

# Lorentz violations: from quantum gravity to black holes

Thomas P. Sotiriou



The University of  
**Nottingham**

UNITED KINGDOM • CHINA • MALAYSIA



European Research Council

Established by the European Commission

**Supporting top researchers  
from anywhere in the world**



# Motivation

**Test Lorentz symmetry!**



# Motivation

Old problem: Can we make gravity renormalizable?

Possible solution: Add higher-order curvature invariants

- Higher order term contain higher order time derivatives
- This introduces ghosts!

Simple solution: give up Lorentz invariance. Then

- Higher order spatial derivatives without higher order time derivatives, i.e. no ghosts
- Renormalizable theory (at the power counting level)

Well, maybe not that simple...



# Lifshitz scalar

Consider the action

$$S = \int dt dx^d \left( \dot{\phi}^2 - a_m \phi (-\Delta)^m \phi + g_n \phi^n \right)$$

It is then natural to choose the scaling dimensions

$$[dt] = [\kappa]^{-z} \quad [dx] = [\kappa]^{-1}$$

which implies that

$$[\phi] = [\kappa]^{(d-z)/2} \quad [a_m] = [\kappa]^{2(z-m)} \quad [g_n] = [\kappa]^{d+z-n(d-z)/2}$$

So long as

$$z \geq d$$

the theory is power counting renormalizable.



# Lifshitz gravity

We first need to split spacetime into space and time by introducing a preferred foliation

$$ds^2 = -N^2 c^2 dt^2 + g_{ij}(dx^i - N^i dt)(dx^j - N^j dt)$$

Remaining symmetry: “foliation preserving diffeomorphisms”

- Time reparametrization:  $t \rightarrow \tilde{t}(t)$
- Spacetime-dependent 3-diffeos:  $x^i \rightarrow \tilde{x}^i(t, x^i)$

Most general action

$$S = \frac{M_{\text{pl}}^2}{2} \int d^3x dt N \sqrt{g} \left\{ K^{ij} K_{ij} - \lambda K^2 - V(g_{ij}, N) \right\}$$

P. Hořava, Phys. Rev. D 79, 084008 (2009)



# General features

- The potential should be at least 6th order in spatial derivatives

$$V = -\xi R - \eta a_i a^i + \frac{1}{M_\star^2} L_4 + \frac{1}{M_\star^4} L_6 \quad a_i = \partial_i \ln N$$

- There are 2 types of Lorentz-violating terms: those that come at lower order and those that come at higher order
- The theory propagates a scalar mode
- Generically there will be more than 60 couplings!
- So far perfectly consistent and viable theory (with certain assumptions about matter coupling)

D. Blas, O. Pujolas and S. Sibiryakov, Phys. Rev. Lett. 104, 181302 (2010)



# Projectable version

P. Hořava, Phys. Rev. D 79, 084008 (2009)  
T.P.S., M. Visser, S. Weinfurtner, PRL 102, 064035 (2009)  
JHEP 0910, 033 (2009)

- The lapse is space-independent
- Drastic simplification - 9 couplings only
- Same degrees of freedom
- Severe infrared viability issues

T.P.S., M. Visser, S. Weinfurtner, JHEP 0910, 033 (2009)  
D. Blas, O. Pujolas, S. Sibiryakov, JHEP 0910, 029 (2009)  
K.Koyama, F. Arroja, JHEP 1003, 061 (2010)

- Shown to be renormalizable!

A. O. Barvinsky et al., PRD 93, 064022 (2016)



# Einstein-aether theory

The action of the theory is

$$S_{\text{ae}} = \frac{1}{16\pi G_{\text{ae}}} \int d^4x \sqrt{-g} (-R - M^{\alpha\beta\mu\nu} \nabla_\alpha u_\mu \nabla_\beta u_\nu)$$

where

$$M^{\alpha\beta\mu\nu} = c_1 g^{\alpha\beta} g^{\mu\nu} + c_2 g^{\alpha\mu} g^{\beta\nu} + c_3 g^{\alpha\nu} g^{\beta\mu} + c_4 u^\alpha u^\beta g_{\mu\nu}$$

and the aether is implicitly assumed to satisfy the constraint

$$u^\mu u_\mu = 1$$

- Most general theory with a unit timelike vector field which is second order in derivatives

T. Jacobson and D. Mattingly, Phys. Rev. D 64, 024028 (2001).



# Hypersurface orthogonality

Now assume

$$u_\alpha = \frac{\partial_\alpha T}{\sqrt{g^{\mu\nu} \partial_\mu T \partial_\nu T}}$$

and choose  $T$  as the time coordinate

$$u_\alpha = \delta_{\alpha T} (g^{TT})^{-1/2} = N \delta_{\alpha T}$$

Replacing in the action and defining one gets

$$S_{\infty}^{ho} = \frac{1}{16\pi G_H} \int dT d^3x N \sqrt{h} \left( K_{ij} K^{ij} - \lambda K^2 + \xi^{(3)} R + \eta a^i a_i \right)$$

