Study of the phase diagram of dense two-color QCD with $N_f = 2$ within lattice simulation

V.V. Braguta[1,2,3], E.-M. Ilgenfritz[4], A.Yu. Kotov[2], A.V. Molochkov[3], A.A. Nikolaev[2,3]

[1]IHEP, Protvino, Russia

[2]ITEP, Moscow, Russia

[3]FEFU, Vladivostok, Russia

[4]JINR, Dubna, Russia

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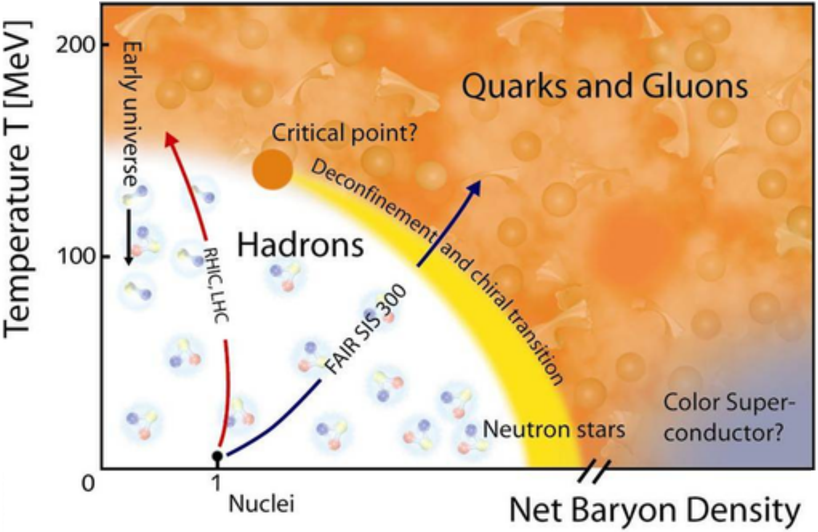
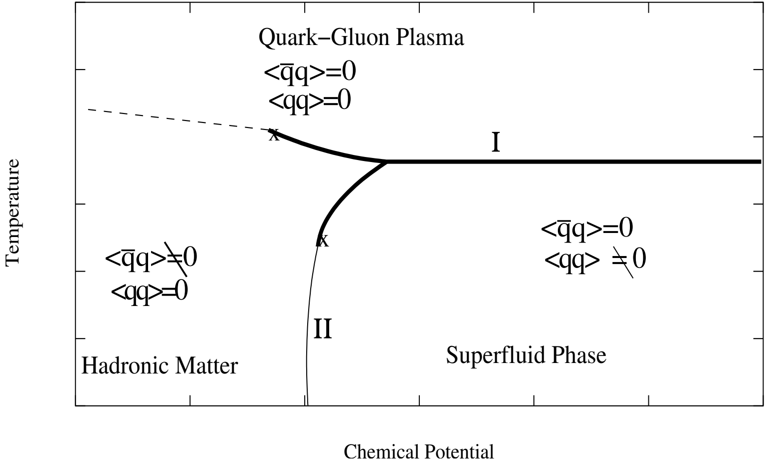
Outline

- Introduction
- Two-color QCD formulation
- Results at small chemical potential
- Results at large chemical potential
- Conclusions

QCD phase diagram



Tentative phase diagram of QC_2D



J.B. Kogut, D. Toublan, D.K. Sinclair, Nucl. Phys. **B642** (2002) 181–209

No sign problem in QC_2D

Case of QC_2D is special:

- $$det\left[M(\mu_q)\right] = det\left[(\tau_2 C\gamma_5)^{-1} M(\mu_q)(\tau_2 C\gamma_5)\right] = \\ = det\left[M(\mu_q^*)\right]^*, \text{ where } C = \gamma_2\gamma_4$$
- In LQC_2D with fundamental quarks $det\left[M(\mu_q)\right]$ is positive definite at real μ_q [see S. Hands, I. Montvay, S. Morrison, M. Oevers, L. Scorzato, and J. Skullerud, Eur. Phys. J. **C17**, 285 (2000)]

At real μ_q
$det\left[M(\mu_q)\right]$ is real, $det\left[M^\dagger(\mu_q)M(\mu_q)\right] > 0$ at $m_q \neq 0$.

