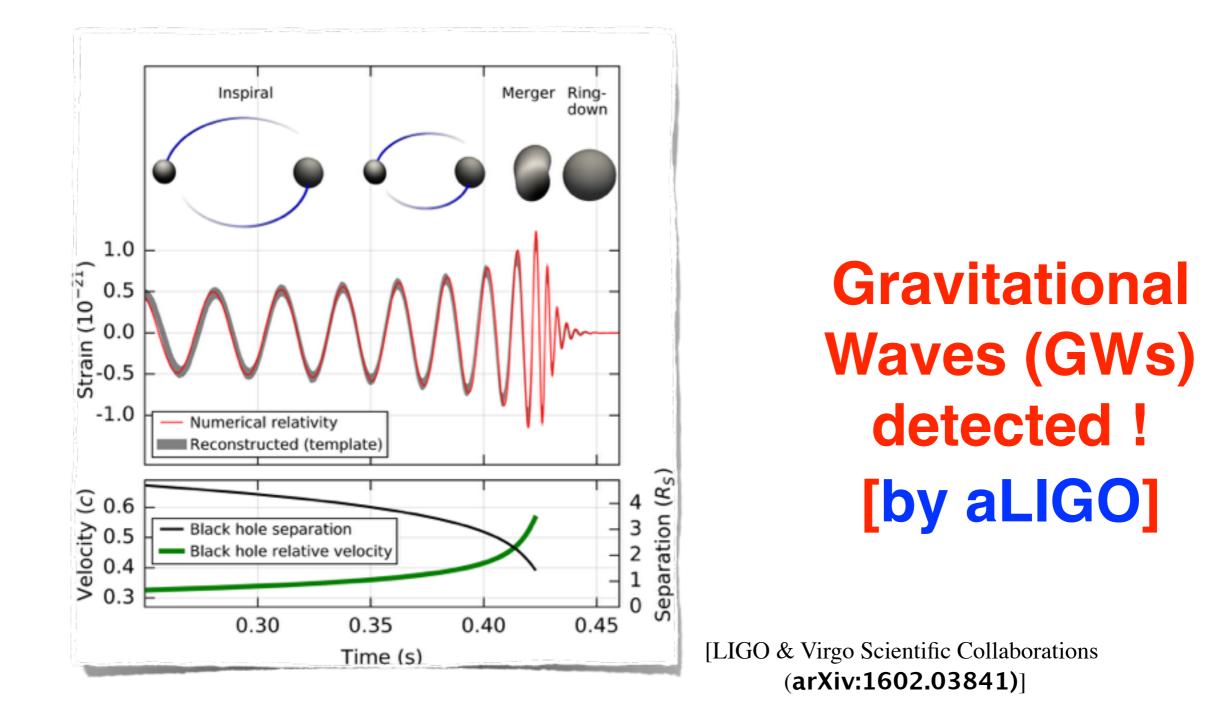
Gravitational Waves as a probe of fundamental Physics

YETI 2017, IPPP, Durham

Daniel G. Figueroa CERN, Theory Division



Einstein 1916 ... aLIGO 2015/16

* O(10) Solar mass Black Holes (BH) exist

* We can test the BH's paradigm (or Neutron Star physics)

* We can further test General Relativity (GR) [so far no deviation]

* We can observe the Universe through GWs Gravitational Waves (GWs) detected ! [by aLIGO]

