangle = 0 \qquad \langle n_Q \rangle = 0.4 \langle n_B \rangle$$

Connection to experiment

- Consider the number of electrically charged particles N_Q
- Its average value over the whole ensemble of events is $\langle N_Q \rangle$
- In experiments it is possible to measure its event-by-event distribution



STAR Collab., PRL (2014)

“Baryometer and Thermometer”

Let us look at the Taylor expansion of R^B_{31}

$$R^B_{31}(T,\mu_B) = \frac{\chi_3^B(T,\mu_B)}{\chi_1^B(T,\mu_B)} = \frac{\chi_4^B(T,0) + \chi_{31}^{BQ}(T,0)q_1(T) + \chi_{31}^{BS}(T,0)s_1(T)}{\chi_2^B(T,0) + \chi_{11}^{BQ}(T,0)q_1(T) + \chi_{11}^{BS}(T,0)s_1(T)} + \mathcal{O}(\mu_B^2)$$

- To order μ^2_B it is independent of μ_B: it can be used as a thermometer

- Let us look at the Taylor expansion of R^B_{12}

$$R^B_{12}(T,\mu_B) = \frac{\chi_1^B(T,\mu_B)}{\chi_2^B(T,\mu_B)} = \frac{\chi_2^B(T,0) + \chi_{11}^{BQ}(T,0)q_1(T) + \chi_{11}^{BS}(T,0)s_1(T)}{\chi_2^B(T,0)} \frac{\mu_B}{T} + \mathcal{O}(\mu_B^3)$$

- Once we extract T from R^B_{31}, we can use R^B_{12} to extract μ_B

Things to keep in mind

- Effects due to volume variation because of finite centrality bin width
  - Experimentally corrected by centrality-bin-width correction method

V. Skokov et al., PRC (2013)

- Finite reconstruction efficiency
  - Experimentally corrected based on binomial distribution A.Bzdak,V.Koch, PRC (2012)
- Spallation protons
  - Experimentally removed with proper cuts in p_T
- Canonical vs Gran Canonical ensemble
  - Experimental cuts in the kinematics and acceptance V. Koch, S. Jeon, PRL (2000)
- Proton multiplicity distributions vs baryon number fluctuations
  - Recipes for treating proton fluctuations

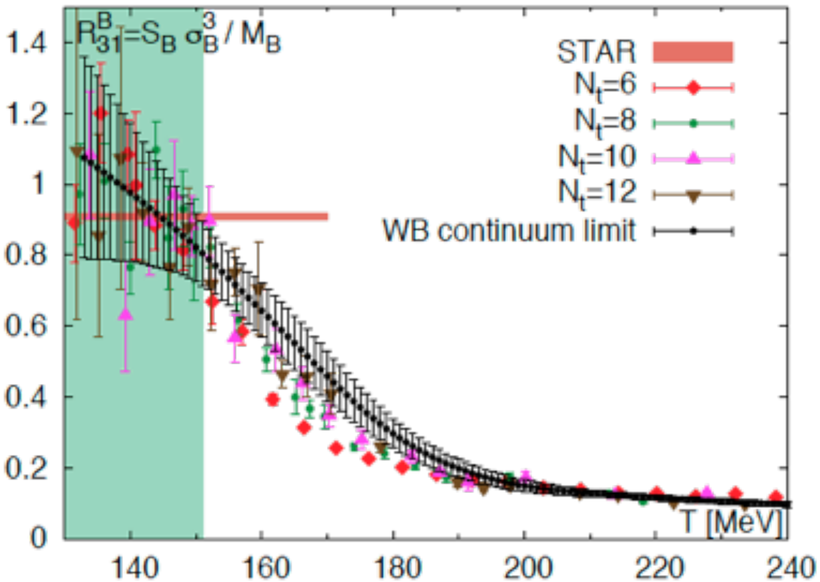
M. Asakawa and M. Kitazawa, PRC(2012), M. Nahrgang et al., 1402.1238

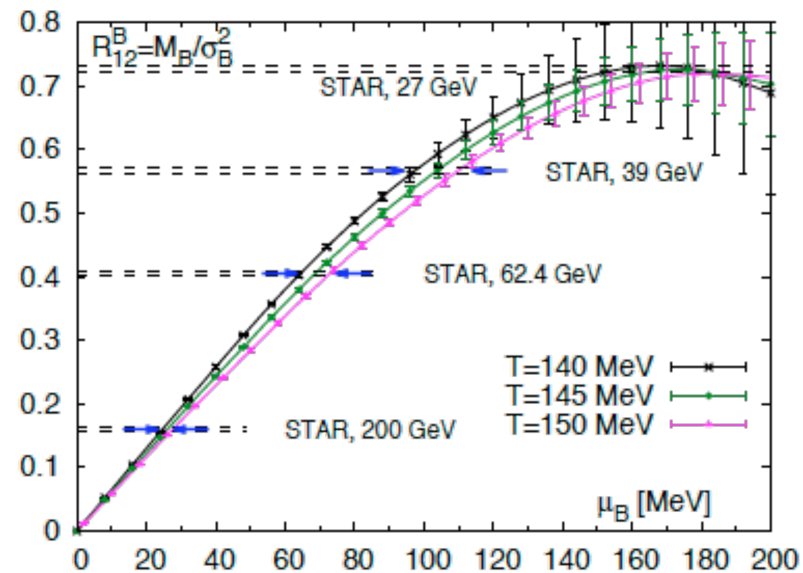
- Final-state interactions in the hadronic phase
  - Consistency between different charges = fundamental test

J.Steinheimer et al., PRL (2013)

Freeze-out parameters from B fluctuations

- Thermometer: $\frac{\chi_3^B(T,\mu_B)}{\chi_1^B(T,\mu_B)}$=$S_B\sigma_B^3/M_B$ Baryometer: $\frac{\chi_1^B(T,\mu_B)}{\chi_2^B(T,\mu_B)}$=$\sigma_B^2/M_B$





WB: S. Borsanyi et al., PRL (2014)
STAR collaboration, PRL (2014)

- Upper limit: $T_f \leq$ 151±4 MeV
- Consistency between freeze-out chemical potential from electric charge and baryon number is found.

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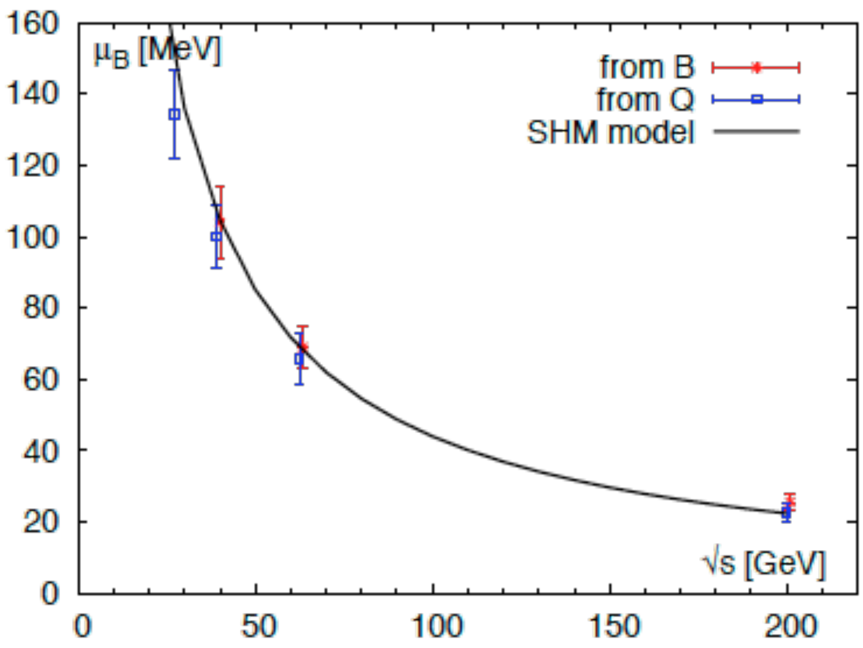

$\sqrt{s}[GeV]$	μ_B^f [MeV] (from B)	μ_B^f [MeV] (from Q)
200	25.8±2.7	22.8±2.6
62.4	69.7±6.4	66.6±7.9
39	105±11	101±10
27	-	136±13.8

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- Consistency between freeze-out chemical potential from electric charge and baryon number is found.

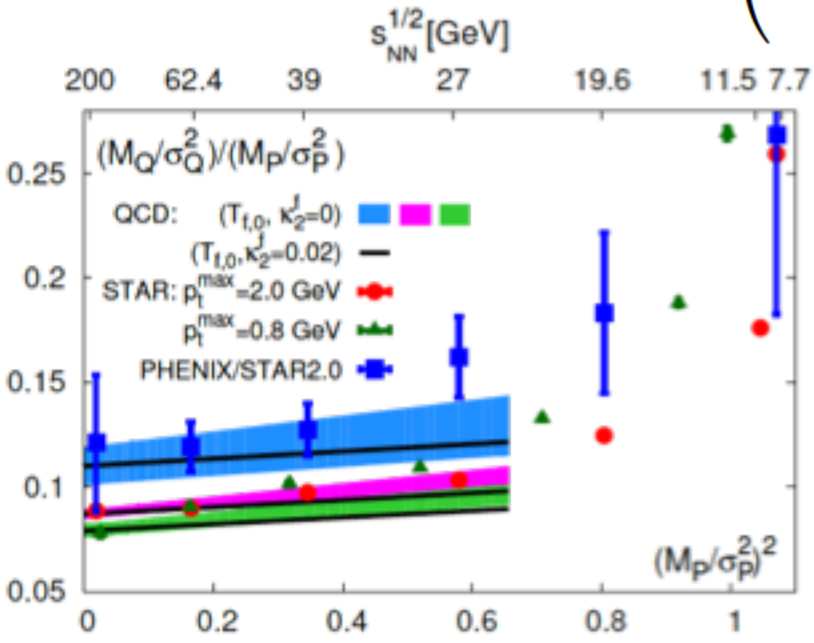
Curvature of the freeze-out line

- Parametrization of the freeze-out line:

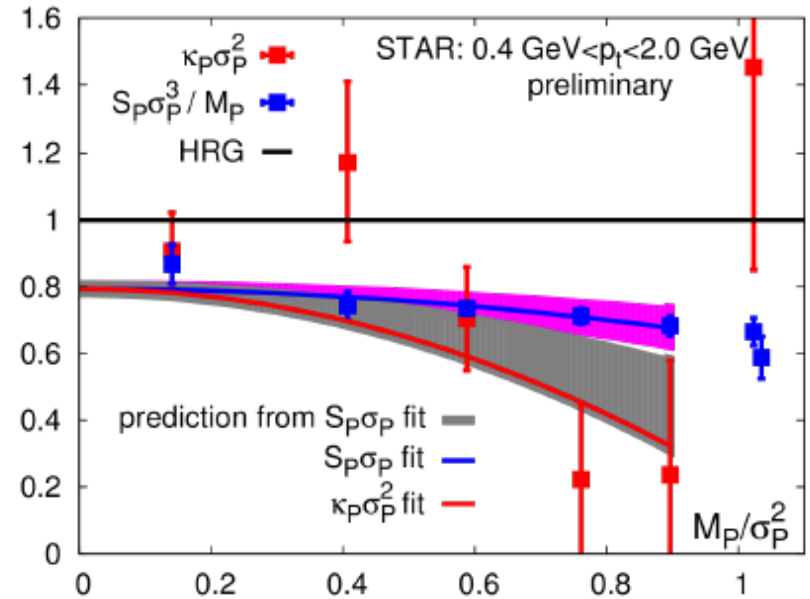
$$T_f(\mu_B) = T_{f,0}\left(1 - \kappa_2^f \bar{\mu}_B^2 - \kappa_4^f \bar{\mu}_B^4\right)$$

- Taylor expansion of the "ratio of ratios" $R_{12}^{QB} = [M_Q/\sigma_Q^2]/[M_B/\sigma_B^2]$

$$R_{12}^{QB} = R_{12}^{QB,0} + \left(R_{12}^{QB,2} - \kappa_2^f T_{f,0} \frac{\mathrm{d}R_{12}^{QB,0}}{\mathrm{d}T}\bigg|_{T_{f,0}} \right) \hat{\mu}_B^2$$



