Critical Phenomena in 8-Flavor QCD

In The Light of Physics Beyond The Standard Model

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Motivation

The Higgs boson with $M_{\mathrm{H}} \simeq 125$ GeV is discovered (2012, LHC-CERN)!

The LHC second-run has started!

Why Not Investigate Higgs & Electroweak Symmetry Breaking(EWSB)?

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Strong Dynamics: Origin of EWSB

Questions/ Answers: New Strong Dynamics $\ni (F_i^a, G^{a\bar{a}})$

1. Physical Content of Higgs?
   Composite Higgs $\bar{F}F$ (c.f. σ_{QCD}, Cooper-Pair).
2. Physics of Electroweak (EW) Symmetry Breaking?
   Chiral Symmetry Breaking.
3. Fine-Tuning Problem for $M_{\mathrm{H}} = 125$ GeV?
   Log (partly Power-Low) corrections for $M_{\mathrm{H}} = 125$ GeV.

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Higgs vs QCD-Scalar Singlet σ_{QCD}

QCD Strong Interaction

- Chiral SSB: $\mathrm{SU_L}(3) \times \mathrm{SU_R}(3) \to \mathrm{SU_V}(3)$.
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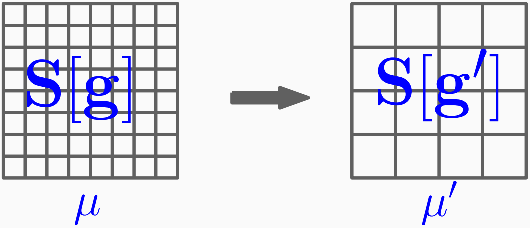
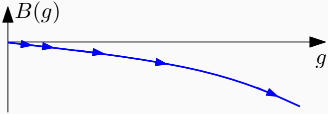
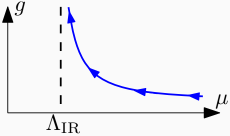
Beta-Function: Usual QCD

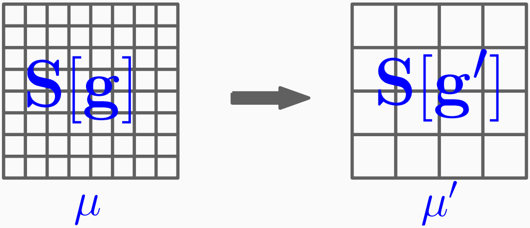


$$B(g, N_c, N_f \simeq 3) = \mu \frac{dg}{d\mu} \ . \tag{1}$$

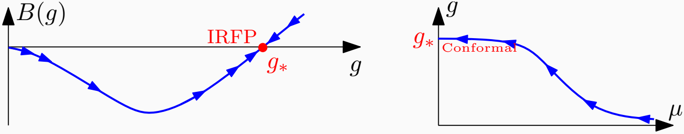




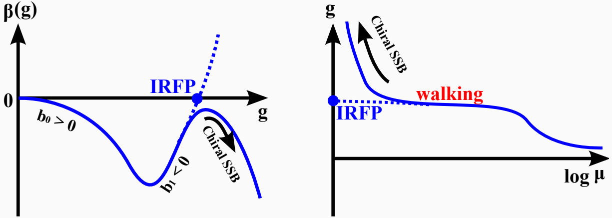
2-Loop Perturv for Many Flavor: Banks-Zaks IRFP (1982)



$$B(g, N_c, N_f \gg 3) = \mu \frac{dg}{d\mu} \; . \tag{2}$$



Walking Dynamics in Many-Flavor QCD



$$B(g) = dg(\mu)/d\mu = -b_0({\color{red}N_f}, N_c)g^3 - b_1({\color{red}N_f}, N_c)g^5 + \cdots \ . \qquad (3)$$

Schwinger-Dyson Equation

- Walking Dynamics in $8 \lesssim N_f \lesssim 12$, (Yamawaki et.al.('86), Holdom ('85))
- Stable (light) Higgs: Techni-Dilaton $\bar{F}F$ (PNGB for Scale Sym Breaking).

Lattice Beta-Func.: Appelquist et.al. (2008-10), LatHC (2016), A.Hasenfratz et.al. (2014-16), ...

Subject of Our Study

We do not know much about Many Flavor QCD and Walking...

We Investigate...

- 8-flavor QCD $\ni (F_{i=1,\cdots 8}^{a=\mathrm{rgb}}, G^{a\bar{a}})$ with
  Lattice Gauge Theory with Monte Carlo Simulations.
  c.f. One Family Model (Farhi-Susskind '79, Dimopoulos '80).
- Chiral symmetry breaking $\mathrm{SU_L}(N_f = 8) \times \mathrm{SU_R}(N_f) \to \mathrm{SU_V}(N_f)$ or
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- The Critical Phenomena near IRFP with Mass Anomalous Dimension γ.

Many Flavor QCD and Critical Phenomena: Miransky-Yamawaki (SD 1997), Braun et.al. (FRG 2006 - 11), Deuzemann et.al. ($N_f = 8, 12$ lattice, 2008 - 11), Kogut-Sinclair (2010), K.Miura et.al. ($N_f = 6, 8$ lattice, 2011-13), Iwasaki et.al. ($N_f = 2, 7$ lattice, 2014).

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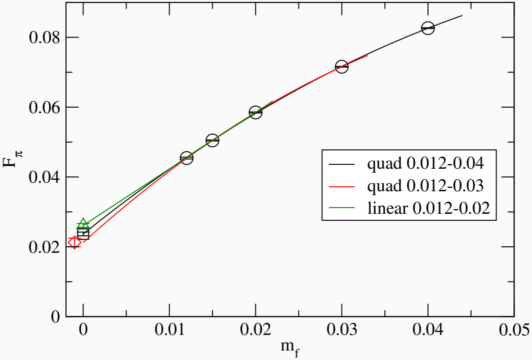
Measurements and Analyses

- **Observable:**
  Hadron Correlators and Masses $M_H(a, L, m_f)$
- **Analyses:**
  - Chiral Perturbation (ChPT) Ansatz.
  - Finite-size Hyperscaling (FSHS) Ansatz.

$N_f = 8$ Techni-Pion Decay Cnst. F_π (Update of LatKMI PRD 2013)



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- ChPT: $F_\pi = F_\pi^0 + C_1^F m_f + C_2^F m_f^2 + \cdots$, $\chi^2/\mathrm{dof} \sim \mathcal{O}(1)$.
- Scale Setting: $F_\pi^0 = 0.0212(12)(^{+49}_{-70}) \to 246/\sqrt{2}$ GeV.

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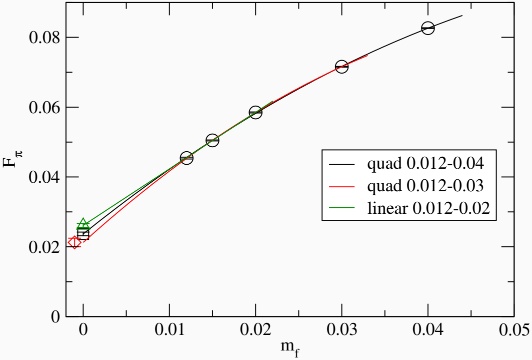

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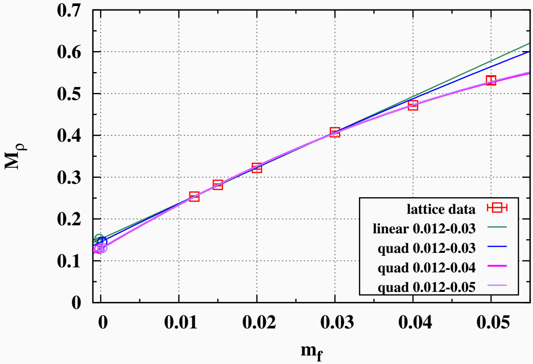

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- ATLAS(1506.00962): An excess of around 2TeV in the diboson channel, but disappearing...