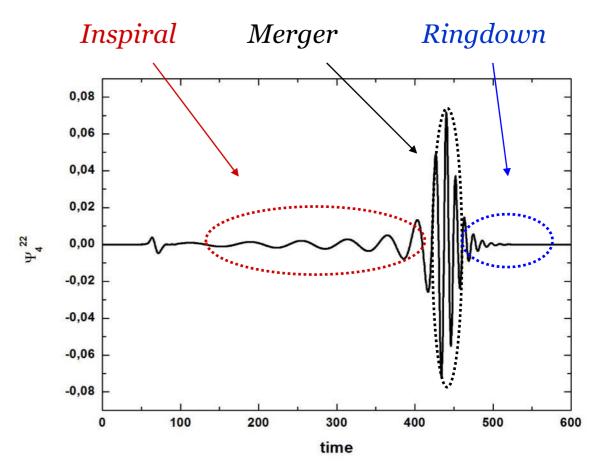
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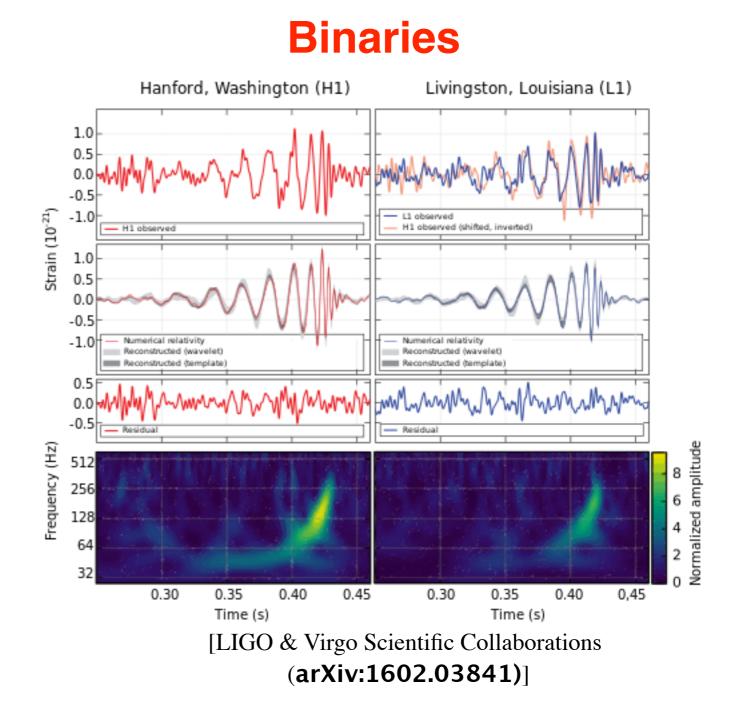
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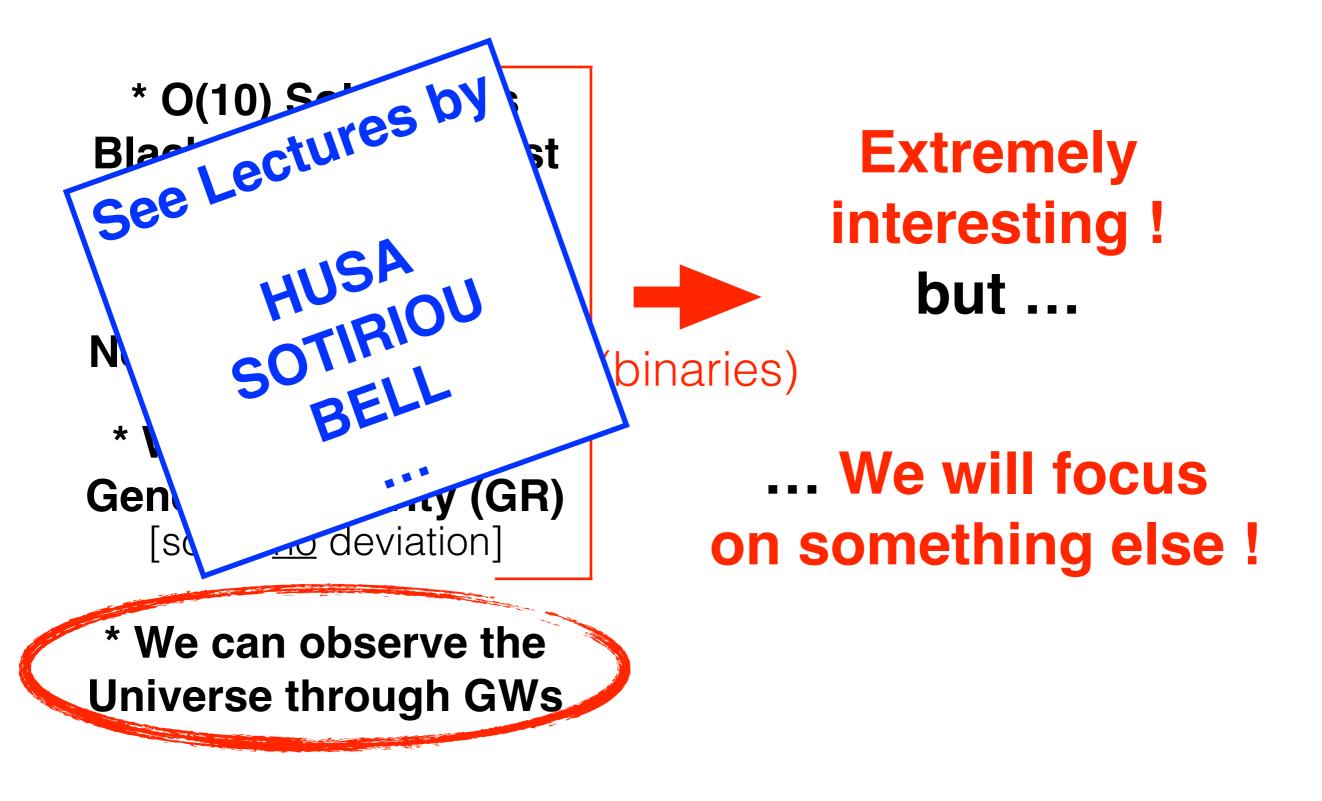
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Extremely interesting ! but ... (binaries)

... We will focus on something else !



