θ dependence in deconfined QCD

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Work in collaboration with

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Based on

JHEP **1603** (2016) 155 (arXiv:1512.06746).

XQCD 2016, Aug. 1-3 2016, UK

[1] That's me

Outline

1. θ dependence in QCD and axions
2. Analytic approaches
3. Lattice results
4. Conclusions

The θ term

The CP-violating term in QCD lagrangian

$$\mathcal{L}_{\theta\,\mathrm{term}} = i\theta q(x), \qquad q(x) \equiv \frac{g^2}{64\pi^2} F^{a\,\mu\nu}\tilde{F}^{a\,\mu\nu},$$

$$q(x) = \partial_\mu K^\mu(x), \qquad Q = \int q(x)\mathrm{d}x \in \mathbb{Z}$$

Behaviour under $U(1)_A$

$$\begin{cases} \psi_j \to e^{i\alpha\gamma_5}\psi_j \\ \bar{\psi}_j \to \bar{\psi}_j e^{i\alpha\gamma_5} \end{cases} \to \begin{cases} \theta \to \theta - 2\alpha N_f \\ m_j \to m_j e^{2i\alpha} \end{cases}$$

(if $m_j = 0$: no θ dependence at all).

θ dependence of the Free Energy

Assuming analyticity in θ

$$F(\theta, T) - F(0, T) = \frac{1}{2}\chi(T)\theta^2\left[1 + b_2(T)\theta^2 + b_4(T)\theta^4 + \cdots\right]$$

Topological Susceptibility

$$\chi = \lim_{V_4\to\infty} \frac{1}{V_4}\langle Q^2\rangle_0$$

Higher moments

$$b_2 = -\frac{\langle Q^4\rangle_0 - 3\langle Q^2\rangle_0^2}{12\langle Q^2\rangle_0}, \quad b_4 = \frac{\langle Q^6\rangle_0 - 15\langle Q^2\rangle_0\langle Q^4\rangle_0 + 30\langle Q^2\rangle_0^3}{360\langle Q^2\rangle_0}, \quad \ldots$$

Parametrize **deviations** of $P(Q)$ from a Gaussian in the theory at $\theta = 0$.

θ dependence of the Free Energy

$F(\theta, T) \geq F(0, T)$

$$Z(-\theta, T) = Z(\theta, T) = \int [\mathrm{d}A] e^{-S_E[A] - i\theta Q} = \left| \int [\mathrm{d}A] e^{-S_E[A] - i\theta Q} \right| \leq$$

$$\leq \int [\mathrm{d}A] \, |\cdots| = \int [\mathrm{d}A] e^{-S_E[A]} = Z(0, T)$$

Phenomenology

Witten-Veneziano formula: θ-dependence matters even if $\theta = 0$

$$m_{\eta'}^2 = \frac{2N_f}{f_\pi^2} \chi^{N_f=0}$$

Strong *CP* problem: Experimentally $|\theta| \lesssim 10^{-9}$ from EDM measurement.

Strong CP problem

Possible solutions

1. At least one massless quark ($m_u = 0$)
2. CP symmetric lagrangian $\rightarrow$ present CP violation induced by SSB
3. **"Dynamical" θ angle.**

"Dynamical" θ angle - The Axion field

- $\mathcal{L}_a = \frac{a}{f_a} F\tilde{F}$, pseudoscalar field "$a$" with only derivative interactions
- As $F(\theta, T) \geq F(0, T)$ the field acquires a VEV such that $\theta + \frac{\langle a \rangle}{f_a} = 0$.

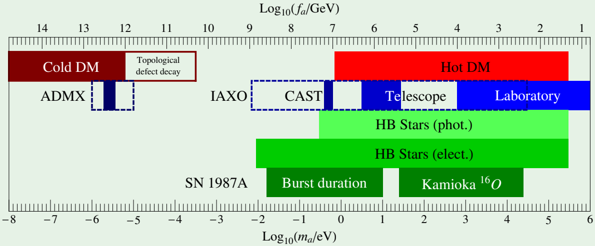
? GB of $U(1)$ axial symmetry (Peccei-Quinn symmetry) ?

