Progress in perturbative QCD

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Disclaimer

A lot of progress in pQCD in the last year

- •Impossible / useless to cover everything in 45 minutes
- In the following: more or less coherent overview of some key ingredients needed for precision physics at the LHC, with CHERRY-PICKED EXAMPLES OF NEW (=AFTER ANNUAL THEORY MEETING 2015) RESULTS

• Apologies if your favorite topic is not covered...

Physics at the LHC: need for precision

- •Despite the standard model being 'complete', strong indications that new physics may be present at the LHC
- •Before the LHC, some expectation of new physics beyond the corner (naturalness, fine tuning, WIMP miracle...): SUSY, extra dimensions... So far, this has not happened
- Discovering new physics turned out to be more challenging. No spectacular new signatures ⇒ new physics can be hiding in small deviations from SM behavior, or in unusual places
- To single them out: TEST THE (IN)CONSISTENCY OF THE SM AT THE LHC, as best as we can

PRECISION QCD IS NOW A PRIVILEGED TOOL FOR DISCOVERY AT THE LHC

Also, pushing the frontier of pQCD forward, we keep learning about the structure of a REAL-WORLD QFT.

Precision goals: some (rough) estimates

Imagine to have new physics at a scale Λ

- •if Λ small \rightarrow should see it directly, bump hunting
- if Λ large, typical modification to observable w.r.t. standard model prediction: $\delta O \sim Q^2 / \Lambda^2$
- standard observables at the EW scale: to be sensitive to ~ TeV new physics, we need to control δO to few percent
- •high scale processes (large p_T , large invariant masses...): sensitive to ~TeV if we control δO to 10-20%

THESE KINDS OF ACCURACIES ARE WITHIN REACH OF LHC EXPERIMENT CAPABILITIES.

WE SHOULD PUSH OUR UNDERSTANDING OF PQCD TO MATCH THEM ON THE THEORY SIDE



"Few percent": the theory side $d\sigma = \int dx_1 dx_2 f(x_1) f(x_2) d\sigma_{part}(x_1, x_2) F_J(1 + \mathcal{O}(\Lambda_{QCD}/Q))$ Input parameters: ~few percent. In principle improvable NP effects: ~ few percent No good control/understanding of them at this level. LIMITING FACTOR FOR FUTURE DEVELOPMENT

HARD SCATTERING MATRIX ELEMENT

- large $Q \rightarrow$ most interesting and theoretically clean
- • $\alpha_{s} \sim 0.1 \rightarrow$ For TYPICAL PROCESSES, we need NLO for ~ 10% and NNLO for ~ 1% accuracy. Processes with large perturbative corrections (Higgs): N³LO
- •Going beyond that is neither particularly useful (exp. precision) NOR POSSIBLE GIVEN OUR CURRENT UNDERSTANDING OF QCD

NLO computations: status and recent progress

NLO computations: where do we stand

Thanks to a very good understanding of one-loop amplitudes and to significant development in MC tools (→ real emission) now NLO IS THE STANDARD FOR LHC ANALYSIS

- •Many publicly available codes allow anyone to perform NLO analysis for reasonably arbitrary [~ 4 particles (~ 3 colored) in the final state] LHC processes: MADGRAPH5_AMC@NLO, OPENLOOPS(+SHERPA), GOSAM(+SHERPA), RECOLA, HELAC...
- The next step for automation: NLO EW (basically there), arbitrary BSM

Dedicated codes allow for complicated final states, e.g.:

- V(V)+jets [BLACKHAT+SHERPA], jets [NJET+SHERPA], tt+jets [Höche et al. (2016)] → also allow for interesting theoretical analysis (mult. ratios predictions...)
- •H+jets [GOSAM+SHERPA]. Recently: up to 3-jets at LO with full top-mass dependence [Greiner et al. (2016)] → investigate the high-pt Higgs spectrum
- •Off-shell effects in ttX processes: ttH [Denner and Feger (2015)], ttj [Bevilacqua et al. (2015)]

NLO computations: where do we stand

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NLO RESULTS: SOME THEORETICAL SURPRISE

