# Scale invariance of $QED_3$

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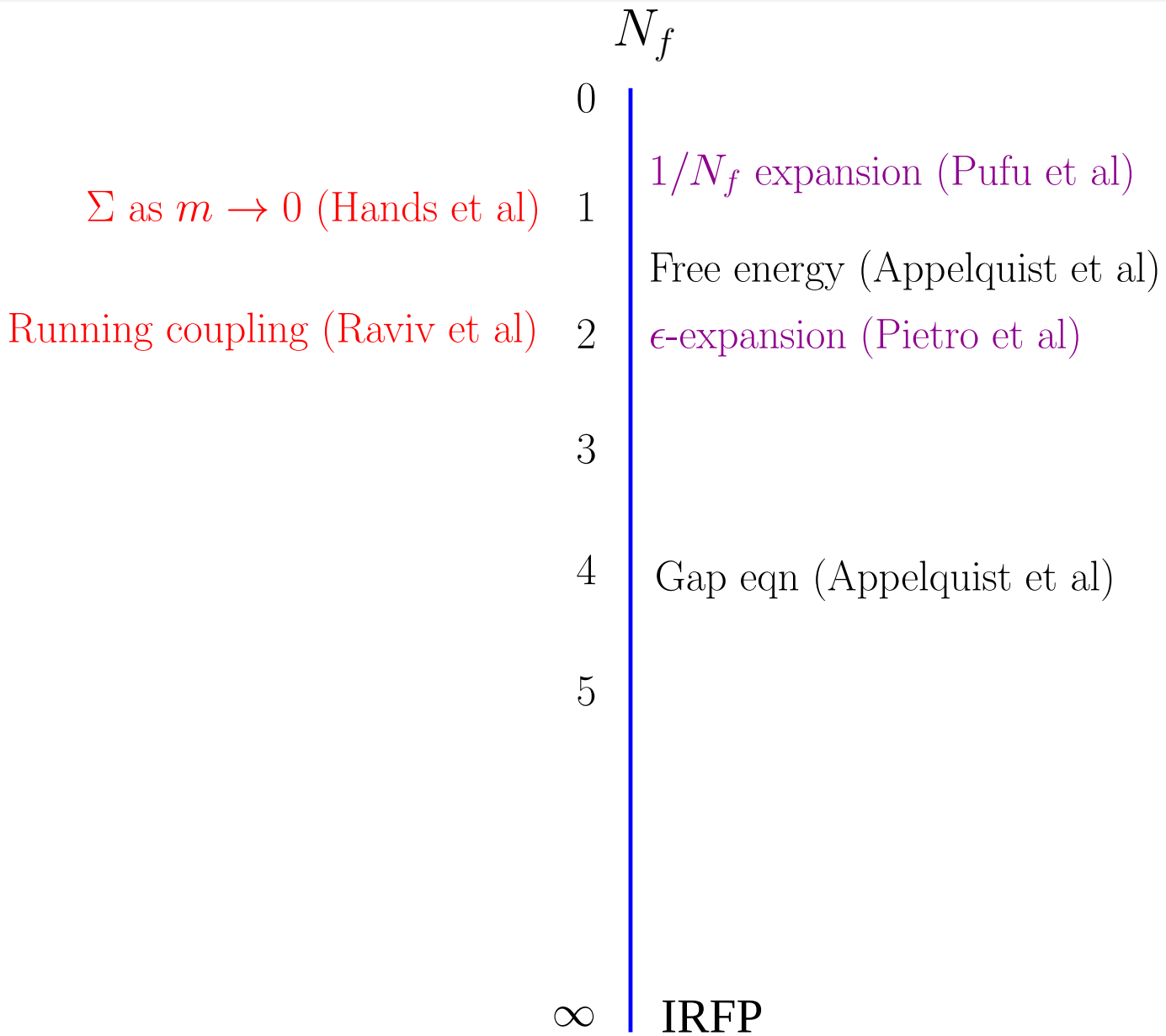
# Non-compact parity-invariant $QED_3$ on $\ell^3$ Euclidean torus

$$L = \sum_{i=1}^{N_f} \left\{ \overline{u}_i C_{\rm reg} u_i - \overline{d}_i C_{\rm reg}^\dagger d_i \right\} + \frac{1}{4g^2} \left( \partial_\mu A_\nu - \partial_\nu A_\mu \right)^2$$

- $u, d \quad \rightarrow \quad$ 2-component fermion field.
- Massless regulated Dirac operator: $\quad C_{\rm reg}$
- $g^2 \quad \rightarrow \quad$ Coupling constant of dimension $[\text{mass}]^1$
  Scale setting: $\quad g^2 = 1 \leftrightarrow$ specify $\ell$.
- $U(2N_f)$ flavor symmetry in the continuum limit since $C = -C^\dagger$.
- $N_f \rightarrow \infty$ has an IR fixed point. What is the effect of finite $N_f$ corrections? Spontaneously break $U(2N_f) \longrightarrow U(N_f) \times U(N_f)$?

# Transition from massive to conformal phase *plausible*



# Tell-tale signs for bilinear condensate $\Sigma$

(Shuryak and Verbaarschot '93) Spontaneous flavor symmetry breaking $\Rightarrow$ Chiral lagrangian at finite $\ell \Rightarrow$ Random matrix theory for low eigenvalues ($z = \Sigma\lambda\ell^3$)

- Finite-size scaling of low-lying eigenvalues of the Dirac operator:

$$\lambda\ell \sim \frac{1}{\ell^2}.$$

- IPR: eigenvectors $\Psi_\lambda$ of the Dirac operator are completely delocalized $\Rightarrow$

$$I_2 \equiv \int \langle |\psi(x)|^4 \rangle d^3x \sim \frac{1}{\ell^3}$$

- Ergodic behaviour of number-variance $\Sigma_2(n)$, the variance in the number of eigenvalues $n$ below a value $\lambda$.

$$\Sigma_2(n) \sim \log(n)$$

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# ... Instead if $QED_3$ is scale-invariant

- Gauge-invariant zero spatial-momentum scalar and vector correlators

$$G_\Sigma(t) \sim \frac{1}{t^{2-2\gamma_m}} \qquad \text{and} \qquad G_V(t) \sim \frac{1}{t^{2-2\gamma_V}}$$

- Mass-anomalous dimensions from finite-size scaling of low-lying eigenvalues of the Dirac operator:

$$\lambda\ell \sim \frac{1}{\ell^{1+\gamma_m}}; \qquad \gamma_m < 1.$$

- True at least in the Anderson type criticality: Eigenvectors $\Psi_\lambda$ of the Dirac operator are delocalized in a "fractal-way"

$$\text{IPR} \sim \frac{1}{\ell^{3-\eta}}.$$

- Critical behavior of number-variance $\Sigma_2(n)$

$$\Sigma_2(n) \sim \frac{\eta}{6} n.$$

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# Parity-invariant Wilson-Dirac fermions

Regularize at the level of two-component fermions (as opposed to the equivalent four component fermions):

$$L = \overline{u} C_w u - \overline{v} C_w^\dagger v; \qquad C_w = C_n + B - m$$

Corresponding 4-component Hermitian Wilson-Dirac operator:

$$H_w = \begin{bmatrix} 0 & C_w(m) \\ C_w^\dagger(m) & 0 \end{bmatrix} \longrightarrow \text{eigenvalues } \lambda.$$

m $\rightarrow$ tune mass to zero as Wilson fermion has additive renormalization.

Advantage: All even flavors $2N_f$ can be simulated without involving square-rooting.

# Parity-invariant overlap fermions

Start from multi-particle Hamiltonians $\mathcal{H}_\pm = -a^\dagger H_\pm a$ where
$H_+ = H_w; \qquad H_- = \gamma_5.$

With one choice of phase, the gauge-invariant overlap has an explicit formula in 3d:

$$\langle +|-\rangle = \det\left(\frac{1+V}{2}\right); \quad V = \frac{1}{\sqrt{C_w C_w^\dagger}} C_w.$$

Parity-invariant fermion determinant: $\{\langle +|-\rangle\}_u \{\langle -|+\rangle\}_v$ .

