Resonances and scattering in QCD – progress and prospects

Christopher Thomas, University of Cambridge

c.e.thomas@damtp.cam.ac.uk

Annual Particle Theory Meeting,
Durham, 19 – 21 December 2016

UNIVERSITY OF CAMBRIDGE

Hadron Spectrum Collaboration

Exotic particles at the LHC?

breaking

www.symmetrymagazine.org

April 14, 2014

CERN's LHCb experiment sees exotic particle

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PRL **112,** 222002 (2014) PHYSICAL REVIEW LETTERS week ending 6 JUNE 2014

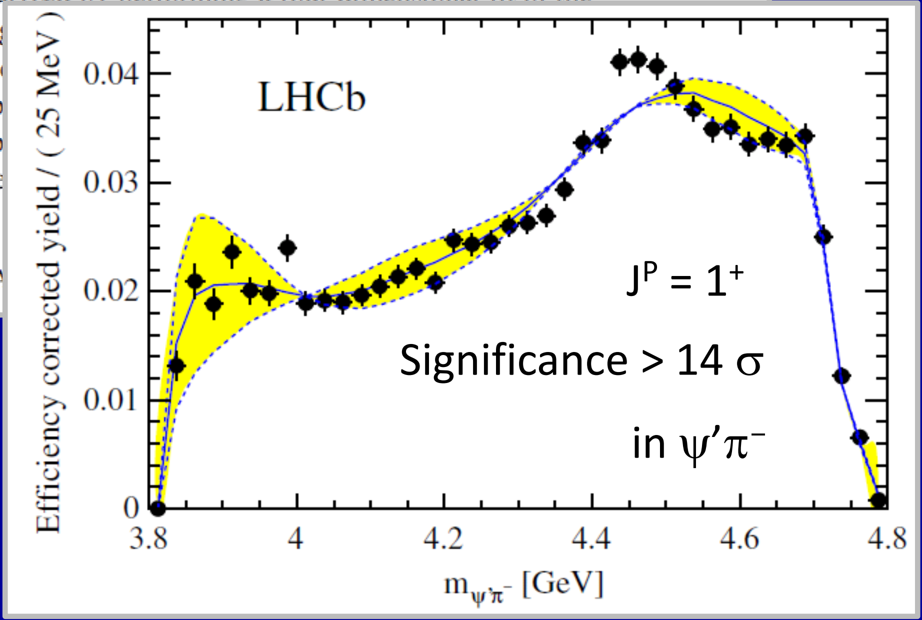
Observation of the Resonant Character of the Z(4430)$^-$ State

R. Aaij *et al.**
(LHCb Collaboration)

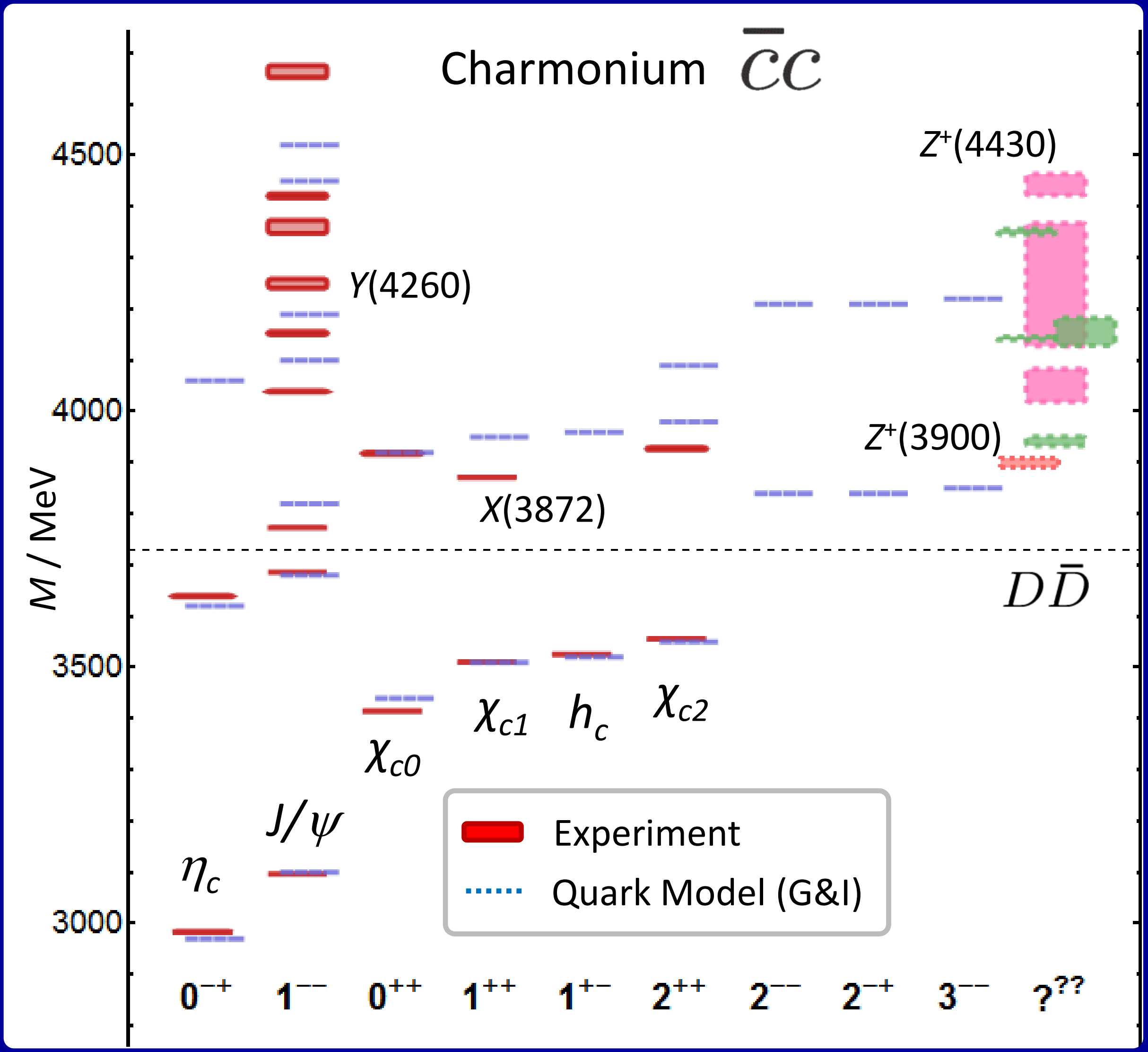
(Received 7 April 2014; published 4 June 2014)

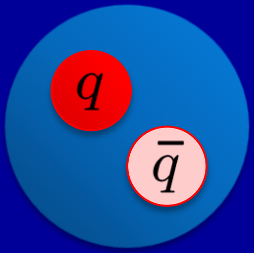
Resonant structures in $B^0 \to \psi' \pi^- K^+$ decays are analyzed by performing a four-dimensional fit of the decay amplitude, using pp collision data correspondin
data cannot be described with $K^+\pi^-$ resonances alon
approach. A highly significant $Z(4430)^- \to \psi'\pi^-$ comp
this state. The observed evolution of the $Z(4430)^-$ amp
nature of this particle. The mass and width measureme
determined unambiguously to be 1^+.

DOI: 10.1103/PhysRevLett.112.222002 PA



Meson spectroscopy





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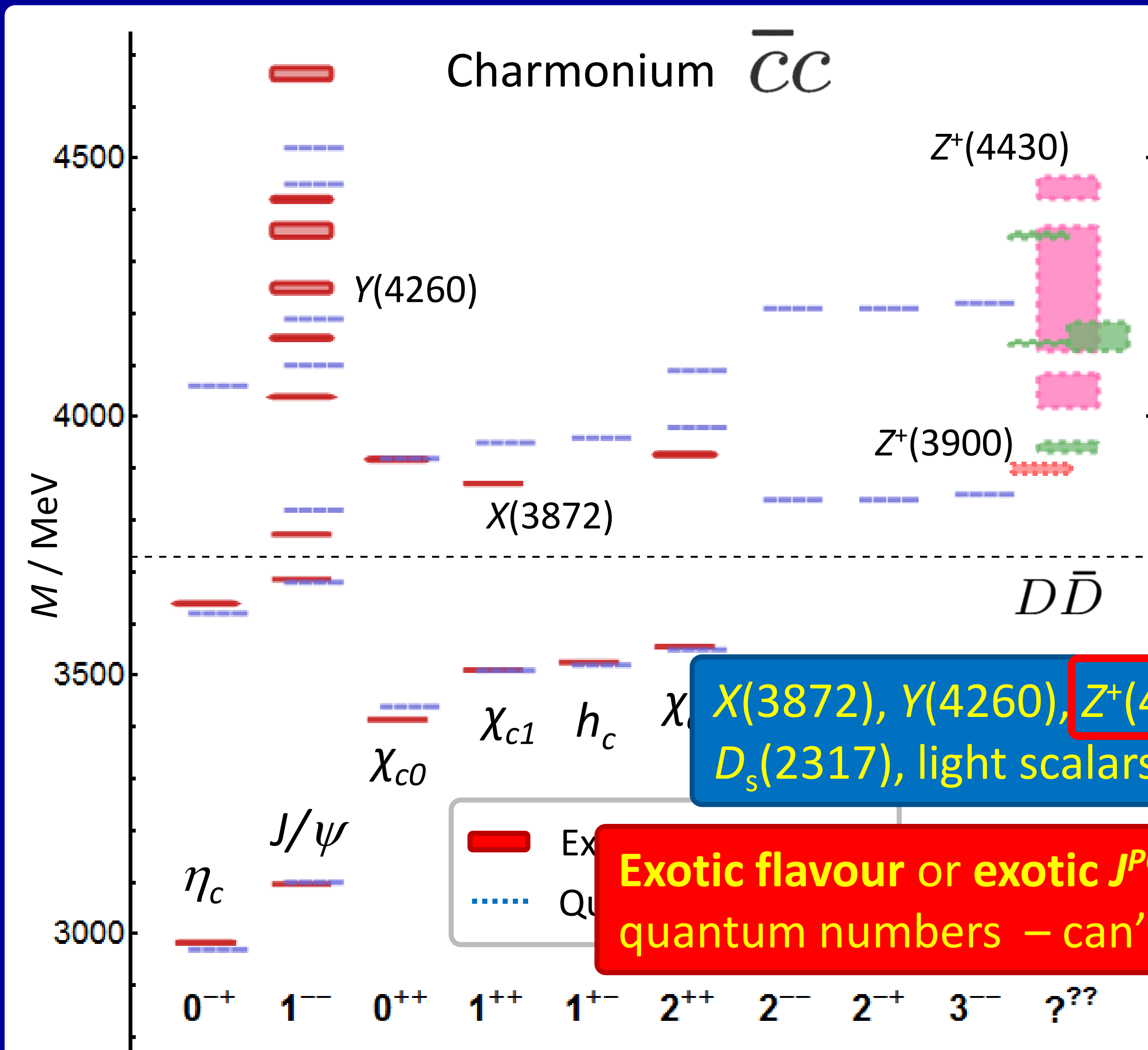
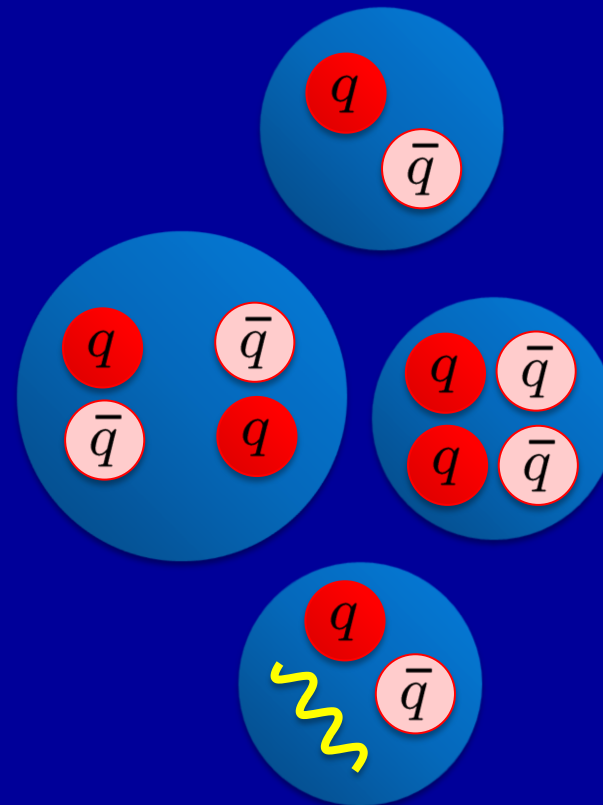
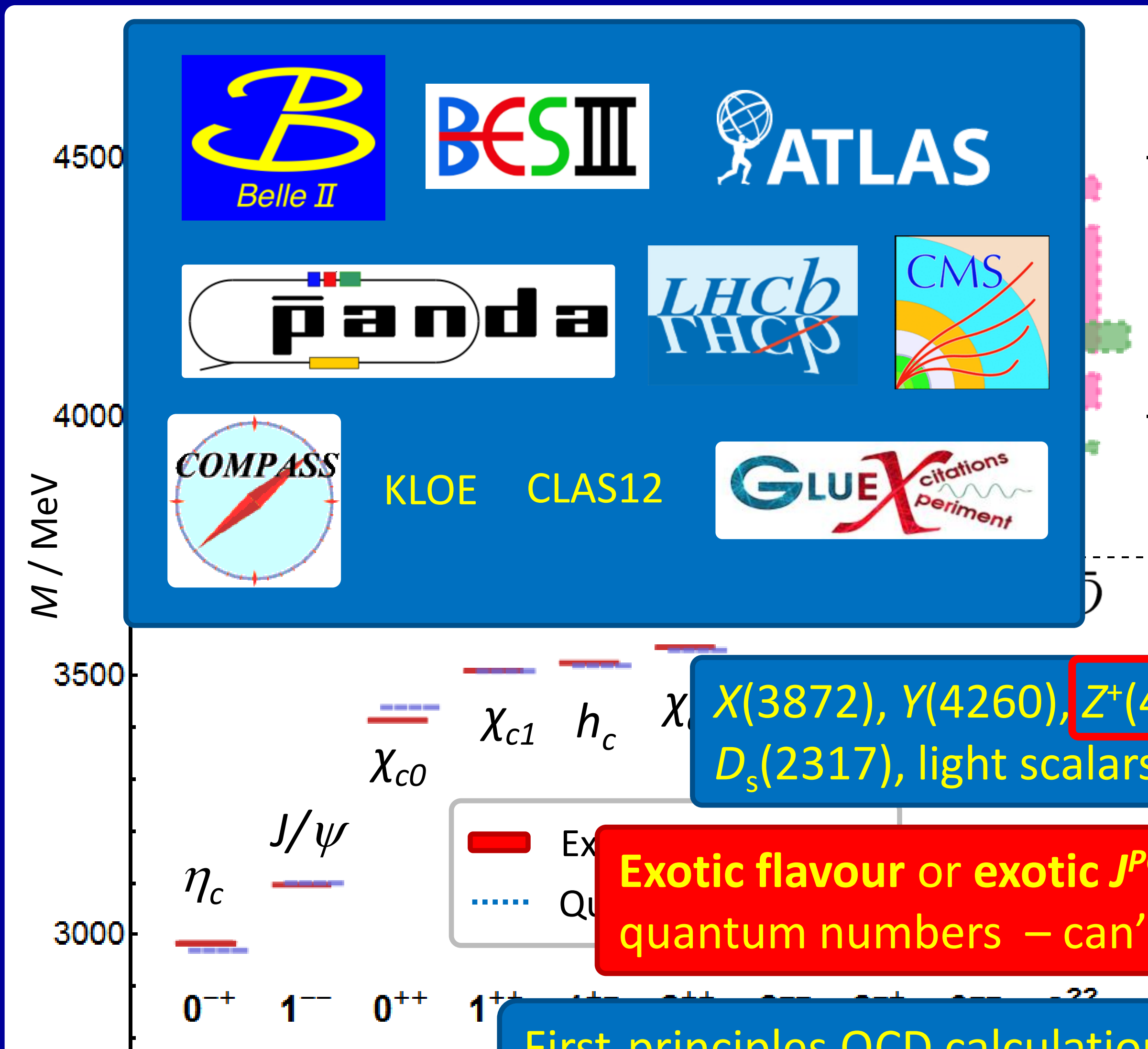


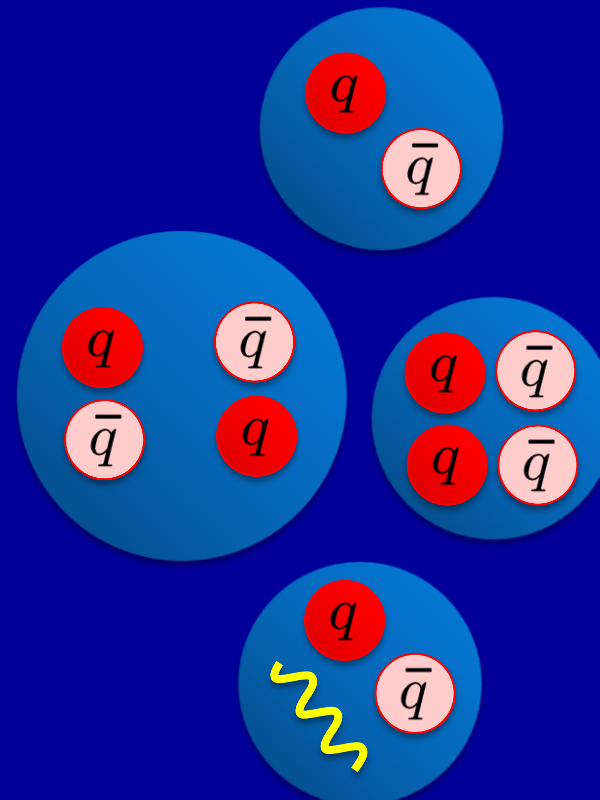


$X(3872)$, $Y(4260)$, $Z^+(4430)$, $Z_c^+(3900)$; Z_b^+, $D_s(2317)$, light scalars, $\pi_1(1600)$ [$J^{PC} = 1^{-+}$], ...

Exotic flavour or **exotic** J^{PC} **(**0^{--}, 0^{+-}, 1^{-+}, 2^{+-}, ...**)** quantum numbers – can't just be a $q\bar{q}$ pair

Meson spectroscopy





X(3872), *Y*(4260), Z^+(4430), Z_c^+(3900); Z_b^+,
D_s(2317), light scalars, π_1(1600) [$J^{PC} = 1^{-+}$], ...

