

asymptotic safety a pathway to quantum gravity

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**Lecture at YETI 2017
IPPP
10 Jan 2017**

standard model

local QFT for fundamental interactions

strong nuclear force

weak force

electromagnetic force

degrees of freedom

spin 0 (the **Higgs** has finally arrived)

spin 1/2 (quite a few)

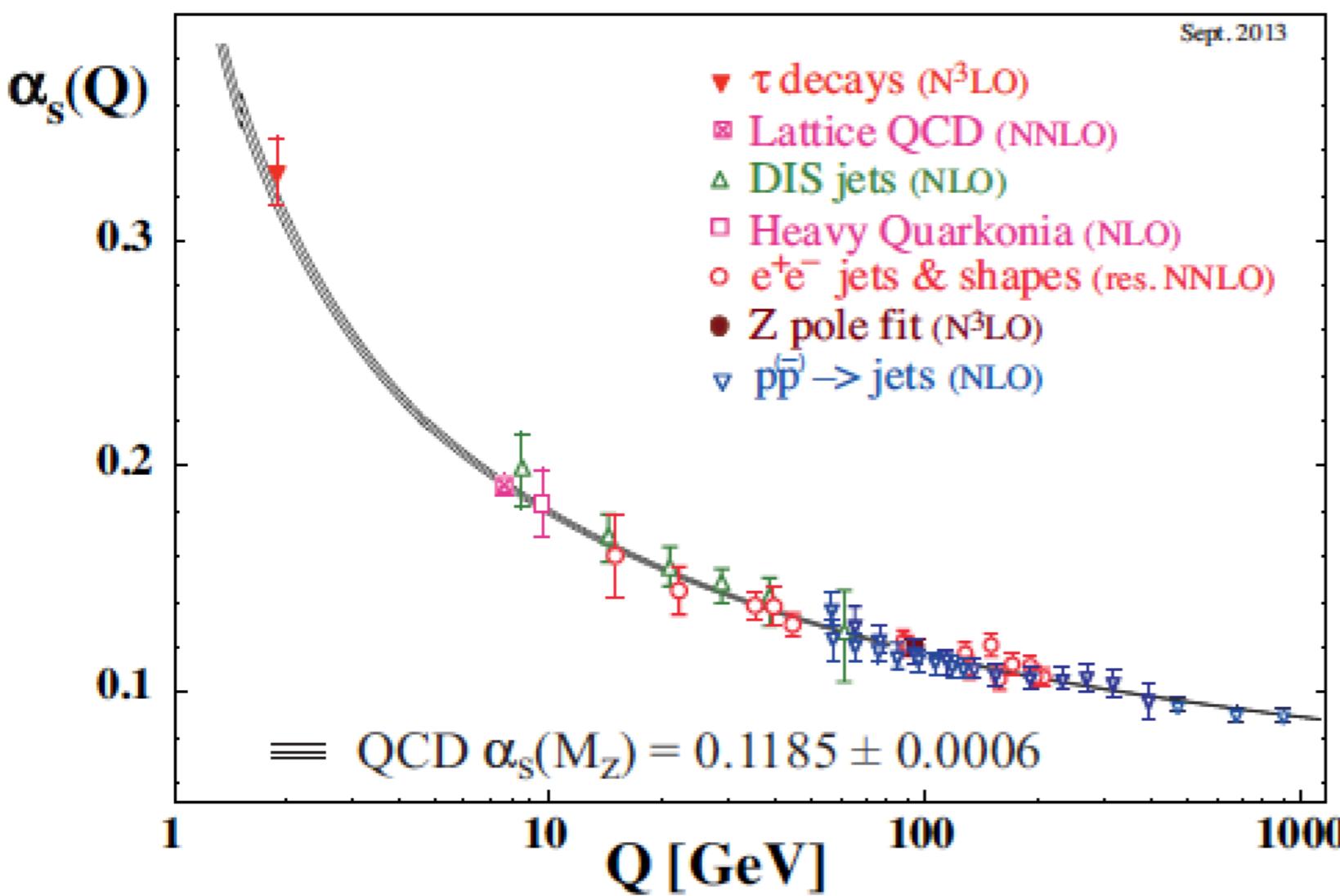
spin 1

perturbatively renormalisable & **predictive**

success story:

running couplings

quantum fluctuations modify interactions
couplings depend on energy or distance



triumph of QFT

asymptotic freedom

't Hooft '74
Gross, Wilczek '74
Politzer '74

gravitation

physics of classical gravity

Einstein's theory of general relativity

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -\Lambda g_{\mu\nu} + 8\pi G_N T_{\mu\nu}$$

Newton's coupling

$$G_N = 6.7 \times 10^{-11} \frac{\text{m}^3}{\text{kg s}^3}$$

cosmological constant

$$\Lambda \approx 10^{-35} \text{s}^{-2}$$

gravitation

physics of classical gravity

Einstein's theory of general relativity

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -\Lambda g_{\mu\nu} + 8\pi G_N T_{\mu\nu}$$

long distances

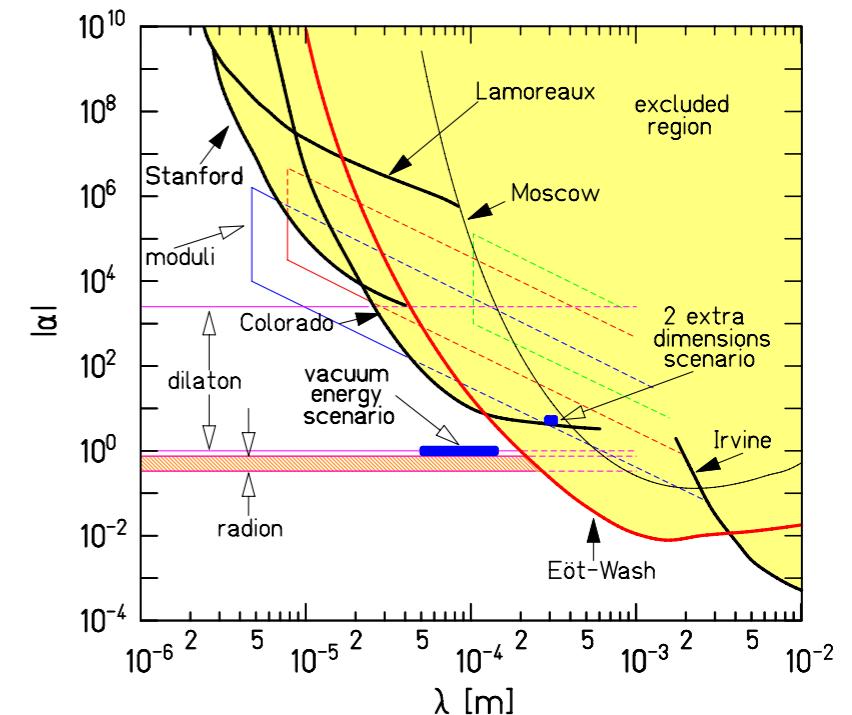
gravity not tested beyond 10^{28} cm

short distances

gravity not tested below 10^{-2} cm

ample space for “new” physics

e.g. dark matter, dark energy,
extra dimensions,
modifications of GR



gravitation

why quantum gravity?

classical general relativity is incomplete

singularities - black holes, early universe

homogeneity of CMB, small CC

classical general relativity is sourced by **quantum** matter

expect **quantum effects** due to quantum matter $\propto T_{\mu\nu}$

gravitation

physics of quantum gravity

action

$$S = \frac{1}{16\pi G_N} \int d^d x (-R + 2\Lambda)$$

dimensional analysis

Planck length

$$\ell_{\text{Pl}} = \left(\frac{\hbar G_N}{c^3} \right)^{1/2} \approx 10^{-33} \text{ cm}$$

Planck mass

$$M_{\text{Pl}} \approx 10^{19} \text{ GeV}$$

Planck time

$$t_{\text{Pl}} \approx 10^{-44} \text{ s}$$

Planck temperature

$$T_{\text{Pl}} \approx 10^{32} \text{ K}$$

expect **quantum modifications** at energy scales M_{Pl}

the trouble with gravity...

- **structure of UV divergences**

gravity: $[g_{\mu\nu}] = 0$, $[\text{Ricci}] = 2$, $[G_N] = 2 - d$

effective expansion parameter: $g_{\text{eff}} \equiv G_N E^2 \sim \frac{E^2}{M_{\text{Pl}}^2}$

N-loop Feynman diagram $\sim \int dp p^{A-[G]N}$

$[G] > 0$: superrenormalisable

$[G] = 0$: renormalisable

$[G] < 0$: **dangerous** interactions

- **perturbative non-renormalisability**

gravity with matter interactions

pure gravity (Goroff-Sagnotti term)

... yet some hints from PT

- **effective theory for gravity** (Donoghue '94)
quantum corrections computable for energies $E^2/M_{\text{Pl}}^2 \ll 1$
knowledge of UV completion not required
- **higher derivative gravity I** (Stelle '77)
 R^2 gravity perturbatively renormalisable
unitarity issues at high energies
- **higher derivative gravity II** (Gomis, Weinberg '96)
all higher derivative operators
gravity ‘weakly’ perturbatively renormalisable
no unitarity issues at high energies

[other ways out

strings

replace QFT by string theory, give up on locality

extra benefit: unification with matter

price tag: more dimensions, supersymmetry, more dofs

loops

LQG sticks to EH action, quantisation procedure altered

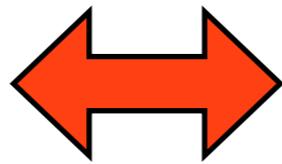
discrete

microscopic building blocks (“lattice”), spin foam, and more

]

today: asymptotic safety

fundamental
definition of QFT

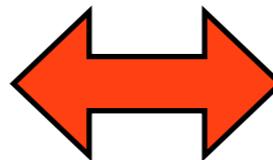


UV fixed point

Wilson '71

today: asymptotic safety

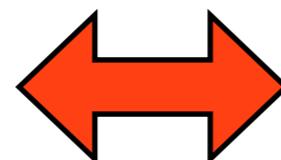
fundamental
definition of QFT



UV fixed point

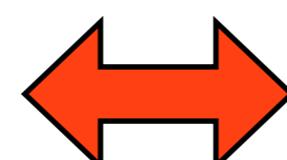
Wilson '71

asymptotic freedom



theory becomes **free**
at highest energies =
Gaussian UV fixed point

asymptotic safety



theory remains **interacting**
at highest energies =
interacting UV fixed point

Weinberg '79

principles of asymptotic safety

renormalisation group

dimension coupling

gravitons

$$D = 2 + \epsilon : \quad \alpha = G_N(\mu)\mu^{D-2}$$

Gastmans et al '78
Christensen, Duff '78
Weinberg '79
Kawai et al '90

renormalisation group

	dimension	coupling	
gravitons	$D = 2 + \epsilon :$	$\alpha = G_N(\mu)\mu^{D-2}$	
			Gastmans et al '78 Christensen, Duff '78 Weinberg '79 Kawai et al '90

energy-dependence: RG beta function of couplings

renormalisation group

gravitons

dimension

coupling

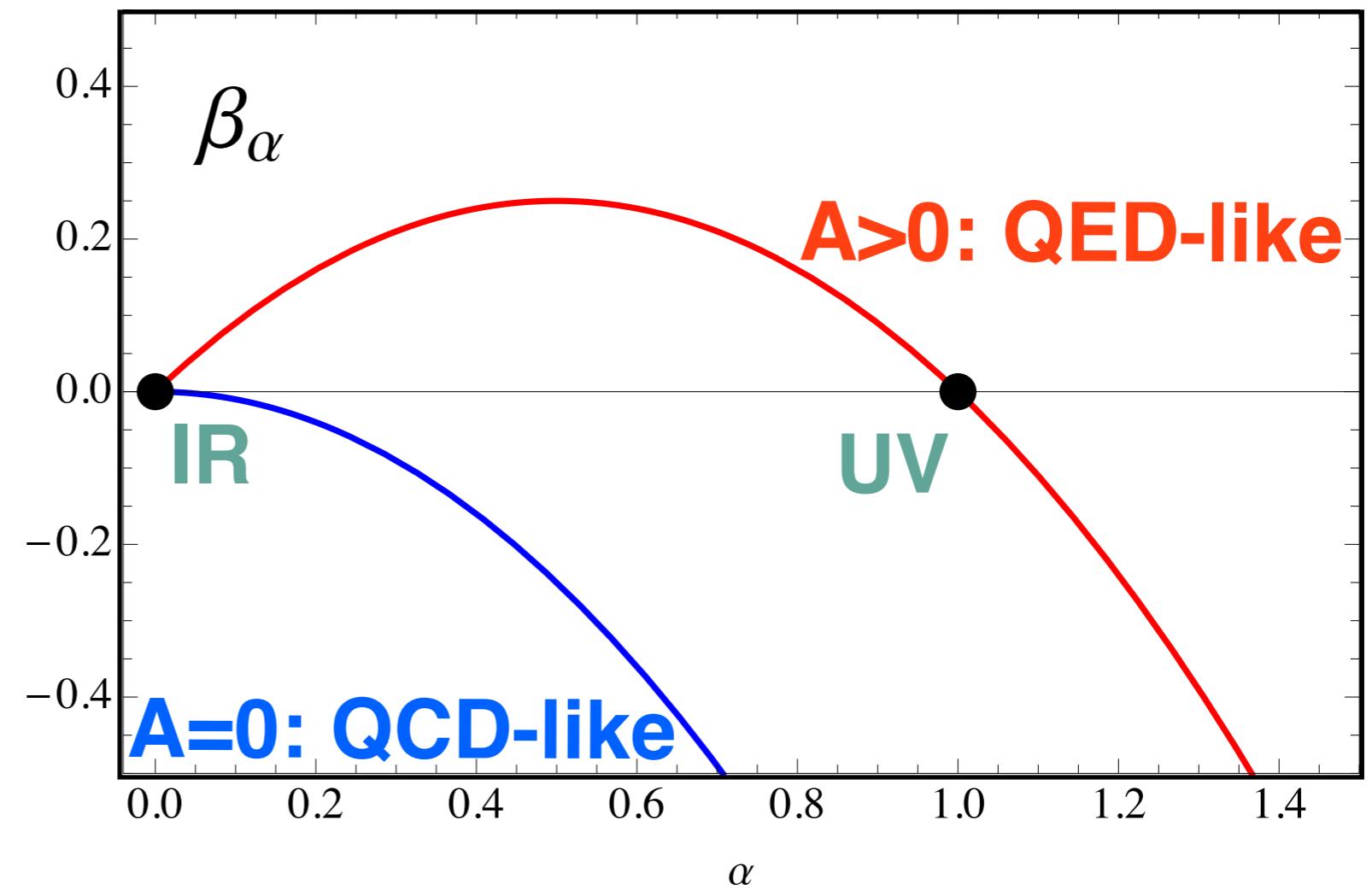
$$D = 2 + \epsilon : \quad \alpha = G_N(\mu) \mu^{D-2}$$