- NLO "revolution" triggered by new ideas for loop amplitude computation → unitarity, on-shell integrand reduction
- •Sophisticated incarnations of traditional "Passarino-Veltman"-like tensor reduction proved to be COMPETITIVE WITH UNITARITY METHODS (COLLIER + OPENLOOPS)
- •Amplitudes computed with numerical methods are fast and stable in degenerate kinematics → can be used in NNLO computations (so far established for color-singlet processes)

NLO: loop-induced processes

In the past year, significant progress for loop-induced processes



- Relevant examples: Higgs pt, gg→VV (especially after qq→VV@NNLO), gg→VH (especially after qq@NNLO), di-Higgs...
- Despite being loop-suppressed, the large gluon flux makes the yield for these processes sizable
- gluon-fusion processes → expect large corrections
- At NLO simple infrared structure, but virtual corrections require complicated two-loop amplitudes
- Real emission: one-loop multi-leg, in principle achievable with 1-loop tools

A small detour: loop amplitudes

Computation of loop-amplitudes in two steps:

- 1. reduce all the integrals of your amplitudes to a minimal set of independent `master' integrals
- 2. compute the independent integrals

At one-loop:

- independent integrals are always the same (box, tri., bub., tadpoles)
- only (1) is an issue. Very well-understood (tensor reduction, unitarity...)



Beyond one-loop: reduction not well understood, MI many and process-dependent (and difficult to compute...)

Two-loop: reduction

- •So far: based on SYSTEMATIC ANALYSIS OF SYMMETRY RELATIONS between different integrals (IBP-LI RELATIONS [Tkachov; Chetyrkin and Tkachov (1981); Gehrmann and Remiddi (2000)] / LAPORTA ALGORITHM [Laporta (2000)])
- •State of the art for phenomenologically relevant amplitudes
 - 2 \rightarrow 2 with massless internal particles (di-jet, H/V+jet, VV)
 - 2 → 2 with two mass scales: ttbar [Czakon et al. (2007)], H+JET WITH FULL TOP MASS DEPENDENCE [Melnikov et al. (2016)]
- •Going beyond: significant improvements of tools, NEW IDEAS
- •Motivated by the one-loop success, many interesting attempts to generalize unitarity ideas / OPP approach to two-loop case
- •We are still not there, but a lot of progress
- Interesting proof-of-concept for unitarity-based approaches: 5/6gluon all-plus amplitudes at two-loops [Badger, Frellesvig, Zhang (2013); Badger, Mogull, Ochiruv, O'Connell (2015); Badger, Mogull, Peraro (2016)]

Two-loop: master integrals

- For a large class of processes (~ phenomenologically relevant scattering amplitudes with massless internal lines) we think we know (at least in principle) how to compute the (very complicated) MI. E.g.: DIFFERENTIAL EQUATIONS [Kotikov (1991); Remiddi (1997); HENN (2013); Papadopoulos (2014)]
- Recent results for very complicated processes: planar 3-jet [Gehrmann, Henn, Lo Presti (2015)], towards planar Vjj/Hjj [Papadopoulos, Tommasini, Wever (2016)]
- In these cases, the basis function for the result is very well-known (Goncharov PolyLogs) and several techniques allow to efficiently handle the result (symbol, co-products...) and numerically evaluate it



Two-loop: master integrals

- Unfortunately, we know that GPL are not the end of the story. For phenorelevant processes, we typically exit from this class when we consider amplitudes with internal massive particles (e.g. ttbar, H+J)
- Progress in this cases as well (e.g. [Tancredi, Remiddi (2016); Adams, Bogner, Weinzierl (2015-16)]) but we are still far from a satisfactory solution → real conceptual bottleneck for further development
- •FIRST STEP TOWARDS A SOLUTION: planar results for H+J with full top mass effects. Solution as 1-fold integrals. Elliptic functions. [Bonciani et al. (2016)]
- Side note: some times physics come and help you. b-quark mass effects for Higgs p_t relevant in the region $m_b \ll p_t \ll m_{H_c}$ Using this condition massively simplify the computation of integrals \rightarrow AMPLITUDE IN THIS REGIME RECENTLY COMPUTED [Melnikov et al. (2016)]. But result cannot be extended for $p_t \gg m_H$