with  $a_i = \partial_i \ln N$  and the parameter correspondence

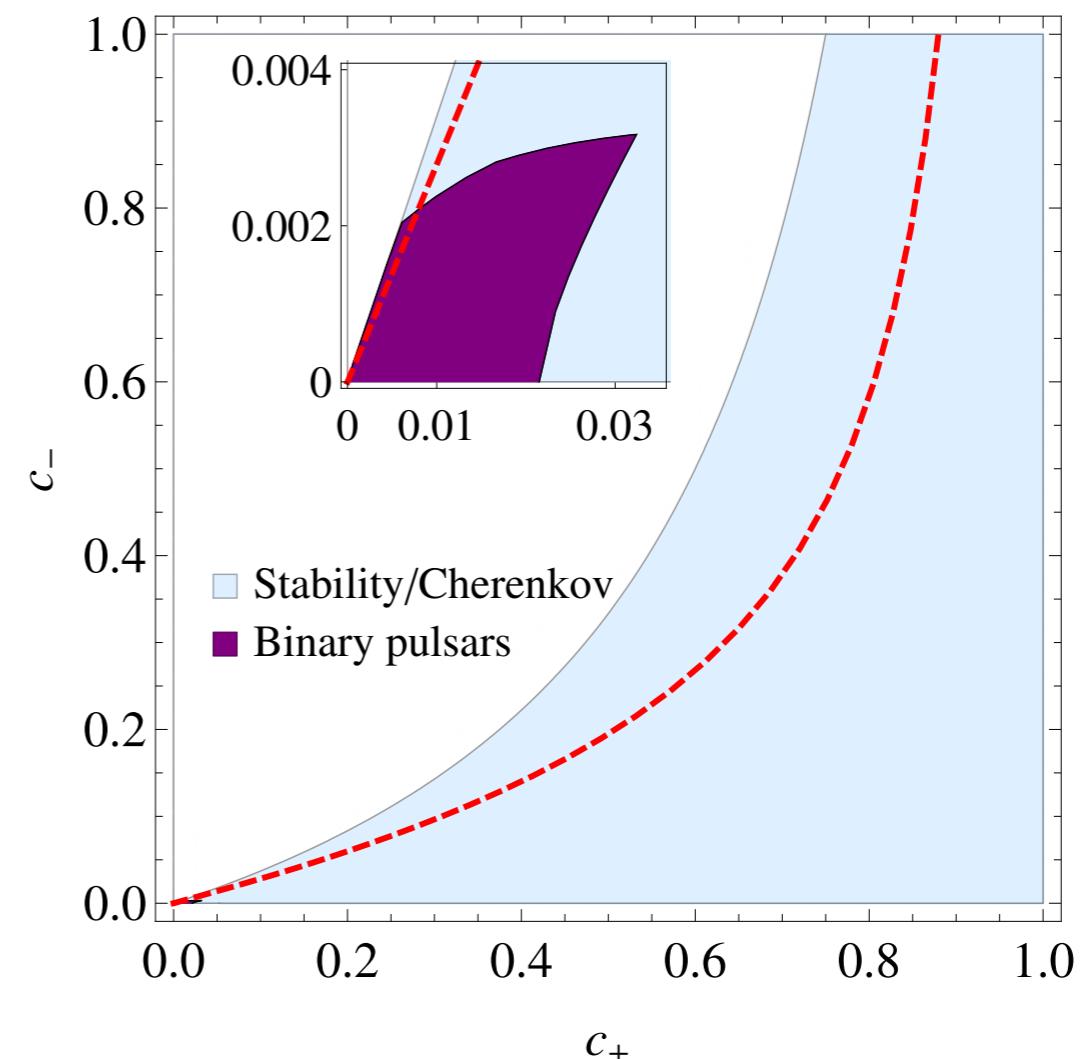
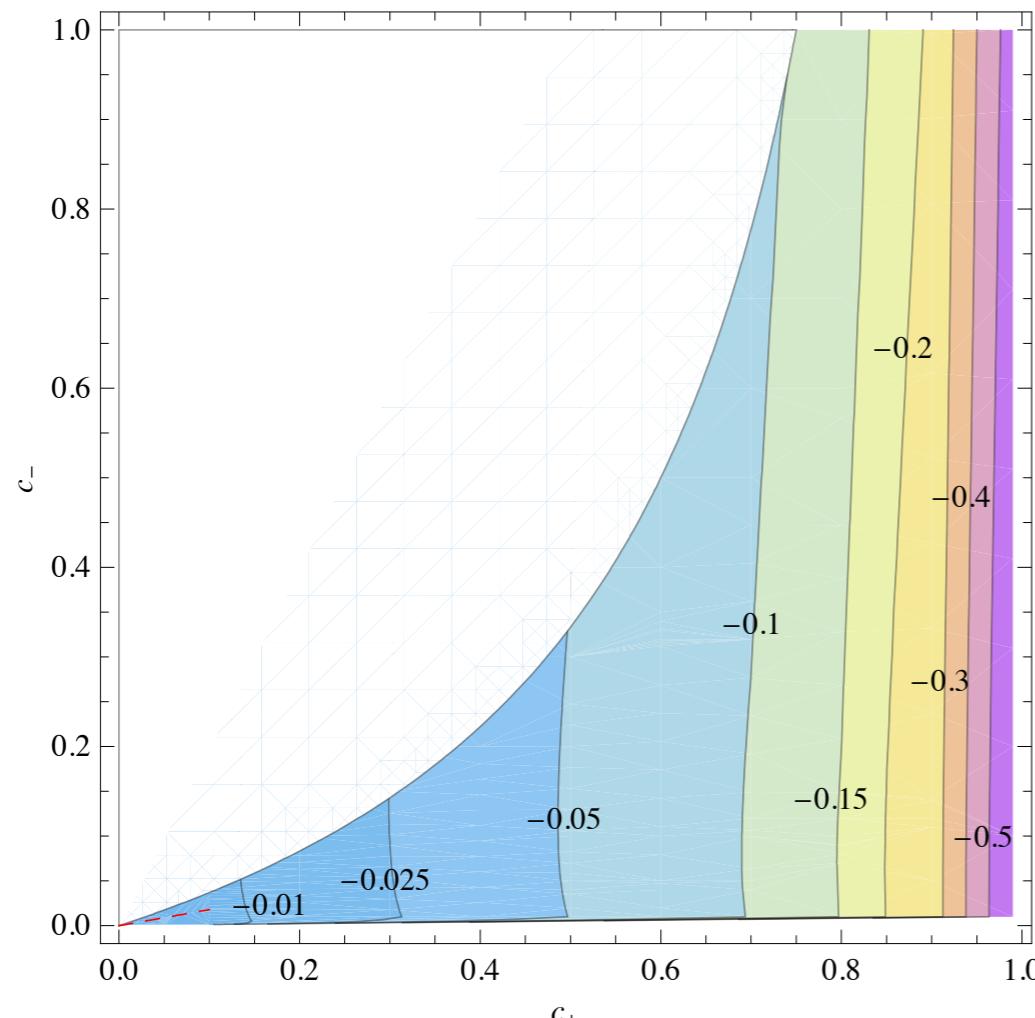
$$\frac{G_H}{G_\infty} = \xi = \frac{1}{1 - c_{13}} \quad \lambda = \frac{1 + c_2}{1 - c_{13}} \quad \eta = \frac{c_{14}}{1 - c_{13}}$$