Gastmans et al '78
 Christensen, Duff '78
 Weinberg '79
 Kawai et al '90

$$\partial_t \alpha = A \alpha - B \alpha^2$$

$$\alpha_* = A/B \ll 1$$

$$t = \ln \mu$$



exact asymptotic safety

gravitons

dimension

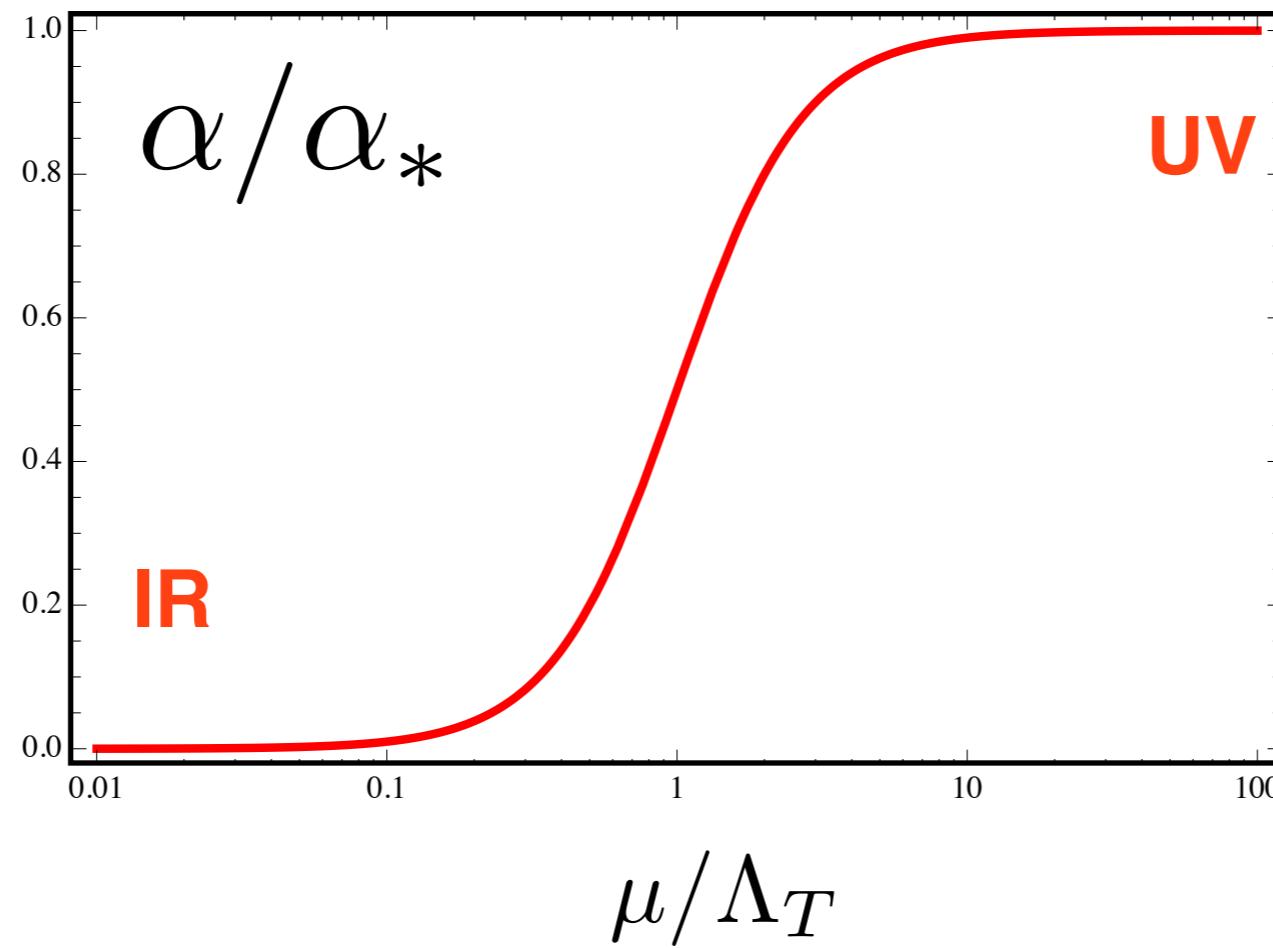
$$D = 2 + \epsilon : \quad \alpha = G_N(\mu)\mu^{D-2}$$

coupling

Gastmans et al '78
Christensen, Duff '78
Weinberg '79
Kawai et al '90

$$G(\mu) \approx G_N$$

classical GR



$$G(\mu) \approx \frac{\alpha_*}{\mu^{D-2}}$$

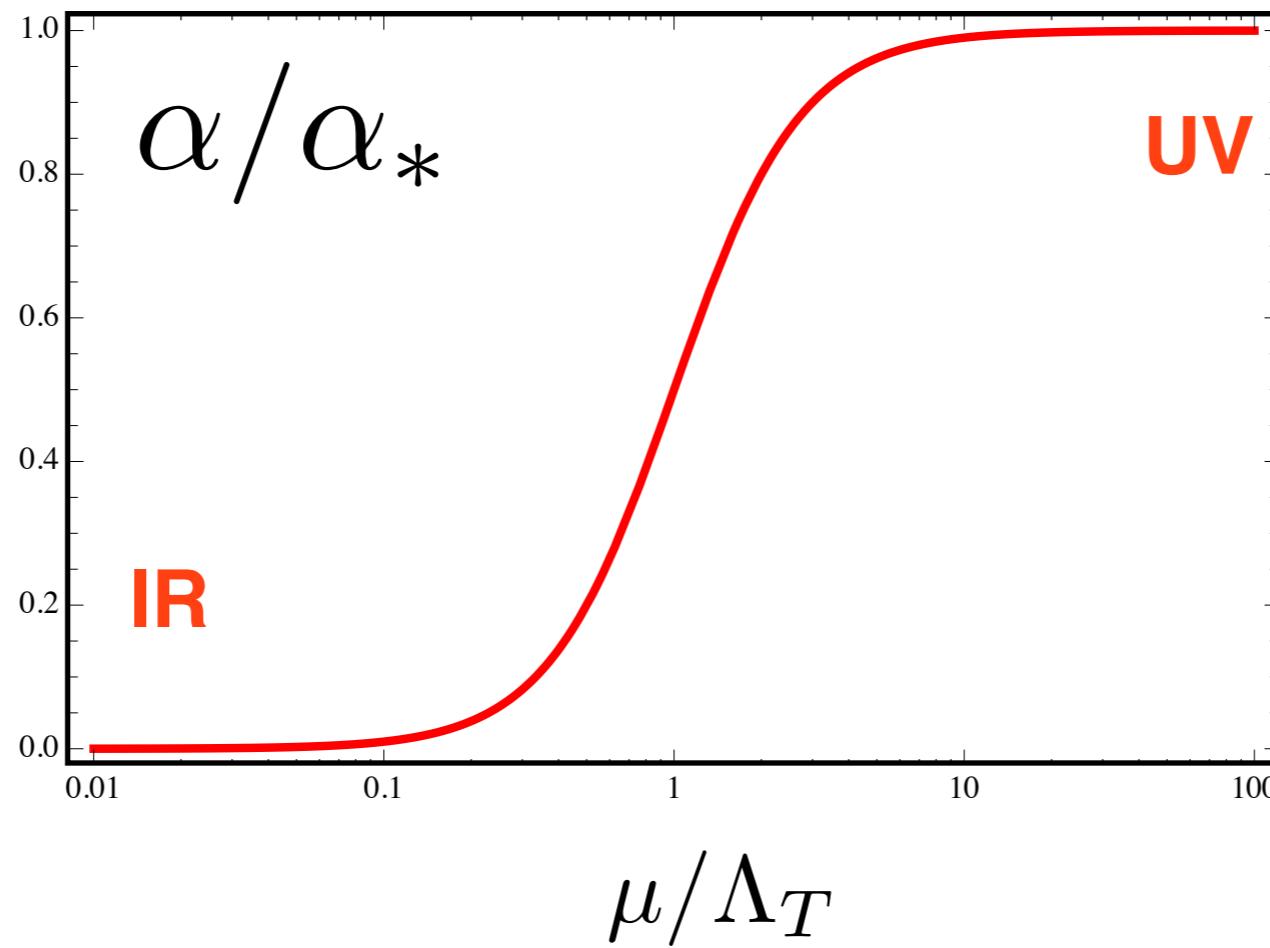
gravity weakens

exact asymptotic safety

UV fixed point implies weakly coupled gravity at high energies

$$\mu \rightarrow \infty : \quad G(\mu) \rightarrow g_* \mu^{2-D} \ll G_N$$

$G(\mu) \approx G_N$
 classical GR



$$G(\mu) \approx \frac{\alpha_*}{\mu^{D-2}}$$

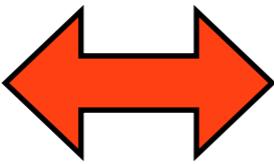
gravity weakens

how is this predictive?

UV: interactions are **softened by fluctuations**

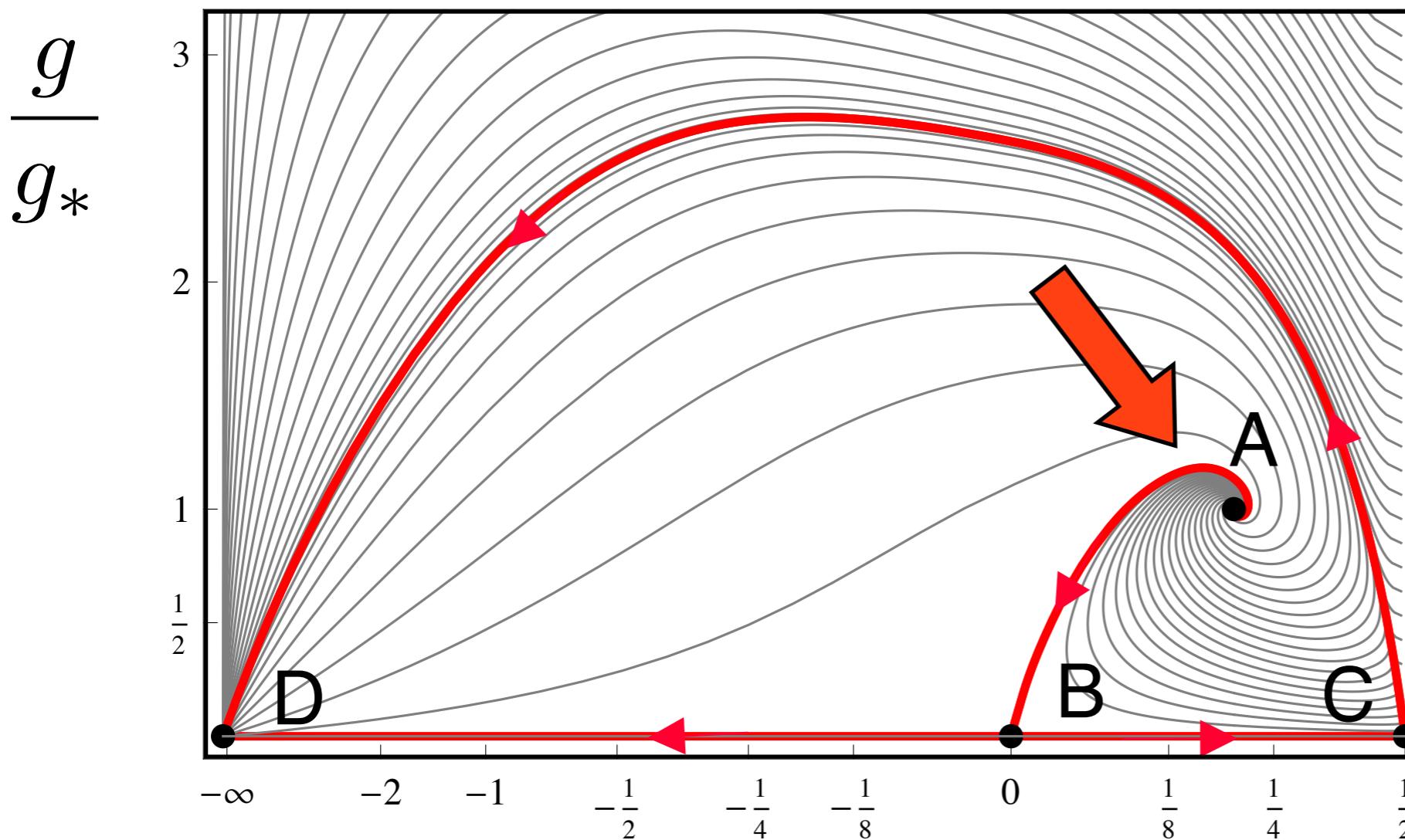
UV behaviour characterised by

relevant, marginal, irrelevant invariants

predictivity  **finitely many relevant invariants**

how is this predictive?

example: gravitational phase diagram in 4D



here:
two relevant
couplings

$$g = G_k \cdot k^2$$
$$\lambda = \Lambda_k / k^2$$

λ

plot taken from
Litim, Satz 1205.4218

exact asymptotic safety

	dimension	coupling	
gravitons	$D = 2 + \epsilon :$	$\alpha = G_N(\mu)\mu^{D-2}$	Gastmans et al '78 Christensen, Duff '78 Weinberg '79 Kawai et al '90
fermions	$D = 2 + \epsilon :$	$\alpha = g_{\text{GN}}(\mu)\mu^{2-D}$	Gawedzki, Kupiainen '85 de Calan et al '91
gluons	$D = 4 + \epsilon :$	$\alpha = g_{\text{YM}}^2(\mu)\mu^{4-D}$	Peskin '80 Morris '04
scalars	$D = 2 + \epsilon :$	$\alpha = g_{NL}(\mu)\mu^{D-2}$	Brezin, Zinn-Justin '76 Bardeen, Lee, Shrock '76
classes of gauge-Yukawa theories	$D = 4 :$	several α_i	Litim, Sannino 1406.2337 Bond, Litim, 1608.00519

asymptotic safety in 4d quantum gravity

evidence for UV fixed point in 4d

overviews: DL 0810.3675 and 1102.4624

gravitation

Einstein-Hilbert

(Reuter '96, Souma '99, Reuter, Lauscher '01, DL '03)

higher dimensions, dimensional reduction (DL '03, Fischer, DL '05)

f(R), polynomials in R

(Lauscher, Reuter, '02, Codello, Percacci, Rahmede '08, Machado, Saueressig '09
Benedetti, Caravelli '12, Dietz, Morris '12, Falls, DL, Nikolopoulos, Rahmede '13)

local potential approximation

(Benedetti, Caravelli '12, Dietz, Morris, '12, Demmel, Saueressig,
Zanusso '12, Falls, DL, Nikolopoulos, Rahmede '13,
Benedetti '13, Benedetti, Guarnieri '13)

higher-derivative gravity

(Codello, Percacci '05)
(Benedetti, Saueressig, Machado '09, Niedermaier '09)

conformally reduced gravity

(DL, Rahmede, in prep.)
(Reuter, Weyer '09, Machado, Percacci '10, DL, Satz '12)

Holst action + Immirzi parameter (Daum, Reuter '10, Benedetti, Speciale '11)

signature effects (Manrique, Rechenberger, Saueressig '11)

gravitation + matter

matter

(Percacci '05, Perini, Percacci '05, Narain, Percacci '09, Narain, Rahmede '09,
Codello '11, Eichhorn et al '13)

Yang-Mills gravity

1-loop: (Robinson, Wilczek '05, Pietrokowski, '06, Toms '07, Ebert, Plefka, Rodigast '08)

beyond: (Manrique, Reuter, Saueressig '09, Folkerts, DL, Pawłowski, 11, Harst, Reuter '11)