Exotic flavour or **exotic J^{PC}** ($0^{--}, 0^{+-}, 1^{-+}, 2^{+-}$, ...) quantum numbers – can't just be a $q\bar{q}$ pair

First-principles QCD calculations → Lattice QCD

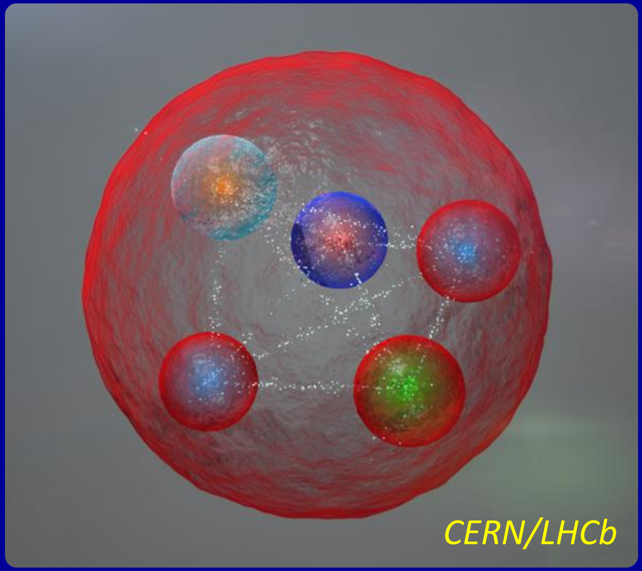
Exotic baryons

PRL **115,** 072001 (2015) Selected for a Viewpoint in *Physics* PHYSICAL REVIEW LETTERS week ending 14 AUGUST 2015

Observation of $J/\psi p$ Resonances Consistent with Pentaquark States in $\Lambda_b^0 \to J/\psi K^- p$ Decays

R. Aaij *et al.*[*]
(LHCb Collaboration)

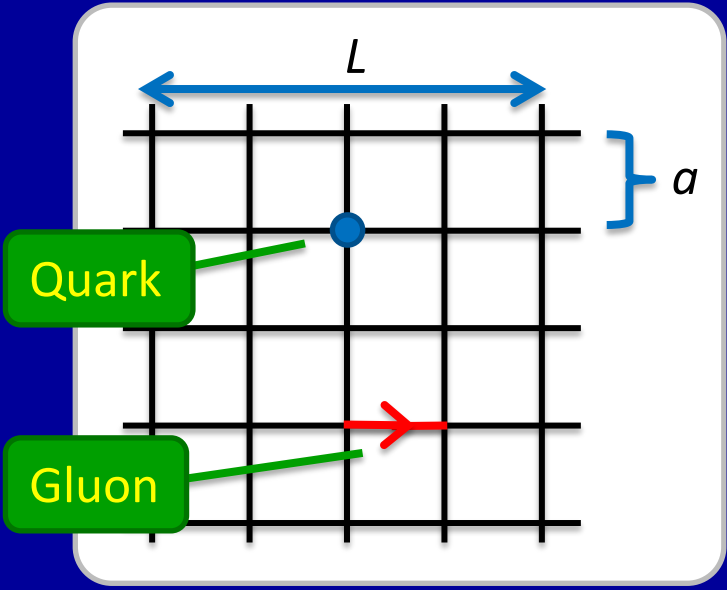
(Received 13 July 2015; published 12 August 2015)



Outline

- Introduction
  - Lattice QCD, scattering and resonances
- Some examples (focus on mesons):
  - The ρ resonance
  - Light scalar mesons
  - Charm-light mesons
- Summary

Lattice QCD Spectroscopy



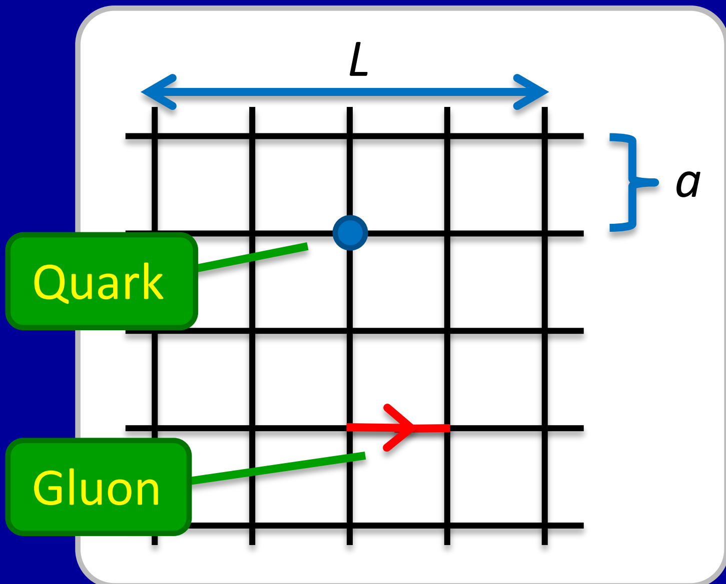
- **Discretise** spacetime in a **finite volume**
- Compute correlation fns. numerically (Euclidean time, $t \rightarrow i\,t$)

$$\langle f \rangle \sim \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}U \, f \, e^{-\tilde{S}[\psi,\bar{\psi},U]}$$

Note: finite *a* and *L*; possibly unphysical m_π

DiRAC

Lattice QCD Spectroscopy



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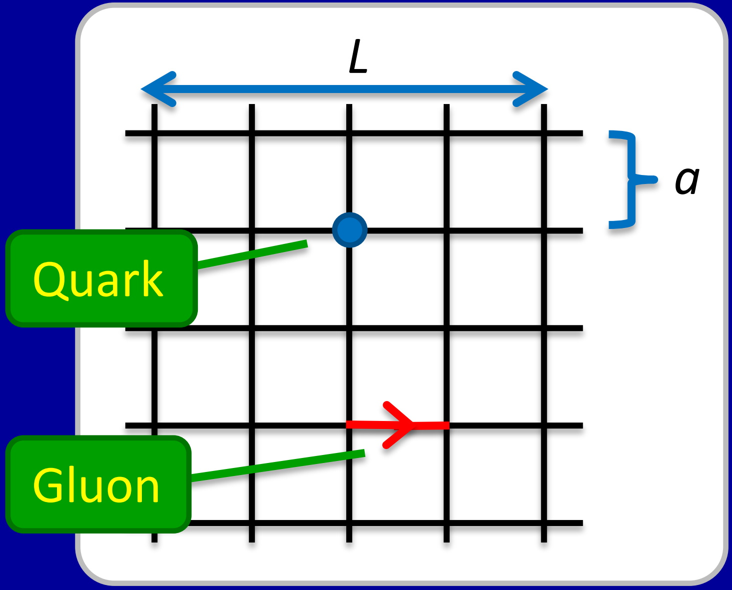
Finite-volume energy eigenstates from:

$$C_{ij}(t) = < 0|\mathcal{O}_i(t)\mathcal{O}_j^\dagger(0)|0 >$$

DiRAC

Lattice QCD Spectroscopy



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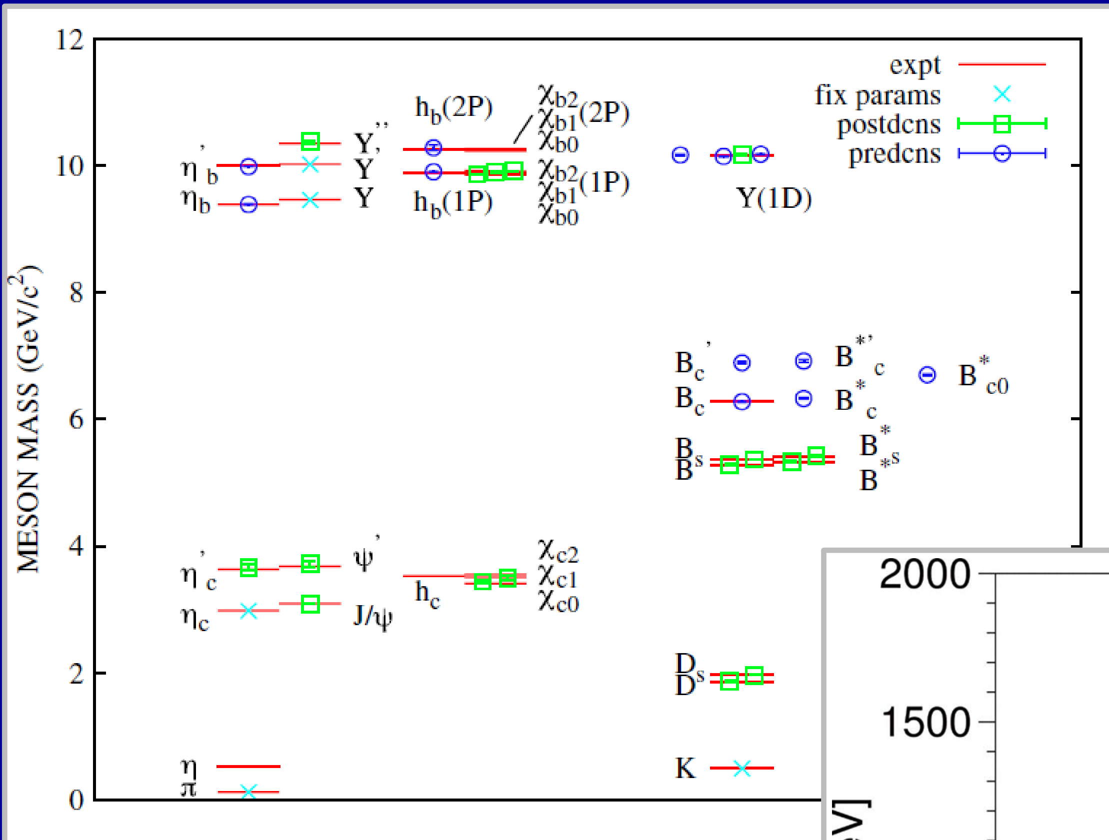
Finite-volume energy eigenstates from:

$$C_{ij}(t) = < 0|\mathcal{O}_i(t)\mathcal{O}_j^\dagger(0)|0 >$$

$$= \sum_n \frac{e^{\boxed{-E_n} t}}{2 E_n} Z_i^{(n)} Z_j^{(n)*}$$

DiRAC

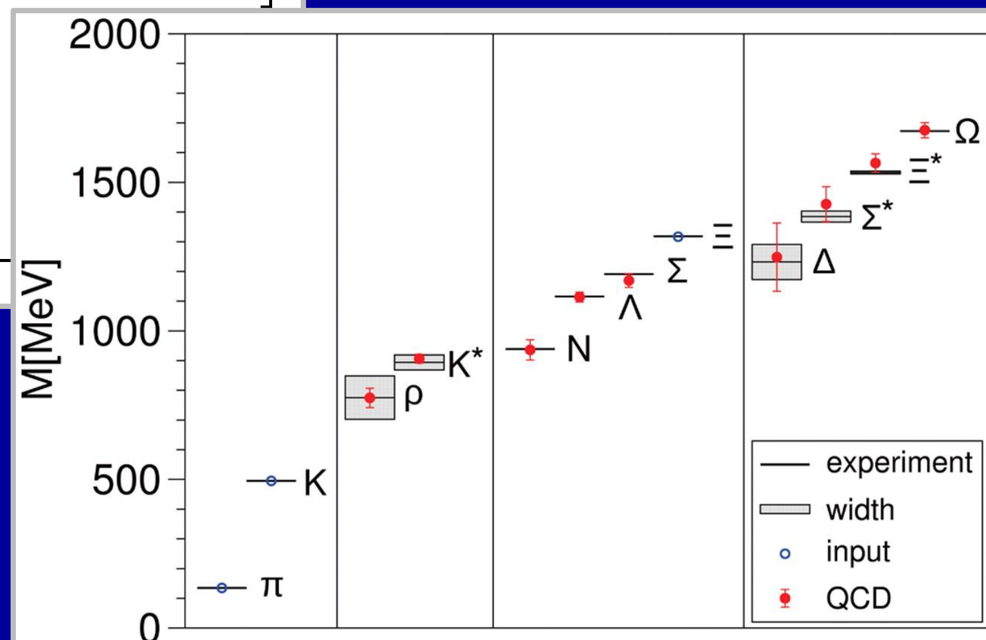
Lower-lying mesons and baryons



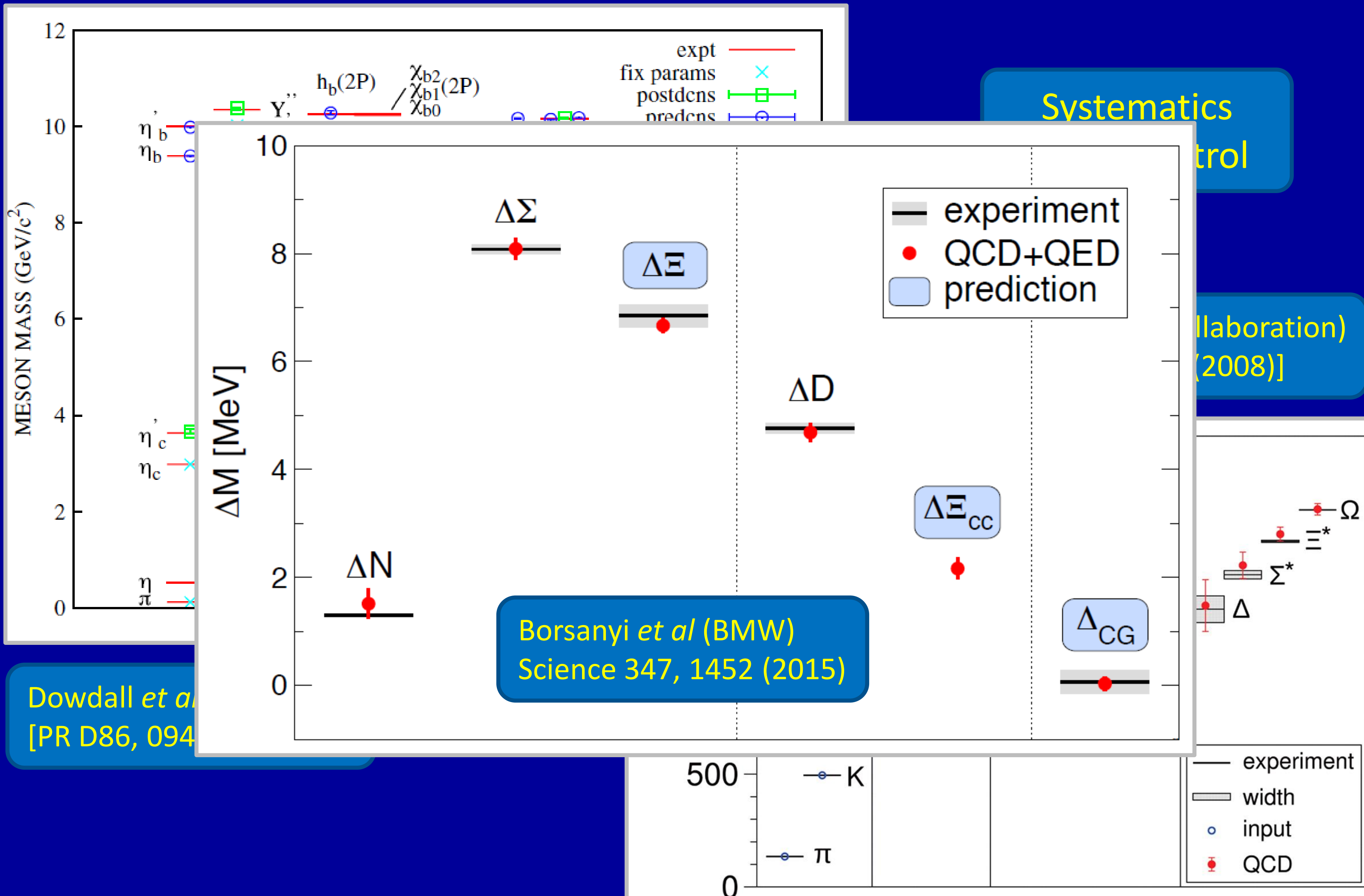
Dowdall *et al* (HPQCD)
[PR D86, 094510 (2012)]

Systematics under control

Durr *et al* (BMW Collaboration)
[Science 322, 1224 (2008)]



Lower-lying mesons and baryons



Borsanyi *et al* (BMW)
Science 347, 1452 (2015)

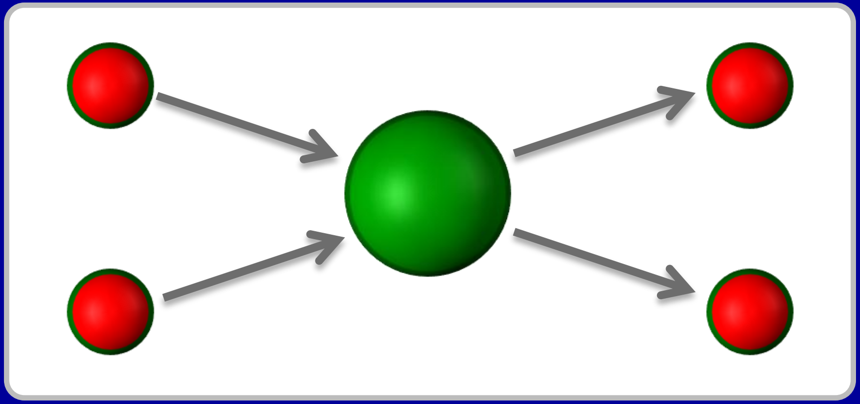
Systematics
trol

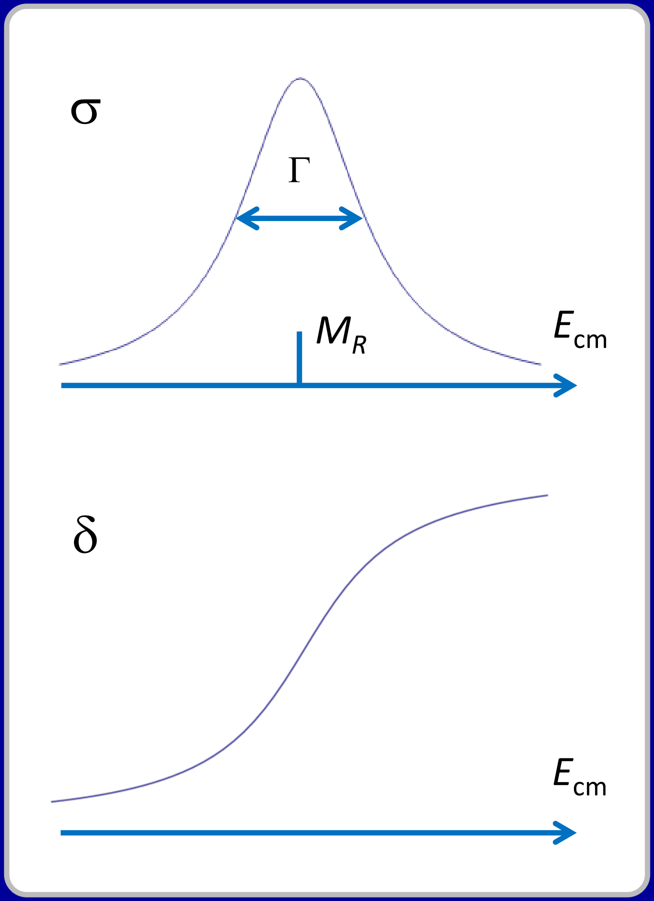
llaboration)
(2008)]

Dowdall *et al*
[PR D86, 094

Scattering and resonances

Most hadrons appear as resonances in scattering of lighter hadrons





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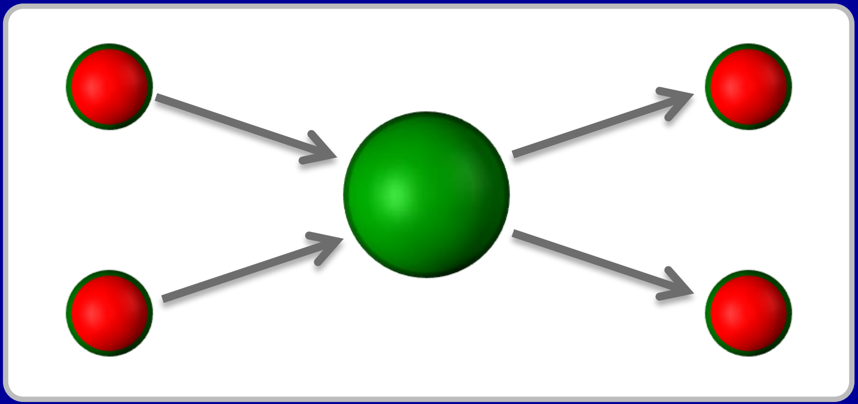

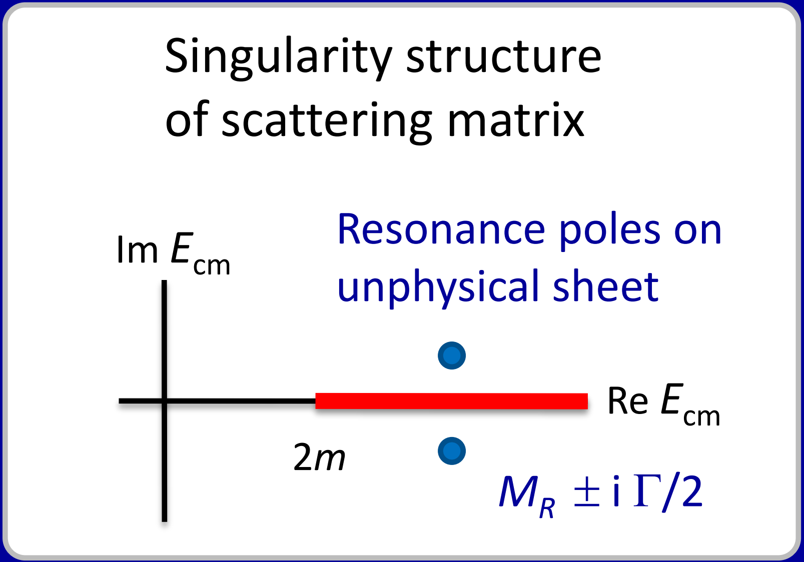

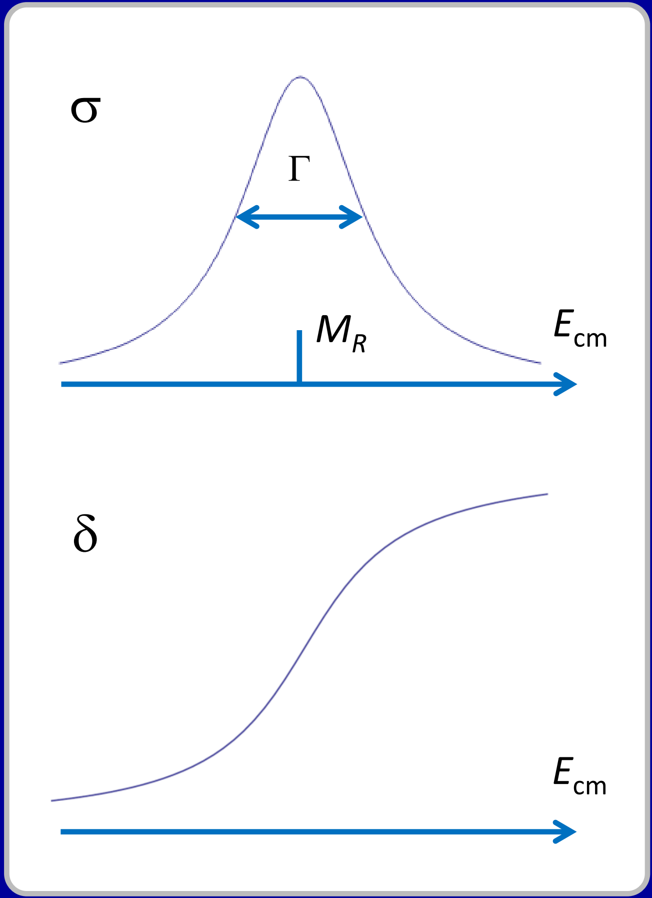

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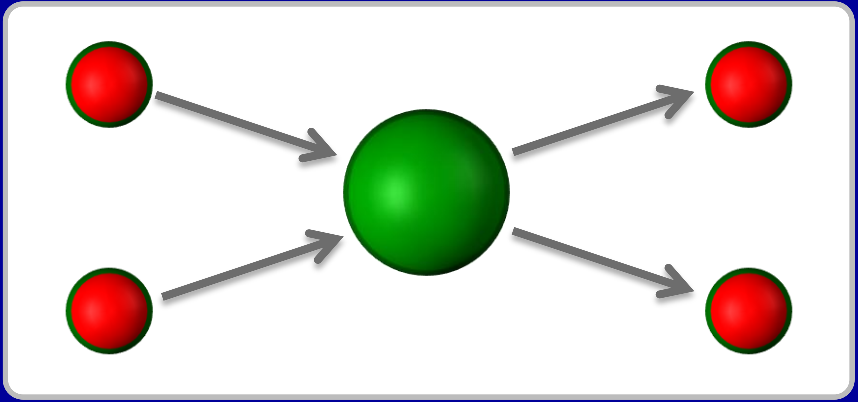
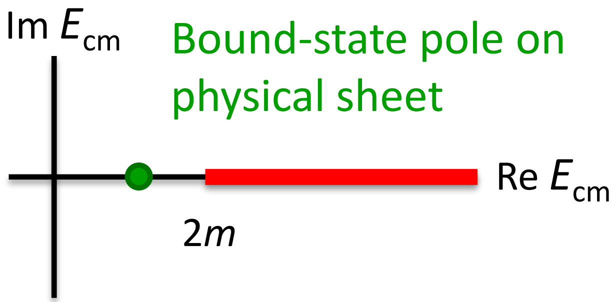


