Hands-on Gravitational Wave Data Analysis

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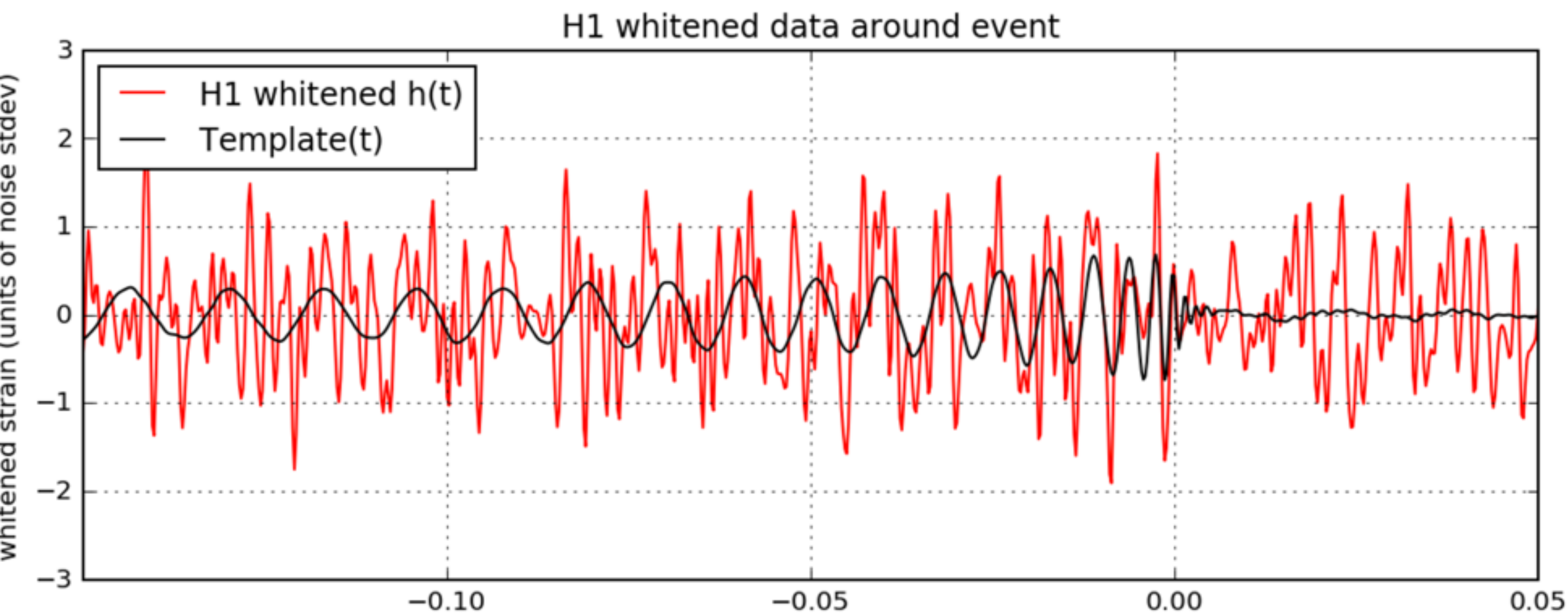


Plan for today: Intro to process open LIGO data

- Use material from LIGO Open Science Center: losc.ligo.org: data, tutorials, publications, documentation.

- Intro to basic data analysis techniques, LIGO data formats, waveform anatomy.

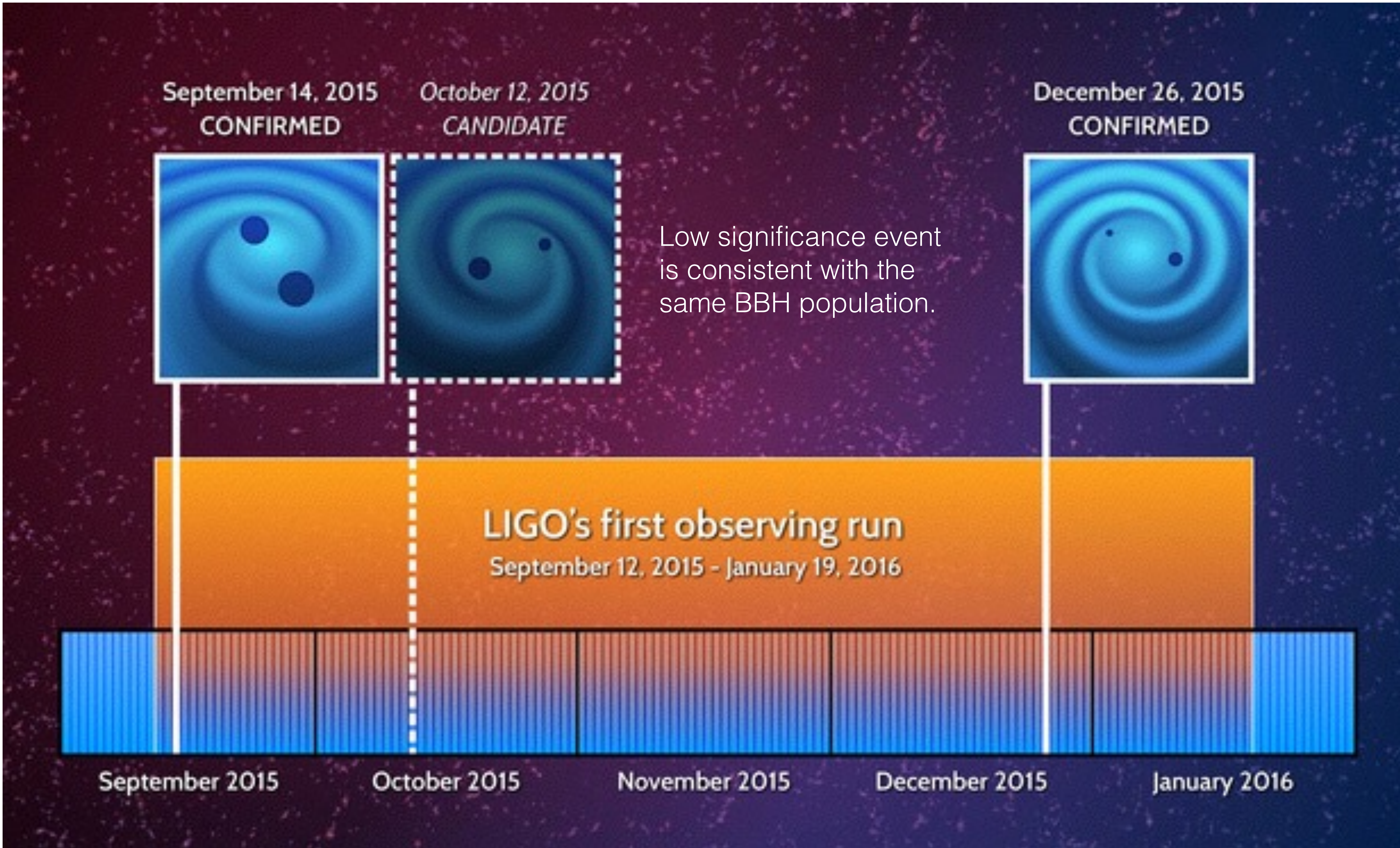
  - Start with a bit of “theory material” & overview of LIGO data + software.