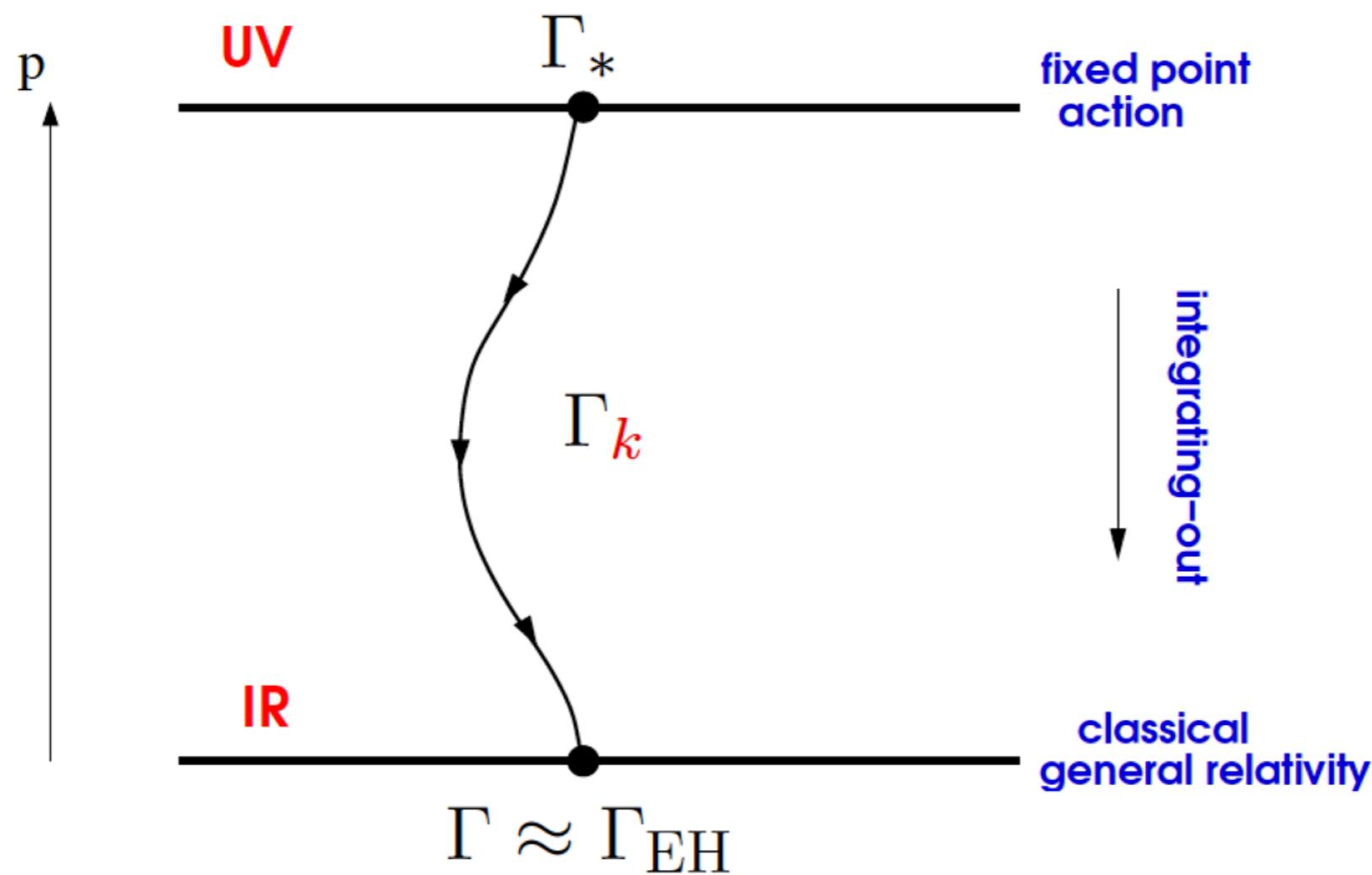
computational methods

4D quantum gravity:

large anom. dimensions, expect large couplings
non-perturbative tools mandatory

functional [Wilsonian] renormalisation

Wetterich '92, Morris '93,
Reuter '96, Dou, Percacci '97,
Litim '00, '03



asymptotic freedom `the knowns'

vs

asymptotic safety `the unknowns'

$$g_* = 0$$

anomalous dimensions

$$\eta_A = 0$$

canonical power counting

$\{\vartheta_{G,n}\}$ are known

F^{256} irrelevant !

$$g_* \neq 0$$

anomalous dimensions

$$\eta_N \neq 0$$

non-canonical power counting

$\{\vartheta_n\}$ are **not** known

$$R^{256}$$

relevant
marginal
irrelevant ?

bootstrap search strategy

hypothesis **relevancy** of invariants **follows**
their canonical **mass** dimension

bootstrap search strategy

hypothesis relevancy of invariants follows their canonical mass dimension

strategy

Step 1 retain invariants up to mass dimension D

Step 2 compute $\{\vartheta_n\}$ (eg. RG, lattice, holography)

Step 3 enhance D, and iterate

convergence (no convergence) of the iteration:

hypothesis supported (refuted)

f(R)

$$\Gamma_k \propto f(R)$$

$$\Gamma_k = \sum_{n=0}^{N-1} \lambda_n k^{d_n} \int d^4x \sqrt{g} R^n$$

Ricci scalars

effective action with invariants up to mass dimension $D = 2(N - 1)$

technicalities: functional renormalisation

$$k \frac{d\Gamma_k}{dk} = \frac{1}{2} \text{Tr} \left[\left(\frac{\delta^2 \Gamma_k[\phi]}{\delta \phi \delta \phi} + \color{red}R_k\right)^{-1} k \frac{d\color{red}R_k}{dk} \right] = \frac{1}{2} \circlearrowleft$$

here:

M Reuter hep-th/9605030

DL [hep-th/0103195](#)
[hep-th/0312114](#)

Falls, DL, Nikolakopoulos, Rahmede

Falls, DL, Nikolakopoulos, Rahmede

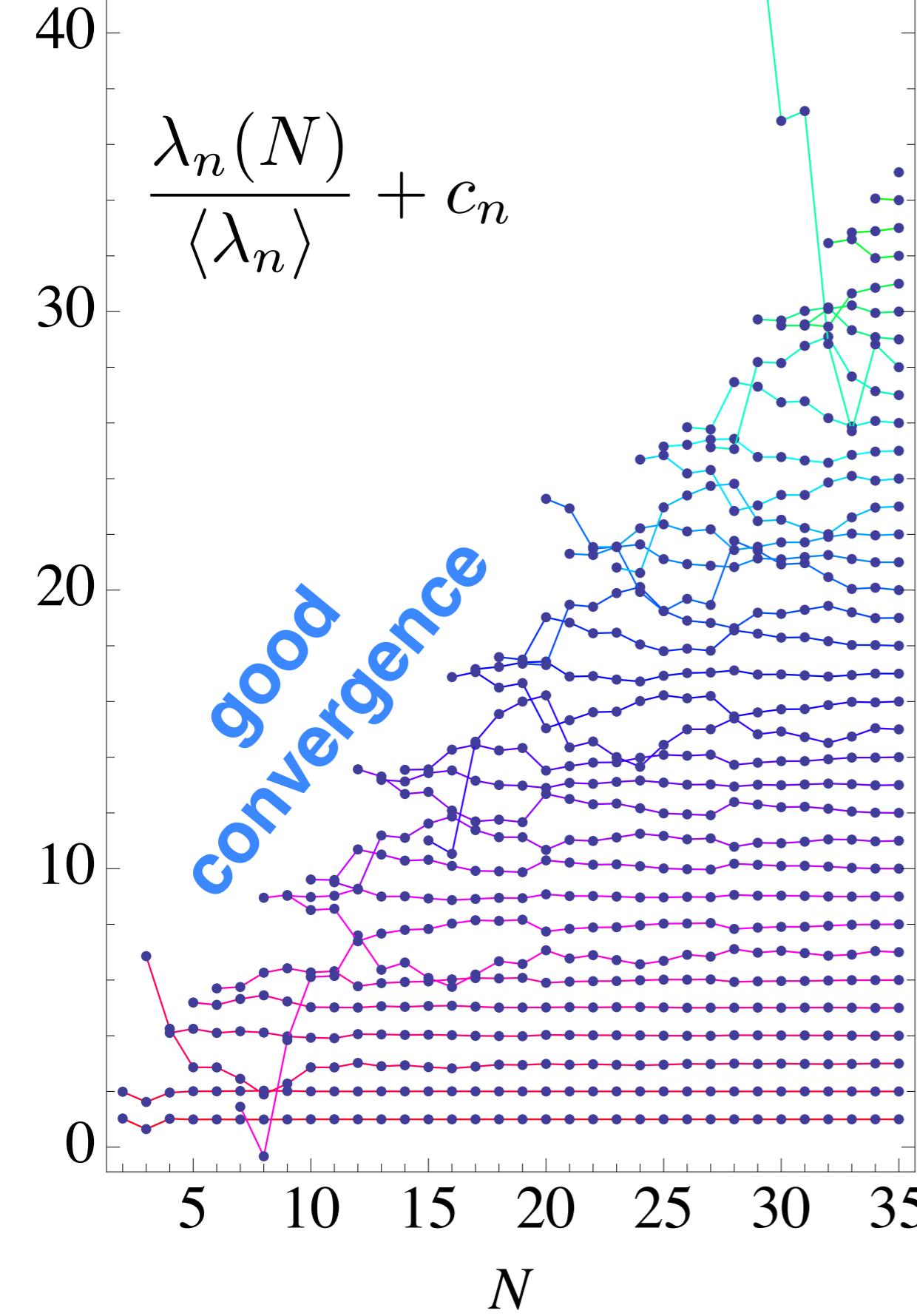
A Codello, R Percacci, C Rahmede 0705.1769, 0805.2909

P Machado, F Saueressig 0712.0445

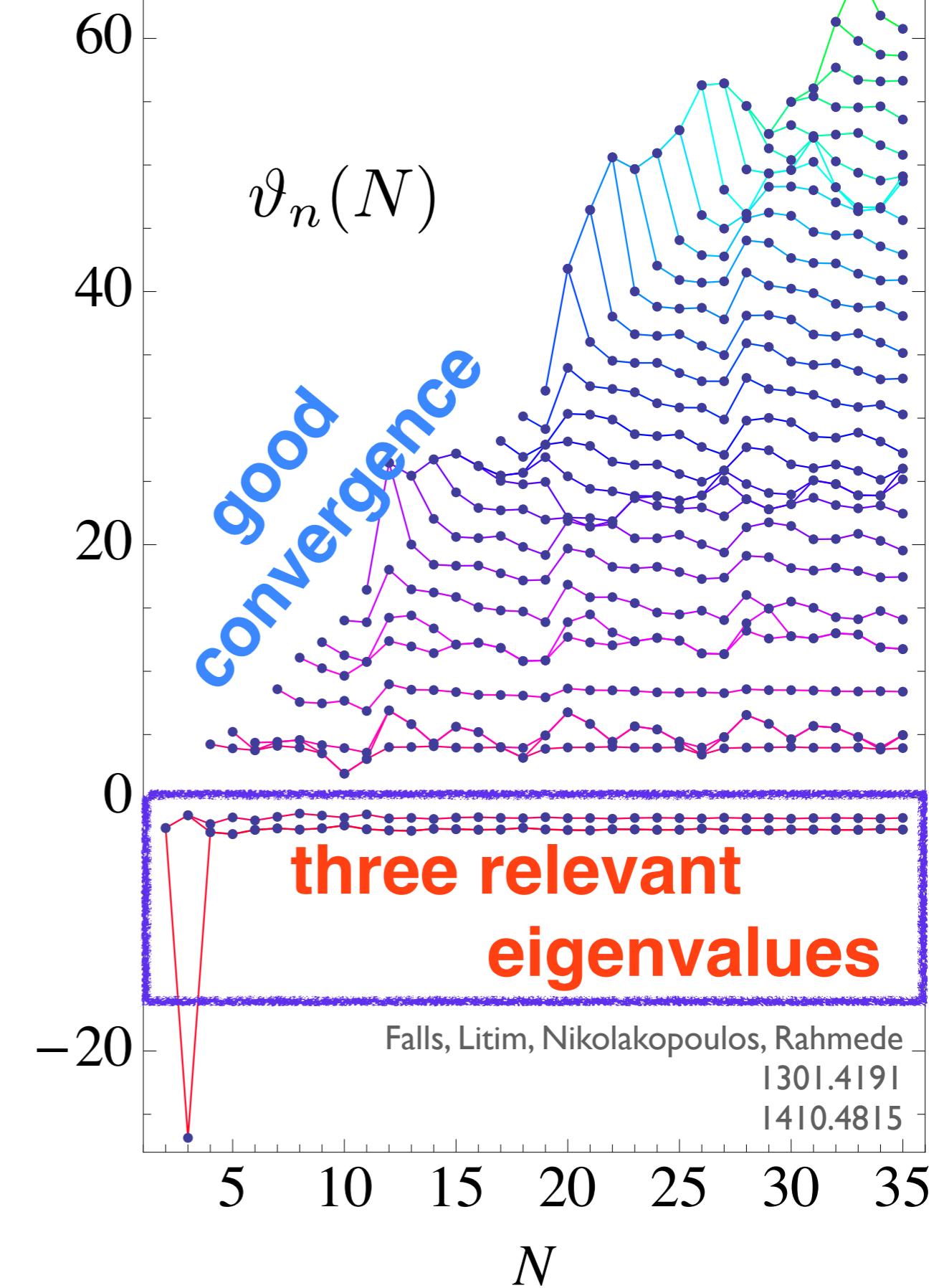
[1301.4191.pdf](#)

1410.4815

UV fixed point



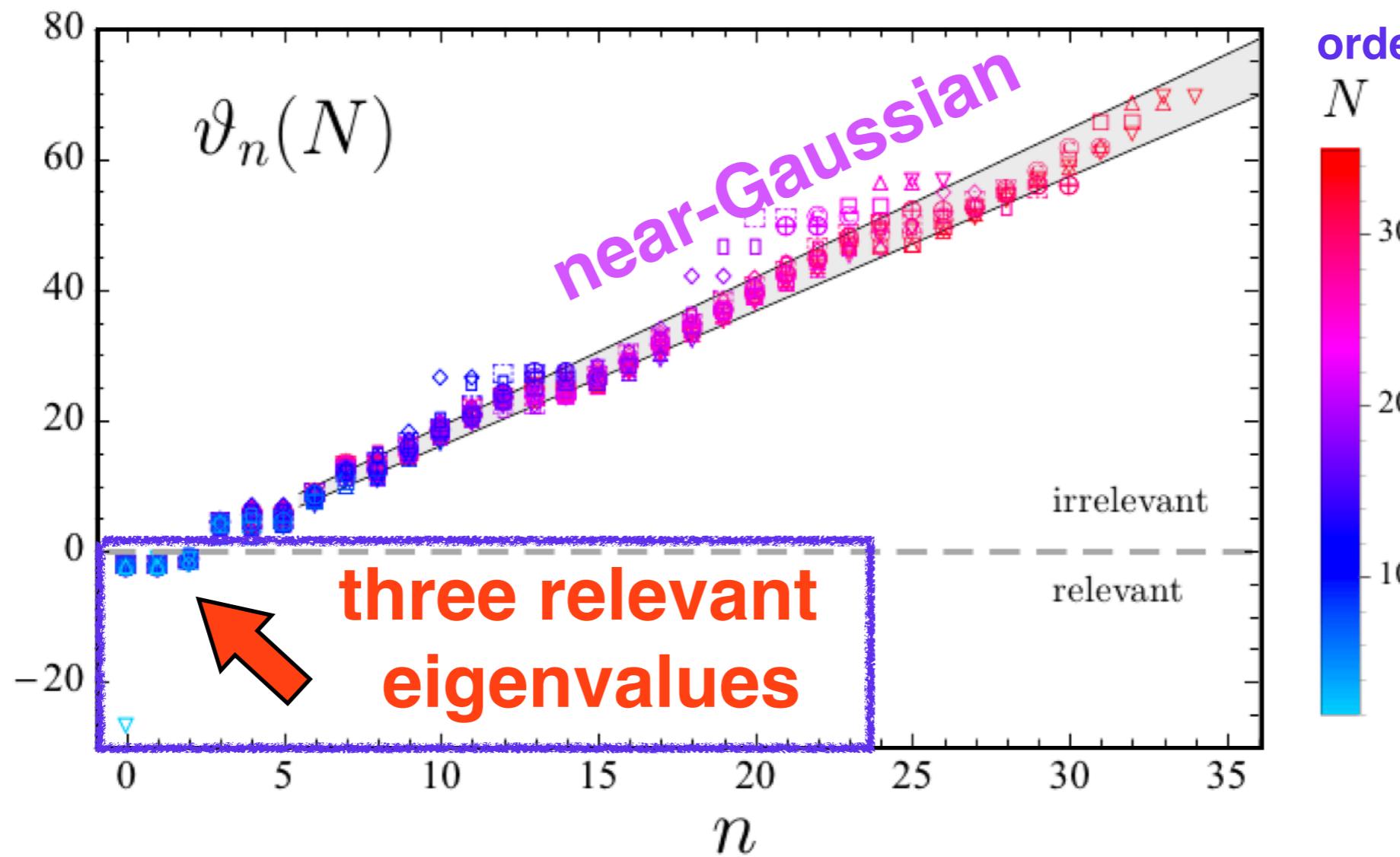
UV scaling exponents



scaling exponents

f(R)-type gravity

$$\Gamma_k = \sum_{n=0}^{N-1} \lambda_n k^{d_n} \int d^4x \sqrt{g} R^n$$



testing asymptotic safety in the physical world

cosmology

early universe and inflation, late-time acceleration
asymptotically safe cosmology