The effective low-energy lagrangian in this case is

$$\mathcal{L} = \mathcal{L}_{QCD} + \frac{1}{2}\partial_\mu a \partial^\mu a + \left(\theta + \frac{a(x)}{f_a}\right) q(x) + \frac{1}{f_a}\left(\substack{model\ \ dependent \\ terms}\right).$$

Axions and QCD vacuum

Bounds on Axion mass [Davide Cadamuro, arXiv:1210.3196]



Axion mass

- As f_a is very large we can safely neglect axion loops to compute F
- Substitution rule $\theta \to a/f_a$
- Axion mass at tree level:

$$m_a(T) = \frac{\sqrt{\chi(T)}}{f_a}$$

Axions as dark matter

Cosmological sources of axions

1. Thermal production
2. Decay of topological objects
3. **Misalignment mechanism.**

EoM of the axion: $\ddot{a}(t) + 3H(t)\dot{a}(t) + m_a^2(T)a(t) = 0$

At $T \gg \Lambda_{QCD}$ if $\dot{a} \ll H$ the H term dominates: $a(t) \sim \mathrm{const}$

When $m_a \sim H$ the field starts oscillating around the minimum

When $m_a \gg H$ a WKB-like approximation can be used:

$$a(t) \sim A(t) \cos \int^t m_a(\tilde{t})\mathrm{d}\tilde{t}; \qquad \frac{\mathrm{d}}{\mathrm{d}t}(m_a A^2) = -3H(t)(m_a A^2)$$

the number of axions in the comoving frame $N_a = m_a A^2/R^3$ is conserved

Overclosure bound: axion density $\leq$ dark matter density

Semiclassical approximation (1)

General expectation

[Coleman "The uses of instantons"]

weak coupling approximation ⇒ semiclassical approximation

But even if the "elementary" coupling is not small the system may described by means of **weakly interacting classical configurations**

Diluted Instanton Gas Approximation

For weakly interacting instantons we have (DIGA, [Gross, Pisarski, Yaffe 1981])

$$Z_\theta = \mathrm{Tr} e^{-H_\theta/T} \approx \sum \frac{1}{n_+! n_-!} (V_4 D)^{n_+ + n_-} e^{-S_0(n_+ + n_-) + i\theta(n_+ - n_-)}$$

$$= \exp\left[2 V_4 D e^{-S_0} \cos\theta\right] \qquad \text{where } 1/\mathrm{D} \text{ is a typical 4-volume.}$$

Free energy

$$F(\theta, T) - F(0, T) \approx \chi(T)(1 - \cos\theta)$$

Semiclassical approximation (2)

Semiclassical behaviour (in the broad sense)

Leading order suppression due to light fermions and zero modes

$$b_2 = -\frac{1}{12} \quad b_4 = \frac{1}{360} \quad b_{2n} = (-1)^n \frac{2}{(2n+2)!}$$

$$\chi(T) \sim T^4 \left(\frac{m}{T}\right)^{N_f} \exp\left[-S_0\right]$$

From perturbation theory

[Gross, Pisarski, Yaffe 1981]

$$S_0 = \frac{8\pi^2}{g^2(T)} \approx \left(\frac{11}{3}N_c - \frac{2}{3}N_f\right) \log\left(T/\Lambda\right)$$

$$\chi(T) \sim m^{N_f}\, T^{4-\frac{11}{3}N_c - \frac{1}{3}N_f}\,.$$

Chiral Perturbation Theory

Ground state energy at $T = 0$ [Di Vecchia, Veneziano 1980]

- θ can be eliminated by an $U(1)_A$ rotation ($\rightarrow$ complex mass matrix)
- χPT can then be applied as usual:

$$E_0(\theta) = -m_\pi^2 f_\pi^2 \sqrt{1 - \frac{4 m_u m_d}{(m_u + m_d)^2} \sin^2 \frac{\theta}{2}}$$

Topological Susceptibility [G.di Cortona et al., JHEP 1601 (2016)]

$$\chi = \frac{z}{(1+z)^2} m_\pi^2 f_\pi^2, \quad b_2 = -\frac{1}{12} \frac{1+z^3}{(1+z)^3}, \quad z = \frac{m_u}{m_d}$$

explicitly

$z = 0.48(3)$	$\chi^{1/4} = 75.5(5)\,\mathrm{MeV}$	$b_2 = -0.029(2)$
$z = 1$	$\chi^{1/4} = 77.8(4)\,\mathrm{MeV}$	$b_2 = -0.022(1)$

Where to trust the approximations?

Instanton calculus can be problematic at $T = 0$

- Needs *ad hoc* procedure to cure divergences in the dominant IR-region
- At $T > T_c$ no confinement scale $\rightarrow$ T works as an infrared regulator.

χPT can be problematic at $T \neq 0$

No chiral symmetry breaking for $T > T_c$.

Topology on the lattice (problem 1)

Measuring the topological charge

Q is well defined only for **smooth enough** gauge configurations

Some of the proposed methods

Field theoretical: perturbative/nonperturbative computation of the renormalization constants

Fermionic: index theorem for Ginsparg-Wilson fermions

Smoothing: cooling, Wilson Flow, Stout, APE, HYP...