Propagator with the full $\mathrm{U}(2N_f)$ symmetry:

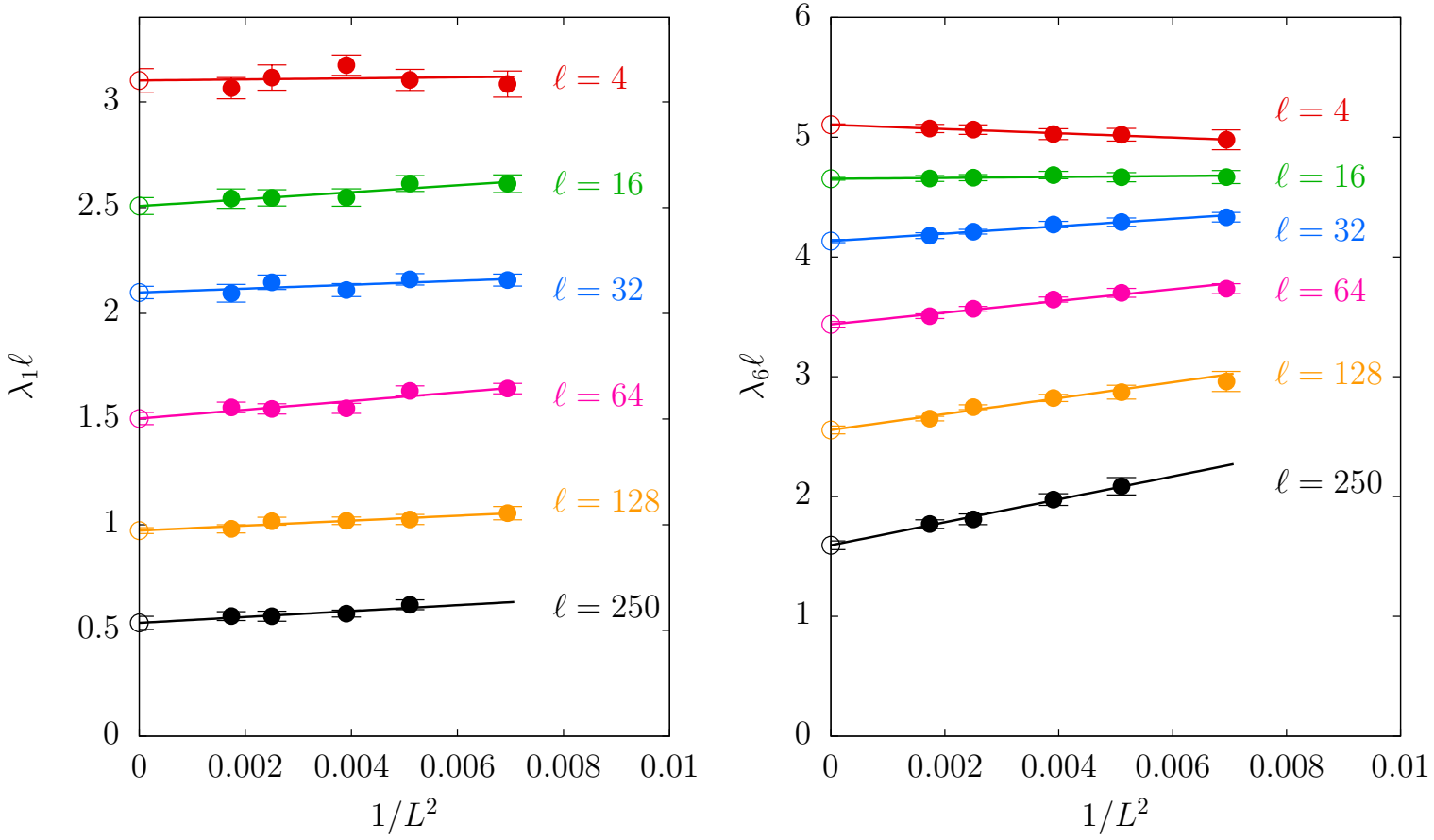
$$\begin{bmatrix} 0 & \frac{1-V}{1+V} \\ \frac{1-V}{1+V} & 0 \end{bmatrix} \to \text{eigenvalues} \frac{1}{i\lambda}.$$

# Continuum limit at fixed $\ell$

- $L^3$ periodic lattice with physical volume $\ell^3$.
- Non-compact gauge-action: $S_g = \frac{L}{\ell} \sum_n \sum_{\mu \neq \nu} (\Delta_\mu \theta_\nu(n) - \Delta_\nu \theta_\mu(n))^2$
- Continuum limit at fixed $\ell$ by taking $L \to \infty$.
- $L$ =12,14,16,20,24 at different $4 < \ell < 250$.
- HYP smeared $\theta$ in Dirac operator.
- Dynamical fermion simulation using standard HMC with both massless Wilson and overlap fermions.

# Continuum limit at fixed $\ell$

$\lambda_j$ $\rightarrow$ eigenvalues of Hermitian Dirac operator



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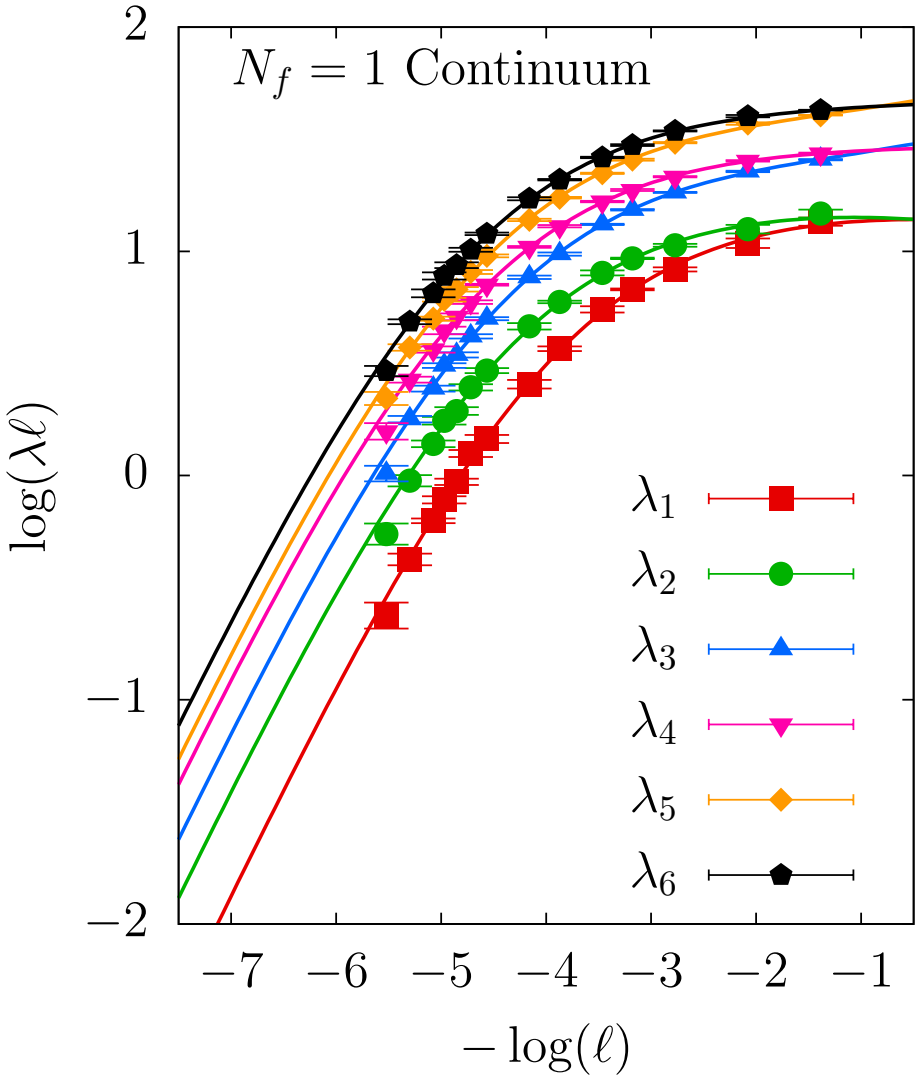
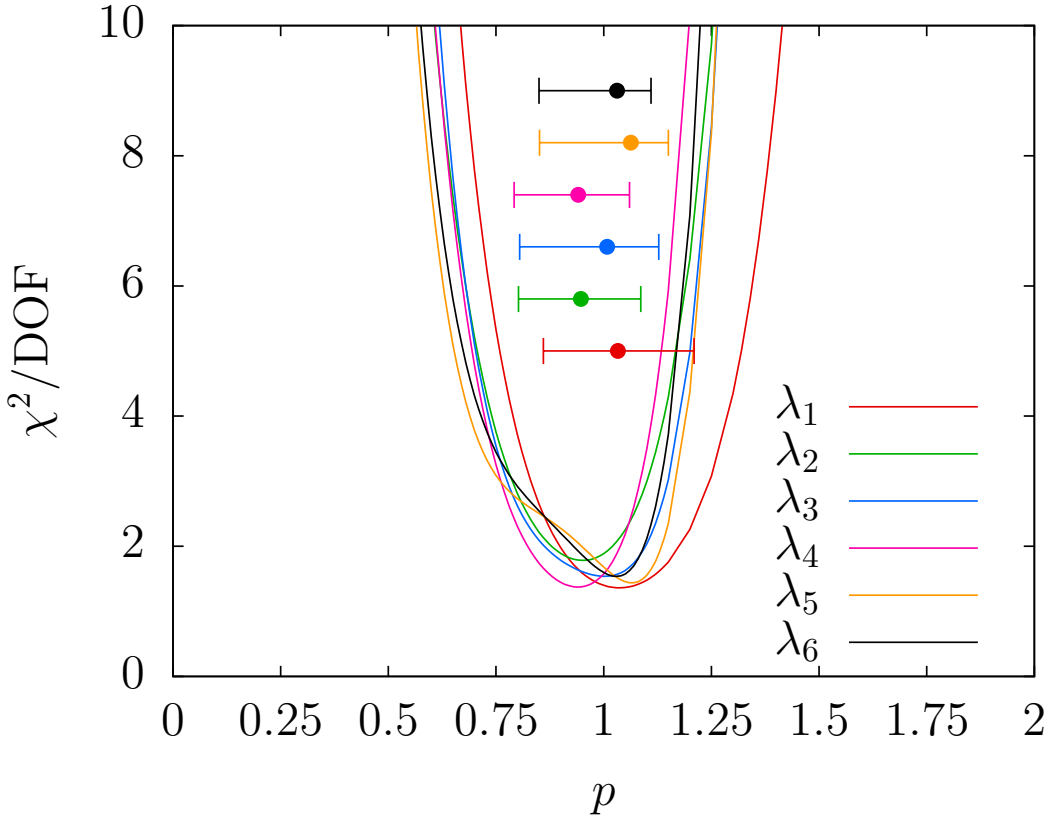
# If bilinear condensate: $\lambda\ell \sim \ell^{-2}$