particle physics

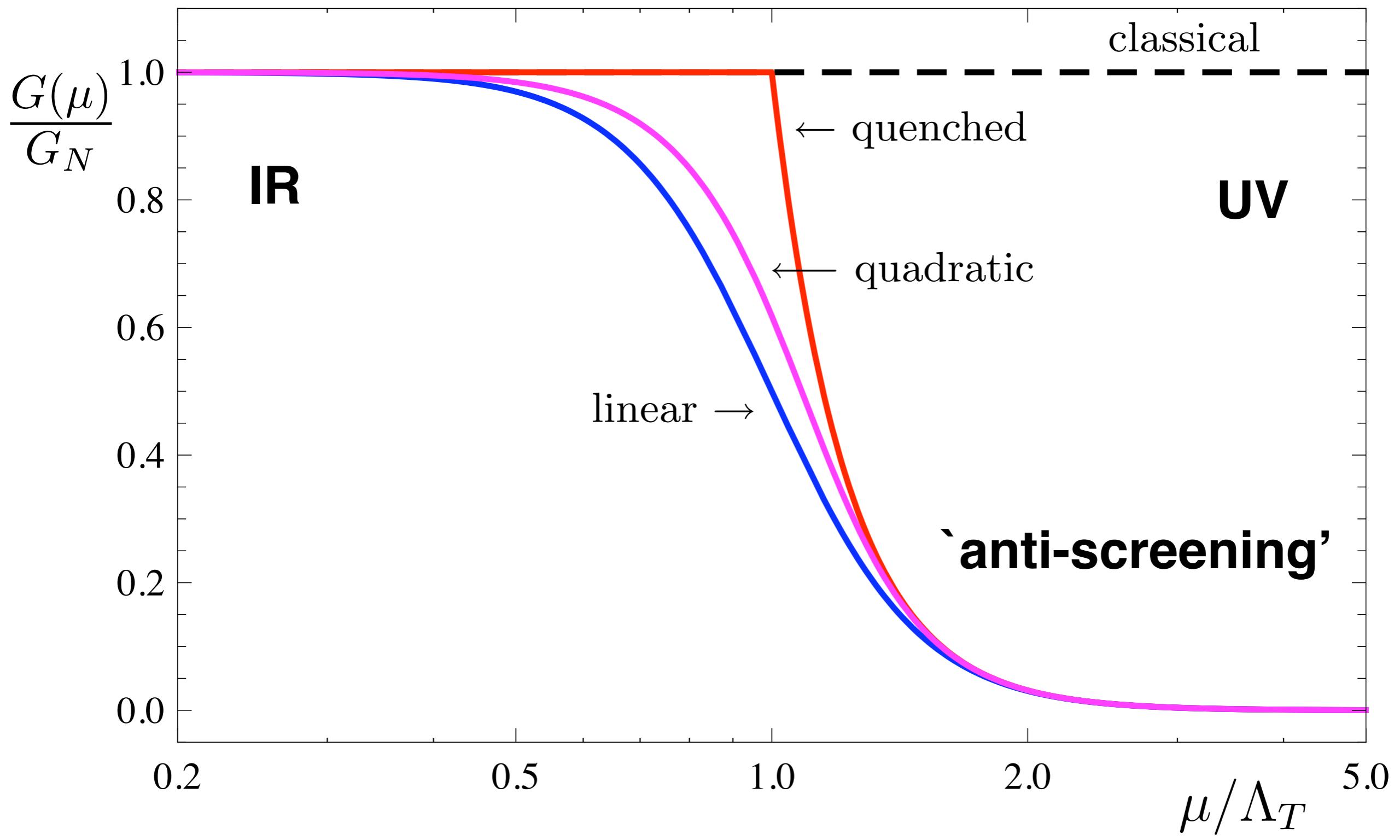
towards a Standard Model including quantum gravity
gravitational scattering: signatures at particle colliders

black holes

quantum corrections to BH space-times
quantum aspects of black hole thermodynamics

low-scale quantum gravity at colliders

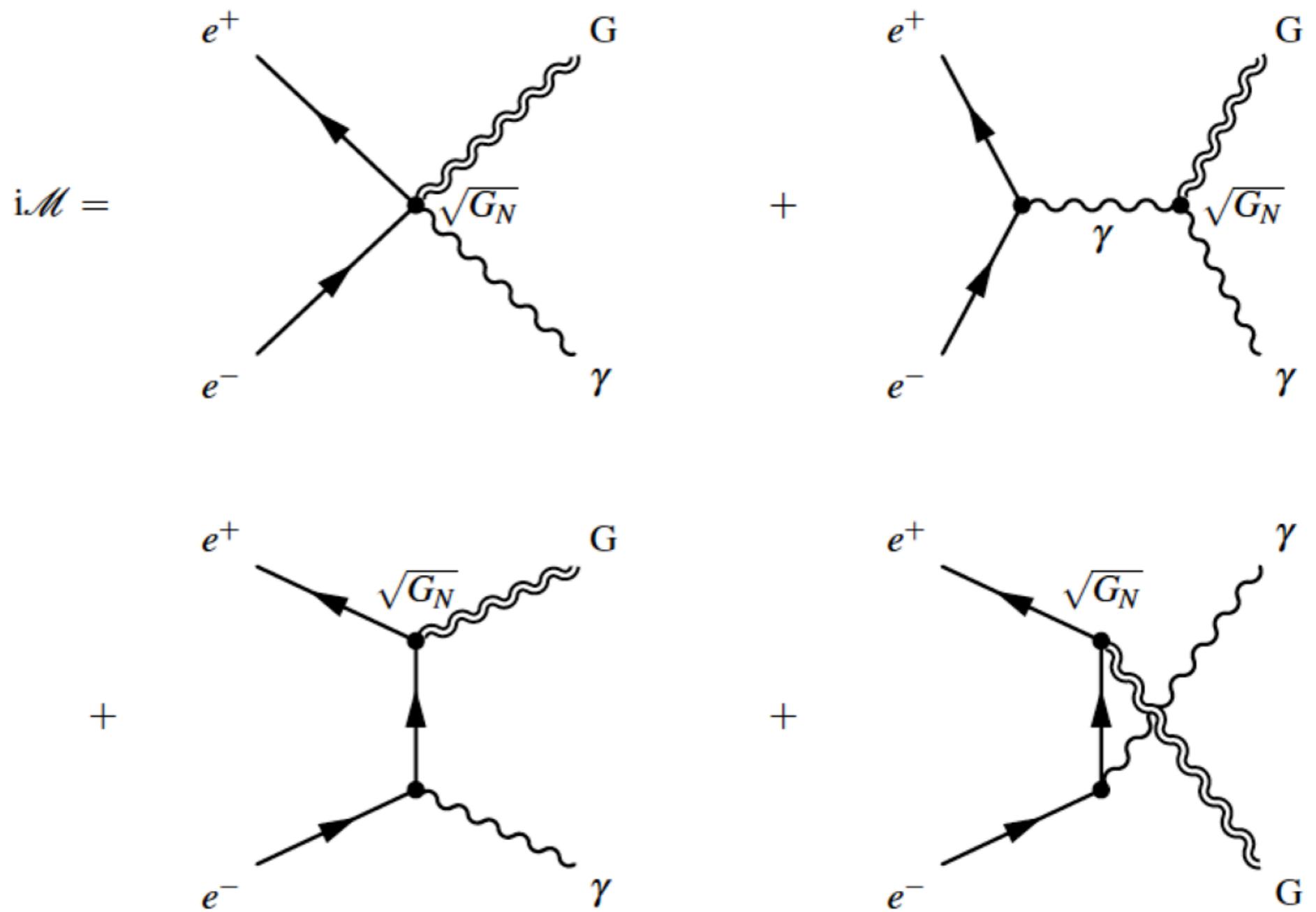
cross-over scale Λ_T



low-scale quantum gravity

- **theory**
fundamental parameters M_D Λ_T g_*
- **experiment**
data to fix or constrain theory parameters

1) missing ET: real gravitons



+ quantum gravity corrections of vertices

real gravitons+jet

bounds from effective theory

			n	2	3	4	5	6	
LEP	0.65 fb^{-1}	$e^+e^- \rightarrow \gamma + E$		1.60	1.20	0.94	0.77	0.66	[27]
CDF	1.1 fb^{-1}	$p\bar{p} \rightarrow \text{jet} + E$		1.31	1.08	0.98	0.91	0.88	[28]
CMS	36 pb^{-1}	$pp \rightarrow \text{jet} + E$		2.29	1.92	1.74	1.65	1.59	[29]
ATLAS	33 pb^{-1}	$pp \rightarrow \text{jet} + E$		2.30	2.00	1.80	n/a	n/a	[30]
ATLAS	1.0 fb^{-1}	$pp \rightarrow \text{jet} + E$		3.16	2.56	2.27	2.10	1.99	[31]
CMS	1.1 fb^{-1}	$pp \rightarrow \text{jet} + E$		3.67	2.96	2.66	2.41	2.25	[32]
CMS	4.7 fb^{-1}	$pp \rightarrow \text{jet} + E$		4.00	3.18	2.78	2.52	2.37	[33]

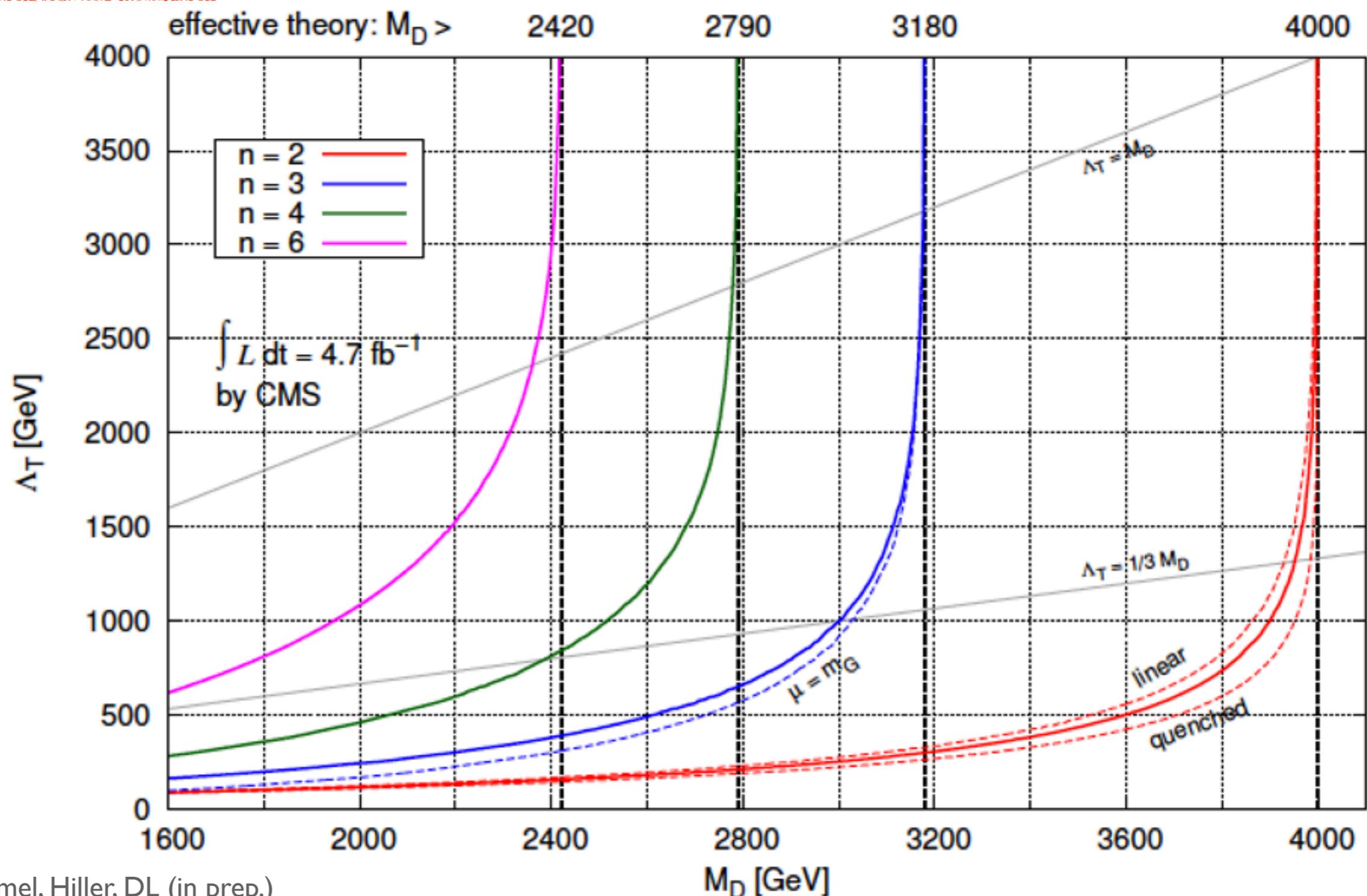
Table 4.1.: The 95% CL lower limits on M_D in effective theory for $n = 2, \dots, 6$ extra dimensions and different datasets collected by LEP, CDF, ATLAS and CMS. Values are given in TeV.