  - Together: work through tutorial using data from first detection,

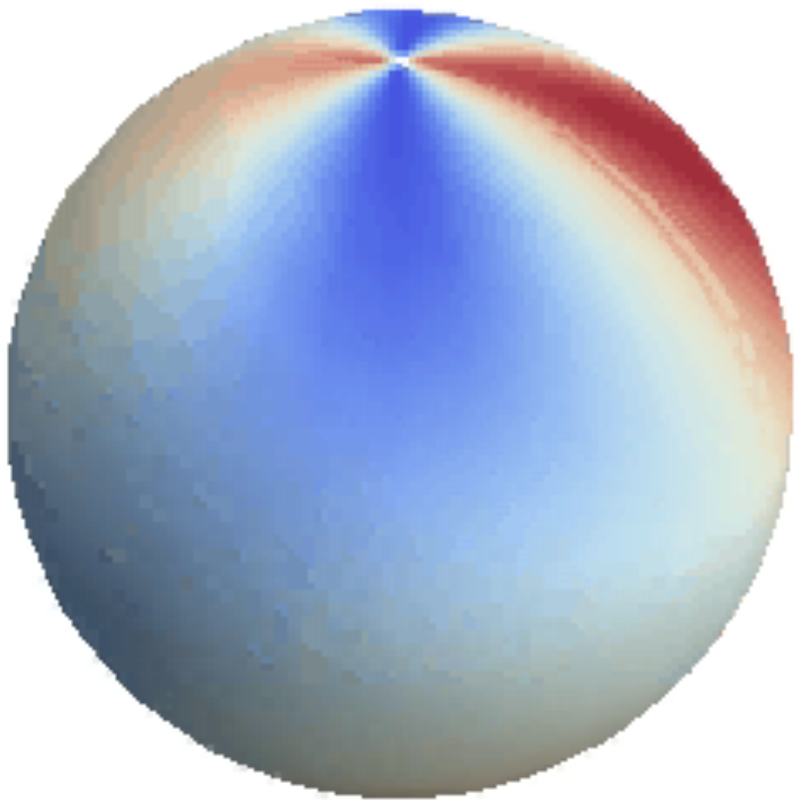
  - Individually: work through 2 more events with suggested additional investigations.

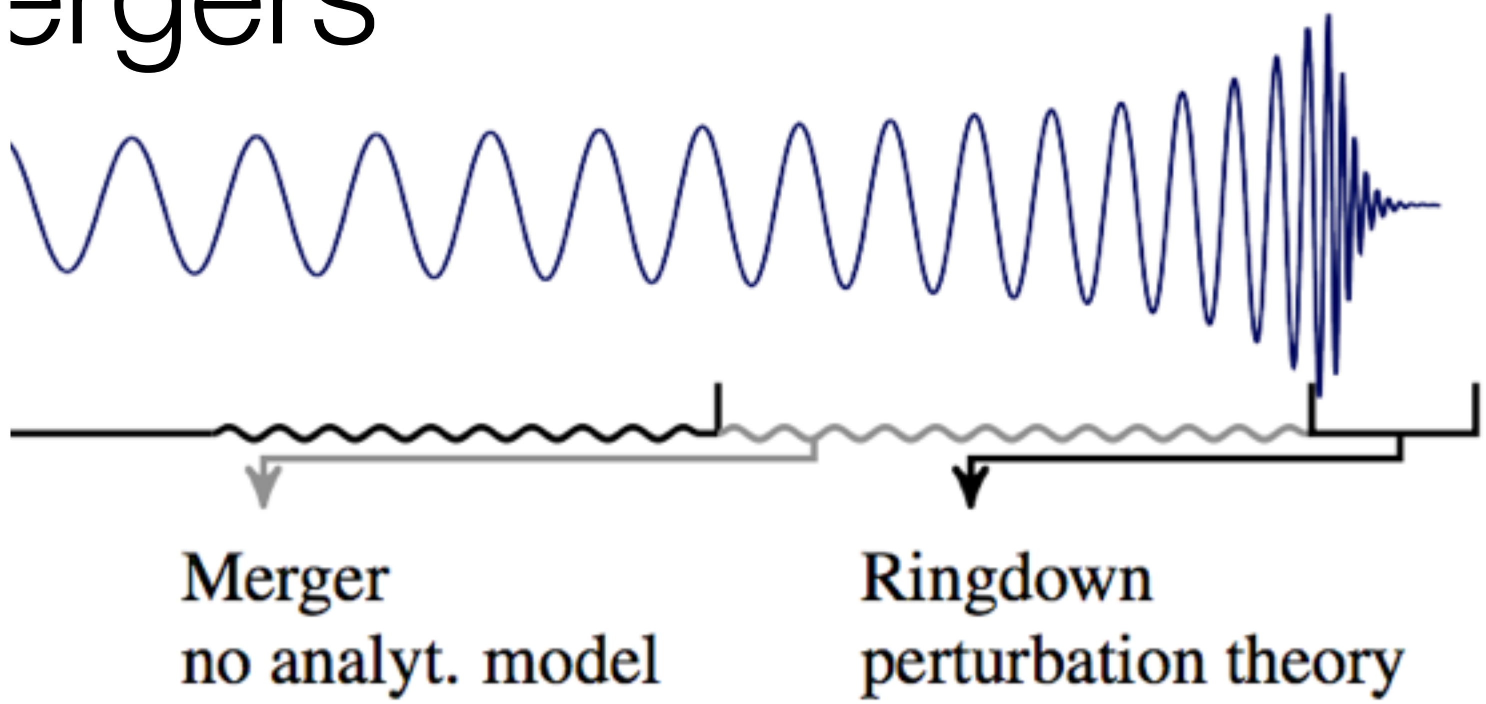
  - Advertisement: phenomenological waveform models, feel free to modify.