A. Bazavov et al., 1509.05786
STAR0.8: PRL (2013)



STAR2.0: X. Luo, PoS CPOD 2014
PHENIX: 1506.07834

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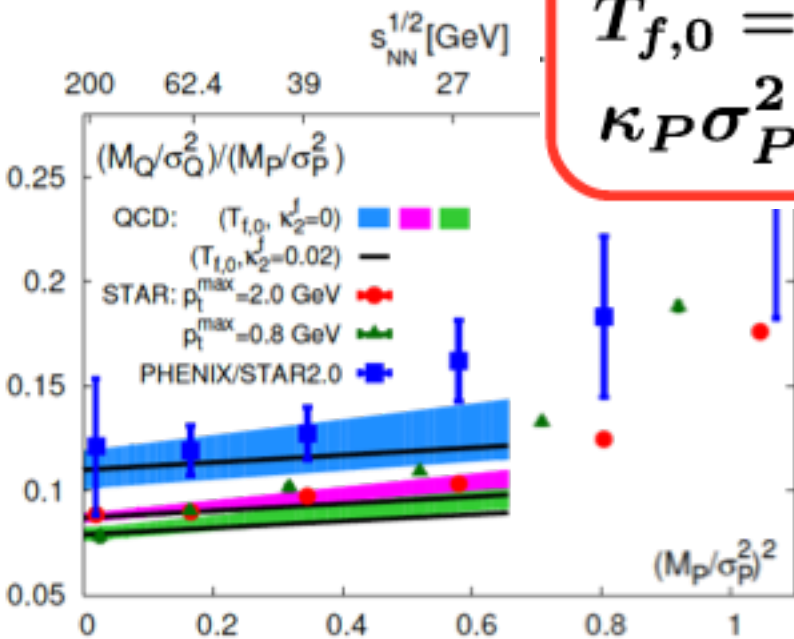
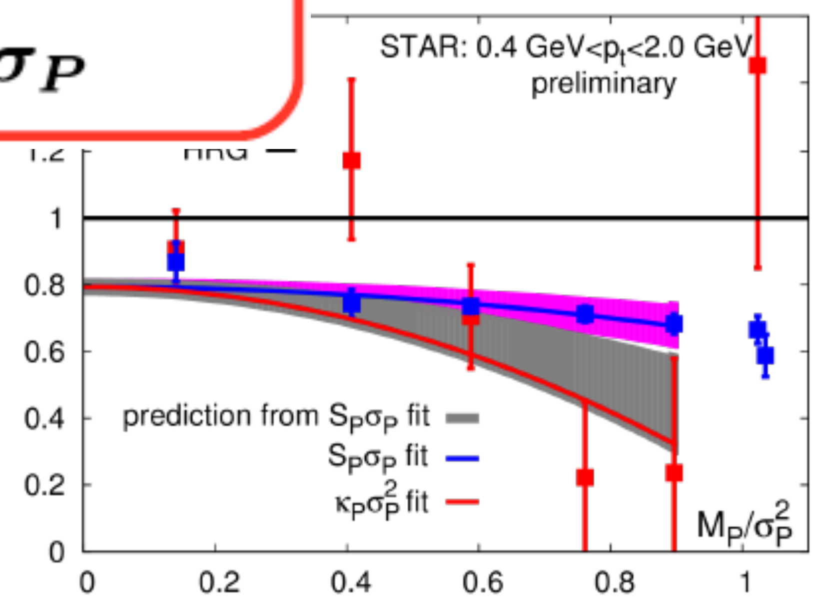
$$R_{12}^{QB} = \quad \ldots \quad \hat{\mu}_B^2$$

$$\boldsymbol{\kappa_2^f < 0.011}$$
$$\boldsymbol{T_{f,0} = (147 \pm 2)\ \mathrm{MeV}}$$
$$\boldsymbol{\kappa_P \sigma_P^2 < S_P \sigma_P}$$





A. Bazavov et al., 1509.05786
STAR0.8: PRL (2013)

STAR2.0: X. Luo, PoS CPOD 2014
PHENIX: 1506.07834

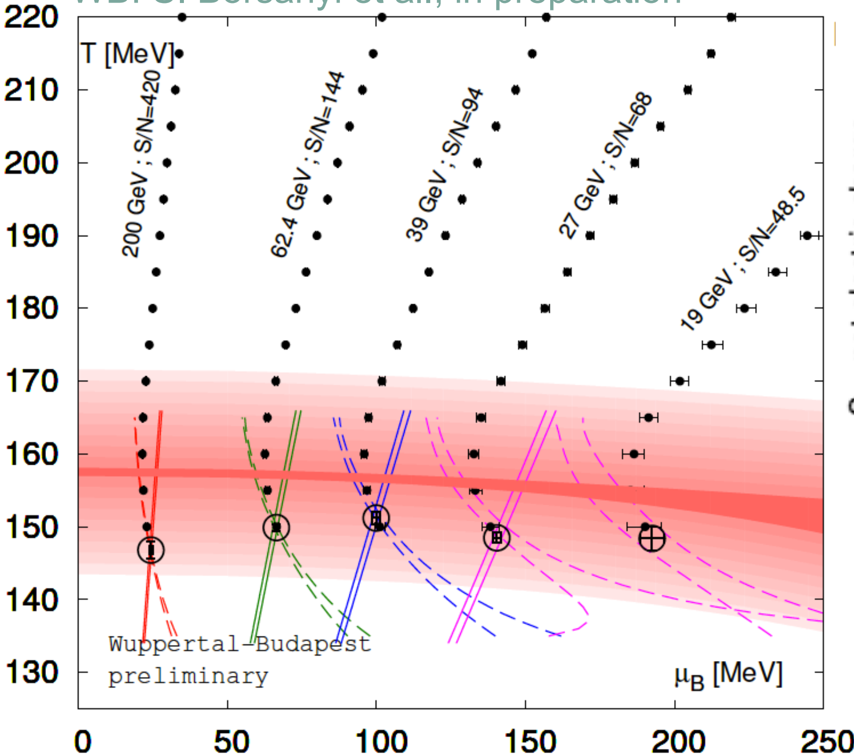
Freeze-out line from first principles

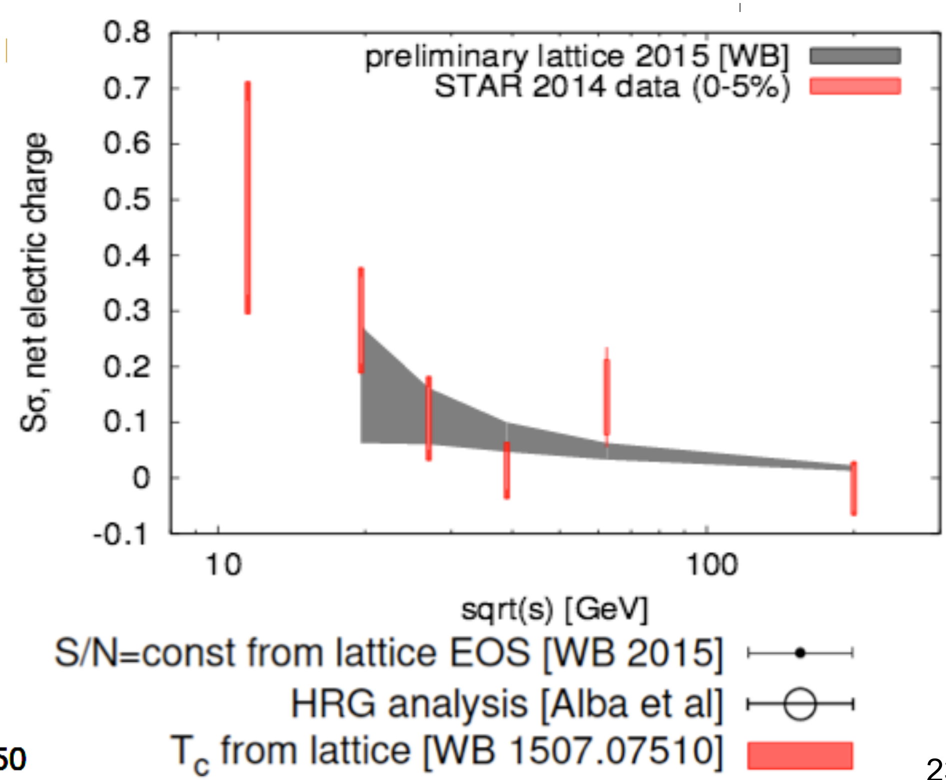
- Use T- and μ_B-dependence of R_{12}^Q and R_{12}^B for a combined fit:

$$R_{12}^Q(T,\mu_B)=\frac{\chi_1^Q(T,\mu_B)}{\chi_2^Q(T,\mu_B)}=\frac{\chi_{11}^{QB}(T,0)+\chi_2^Q(T,0)q_1(T)+\chi_{11}^{QS}(T,0)s_1(T)}{\chi_2^Q(T,0)}\frac{\mu_B}{T}+\mathcal{O}(\mu_B^3).$$

$$R_{12}^B(T,\mu_B)=\frac{\chi_1^B(T,\mu_B)}{\chi_2^B(T,\mu_B)}=\frac{\chi_2^B(T,0)+\chi_{11}^{BQ}(T,0)q_1(T)+\chi_{11}^{BS}(T,0)s_1(T)}{\chi_2^B(T,0)}\frac{\mu_B}{T}+\mathcal{O}(\mu_B^3)$$

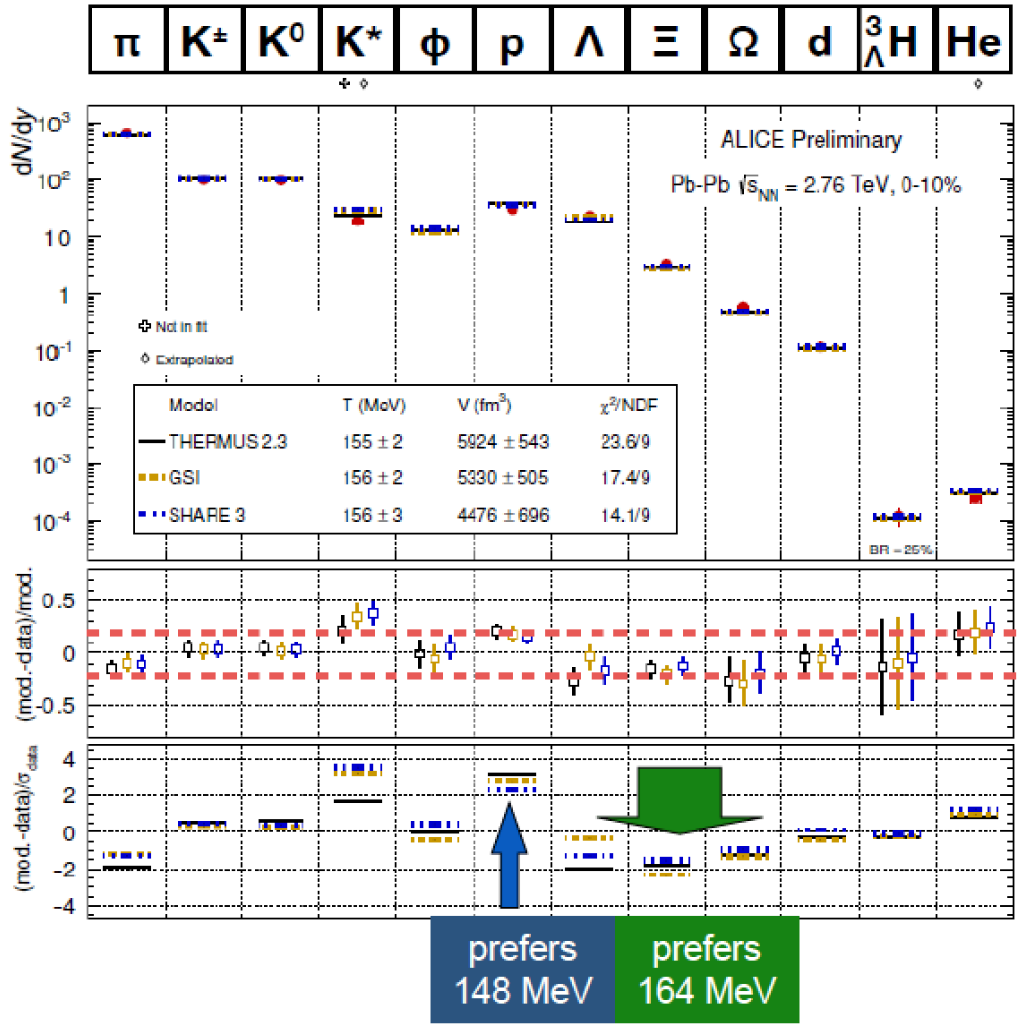
WB: S. Borsanyi et al., in preparation





What about strangeness freeze-out?