$\lambda\ell \sim \ell^{-p} F(1/\ell) \longrightarrow$ approximate by [1/1] Padé



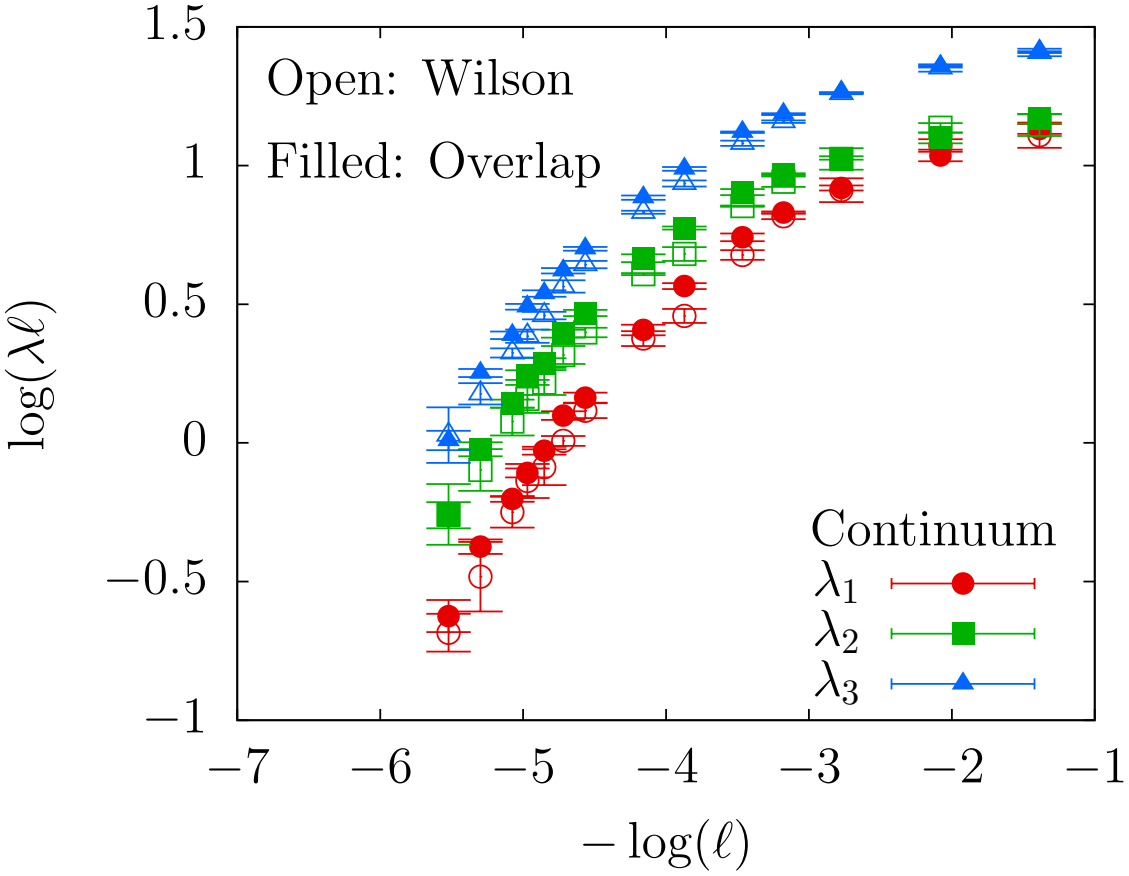
# If bilinear condensate: $\lambda\ell \sim \ell^{-2}$