Diquark source

In QC_2D there is a possibility to add diquark source to the action to study spontaneous breakdown of $U(1)_V$:

$$S_F = \sum_{x,y} \left[\overline{\chi}_x M(\mu_q)_{xy} \chi_y + \frac{\lambda}{2} \delta_{xy} \left(\chi^T \tau_2 \chi + \overline{\chi} \tau_2 \overline{\chi}^T \right) \right],$$

which modifies partition function as follows:

$$Z = \int DU \, det \left[M^\dagger(\mu_q) M(\mu_q) + \lambda^2 \right]^{\frac{1}{2}} e^{-S_G[U]}$$

instead of

$$Z = \int DU \, det \, M(\mu_q) e^{-S_G[U]}.$$

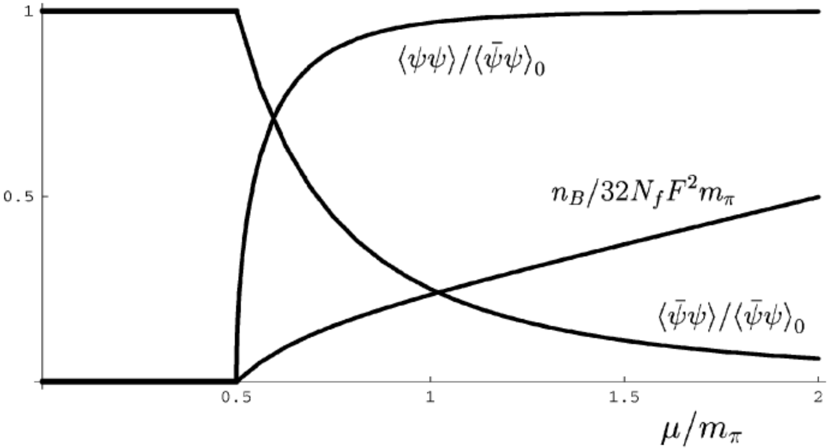
$\langle qq \rangle$ is colorless, gauge invariant and thus may be measured.

Previous studies of dense QC_2D (1)

Chiral perturbation theory

- J. B. Kogut, M. A. Stephanov and D. Toublan, Phys. Lett. **B 464** (1999) 183
- K. Splittorff, D. T. Son and M. A. Stephanov, Phys. Rev. **D 64** (2001) 016003
- J. B. Kogut, M. A. Stephanov, D. Toublan, J. J. M. Verbaarschot and A. Zhitnitsky, Nucl. Phys. **B 582** (2000) 477
- K. Splittorff, D. Toublan and J. J. M. Verbaarschot, Nucl. Phys. **B 620** (2002) 290
- T. Kanazawa, T. Wettig and N. Yamamoto, JHEP 0908 (2009) 003

Predictions of ChPT ($\lambda \to 0$)



Picture from J. B. Kogut, M. A. Stephanov, D. Toublan,
J. J. M. Verbaarschot and A. Zhitnitsky, Nucl. Phys. **B 582** (2000) 477

Previous studies of dense QC_2D (2)

Nambu–Jona-Lasinio model

- C. Ratti and W. Weise, Phys. Rev. **D 70** (2004) 054013
- T. Brauner, K. Fukushima and Y. Hidaka, Phys. Rev. **D 80** (2009) 074035 [Erratum Phys. Rev. D 81 (2010) 119904]
- L. He, Phys. Rev. **D 82** (2010) 096003

Random matrix theory

- B. Vanderheyden and A. D. Jackson, Phys. Rev. **D 64** (2001) 074016
- T. Kanazawa, T. Wettig and N. Yamamoto, Phys. Rev. **D 81** (2010) 081701
- T. Kanazawa, T. Wettig and N. Yamamoto, JHEP 1112 (2011) 007