*





- * Late Universe: Hubble diagram from Binaries
- * Early Universe: High Energy Particle Physics



* Late Universe: Hubble diagram from Binaries

* Early Universe: High Energy Particle Physics



YETI 2017: Gravitational probes of fundamental physics



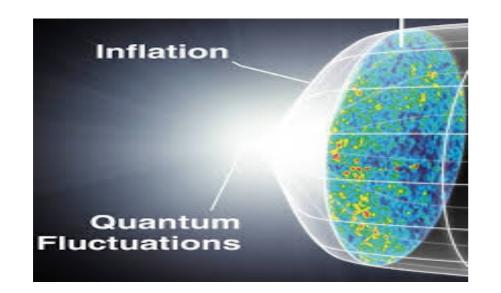


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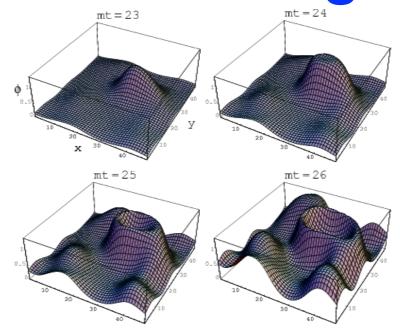


Inflationary Period



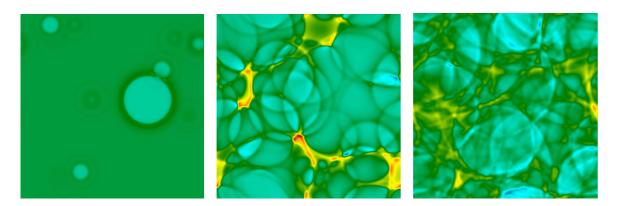
(Image: Google Search)

Preheating



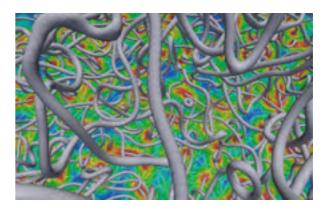
(Fig. credit: Phys.Rev. D67 103501)

Phase Transitions



(Image: PRL 112 (2014) 041301)

Cosmic Defects



(Image: Daverio et al, 2013)

OUTLINE

1) GWs from Inflation

Early Universe

- 2) GWs from Preheating
- 3) GWs from Phase Transitions

4) GWs from Cosmic Defects

OUTLINE

0) GW definition

1) GWs from Inflation

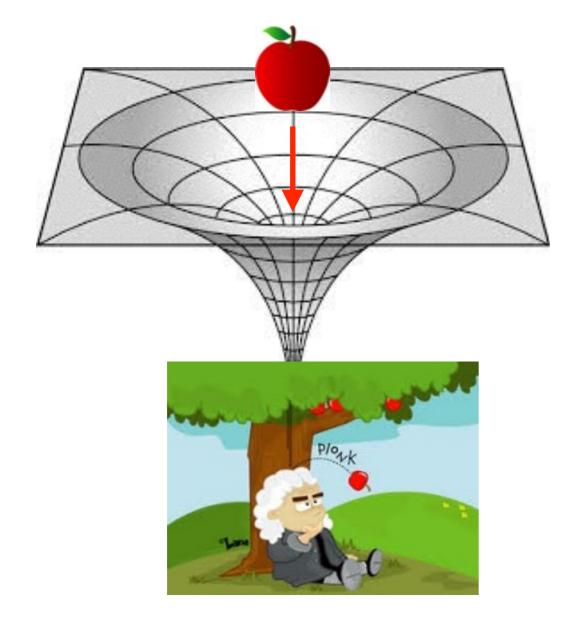
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General Relativity (GR)