Likelihood of different values of $p$ as $\ell \to \infty$



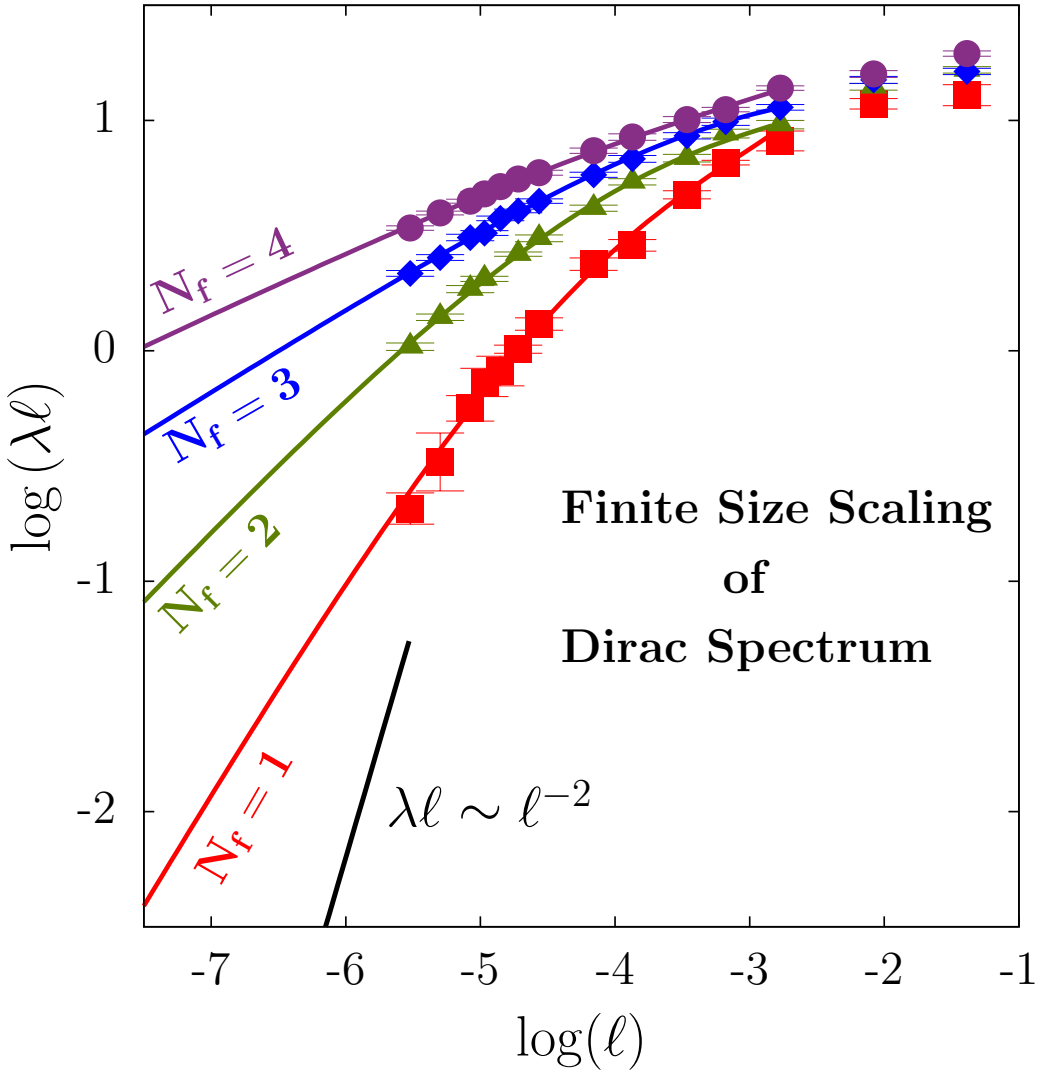

Expectation when condensate: $p = 2 \longrightarrow$ seems to be ruled out.
Conversely, mass anomalous dimension $\gamma_m = p = 1.0(2)$

# Agreement between Wilson and overlap fermion formulations




On lattice, Wilson fermions break $U(2N_f) \rightarrow U(N_f) \times U(N_f)$. Overlap has exact $U(2N_f)$. The agreement shows continuum limits are under control.

# Anomalous dimension decreases with $N_f$




$\gamma_m \approx 1/N_f$

# Anomalous dimension decreases with $N_f$

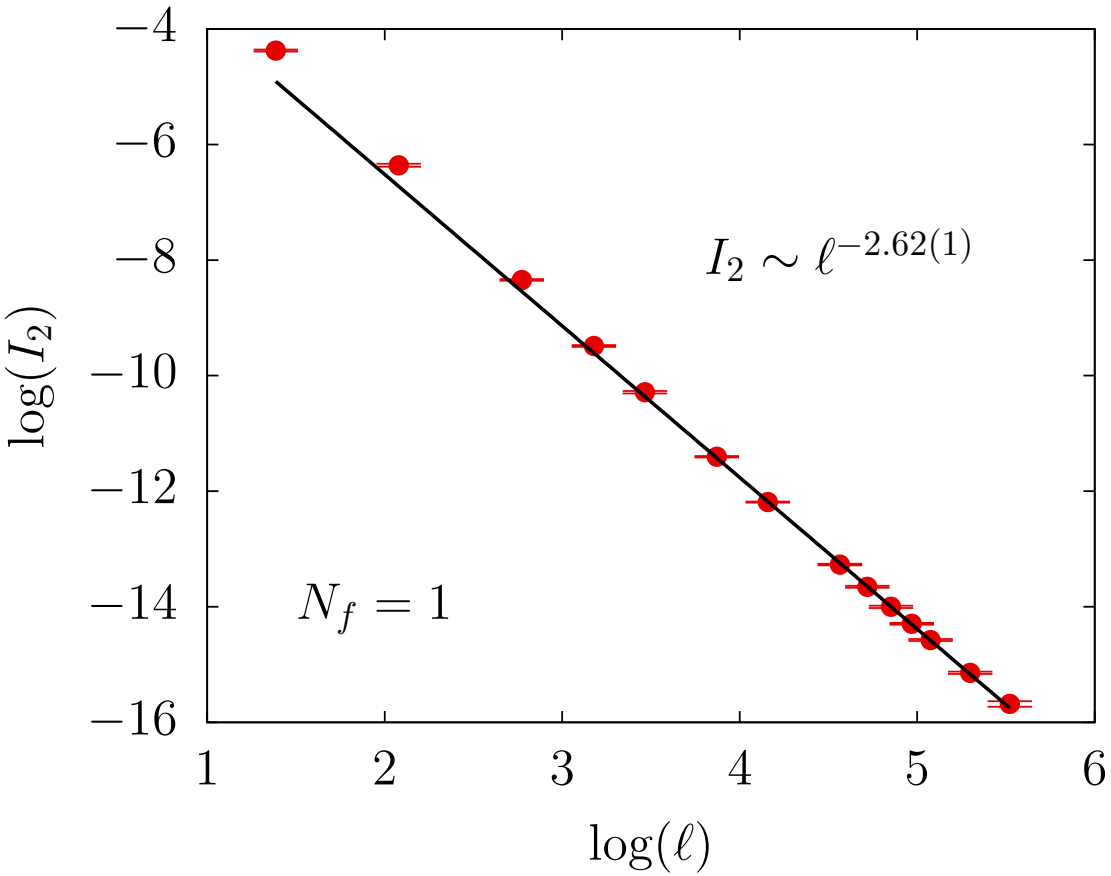
A result by Gusynin *et al.* last week, upto $\mathcal{O}(1/N_f^2)$:

| $N_f$ | $\gamma_m$ (our work) | $\gamma_m$ ($1/N_f$ expansion) |
|---|---|---|
| 1 | 1.0(2) | |
| 2 | 0.63(15) | condensate ($\Sigma \approx 10^{-12}$) |
| 3 | 0.37(5) | 0.37 |
| 4 | 0.28(5) | 0.28 |

- Agreement between our lattice method and $1/N_f$-calculation when we both agree on $N_f > N_{\mathrm{crit}}$.
- Based on our first principle calculation, we can only speculate that higher order $1/N_f$-corrections are important to decide on the existence of condensate.

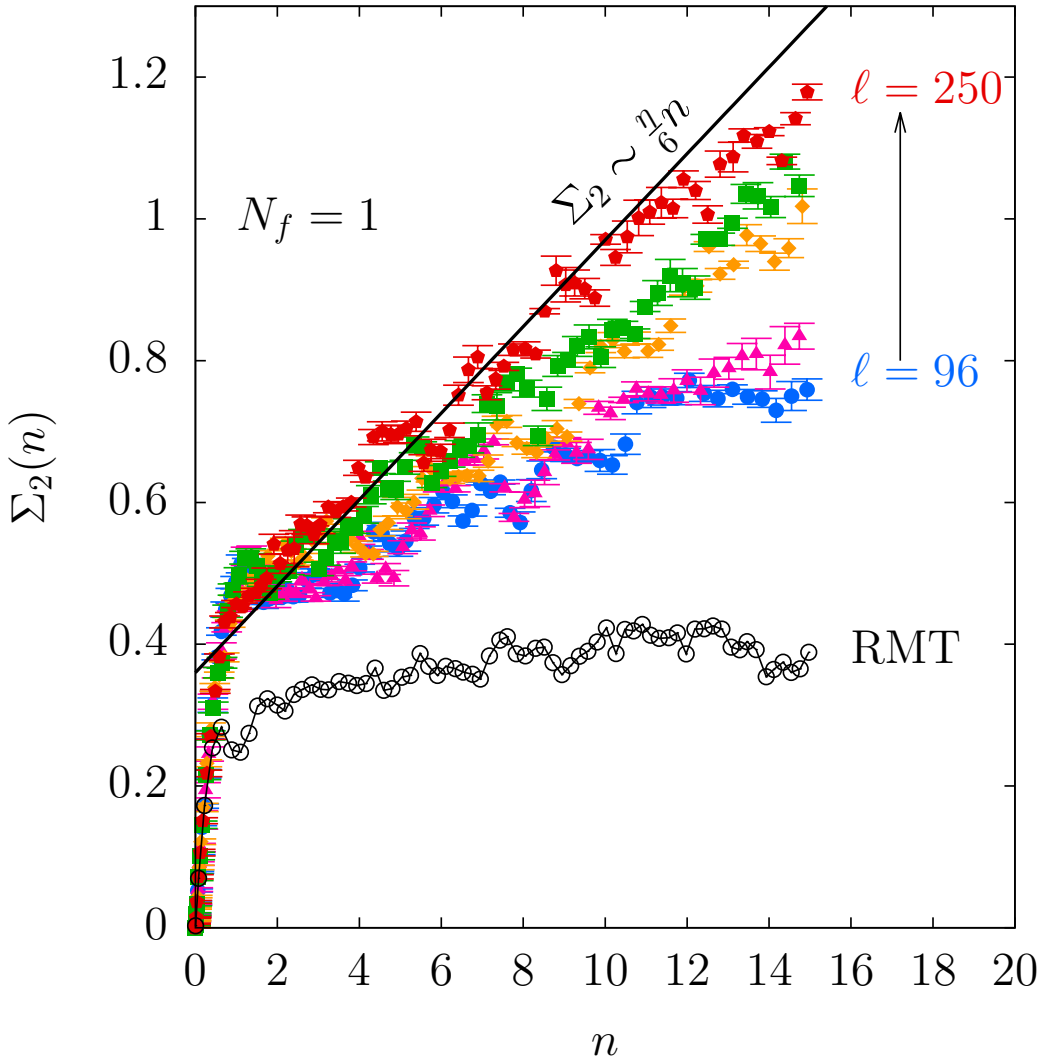
# Fractal behavior of Inverse Participation Ratio (IPR)