Back to loop induced: NLO for $gg \rightarrow VV$

Thanks to the progress in loop-amplitude computations, NLO corrections to $gg \rightarrow WW/ZZ$ and to $gg \rightarrow (H) \rightarrow VV$ signal/background interference [FC, Melnikov, Röntsch, Tancredi (2015-16); Campbell, Ellis, Czakon, Kirchner (2016)]



- Large corrections (relevant especially for precision $pp \rightarrow ZZ$ cross-section)
- Higgs interference: large, but as expected (*K*_{sig}~*K*_{bkg}~*K*_{int})
- Top mass effects (important for interference) through 1/mt expansion → reliable only below threshold (although some hope for past-threshold extension via Padé approximations)

Loop induced: di-Higgs@NLO

[Borowka, Greiner, Heinrich, Jones, Kerner, Schlenk, Schubert, Zirke (2016)]



- 2-loop amplitude beyond current reach (reduction and for MI)
- Completely different approach: *FULLY NUMERICAL INTEGRATION OF EACH INDIVIDUAL INTEGRAL*
- Table of 665 phase-space points
- Highly non-trivial computerscience component (GPUs, very delicate numerical integration...)

LESSONS FROM THE EXACT COMPUTATION:

- Reasonable approximations to extend 1/m_t result beyond the top threshold (rescaled Born, exact real radiation) can fail quite significantly
- Exact K-factor much less flat than for m_t approximations

Loop induced: di-Higgs@NLO

[Borowka, Greiner, Heinrich, Jones, Kerner, Schlenk, Schubert, Zirke (2016)]

Now that we know the exact result, many interesting questions:

- do we understand why the approximate m_t result fails so miserably (high energy matching, genuinely large two-loop components...)?
 - ideal playground for approximation testing. Can we find something which works? Can we study e.g. the Padé approximation used to extend the $1/m_t$ expansion in gg \rightarrow VV?
 - especially relevant because we now know FULLY DIFFERENTIAL NNLO CORRECTIONS IN THE $M_T \rightarrow \infty$ LIMIT [de Florian et al (2016)] \rightarrow Would like to know best way to combine the results

CAN THIS FULLY NUMERICAL APPROACH BE APPLIED TO MORE GENERAL CASES?
● processes with more than two (m_{HH}, y_{HH}) variables (gg→4l)
● processes with a more complicated tensor structure (H+J)

NLO computations: NLO+PS

Thanks to a *understanding of one-loop amplitudes* and to *significant development in MC tools* (→ *real emission, all order soft/collinear emission*) now NLO + PS IS THE STANDARD FOR LHC ANALYSIS



PARTON SHOWER EVOLUTION

- •All order-emission of soft/collinear partons
- •Does not capture hard emission/ virtual corrections
- As such, Irrelevant for High-Q Physics
- CAN GENERATE FULL EVENTS → HADRONIZATION → DETECTOR SIMULATIONS
- •Also, although in the (N)LL approximation only, capture multiparton dynamics (e.g. jet structure...)

NLO computations: NLO+PS

Ideally: combine NLO and PS

- •Methods to combine NLO computations and fixed order ("matching") now standard: MC@NLO (~exponentiate soft radiation), POWHEG (~exponentiate full real emission), GENEVA (~SCET matching) NEW KID IN TOWN: KrkNLO [Jadach et al (2016)] (~redefine PDF to contain "nasty" universal bits of NLO)
- •Improved accuracy pushed for improvement in parton shower
 - better control of evolution, e.g. DIRE [Höche, Prestel (2015)]
 - •better control of some logarithmic structure, e.g. HEJ for highenergy logs [Andersen, Smillie (2011-...)], DEDUCTOR [Nagy, Soper (2016)] for threshold logs
 - •beyond purely classical evolution (try and introduce some quantum corrections), e.g. [Nagy, Soper (2014-...)]
 - better control of resonance structure of the process [Ježo, Nason (2015), Frederix et al. (2016)]