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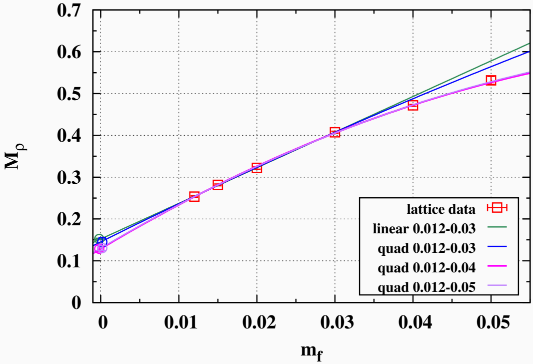

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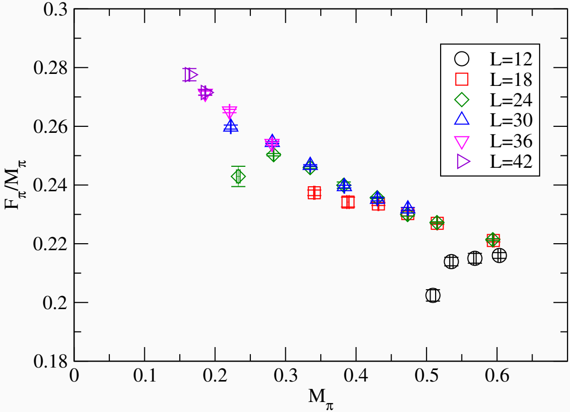
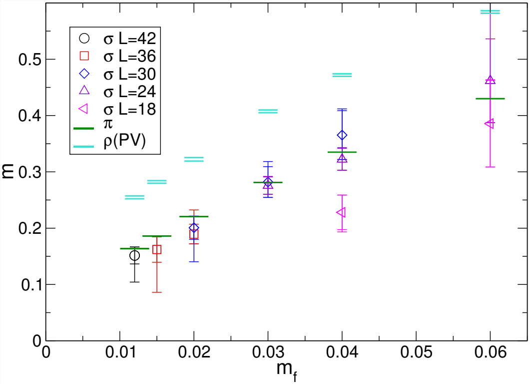
The Ratio F_π/M_π, (Update of LatKMI PRD2013)



The pion becomes lighter, indicating its pNGB nature.

(c.f. LSD Collaboration 2016.)

$N_f = 8$ Flavor Singlet Scalar σ (Update of LatKMI PRD '14)



Light $\sigma \sim$ Dilaton (PNGB for Broken Scale Symm.)

c.f. LatKMI 2012, LSD Collaboration 2016, Lattics Higgs Collaboration 2012, Athenodorou et.al. 2014.

Dilaton ChPT

Ref.: Matsuzaki-Yamawaki PRL 2014. (c.f. Crewther et.al. PRD 2015).

$$\begin{aligned} Z &= \int \mathcal{D}[\psi, \bar{\psi}, A_\mu] \; e^{iS[\psi,\bar{\psi},A_\mu]} \\ &\sim \int \mathcal{D}[U, V] \; e^{iS_{\rm eff}[U,V]} \, , \\ (U, V) &= (e^{2i\pi^a(x)T^a/F_\pi}, e^{\sigma(x)/F_\sigma}) \, . \end{aligned} \tag{4}$$

We expand $S_{\rm eff}$ in terms of $(\pi(x), \sigma(x))$ and read off coefficients of their quadratic terms, giving mass terms of them:

$$M_\sigma^2 = M_\sigma^2|_{m_f \to 0} + D \cdot M_\pi^2 \, , \tag{5}$$

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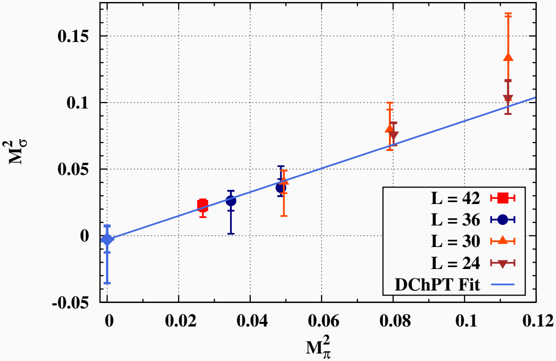

$$M_\sigma^2|_{m_f \to 0} = -0.0028(98)\left(^{36}_{354}\right) \ni 0.0002 \, , \qquad (7)$$

$$(c.f. M_\sigma^2|_{m_f \to 0} = 0.0002 \text{ gives } M_\sigma = 125 \text{ GeV}) \, ,$$

$$F_\sigma \sim 522 \text{ GeV} \, . \qquad (8)$$

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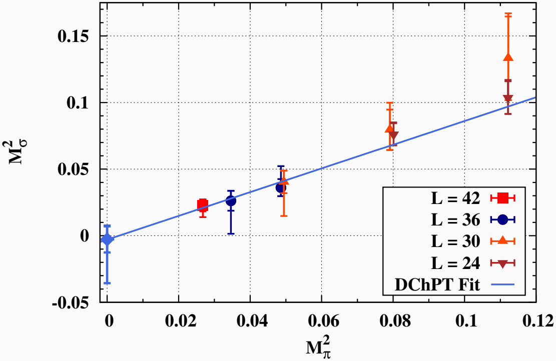

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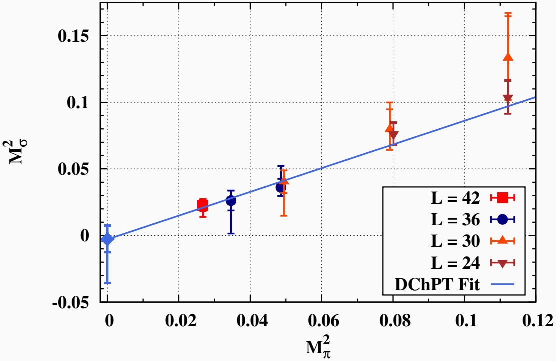
DChPT Fit



$$M_\sigma^2|_{m_f \to 0} = -0.0028(98)(^{36}_{354}) \ni 0.0002 \ , \qquad (7)$$

$$(c.f. M_\sigma^2|_{m_f \to 0} = 0.0002 \text{ gives } M_\sigma = 125 \text{ GeV}) \ ,$$

$$F_\sigma \sim 522 \text{ GeV} \ . \qquad (8)$$

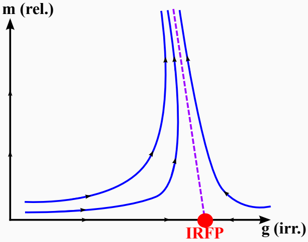
Measurements and Analyses

- **Observable:**
  Hadron Correlators and Masses $M_H(a, L, m_f)$
- **Analyses:**
  - Chiral Perturbation (ChPT) Ansatz.
  - Finite-size Hyperscaling (FSHS) Ansatz.