Singularity structure of scattering matrix

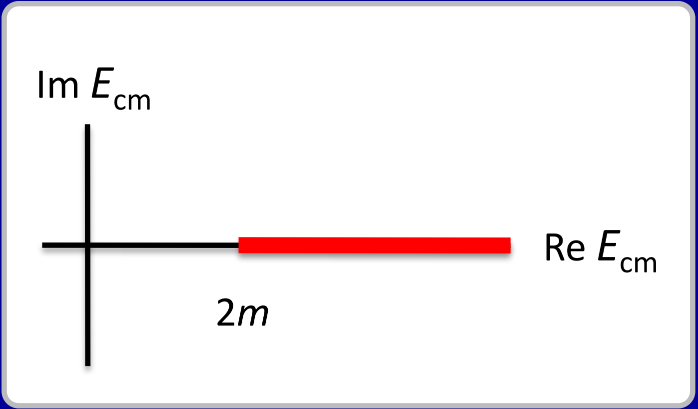


Scattering in Lattice QCD

Don't have direct access to scattering physics

Scattering in Lattice QCD

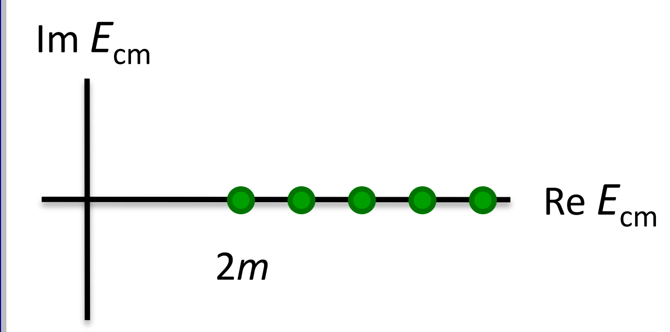
Infinite volume – continuous spectrum above threshold

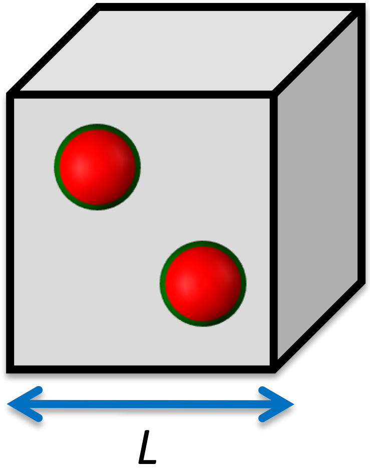


Scattering in Lattice QCD

Infinite volume – continuous spectrum above threshold

Finite volume – discrete spectrum





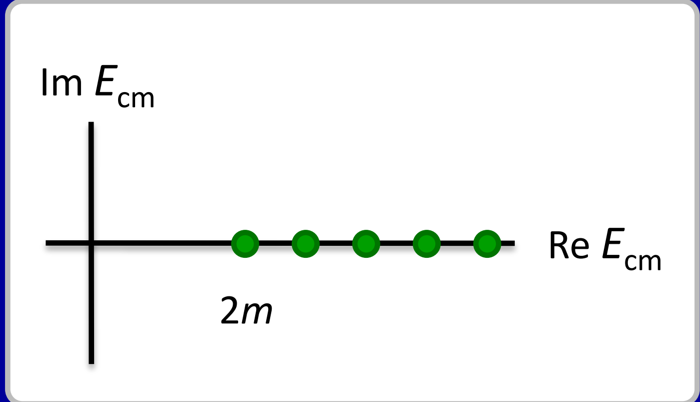
Non-interacting: $\vec{k}_{A,B} = \frac{2\pi}{L}(n_x, n_y, n_z)$

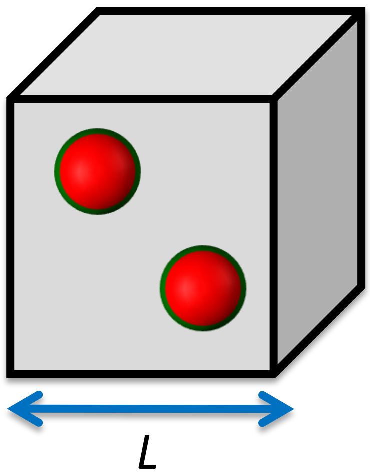
[periodic b.c.s]

Scattering in Lattice QCD

Infinite volume – continuous spectrum above threshold

Finite volume – discrete spectrum





[periodic b.c.s]

Non-interacting: $\vec{k}_{A,B} = \frac{2\pi}{L}(n_x, n_y, n_z)$

Interacting: $\vec{k}_{A,B} \neq \frac{2\pi}{L}(n_x, n_y, n_z)$

c.f. 1-dim: $k = \frac{2\pi}{L} n + \frac{2}{L}\delta(k)$

scattering phase shift

Scattering in Lattice QCD

Lüscher method [NP B354, 531 (1991)] extended by many others:
relate **finite-volume energy levels** $\{E_{\mathrm{cm}}\}$ to **infinite-volume scattering *t*-matrix**

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Elastic scattering: from E_{cm} get $t(E_{cm})$ or equivalently $\delta(E_{cm})$

$$\rho = \frac{2k_{\text{cm}}}{E_{\text{cm}}}$$

Scattering *t*-matrix: $S = I + 2i\rho\ t$

$$t^{(\ell)} = \frac{1}{\rho} e^{i\delta_\ell} \sin \delta_\ell$$

Larger set of E_{cm} by e.g. overall non-zero mom., twisted b.c.s, different vols.

[Complication: reduced symmetry of lattice volume → mixing of partial waves]

The ρ resonance in $\pi\pi$ scattering

($J^{PC} = 1^{--}$, I = 1)

Experimentally

$\mathrm{BR}(\rho \to \pi\pi) \sim 100\%$

The ρ resonance in ππ scattering

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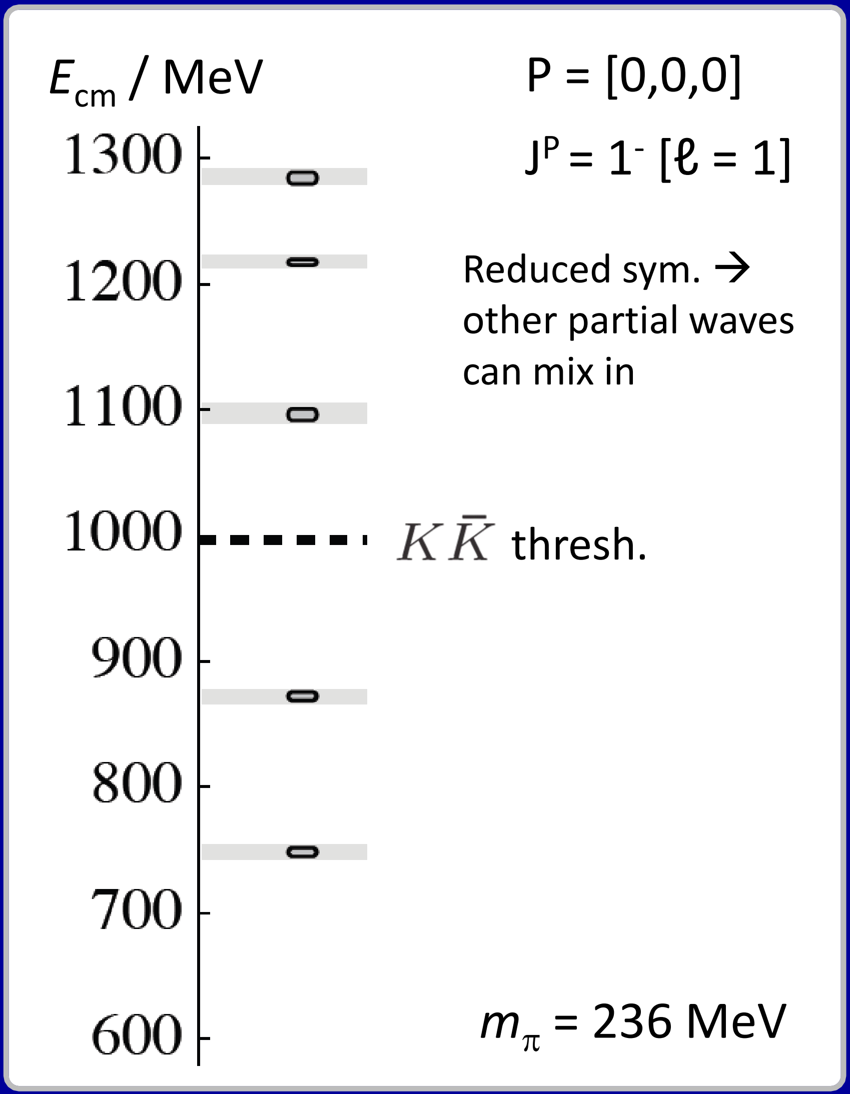


Experimentally

$\mathrm{BR}(\rho \to \pi\pi) \sim 100\%$

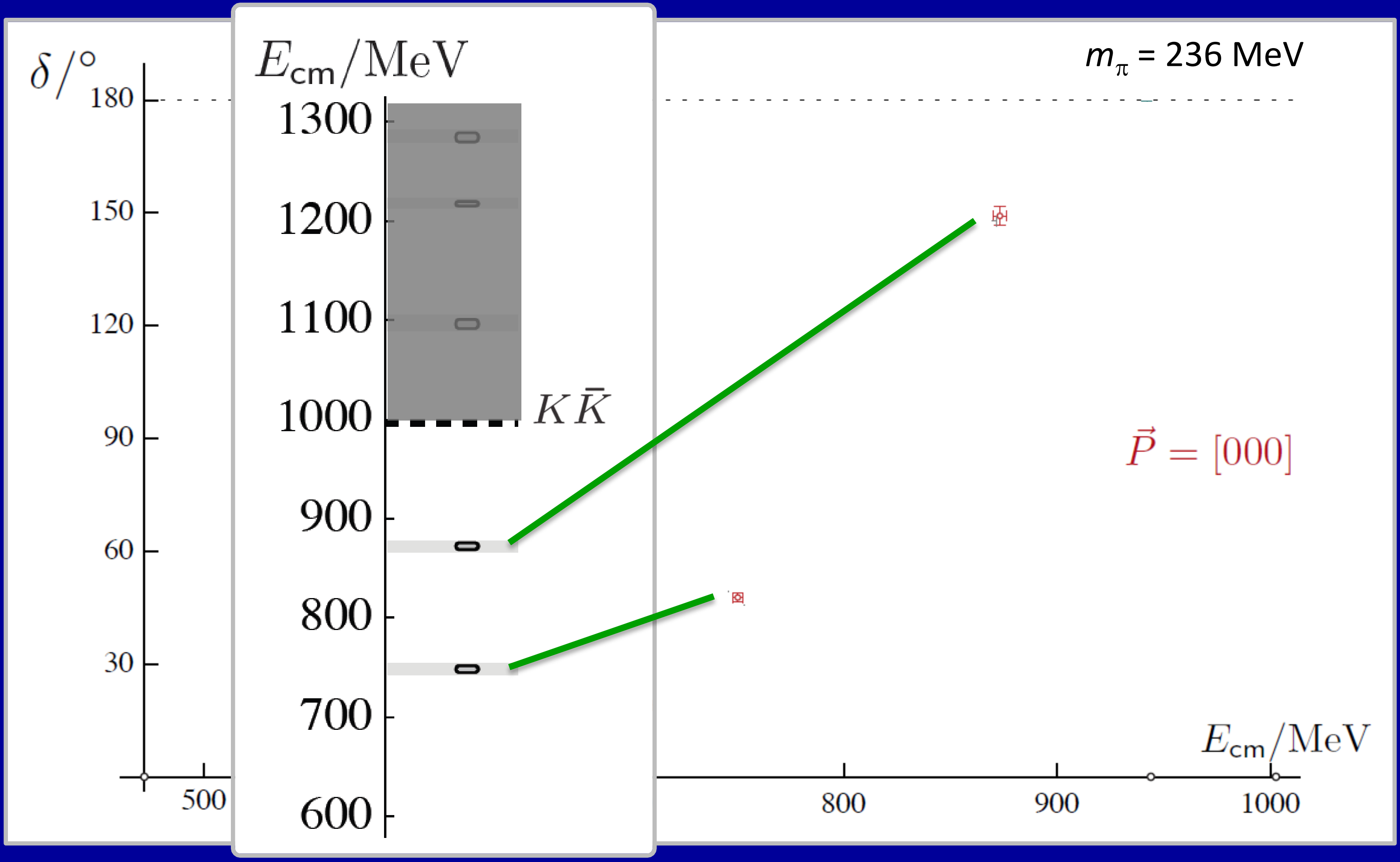
Finite volume spectrum from:

$$C_{ij}(t) = < 0|\mathcal{O}_i(t)\mathcal{O}_j^\dagger(0)|0 >$$

Use many different operators

Wilson *et al* (HadSpec) [PR D92, 094502 (2015)] and Dudek, Edwards, CT (HadSpec) [PR D87, 034505 (2013)]

The ρ resonance: **elastic** $\pi\pi$ scattering



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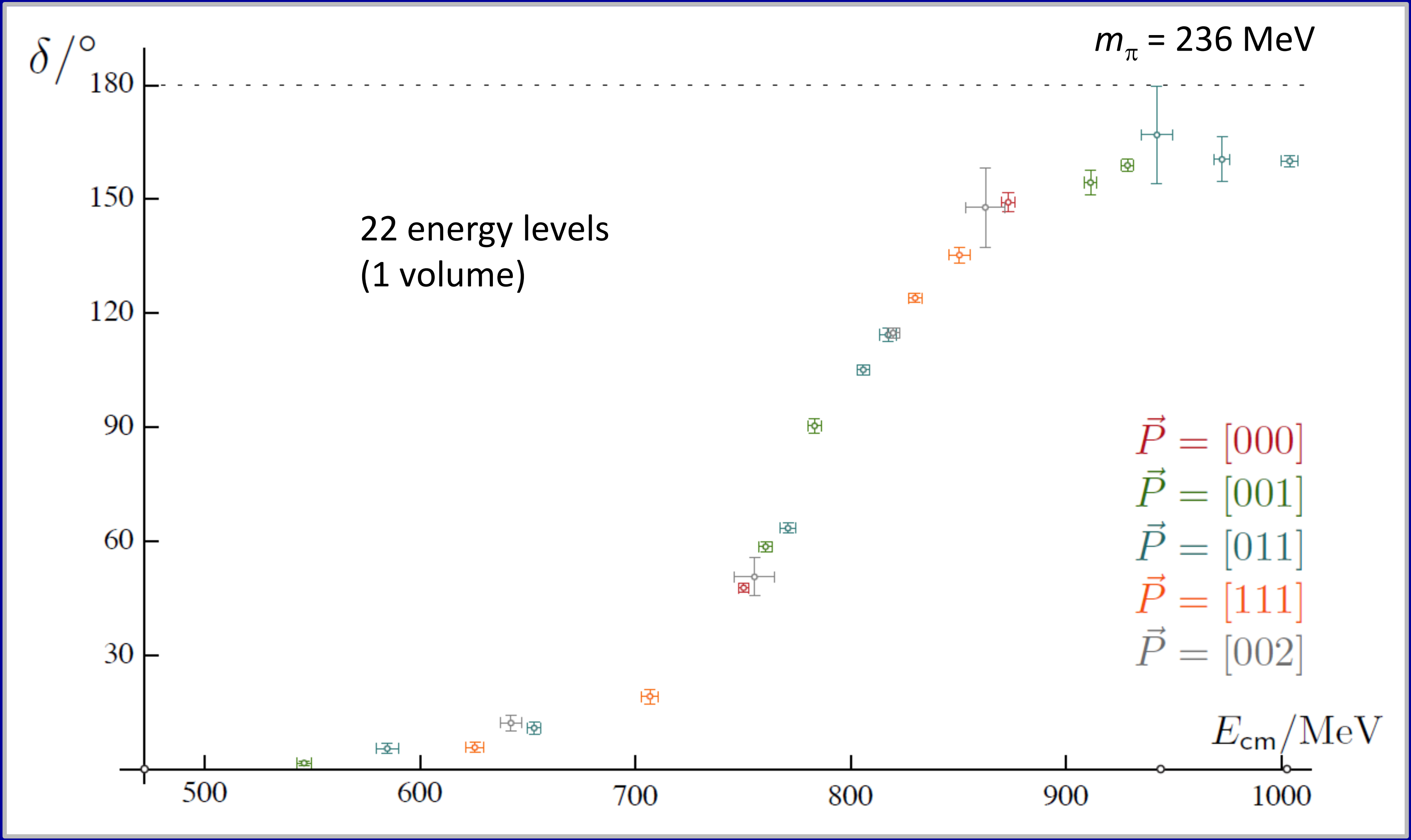

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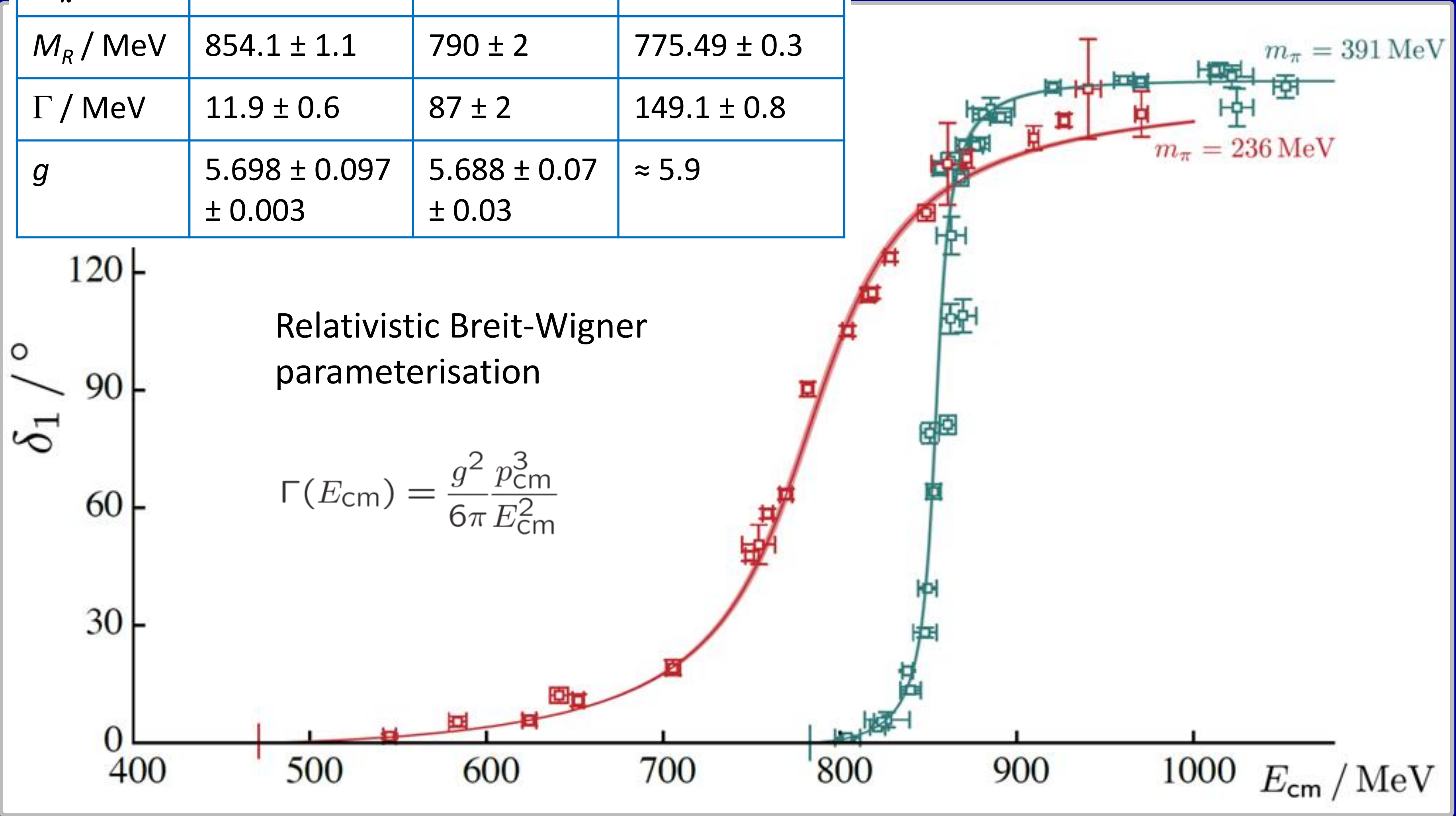
The ρ resonance: **elastic ππ scattering**

m_π / MeV	391	236	Experimental
M_R / MeV	854.1 ± 1.1	790 ± 2	775.49 ± 0.3
Γ / MeV	11.9 ± 0.6	87 ± 2	149.1 ± 0.8
g	5.698 ± 0.097 ± 0.003	5.688 ± 0.07 ± 0.03	≈ 5.9



Relativistic Breit-Wigner parameterisation

$$\Gamma(E_{cm}) = \frac{g^2}{6\pi}\frac{p_{cm}^3}{E_{cm}^2}$$

(HadSpec) [PR D87, 034505 (2013); PR D92, 094502 (2015)]

The ρ resonance: **elastic** $\pi\pi$ scattering: other calcs.