Bottom line

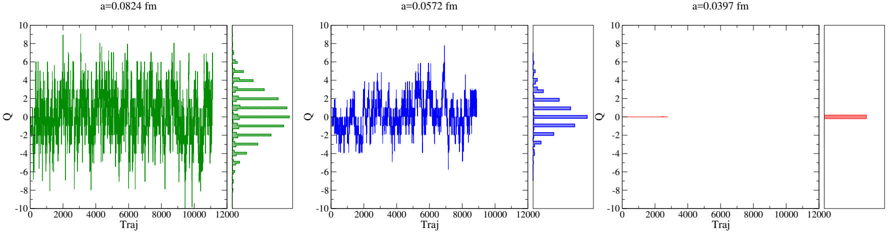
All reasonable methods give **equivalent** results
(close to the continuum limit)

[Panagopoulos, Vicari, Phys.Rept. 470 (2009) 93-150]
[C. Bonati, M. D'Elia PRD89 (2014) no.10, 105005]

Topology on the lattice (problem 2)

Sampling

As the continuum limit is approached it gets **increasingly difficult** to correctly sample the different topological sectors.



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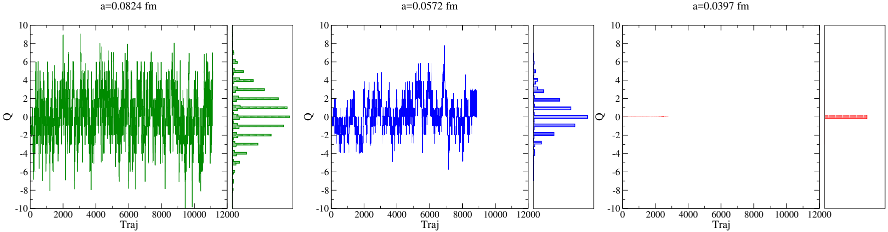
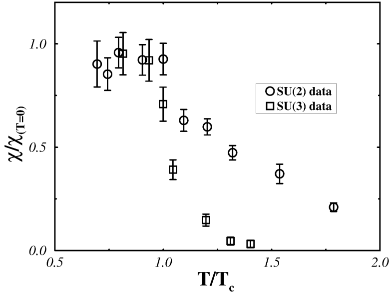


As we like to say the topological charge gets...

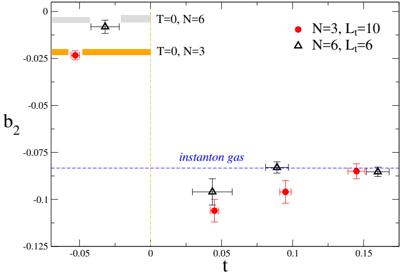
FROZEN

A glimpse of YM theory

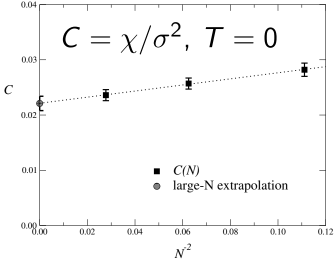
B.Alles et al., PLB412 (1997)



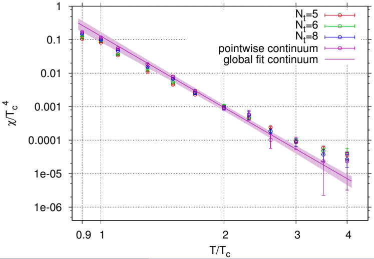
C.Bonati et al., PRL110 (2013)



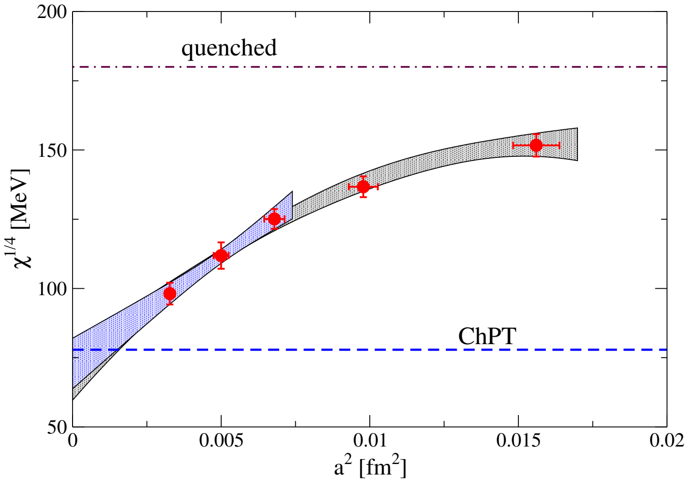
L.Del Debbio et al., JHEP 0208 (2002)