Condensate: $I_2 \sim \ell^{-3}$; Critical: $I_2 \sim \ell^{-3+\eta}$




$\eta = 0.38(1)$ (Critical!)

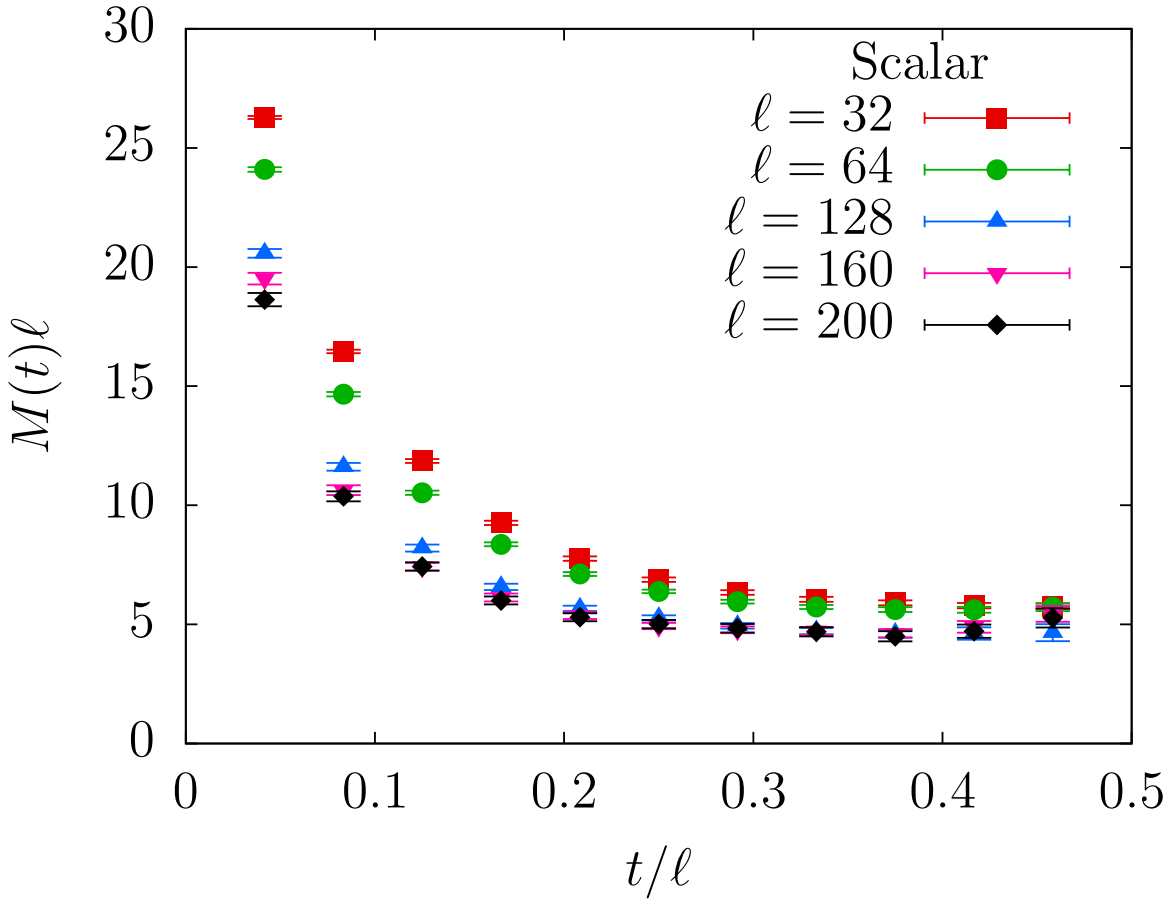
# Number variance shifts away from RMT towards criticality




(Altshuler *et al.* '88) Critical relation: $\Sigma_2 \sim \dfrac{\eta}{6} n \quad \longrightarrow \quad \eta$ from IPR

# Further evidence for scale-invariance: Absence of mass-gap

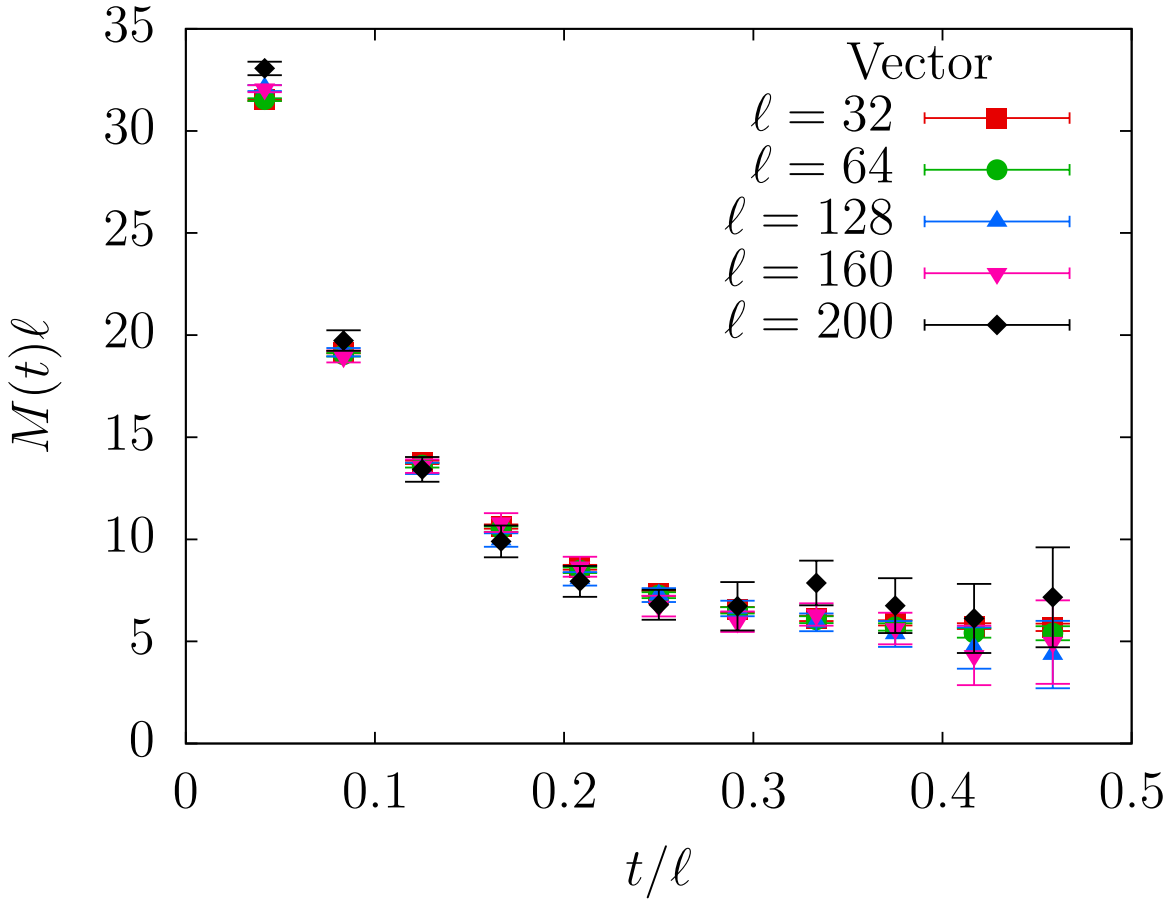
Scalar: $\overline{u}u(t) - \overline{v}v(t)$