Previous and ongoing lattice studies of QC_2D at $\mu_q \neq 0$

$N_f = 8$, staggered fermions without rooting

S. Hands, J. B. Kogut, M. P. Lombardo, S. E. Morrison, Nucl. Phys. **B 558**, 327—346 (1999)

$N_f = 4$, staggered fermions with rooting

J. B. Kogut, D. Toublan, and D. K. Sinclair, Phys.Lett. **B514**, 77–87 (2001); Nucl. Phys. **B 642**, 181–209 (2002)

$N_f = 2$, Wilson fermions

S. Cotter, P. Giudice, S. Hands, and J. I. Skullerud, Phys. Rev. D **87**, 034507 (2013)
T. Makiyama *et al.*, Phys. Rev. **D 93**, 014505 (2016)

$N_f = 1$, adjoint fermions

S. Hands, I. Montvay, S. Morrison, M. Oevers, L. Scorzato, and J. Skullerud, Eur. Phys. J. **C17**, 285 (2000)

Previous and ongoing lattice studies of QC_2D

Ongoing studies with $N_f = 2$

- V.V. Braguta, E.-M. Ilgenfritz, A.Yu. Kotov, A.V. Molochkov, A.A. Nikolaev (this talk, see also hep-lat/1605.04090)
- L. Holicki, J. Wilhelm, D. Smith, B. Wellegehausen, and L. von Smekal (talk by Lukas Holicki on Lattice 2016)
- T. Boz, P. Giudice, S. Hands, and J.-I. Skullerud (poster by Pietro Giudice on xQCD 2016)
- J. Rantaharju, V. Drach, C. Pica, and F. Sannino (talk by Jarno Rantaharju on xQCD 2016, Wed.)
- Jong-Wan Lee *et al.* (talk by Jong-Wan Lee on xQCD 2016, Wed.)

Action and lattice set-up

We consider $N_f = 2$ of staggered fermions with rooting:

$$Z = \int DU \, det\left[M^\dagger(\mu_q)M(\mu_q) + \lambda^2\right]^{\frac{1}{4}} e^{-S_G[U]} \,,$$

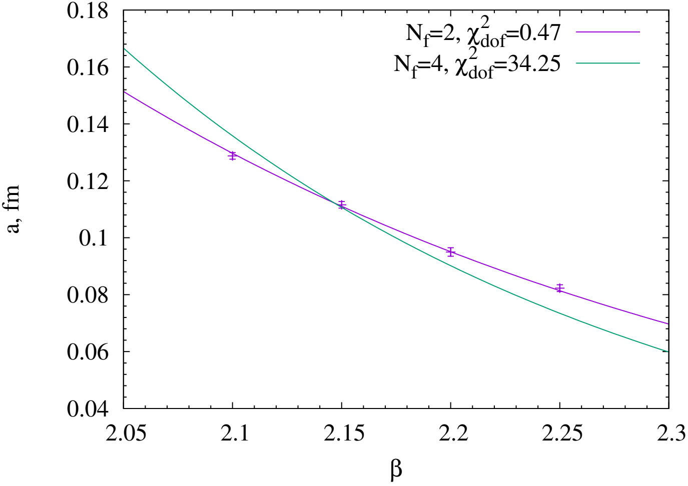
where $S_G[U]$ is the unimproved Wilson gauge action and

$$M_{xy}(\mu_q) = m_q a \delta_{xy} + \frac{1}{2}\sum_{\mu=1}^{4} \eta_\mu(x)\Big[U_{x,\mu}\delta_{x+\hat{\mu},y}e^{\mu_q a \delta_{\mu,4}} - \\ - U^\dagger_{x-\hat{\mu},\mu}\delta_{x-\hat{\mu},y}e^{-\mu_q a \delta_{\mu,4}}\Big] \,.$$