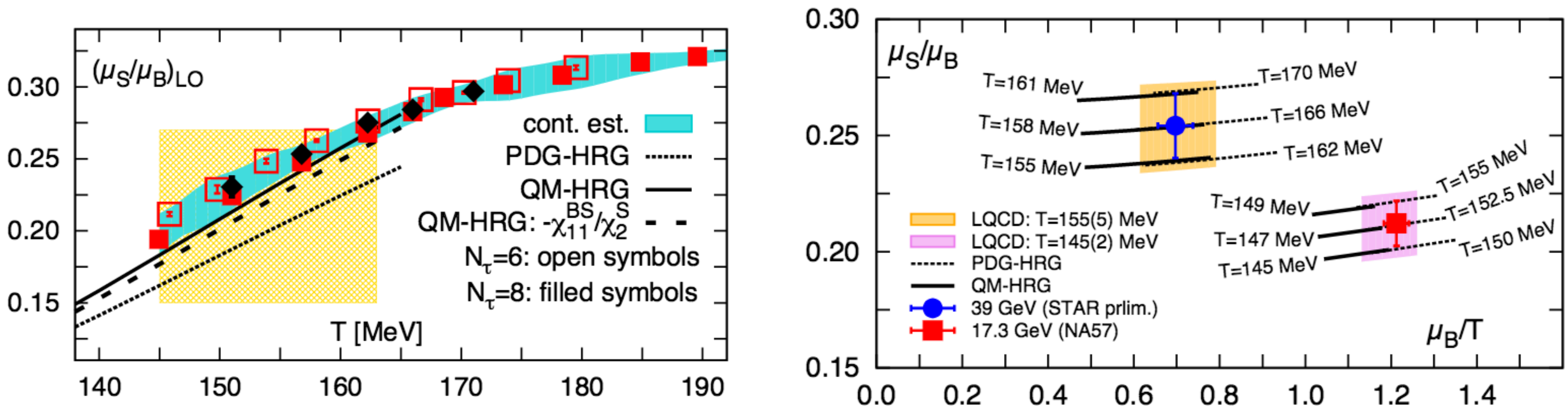
- Yield fits seem to hint at a higher temperature for strange particles



M. Floris: QM 2014

Missing strange states?

- Quark Model predicts not-yet-detected (multi-)strange hadrons



- QM-HRG improves the agreement with lattice results for the baryon-strangeness correlator:

$$(\mu_S/\mu_B)_{LO} = -\chi_{11}^{BS}/\chi_2^S + \chi_{11}^{QS}\mu_Q/\mu_B$$

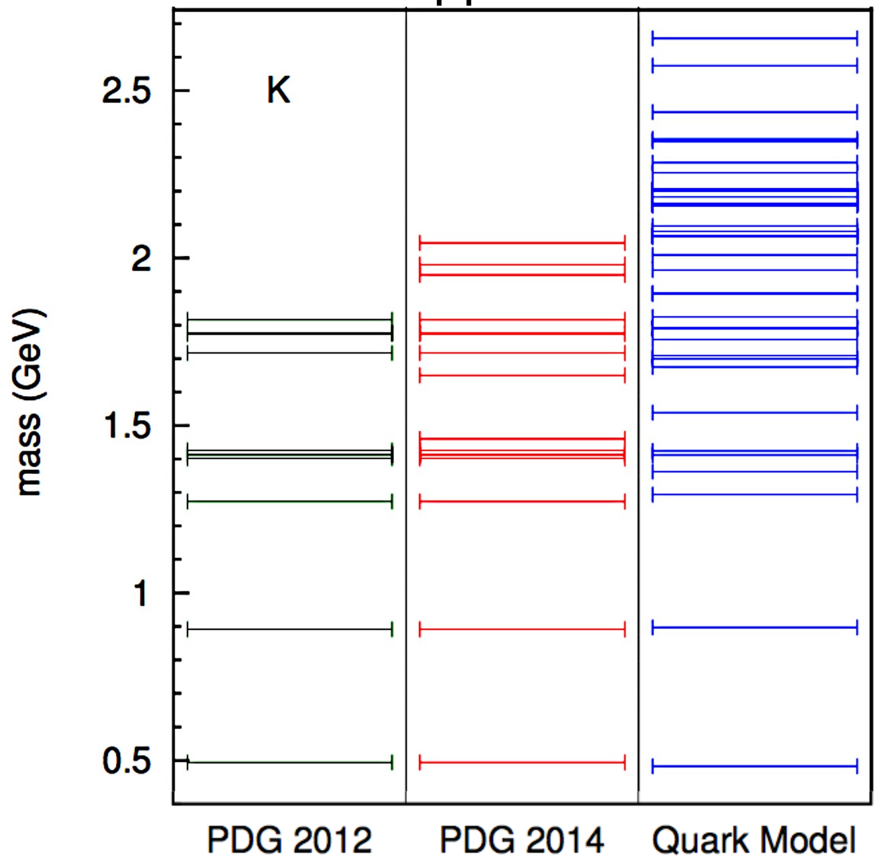
- The effect is only relevant at finite μ_B

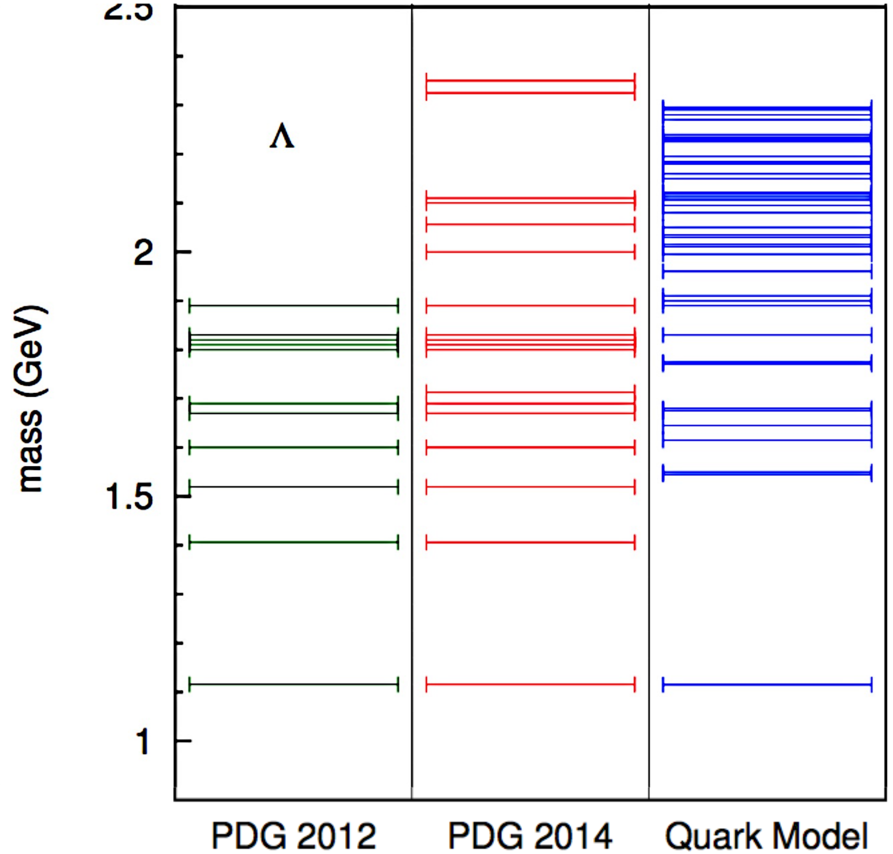
- Feed-down from resonance decays not included

A. Bazavov et al., PRL (2014)

Missing strange states?

- New states appear in the 2014 version of the PDG





WB collaboration, in preparation

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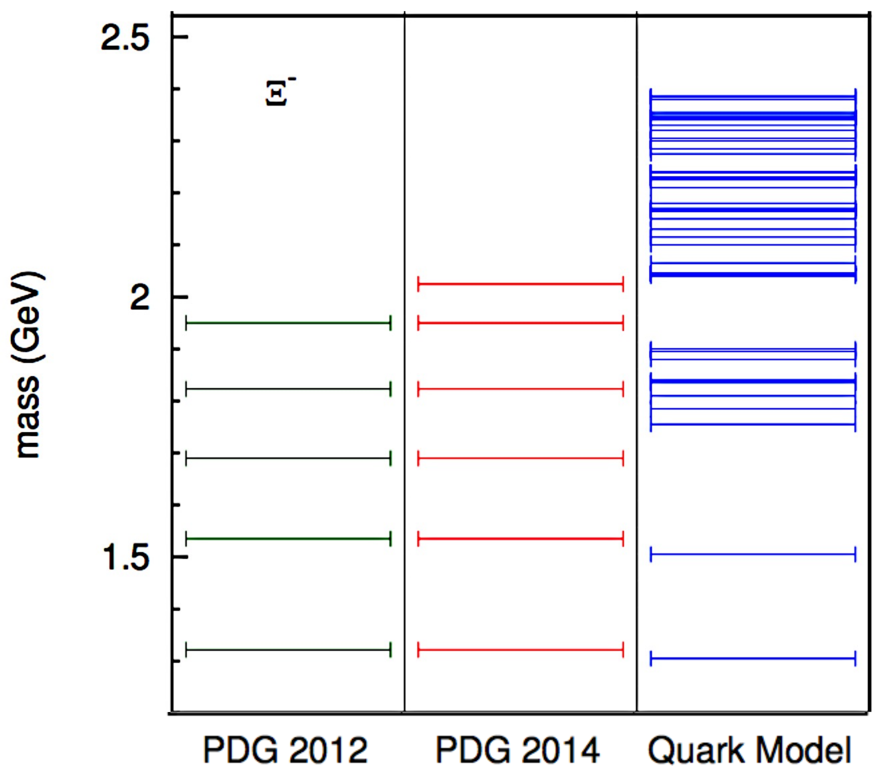
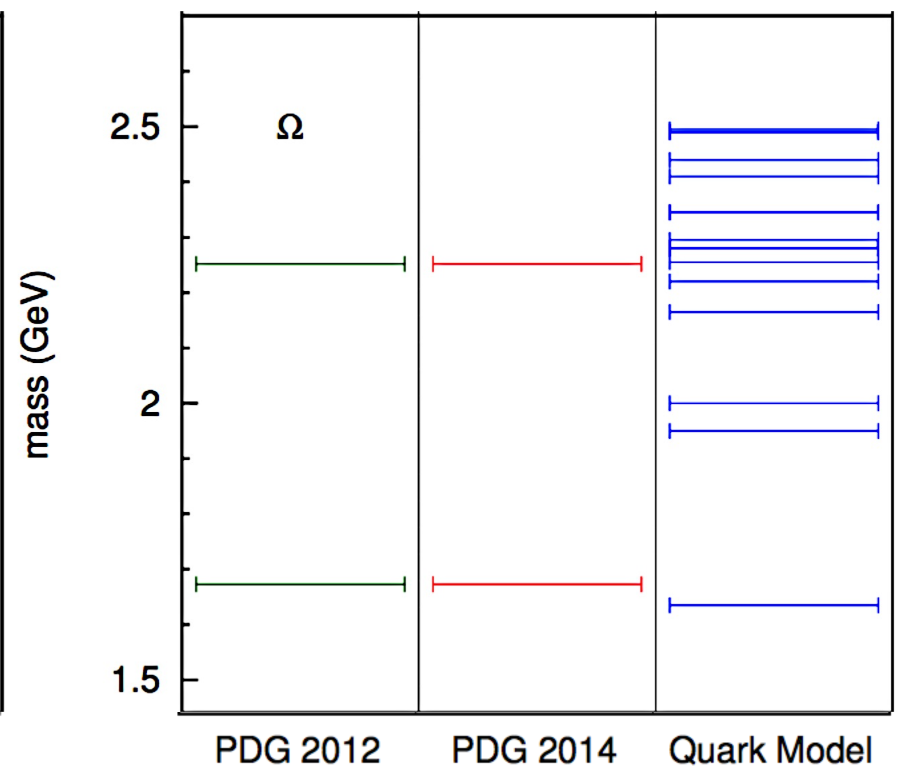
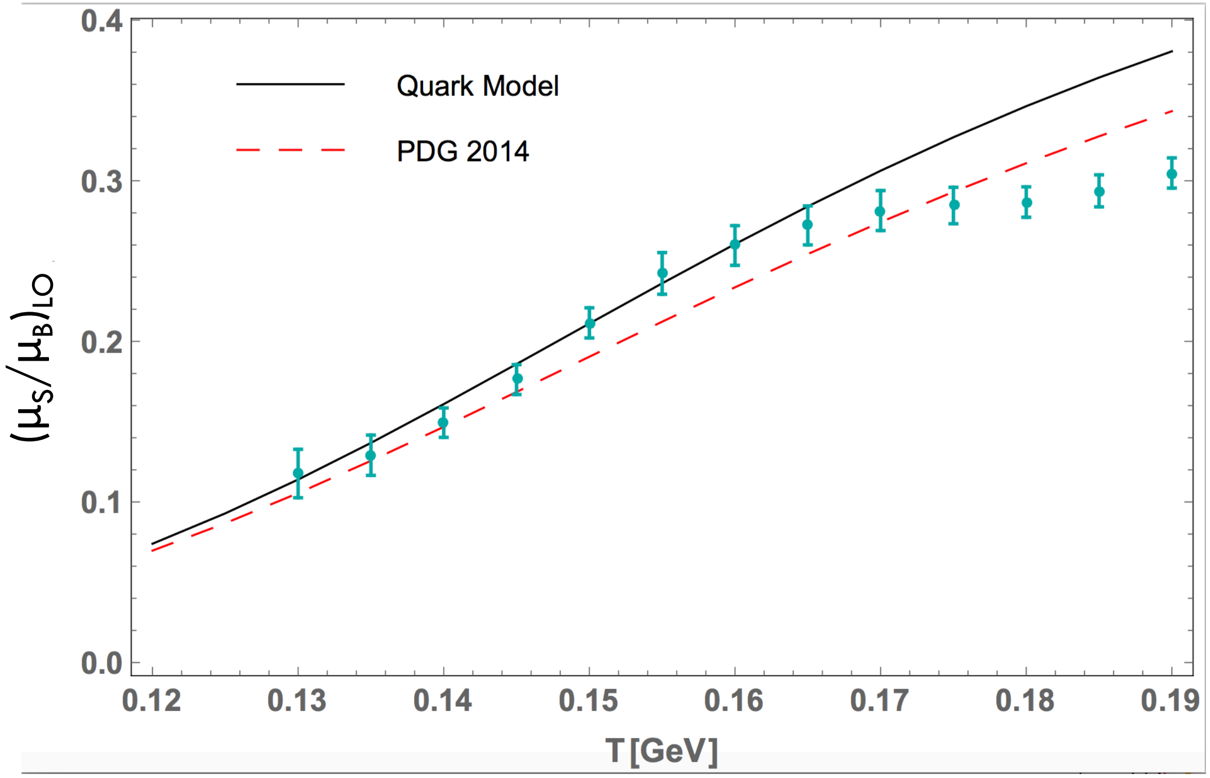