Some other recent lattice QCD calculations:

- Bali *et al* (RQCD) [PR D93, 054509 (2016)]: $m_\pi \approx 150$ MeV ($N_f = 2$)
- Bulava *et al* [NP B910, 842 (2016)]: $m_\pi \approx 240$ MeV
- Guo *et al* [PR D94, 034501 (2016)]: $m_\pi = 315, 226$ MeV ($N_f = 2$)

Scattering in Lattice QCD

Lüscher method [NP B354, 531 (1991)] extended by many others:
relate **finite-volume energy levels** $\{E_{cm}\}$ to **infinite-volume scattering *t*-matrix**

Coupled-channel scattering:

E.g. $$\mathrm{t}(E_{\mathrm{cm}}) = \begin{pmatrix} t_{\pi\pi\to\pi\pi}(E_{\mathrm{cm}}) & t_{\pi\pi\to K\bar{K}}(E_{\mathrm{cm}}) \\ t_{K\bar{K}\to\pi\pi}(E_{\mathrm{cm}}) & t_{K\bar{K}\to K\bar{K}}(E_{\mathrm{cm}}) \end{pmatrix}$$

Determinant equation for $\mathbf{t}(E_{cm})$ at each E_{cm}

- Given $\mathbf{t}(E_{cm})$: solns. of equ. → finite-vol. spec. $\{E_{cm}\}$
  But we need: spectrum → $\mathbf{t}(E_{cm})$
- **Under-constrained problem** (e.g. 2 channels: 3 unknowns but 1 equ.)

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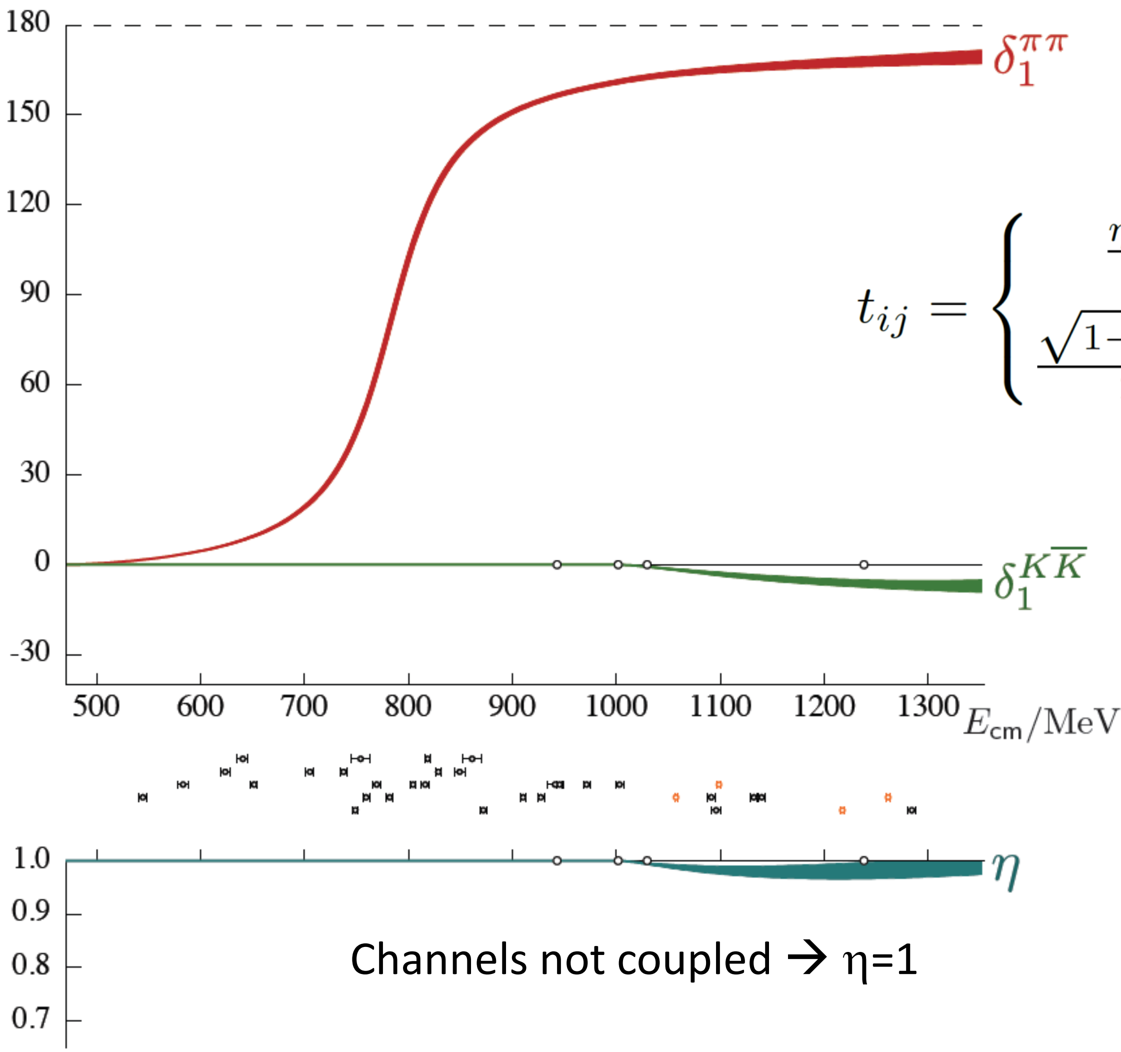
  But we need: spectrum → $\mathbf{t}(E_{cm})$
- **Under-constrained problem** (e.g. 2 channels: 3 unknowns but 1 equ.)
  → Parameterize E_{cm} dependence of *t*-matrix and fit $\{E_{lattice}\}$ to $\{E_{param}\}$

Try different parameterizations, e.g. various *K*-matrix forms
(for elastic scattering also Breit Wigner, effective range expansion).

The ρ resonance: **coupled-channel** $\pi\pi, K\bar{K}$



m_π = 236 MeV

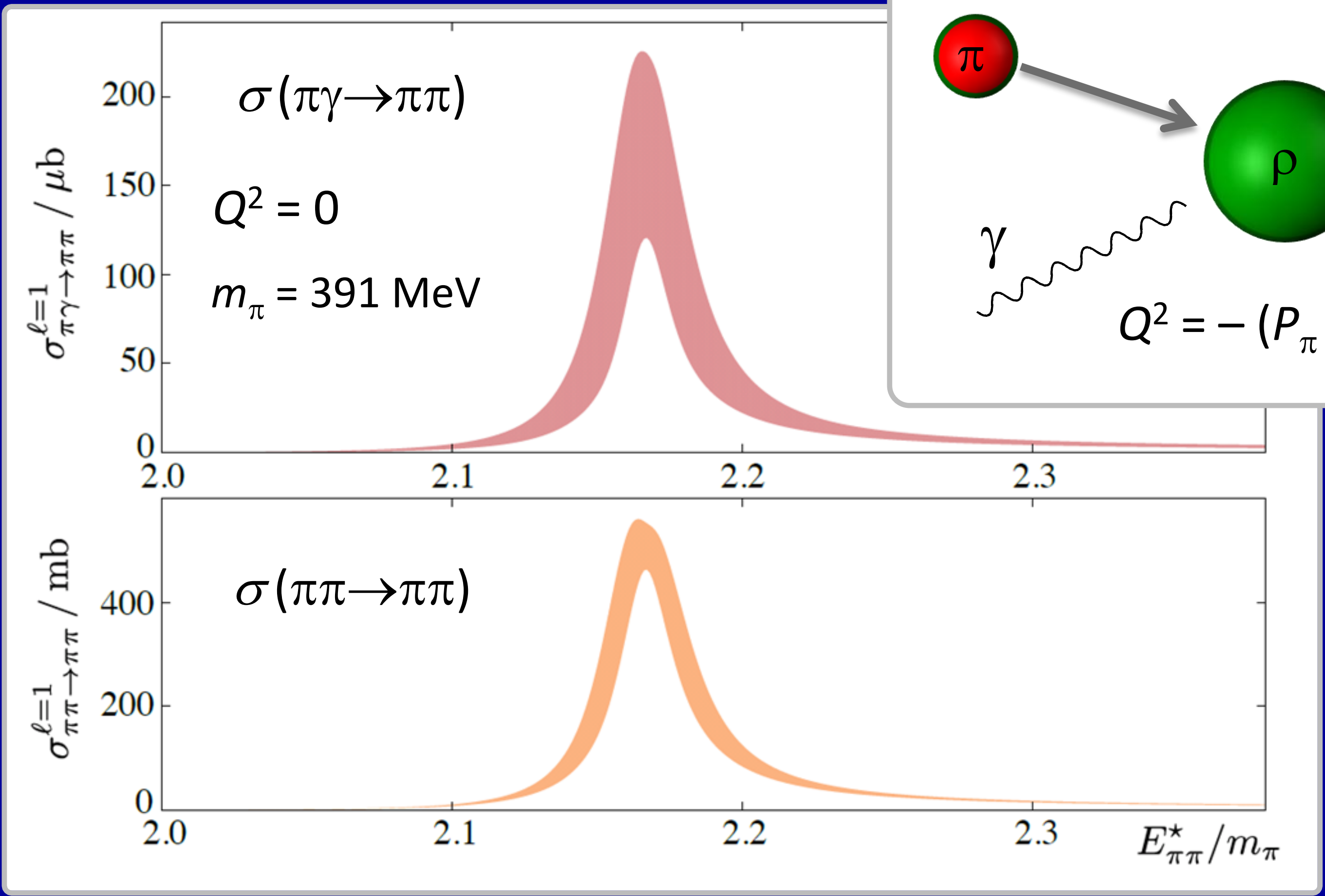
$$t_{ij} = \begin{cases} \dfrac{\eta\, e^{2i\delta_i} - 1}{2i\,\rho_i} & (i = j) \\ \dfrac{\sqrt{1-\eta^2}\, e^{i(\delta_i+\delta_j)}}{2\sqrt{\rho_i\,\rho_j}} & (i \neq j) \end{cases}$$

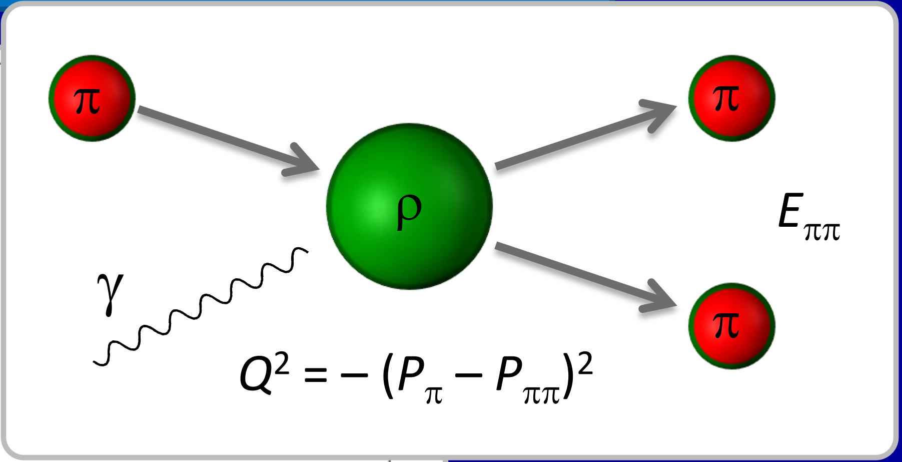
34 energy levels
(1 volume)

(HadSpec) [PR D92, 094502 (2015)]

Resonant $\pi^+ \gamma \to \rho \to \pi^+ \pi^0$ amplitude

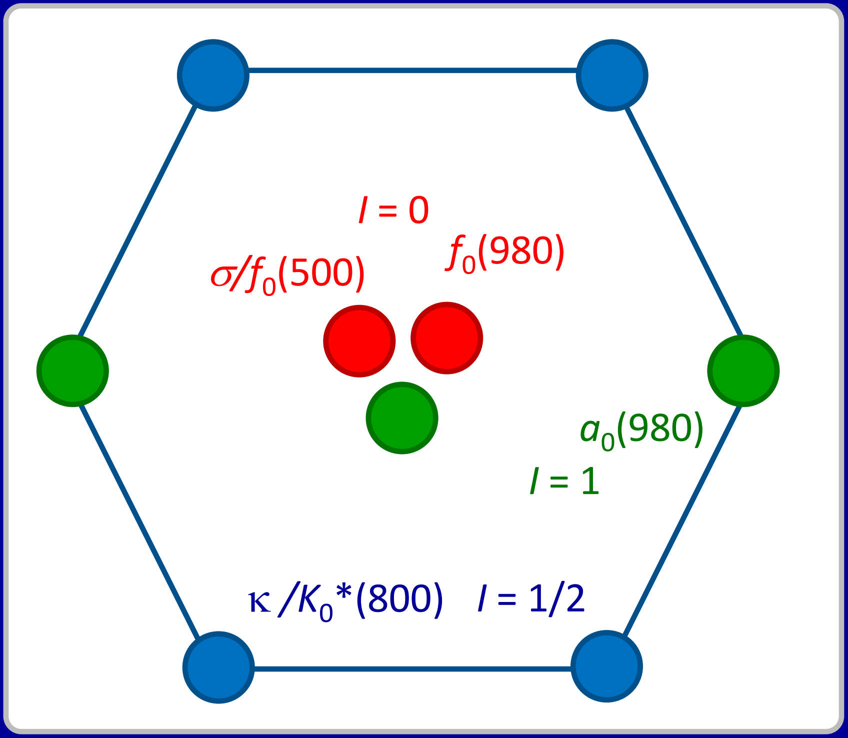
Need: $C_{ij}(t_f, t, t_i) =< 0|O_i(t_f)\ \bar{\psi}(t)\gamma^\mu\psi(t)\ O_j(t_i)|0 >$





Briceño *et al* (HadSpec) [PRL 115, 242001 (2015); PRD 93, 114508 (2016)]

Light scalar mesons (< 1 GeV)



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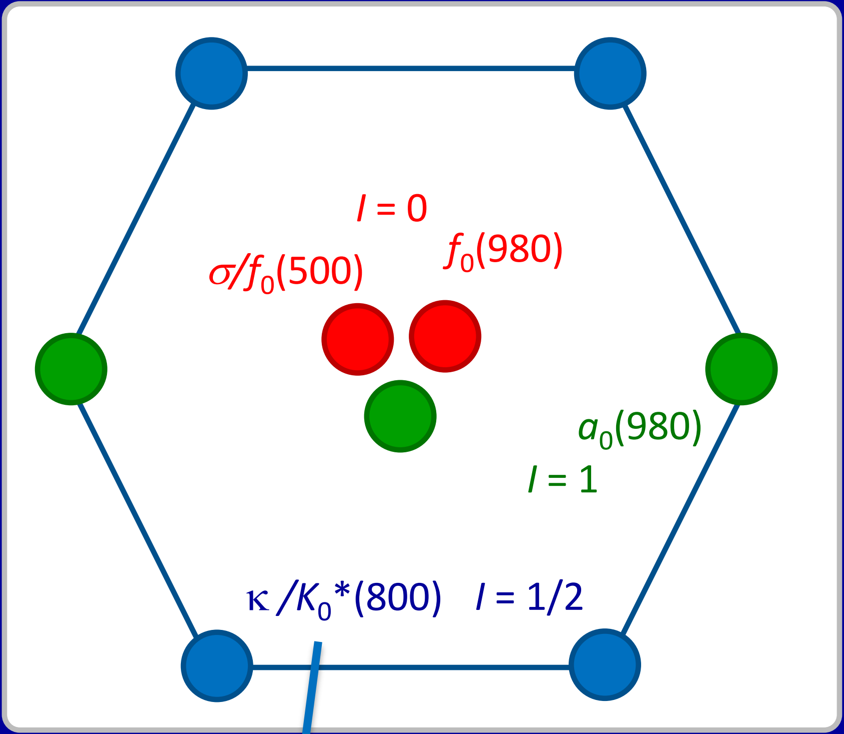

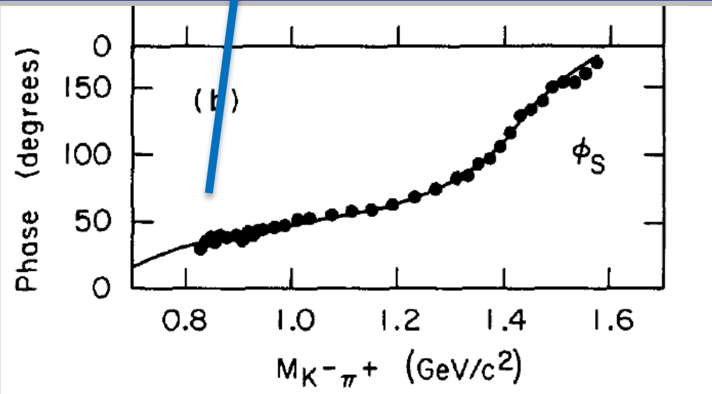

Aston *et al* (LASS) [NP B296, 493 (1988)]

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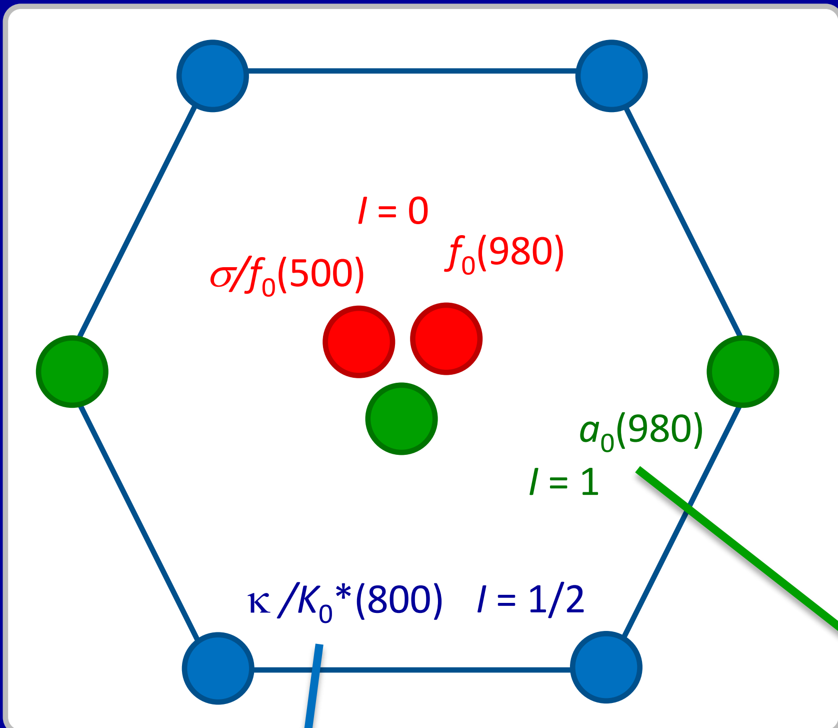