2 high significance events detected, 1 at low significance (based on loudest event statistic): GW150914, LVT151012, GW151226



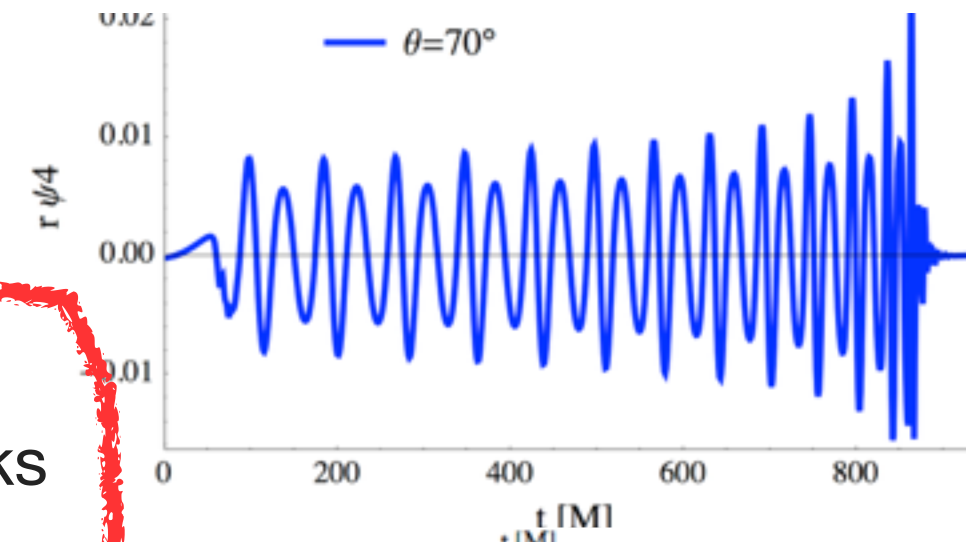
ergers



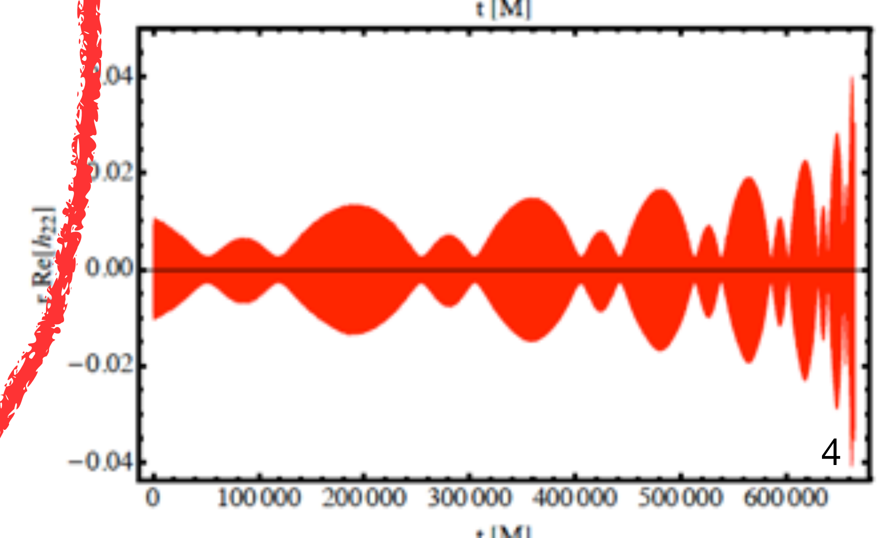


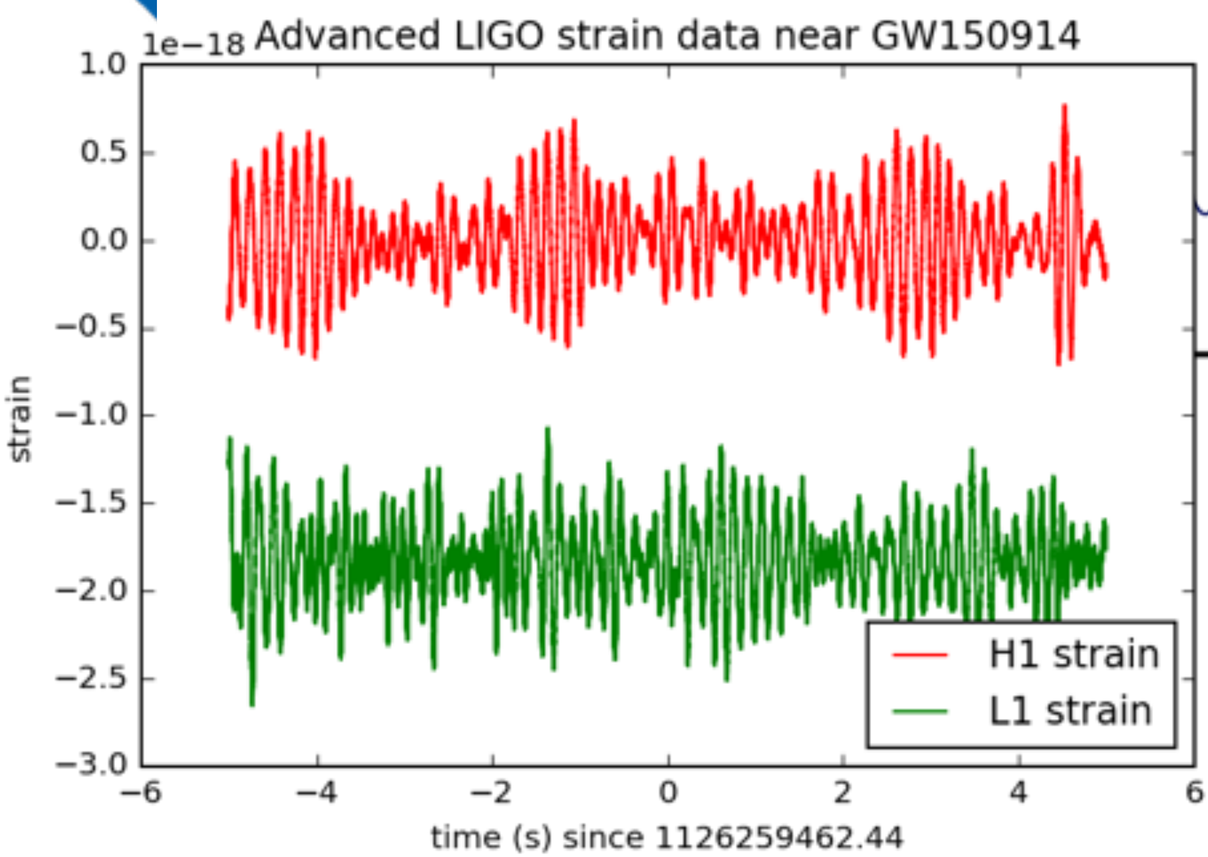
Numerical Relativity (NR)