interpreting LHC data

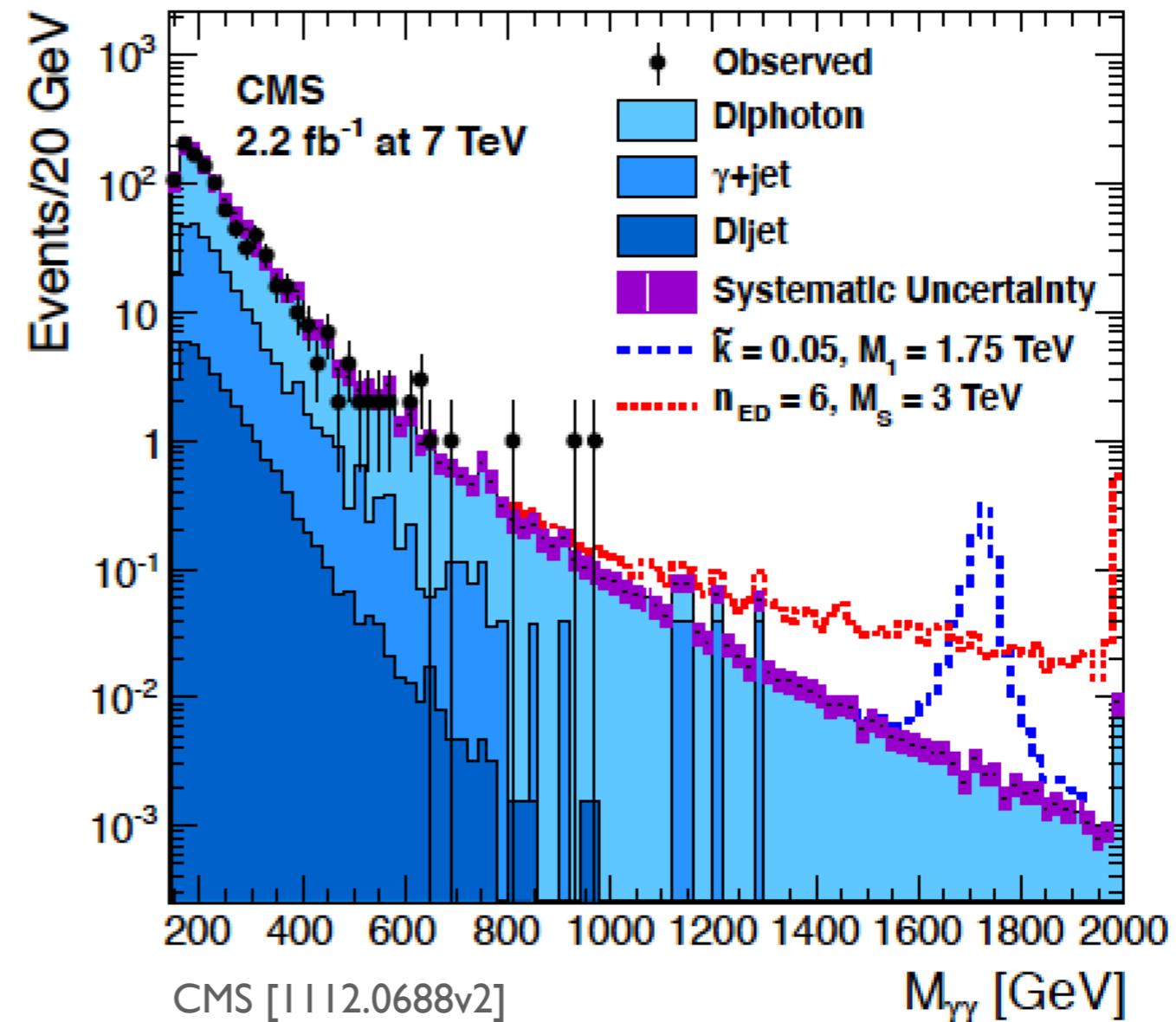
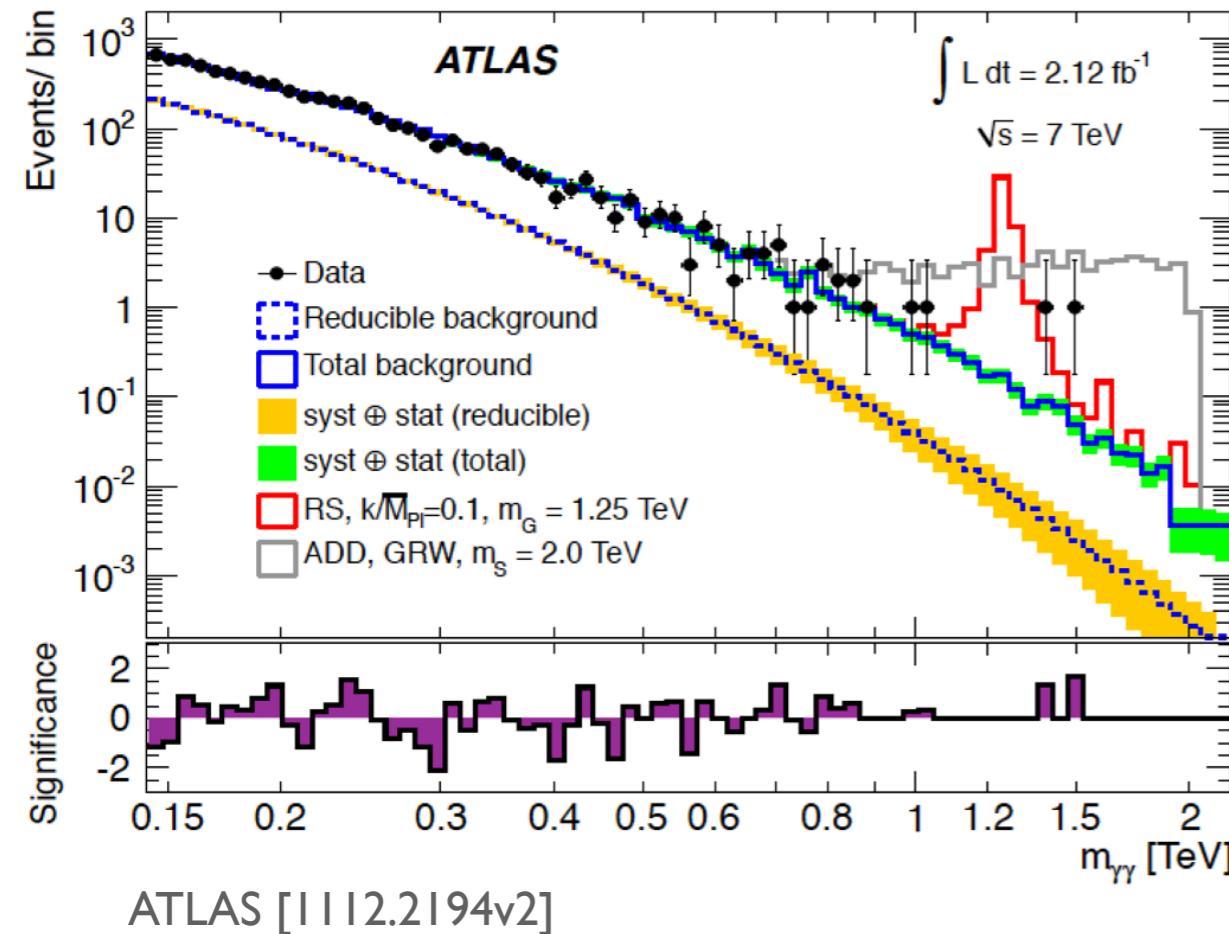
graviton+jet (MET)

Pythia v8.153

$\sqrt{s} = 7 \text{ TeV}$, $\mu = E_G$, quadratic approximation



2) virtual gravitons + diphotons



No significant excess $\rightarrow 95\%$ confidence level lower limits

k	$\Lambda_{\text{eff.Th.}}$
1	3.05 TeV
1.7	3.29 TeV

ATLAS (Oct 2012)

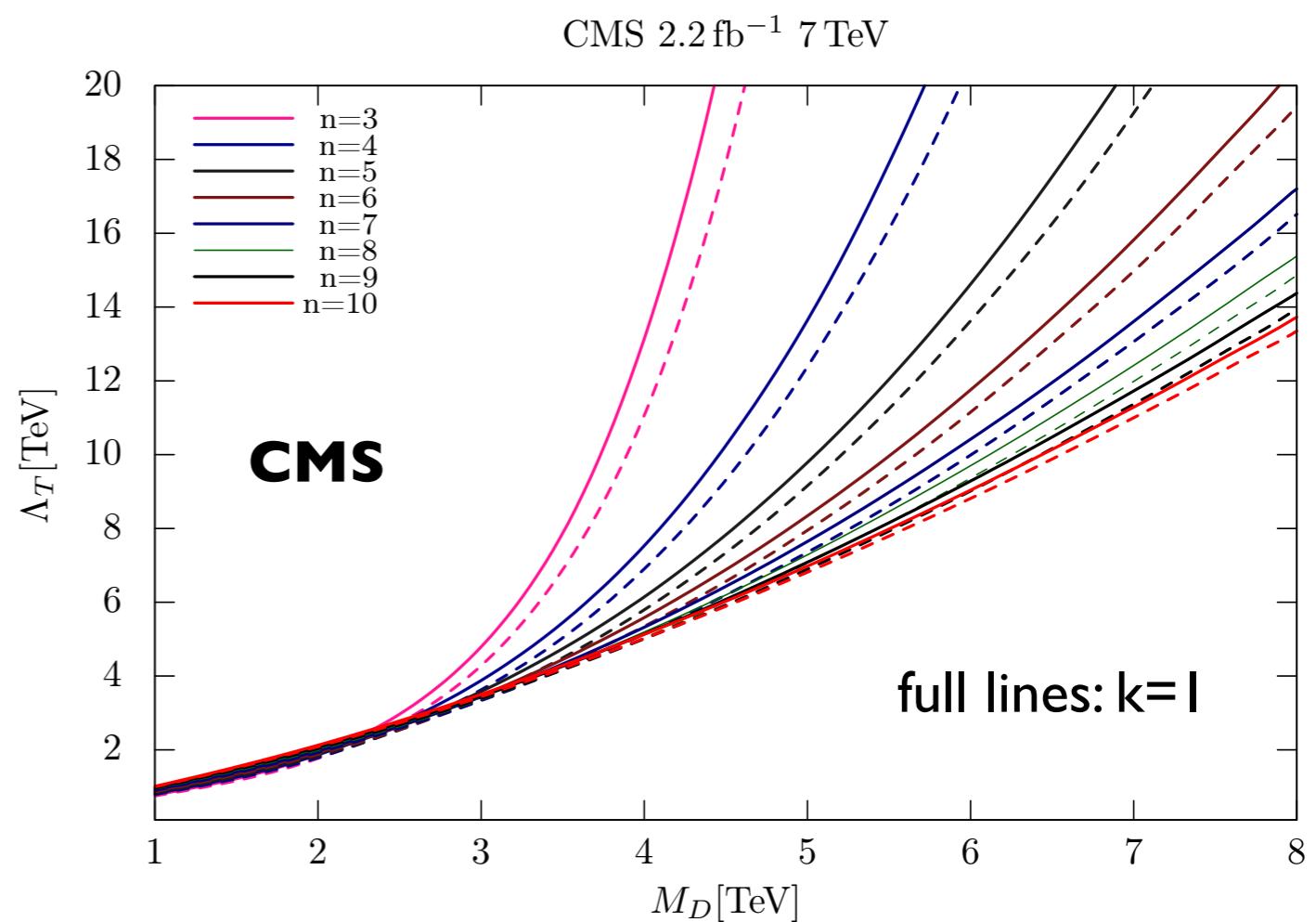
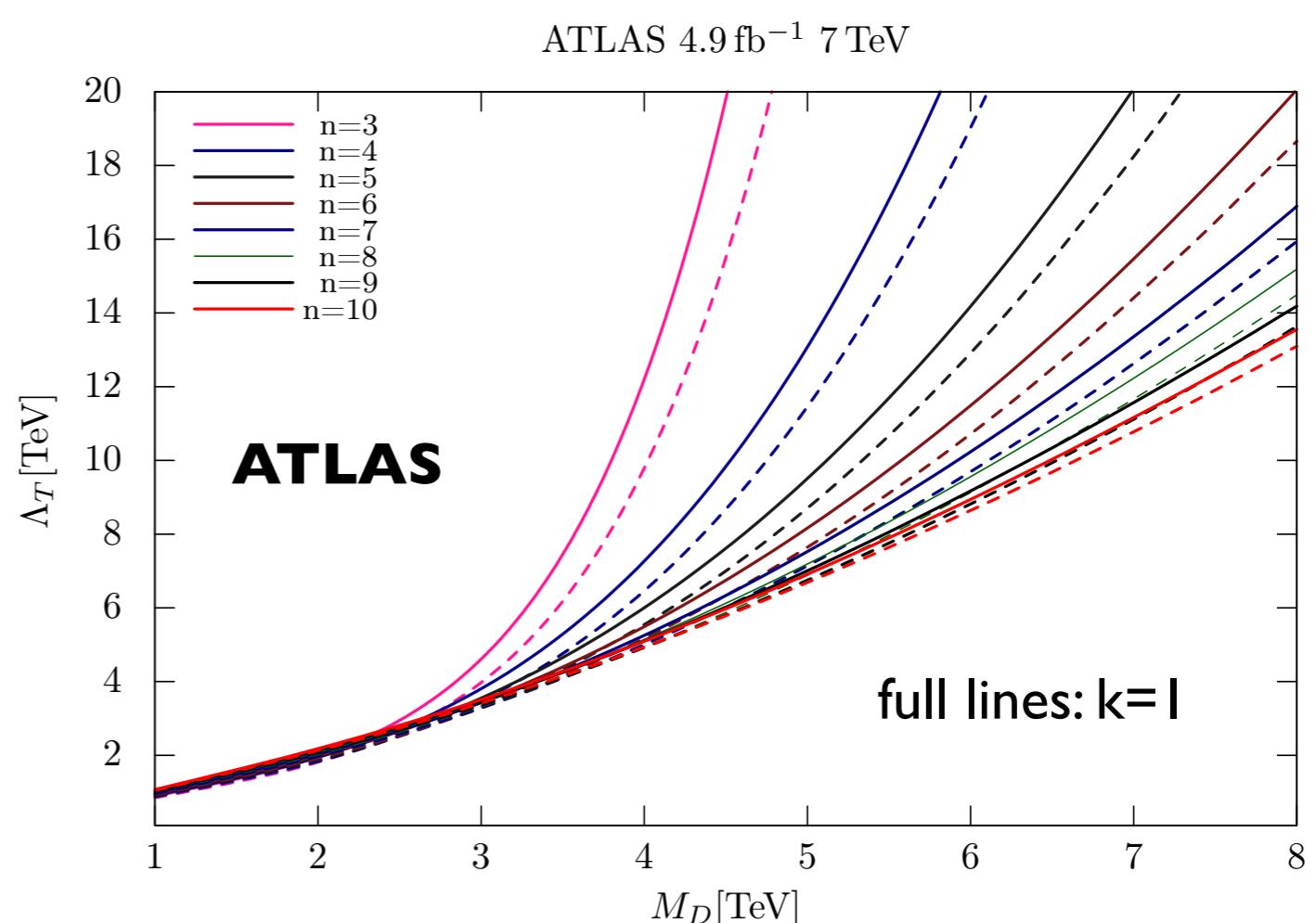
k	$\Lambda_{\text{eff.Th.}}$
1	2.94 TeV
1.6	3.18 TeV

CMS (Dec 2011)