T. Jacobson, Phys. Rev. D 81, 101502 (2010).



# Combined Constraints

## Einstein-aether theory



$$\omega_{\text{ISCO}} r_g$$

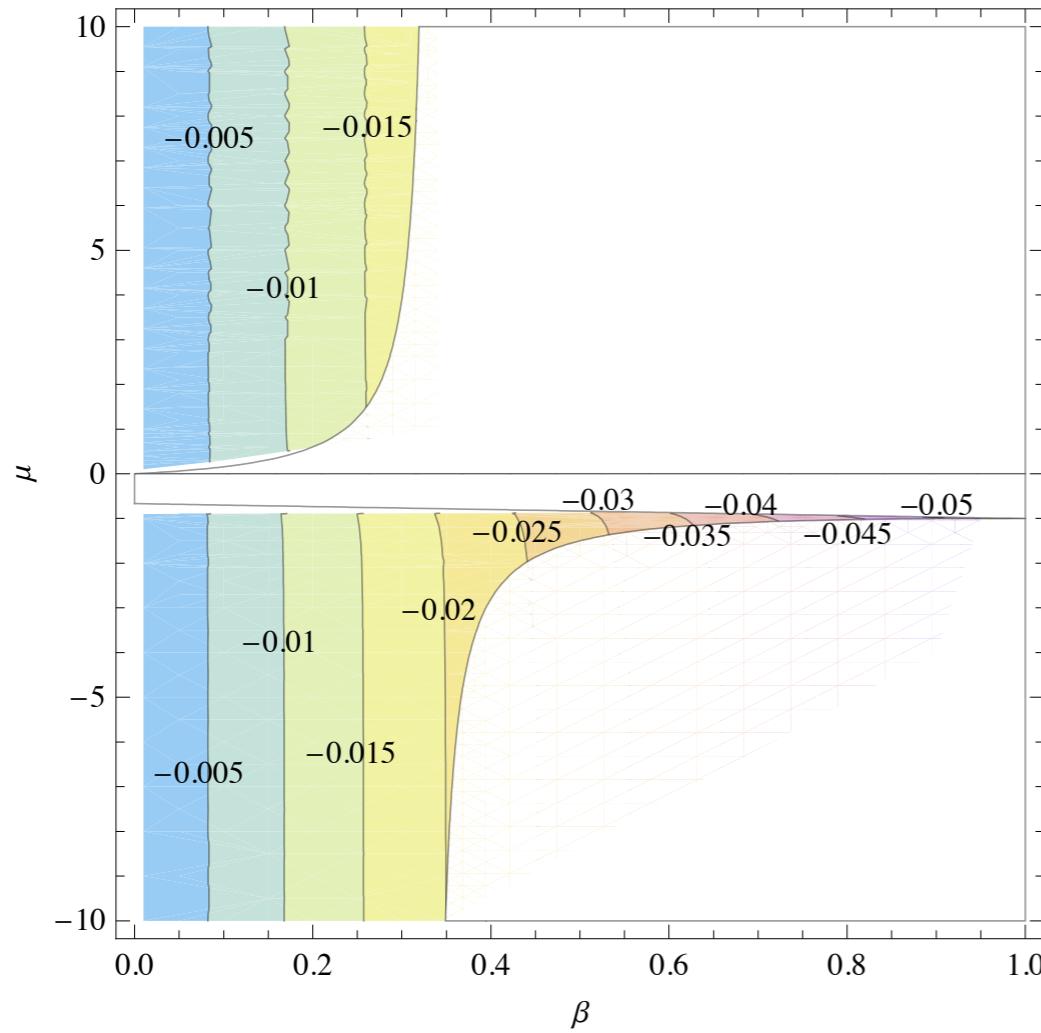
E. Barausse, T. Jacobson and T.P.S., Phys. Rev. D 83, 124043 (2011)