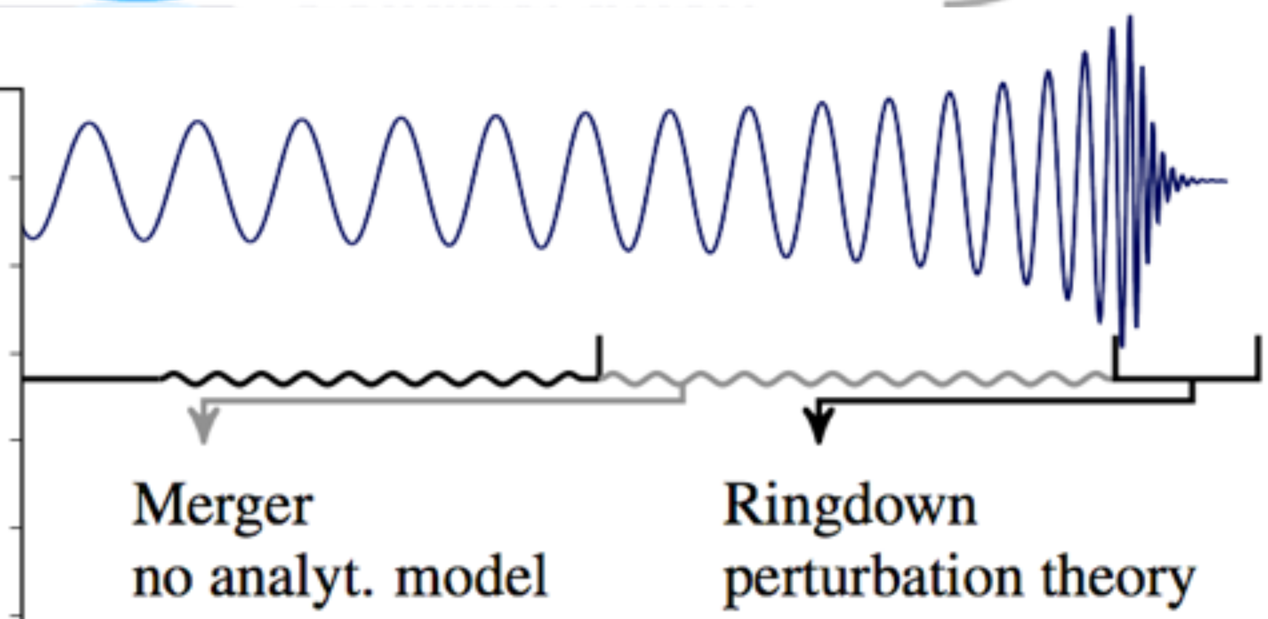
adiabatic inspiral,
rturbation theory.



- Late inspiral & merger: post-Newtonian expansion breaks
  - solve full Einstein equations numerically as PDEs, "match" to post-Newtonian inspiral.
  - Most of the energy released (< 12 % of the mass).
- Ringdown: superposition of damped harmonics.







Numerical Relativity (NR)

How to find a signal buried in the detector noise?

$$h(t,\omega) = e^{i\omega t}$$

Simplest case: sinusoidal -> Idea: Fourier transform!

$$\tilde{x}(f) = \int_{-\infty}^{\infty} x(t) \mathrm{e}^{-2\pi i f t} dt$$

$$\delta(t-t_0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{i\omega t} e^{-i\omega t_0} d\omega$$

$$\delta(\omega-\omega_0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{i\omega t} e^{-i\omega t_0} dt$$

Universitat de les Illes Balears

LSC LIGO Scientific Collaboration

LIGO@UIB

Optimal analysis of data from GW detectors relies on matched filtering with template waveforms <= model solutions of GR

scalar product:

$$(h_1, h_2) = \max_{t_0,\phi_0} 2\,Re \int_{-\infty}^{\infty} \frac{\tilde{h}_1(f)\,\tilde{h}_2^*(f)}{S_n(f)} df$$

signal-to-noise ratio:

$$\sigma = ||h|| = \sqrt{(h,h)}$$

match:

$$M = \max_{t_0,\phi_0} \frac{(h_1, h_2)}{||h_1||\,||h_2||} = \frac{\max_{t_0} |\int \tilde{h}_1 \tilde{h}_2^* e^{i2\pi f t_0}|}{||h_1||\,||h_2||}$$

likelihood:

$$\mathcal{L}(\vec{d}|\vec{\vartheta}) \propto \exp\left[-\frac{1}{2}\sum_{k=1,2}\langle h_k^M(\vec{\vartheta}) - d_k | h_k^M(\vec{\vartheta}) - d_k\rangle\right]$$

fitting factor: $F = \max_{\vec{\theta}} M$

We define the correlation function between two time series $x(t)$ and $y(t)$ for a time shift τ as:

$$R_{xy}(\tau) = \int_{-\infty}^{\infty} x(t) y^\star(t-\tau) dt \,, \tag{1}$$

where a * denotes complex conjugation. Working with Fourier transforms the correlation function can be written as he inverse Fourier transform of $\tilde{x}(f)\tilde{y}^\star(f)$:

$$R_{xy}(\tau) = \int_{-\infty}^{\infty} \tilde{x}(f)\tilde{y}^\star(f) e^{2\pi i f \tau} df \,, \qquad x(t) = \int_{-\infty}^{\infty} \tilde{x}(f) e^{2\pi i f t} df \tag{2}$$

The value of τ for which R_{xy} is maximal determines the time shift required to get the maximum correlation between $x(t)$ and $y(t)$. The self-correlation R_{xx} is maximal for $\tau = 0$:

$$R_{xx}(0) = (x|x) := \int_{-\infty}^{\infty} |\tilde{x}(f)|^2 df \,. \tag{3}$$

More generally we can define the scalar product

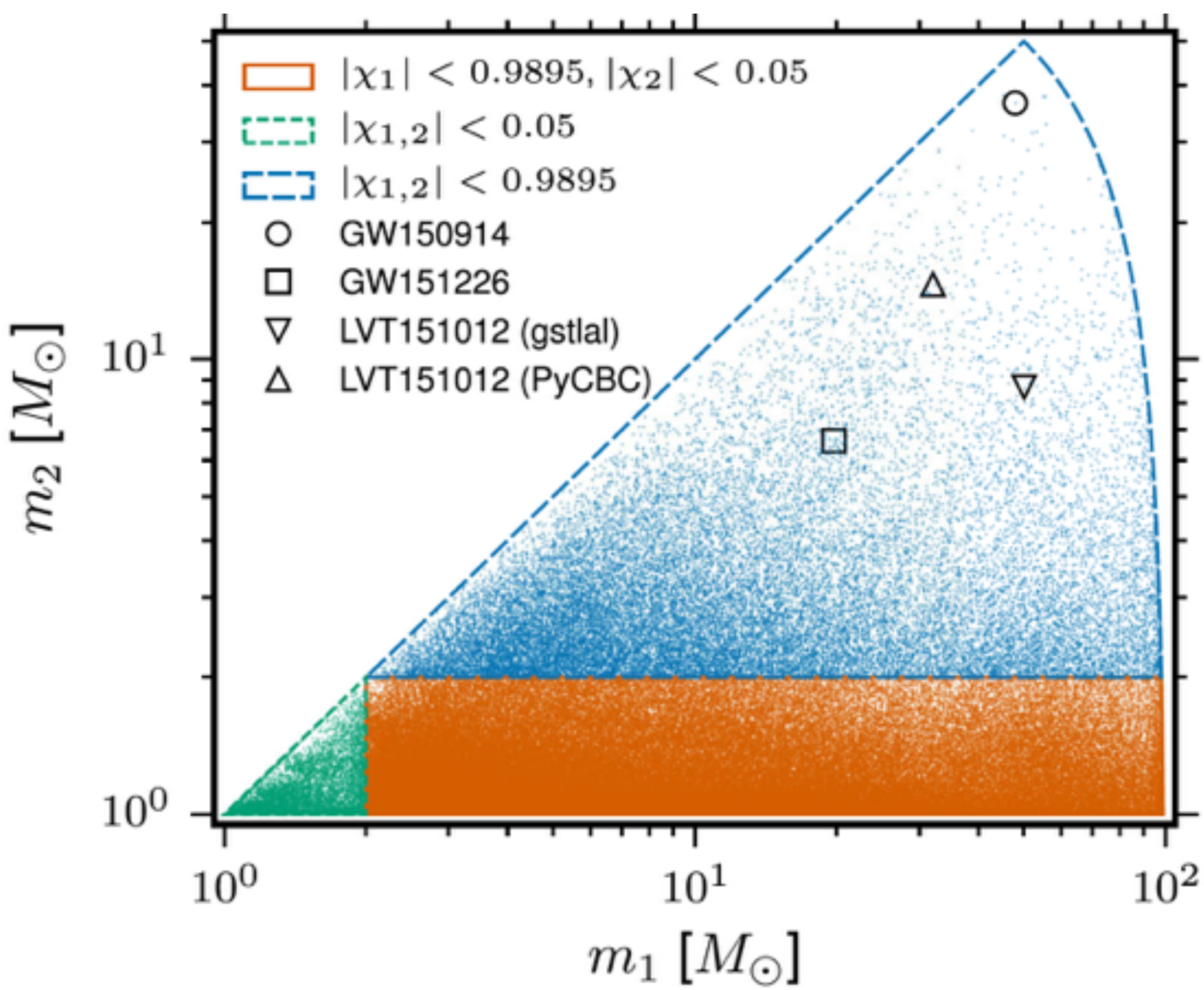
$$(x|y) := \int_{-\infty}^{\infty} \tilde{x}(f)\tilde{y}^\star(f) df \,. \tag{4}$$