WB collaboration, in preparation

Missing strange states?

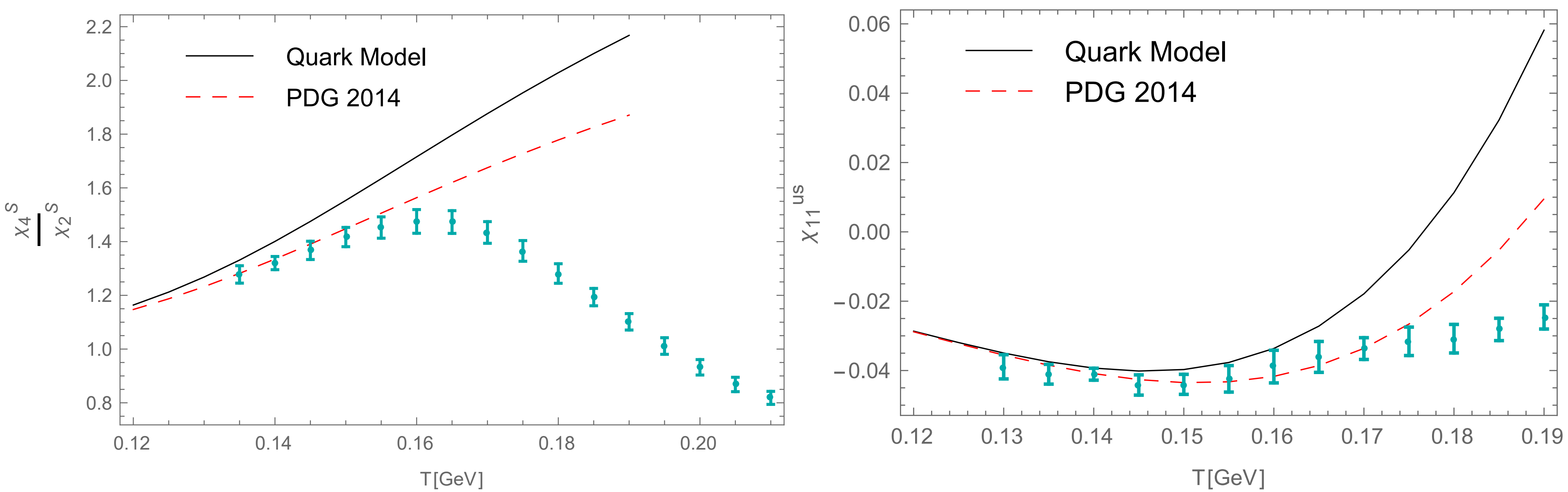
- The comparison with the lattice is improved for the baryon-strangeness correlator:



WB collaboration, in preparation

Missing strange states?

- Some observables are in agreement with the PDG 2014 but not with the Quark Model:



- χ_4^S/χ_2^S is proportional to <S^2> in the system
- It seems to indicate that the quark model predicts too many multi-strange states

WB collaboration, in preparation

Missing strange states?

- Idea: define linear combinations of correlators which receive contributions only from particles with a given quantum number

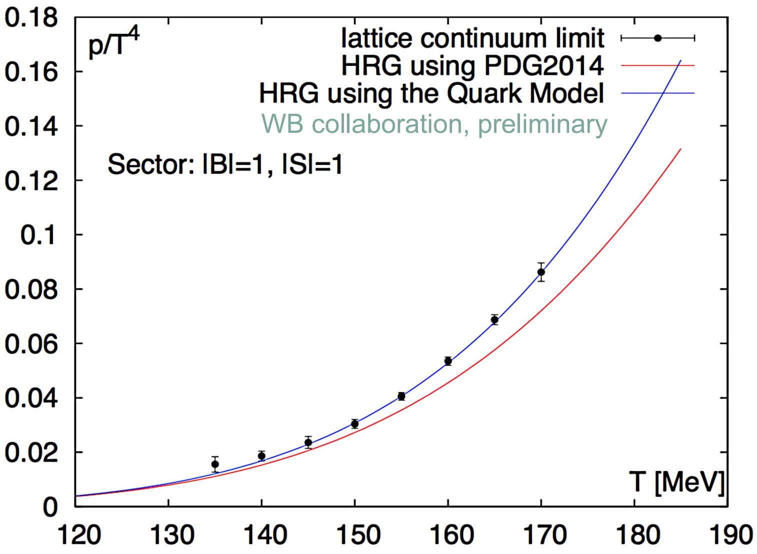
- They allow to compare PDG and QM prediction for each sector separately

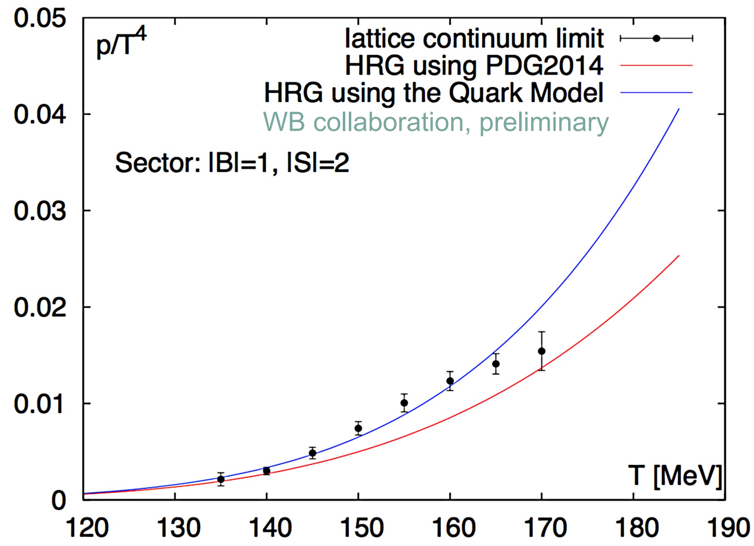
$$
\begin{aligned}
P_S(\hat{\mu}_B, \hat{\mu}_S) &= P_{0|1|} \cosh(\hat{\mu}_S) \\
&+ P_{1|1|} \cosh(\hat{\mu}_B - \hat{\mu}_S) \\
&+ P_{1|2|} \cosh(\hat{\mu}_B - 2\hat{\mu}_S) \\
&+ P_{1|3|} \cosh(\hat{\mu}_B - 3\hat{\mu}_S)
\end{aligned}
$$

$$
\begin{aligned}
P_{0|1|} &= \chi_2^S - \chi_{22}^{BS} \\
P_{1|1|} &= \frac{1}{2}\left(\chi_4^S - \chi_2^S + 5\chi_{13}^{BS} + 7\chi_{22}^{BS}\right) \\
P_{1|2|} &= -\frac{1}{4}\left(\chi_4^S - \chi_2^S + 4\chi_{13}^{BS} + 4\chi_{22}^{BS}\right) \\
P_{1|3|} &= \frac{1}{18}\left(\chi_4^S - \chi_2^S + 3\chi_{13}^{BS} + 3\chi_{22}^{BS}\right)
\end{aligned}
$$

A. Bazavov et al., PRL (2013)

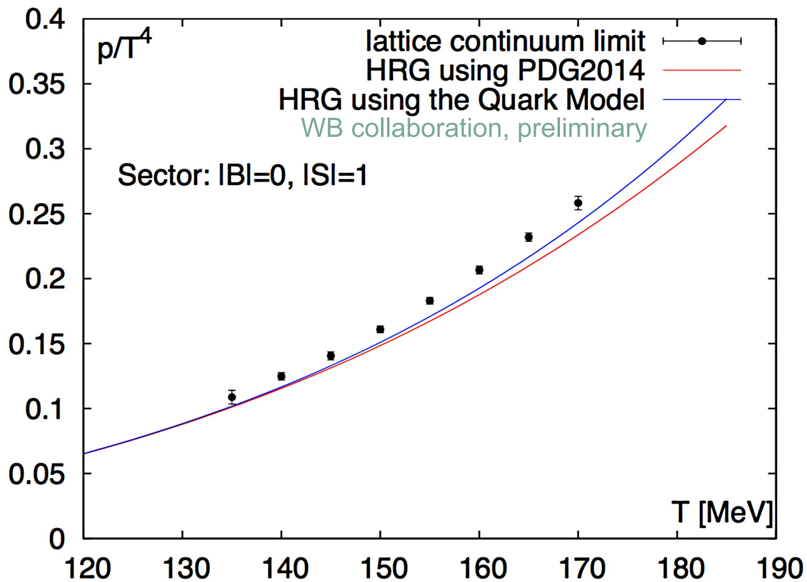
Missing strange states?