Simulation parameters

$16^3 \times 32$ lattice (zero-temperature scan), $\beta = 2.15$, $am = 0.005$
$a = 0.112(1)$ fm, $M_\pi = 378(4)$ MeV; $M_\pi L_s \approx 3.5$, $L_s \approx 1.8$ fm
Diquark source: $\lambda = 0.001$, 0.00075 and 0.0005

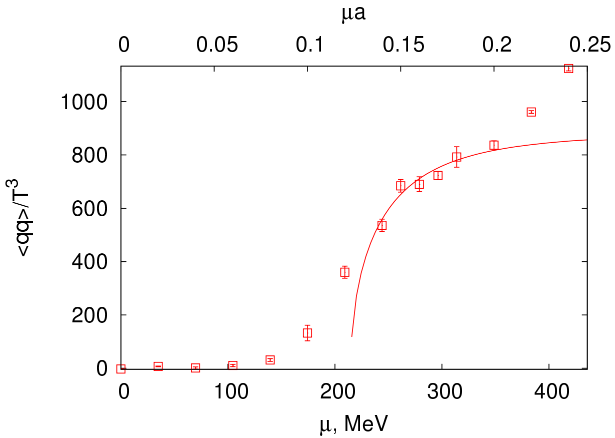
β-function: fit by the two-loop formula



Good fit for two-loop formula with $N_f = 2$

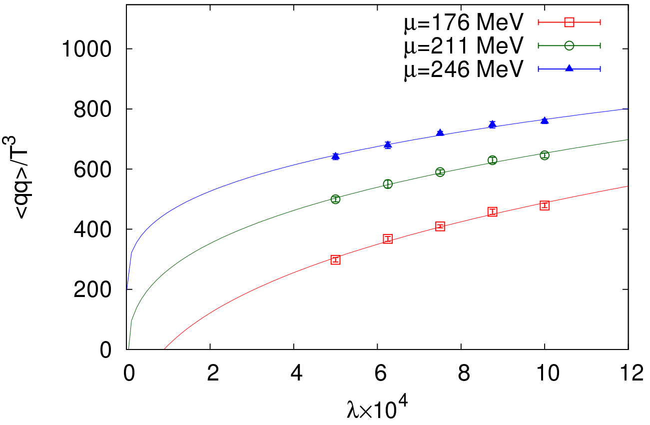
Small chemical potential region

Diquark condensate ($\lambda \to 0$ extrapolation)



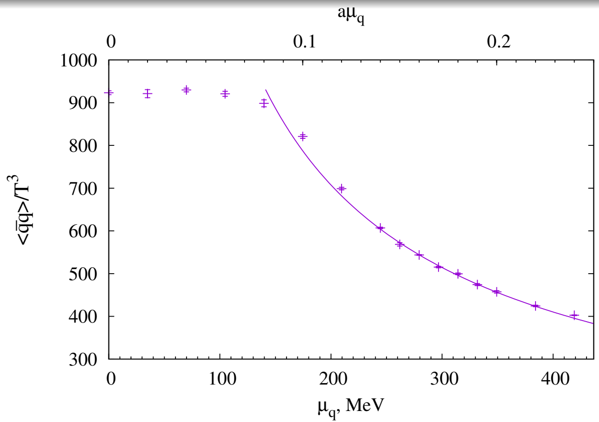
- Reasonable agreement with ChPT: $\langle qq \rangle / \langle \bar{q} q \rangle_0 = \sqrt{1 - \mu_c^4/\mu^4}$
- Phase transition at $\mu_c = 215(10)$ MeV $\simeq m_\pi/2$
- Bose Einstein condensate (BEC) phase $\mu \in (200; 350)$ MeV