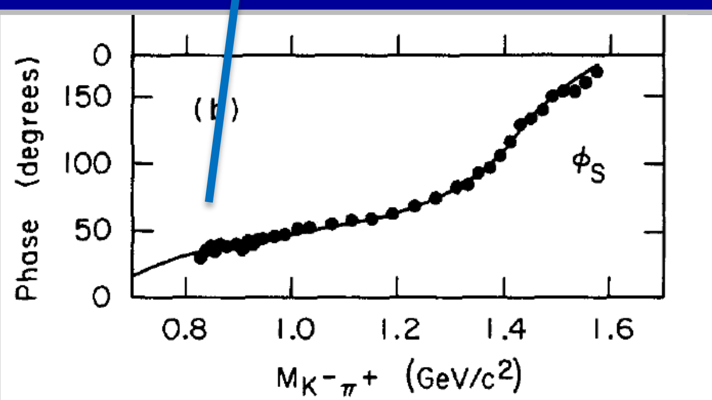

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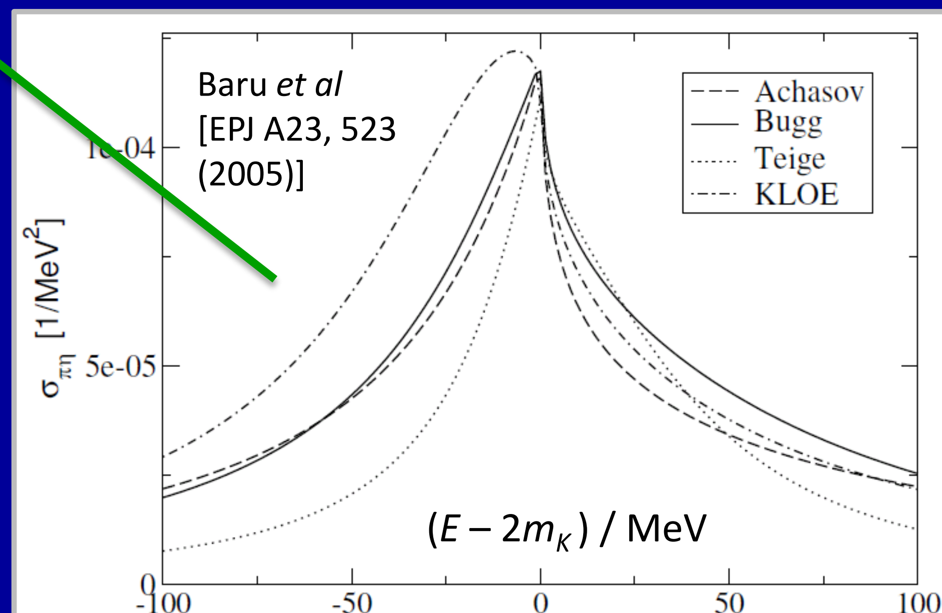

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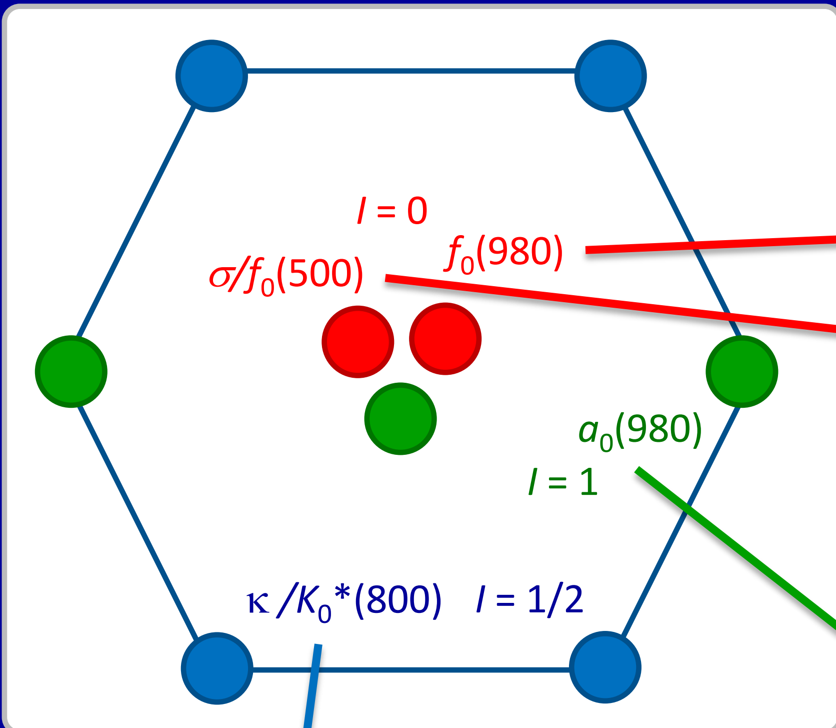

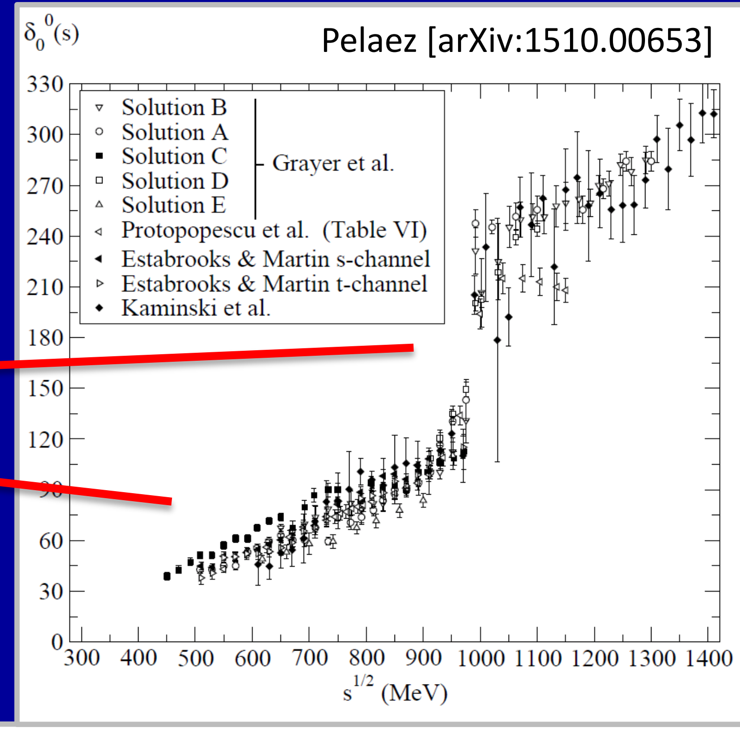

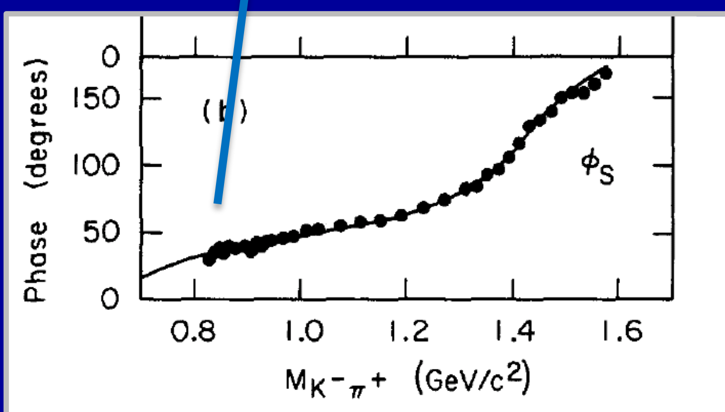

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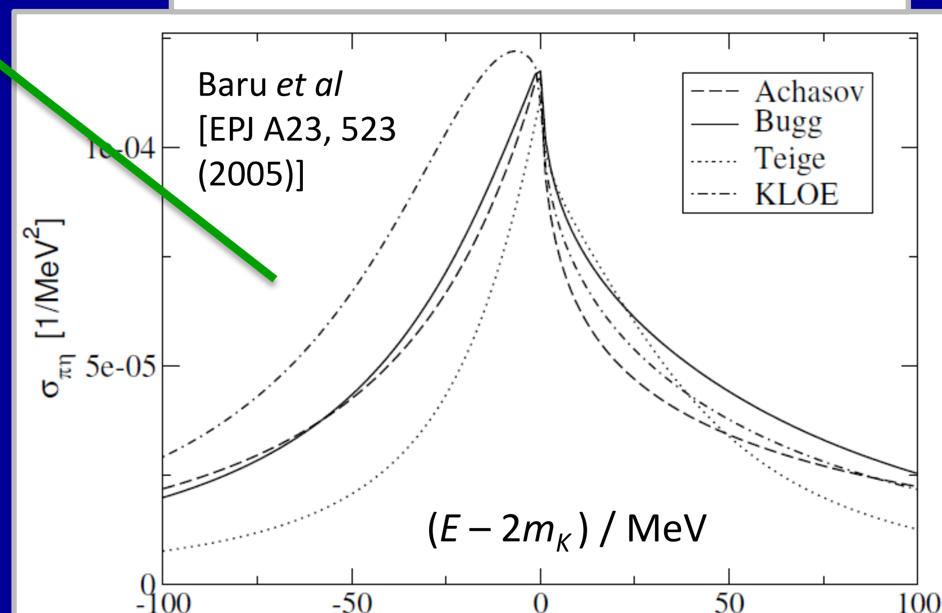
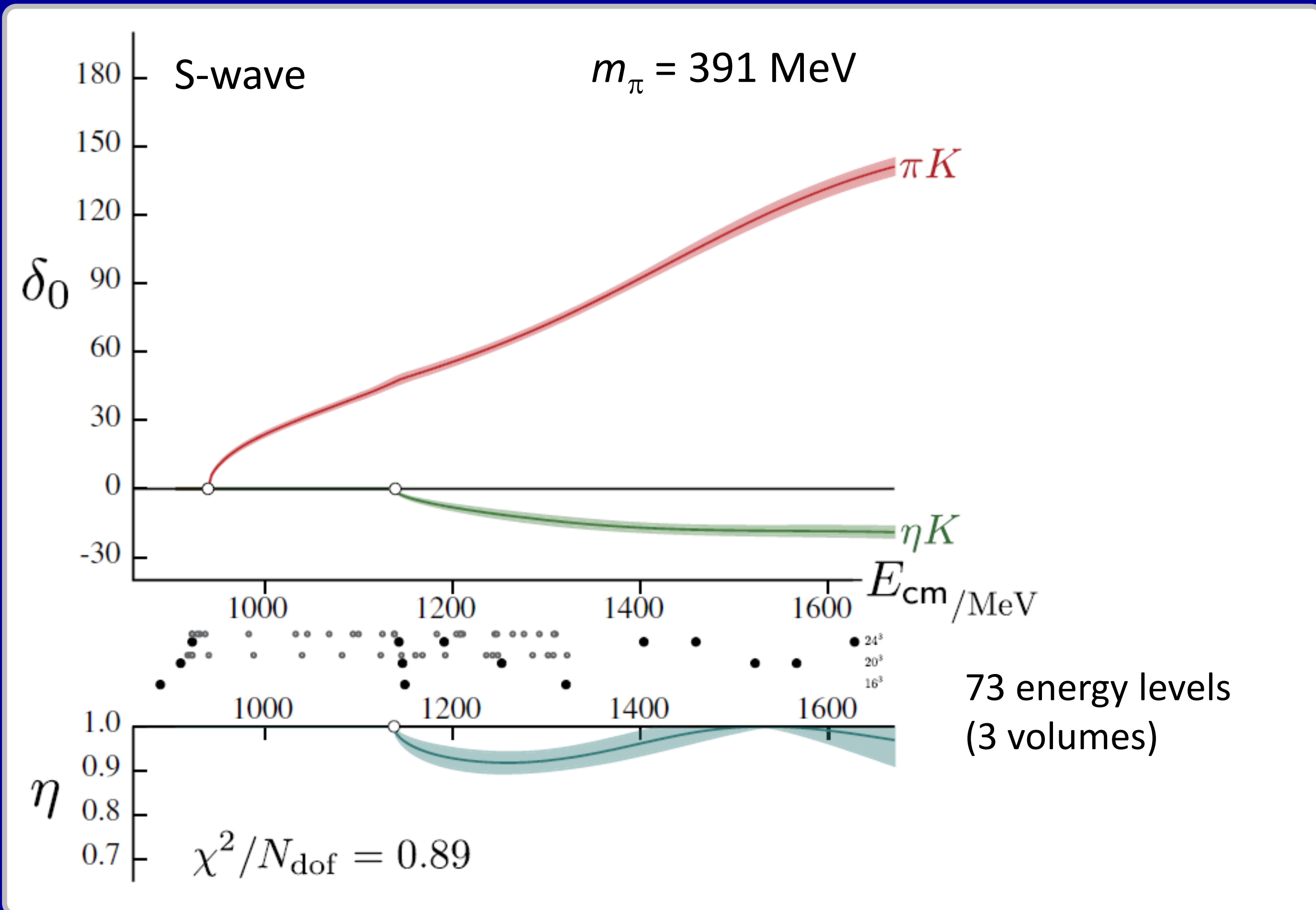


κ in πK, ηK

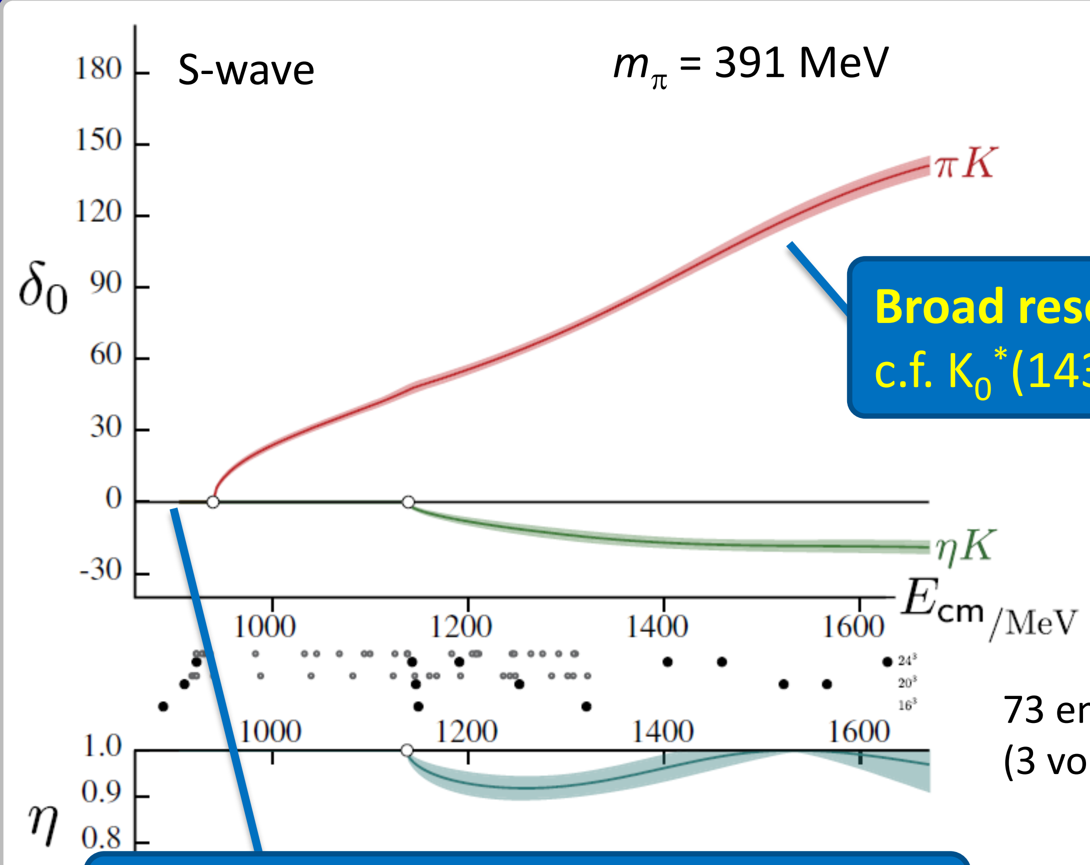
$J^P = 0^+$, Isospin = ½, Strangeness = 1



Wilson, Dudek, Edwards, CT (HadSpec) [PRL 113, 182001 2014); PR D91, 054008 (2015)]

κ in πK, ηK

$J^P = 0^+$, Isospin = ½, Strangeness = 1



Broad resonance
c.f. $K_0^*(1430)$

73 energy levels
(3 volumes)

Virtual bound state [pole on real axis below threshold on unphysical sheet]

c.f. κ in unitarised χpt [Nebreda & Pelaez, PR D81, 054035 (2010)]

Wilson, Dudek, Edwards, CT (HadSpec) [PRL 113, 182001 2014); PR D91, 054008 (2015)]

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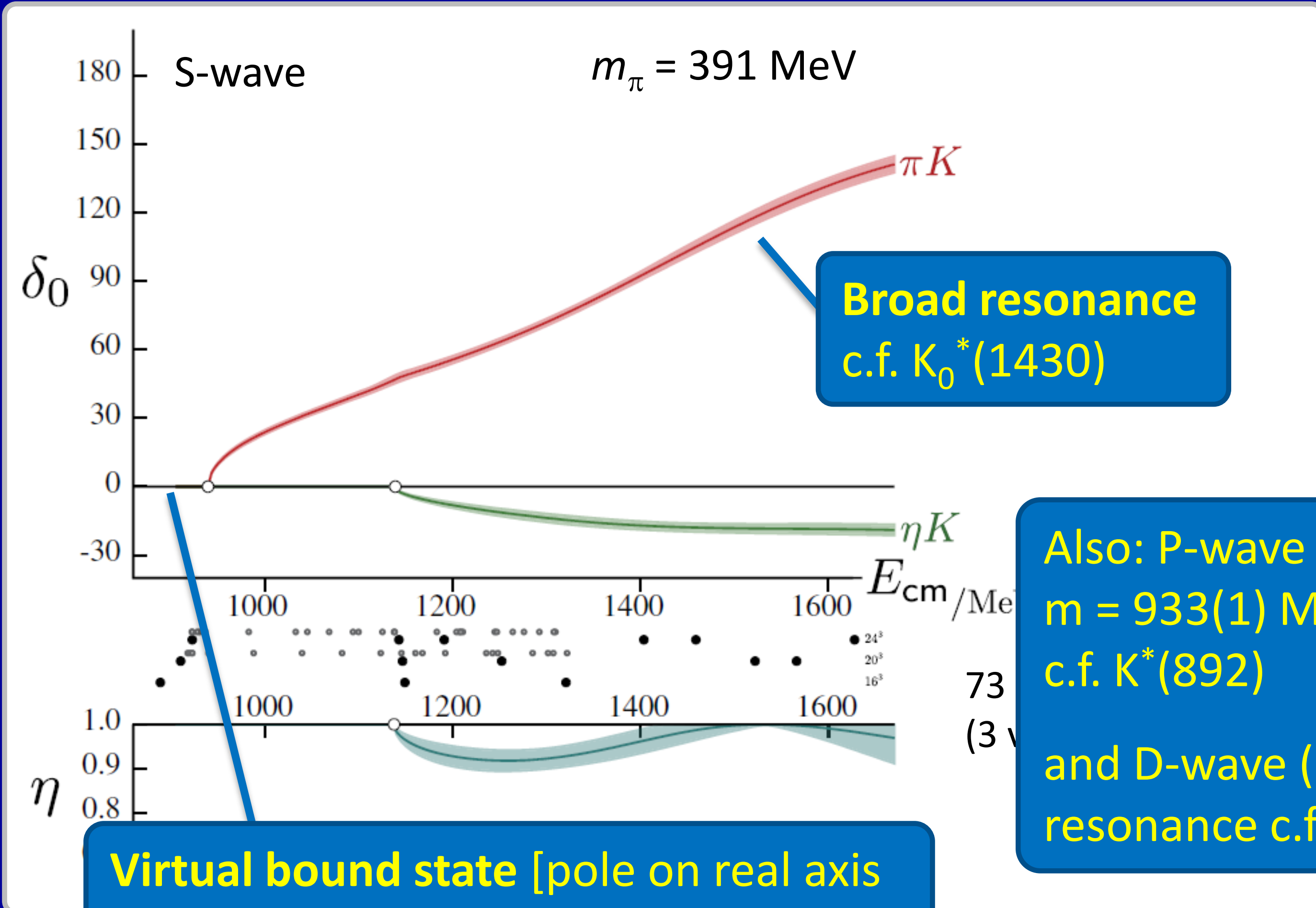