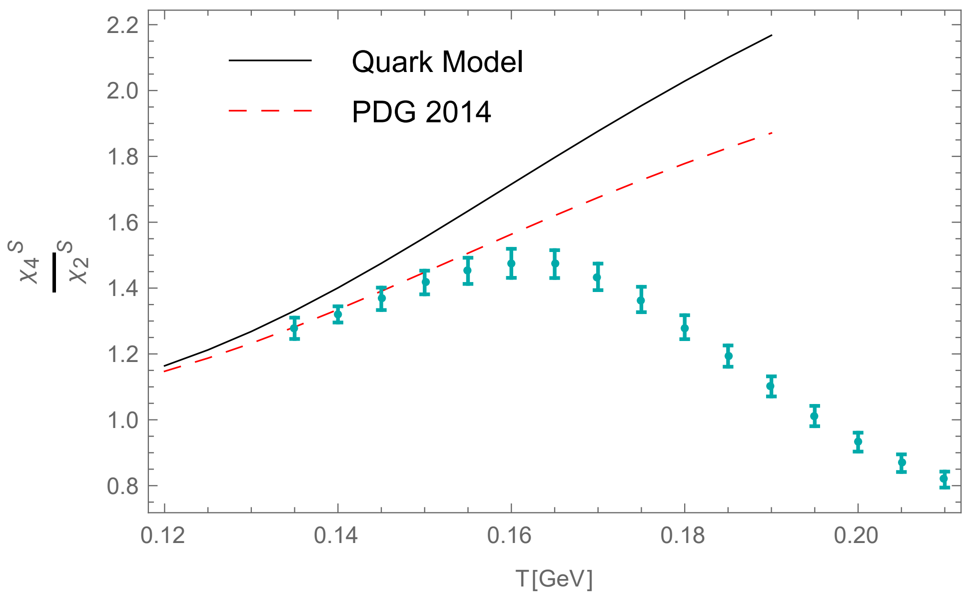




- The precision in the lattice results can allow to distinguish between the two scenarios
- Quark model yields better agreement with the data for the strange baryons

Not enough strange mesons





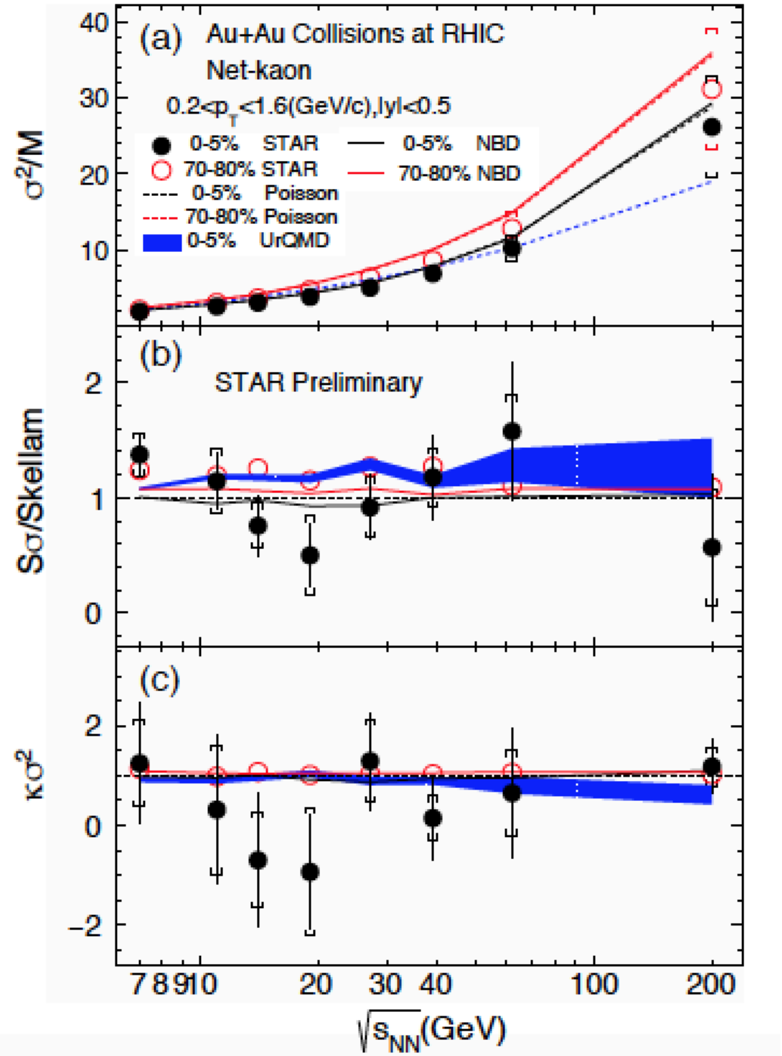
- Both Quark Model and PDG 2014 underestimate the partial pressure due to strange mesons
- This explains why the QM overestimates χ_4^S/χ_2^S: more strange mesons would bring the curve down

P.M. Lo, K. Redlich, C. Sasaki, PRC (2015)

Kaon fluctuations

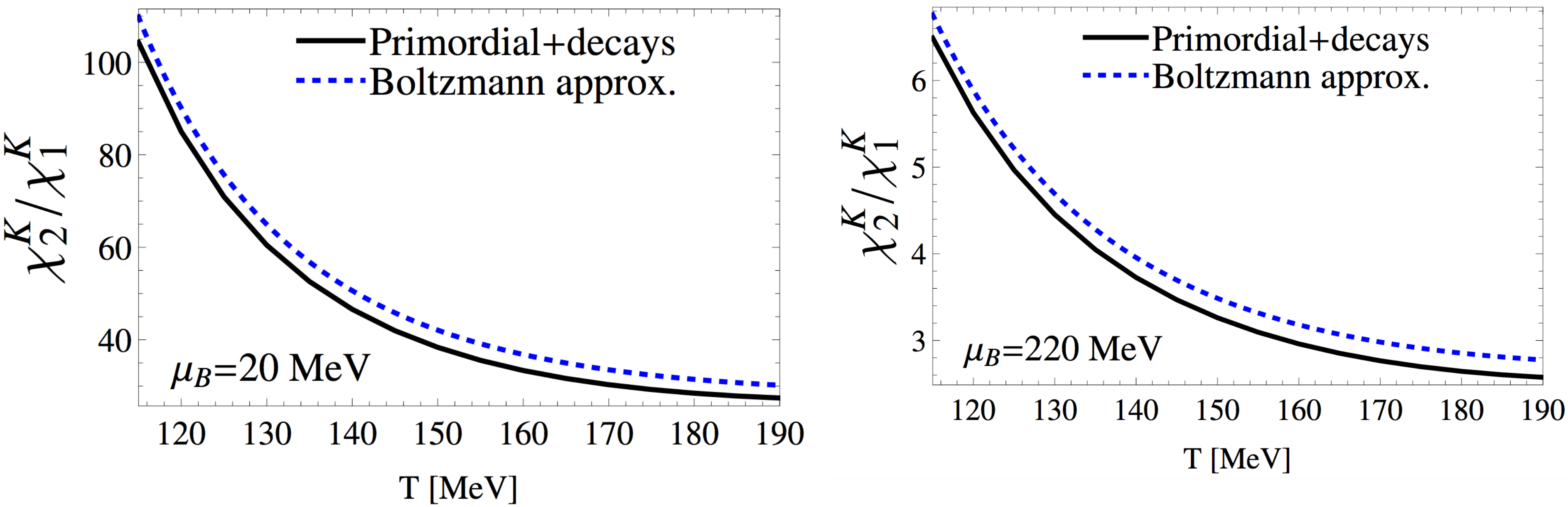
Talk by Ji XU at SQM 2016

- Experimental data are becoming available.
- Exciting result but presently hampered by systematic errors
- BES-II will help
- Kaon fluctuations from HRG model will be affected by the hadronic spectrum and decays



Kaon fluctuations on the lattice?

J. Noronha-Hostler, C.R. et al., 1607.02527



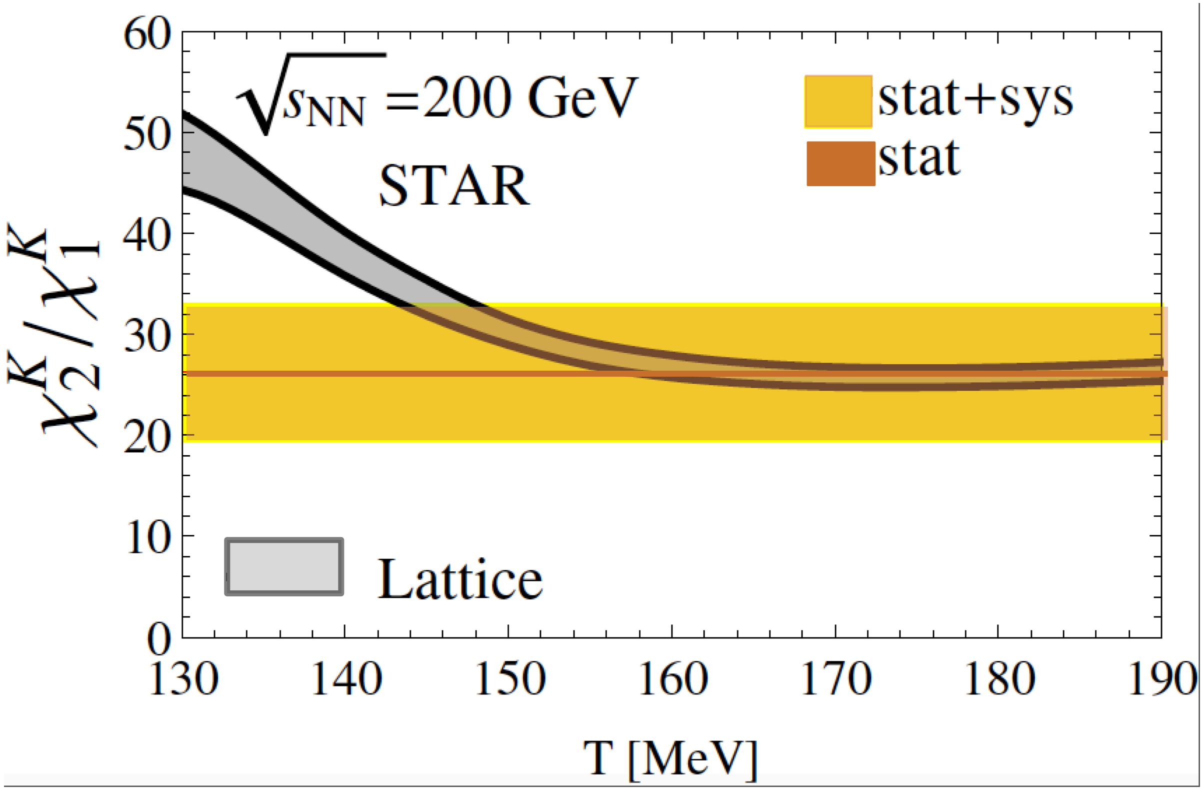
- Boltzmann approximation works well for lower order kaon fluctuations

$$\frac{\chi_2^K}{\chi_1^K} = \frac{\cosh(\hat{\mu}_S + \hat{\mu}_Q)}{\sinh(\hat{\mu}_S + \hat{\mu}_Q)}$$

- χ_2^K/χ_1^K from primordial kaons + decays is very close to the one in the Boltzmann approximation

Kaon fluctuations on the lattice?