$$\Rightarrow \quad M \sim \ell^{-1}$$

# Further evidence for scale-invariance: Absence of mass-gap

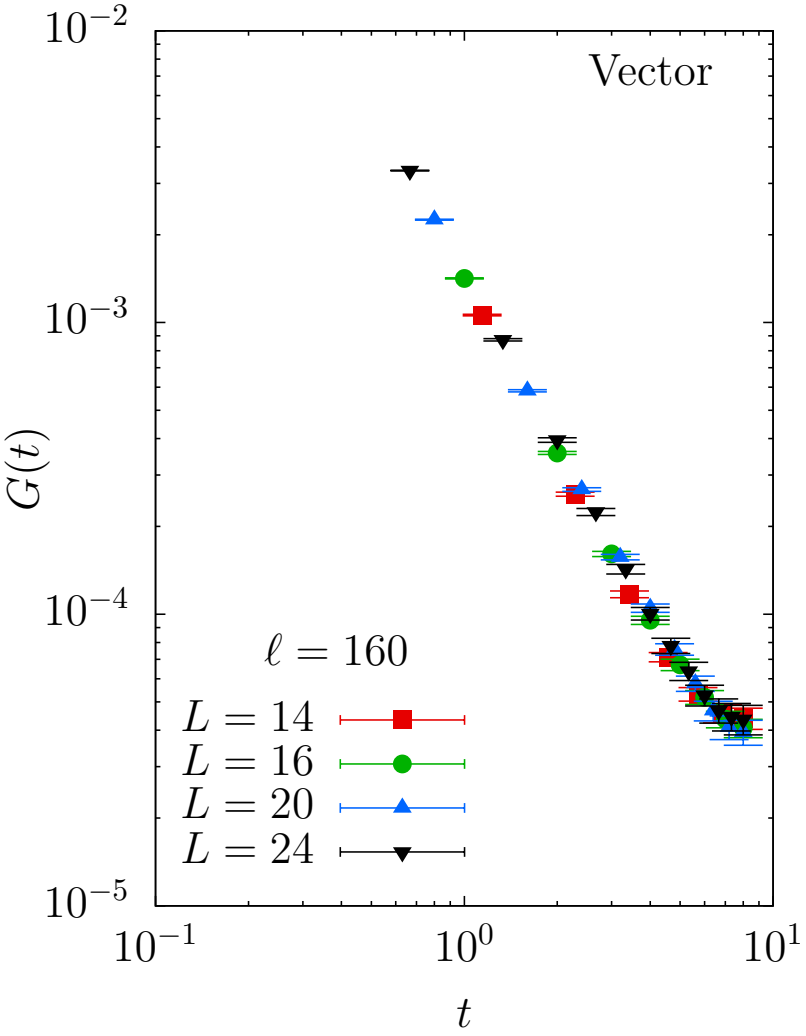
Vector: $\overline{u}\sigma_i u(t) - \overline{v}\sigma_i v(t)$

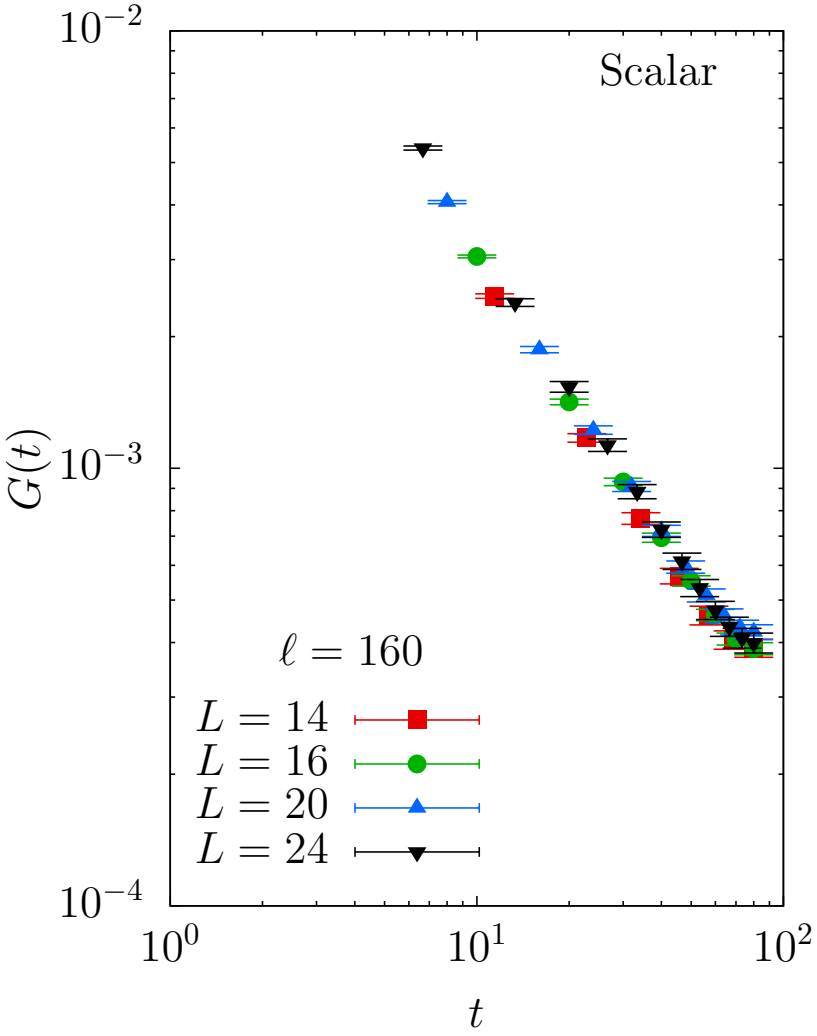



$$\Rightarrow \quad M \sim \ell^{-1}$$

# Long-distance behavior of correlators

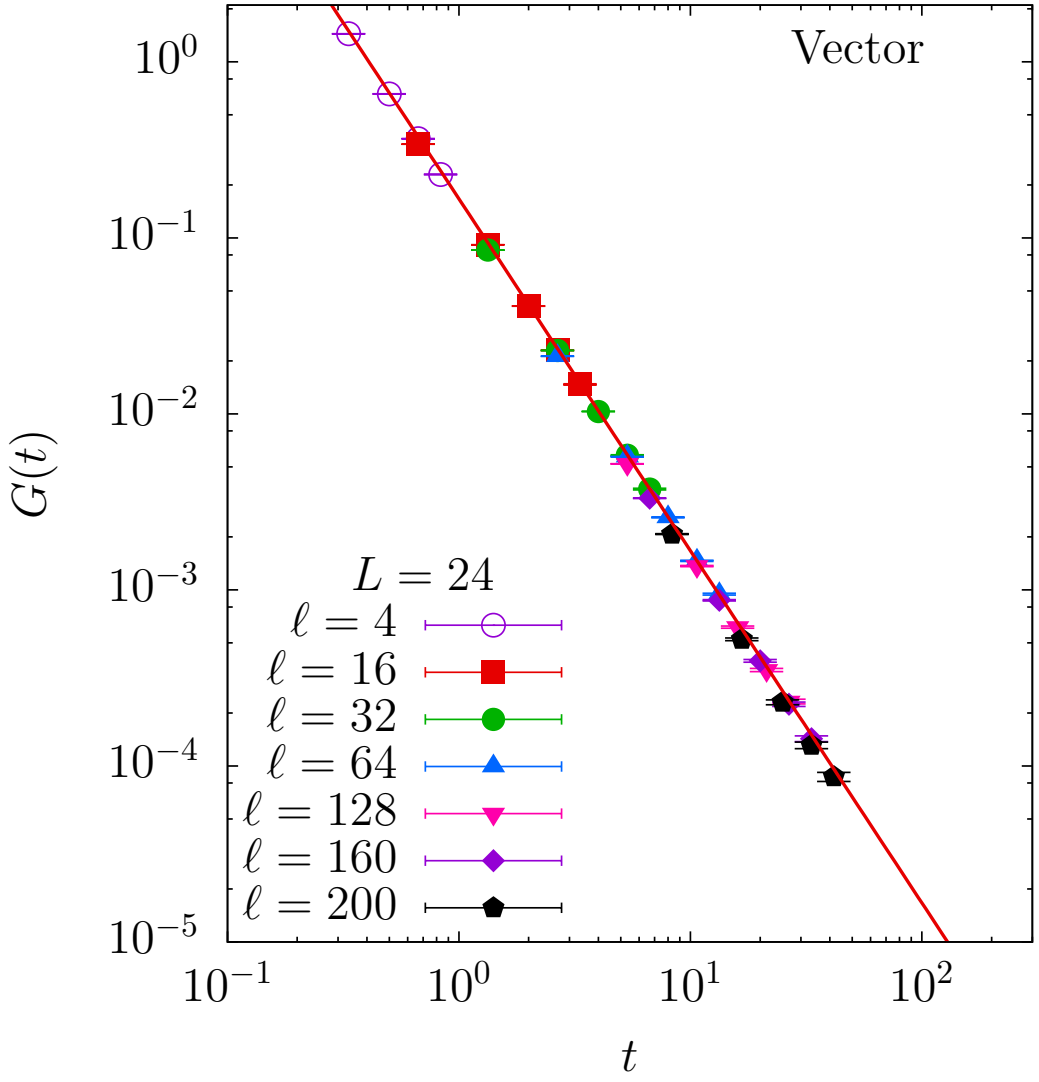
Lattice spacing effects are small → put together data from different $\ell$ on same $L^3$ lattice to fill-up "gaps"