interpreting LHC data

virtual gravitons + diphotons

implementation in Pythia8
weak PDF dependence
weak scheme dependence



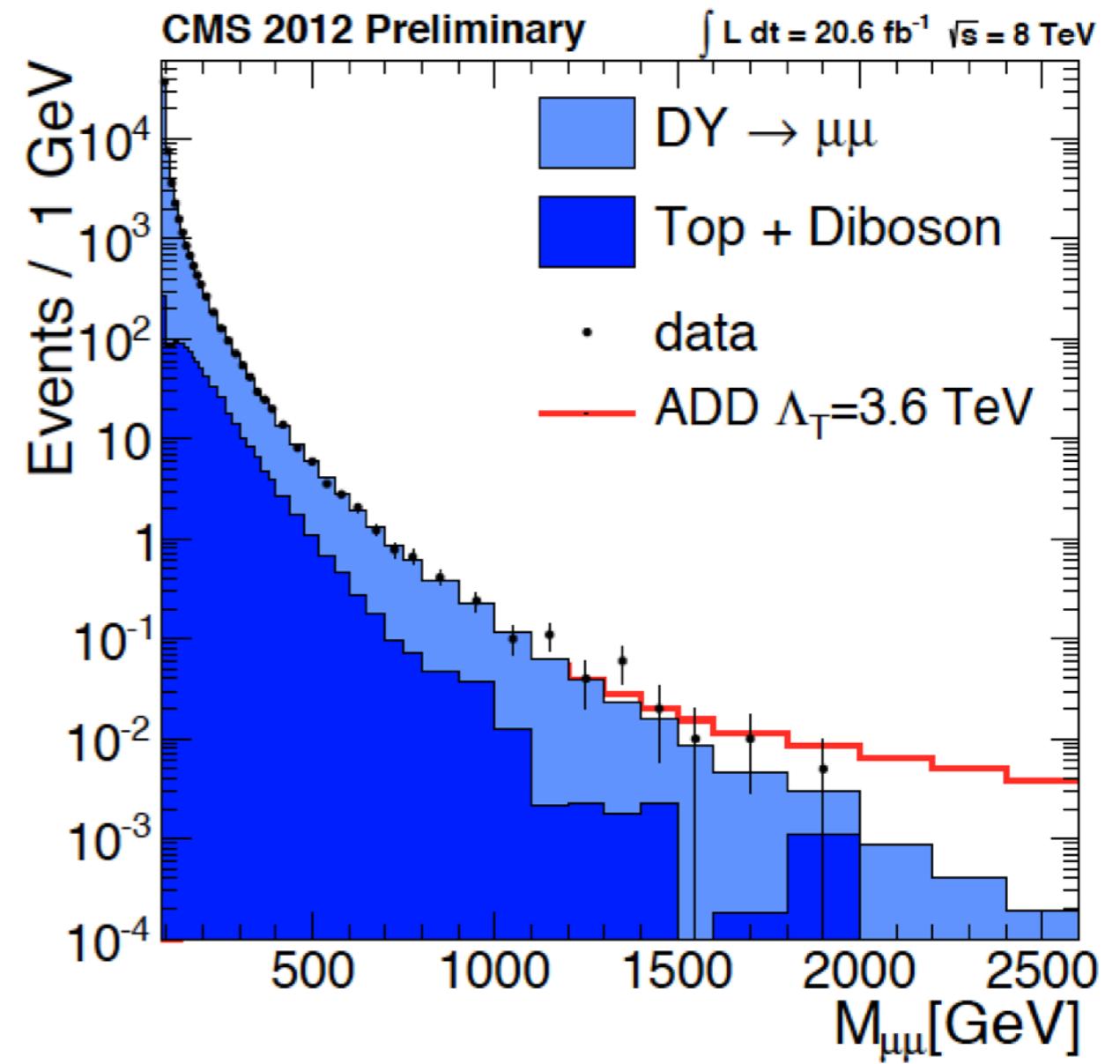
3) virtual gravitons + dileptons

$$S_{\text{eff}} = -\frac{4\pi}{\Lambda_{\text{eff}}^4}$$

combined 95% CL

$\Lambda_{\text{eff}} > 4.15 \text{ TeV}$

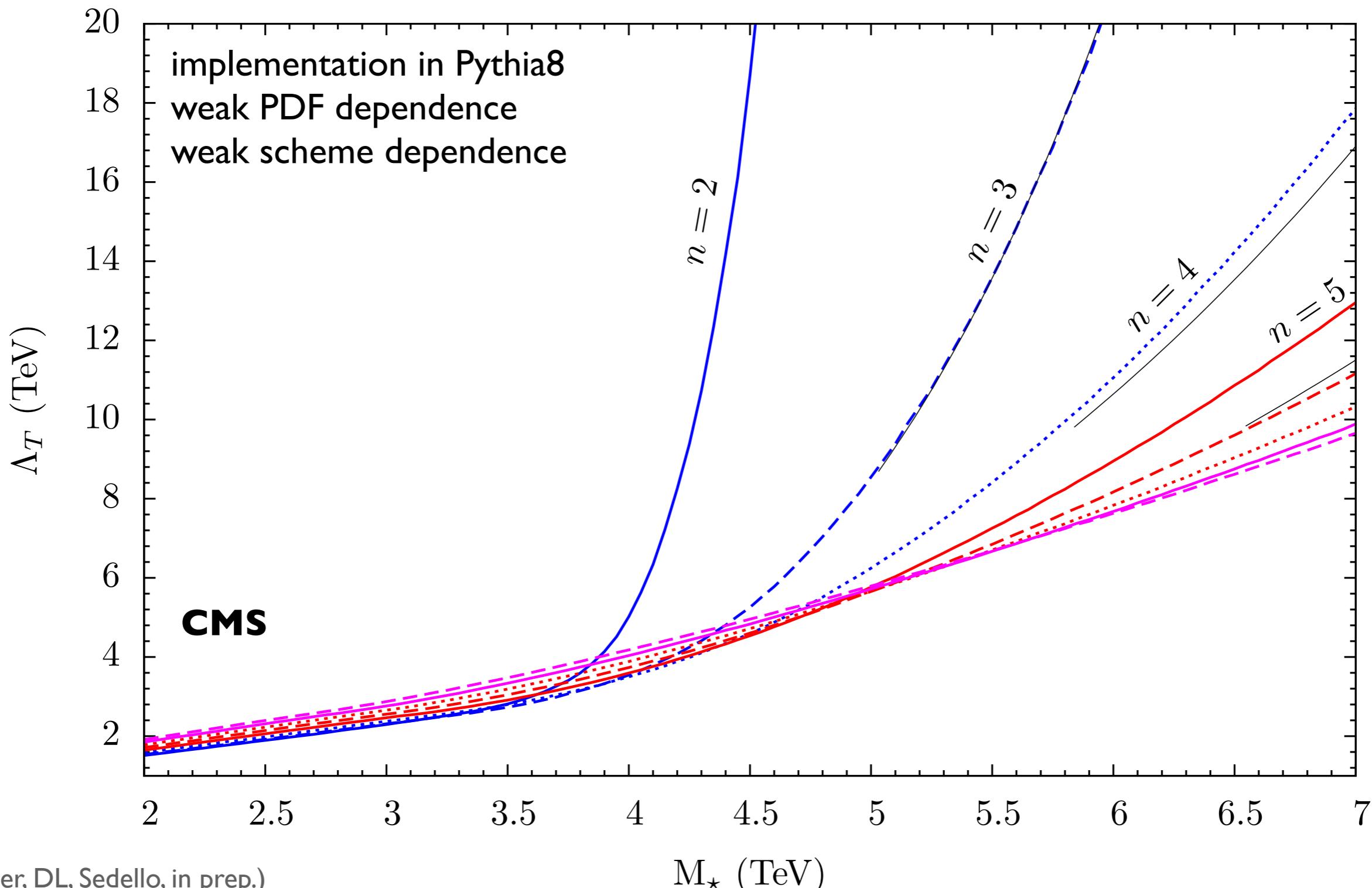
based on 20.6 fb^{-1}



CMS [EXO-12-027]

interpreting LHC data

virtual gravitons + dileptons



black holes and asymptotic safety

classical black holes

space-time classical

- **classical Schwarzschild black holes**

metric

$$ds^2 = -f(r) dt^2 + f^{-1}(r) dr^2 + r^2 d\Omega_{d-2}^2, \quad f = 1 - \frac{G_N M}{r^{d-3}}$$

classical Schwarzschild radius

$$r_{\text{cl}} = (G_N M)^{1/(d-3)}$$

classical black holes

space-time classical

- **classical Schwarzschild black holes**

metric

$$ds^2 = -f(r) dt^2 + f^{-1}(r) dr^2 + r^2 d\Omega_{d-2}^2, \quad f = 1 - \frac{G_N M}{r^{d-3}}$$

classical Schwarzschild radius

$$r_{\text{cl}} = (G_N M)^{1/(d-3)}$$

- **features**

event horizon

curvature singularity at origin

temperature, Hawking radiation, information paradox

black hole thermodynamics

quantum black holes

space-time fluctuates

‘hoped-for’ features

(some version of an) event horizon

absence of curvature singularities

finite temperature

resolution to information paradox

origin of black hole thermodynamics

quantum black holes I

‘effective’ space-time

- **improved Schwarzschild black holes**

metric

$$ds^2 = -f(r) dt^2 + f^{-1}(r) dr^2 + r^2 d\Omega_{d-2}^2, \quad f = 1 - \frac{G_N M}{r^{d-3}}$$

$$G_N \rightarrow G_N(\mu(r))$$

asymptotic safety:

$$\frac{g_*}{\mu^{d-2}} \leq G(\mu) \leq G_N$$

quantum black holes I

‘effective’ space-time

- **improved Schwarzschild black holes**

metric

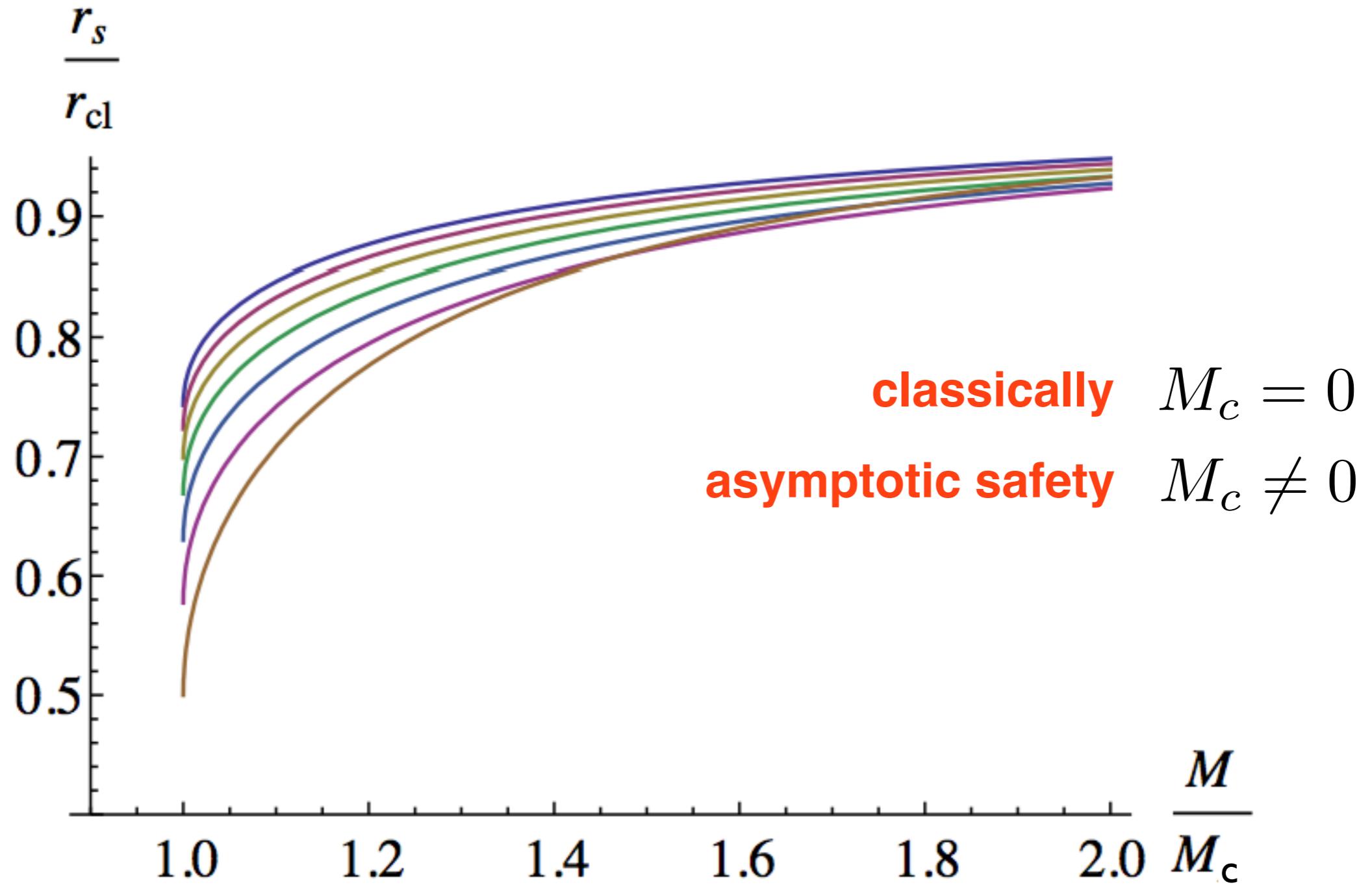
$$ds^2 = -f(r) dt^2 + f^{-1}(r) dr^2 + r^2 d\Omega_{d-2}^2, \quad f = 1 - \frac{G_N M}{r^{d-3}}$$

$$G_N \rightarrow G_N(\mu(r))$$

- **improved horizon radius**

$$(r_s)^{d-3} = G_N(r_s) \cdot M$$

horizon radius

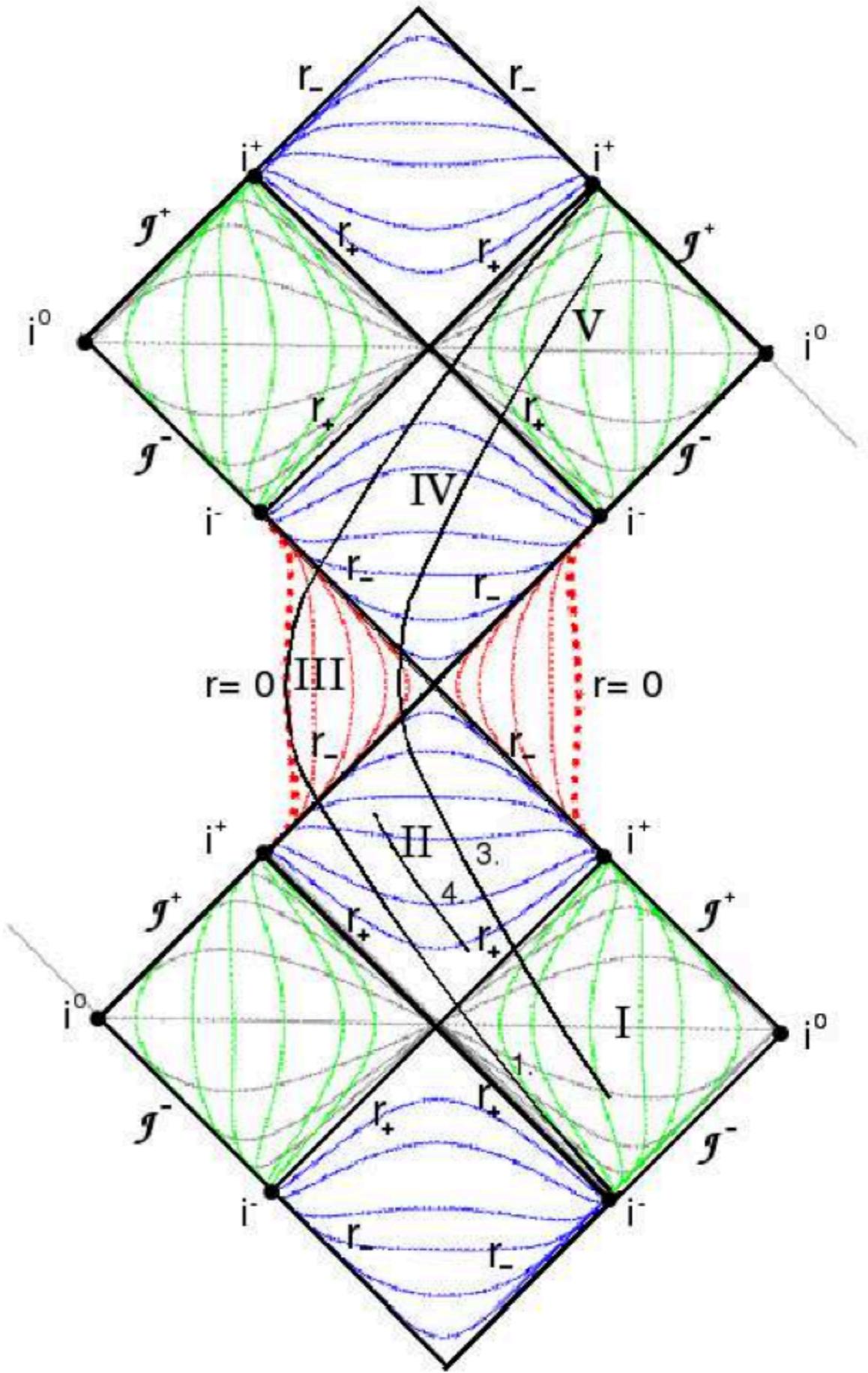


Penrose diagram

classically
geodesically incomplete
curvature singularity

asymptotic safety
geodesically complete
singularity reduced or absent

The Penrose diagram of a quantum black hole with $M > M_c$. The black curves in regions I and V are curves of constant t . The green (blue) [red] curves are curves of constant r in region I and V (II and IV) [III], respectively. In- and outgoing radial null geodesics are at 45° . Curves 1., 3. and 4. correspond to schematic plots of various solutions to the equations of motion. The points i^0 , i^+ and i^- denote spatial infinity, future infinity and past infinity, respectively. \mathcal{J}^- and \mathcal{J}^+ denote past and future null infinity (see text).