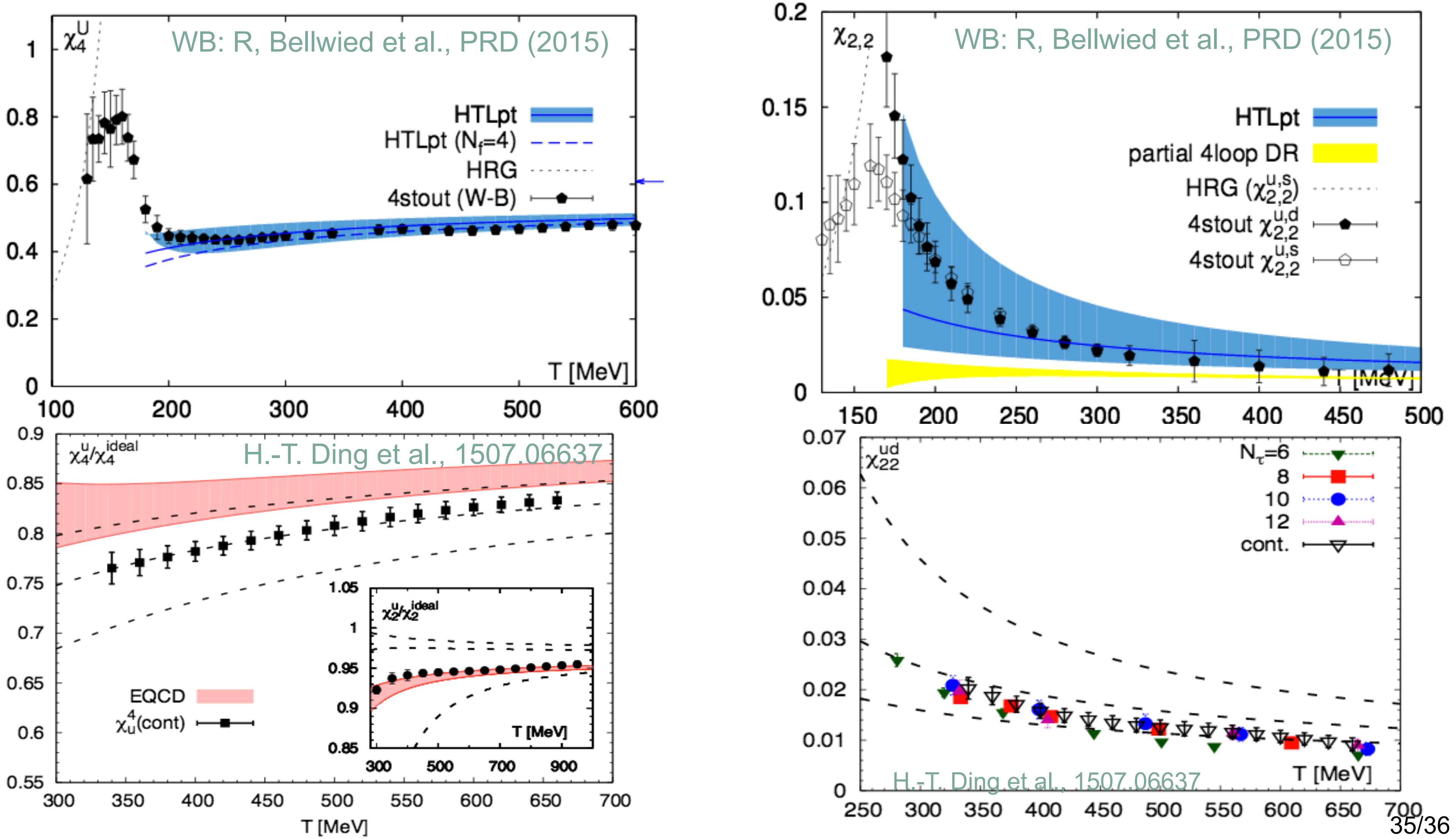
J. Noronha-Hostler, C.R. et al., 1607.02527



- Experimental uncertainty does not allow a precise determination of T_f^K

Fluctuations at high temperatures

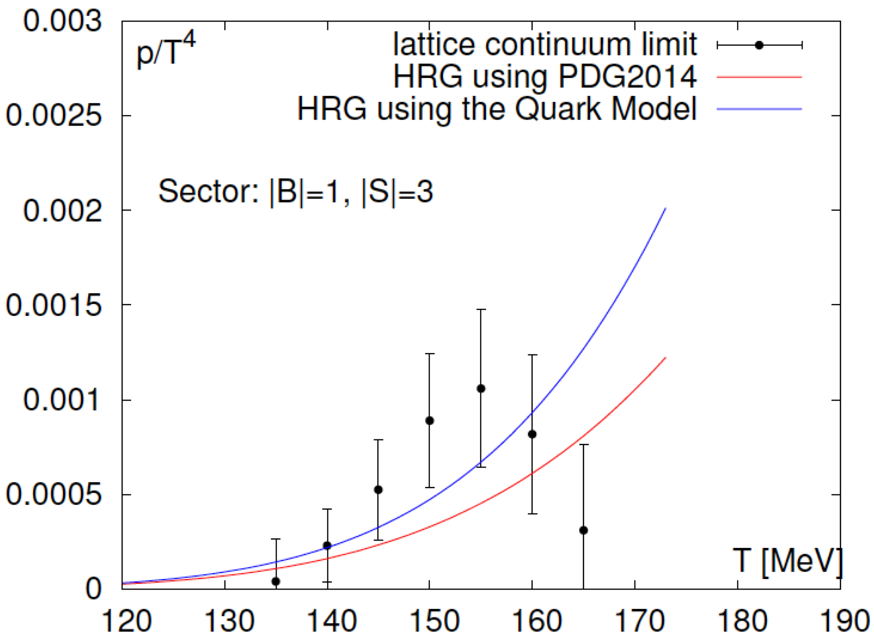
HTL: N. Haque et al., JHEP (2014); DR: S. Mogliacci et al., JHEP (2013)

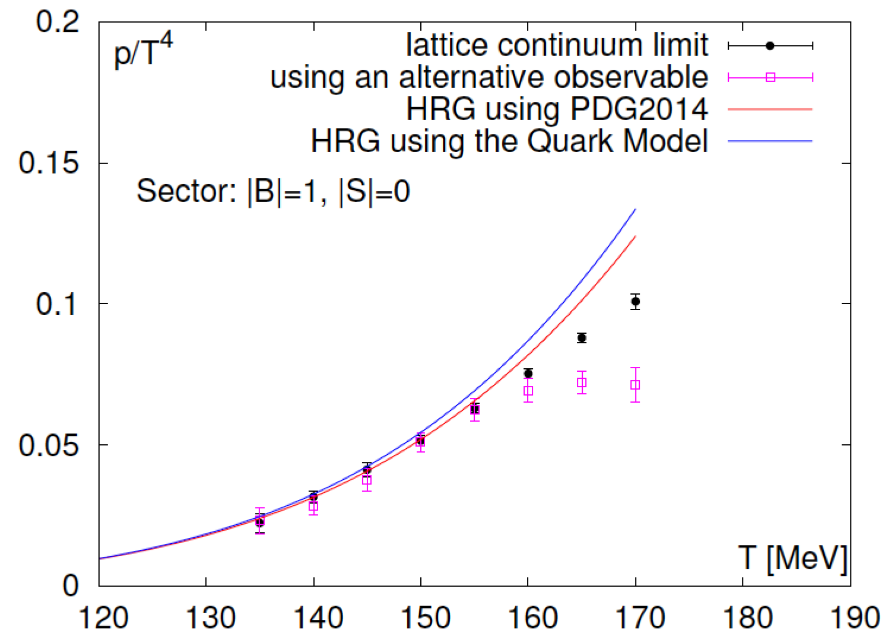


Conclusions

- Unprecedented precision in lattice QCD data allows a direct comparison to experiment for the first time
- QCD thermodynamics at μ_B=0 can be simulated with high accuracy
- Extensions to finite density are under control up to $O(\mu_B^6)$
- Comparison with experiment allows to determine properties of strongly interacting matter from first principles
- Perturbative QCD valid starting from T~250 MeV

Missing strange states?