Finite-Size Hyper-Scaling (FSHS)

Finit-Size Hyperscaling (FSHS) via Renormalization Group

- **FSHS:** $LM_H = F(X, X_{\rm irr})$, (D.Debbio et.al. '10)
  $X = Lm_f^{1/(1+\gamma)}$ = relevant operator with mass anomalous dimension γ
  $X_{\rm irr} = gm^{\omega}, \cdots$ = irrelevant operator (A.Hasenfratz et.al. '14).
- **Large X around IRFP:** $F(X, X_{\rm irr}) \to F(X) \to C_0^H + C_1^H X$.
- **Irr. OP Correction:** $F(X, X_{\rm irr}) \to (1 + C_2^H(g)m^{\omega})(C_0^H + C_1^H X)$.



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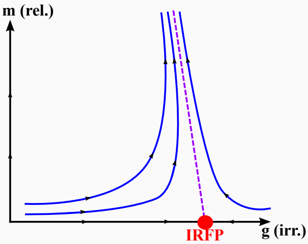

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Does mass spectra in 8-flavor QCD respect FSHS with universal γ?

- **Yes:** The theory is Conformal,
- **No:** Chirally Broken (c.f. $N_f = 4$),
  or Too small X, too large m_f, some $X_{\rm irr}$ effects.

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Naive FSHS for Each Observables I (Update of LatKMI PRD 2013)

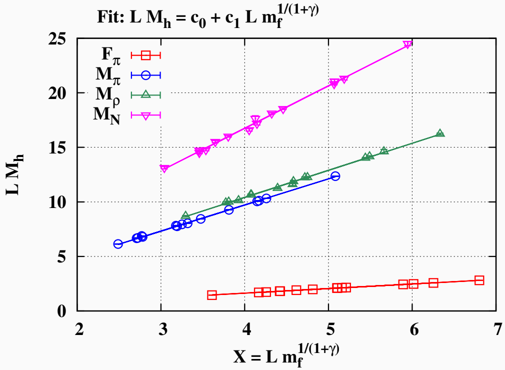
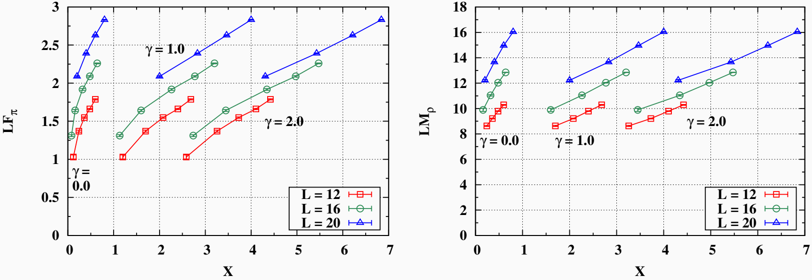


Figure: FSHS Fit for $N_f = 8$ Spectra

FSHS in 4-flavor QCD, Preliminary



In the $N_f = 4$ case, data points never distribute around a single line within the unitality band $0 \leq \gamma \leq 2$, and indicates no IRFP dynamics at all.

Naive FSHS for Each Observables II (Preliminary)

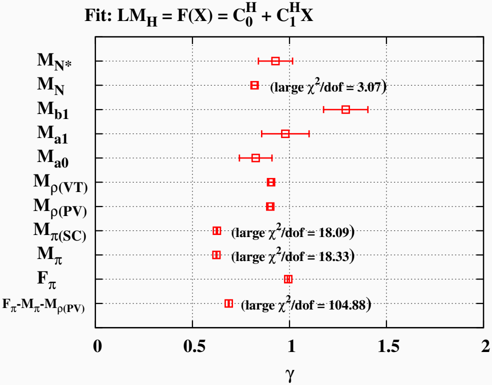
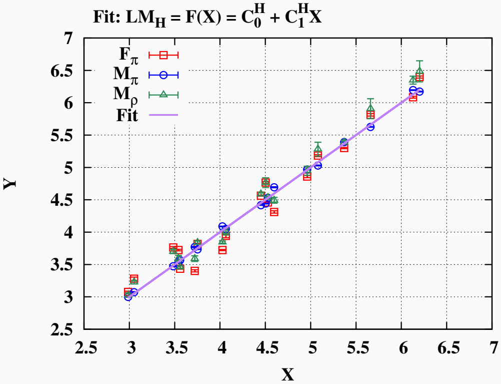


Figure: Mass anomalous dimension γ for $N_f = 8$.

c.f. LSD Collaboration (2014).

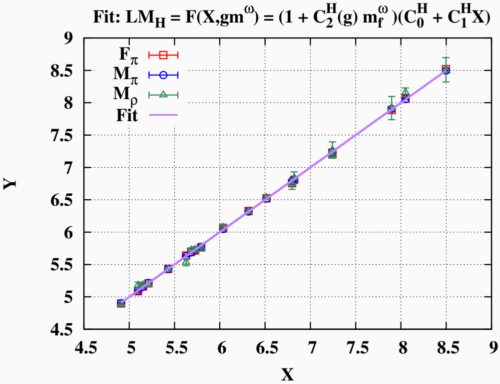
Naive FSHS w. Common γ Imposed (Update of LatKMI PRD'13)



$$Y = (F(X) - C_0^H)/C_1^H, \text{ Fit Line: } Y = X.$$

$$(\gamma, \chi^2/\mathrm{dof}) = (0.687(02), 104.88).$$

FSHS with With Collection, Preliminary



$$Y = \{F(X)/(1 + C_2^H m_f^{\omega}) - C_0^H\}/C_1^H, \text{ Fit Line: } Y = X.$$
$$(\gamma, \omega, \chi^2/\mathrm{dof}) = (1.108(48), 0.347(14), 1.05)$$

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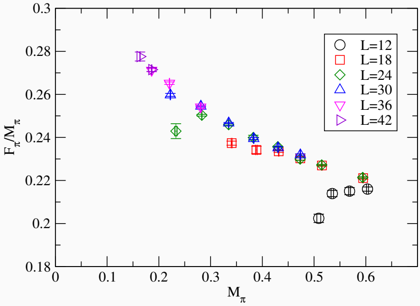
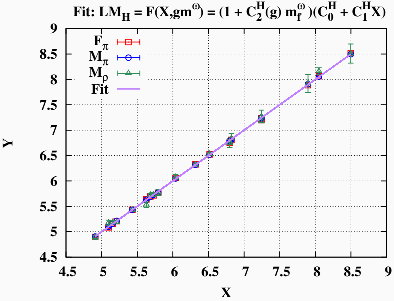
Chirally Broken or Conformal in $N_f = 8$??





Chirally Broken or **Conformal**?

How these two figures are compatible?