S. Borsanyi et al., PLB752 (2016)

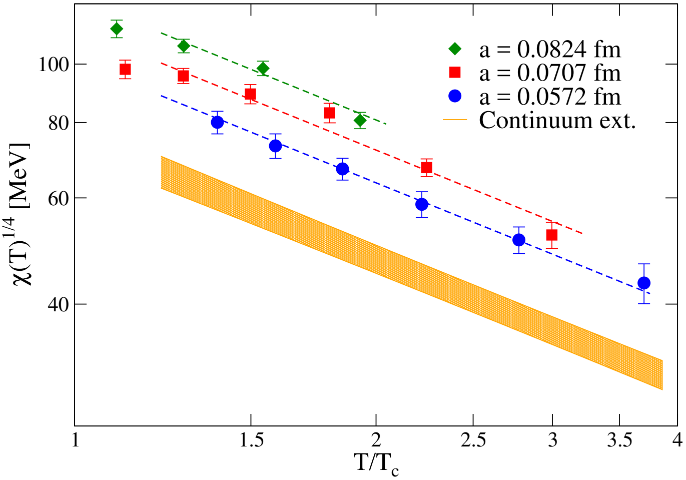


QCD at $T = 0$



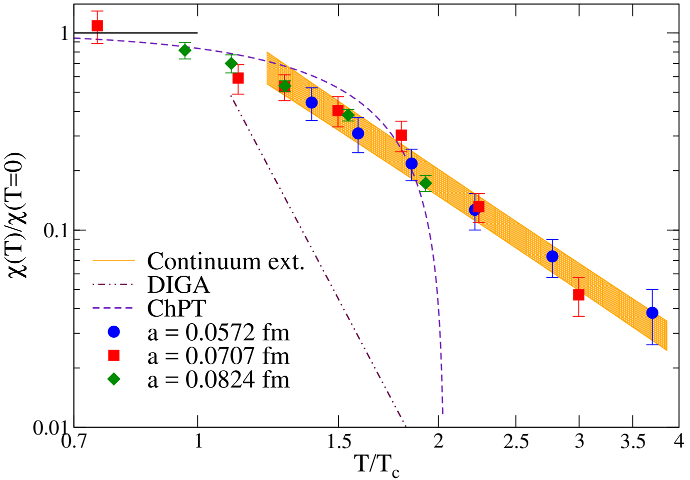
- Large cut-off effects
- Continuum limit compatible with ChPT ($73(9)\mathrm{MeV}$ vs $77.8(4)\mathrm{MeV}$)

QCD at $T \gtrsim T_c$ (our work)



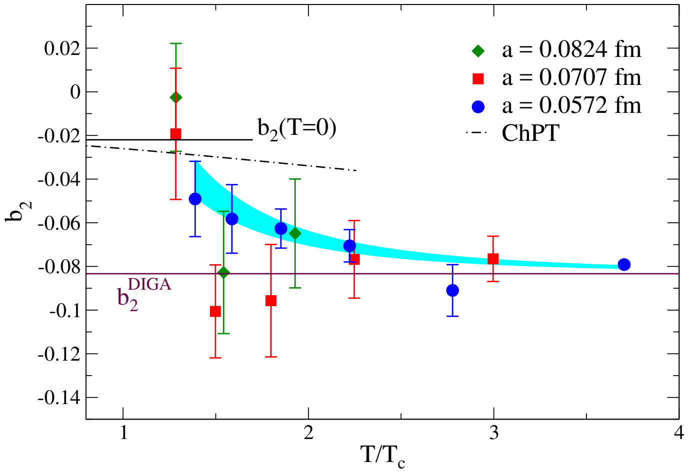
- Three lattice spacings (up to $T \sim 2T_c$)
- Global fit assuming behaviour: $\chi^{1/4} = C\left(1 + Da^2\right) T^E$

Cut-off effects-suppressing ratio



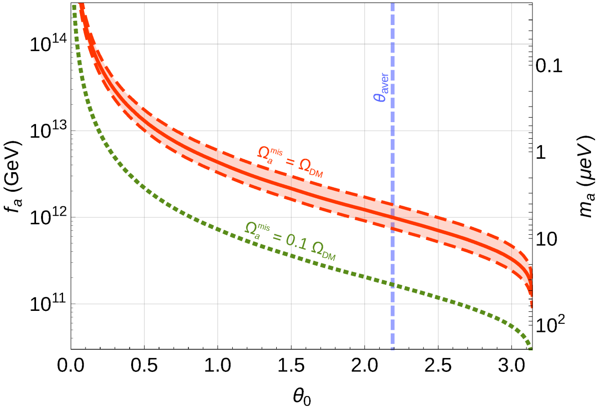
- Cut-off effect strongly reduced in the ratio $\chi(T)/\chi(T=0)$
- $\chi(T) \propto 1/T^b$ with $b = 2.90(65)$ (DIGA prediction: $b = 7.66 \div 8$)

Deviations from DIGA



- Larger deviations than in pure gauge theories and opposite in sign
- Quark mediated instanton interactions?