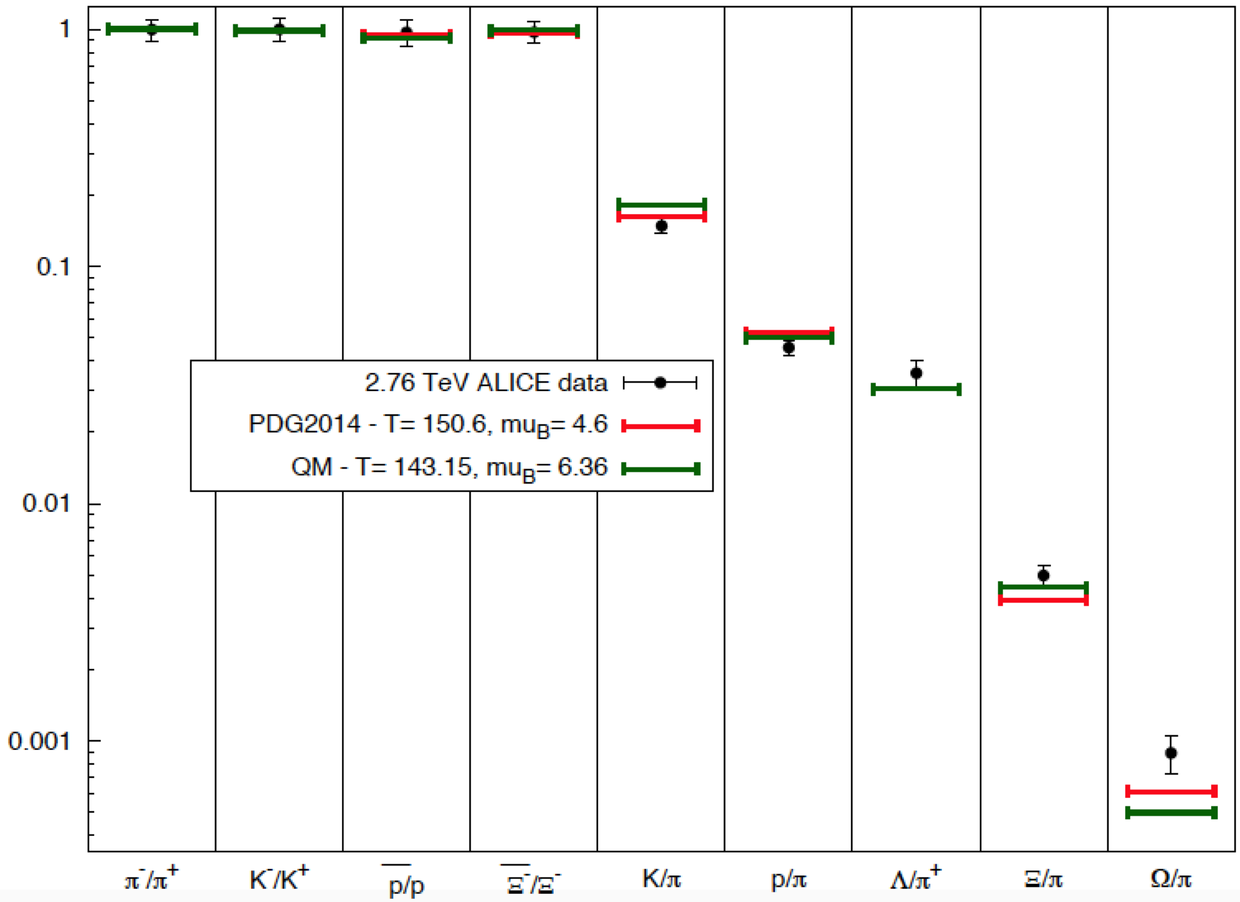




WB collaboration, in preparation

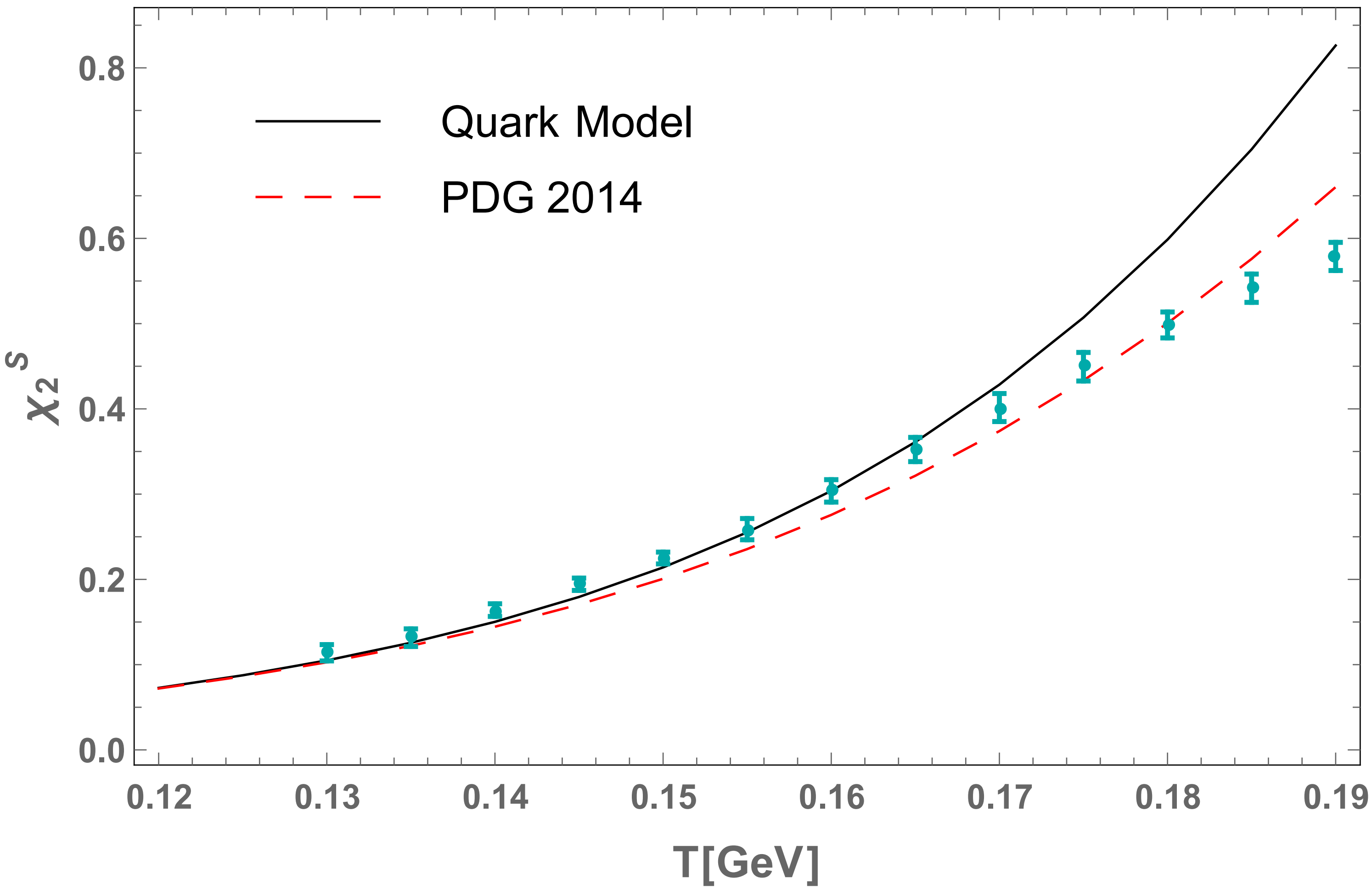
Effect of resonance decays

- The decays have a big effect on the freeze-out parameters



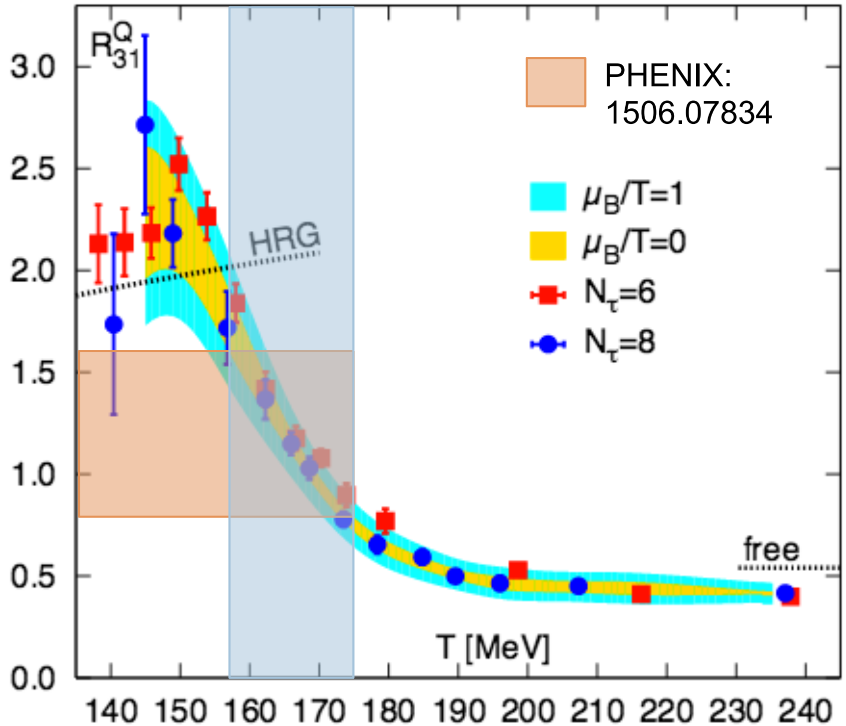
P. Alba et al., in preparation

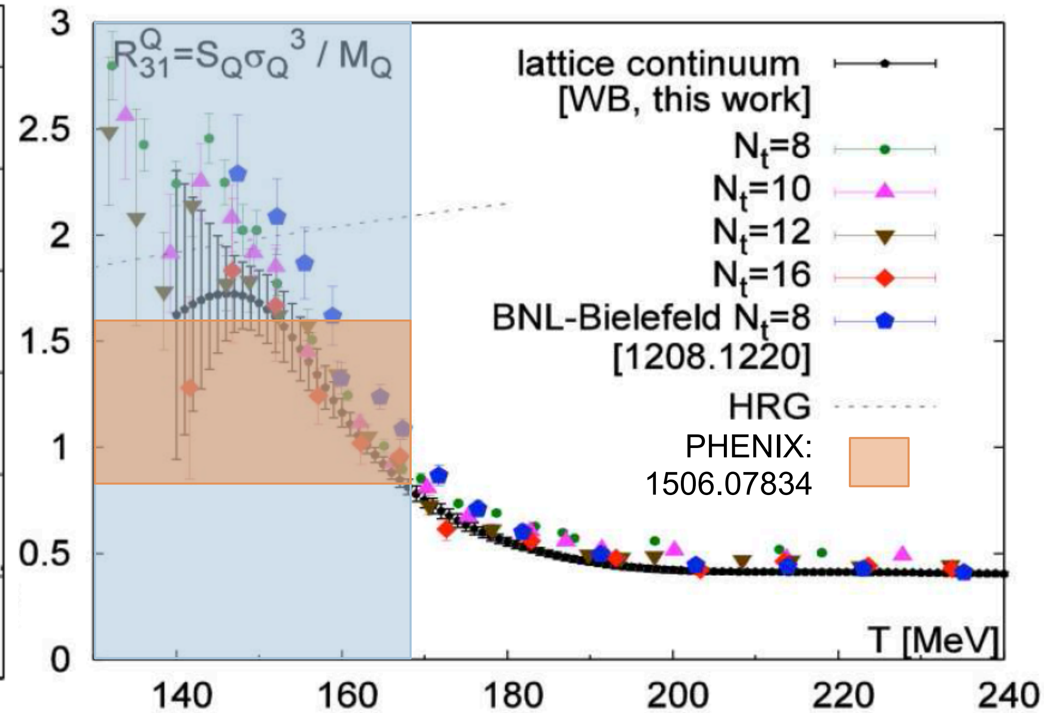
Missing strange states?



WB collaboration, in preparation

Freeze-out parameters from Q fluctuations





A. Bazavov et al. (2014)

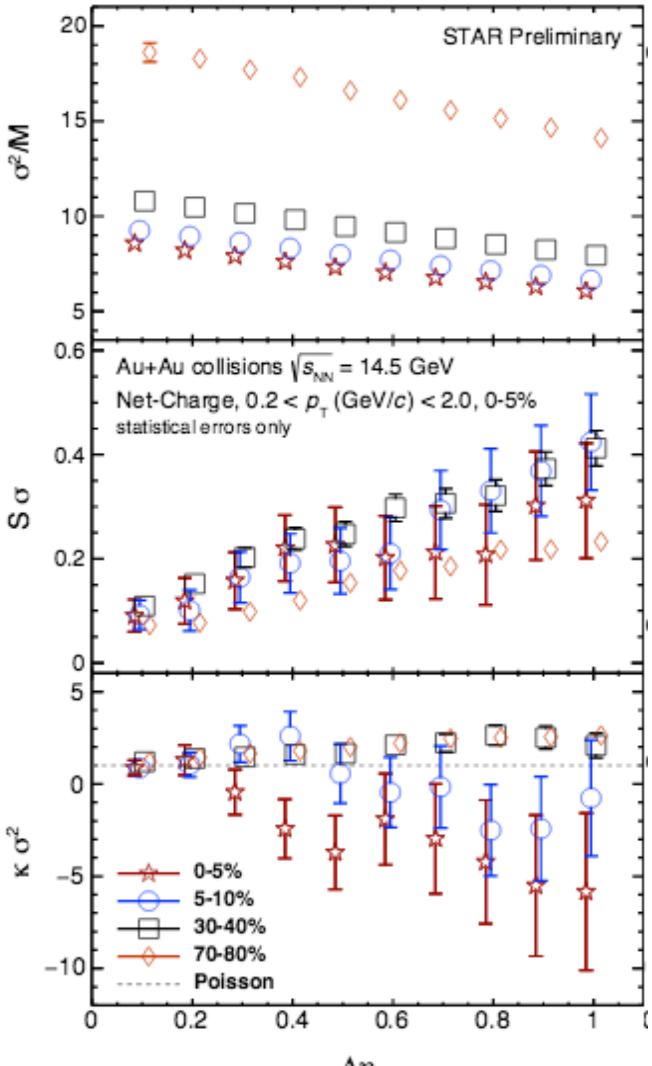
WB: Borsanyi et al. PRL (2013)

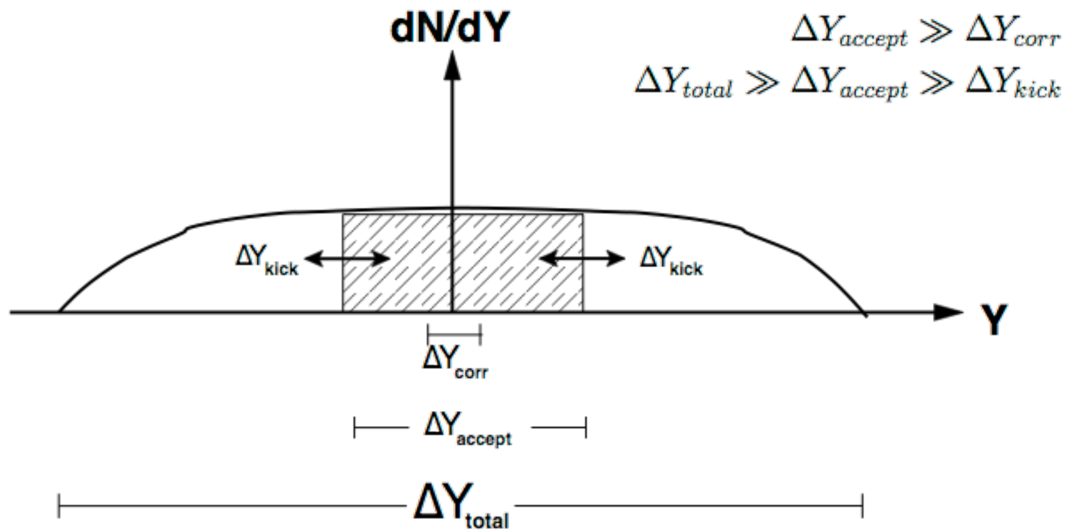
- Studies in HRG model: the different momentum cuts between STAR and PHENIX are responsible for more than 30% of their difference

F. Karsch et al., 1508.02614

- Using continuum extrapolated lattice data, lower values for T_f are found

Effects of kinematic cuts





V. Koch, 0810.2520

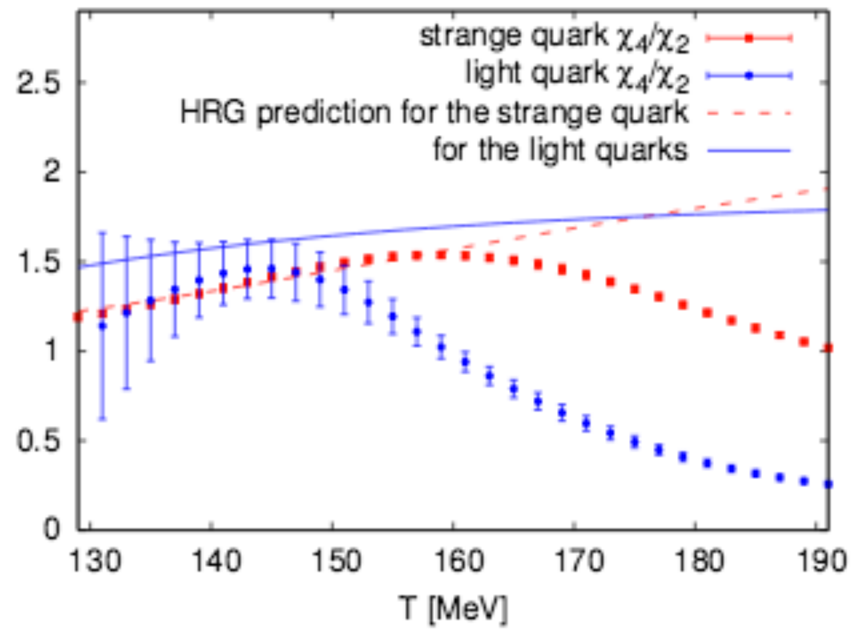
- Rapidity dependence of moments needs to be studied for 1<Δη<2
- Difference in kinematic cuts between STAR and PHENIX leads to a 5% difference in T_f