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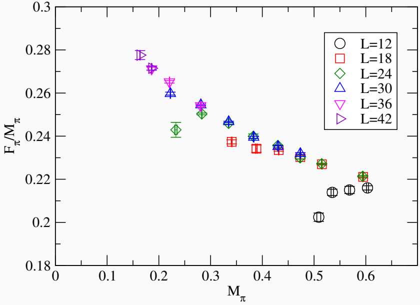

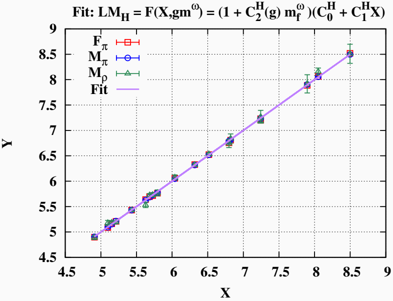

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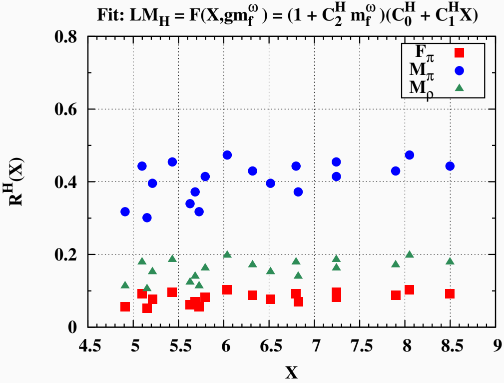
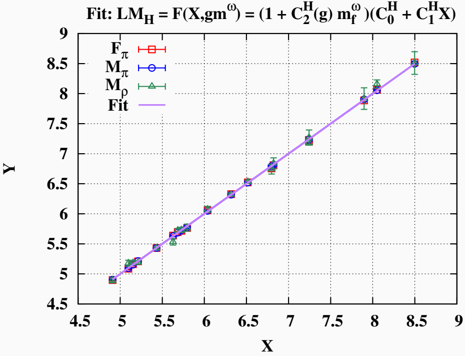
Critical Phenomena around IRFP with IRR-CORR. I



$$R^H(X) = \frac{C_2^H m_f^{\omega}(C_0^H + C_1^H X)}{(1 + C_2^H m_f^{\omega})(C_0^H + C_1^H X)} . \qquad (9)$$

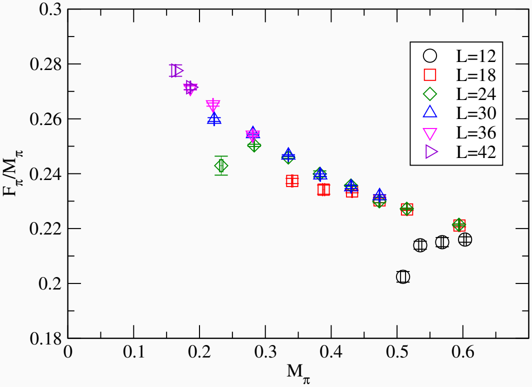
Really Conformal? Maybe No...



50% Correction to M_π Scaling: Broken Chiral Symmetry?

- The IRFP scaling variable $X = Lm^{1/(1+\gamma)}$ is not dominant any more.
- The M_π, specifically, scales differently from the others.

Really Chirally Broken?



The pion becomes lighter, indicating its pNGB nature.

It's premature to conclude this!

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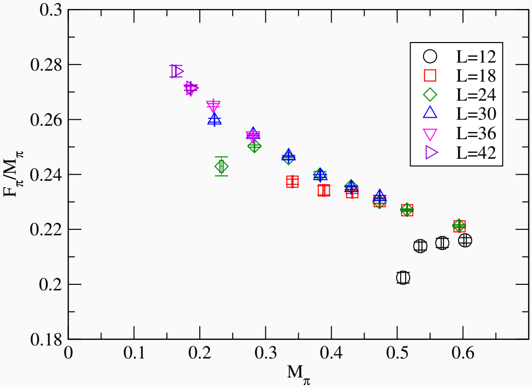

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Critical Phenomena around IRFP with IRR-CORR. II

The increasing ratio $(F_\pi/M_\pi)(m_f \to 0)$ may result from the effect of the irrelevant operator:

$$\frac{F_\pi}{M_\pi}(m_f) = \frac{LF_\pi}{LM_\pi}(m_f) = \frac{(1 + C_2^F m_f^\omega)(C_0^F + C_1^F X)}{(1 + C_2^\pi m_f^\omega)(C_0^\pi + C_1^\pi X)} \,, \tag{10}$$

$$X = L m_f^{1/(1+\gamma)} = \text{large and fixed} \,,$$

$$\begin{aligned} &\frac{d}{dm_f}\frac{F_\pi}{M_\pi}(m_f \to 0) \\ &= \frac{F_\pi}{M_\pi}(0)\Bigg[\omega\left(\frac{C_2^F}{1+C_2^F} - \frac{C_2^\pi}{1+C_2^\pi}\right) m_f^{(\omega-1)<0} \\ &\quad + \frac{X^{-1}}{1+\gamma}\left(\frac{1}{1+(C_0^F/C_1^F)X^{-1}} - \frac{1}{1+(C_0^\pi/C_1^\pi)X^{-1}}\right) m_f^{-\gamma/(1+\gamma)<0}\Bigg] \end{aligned} \tag{11}$$

We cannot exclude the possibility of the conformal scenario.

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Summary and Perspective

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- near the boarder of the chiral broken and conformal phases with the light σ and the (would-be) mass anomalous dimension $\gamma \sim 1$.
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- theoretically interesting as a frontier of the gauge theory.

Future Perspective

- Smaller fermion mass, larger lattice, several lattice spacings.
- Work in progress: S-parameter $S \sim 0.25 - 0.275$, η' (Y.Aoki, lattice-2016 talk, $M_{\eta'}/M_\rho \simeq 3.5$), finite T (K.M. Proceedings SCGT2015, chiral transition/crossover signal?), DM, topology.

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Setups

Lattice Action:	$N_f = 8$ HISQ Action $+$ Tree-level Symanzik Gauge Action.
Argolithm:	HMC with Hasenbush pre-conditioning.
Configurations:	$\mathcal{O}(10^4)$ Configs., $\beta = 3.8,\ L \in [12, 42],\ m_f \in [0.012, 0.16]$
Observables:	$F_\pi,\ M_\pi,\ M_\rho,\ M_{a1},\ M_N$, VPF, $\cdots$.
Code etc:	MILC ver.7.6.3 with some modifications, SciDac Libraly.
Computer:	KMI HPC Cluster φ, Nagoya-Univ-ITC CX400, Kyushu-Univ-RIIT CX400/HA8000.

Setups II

$\color{blue}\bigstar$: New Configs. $\color{red}\bigstar$: Updates, $\mathcal{O}(10^4)$ Configs.

$m_f \diagdown L$	42	36	30	24	18	12
0.012	$\color{blue}\bigstar$					
0.015	$\color{blue}\bigstar$	$\color{red}\bigstar$				
0.02		$\color{red}\bigstar$	$\color{red}\bigstar$	$\bigstar$		
0.03		$\color{blue}\bigstar$	$\color{red}\bigstar$	$\color{red}\bigstar$		
0.04			$\color{red}\bigstar$	$\color{red}\bigstar$	$\color{red}\bigstar$	
0.05			$\bigstar$	$\bigstar$	$\bigstar$	$\bigstar$
0.06			$\bigstar$	$\color{red}\bigstar$	$\color{red}\bigstar$	
0.07			$\bigstar$	$\bigstar$	$\bigstar$	$\bigstar$
0.08				$\color{red}\bigstar$	$\color{red}\bigstar$	$\bigstar$
0.09						$\bigstar$
0.10				$\bigstar$	$\bigstar$	$\bigstar$
0.12						$\bigstar$
0.14						$\bigstar$
0.16						$\bigstar$

Superconductivity (S.C.) vs Higgs Sector

Why Composite Higgs?