 $G_{\mu\nu} = \frac{1}{m_p^2} T_{\mu\nu}$ geometry matter



$$ds^2 = g_{\mu\nu}(x)dx^{\mu}dx^{\nu}$$

DIFF:
$$x^{\mu} \to x'^{\mu}(x)$$

symmetry

1st approach to GWs

 $\begin{array}{l} \text{Minkowski} \\ g_{\mu\nu} = \overset{\uparrow}{\eta_{\mu\nu}} + h_{\mu\nu}(x) & \underset{\text{fixed}}{\text{fixed}} \\ (\ |h_{\mu\nu}| \ll 1 \) & \end{array}$

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$$\begin{array}{l} \text{Minkowski} \\ f \\ g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}(x) & \begin{array}{c} \text{fixed} \\ f \\ (\left| h_{\mu\nu} \right| \ll 1 \end{array} \end{array}$$

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DIFF:
$$x^{\mu} \not\prec x'^{\mu}(x)$$

 $x^{\mu} \rightarrow x'^{\mu} = x^{\mu} + \xi^{\mu}(x)$
 $(|\partial_{\mu}\xi_{\nu}(x)| \lesssim |h_{\mu\nu}|)$
 $residual$
 $symm.$
 $h_{\mu\nu}(x) \rightarrow h'_{\mu\nu}(x') = h_{\mu\nu}(x) - \partial_{[\mu}\xi_{\nu]}$

1st approach to GWs

×

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Trace-reversed

$$\bar{h}_{\mu\nu} \equiv h_{\mu\nu} - \frac{1}{2}h\eta_{\mu\nu} \longrightarrow \partial^{\alpha}\partial_{\alpha}\bar{h}_{\mu\nu} + \eta_{\mu\nu}\partial^{\alpha}\partial^{\beta}\bar{h}_{\alpha\beta} - \partial^{\alpha}\partial_{(\mu}\bar{h}_{\alpha\nu)} = -\frac{2}{m_{p}^{2}}T_{\mu\nu}$$

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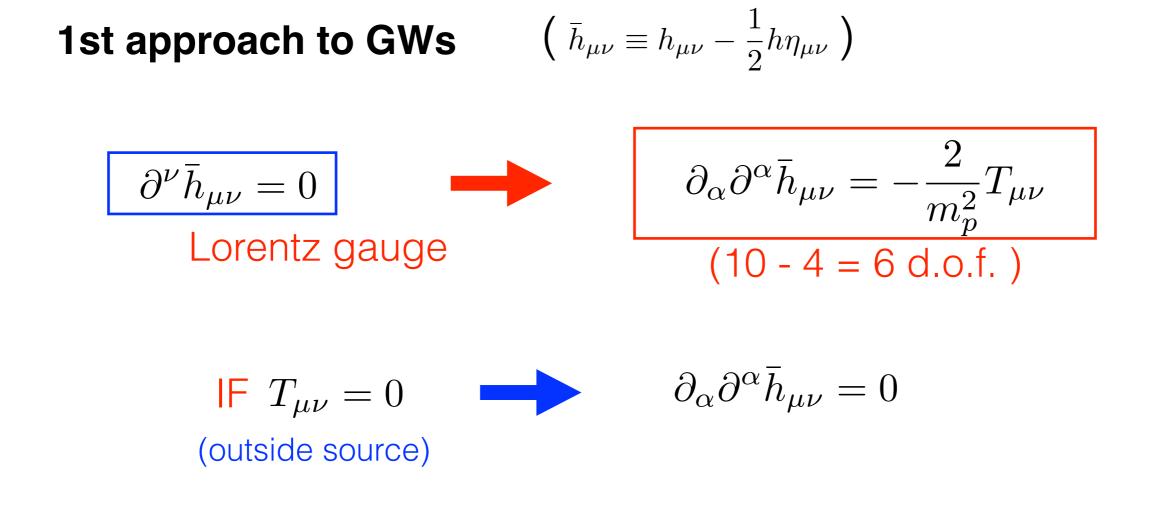
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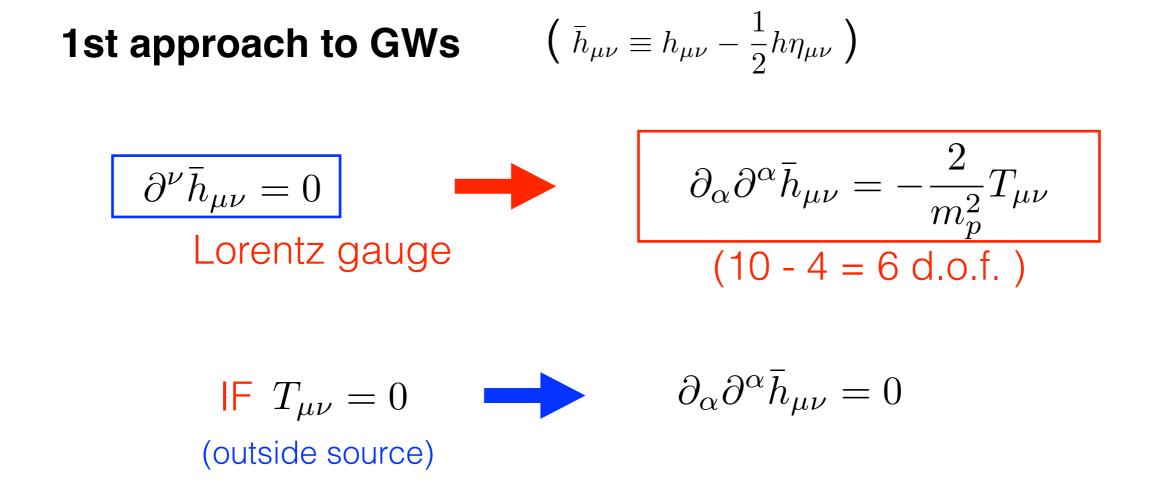
$$\begin{aligned} x^{\mu} \to x'^{\mu} &= x^{\mu} + \xi^{\mu}(x) \\ (\left| \partial_{\mu} \xi_{\nu}(x) \right| \lesssim \left| h_{\mu\nu} \right|) & \text{residual} \\ \text{symm.} \\ h_{\mu\nu}(x) \to h'_{\mu\nu}(x') &= h_{\mu\nu}(x) - \partial_{[\mu} \xi_{\nu]} \end{aligned}$$