K. Yagi, D. Blas, N. Yunes and E. Barausse Phys. Rev. Lett. 112, 161101 (2014)

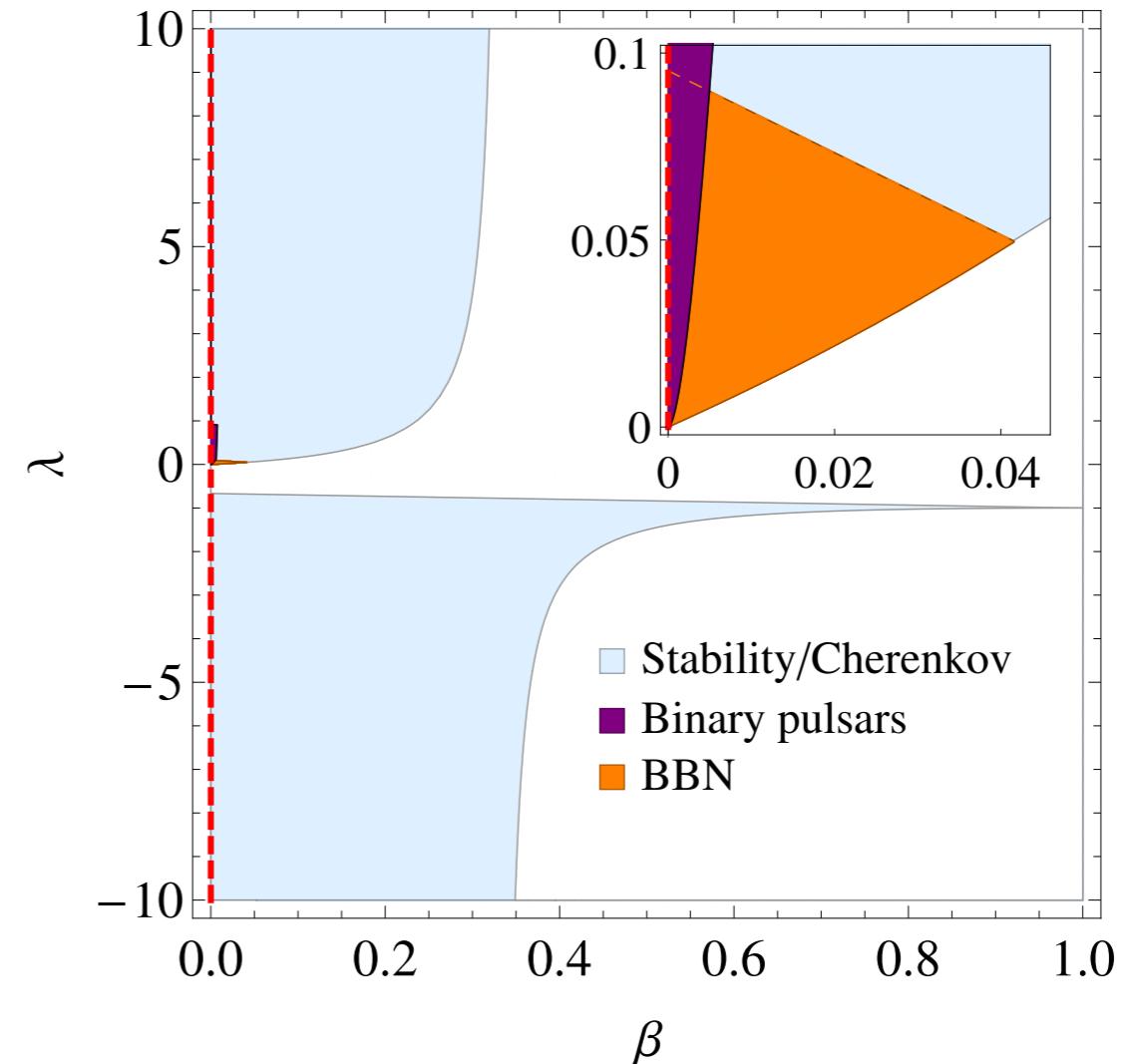


# Combined Constraints

## Horava gravity



$$\omega_{\text{ISCO}} r_g$$



E. Barausse, T. Jacobson and T.P.S., Phys. Rev. D 83, 124043 (2011)

K. Yagi, D. Blas, N. Yunes and E. Barausse Phys. Rev. Lett. 112, 161101 (2014)



# Percolations of LV

But what about the matter sector and lower order operators?

- Different speeds for different fields in the IR, with logarithmic running!

R. Iengo, J. G. Russo and M. Serone, JHEP 0911, 020 (2009)

Possible ways out:

- Some extra symmetry, e.g. supersymmetry

S. Groot Nibbelink and M. Pospelov, Phys. Rev. Lett. 94, 081601 (2005)

- Assume Lorentz symmetry in matter and let the weak coupling to gravity (the Lorentz-violating sector) do the rest

M. Pospelov and Y. Shang, Phys. Rev. D 85, 105001 (2012)



# Strong coupling

The low energy action exhibits strong coupling at energy

$$M_{\text{sc}} = f(|\lambda - 1|, \eta) M_{\text{pl}}$$

Can be a large scale, but problem with renormalizability!

- A. Papazoglou and T. P. S., Phys. Lett. B 685, 197 (2010)
- I. Kimpton and A. Padilla, JHEP 1007, 014 (2010)

Strong coupling problem can be circumvented if

$$M_{\text{sc}} > M_{\star}$$

- D. Blas, O. Pujolas and S. Sibiryakov, Phys.Lett. B 688, 350 (2010)

But then potential tension with observations!

$$10^{16} \text{ GeV} > M_{\star} > M_{\text{obs}}$$

- A. Papazoglou and T. P. Sotiriou, Phys. Lett. B 685, 197 (2010)



# Hierarchy of scales

Consider the dispersion relation

$$E^2 = m^2 + p^2 + \eta_4 \frac{p^4}{M_{LV}^2} + \mathcal{O}\left(\frac{p^6}{M_{LV}^4}\right)$$

Assume that there is a universal LV scale, so

$$M_{LV} \sim M_\star$$

Constraint from synchrotron radiation from the Crab Nebula:

$$M_{\text{obs}} > 2 \times 10^{16} \text{ GeV}$$

$M_\star$  cannot be a universal scale!

S. Liberati, L. Maccione and T. P. S., Phys. Rev. Lett. 109, 151602 (2012)



# Pertinent questions

- Renormalization group flow: Do couplings get the values we want them to get?
- Quantization: some real quantum gravity predictions
- Matter coupling and relevant (possibly worrisome) phenomenology
  - I. Kimpton and A. Padilla, JHEP 1304, 133 (2013)
  - M. Colombo, A. E. Gumrukcuoglu, and T.P.S., Phys. Rev. D 91, 044021 (2015);  
Phys. Rev. D 92, 064037 (2015)
  - A. Coates, M. Colombo, A. E. Gumrukcuoglu, and T.P.S., Phys. Rev. D 94, 084014 (2016)
- Vacuum energy
- Causal structure: do we understand it?
- Black holes and singularities: are there black holes? Are singularities resolved?