	S.C. (Ginzburg-Landau)	Higgs Sector
Theory	U(1) Gauged σ Model	$\mathrm{SU_W}(2) \times \mathrm{U_Y}(1)$ Gauged σ Model
Condensate	$\sigma \sim ee$	$\sigma = h \sim \bar{\Psi}\Psi$??
Meissner	$M_{\mathrm{mag}}^2 \sim (2e^2/m_e^2)\sigma$	$M_{\mathrm{W}}^2 = g^2\sigma^2/4$
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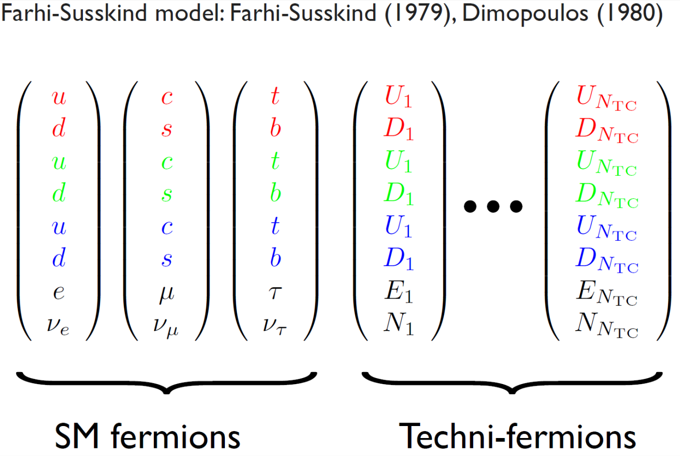
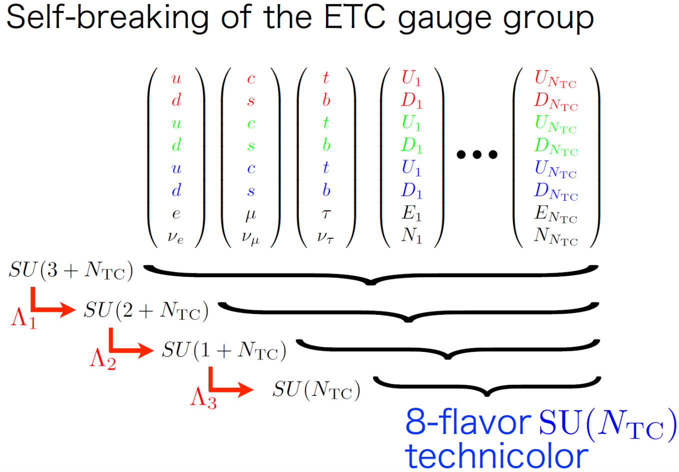
One Family Model



Farhi-Susskind model: Farhi-Susskind (1979), Dimopoulos (1980)

$$\underbrace{\begin{pmatrix} \color{red}{u} \\ \color{red}{d} \\ \color{green}{u} \\ \color{green}{d} \\ \color{blue}{u} \\ \color{blue}{d} \\ e \\ \nu_e \end{pmatrix} \begin{pmatrix} \color{red}{c} \\ \color{red}{s} \\ \color{green}{c} \\ \color{green}{s} \\ \color{blue}{c} \\ \color{blue}{s} \\ \mu \\ \nu_\mu \end{pmatrix} \begin{pmatrix} \color{red}{t} \\ \color{red}{b} \\ \color{green}{t} \\ \color{green}{b} \\ \color{blue}{t} \\ \color{blue}{b} \\ \tau \\ \nu_\tau \end{pmatrix}}_{\text{SM fermions}} \quad \underbrace{\begin{pmatrix} \color{red}{U_1} \\ \color{red}{D_1} \\ \color{green}{U_1} \\ \color{green}{D_1} \\ \color{blue}{U_1} \\ \color{blue}{D_1} \\ E_1 \\ N_1 \end{pmatrix} \bullet\bullet\bullet \begin{pmatrix} \color{red}{U_{N_{\mathrm{TC}}}} \\ \color{red}{D_{N_{\mathrm{TC}}}} \\ \color{green}{U_{N_{\mathrm{TC}}}} \\ \color{green}{D_{N_{\mathrm{TC}}}} \\ \color{blue}{U_{N_{\mathrm{TC}}}} \\ \color{blue}{D_{N_{\mathrm{TC}}}} \\ E_{N_{\mathrm{TC}}} \\ N_{N_{\mathrm{TC}}} \end{pmatrix}}_{\text{Techni-fermions}}$$

Tambling



Self-breaking of the ETC gauge group

$$\begin{pmatrix} u \\ d \\ u \\ d \\ u \\ d \\ e \\ \nu_e \end{pmatrix} \begin{pmatrix} c \\ s \\ c \\ s \\ c \\ s \\ \mu \\ \nu_\mu \end{pmatrix} \begin{pmatrix} t \\ b \\ t \\ b \\ t \\ b \\ \tau \\ \nu_\tau \end{pmatrix} \begin{pmatrix} U_1 \\ D_1 \\ U_1 \\ D_1 \\ U_1 \\ D_1 \\ E_1 \\ N_1 \end{pmatrix} \cdots \begin{pmatrix} U_{N_{\mathrm{TC}}} \\ D_{N_{\mathrm{TC}}} \\ U_{N_{\mathrm{TC}}} \\ D_{N_{\mathrm{TC}}} \\ U_{N_{\mathrm{TC}}} \\ D_{N_{\mathrm{TC}}} \\ E_{N_{\mathrm{TC}}} \\ N_{N_{\mathrm{TC}}} \end{pmatrix}$$

$SU(3+N_{\mathrm{TC}})$

$\Lambda_1 \rightarrow SU(2+N_{\mathrm{TC}})$

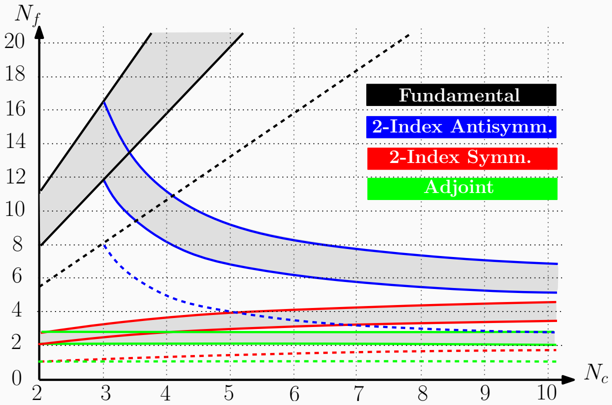
$\Lambda_2 \rightarrow SU(1+N_{\mathrm{TC}})$

$\Lambda_3 \rightarrow SU(N_{\mathrm{TC}})$

8-flavor $\mathrm{SU}(N_{\mathrm{TC}})$ technicolor

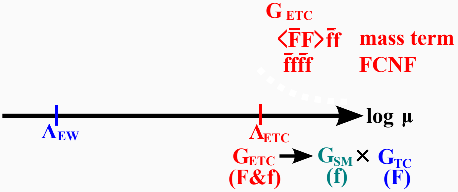
$N_f - N_c$ Phase Diagram

Ref.: Dietrich-Sannino PRD 2007.



Extended Technicolor Model

Ref.: Hill-Simmons Phys. Rept. 381 (2003).



- $\Lambda_{\rm ETC} \gtrsim 10^3$ TeV to suppress FCNC $\propto 1/\Lambda^2_{\rm ETC}$.
- SM Mass, $M_{q,l}|_{\mu=\Lambda_{\rm EW}} \propto \Lambda^3_{\rm EW}/\Lambda^2_{\rm ETC}$ gets too small.