Broad resonance
c.f. $K_0^*(1430)$

Also: P-wave (1^-) bound state, m = 933(1) MeV, g = 5.93(26)
c.f. $K^*(892)$

and D-wave (2^+) narrow resonance c.f. $K_2^*(1430)$

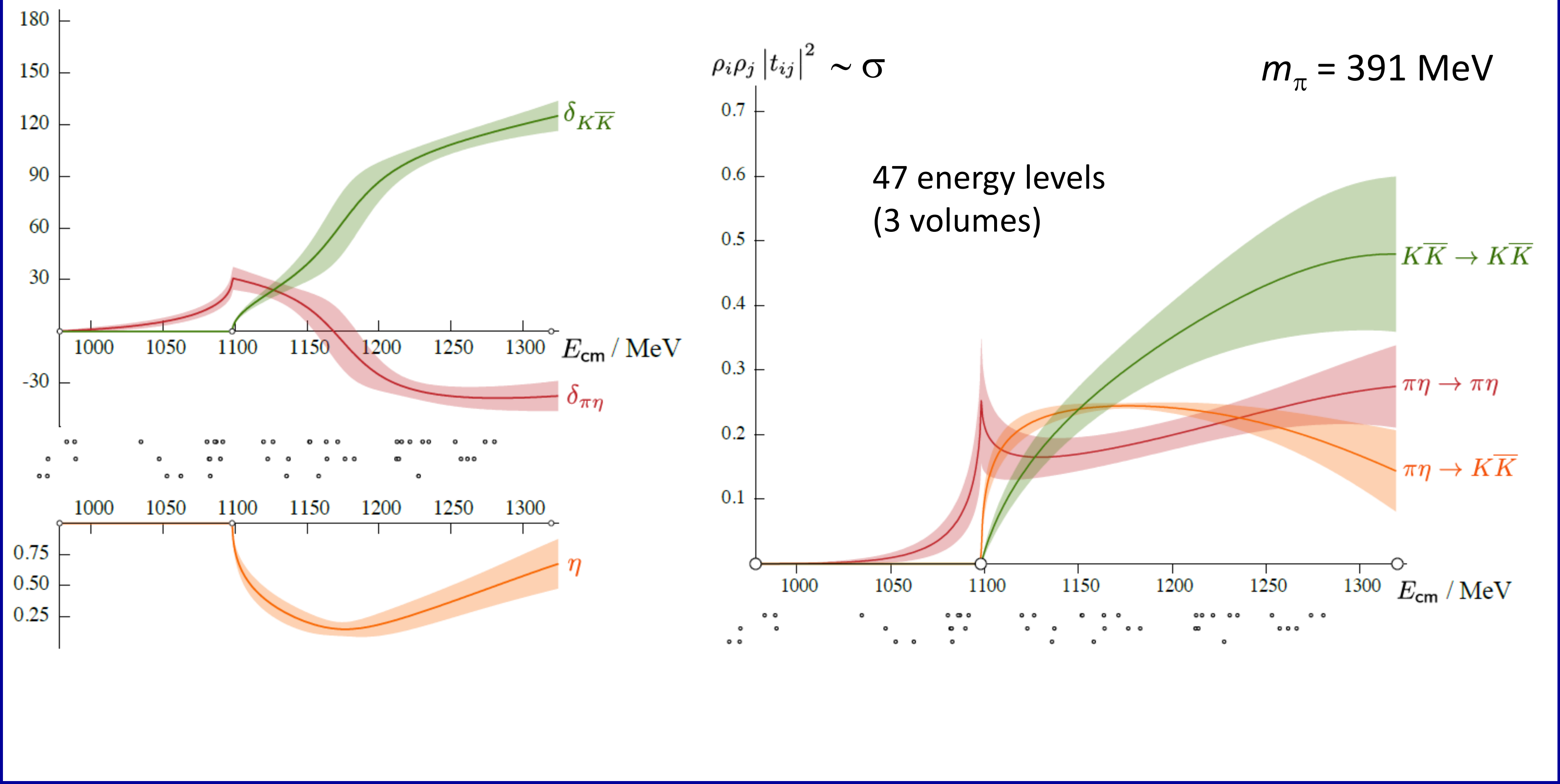
Virtual bound state [pole on real axis below threshold on unphysical sheet]

c.f. κ in unitarised χpt [Nebreda & Pelaez, PR D81, 054035 (2010)]

Wilson, Dudek, Edwards, CT (HadSpec) [PRL 113, 182001 2014); PR D91, 054008 (2015)]

a_0 resonance in $\pi\eta$, $K\bar{K}$

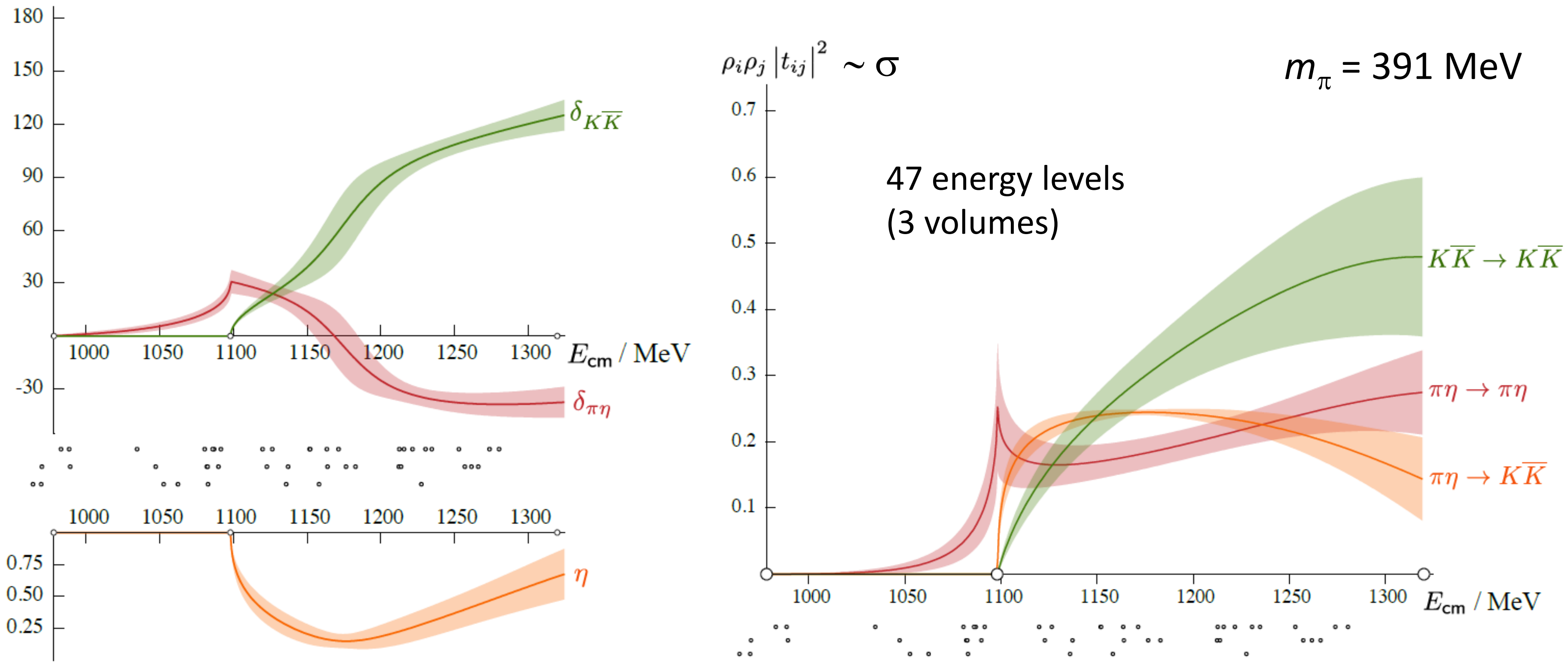
$J^P = 0^+$, I = 1



Dudek, Edwards, Wilson (HadSpec)
[PR D93, 094506 (2016)]

a_0 resonance in $\pi\eta,\ K\bar{K}$

$J^P = 0^+, I = 1$



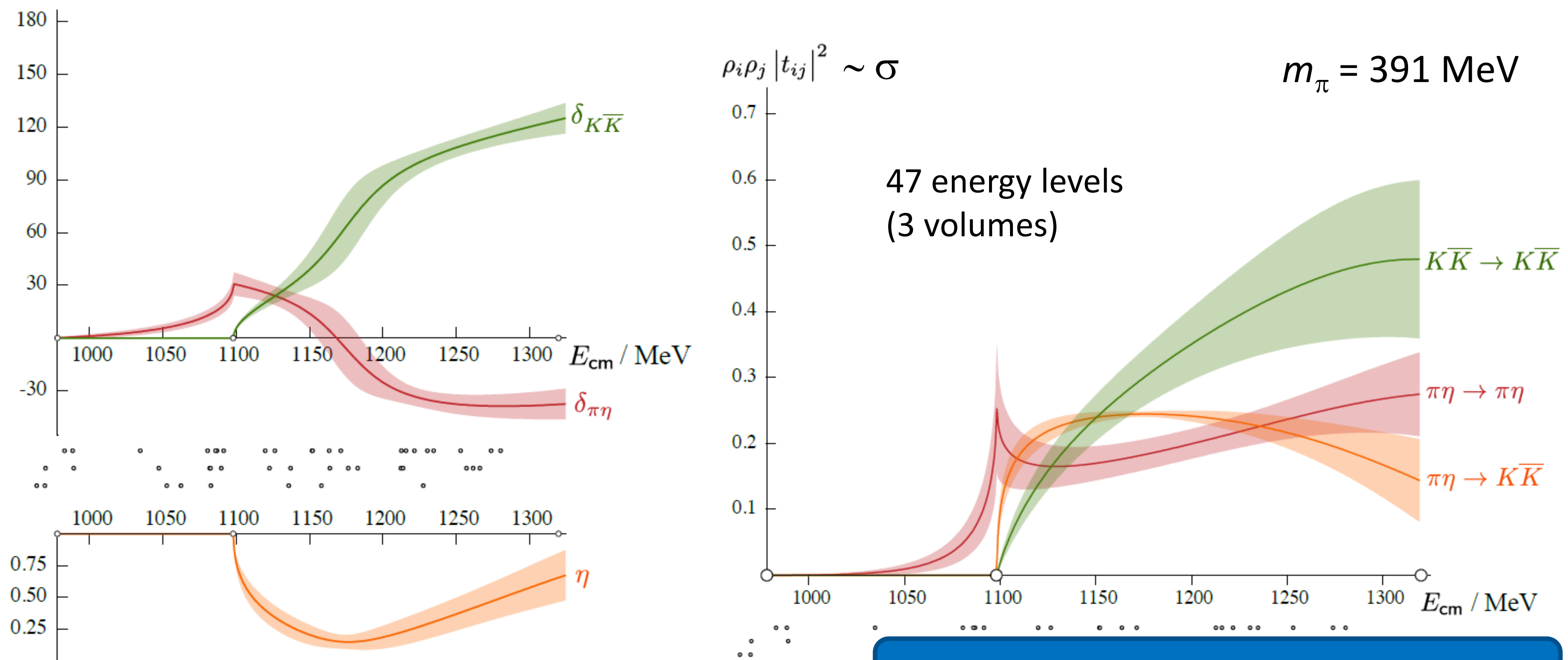
Pole (sheet IV) at: $\sqrt{s_0} = \left((1177 \pm 27) + \frac{i}{2}(49 \pm 33)\right)$ MeV, $\left|c_{K\bar{K}}/c_{\pi\eta}\right| = 1.30(37)$

Resonance strongly coupled to both $\pi\eta$ and $K\bar{K}$

Dudek, Edwards, Wilson (HadSpec) [PR D93, 094506 (2016)]

a_0 resonance in $\pi\eta,\ K\bar{K}$

$J^P = 0^+, I = 1$



Pole (sheet IV) at: $\sqrt{s_0} = \left((1177 \pm 27) + \frac{i}{2}(49 \pm 33) \right)$

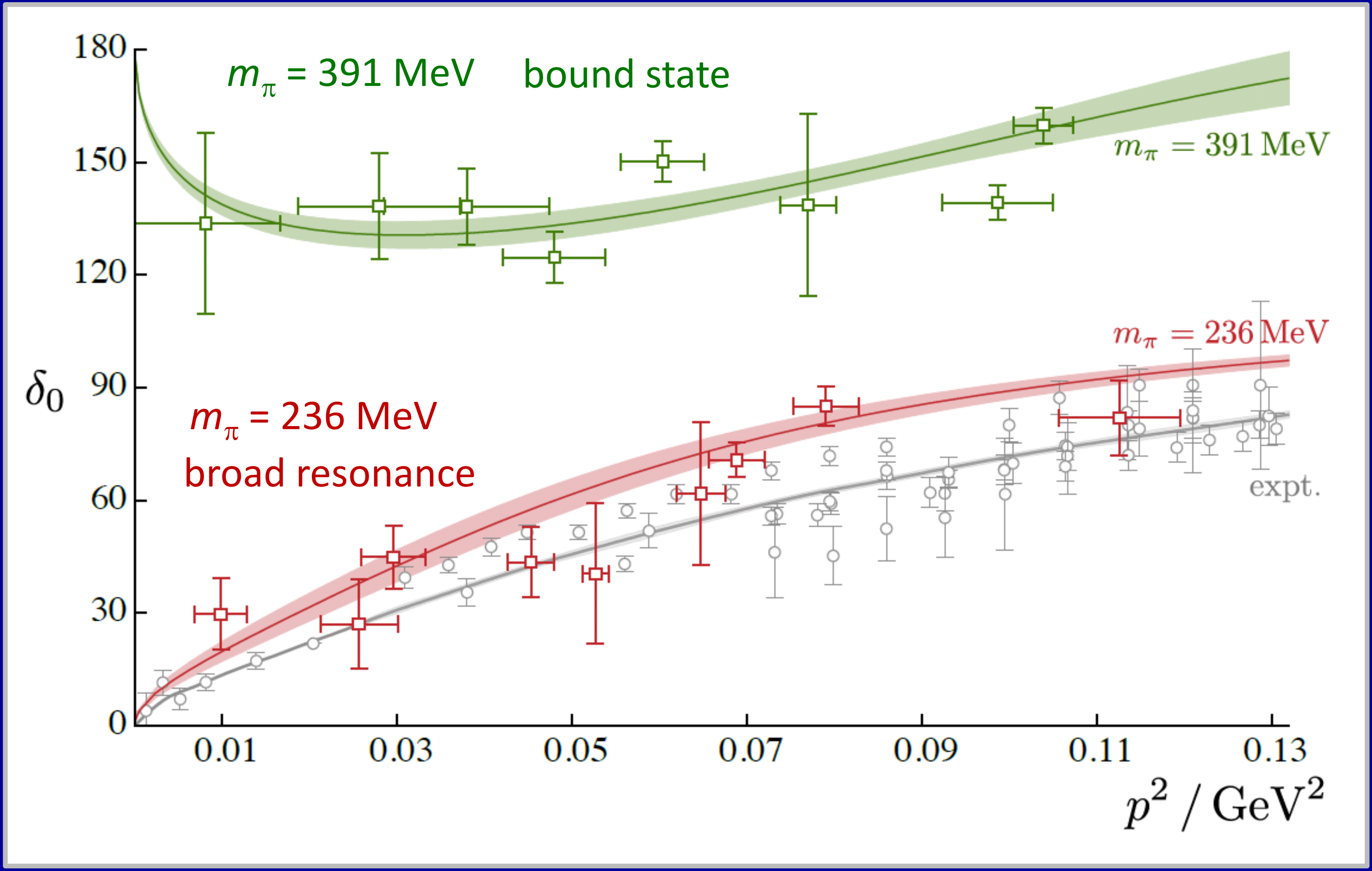
Also: including πη′ in S-wave, and a D-wave (2^+) resonance c.f. a_2

Resonance strongly coupled to both $\pi\eta$ and $K\bar{K}$

Dudek, Edwards, Wilson (HadSpec) [PR D93, 094506 (2016)]

$f_0(500)/\sigma$ in $\pi\pi$ scattering

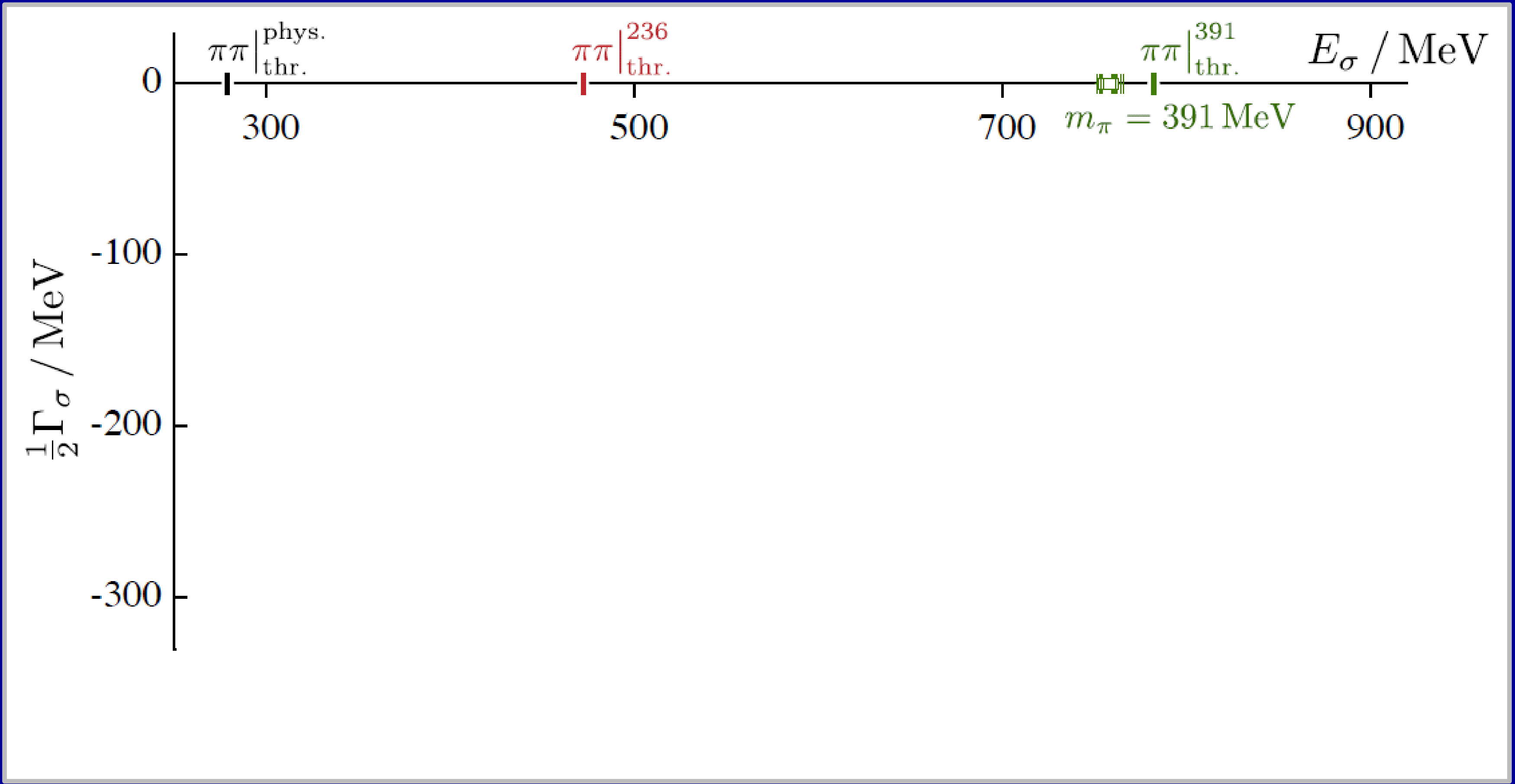
$J^P = 0^+, I = 0$



Briceño, Dudek, Edwards, Wilson (HadSpec) [arXiv:1607.05900]

$f_0(500)/\sigma$ in $\pi\pi$ scattering

$J^P = 0^+, I = 0$



Briceño, Dudek, Edwards, Wilson (HadSpec) [arXiv:1607.05900]

$f_0(500)/\sigma$ in $\pi\pi$ scattering

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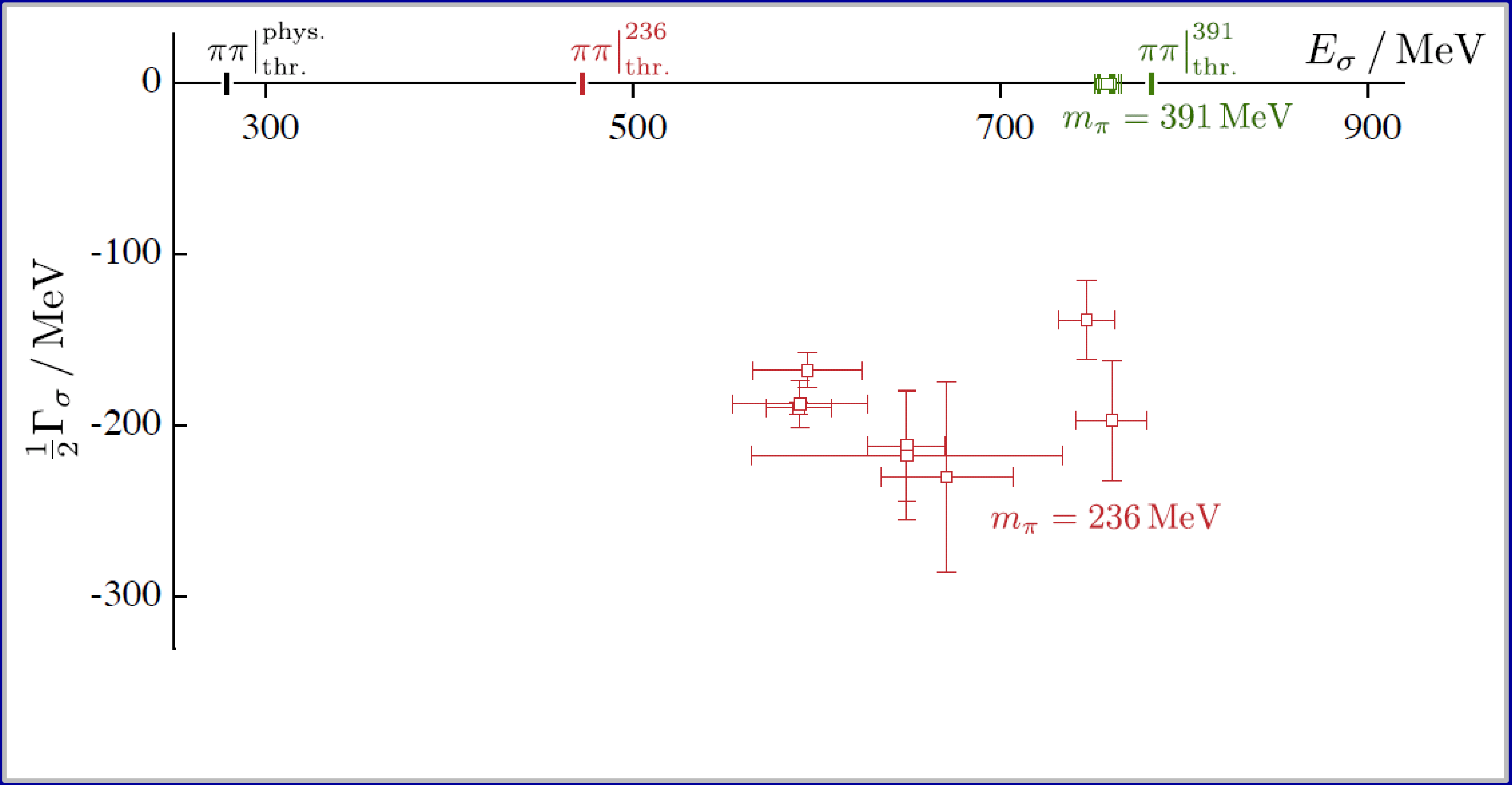