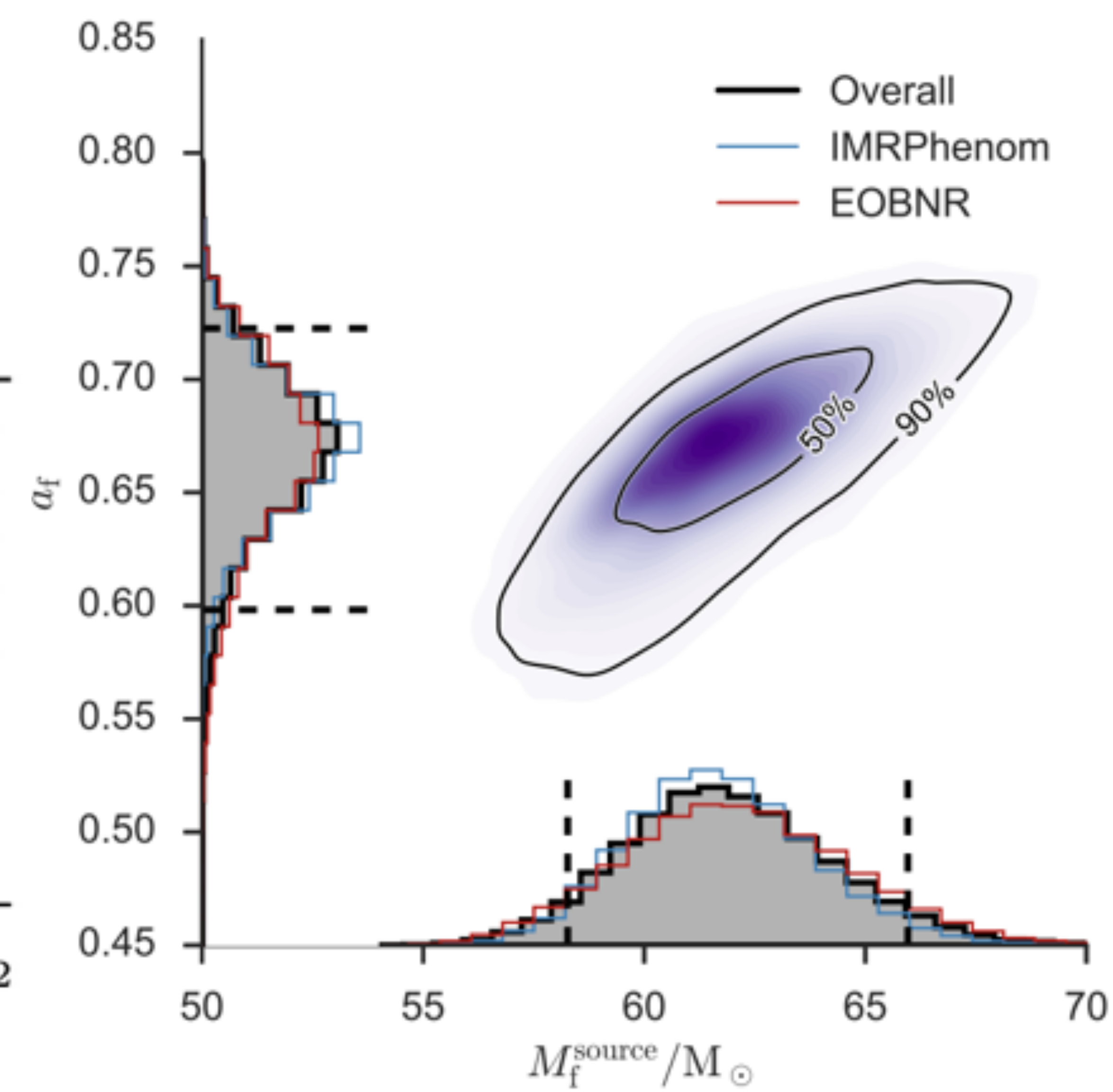
The "match" which determines the efficiency of a template y to identify a signal x is defined as

$$M = \max_\tau \frac{|R_{xy}(\tau)|}{\sqrt{(x|x)(y|y)}}. \tag{5}$$

Typical for GW matched filter analysis: split data analysis problem into 2 parts:

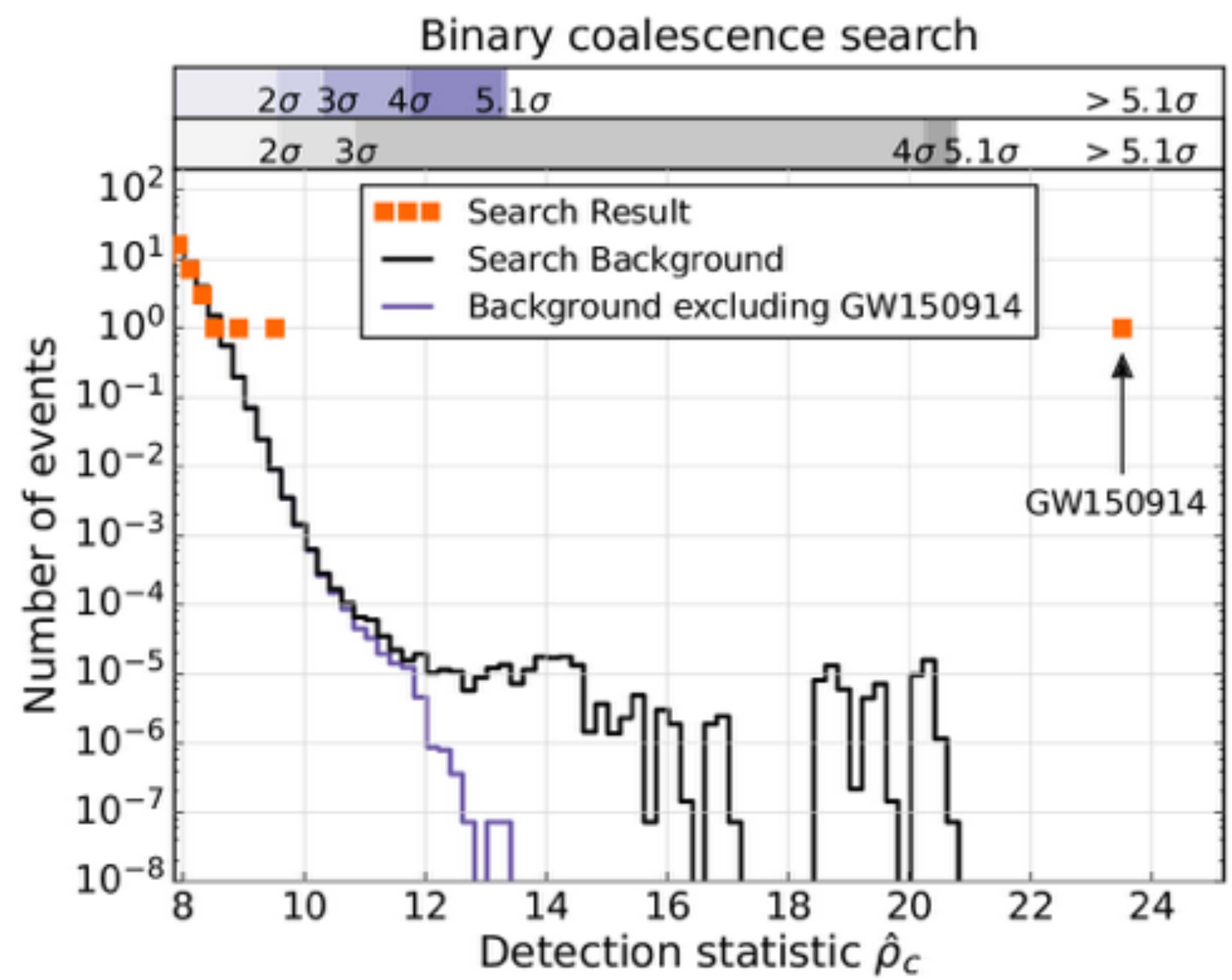
- detection: what is the statistical evidence of seeing a signal above background, fixed template bank [rough parameter estimation].
- Bayesian parameter estimation: vary templates with random walks in parameter space, using MCMC etc., test consistency in waveform models.





Background calculation: GW150914

- Calculate background with 0.1 s time shifts L1 vs H1: 16 days -> 608 000 years.
- trials factor of 3 for 3 bins.
- No equally strong triggers found in time shifted data:



- FAR < 5×10^{-6} yr^{-1}

number of time shifts

- FAP < $2 \times 10^{-7} \simeq 3/(16*24*3600*10) \simeq 5.1\sigma$

$$5.1 = -\sqrt{2}\,\mathrm{erf}^{-1}[1 - 2(1 - \mathrm{FAP})]$$

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Other freely available software for GW science

- LAL/LALApps (LIGO Algorithms Libary)
  - C-library for analysing data from the ground-based interferometer network.
  - Includes search codes for CBC, Continuous Waves (pulsars), unmodelled burst sources, stochastic sources; waveform models; ….
- Einstein Toolkit: open source parallel programming environment for numerical relativity, includes examples for production-level simulations (e.g. reproducing GW150914).

BBH coalescence to leading order:

Newtonian conservative dynamics + leading order radiation reaction.

- Start with energy, e.g. as function of separation R or orbital frequency ω: $E(R)$, $E(\omega)$. Kepler: $\omega^2 R^3 = G M$.

$$E(R) = m_1 + m_2 - M \frac{\eta}{2} \frac{M}{R} \qquad \eta = \frac{m_1 m_2}{(m_1 + m_2)^2}$$

- To compute the rate of change of any quantity X (e.g. $X=\omega$, R) we write

$$\frac{dE}{dt} = -\frac{32}{5} \frac{G^4}{c^5} \eta^2 \left(\frac{v}{c}\right)^{10} \left(1 + O(v^2) + \dots\right) \qquad \frac{dX}{dt} = \frac{\frac{dE}{dt}}{\frac{dE}{dX}}$$

$$v = (GM\omega)^{1/3}$$

$$R(t) = \left(\frac{256}{5} \eta M^3\right)^{\frac{1}{4}} (t_c - t)^{\frac{1}{4}}$$

Fourier domain waveform

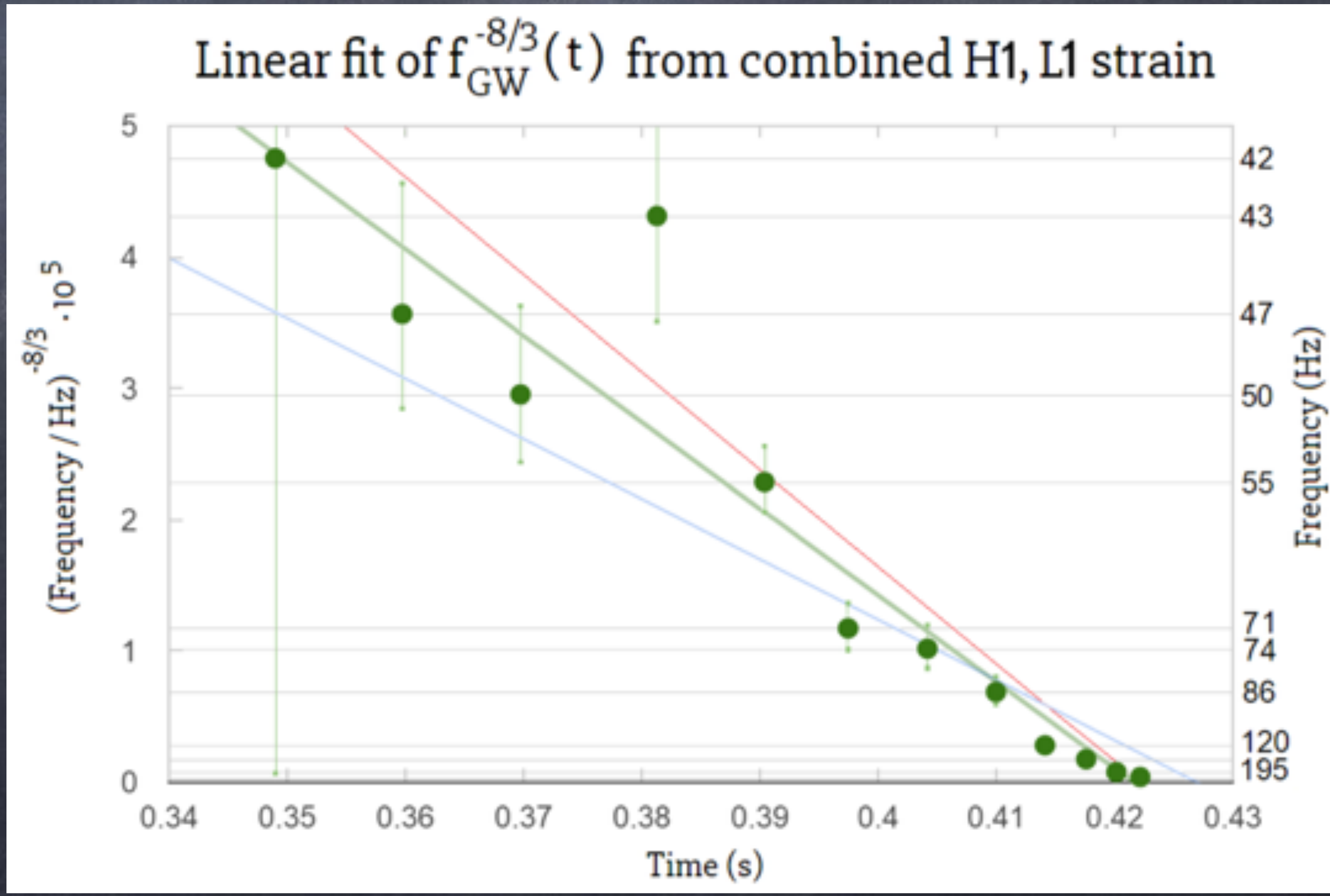
- Stationary phase approximation (SPA)