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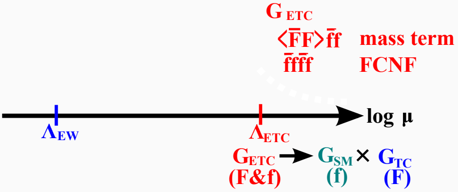

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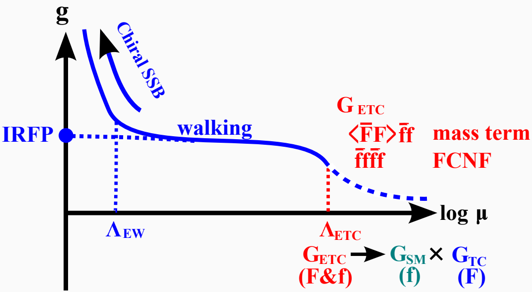
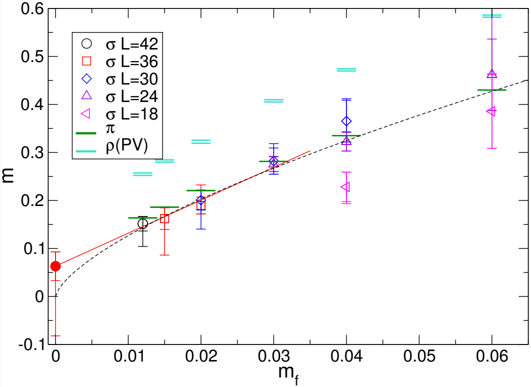
Waling Technicolor Model

Refs.: Yamawaki et.al. ('86), Appelquist et.al. ('86).



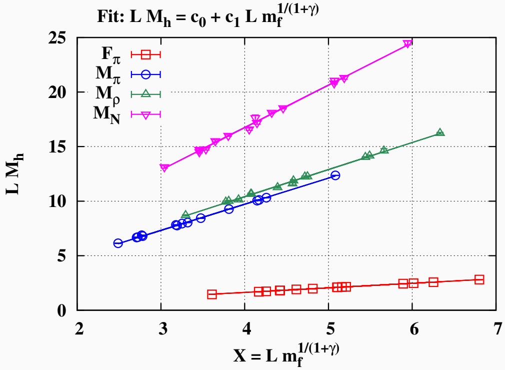
$$M_{q,l}\big|_{\mu=\Lambda_{\rm EW}} \propto \frac{\Lambda_{\rm EW}^3}{\Lambda_{\rm ETC}^2} \times \left[\frac{\Lambda_{\rm ETC}}{\Lambda_{\rm EW}}\right]^{\gamma} . \tag{12}$$

$N_f = 8$ Flavor Singlet Scalar σ II (Update of LatKMI PRD 2014)



Light $M_\sigma|_{m_f\to 0} \sim 0 - 780$ GeV (c.f. $F_\pi|_{m_f\to 0} = 246/\sqrt{2}$ GeV).
(c.f. LatHC Collab. ('14), Hietanen et.al. ('14), Athenodorou et.al. ('15)).

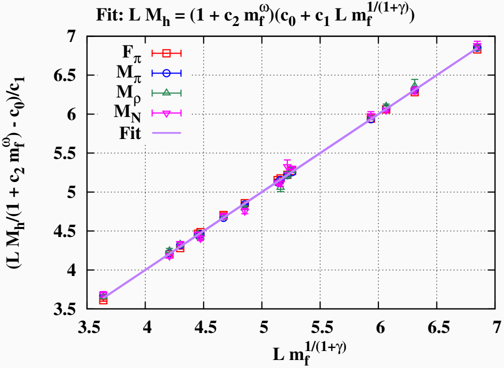
FSHS: Individual Fits



	F_π	M_π	M_ρ	M_N
γ	1.003(5)	0.627(2)	0.896(11)	0.810(11)
χ^2/dof	2.34	15.26	1.41	2.58

FSHS: Simultaneous Fits with A-Hasenfratz Type Collection

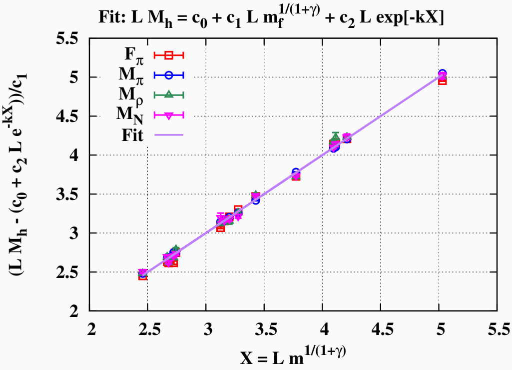
Fit Ansatz: Cheng-Hasenfratz-Liu ('14).



$$(\gamma, \chi^2/\mathrm{dof}) = (1.014(35), 2.46) \text{ for } \omega = 0.35.$$

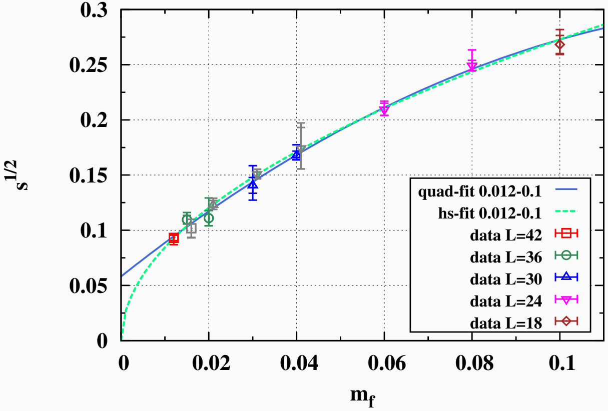
FSHS: Simultaneous Fits with Frascati-Groningen Type Collection

Fit Ansatz: Lombardo-Miura-Silva-Pallante ('14).



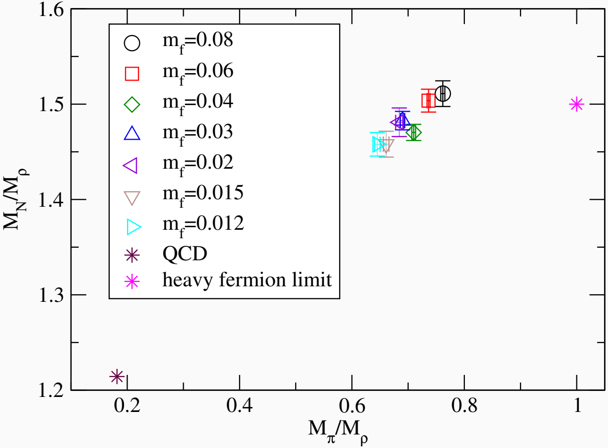
$(\gamma, \chi^2/\mathrm{dof}) = (0.617(2), 12.80)$ for $k = 0.1$.