Example: unified treatment of WWbb "Single-top" "Top-pair" "WW"

These 3 "processes" share the same initial/final state \rightarrow THEIR SEPARATION IS UNPHYSICAL (quantum interference)

- in the past: we were unable to properly generate the WWbb final state
- more or less ad-hoc ways of separating the three (IDEA: selection cuts should clearly select one of the 3 topologies)
- thanks to recent advance we can consider WWbb as a whole, putting these analysis on solid theoretical grounds

Example: unified treatment of WWbb

[Ježo, Lindert, Nason, Oleari, Pozzorini (2016)]



- Radiation in the decay crucial for the reconstructed top mass
- After top selection cuts, naive expectation WWbb~ top production \otimes decay works well ($\Gamma_t \ll m_t \rightarrow$ factorization) \rightarrow NNLO!
- •Shift in reconstructed top mass: ~ 100 MeV (WWbb vs top prod⊗decay)



Bulk of corrections ~ trivial (= no loop, LO at higher multiplicity). CAN WE CAPTURE THEM?

• Parton shower MC provide an ideal trastities work to perform such

- combination
- "Merge" together samples of different multiplicities. Well established techniques to LO (CKKW, MLM), and a lot of different approaches to NLO accuracy (NLOPS, MEPs, MENLOPS, MEPs@NLO, FxFx, MINLO, GENEVA...)



Merging: Higgs p_t with finite top mass effects Complete NLO corrections with full top-quark mass dependence: still unavailable (2-loop amplitudes) (*NNLO in the HEFT*)



- At high pt merged samples can give a good idea of the corrections [Frederix et al (2016), Greiner et al (2016)]
- Give similar result of approximate NLO of [Neumann, Williams (2016)]
- Same behavior as predicted by high energy resummation [Muselli et al (2016)]
- COHERENT PICTURE (waiting for the NLO result...)

From merging to NNLOPS Merged sample close to full NNLO computation (~right real emission, missing virtual corrections). For color-singlet processes, extension of merging ideas led to combination of NNLO + PS



Logs beyond Parton Shower: progress in resumation

Logs and resummation

- •Often, at the LHC we are dealing with multi-scale processes → large ratios → large soft/collinear logs, resummation at least desirable
- AT HIGH Q, VERY FAR FROM SOFT/COLLINEAR REGIONS \rightarrow EFFECTS SHOULD BE IRRELEVANT
- •BUT: often in intermediate regions (statistics...)
- •Also, often fiducial cuts / analysis strategies force us into softsensitive regions (jet veto, jet substructure...)
- FINALLY, understanding all-order structure of perturbative soft/ collinear emission can give glimpse into non-perturbative regime of QCD (and help singling out genuine non perturbative effects, hadronization, UE...)

Resummation: recent progress

The past year saw many interesting development, obtained with different frameworks (SCET, ordinary QCD...). Impossible to summarize in few slides (even to enumerate...).

SOME examples

- •Forward scattering and Glauber gluons, next-to-leading-power resummation, non-global logarithms, automatic NNLL for IRC observables, jet radius logs, Higgs quark mass logs...
- •Progress in automation / resummation for generic observables (two-loop soft function, ttH, ttW resummations...), high precision phenomenology (Higgs/DY pt, Jet Veto...)
- •One loop soft function with arbitrarily many soft gluons, threeloop soft anomalous dimension, three-loop double differential soft function/rapidity an. dim. (and N³LL p_t resummation)
- Jet substructure, better understanding, better observables...