Trace-reversed $\bar{h}_{\mu\nu} \equiv h_{\mu\nu} - \frac{1}{2}h\eta_{\mu\nu} \longrightarrow \partial^{\alpha}\partial_{\alpha}\bar{h}_{\mu\nu} + \eta_{\mu\nu}\partial^{\alpha}\partial^{\beta}\bar{h}_{\alpha\beta} - \partial^{\alpha}\partial_{(\mu}\bar{h}_{\alpha\nu)} = -\frac{2}{m_{p}^{2}}T_{\mu\nu}$ $\partial^{\nu}\bar{h}_{\mu\nu} = 0$ $\partial_{\alpha}\partial^{\alpha}\bar{h}_{\mu\nu} = -\frac{2}{m_{p}^{2}}T_{\mu\nu}$ Lorentz gauge

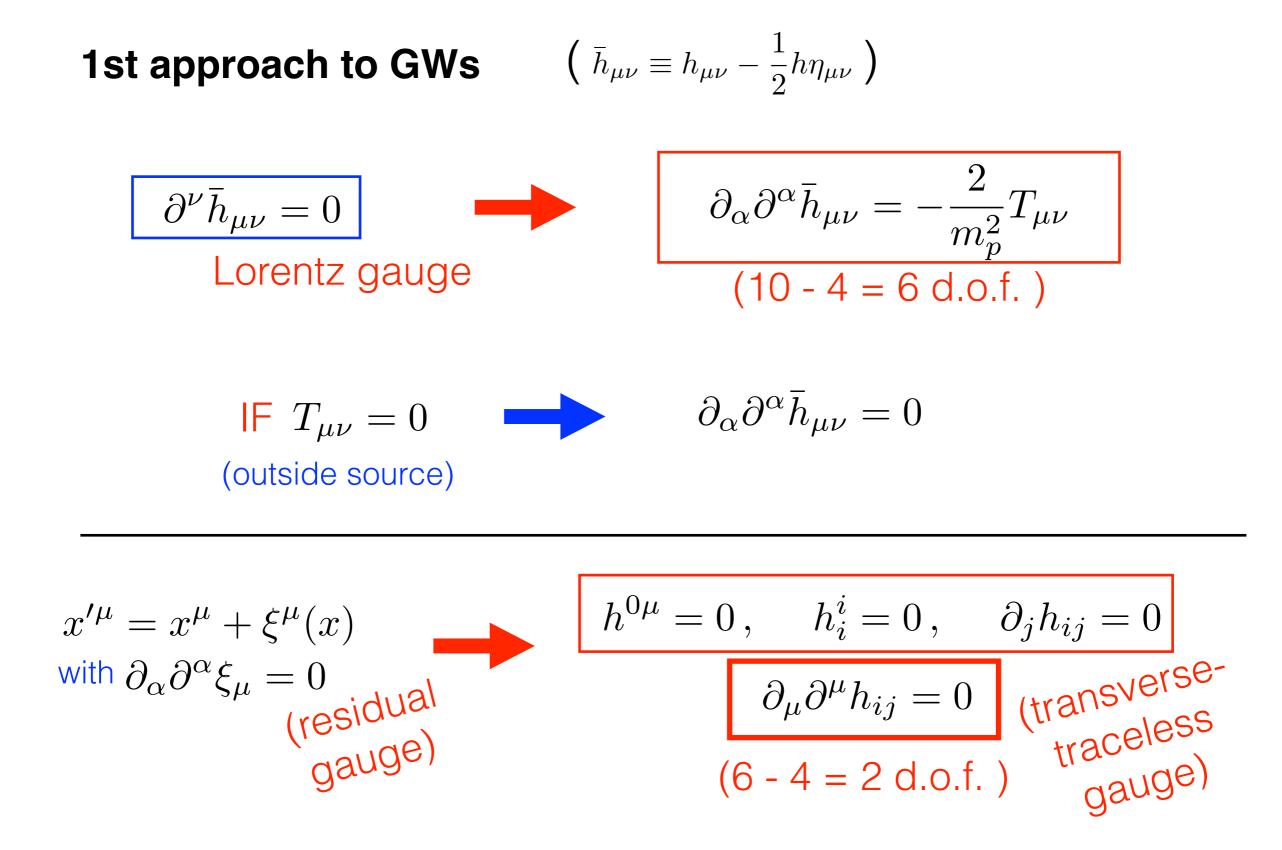
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$$\left(\bar{h}_{\mu\nu} \equiv h_{\mu\nu} - \frac{1}{2}h\eta_{\mu\nu} \right)$$