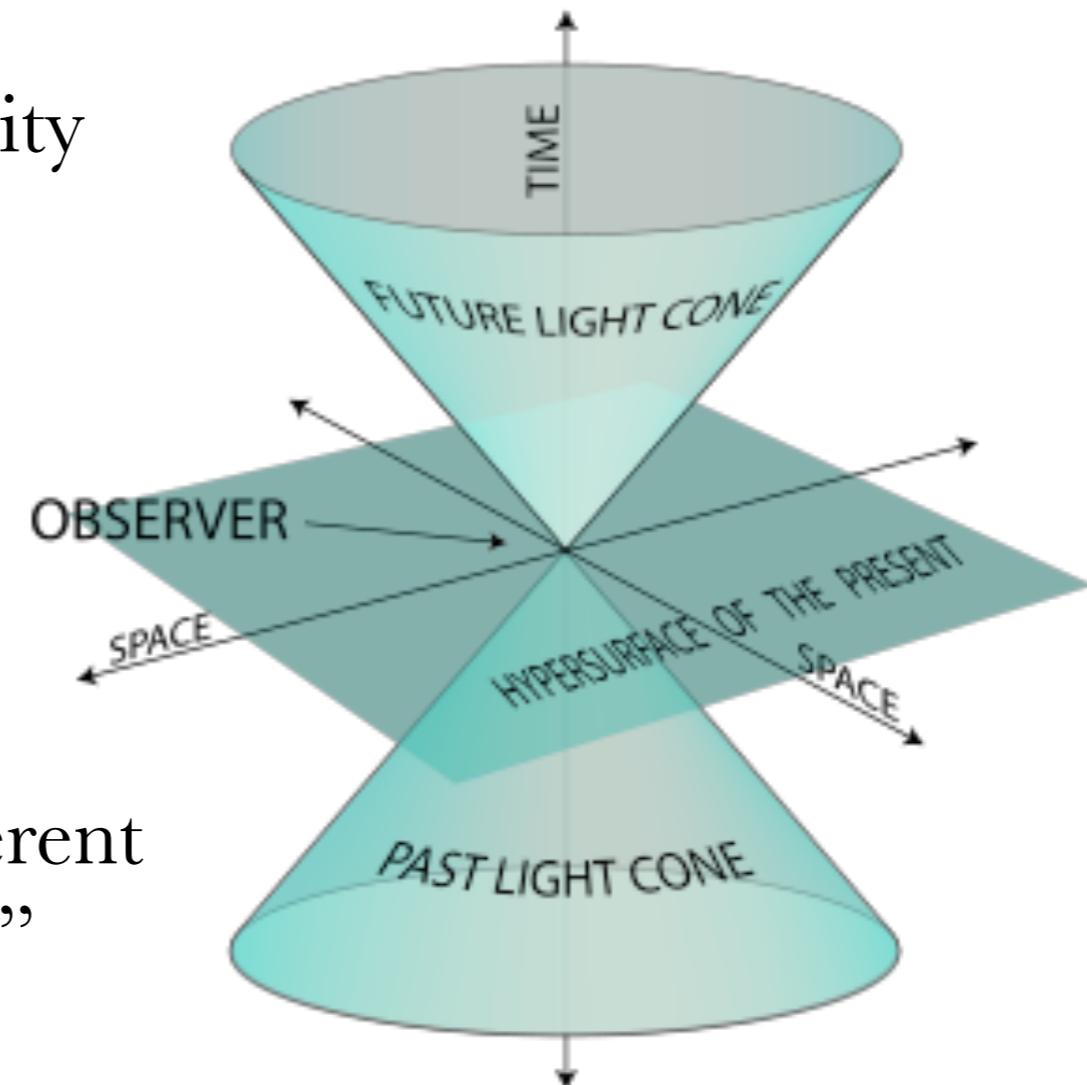
# LV and causal structure

Causal structure in special relativity

- LV with linear dispersion relations

$$\omega \propto k$$

- Different modes have different speeds and different “light” cones
- But there are still “light” cones!