Non global logs

If observable sensitive only to radiation in PART OF THE PHASE SPACE: complicated "non global" logarithmic structure, nonexponentiation [Dasgupta, Salam (2001)]

•Example: hemisphere jet mass



PROBLEMATIC TO RESUM BEYOND LL

de with m gard partons and an arbitrary manuage office with m h soft partons is of course well known. Each of the Each of the hard partons get dressed with a Wilson line along its direction. In analogy to factorization o factorization for amplitudes with coft particles [32], we have $n_2) \dots S_m(n_m) = \sum_{m \in \mathbb{Z}^m} \langle \mathcal{H}_m(\{\underline{n}\}, Q, \mu) \otimes S_m(\underline{n}\} S_m(\underline{n}) \otimes S_m(\underline{n}) \rangle \rangle \rangle S_m(n_m) \otimes S_m(\underline{n}) \otimes S_m(\underline$ where $n_i^{\mu} = p_i^{\mu} / E_i$ and $\{p\} = \{p_1, p_2, \dots, p_m\}$, $p_2, \dots, p_m\}$, but while the coff case involved quark ides, we are now dealing with or ides, we are now dea ields, perform the decoupling transformation and then element with exactly one collinea y one collinear particle in each sector, such the sector, such the sector, such the sector, such the sector, and the sector of errescerin scaling poly in attens is the expansion of the phase-space constraints.) To get the amplitude w "Simplitude with an arbitrary **Expension Soft** particles in the final state, oggi bances can be part of the amplitude e Batomatrix elementiof the Wilson-Hine öperatör (2.4). Doing so, the cross section takes the form

 $\mathcal{C}_{\mathbf{C}}$



Jet radius logs

[Dasgupta et al (2016), Chen et al (2015) Kolodrubetz et al (2016), Kang et al (2016)]

•Clustering logs now to all orders, at LL \rightarrow small R accessible •LL ↔ PS... but here disentangled

3 effects:

- ▶ perturbative ($\sim \ln R$)
- ► hadronisation (~ 1/R)
- ► MPI/UE (~ \mathbb{R}^2)

To disentangle them, need $\geq 3 \text{ R}$ values:

- ► 0.6–0.7: large MPI/UE
- ► 0.4: non-pert. effects cancel?
- ► 0.2–0.3: large hadronisation



Highest precision for standard candles: N³LO/NNLO predictions



(inclusive VBF@N³LO: [Dreyer, Karlberg (2016)]

Beyond fully inclusive: NNLO differential

Apart from complicated multi-loop amplitudes, the big problem of higher order computations is how to consistently handle IR singularities



COMPLICATED IR STRUCTURE HIDDEN IN THE PHASE SPACE INTEGRATION

The problems with NNLO computations

Apart from complicated two-loop amplitudes, the big problem of NNLO computations is how to consistently handle IR singularities



- IR divergences hidden in PS integrations
- After integrations, all singularities are manifest and cancel (KLN)
- •We are interested in realistic setup (arbitrary cuts, arbitrary observables) → we need fully differential results, we are not allowed to integrate over the PS
- The challenge is to EXTRACT PS-INTEGRATION SINGULARITIES WITHOUT ACTUALLY PERFORMING THE PS-INTEGRATION

NNLO differential: solutions

Thanks to multi-year effort of the whole community: we now have DIFFERENT WAYS TO DEAL WITH THIS PROBLEM. Each has its own merits / problems.

Local subtractions (cancellations point by point in the phase-space)

- antenna [Gehrmann-de Ridder, Gehrmann, Glover] → jj, Hj, Vj
- Sector-decomposition+FKS [Czakon; Boughezal, Melnikov, Petriello;
 Czakon, Heymes] → ttbar, single-top, Hj
- P2B [Cacciari, Dreyer, Karlberg, Salam, Zanderighi] → VBF_H, single-top
- Colorful NNLO [Del Duca, Somogyi, Tocsanyi, Duhr, Kardos]: only e+e- so far

Non-local subtractions (cancellation globally after integration)

- q_t subtraction [Catani, Grazzini] \rightarrow H, V, VH, VV, HH
- N-jettiness [Boughezal et al; Gaunt et al] \rightarrow H, V, $\gamma\gamma$, VH, Vj, Hj, singletop