$$\begin{array}{l} x'^{\mu} = x^{\mu} + \xi^{\mu}(x) \\ \text{with } \partial_{\alpha} \partial^{\alpha} \xi_{\mu} = 0 \\ (\text{residual} \\ \text{gauge}) \end{array}$$



1st approach to GWs

(TT gauge: 6 - 4 = 2 d.o.f.)
$$h^{0\mu} = 0, \quad h^i_i = 0, \quad \partial_j h_{ij} = 0$$

$$\partial_{\mu}\partial^{\mu}h_{ij} = 0$$
 Wave Eq. \rightarrow Gravitational Waves !

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2 dof = 2 polarizations
$$h_{ab}(t, \mathbf{x}) = \int_{-\infty}^{\infty} df \int d\hat{n} h_{ab}(f, \hat{n}) e^{-2\pi i f(t-\hat{n}\mathbf{x})}$$
 (plane wave)

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$$h_{ab}(f,\hat{n}) = \sum_{A=+,\mathbf{x}} h_A(f,\hat{n})\epsilon_{ab}^{(A)}(\hat{n}) = \begin{pmatrix} h_+ & h_x & 0 \\ h_x & -h_+ & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Transverse-Traceles (2 dof)

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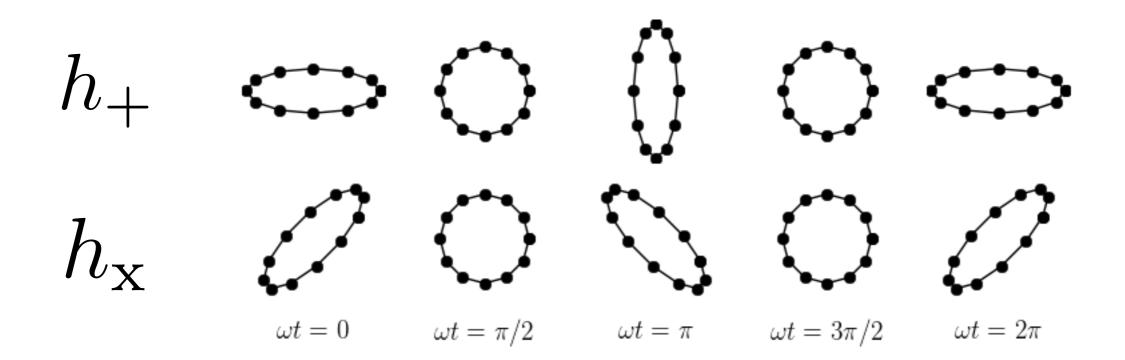
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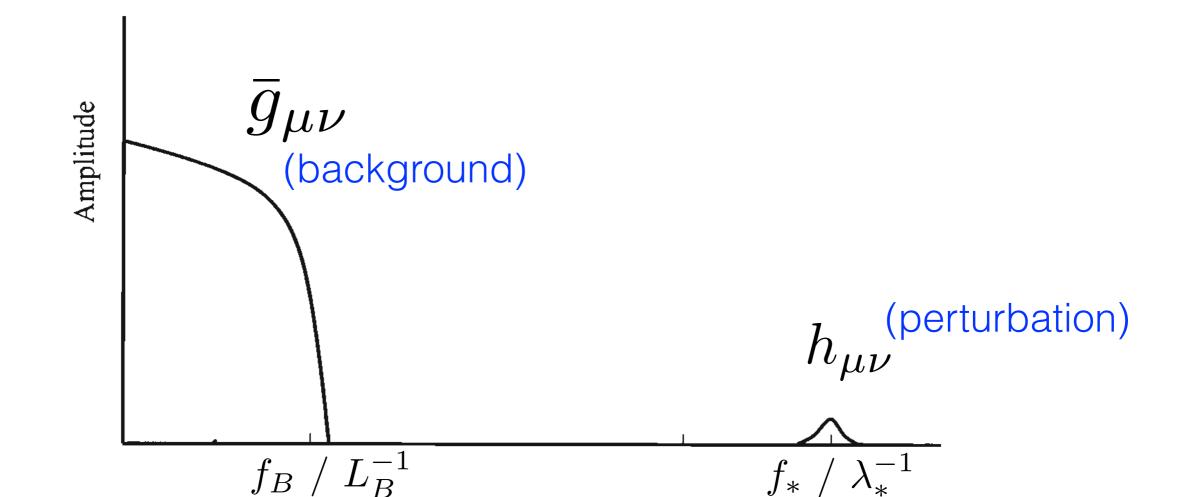
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$$R_{\mu\nu} = \frac{8\pi G}{c^4} \left(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T \right) \longrightarrow R_{\mu\nu} = \bar{R}_{\mu\nu} + R^{(1)}_{\mu\nu} + R^{(2)}_{\mu\nu} + \dots ,$$

(background) $\mathcal{O}(h) \quad \mathcal{O}(h^2)$

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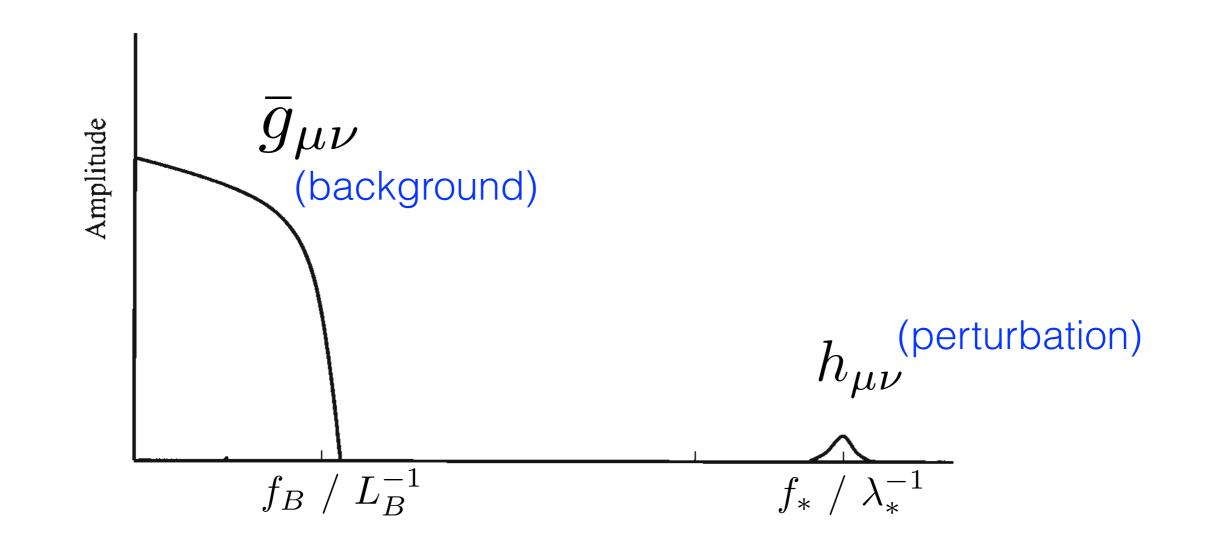
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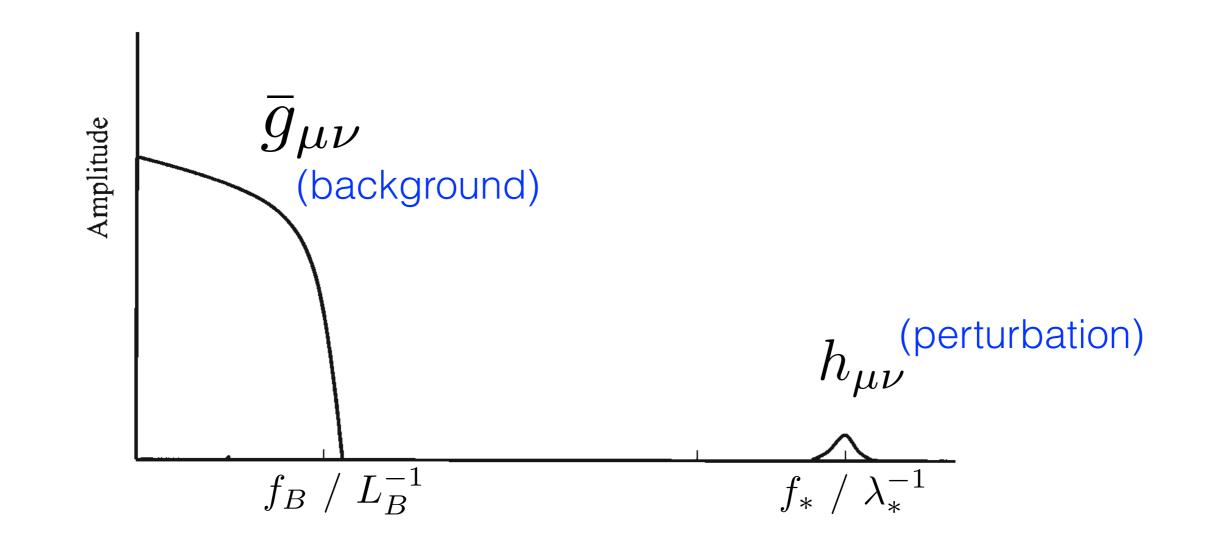
Low Freq. / Long Scale: $\bar{R}_{\mu\nu} = -[R^{(2)}_{\mu\nu}]^{\text{Low}} + \frac{8\pi G}{c^4} \left(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T\right)^{\text{Low}}$