$$|\tilde{h}| = \frac{2t_{sol}}{t_{dist}} \mathcal{M}^{5/6} \sqrt{\frac{5}{96\pi}} \pi t_{sol} (\pi t_{sol} f)^{-7/6}$$

See also: “The basic physics of the binary black hole merger GW150914”, LSC+Virgo, https://arxiv.org/abs/1608.01940

$$\mathcal{M} = \frac{c^3}{G}\left(\left(\frac{5}{96}\right)^3 \pi^{-8} (f_{\mathrm{GW}})^{-11} (\dot{f}_{\mathrm{GW}})^3\right)^{1/5}$$

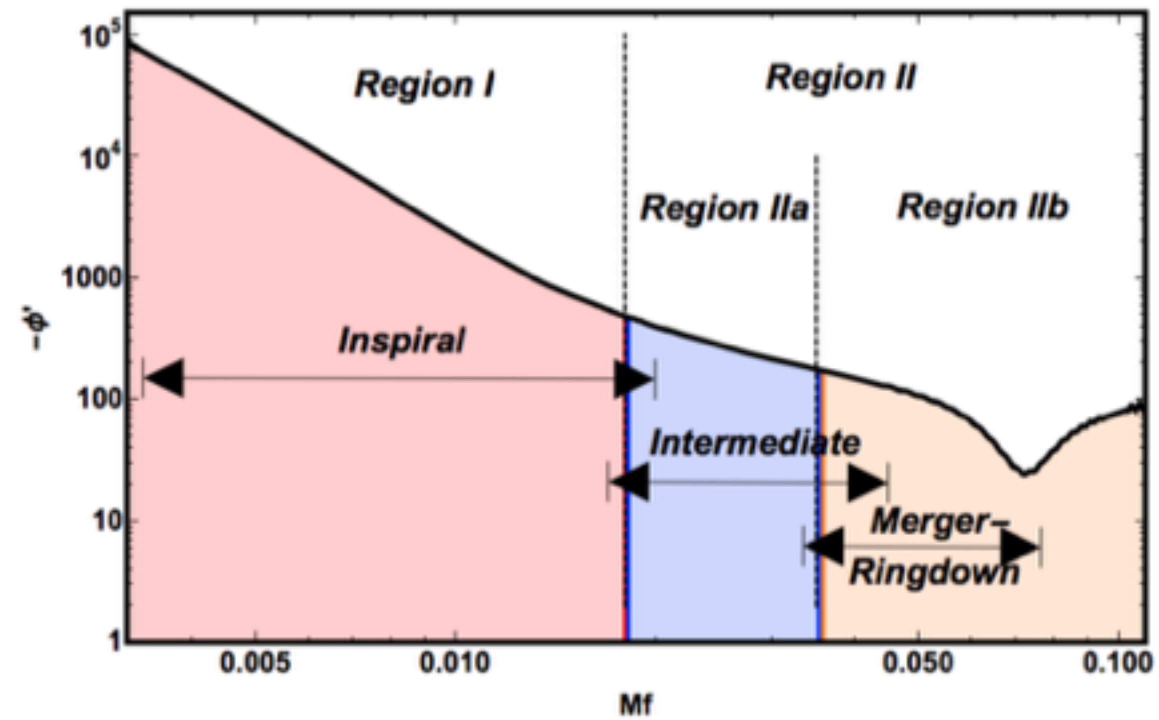
$$f_{\mathrm{GW}}^{-8/3}(t) = \frac{(8\pi)^{8/3}}{5}\left(\frac{G\mathcal{M}}{c^3}\right)^{5/3}(t_c - t)$$