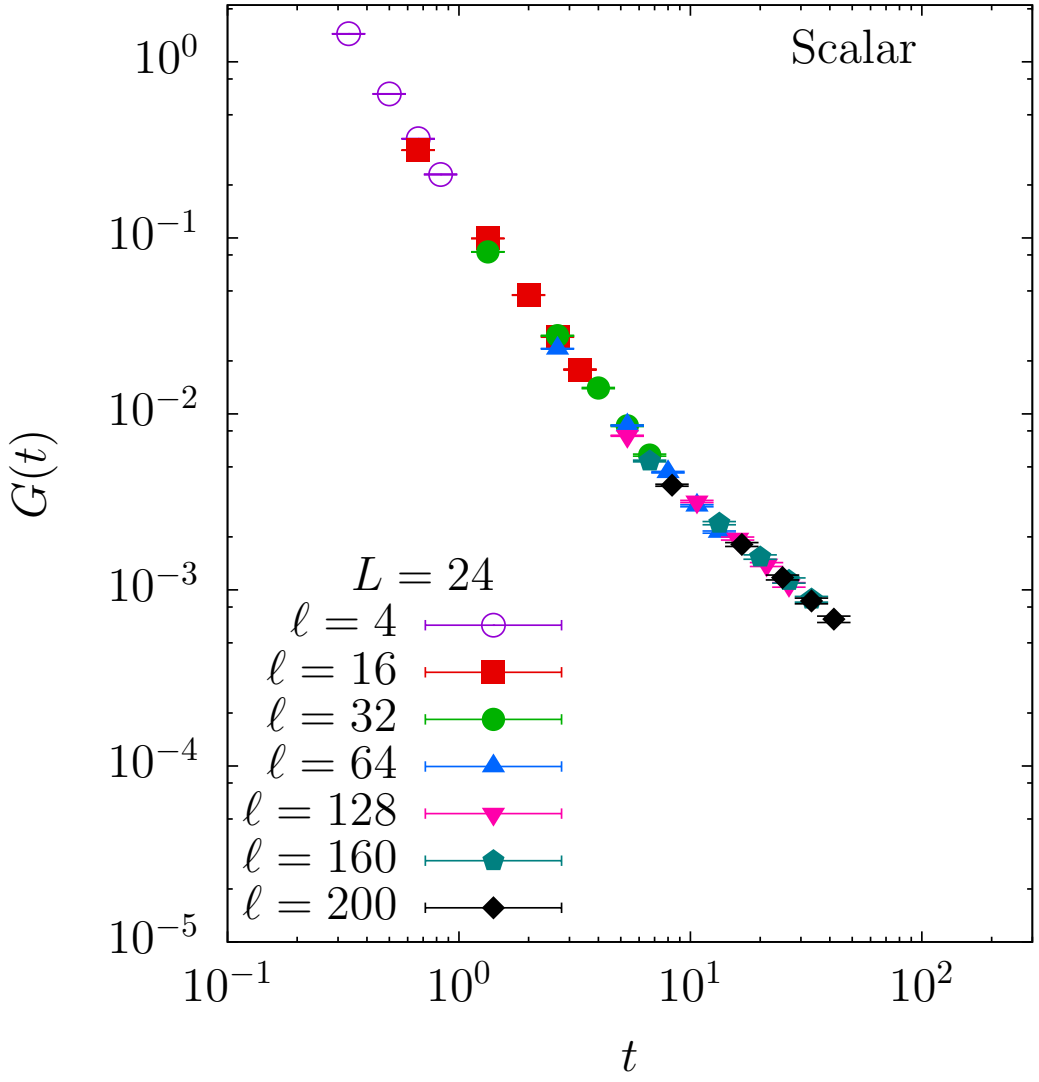
# Long-distance behavior of **vector** correlator

Vector correlator by putting together data from different $\ell$ on $24^3$ lattice.



# Long-distance behavior of **scalar** correlator

Scalar correlator by putting together data from different $\ell$ on $24^3$ lattice.