Diquark condensate: critical index



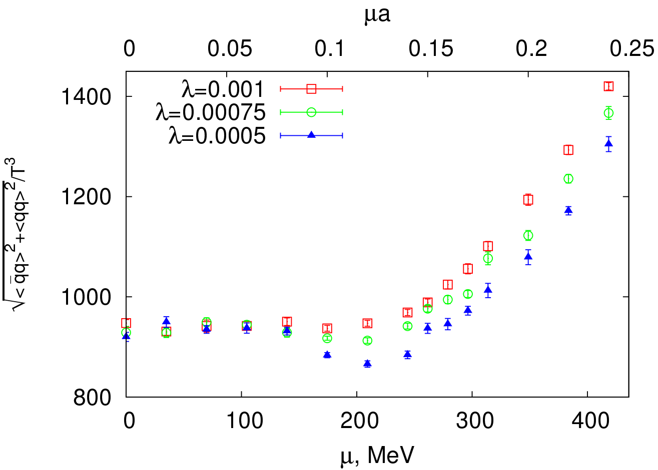
- Fit $\langle qq \rangle = A + B\lambda^{1/3}$ with $\chi^2_{dof} \simeq 1$
- $\langle qq \rangle_{\lambda=0} = -0.0021(12)$ at $a\mu = 0.12\,(\mu = 211\,\mathrm{MeV})$

Chiral condensate ($\lambda \to 0$ extrapolation)



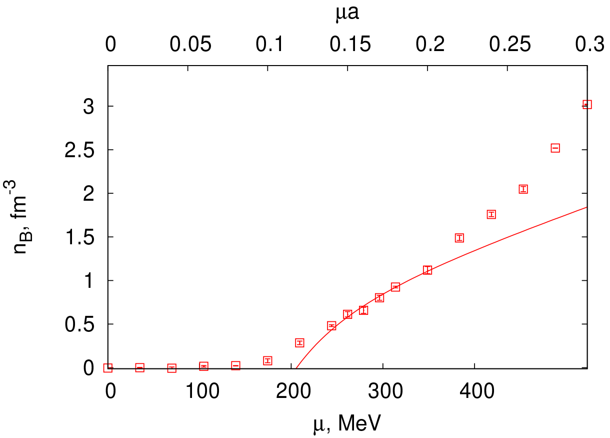
- Good fit $\langle \bar{q}q \rangle = A/\mu^{\alpha}$ with $\alpha = 0.78(2)$, $\chi^2_{dof} = 0.3$
- LO ChPT predicts $\langle \bar{q}q \rangle / \langle \bar{q}q \rangle_0 = \mu_c^2/\mu^2$
- Similar slower decrease with $\alpha = 1 \ldots 1.3$ was observed in Nucl. Phys. **B 642**, 181 (2002) and PRD **87**, 034507 (2013)

Chiral and diquark condensates



Check of the LO ChPT prediction $\langle\bar{q}q\rangle^2 + \langle qq\rangle^2 = const$

Baryon density ($\lambda \to 0$)



- Good agreement with ChPT: $n_B \sim \mu - \mu_c^4/\mu^3$
- Phase transition at $\mu_c = 207(7)\,\text{MeV} \simeq m_\pi/2$
- Deviation from ChPT prediction starts at $n_B \sim 1\ \text{fm}^{-3}$