$N_f = 8$ String Tension

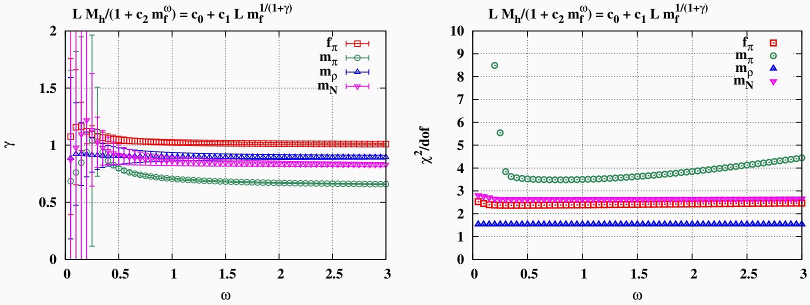


$$\sqrt{\text{string}} \cdot a = C(m_f a)^{1/(1+\gamma)},$$
$$(\gamma, \chi^2/\text{dof}) = (0.97(4), 0.68).$$

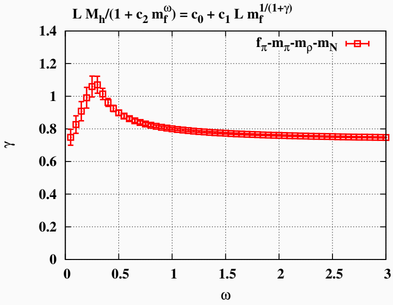
Edinburgh-Type Plot

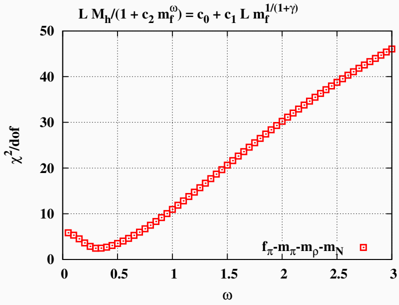


FSHS: Individual Fits with A-Hasenfratz Collection

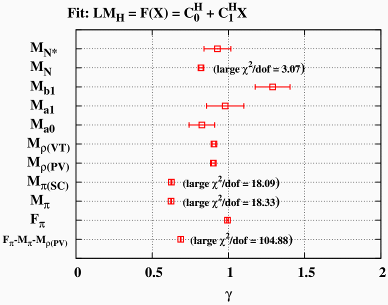


FSHS: Simultaneous Fits with A-Hasenfratz Collection

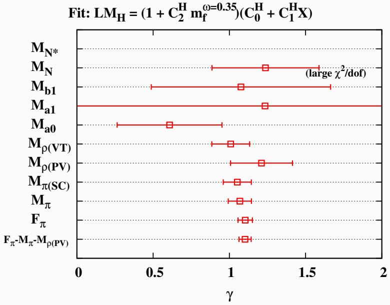




Summary of γ in 8-flavor QCD



Fit: $LM_H = F(X) = C_0^H + C_1^H X$

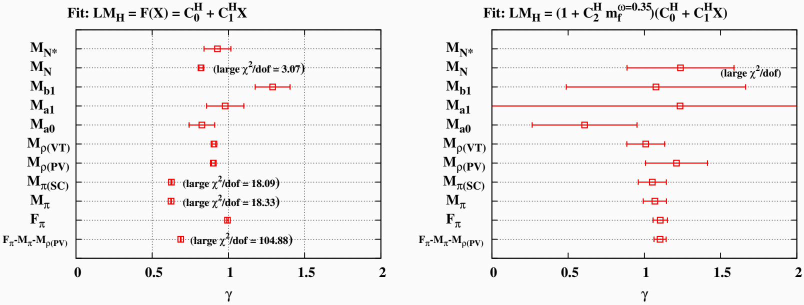


Fit: $LM_H = (1 + C_2^H m_f^{\omega=0.35})(C_0^H + C_1^H X)$

Dose mass spectra in 8-flavor QCD scale with universal γ?

Barely Yes: $\gamma \sim 1$ with large uncertainties.

Summary of γ in 8-flavor QCD



Dose mass spectra in 8-flavor QCD scale with universal γ?

Barely Yes: $\gamma \sim 1$ with large uncertainties.

γ in 12-flavor QCD

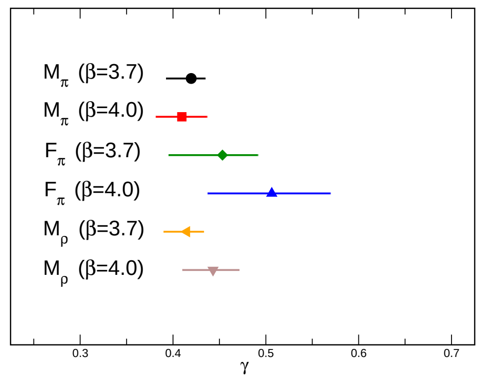


Figure: γ in 12-flavor QCD, for comparison. Update of LatKMI PRD 2012.

Naive Hyper-Scaling Fit for $m_f \in [0.012, 0.03]$

$$M_h = C_h m_f^{1/(1+\gamma)} \ , \tag{13}$$

OP	g	dg	χ^2/dof
F_π	$9.950105e-01$	$1.486191e-02$	0.650459
M_π	$6.819529e-01$	$5.860037e-03$	1.739950
$\langle\bar{\psi}\psi\rangle$	$5.143576e-02$	$7.617876e-04$	1.821225
M_ρ	$9.237836e-01$	$3.427526e-02$	2.982169
M_{a0}	$8.089052e-01$	$1.294329e-01$	0.075278
M_{a1}	$1.031132e+00$	$2.186141e-01$	0.888295
M_{b1}	$9.199789e-01$	$2.691208e-01$	0.150320
M_N	$8.374501e-01$	$2.432380e-02$	3.377506
M_{N*}	$8.932025e-01$	$1.160504e-01$	0.467040

Quadratic Fit for $m_f \in [0.012, 0.03]$

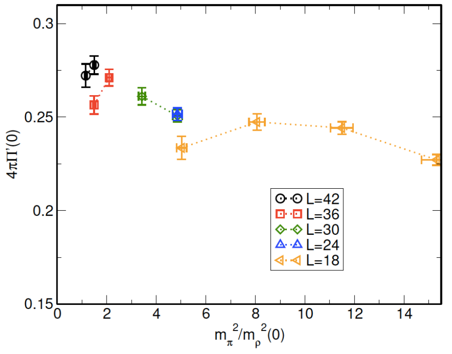
$$M_h = C_0 + C_1 m_f + C_2 m_f^2 \ , \tag{14}$$

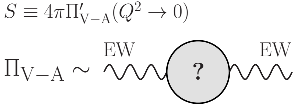
$$x \equiv N_f \left(\frac{M_\pi}{(4\pi F/\sqrt{2}} \right)^2 \ , \tag{15}$$

$$x(mf = 0.012) = 6.01(70) \ , \quad x(mf = 0.03) = 17.8(2.1) \ . \tag{16}$$

OP	C_0	χ^2/dof
F_π	0.0212(12)	0.31
M_π	1.866(57)	0.04
$\langle\bar{\psi}\psi\rangle$	0.000221(43)	0.54
M_ρ	0.1520(30)	0.36
M_{a0}	0.162(14)	0.12
M_{a1}	0.217(22)	1.81
M_{b1}	0.200(29)	0.52
M_N	0.2148(35)	0.40
M_{N*}	0.272(18)	0.03

$N_f = 8$ S-Parameter



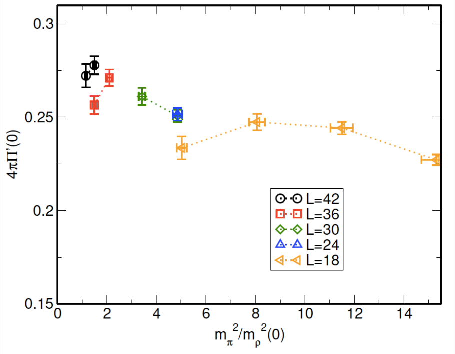


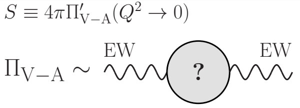
$$S \equiv 4\pi\Pi'_{V-A}(Q^2 \to 0)$$

The result $S \sim 0.25 - 0.275$ is smaller than that $S_{\mathrm{QCD},N_f=2} \sim 0.43$.

For the latter, Ref.: JLQCD PRL 2008, P. Boyle et.al. PRD 2010, LSD-Collab. PRD 2014.