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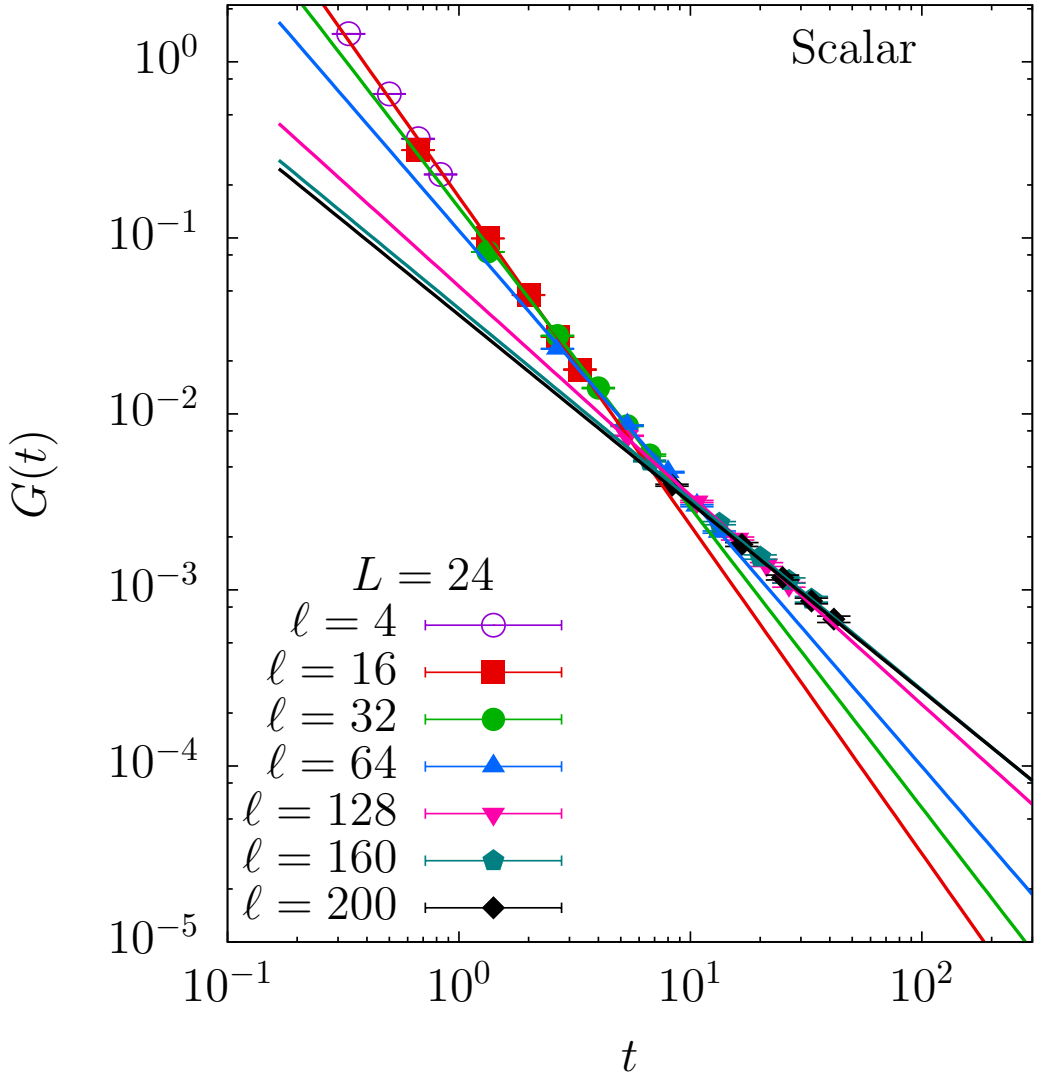
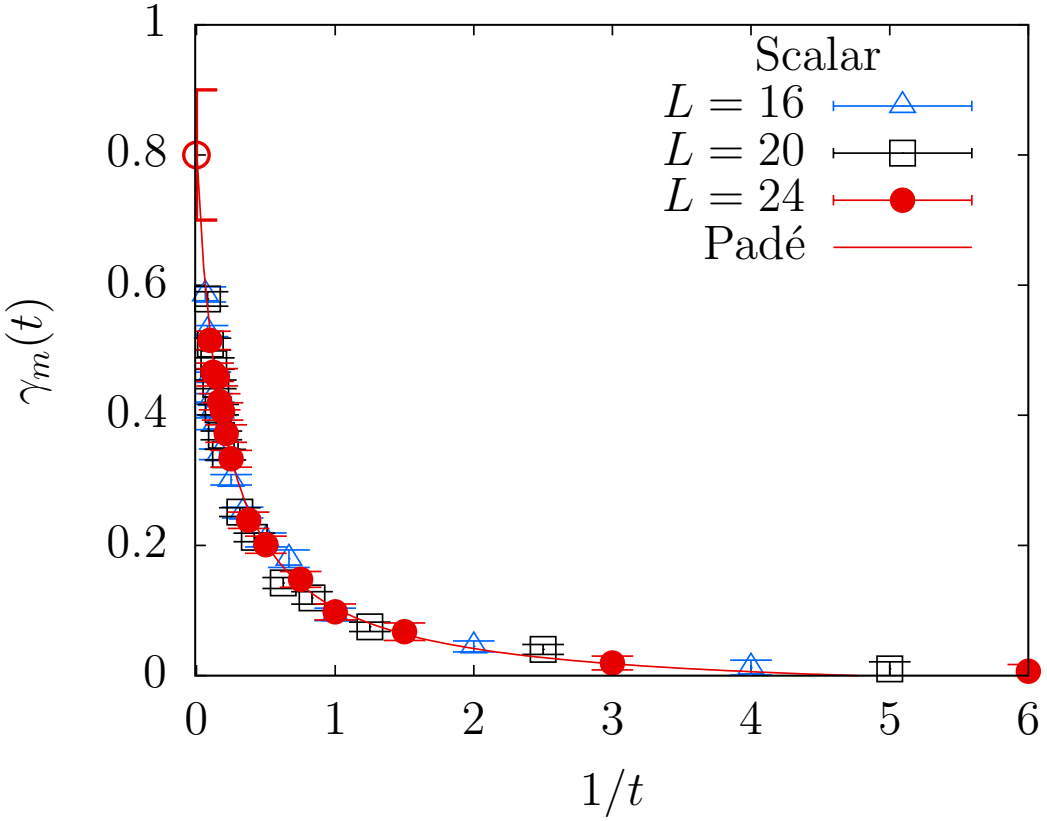
Scalar correlator by putting together data from different $\ell$ on $24^3$ lattice.



# Long-distance behavior of **scalar** correlator

Flow of scale-dependent exponent $\gamma_m(t)$ from short-distance (Asymptotic freedom $\Rightarrow \gamma_m = 0$) to long distance (non-trivial IR fixed point) where $\gamma_m \approx 0.8$.



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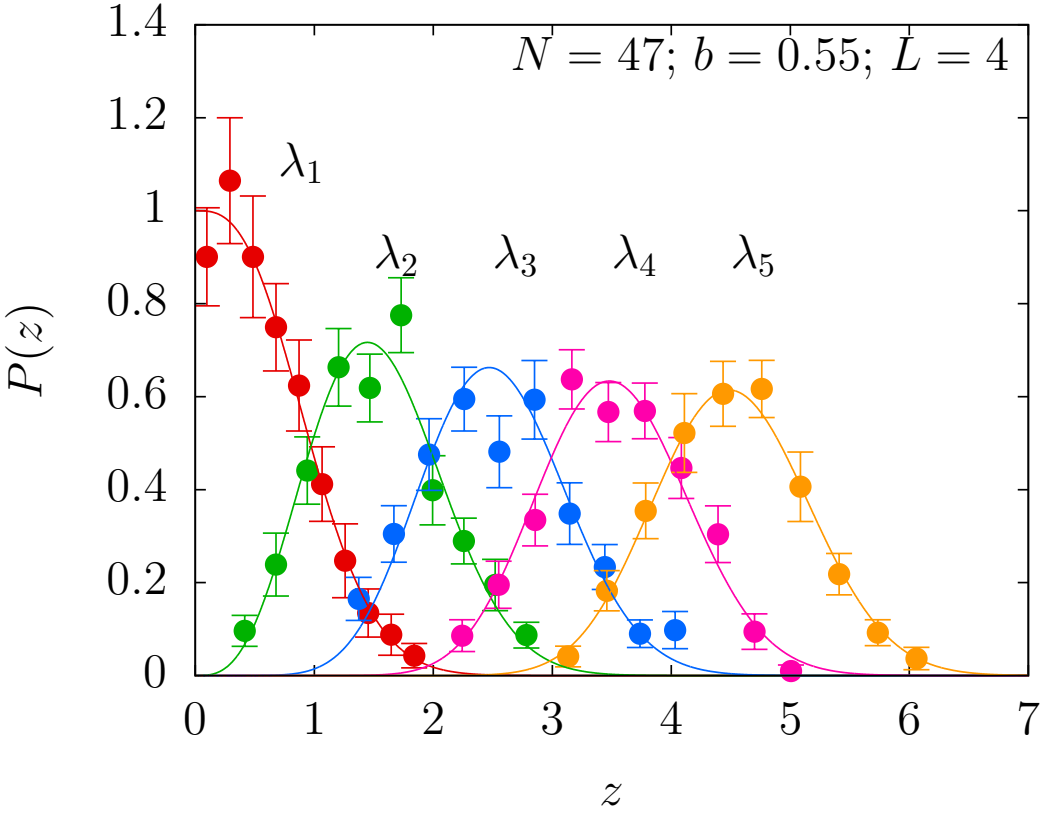
# Exploring the $(N_f, N_c)$ plane as a possibility




Agreement with Non-chiral random matrix model in large-$N_c$ $\Rightarrow$ condensate. $\Sigma/\sigma = 0.10(1)$.

# Conclusions

- Even for $N_f = 1$, the low-lying eigenvalues of the Dirac operator do not scale as $\ell^{-3}$ $\Rightarrow$ No bilinear condensate.
- Converse: $\lambda \sim \ell^{1+\gamma_m}$ for a scale-invariant theory $\Rightarrow$ $\gamma_m \approx 1$ for $N_f = 1$ (upper bound for CFTs).
- Inverse Participation Ratio does not scale as $\ell^{-3}$.
- The number variance $\Sigma_2(n)$ does not agree with the ergodic random matrix theory behavior. Instead, the behavior is critical.
- No mass scale in the long-distance behavior of scalar and vector correlators.
- We also established the presence of condensate using the same methods in the large-$N_c$ theory. Exploring the $(N_f, N_c)$-plane for a line of transition from scale-invariant to broken phase seems to the interesting next step.