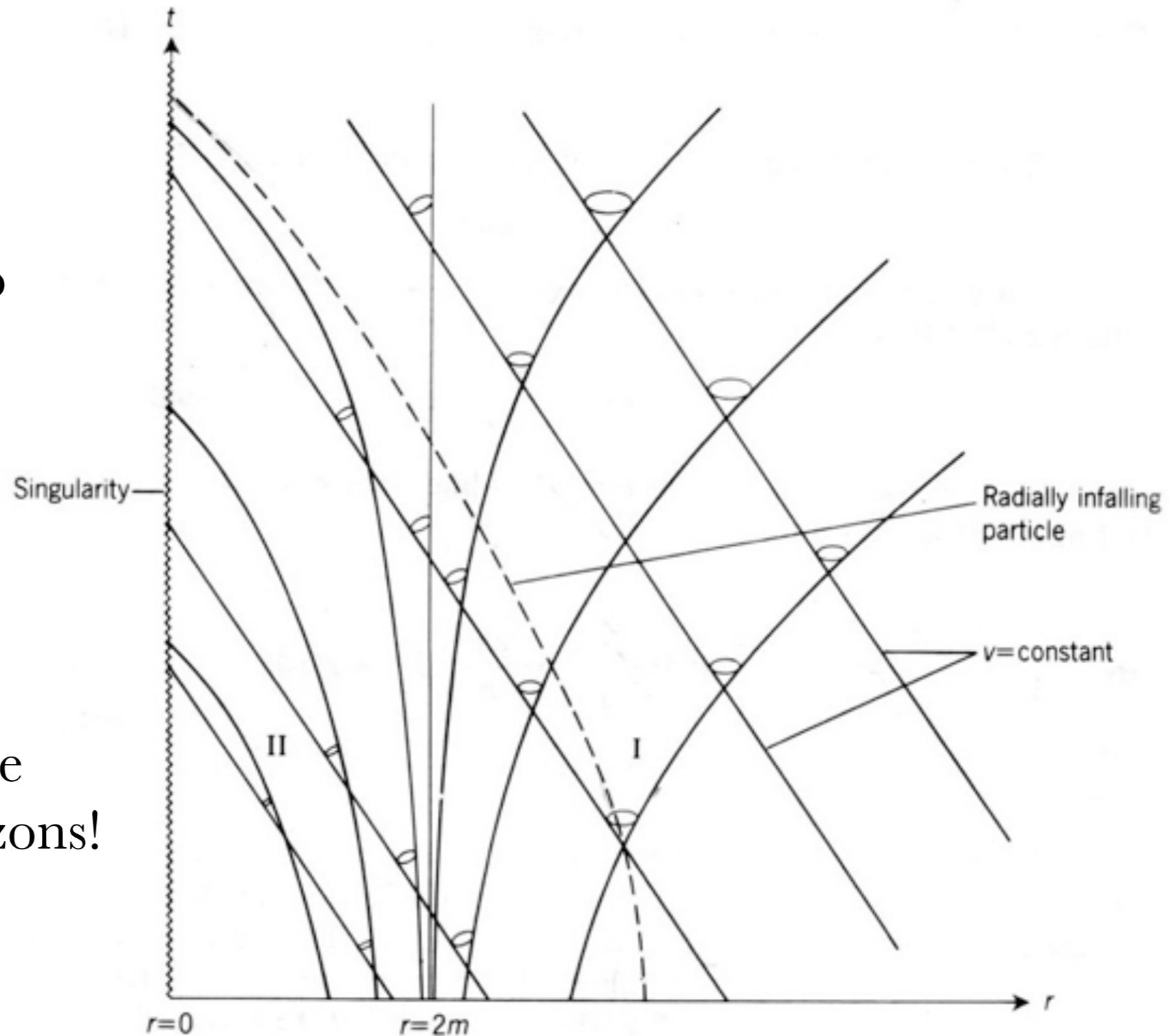




# LV and black hole structure

What happens to  
black holes?

- They will have multiple horizons!





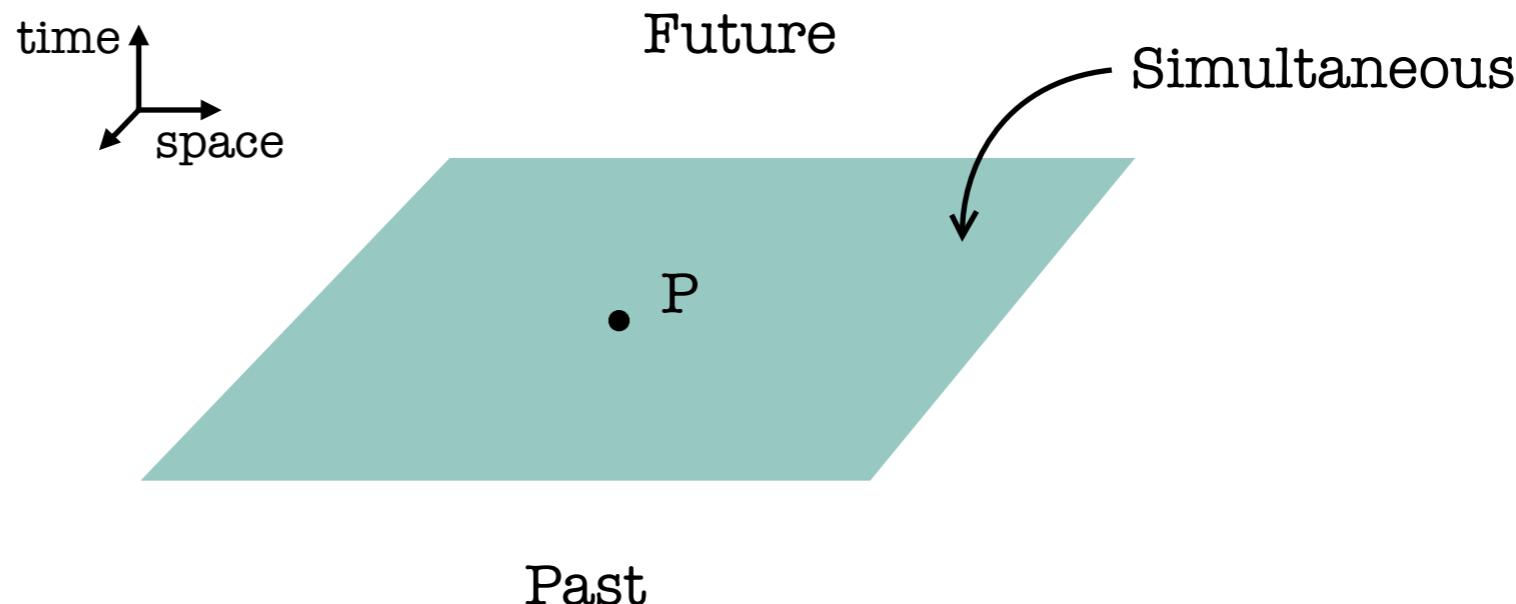
# LV and black hole structure

- LV with non-linear dispersion relations

$$\omega^2 \propto k^2 + ak^4 + \dots$$

- No light cones!