$N_f = 8$ S-Parameter II





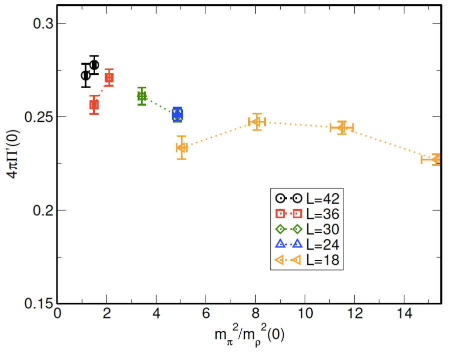
$$S \equiv 4\pi\Pi'_{\mathrm{V-A}}(Q^2 \to 0)$$

$$\Pi_{\mathrm{V-A}} \sim$$

Dispersion Relation (c.f. Knecht et.al.(Large N_c '98), LSD-Collab.('14))

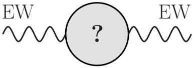
- $\Pi_{\mathrm{V-A}}(Q^2) = -F_\pi + \frac{Q^2}{12\pi}\int_0^\infty \frac{ds}{\pi}\frac{R_{\mathrm{V}} - R_{\mathrm{A}}}{s + Q^2}$.
- $S \propto \Pi'_{\mathrm{V-A}}(Q^2 \to 0)$ = small: Parity Doubling $R_{\mathrm{V}} \sim R_{\mathrm{A}}$.
- $(F_{\mathrm{V}}, M_{\mathrm{V}}) \simeq (F_{\mathrm{A}}, M_{\mathrm{A}})$ with $R_{\mathrm{V/A}} \simeq 12\pi^2 F^2_{\mathrm{V/A}}\delta(s - M^2_{\mathrm{V/A}})$

$\mathrm{N_f} = 8$ S-Parameter III



$$S \equiv 4\pi\Pi'_{\mathrm{V-A}}(Q^2 \to 0)$$

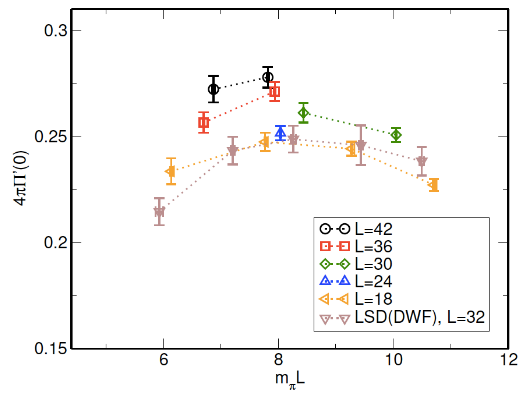
$\Pi_{\mathrm{V-A}} \sim$



Still much larger than $S_{\mathrm{exp}} = 0.03(10)$.

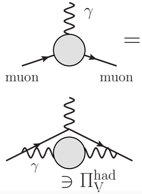
Pions: $(N_f^2 - 1) = 63 = 3 + 60$. Should Be Subtracted (Future Work).

$N_f = 8$ S-Parameter IV



The decreasing *S* at small $m_f \Longleftarrow$ Finite Vox Size Effects?

Sparameter vs $(g-2)$



$$= -ie[\gamma_\nu F_1(q^2) + \frac{i\sigma^{\nu\rho}q_\rho}{2m_{\rm muon}}\boxed{F_2(q^2)}]$$

$$a_\mu = (g-2)/2 \to F_2(0)$$

$$S \equiv 4\pi\Pi'_{\rm V-A}(Q^2 \to 0)$$

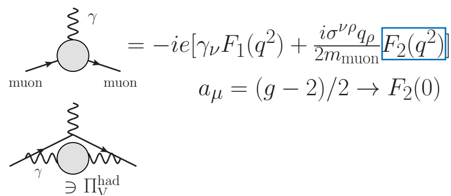
$$\Pi_{\rm V-A} \sim$$

There exists 3.3σ deviation between $a_\mu^{\rm exp}$ and $a_\mu^{\rm SM}$. At present, this may be the unique signal indicating the BSM.

$(g-2)$ vs S-Parameter

- The physics interest is shared: BSM.
- The lattice technology is shared: Vacuum Polalization Function Π. Ward-Takahashi Identity $\Delta_\nu(\Pi_{\nu\rho} - c.t.) = 0$, Fit Analyses of $\Pi(Q^2 \to 0)$, Wave Functional Renormalization $Z_{\rm V/A}$, and many others.

Sparameter vs $(g-2)$



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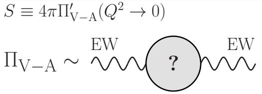

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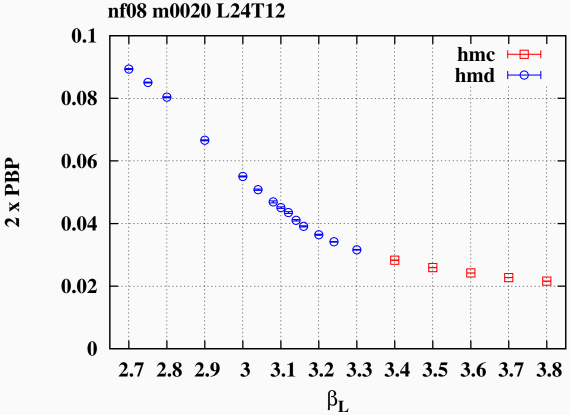
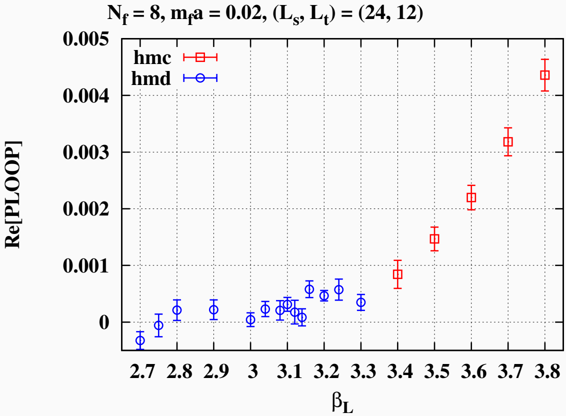
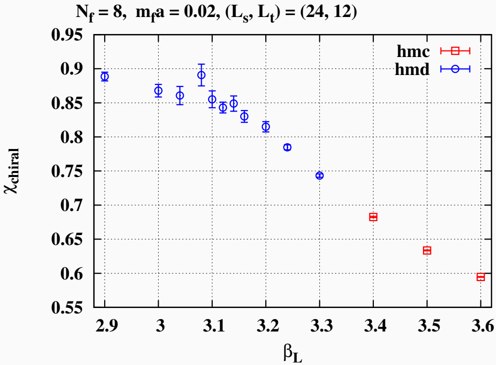
Chiral Condensate as a func. of T



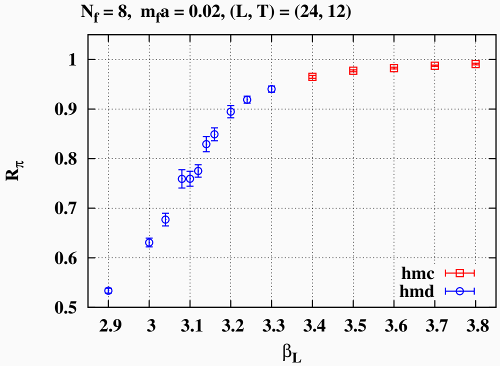
Polyakov Loop as a func. of T



Chiral Susceptibility



Susceptibility Ratio R_π



Increasing Rate of R_π