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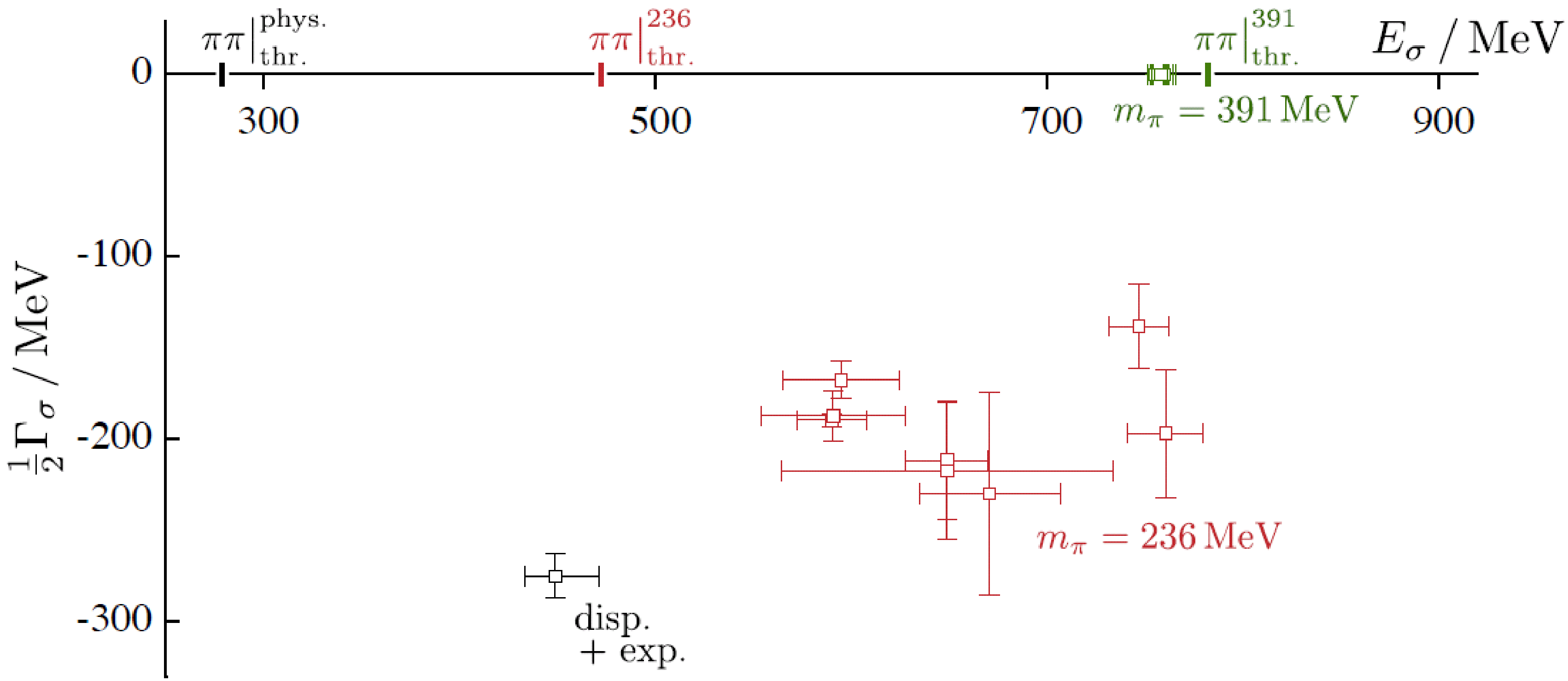

analysis of exp. data, Pelaez [arXiv:1510.00653]

Briceño, Dudek, Edwards, Wilson (HadSpec) [arXiv:1607.05900]

$f_0(500)/\sigma$ in $\pi\pi$ scattering

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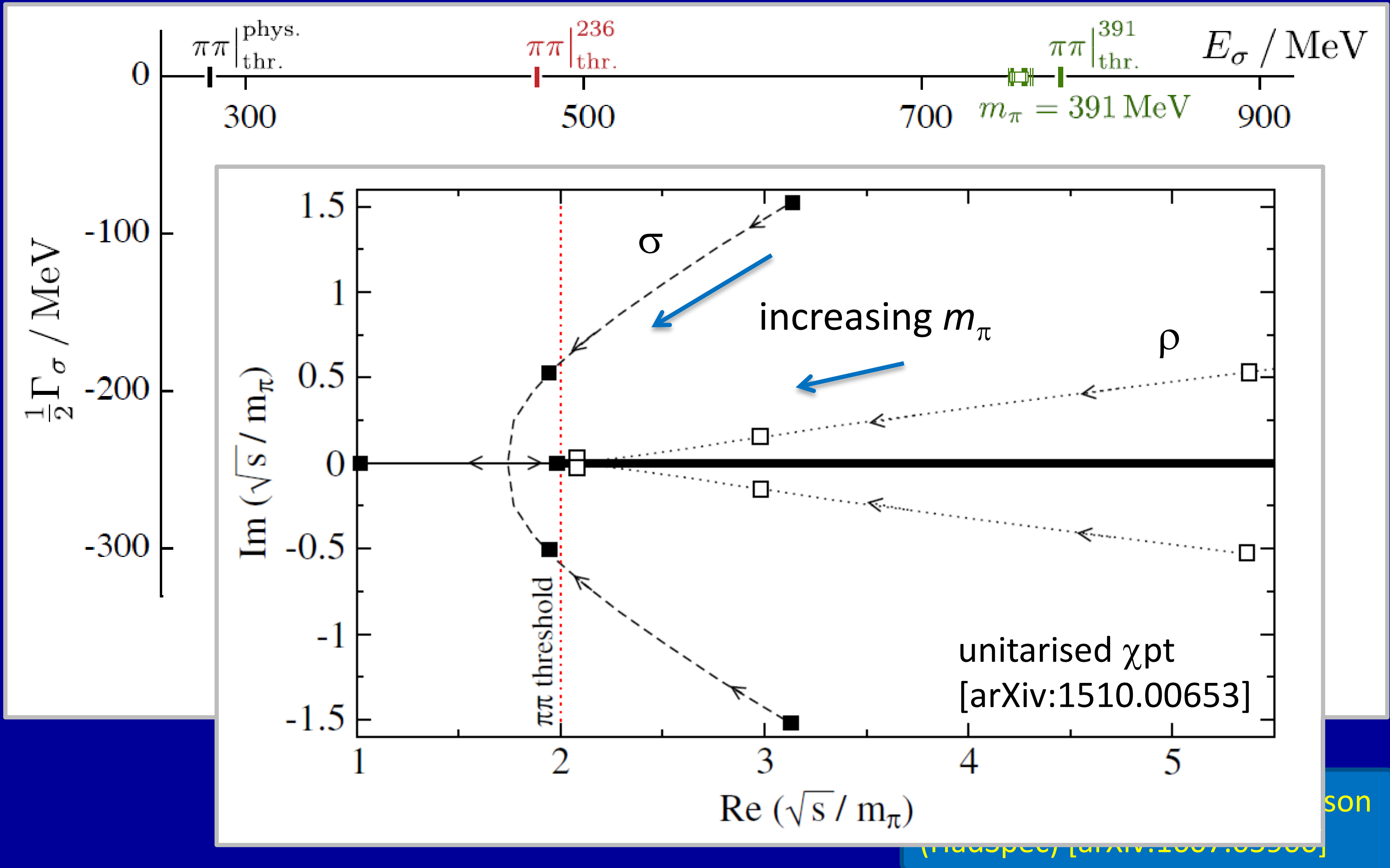
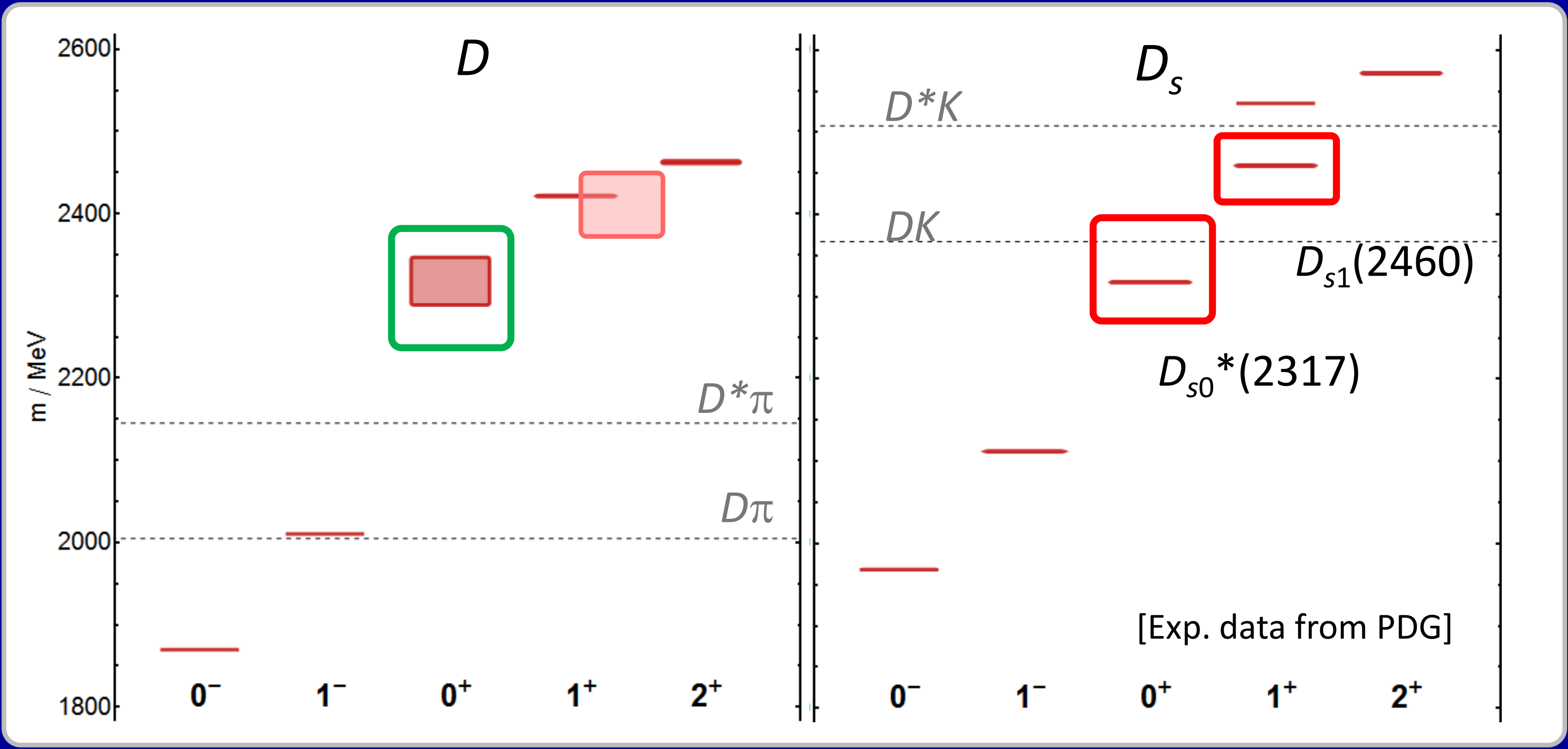


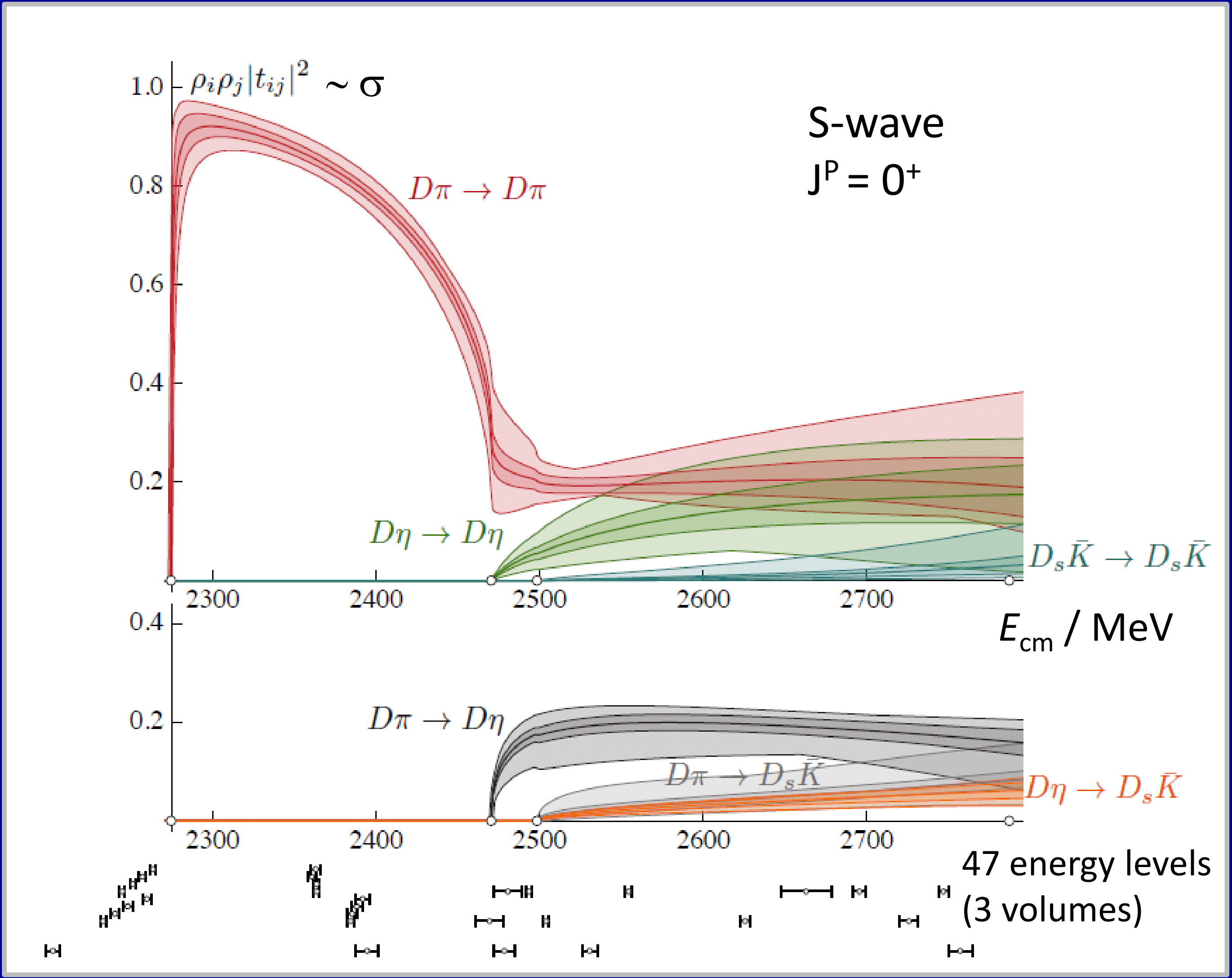
Charm-light (D) and charm-strange (D_s) mesons



Some earlier LQCD studies:

- Mohler *et al* [PR D87, 034501 (2012)] – 0^+ $D\,\pi$ and 1^+ $D^*\,\pi$ resonances
- Mohler *et al* [PRL 111, 222001 (2013)] – 0^+ $D_s(2317)$ below $D\,K$ threshold
- Lang *et al* [PRD 90, 034510 (2014)] – 0^+ $D_s(2317)$ and 1^+ $D_{s1}(2460)$, $D_{s1}(2536)$

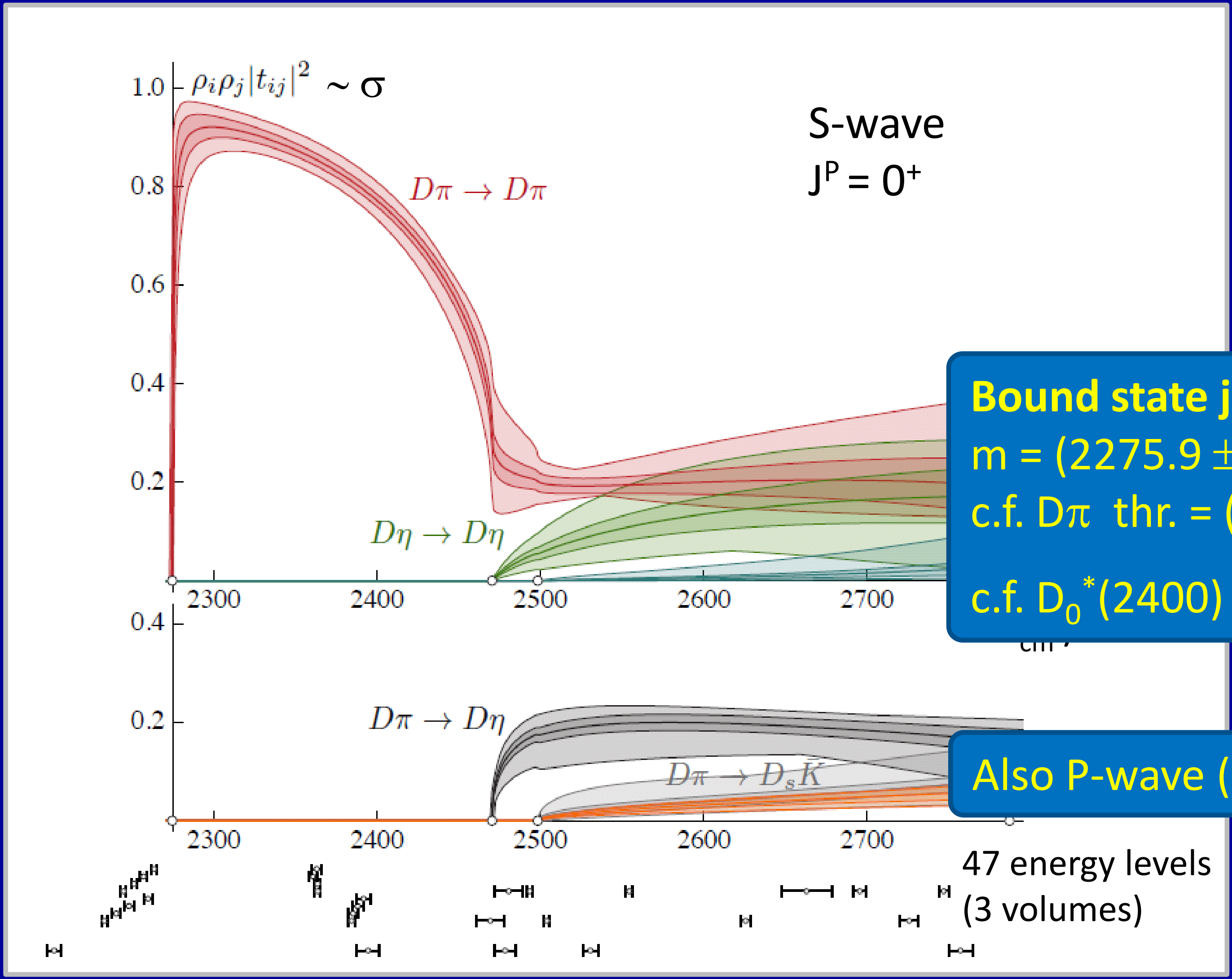
Dπ, Dη, $D_s\bar{K}$ (I=½)



m_π = 391 MeV

Moir, Peardon, Ryan, CT, Wilson (HadSpec) [JHEP 1610, 011 (2016)]

$D\pi$, $D\eta$, $D_s\bar{K}$ (I=½)



m_π = 391 MeV

Bound state just below thresh.
m = (2275.9 ± 0.9) MeV
c.f. $D\pi$ thr. = (2276.4 ± 0.9) MeV

c.f. $D_0^*(2400)$

Also P-wave (1^-) and D-wave (2^+)

Moir, Peardon, Ryan, CT, Wilson (HadSpec) [JHEP 1610, 011 (2016)]

Some other recent work on charmonium(-like) mesons:

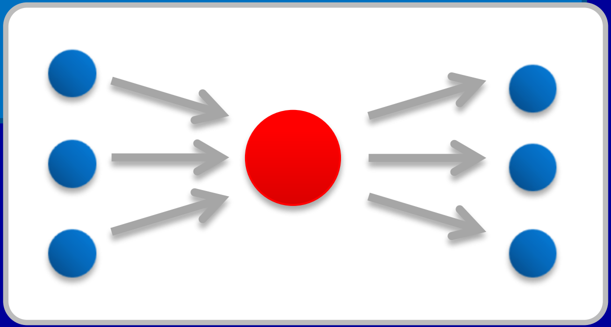
- Ozaki, Sasaki [PR D87, 014506 (2013)] – no sign of *Y*(4140) in J/ψ φ
- Prelovsek & Leskovec [PRL 111, 192001 (2013)] – 1^{++} I=0 near $D\bar{D}^*$ – *X*(3872)?
- Prelovsek *et al* [PL B727, 172; PR D91, 014504 (2015)] – no sign of Z^+(3900) in 1^{+-}
- Chen *et al* (CLQCD) [PR D89, 094506 (2014)] – 1^{++} I=1 $D\bar{D}^*$ weakly repulsive
- Padmanath *et al* [PR D92, 034501 (2015)] – 1^{++} I=0 [X(3872)?]; no I=1 or *Y*(4140)
- Lang *et al* [JHEP 1509, 089 (2015)] – I=0 $D\bar{D}$: 1^{--} ψ(3770) and 0^{++}
- Chen *et al* (CLQCD) [PR D92, 054507 (2015)] – 1^{+-} I=1 $D^*\bar{D}^*$ weakly repulsive?
- Chen *et al* (CLQCD) [PR D93, 114501 (2016)] – 0^{--}, 1^{+-} I=1$D^*\bar{D}_1$ some attraction?
- Ikeda *et al* (HAL QCD) [PRL 117, 242001 (2016)] – π J/ψ, ρ η_c,$D\bar{D}^*$ using HAL QCD method – suggest Z^+(3900) is a threshold cusp
- Albaladejo *et al* [EPJ C76, 573 (2016)] – different scenarios for PR D91, 014504

Bottom mesons:

- Lang *et al* [PL B750, 17 (2015)] – *BK* (0^+) and *B*K* (1^+) I=0 bound states
- Lang *et al* [PR D94, 074509 (2016)] – $B_s\pi$, *BK* (I=1) $J^P = 0^+$ no sign of *X*(5568)

Summary

- Significant progress in using lattice QCD to study resonances etc. Coupled-channel scattering for the first time.
- Extract many energy levels → map out scattering amps.
- Some examples of recent work:
  - ρ resonance (many calculations),
  - Light scalars (κ, a_0(980), σ), charm-light mesons
- Also transitions, e.g. ρ resonance ($\pi\pi$) → $\pi\,\gamma$
- Ongoing work on formalism (e.g. 3-hadron scattering)
- Connections with analysis of experimental data
- Use m_π dependence as a tool



Extra slides

Hadron Spectrum Collaboration

Jefferson Lab, USA:

Jozef Dudek, Robert Edwards, David Richards, **Raul Briceño**

Trinity College Dublin:

Mike Peardon, Sinéad Ryan, **David Wilson**, *Cian O'Hara, David Tims*

University of Cambridge:

CT, **Graham Moir**, *Gavin Cheung, Antoni Woss*

Tata Institute:

Nilmani Mathur

Spectroscopy on the lattice

Energy eigenstates from:

$$C_{ij}(t) = <0|\mathcal{O}_i(t)\mathcal{O}_j^\dagger(0)|0>$$

Interpolating operators

$$\sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}}\ \bar{\psi}(x)\left[\Gamma \overleftrightarrow{D}\overleftrightarrow{D}\ldots\right]\psi(x)$$

$$C_{ij}(t) = \sum_n \frac{e^{-E_n t}}{2E_n} <0|\mathcal{O}_i(0)|n><n|\mathcal{O}_j^\dagger(0)|0>$$

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Large basis of ops → matrix of corrs. – generalised eigenvalue problem

$$C_{ij}(t)v_j^{(n)} = \lambda^{(n)}(t)C_{ij}(t_0)v_j^{(n)}$$

$$\lambda^{(n)}(t) \rightarrow e^{-E_n(t-t_0)}$$

$$v_i^{(n)} \rightarrow Z_i^{(n)} \equiv< 0|\mathcal{O}_i|n >$$

($t >> t_0$)

Scattering on the lattice – ‘Lüscher method’

Lüscher, Nucl. Phys. B354, 531 (1991); extended by many others

$$\det\left[\delta_{ij}\delta_{\ell\ell'}\delta_{mm'} + i\rho_i\ t_{ij}^{(l)}\ \left(\delta_{\ell\ell'}\delta_{mm'} + i\mathcal{M}^{\vec{P}}_{\ell m;\ell' m'}(q_i^2)\right)\right] = 0$$

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$\vec{P} = \text{overall mom.}$

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i, *j* label channels
e.g. Kπ, Kη

$$\rho_i = \frac{2k_{\mathrm{cm},i}}{E_{\mathrm{cm}}}$$

$$\vec{q} = \vec{k}_{\mathrm{cm}} L/2\pi$$

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~ gen. zeta fns. – effect of finite vol.

Reduced symmetry → ℓ mix

Subduce to lattice irrep (Λ) →

$$\mathcal{M}^{\vec{d},\Lambda}_{\ell n;\ell' n'}\delta_{\Lambda\Lambda'}\delta_{\mu\mu'}$$

(ℓ that subduce to Λ mix)

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(ℓ that subduce to Λ mix)

Given *t*: solns → finite-vol. spec. $\{E_{\mathrm{cm}}\}$

We need: spectrum → *t*-matrix

Scattering Parameterizations

Elastic

$$t^{(\ell)} = \frac{1}{\rho} e^{i\delta_\ell} \sin \delta_\ell$$

$$k_i^{2\ell+1} \cot \delta_\ell = \frac{1}{a_\ell} + \frac{1}{2} r_\ell k_i^2 + P_2 k_i^4 + \mathcal{O}(k_i^6)$$

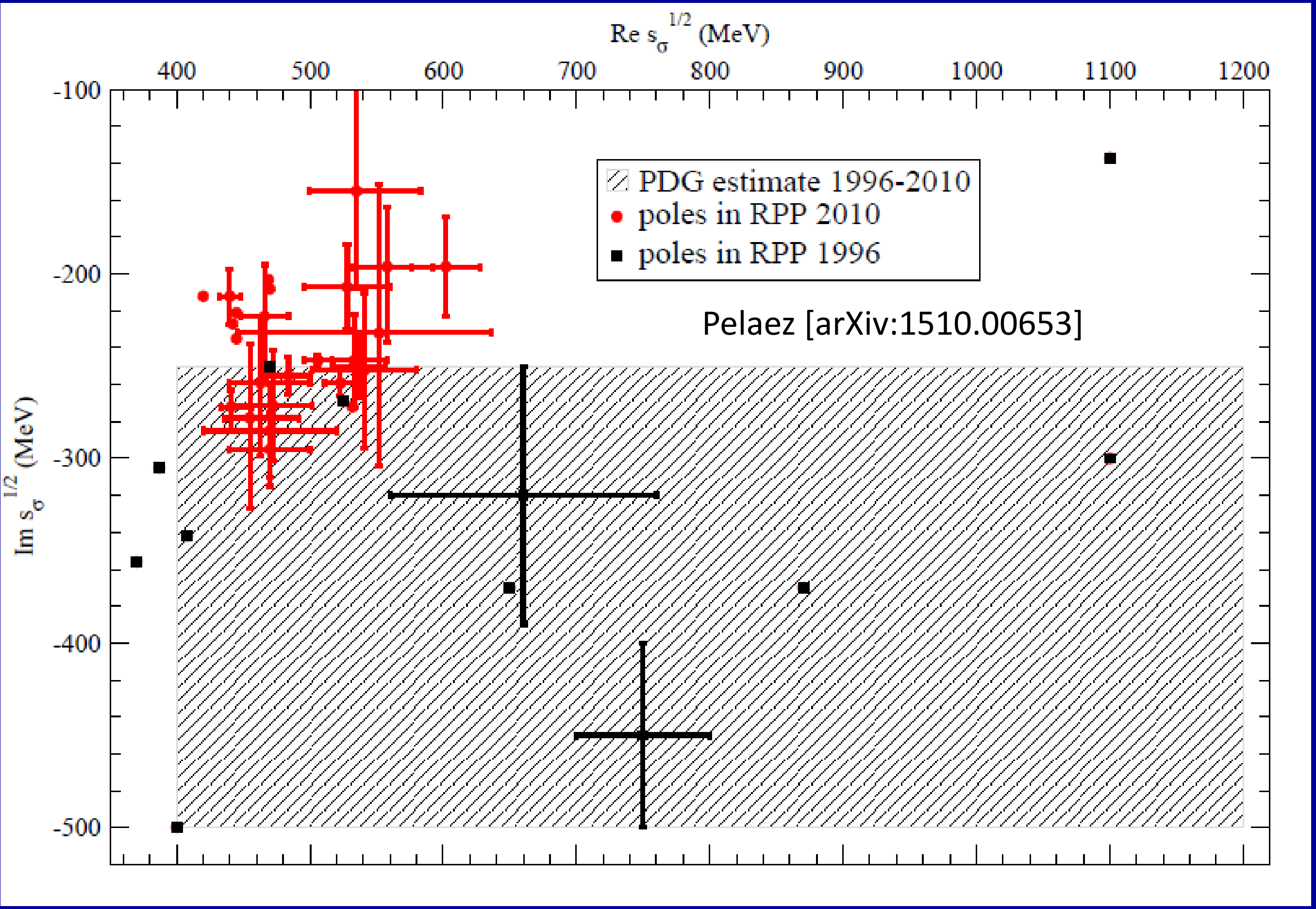
$$t^{(\ell)}(s) = \frac{1}{\rho(s)} \frac{\sqrt{s}\,\Gamma_\ell(s)}{m_R^2 - s - i\sqrt{s}\,\Gamma_\ell(s)} \qquad \Gamma_\ell(s) = \frac{g_R^2}{6\pi} \frac{k^{2\ell+1}}{s\, m_R^{2(\ell-1)}}$$

K-matrix param. – respects unitarity (conserve prob.) and flexible

$$t_{ij}^{-1}(s) = \frac{1}{(2k_i)^\ell} K_{ij}^{-1}(s) \frac{1}{(2k_j)^\ell} + I_{ij}(s) \qquad \mathrm{Im}[I_{ij}(s)] = -\delta_{ij}\rho_i(s)$$

e.g. $K_{ij} = \left(g_i^{(0)} + g_i^{(1)} s\right)\left(g_j^{(0)} + g_j^{(1)} s\right) \frac{1}{m^2 - s} + \gamma_{ij}^{(0)} + \gamma_{ij}^{(1)} s$

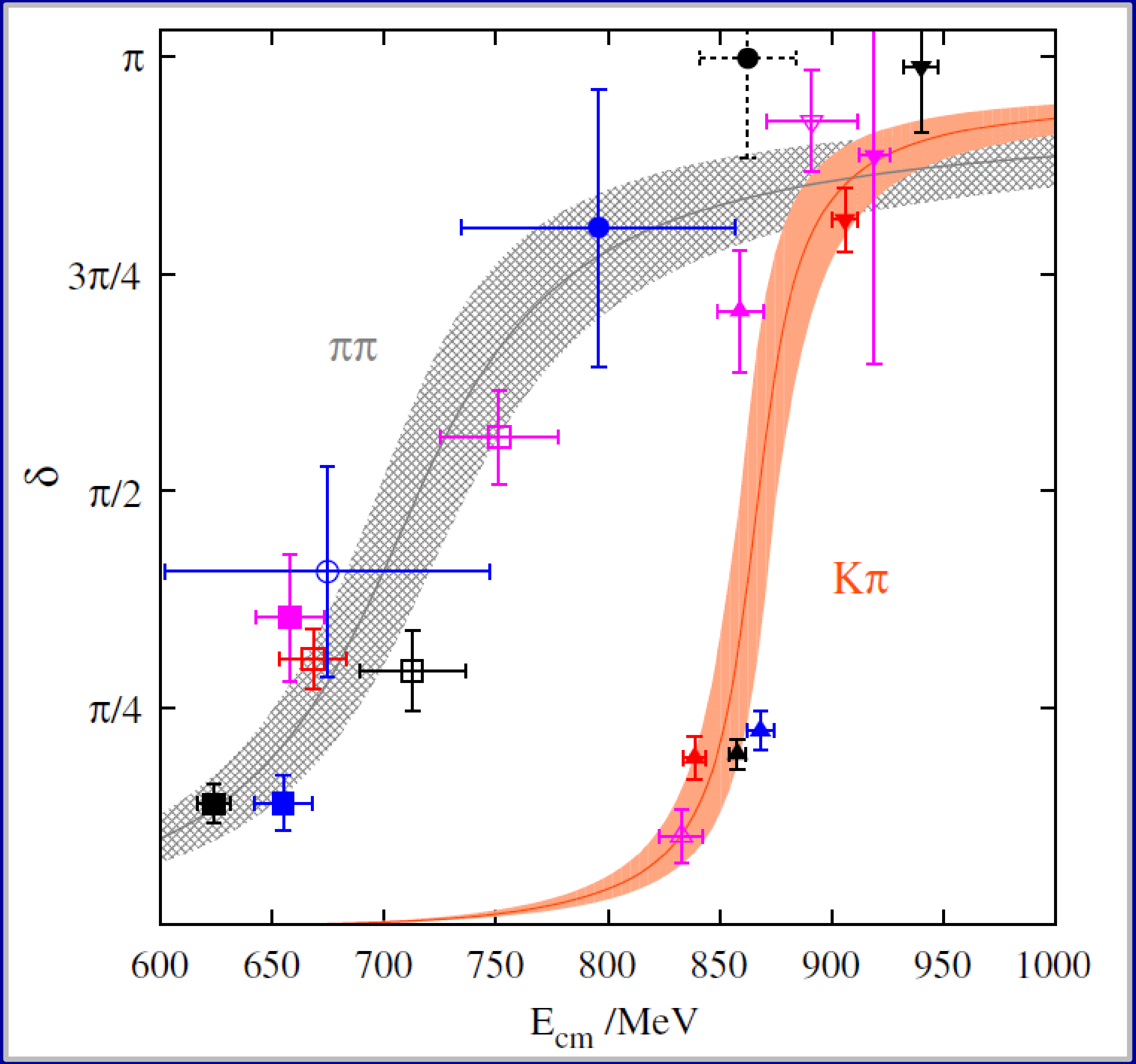
$f_0(500)/\sigma$ in $\pi\pi$ scattering



Pelaez [arXiv:1510.00653]

The ρ: other elastic ππ calcs.

Bali *et al* (RQCD)
[PR D93, 054509 (2016)]

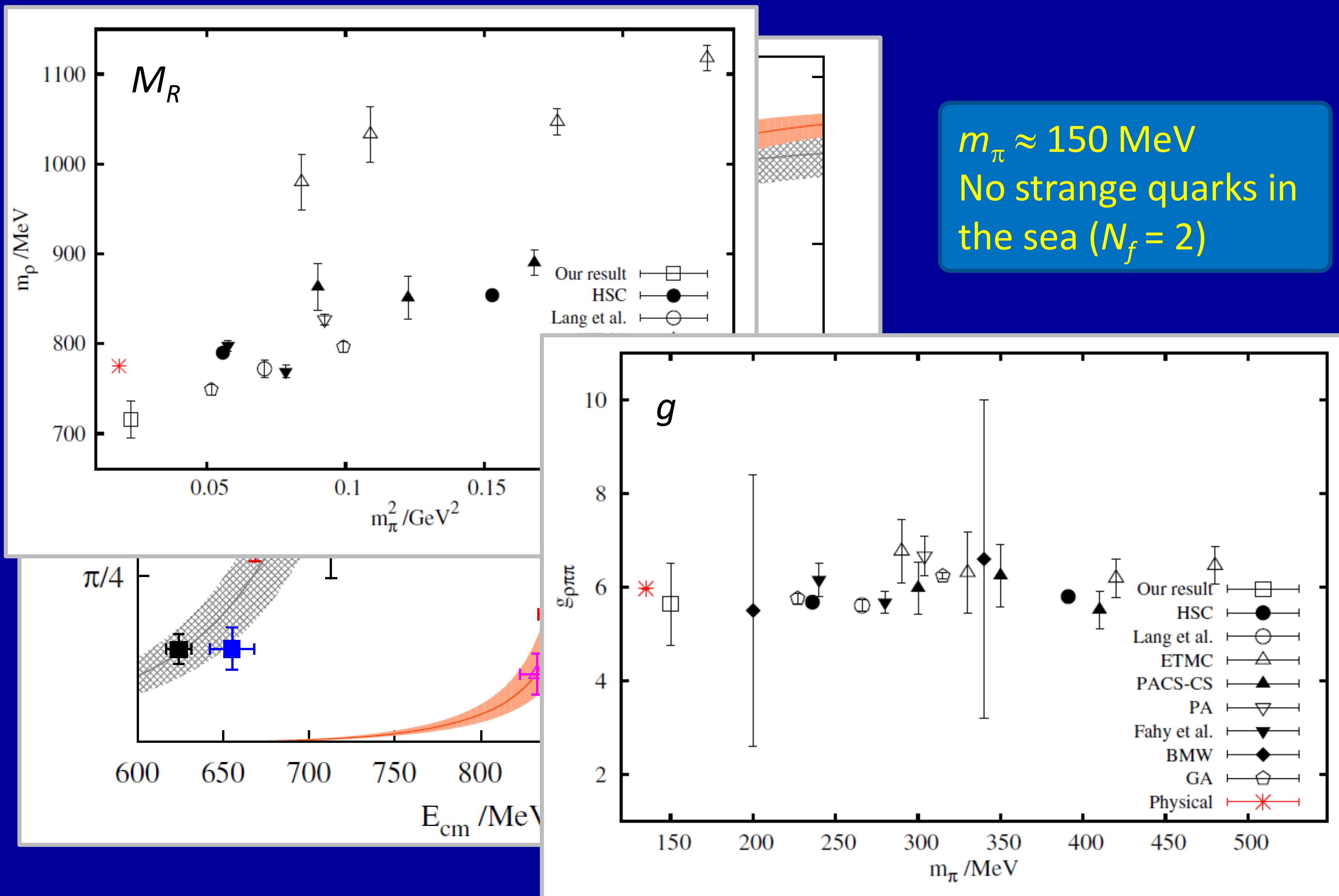


$m_\pi \approx 150$ MeV
No strange quarks in the sea ($N_f = 2$)

$M_R = 716 \pm 21 \pm 21$ MeV
$\Gamma = 113 \pm 35 \pm 3$ MeV
$g = 5.64 \pm 0.87$

The ρ: other elastic ππ calcs.

Bali *et al* (RQCD)
[PR D93, 054509 (2016)]



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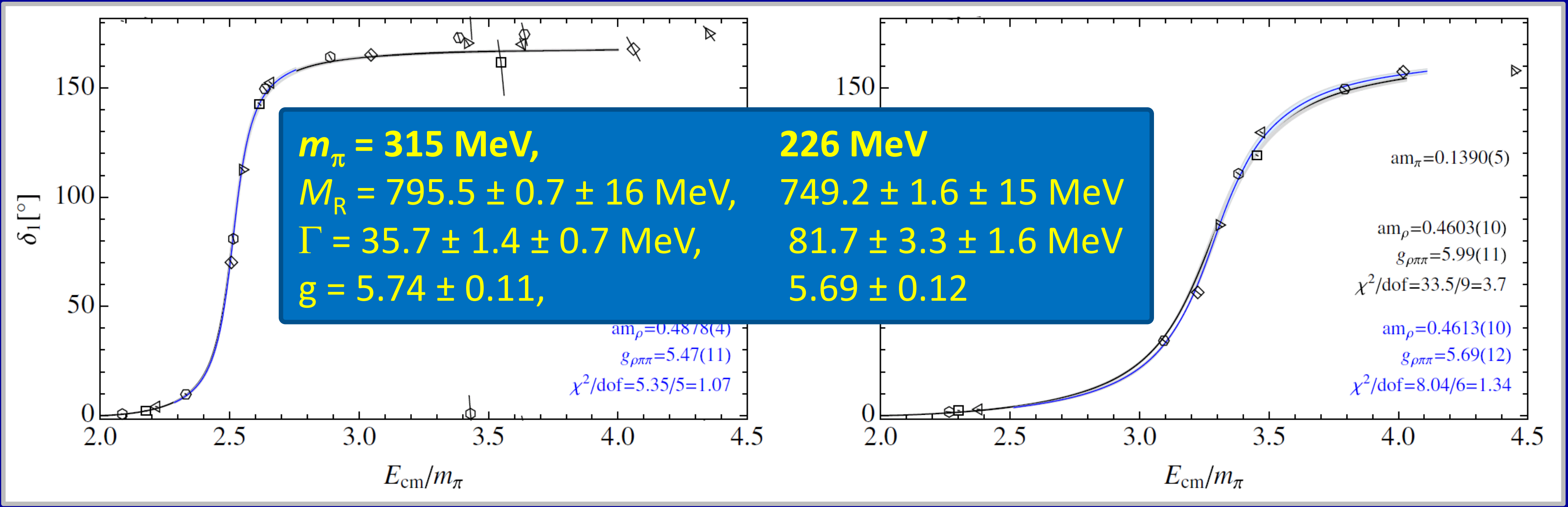
The ρ: other elastic ππ calcs.

Guo *et al* [PR D94, 034501 (2016)],
Hu *et al* [PRL 117, 122001 (2016)]

No strange quarks in the sea (N_f = 2)



m_π = 315 MeV,	**226 MeV**
M_R = 795.5 ± 0.7 ± 16 MeV,	749.2 ± 1.6 ± 15 MeV
Γ = 35.7 ± 1.4 ± 0.7 MeV,	81.7 ± 3.3 ± 1.6 MeV
g = 5.74 ± 0.11,	5.69 ± 0.12

Some other recent calculations:

- Bulava *et al* [NP B910, 842 (2016)]