Axions as dark matter (from 1512.06746)



Initial condition?

- If PQ symmetry breaks before inflation the initial value is constant
- Otherwise an average on the initial value has to be performed.

Results from other groups

We obtain $\chi(T) \propto T^{-3}$, what about the other studies?

P. Petreczky et al., arXiv:1606.03145

$\chi(T) \propto T^{-8}$ with large extrapolation to the continuum

Sz. Borsanyi et al., arXiv:1606.07494

$\chi(T) \propto T^{-8}$ using "thermodynamical integration for $\chi(T)$"

[cfr. J. Frison et al. poster, 1606.07175]

- $\chi \sim 2Z_1/Z_0$, effects of larger Q values, finite size effects?
- zero modes reweighting $n_{zero} \sim Q$ was used

K. Kanaya et al.

See the poster outside...

Conclusions & Outlook

Conclusions

- slower convergence of b_{2n} to DIGA than in YM: attractive interactions between instantons?
- $\chi(T)$ shows strong deviation from DIGA for $T \lesssim 4T_c$
- the limit on f_a increases by almost an order of magnitude

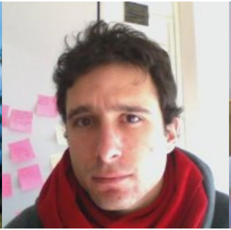
(Near) future outlook

- check for residual discretization effects going closer to the continuum
- use new algorithms to reduce/remove the topological freezing (and check for "small Q" effects)
- go to higher temperature




Thank you for your attention!

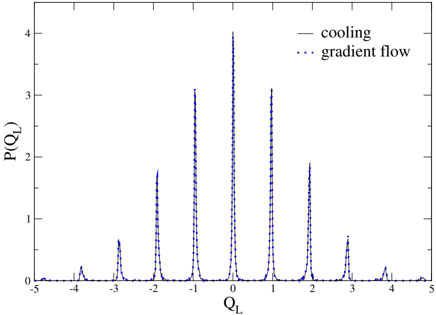





Backup slides with something more

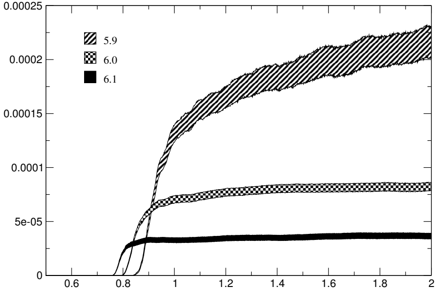
Comparison between smoothing algorithms

Bonati, D'Elia 1401.2441



Topological charge distribution obtained by cooling or gradient flow in $SU(3)$ at $\beta = 6.2$.

Del Debbio, Pica 0309145



Cooling-like picture displaying the values of the top. susceptibility as a function of the mass used in the overlap Dirac operator in $SU(3)$.

Virial-like corrections to DIGA

$F(\theta, T)$ is an even function of period 2π, thus

$$F(\theta, T) - F(0, T) = \sum_{n>0} a_n \left[1 - \cos(n\theta)\right] = \sum_{n>0} c_{2(n-1)} \sin^{2n}(\theta/2)$$

Developing in series we obtain

$$\chi = c_0/2; \quad b_2 = -\frac{1}{12} + \frac{c_2}{8\chi}; \quad b_4 = \frac{1}{360} - \frac{c_2}{48\chi} + \frac{c_4}{32\chi}$$

and c_{2n} contributes only to b_{2m} with $m \geq n$. This is a virial-like expansion and it is reasonable to assume

$$c_{2(n-1)} = d_{2(n-1)} \frac{\chi^n}{\chi^{n-1}(T=0)} \ .$$

The first correction to DIGA is thus

$$F(\theta) = \chi(1 - \cos\theta) + d_2 \frac{\chi^2}{\chi(T=0)} \sin^4(\theta/2)$$

$$b_2 = -\frac{1}{12} + \frac{d_2}{8} \frac{\chi}{\chi(T=0)}, \qquad d_2 = 0.80(16) \ .$$