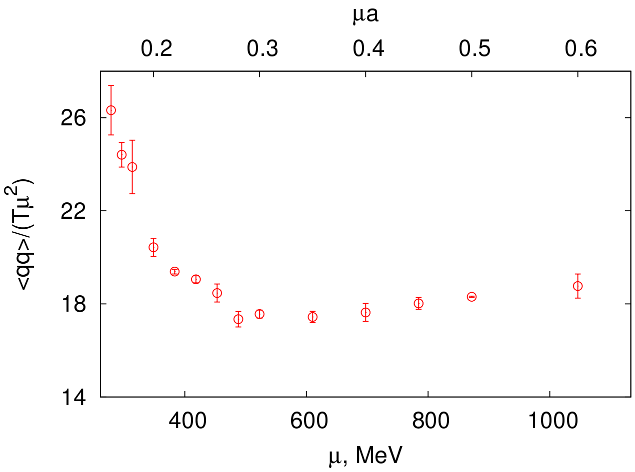
Large chemical potential region

Phase diagram for $N_c \to \infty$

- Hadronic phase at $\mu < M_N/N_c$ ($p \sim O(1)$)
- Dilute baryon gas at $\mu > M_N/N_c$, width $\delta\mu \sim \Lambda_{QCD}/N_c^2$
- Quarkyonic phase at $\mu > \Lambda_{QCD}$ ($p \sim N_c$)
    - Degrees of freedom:
        - Baryons (on the surface)
        - Quarks (inside the Fermi sphere $|k| < \mu$)
    - Chiral symmetry restoration
    - The system is in the confinement phase

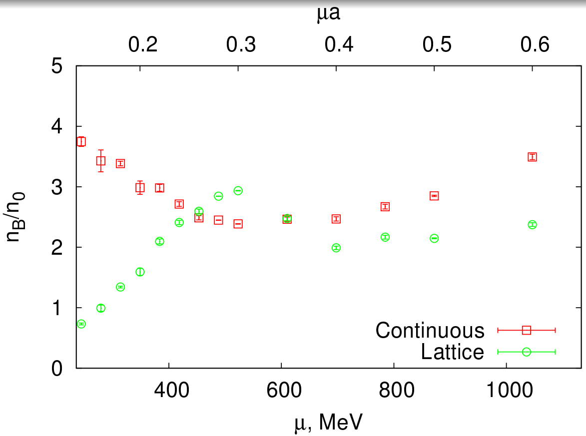
L. McLerran, R.D. Pisarski, *Phases of cold, dense quarks at large N(c)*, Nucl. Phys. **A 796** (2007) 83 [hep-ph/0706.2191]

Diquark condensate ($\lambda \to 0$ extrapolation)



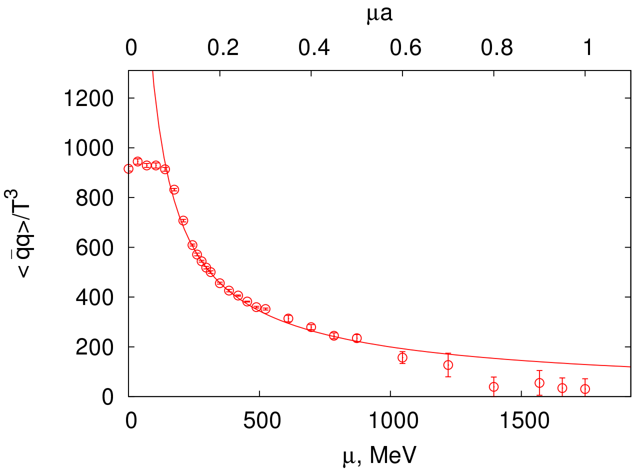
- Bardeen–Cooper–Schrieffer (BCS) phase at $\mu > 500$ MeV
- $\langle qq \rangle \sim \mu^2$: **baryons on the Fermi-surface**

Baryon density ($\lambda \to 0$ extrapolation)

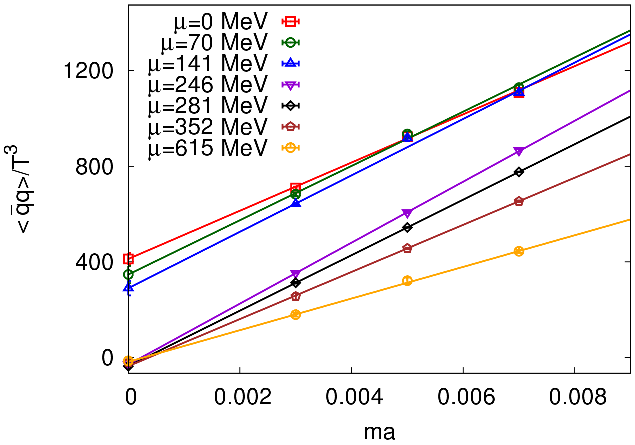


- Free quarks at $T = 0$:
  $n_B^{(0)} = N_f(2s+1)\int \frac{d^3k}{(2\pi)^3}\theta(|k|-\mu) = 2\mu^3/(3\pi^2)$
- **Quarks inside the Fermi sphere** dominate over the surface:
  $\frac{4}{3}\pi\mu^3 > 4\pi\mu^2\Lambda_{QCD} \Rightarrow \mu > 3\Lambda_{QCD}$