Causal structure without relativity

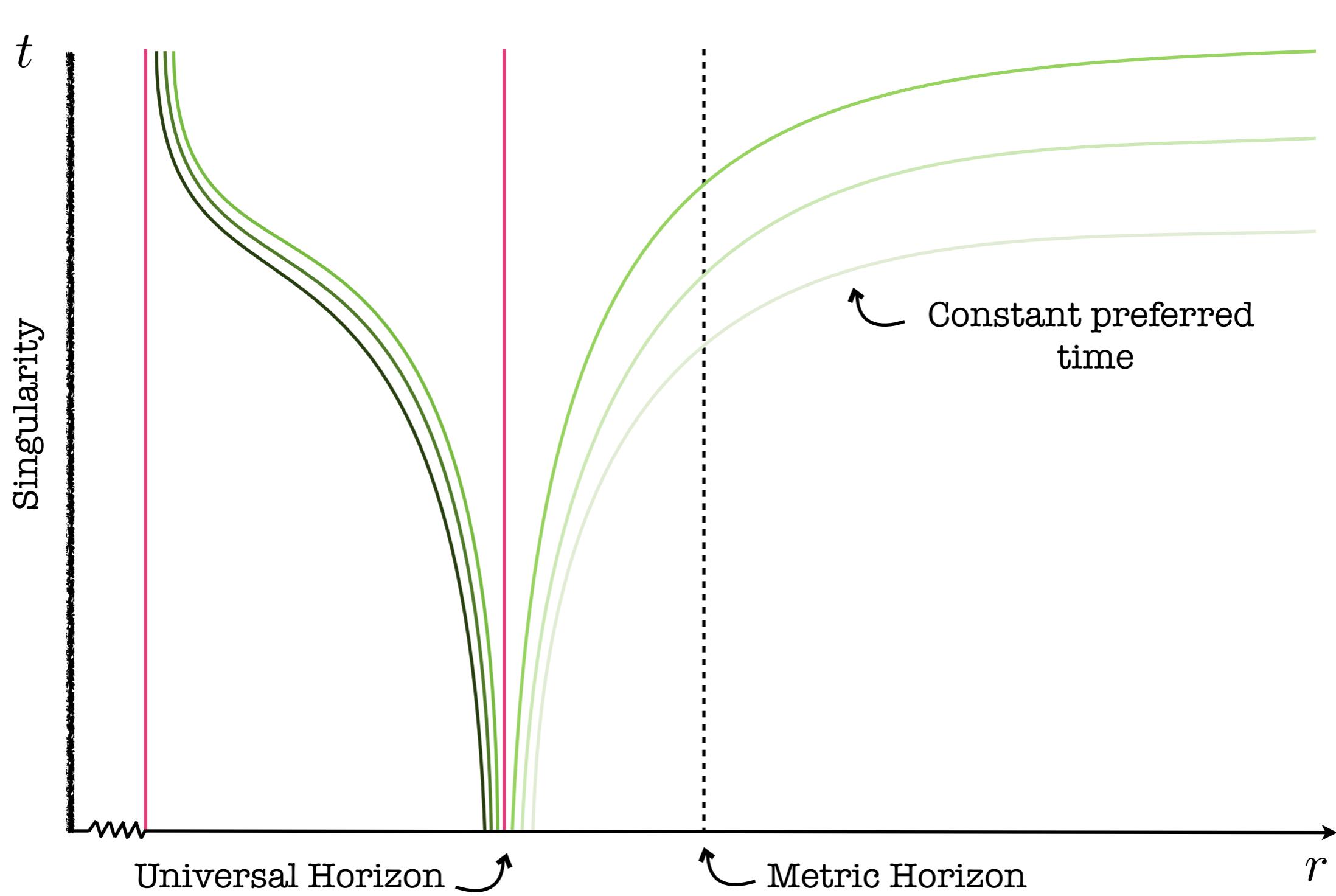


No black holes at all??