quantum black holes @ LHC

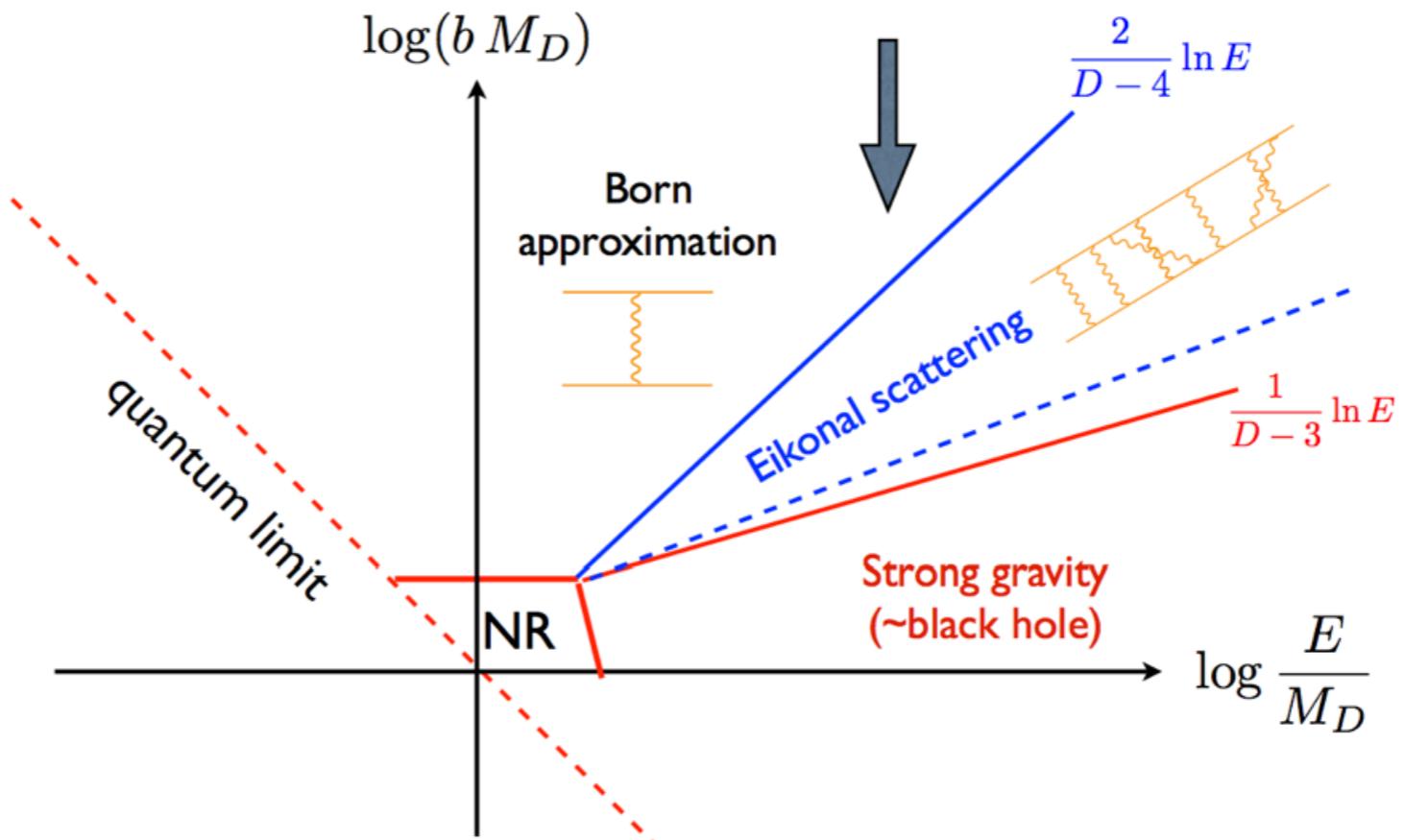


Figure 3: A proposed “phase diagram” of different regimes for gravitational scattering. In particular, we consider the effect of decreasing impact parameter, at fixed ultraplanckian energy, as indicated by the arrow. NR indicates the regime where higher-dimension operators are expected to be important.

(Giddings '11)

black hole production

Dimopoulos, Landsberg ('01)
Giddings, Thomas ('01)

- **classical Schwarzschild black holes**

metric

$$ds^2 = -f(r) dt^2 + f^{-1}(r) dr^2 + r^2 d\Omega_{d-2}^2, \quad f = 1 - \frac{G_N M}{r^{d-3}}$$

classical Schwarzschild radius

$$r_{\text{cl}} = (G_N M)^{1/(d-3)}$$

- **production cross section**

semi-classical $\hat{\sigma} = F \times \pi r_{\text{cl}}^2 (M = \sqrt{s}) \times \theta(\sqrt{s} - M_{\min})$

form factor F

free parameter $M_{\min} = (3 - 5)M_D$

black hole production @ LHC

prediction I

horizon radius decreases

prediction II

lower bound on black hole mass

$$M_c = \mathcal{O}(M_D)$$

prediction III

maximum temperature

partonic production
cross section

$$F(\sqrt{s}) = \left(\frac{r_s}{r_{\text{cl}}} \right) \Big|_{M_{\text{phys}}=\sqrt{s}}$$

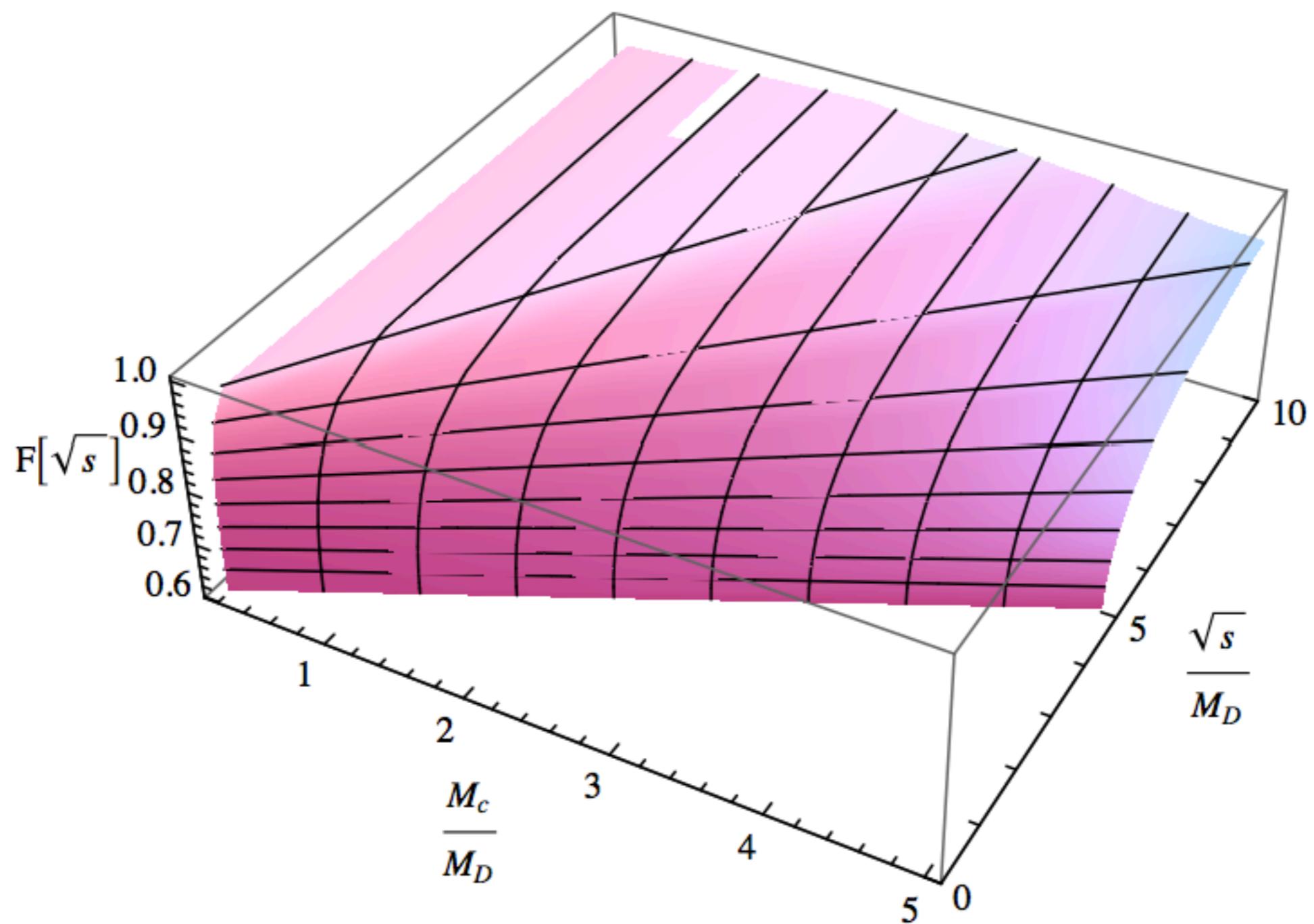


Figure 9: The gravitational form factor $F(\sqrt{s})$ with parameter $\gamma = \gamma_{\text{dS}}$ with $n = 4$ extra dimensions.

black hole production @ LHC

production cross section at the LHC $pp \rightarrow \text{final state}$

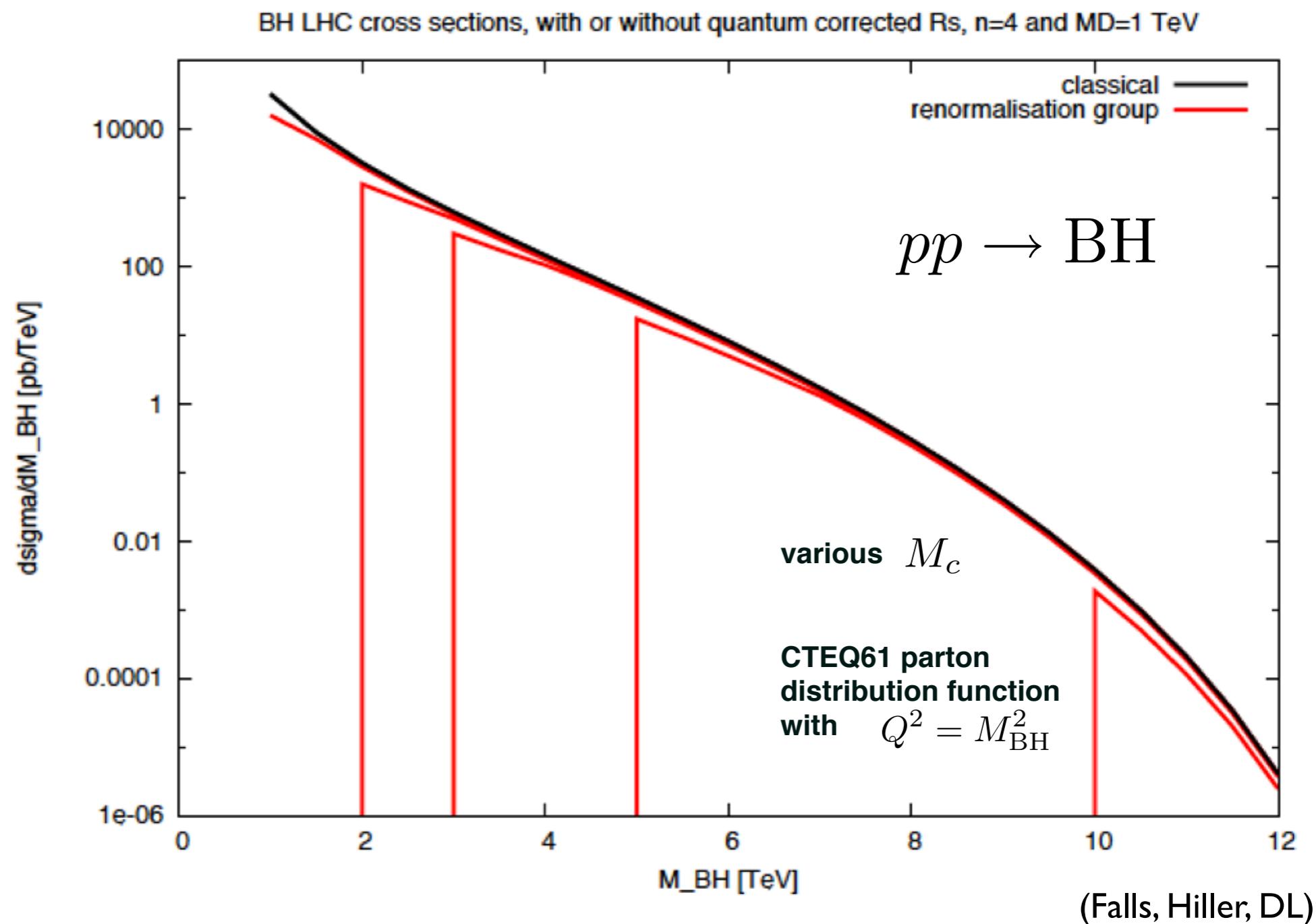
$$\sigma = \sum_{i,j} \int_0^1 dx_1 \int_0^1 dx_2 f_i(x_1) f_j(x_2) \hat{\sigma}(q_i q_j \rightarrow \text{final state})$$

elastic BH production $pp \rightarrow \text{BH}$

$$\frac{d\sigma}{dM} = \frac{2M}{s} \sum_{i,j} \int_{M^2/s}^1 \frac{dx}{x} f_i\left(\frac{M^2}{xs}\right) f_j(x) \hat{\sigma}(q_i q_j \rightarrow \text{BH})|_{\hat{s}=M^2}.$$

black hole production @ LHC

$n = 4$ extra dimensions



conclusions

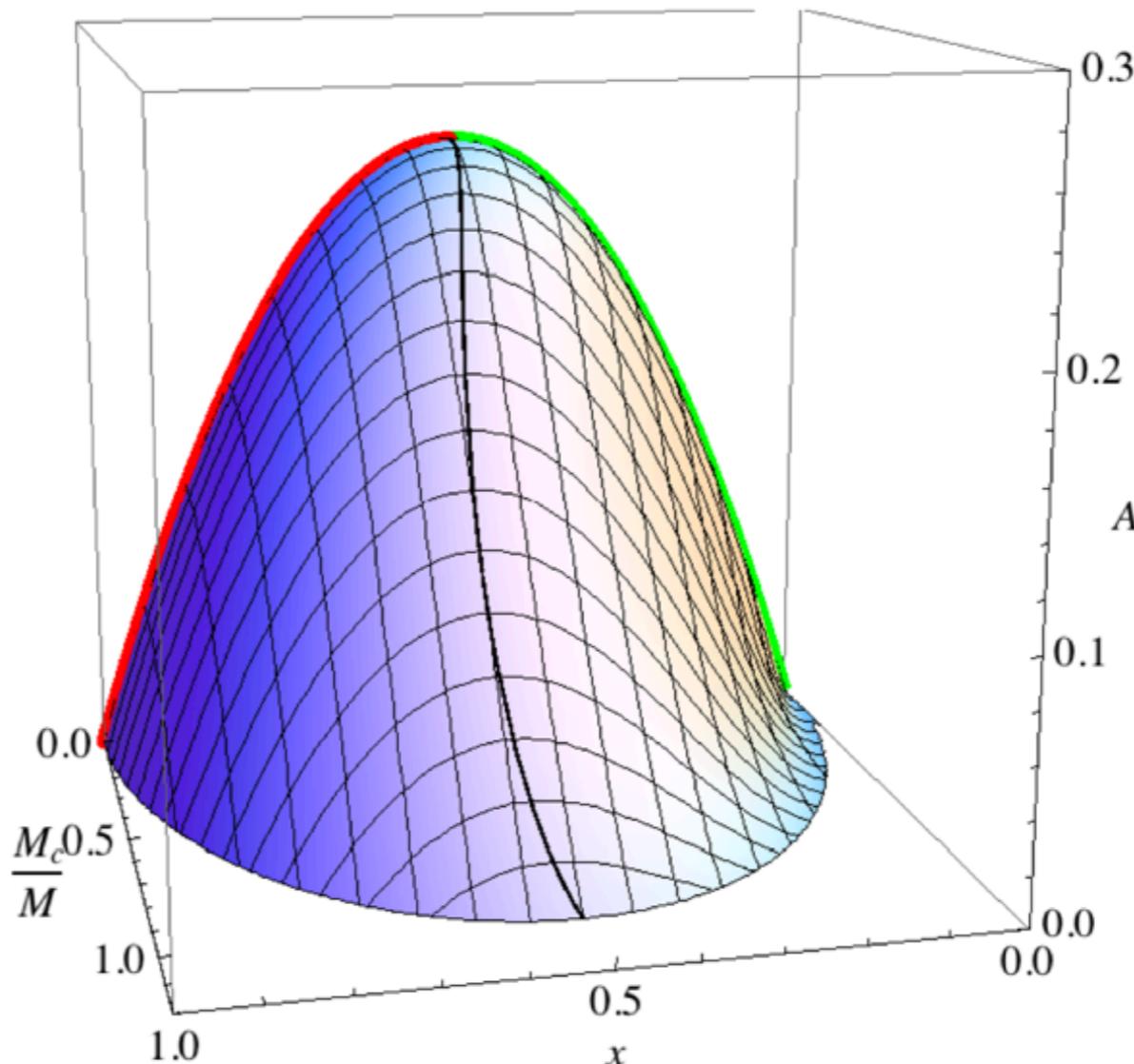
- **asymptotic safety** offers QFT-based description of gravity even at high energies
- **asymptotic safety** can be tested in the physical world (cosmology, particle physics, black holes)
- self-consistent picture, bootstrap test
challenges:
 - better structural insights
 - answers to open riddles (of PP & cosmo)