Chiral condensate ($\lambda = 0.0005$, $ma = 0.005$)

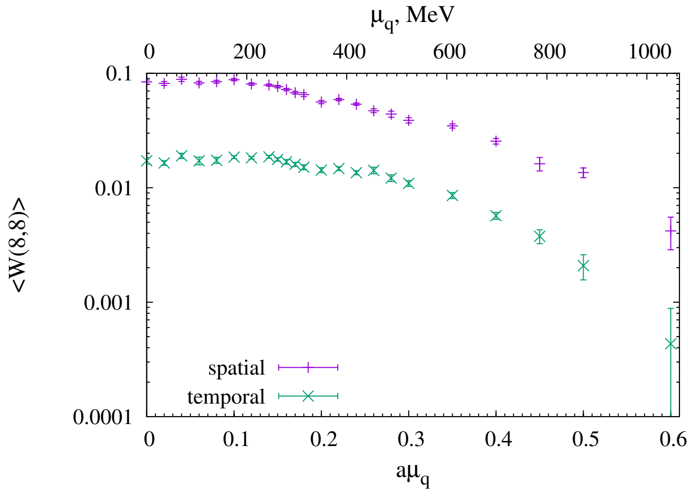


Chiral condensate ($\lambda \to 0$, chiral limit $m \to 0$)



No chiral symmetry breaking at large enough μ

Gluonic observables: 8 × 8 Wilson loops



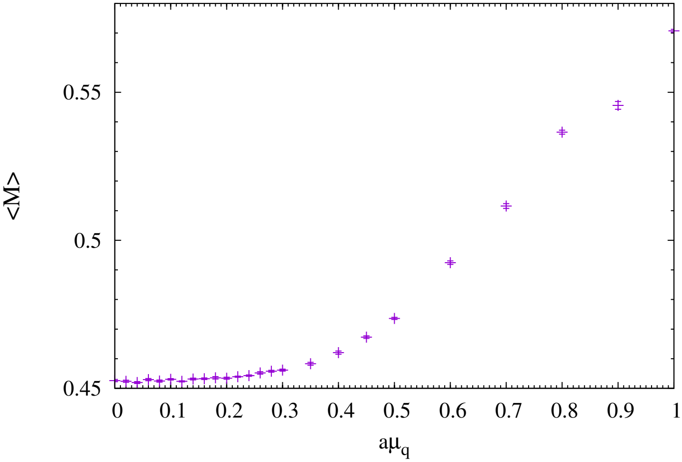
Polyakov loop is zero within the errorbars for all $a\mu_q$.

Conclusions

- For the first time all three phases have been observed in one lattice simulation: **(1)** hadronic phase for $0 < \mu < \mu^c$;
  (2) "baryon onset" with a superfluid condensate due to Bose-Einstein mechanism for $\mu^c < \mu < \mu^d$;
  (3) the phase with diquark condensation due to the Bardeen-Cooper-Schrieffer mechanism for $\mu^d < \mu$
- Good agreement with LO ChPT predictions for all observables except the bare chiral condensate
- Dilute baryon gas at $m_\pi/2 < \mu < m_\pi/2 + 150$ MeV
- BCS phase at $\mu > 500$ MeV ($a\mu > 0.28$)
- BCS phase may be similar to quarkyonic phase