NNLO differential: solutions

- Thanks to multi-year effort of the whole community: we now have DIFFERENT WAYS TO DEAL WITH THIS PROBLEM. Each has its own merits/problems.
- Some of these techniques are quite generic
 - IN PRINCIPLE, they allow for ARBITRARY COMPUTATIONS
 - IN PRACTICE: `genuine' $2 \rightarrow 2$ REACTIONS, with big computer farms
- Non-local subtractions (cancellation globally after integration)
 - 2016: from "PROOF OF CONCEPT" to PHENOMENOLOGY
 - N-jettiness [Boughezal et al; Gaunt et al] → H, V, γγ, VH, Vj, Hj, singletop

Recent NNLO results: dijet [Currie, Glover, Pires (2016)]

~40 partonic channels, highly non-trivial color flow. Realistic jet



Non trivial shape correction (NLO scale choice?), sizable effect
Large effect on PDF? (see also jj in DIS [Niehues, Currie, Gehrmann (2016)])

Recent NNLO results: VJ



NNLO

Z/Wj, γj known. Zj: independent computations
Highly improved theoretical accuracy (~exp error)
Small deviations evident (PDFs? Calibration?)

Recent NNLO results: di-bosons [Grazzini et al. (2015-2016)]

In the last year, the PROGRAM OF COMPUTING FULLY DIFFERENTIAL NNLO CORRECTION TO DI-BOSON PROCESSES HAS BEEN COMPLETED



- Fully exclusive analysis possible. Corrections strongly cut-sensitive
 → FIDUCIAL REGION comparisons (jet veto, gg contribution...)
- •General picture: GOOD AGREEMENT DATA/NNLO (with some possible room for discussion for WW jet-veto, see [Dawson et al (2016)])

Recent NNLO results: top

TTBAR DIFFERENTIAL DISTRIBUTIONS



- Tension in p_{t,top} alleviated
- Allow for precision physics in the top sector

T-CHANNEL SINGLE-TOP PLUS TOP-DECAY (NWA)



- Small inclusive corrections
- LARGE CORRECTIONS in exclusive region
- Similar behavior observed in Higgs in VBF [Cacciari et al (2015)]

Recent NNLO results: MCFM@NNLO

[Campbell, Ellis, Williams (2016); Campbell et al (2016); Boughezal et al (2016)]



•NNLO slicing available for some color-singlet processes in MCFM
•V/H+J will be next?

Recent NNLO results: H+J phenomenology

[Chen et al (2016)]



- Realistic final states → fiducial region
- Important benchmarking between different computations
- •Non-trivial final states possible

Application of f.o. results: H and jet vetoes

[Banfi, FC, Dreyer, Monni, Salam, Zanderighi, Dulat (2015)]



•Combination of f.o. N³LO (Higgs inclusive) and NNLO (H+J exclusive) with NNLL resummation, LL_R resummation, mass effects...

•No breakdown of fixed (high) order till very low scales

• Even more so for Z+jet [Gerhmann-De Ridder et al (2016)]

Application of NNLO results: H pT

[Monni, Re, Torrielli (2016)]



• Matching of NNLO H+J with NNLL Higgs p_T resummation

- •Significant reduction of perturbative uncertainties
- •Again, no breakdown of perturbation theory (resummation effects: 25% at $p_T = 15$ GeV, ~0% at $p_T = 40$ GeV)

Conclusions and outlook

- •LHC is driving amazing progress in perturbative QCD
- "LHC as a precision machine": possible!
 - Sophisticated higher order computations achievable
 - Big progress in multi-loop computations
 - Better understanding of logarithmic structures / PS
 - Reliable theory-experiment comparison possible (fiducial region...)
- Many other aspects not covered here
 - Progress in input parameters: α_s fits, PDFs improvement. Photon PDF [Manohar, Nason, Salam, Zanderighi (2016), Harland, Khoze, Ryskin (2016)]
 - 5-loop evolution of α_s [Baikov, Chetyrkin, Kühn (2016)]
 - Input parameters: the top mass [Beneke et al, Hoang et al (2016)]
 - EW corrections, mixed QCD-EW...
- Going beyond state of the art: quite hard (technical/conceptual problems)

A LOT OF THEORETICAL FUN AHEAD, DIRECTLY RELEVANT FOR LHC PHENOMENOLOGY! Thank you very much for your attention!