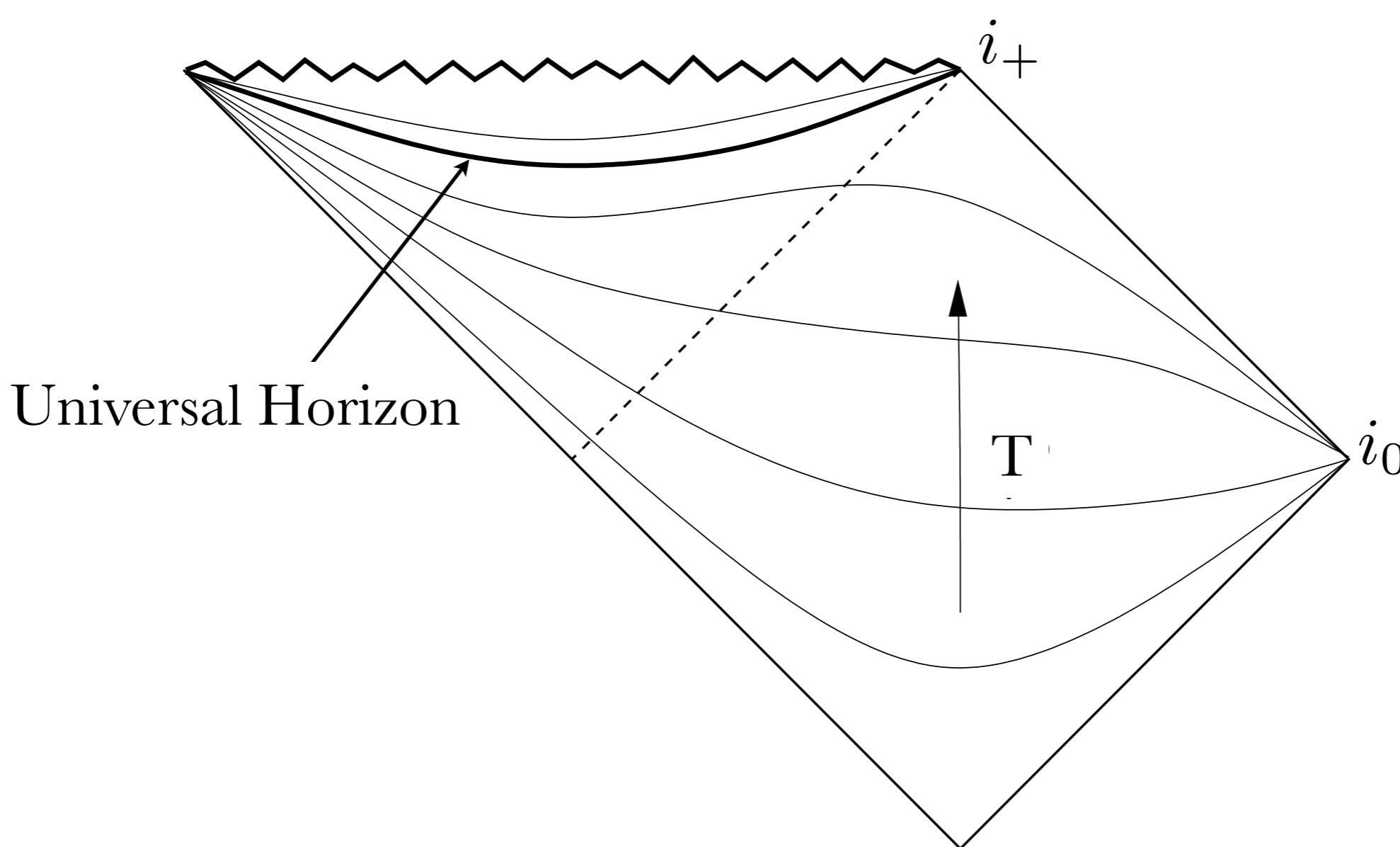
# Spacetime diagram

E. Barausse, T. Jacobson and T.P.S., Phys. Rev. D 83, 124043 (2011)





# Penrose diagram



Taken from D. Blas and S. Sibiryakov, Phys. Rev. D 84, 124043 (2011)



# Rotating black holes

- Slowly rotating BHs in Einstein-aether theory do not have a preferred foliation.
- Slowly rotating BHs in Horava gravity have universal horizons.
  - E. Barausse and T.P.S., Phys. Rev. Lett. 109, 181101 (2012)
  - E. Barausse and T.P.S., Phys. Rev. D 87, 087504 (2013)
  - E. Barausse and T.P.S., Class. Quant. Grav 30, 244010 (2013)
  - E. Barausse, I. Vega and T.P.S., Phys. Rev. D 93, 044044 (2016)
- 3d rotating black holes can have universal horizons even with flat asymptotics.
- Universal horizons can lie “outside” de Sitter horizons.

T.P.S., I. Vega and D. Vernieri, Phys. Rev. D 90, 044046 (2014)



# Beyond exact solutions

- A new “toolkit” is needed

M. Colombo, J. Bhattacharyya, and T.P.S., Class. Quant. Grav. 33, 235003 (2016).

- Can we define this horizon in full generality?

Yes!

- Can we have a local definition when we have less symmetry?

## Theorem

$(u \cdot \chi) = 0, \quad (a \cdot \chi) \neq 0$  form a set of necessary and sufficient conditions for a hypersurface to be a universal horizon

- Is the universal horizon relevant to astrophysics?

No, it lies always behind the usual horizon



# Spherical collapse

J. Bhattacharyya, A. Coates, M. Colombo, and T.P.S., Phys. Rev. D 93, 064056 (2016).

One can make the ansatz

$$ds^2 = -N^2dT^2 + S^2(N^RdT + dR)^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

and then the “ $T$ -equation” takes the form

$$\partial_R \left[ r^2 S N^2 \vec{A}^R \right] = 0 \quad \Leftrightarrow \quad (s \cdot \vec{A}) = \frac{f_{IM}(T)}{r^2 N^2}$$

- Horava gravity has an instantaneous mode!
- The universal horizon corresponds to  $N \rightarrow \infty$
- This foliation was used for simulations

D. Garfinkle, C. Eling and T. Jacobson, Phys. Rev. D 76, 024003 (2007)

- It does not penetrate the horizon!



# Perspectives

- LV QG: Well-defined candidate, testable predictions
- Major IR viability issues resolved (in some versions)
- Some very interesting predictions. I did not mention:
  - Cosmology: scale invariant spectrum, no horizon problem, novel dark energy model, ...  
S. Mukohyama, Class. Quant. Grav. 27, 223101 (2010)  
D. Blas and S. Sibiryakov, JCAP 1107, 026 (2011)
  - Possible contact with discrete QG (CDT)  
P. Hořava, Phys. Rev. Lett. 102, 161301 (2009)  
T. P. Sotiriou, M. Visser and S. Weinfurtner, Phys. Rev. Lett. 107, 131303 (2011)
- Major challenges ahead: renormalization group flow, quantization
- What about quantum predictions?



# Perspectives

- Black holes are of great interest in Lorentz-violating theories. New notion: “universal horizon”
- Non-trivial causal structure
- Is this horizon stable?
- Does it form from collapse?

D. Blas and S. Sibiryakov, Phys. Rev. D 84, 124043 (2011)

M. Saravani, N. Afshordi and R. B. Mann, Phys. Rev. D 89, 084029 (2014)

J. Bhattacharyya, A. Coates, M. Colombo, and T.P.S., Phys. Rev. D 93, 064056 (2016).