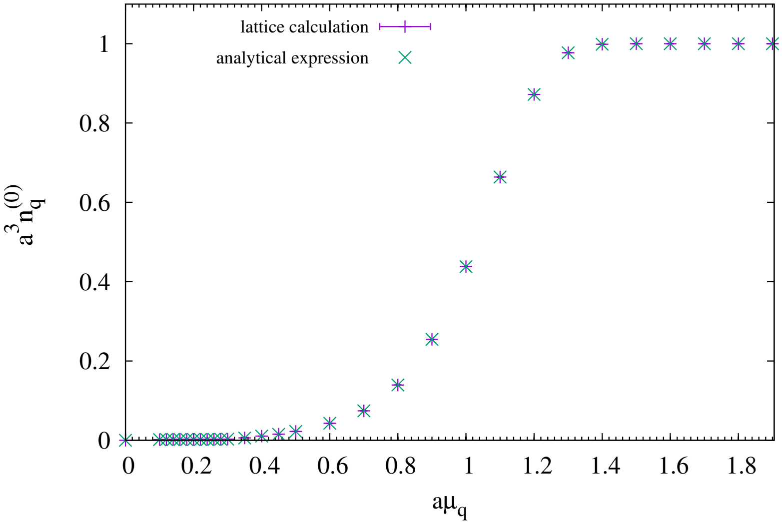
Thank you for attention

Z_2 monopole density



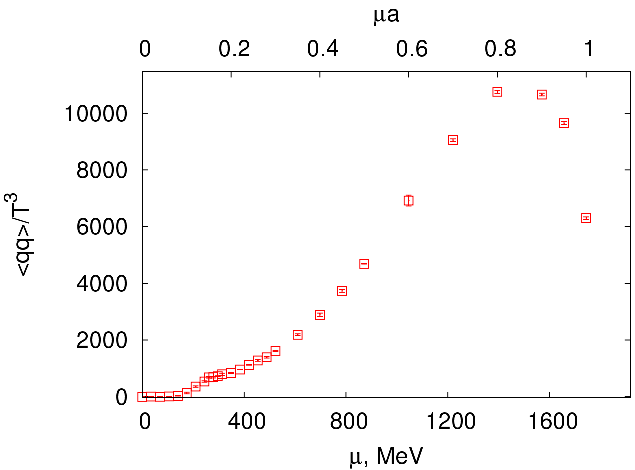
$$M = 1 - (\sum_{cubes} \prod_{P \in \partial C} sign[TrU_P])/N_{cubes}$$

Saturation for the free baryon density



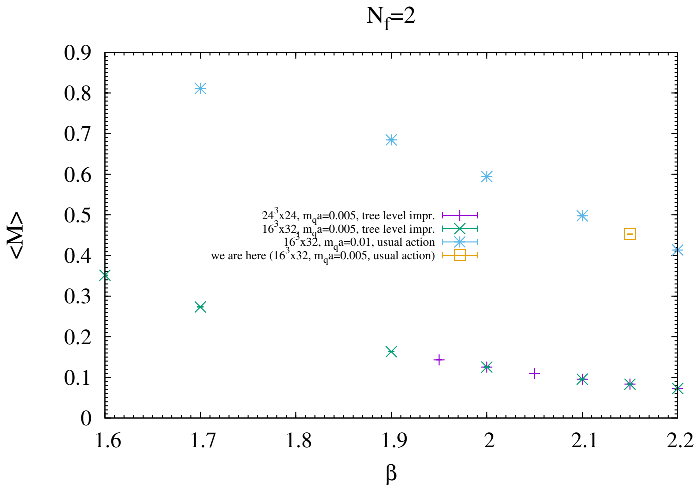
$16^3 \times 32$ lattice, $ma = 0.005$, $\lambda = 0.0005$, free fermions

Saturation for the diquark condensate



$16^3 \times 32$ lattice, $ma = 0.005$, $\lambda = 0.0005$

Z_2 monopole density at $a\mu_q = 0$



$M = 1 - (\sum_{cubes} \prod_{P \in \partial C} sign[TrU_P])/N_{cubes}$
For the details see David Scheffler's PhD thesis