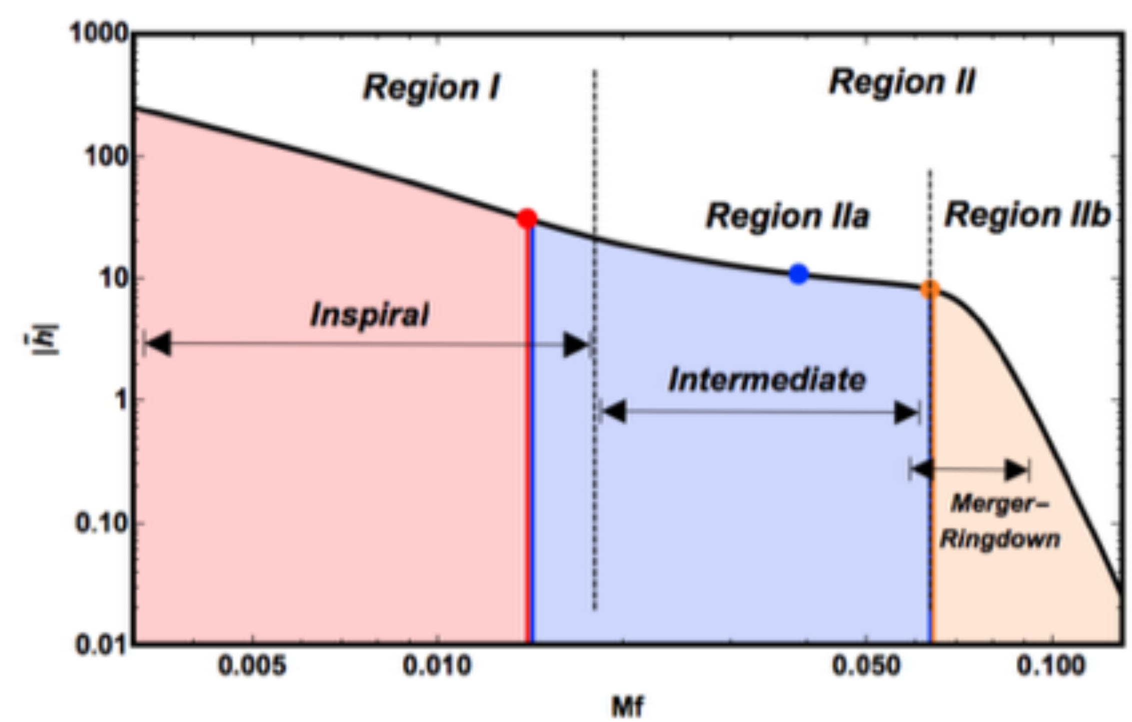


Inspiral-Merger-Ringdown Waveform models

- 2 main families: EOBNR, Phenom
  - calibrated to effective-one-body resumption of post-Newtonian perturbation scheme

    + numerical relativity solutions of full GR
- EOBNR: time domain, integrate ODEs
  - used in our example
- Phenom: frequency domain, piecewise analytic.





IMRPhenom*,
gIMR:

Free implementation in LAL C-library and Mathematica.

Take away the model, and modify it, e.g. to test General Relativity, implement Lorentz violation, alternative gravity models, etc ….

Use it for astrophysics: models final spin, radiated energy, …

Get LIGO data from <u>losc.ligo.org</u> and look at the data yourself!

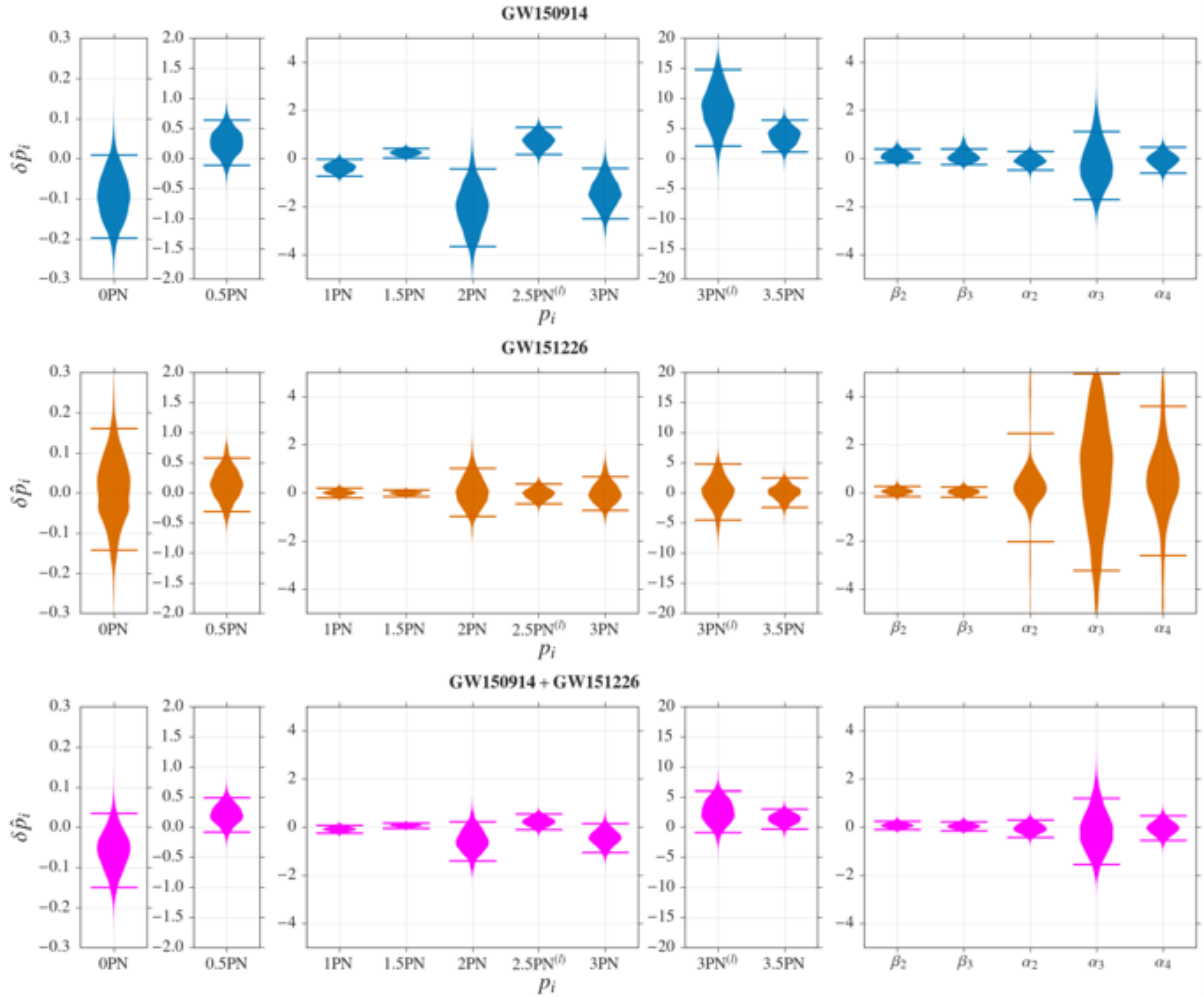


FIG. 6. Posterior density distributions and 90% credible intervals for relative deviations $\delta\hat{p}_i$ in the PN parameters p_i, as well as intermediate parameters β_i and merger-ringdown parameters α_i. The top panel is for GW150914 by itself and the middle one for GW151226 by itself, while the bottom panel shows *combined* posteriors from GW150914 and GW151226. While the posteriors for deviations in PN coefficients from GW150914 show large offsets, the ones from GW151226 are well-centered on zero as well as being more tight, causing the combined posteriors to similarly improve over those of GW150914 alone. For deviations in the β_i, the combined posteriors improve over those of either event individually. For the α_i, the joint posteriors are mostly set by the posteriors from GW150914, whose merger-ringdown occurred at frequencies where the detectors are the most sensitive.

Exercises

- Compare 3 events.
- Print & plot intermediate quantities, plot Newton+Quadrupole SPA waveform together with BBH template and noise. Estimate time delay between detectors.
- Waveform anatomy: determine chirp mass, total mass (assume equal masses, source seen face-on), ringdown frequency, estimate final spin.
- Plot horizon distance as a function of mass.
- Mass scaling of template:construct templates corresponding to different masses, match between templates corresponding to different masses, compare data with template of different mass.
- Plot appropriately time-shifted data of both detectors, plot time-shifted residuals of both detectors.

$$\frac{\mathrm{d}E_{\mathrm{GW}}}{\mathrm{d}t} = \frac{c^3}{16\pi G}\iint |\dot{h}|^2 \mathrm{d}S$$

GW luminosity

Real and imaginary ringdown frequency as a function of final Kerr parameter.