extra slides

rotation I

classically
inner & outer horizon

asymptotic safety
inner & outer horizon
reduced phase space



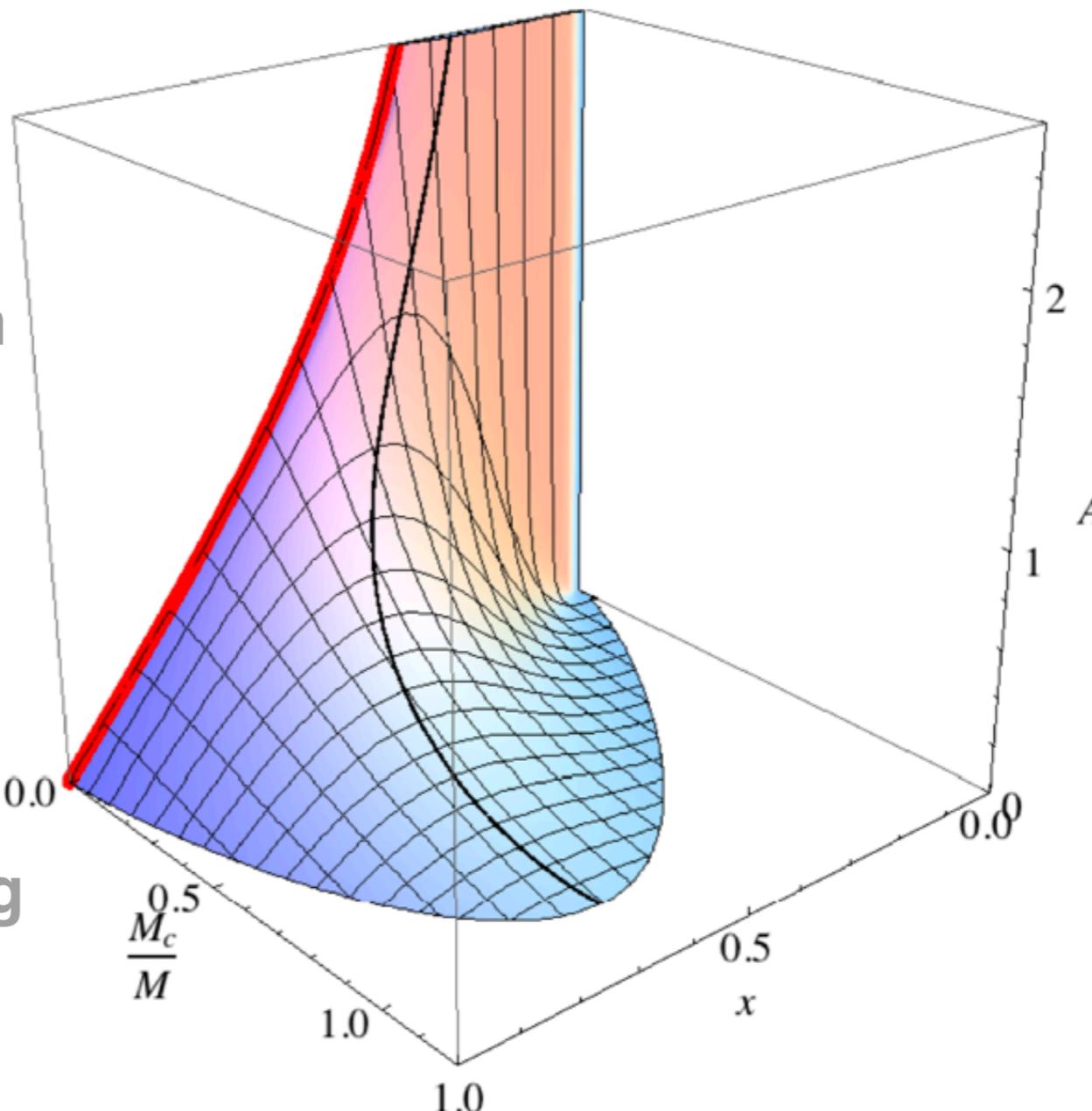
The phase space of black hole solutions in four dimensions. The points on the two-dimensional surface represent horizon radii x (3.8) as a function of angular momentum A (3.9) and the mass ratio M_c/M . The thick black line identifies the radius of the critical horizon $x_c(A, M; M_c)$. For fixed mass, the regions with $x > x_c(A)$ ($x < x_c(A)$) correspond to the event (Cauchy) horizons. The red (green) lines represent the classical solutions (Cauchy) horizon, respectively, in the limit $M_c/M \rightarrow 0$.

(Litim, Nikolopoulos'13)

rotation II

classically
absence of inner horizon
ultra-spinning solutions

asymptotic safety
inner & outer horizon
absence of ultra-spinning
reduced phase space

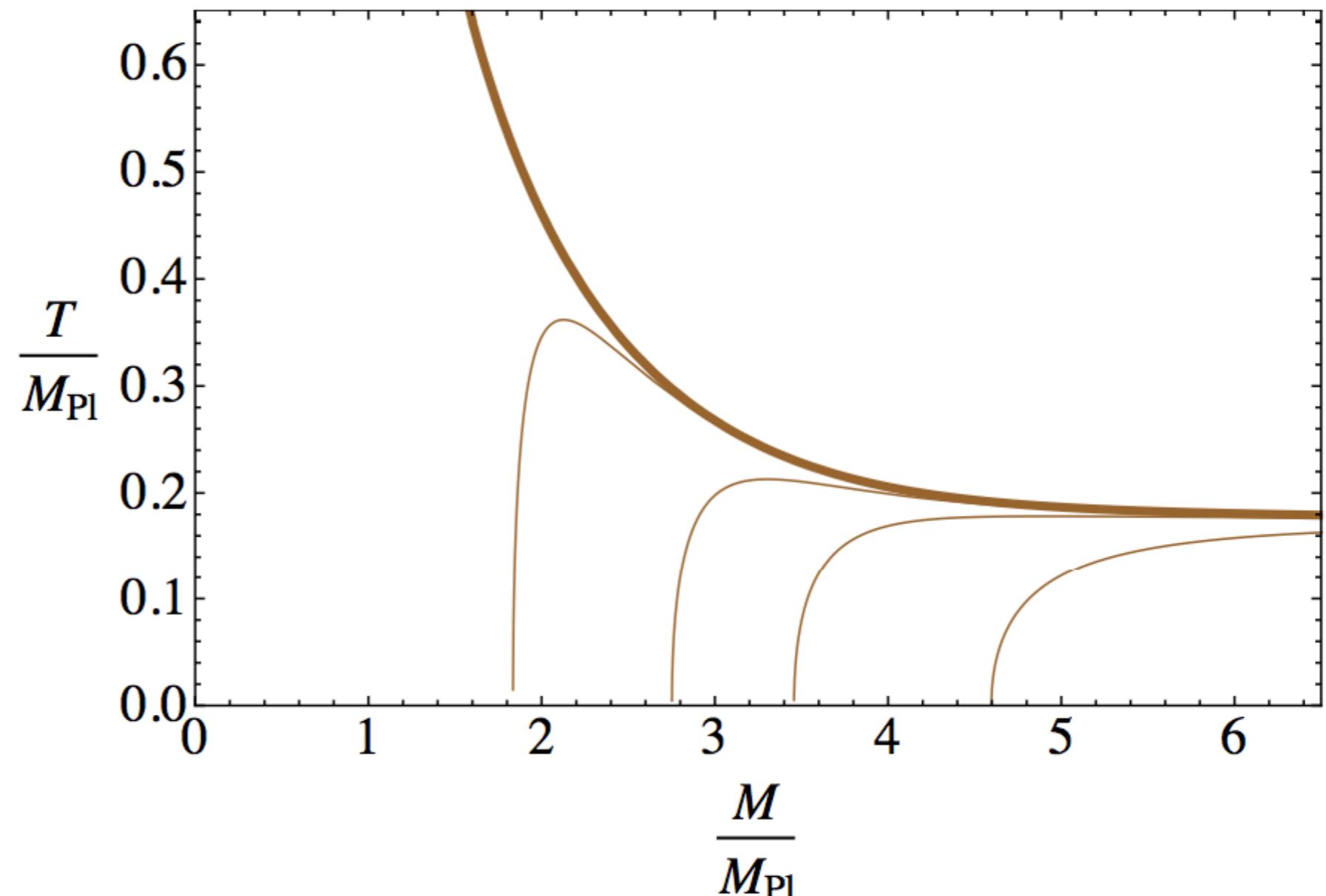


The allowed phase space of black hole solutions in six dimensions. The points of the two-dimensional surface represent the radius of the horizon x , as a function of A and M_c . The thick black line gives the radius of the critical horizon x_c . The regions with $x > x_c(A)$ ($x < x_c(A)$) correspond to event (Cauchy) horizons. The red line at $M_c/M = 0$ represents the classical event horizon.

temperature

classically:
temperature
diverges

asymptotic safety:
maximum
temperature



Reuter, Bonnano, '02
Falls, Litim, Raghuraman, '10
Litim, Nikolopoulos'13

quantum black holes II

black hole thermodynamics

Bardeen, Carter, Hawking '73
Bekenstein '73
Hawking '75
Gibbons, Hawking '77

quantum black holes II

black hole thermodynamics

entropy = horizon area

temperature = surface gravity

Bardeen, Carter, Hawking '73

Bekenstein '73

Hawking '75

Gibbons, Hawking '77

quantum black holes II

classical black hole thermodynamics

classically

$$A = A(M, J, q)$$

$$\frac{\delta Q}{T} = \frac{\delta A}{4G_N}$$

quantum black holes II

effective black hole thermodynamics

classically

$$A = A(M, J, q)$$

$$\frac{\delta Q}{T} = \frac{\delta A}{4G_N}$$

quantum-mechanically:

$$A = A(M, J, q; \kappa)$$

$$\frac{\delta Q}{T} = \frac{\delta A}{4G_\kappa}$$

quantum black holes II

‘effective’ black hole thermodynamics

RG

$$A = A(M, J, q; \kappa)$$

$$\frac{\delta Q}{T} = \frac{\delta A}{4G\kappa}$$

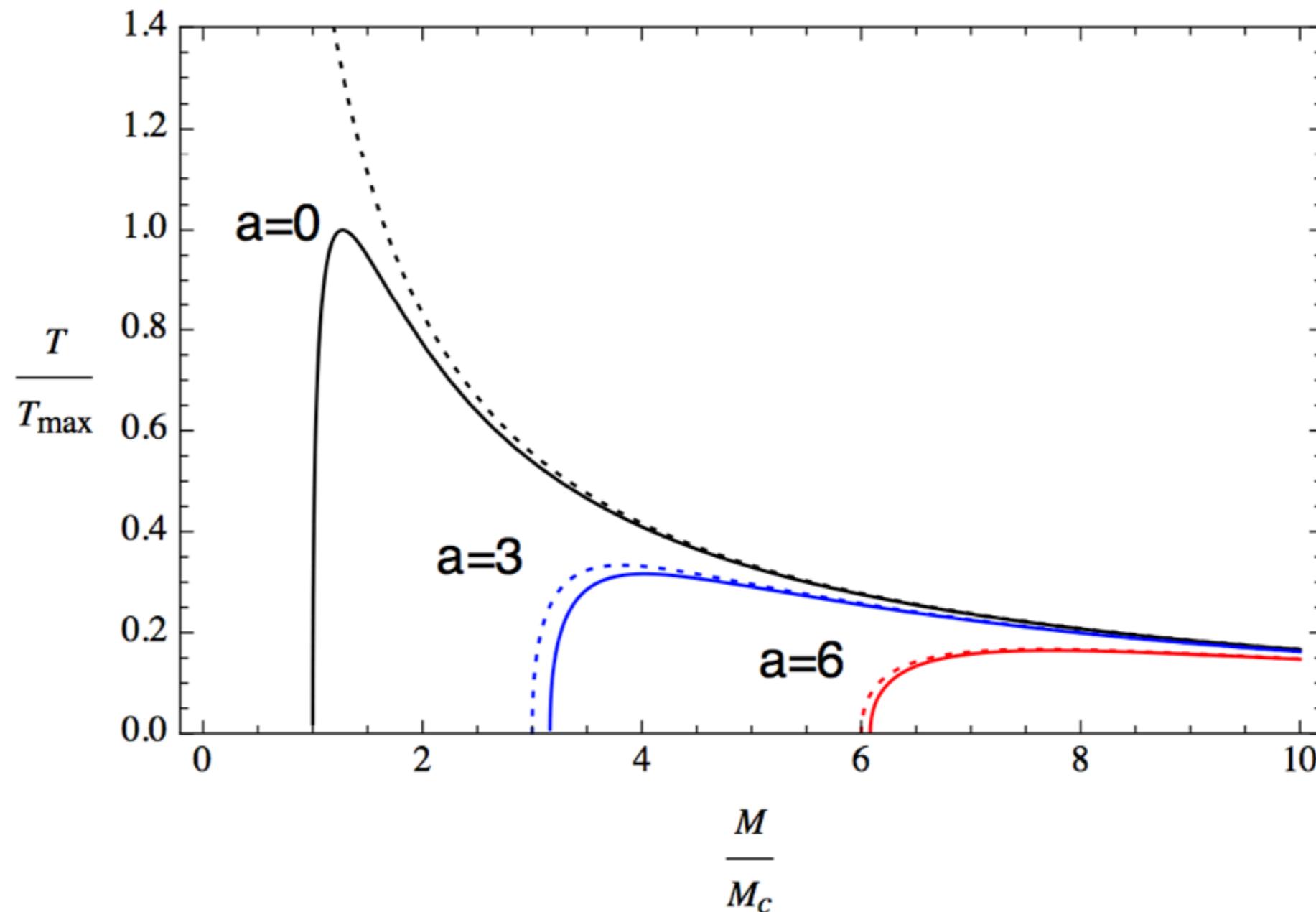
predictions

mass function $M^2 \equiv \frac{4\pi}{A} \left[\left(\frac{A + 4\pi G(A) e^2(A) q^2}{8\pi G(A)} \right)^2 + J^2 \right]$

temperature $T = 4G(A) \frac{\partial M}{\partial A}$

entropy $S = \frac{A}{4G\kappa} \quad k = k(M, J, q) \equiv k(A)$

temperature



Horizon temperature as a function of the black hole mass, comparing classical gravity (dashed lines) with asymptotically safe gravity with $g_* = 1$ (solid lines) for several angular momenta, with a given in units of $1/M_c$. Temperatures are normalised to the maximum temperature of the asymptotically safe Schwarzschild black hole (see text).

quantum black holes @ LHC