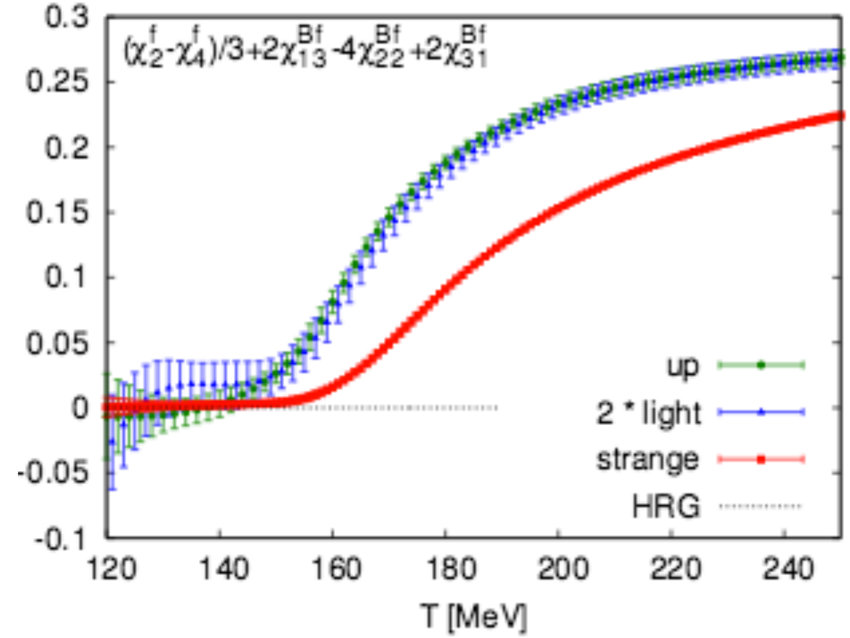
Talk by J. Thaeder on Monday

Talk by F. Karsch on Monday

Strangeness fluctuations

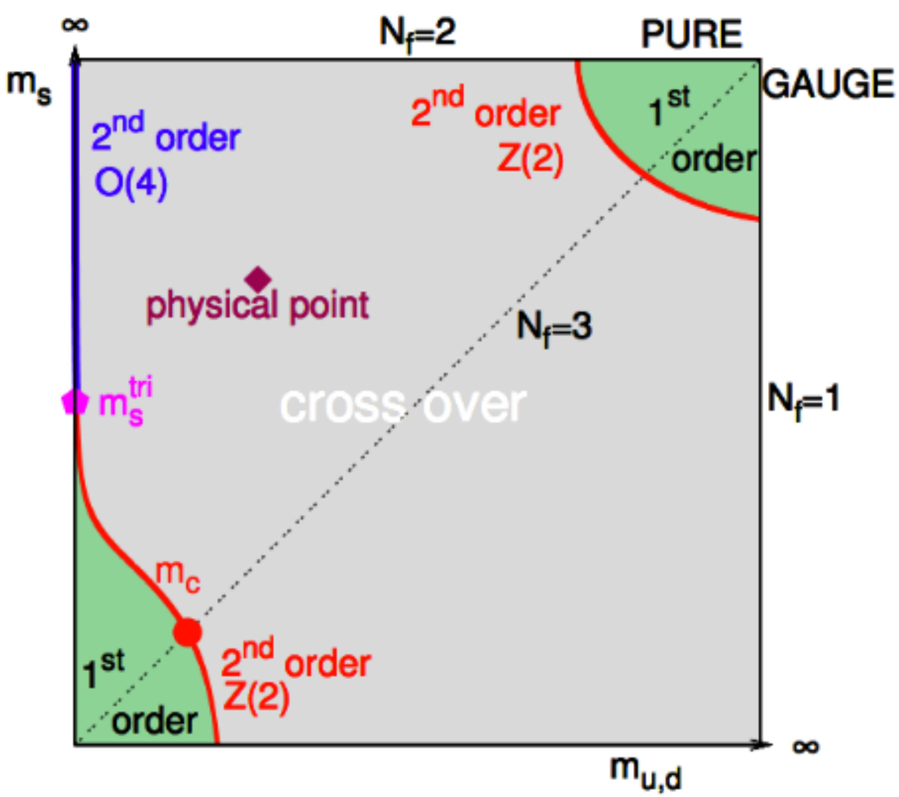
WB: R. Bellwied et al, PRL (2013)





- Lattice data hint at possible flavor-dependence in transition temperature
- Possibility of strange bound-states above T_c?

Columbia plot



- Pure gauge theory: Tc=294(2) MeV
  Francis et al., 1503.05652
- N_f=2 QCD at $m_\pi > m_\pi^{phys}$:
  - O(a) improved Wilson, N_t=16
  - m_π=295 MeV T_c=211(5) MeV
  - m_π=220 MeV T_c=193(7) MeV

  Brandt et al., 1310.8326
  - Twisted-mass QCD

  \- m_π=333 MeV T_c=180(12) MeV

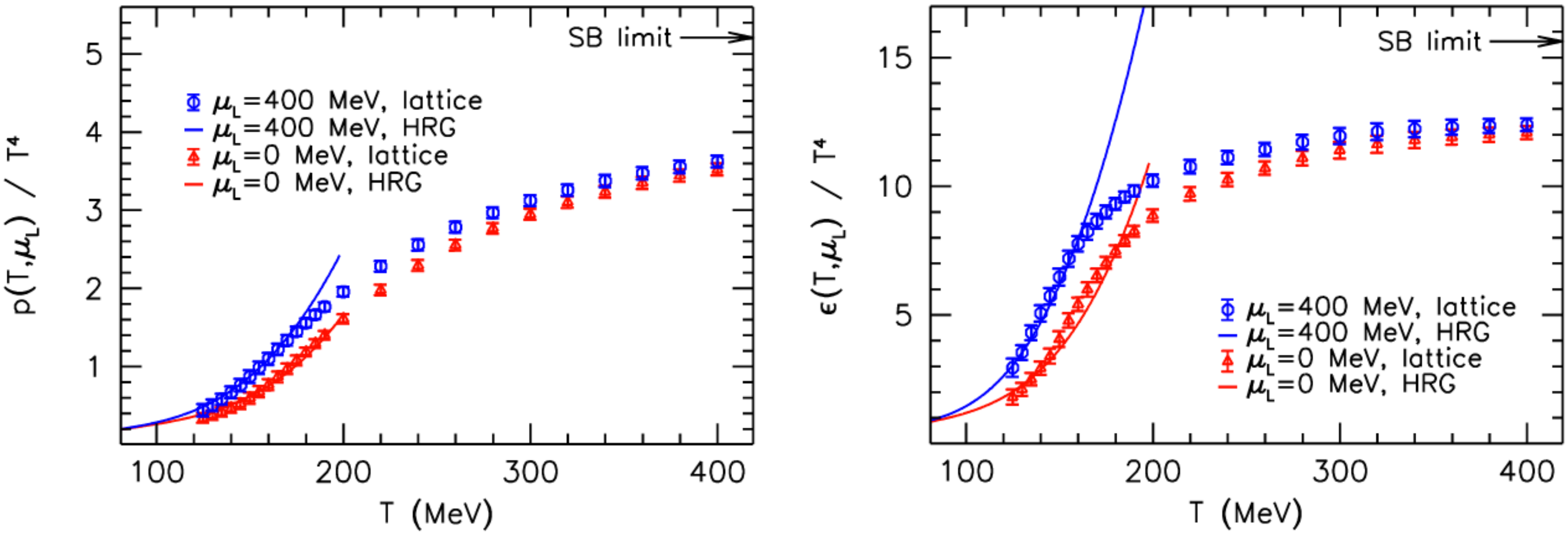
  Burger et al., 1412.6748
- N_f=2+1 O(a) improved Wilson
  - Continuum results

  Borsanyi et al., 1504.03676
- HISQ action, N_t=6, no sign of 1^{st} order phase transition at m_π=80 MeV
  HotQCD, 1312.0119, 1302.5740

Equation of state at $\mu_B>0$

- Expand the pressure in powers of μ_B (or $\mu_L=3/2(\mu_u+\mu_d)$)

$$\frac{p(T,\{\mu_i\})}{T^4}=\frac{p(T,\{0\})}{T^4}+\frac{1}{2}\sum_{i,j}\frac{\mu_i\mu_j}{T^2}\chi_2^{ij} \quad \text{with} \quad \chi_2^{ij}\equiv\frac{T}{V}\frac{1}{T^2}\left.\frac{\partial^2\log\mathcal{Z}}{\partial\mu_i\partial\mu_j}\right|_{\mu_i=\mu_j=0}$$

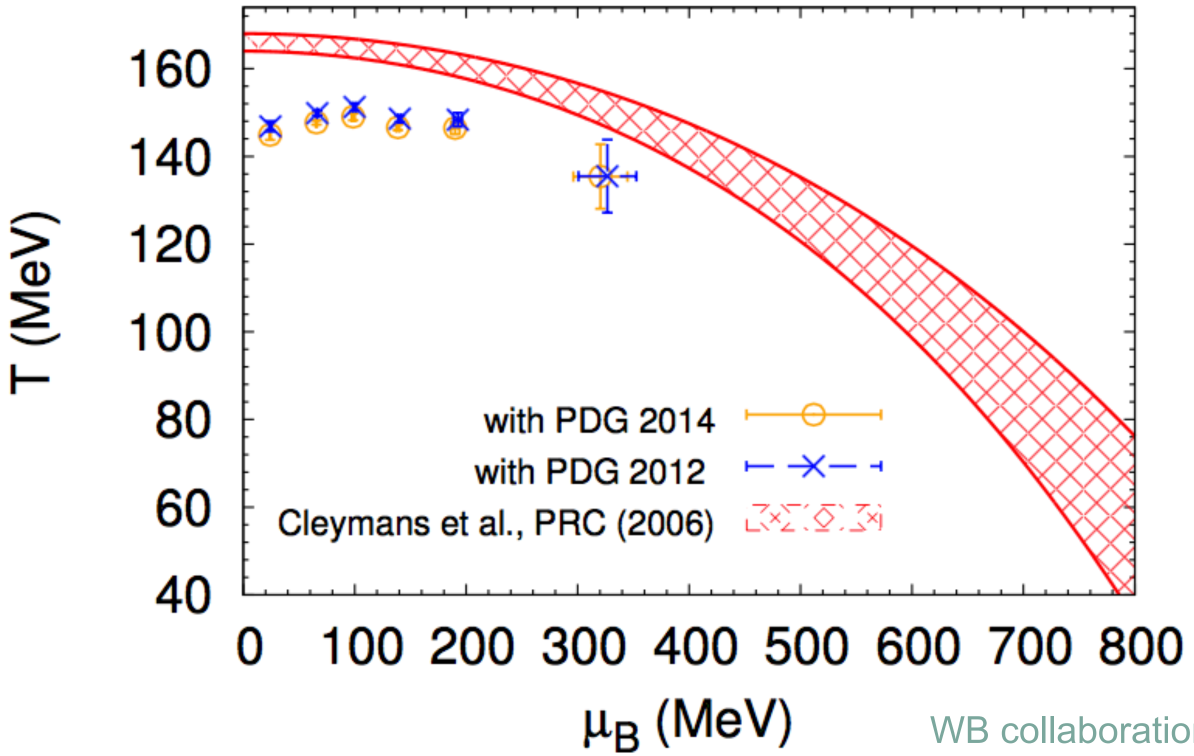


S. Borsanyi et al., JHEP (2012)

- Continuum extrapolated results at the physical mass

Effect of resonance decays

- We used the PDG2014 to estimate the effect of resonance decays on the fit to proton and charge fluctuations
- The results agree with the ones obtained with the PDG2012 within errorbars



WB collaboration, in preparation