High Freq. / Short Scale: $R_{\mu\nu}^{(1)} = -[R_{\mu\nu}^{(2)}]^{\text{High}} + \frac{8\pi G}{c^4} \left(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T\right)^{\text{High}}$

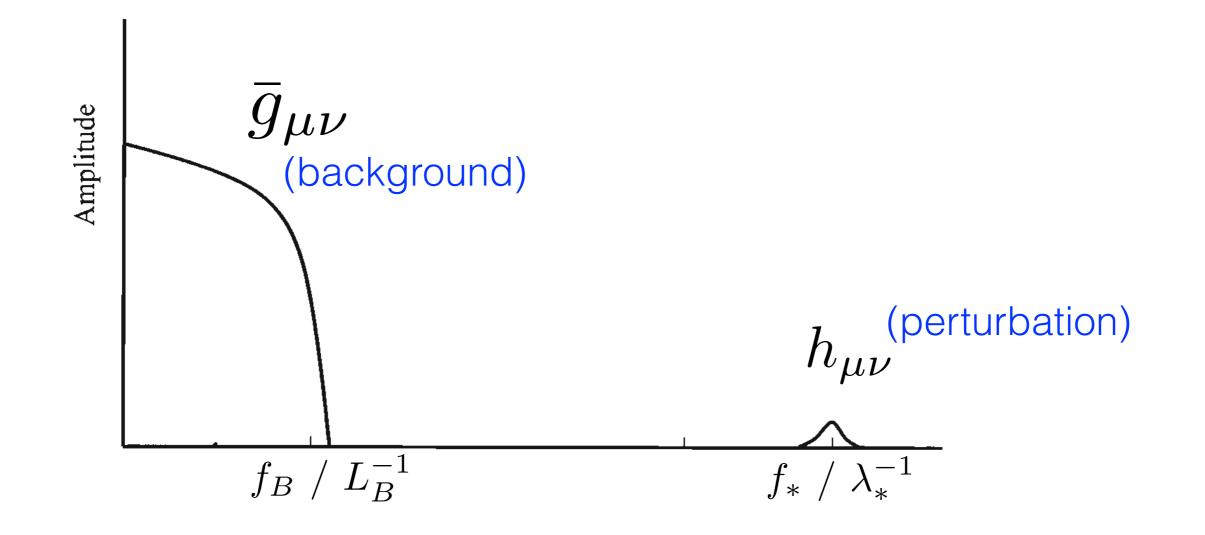
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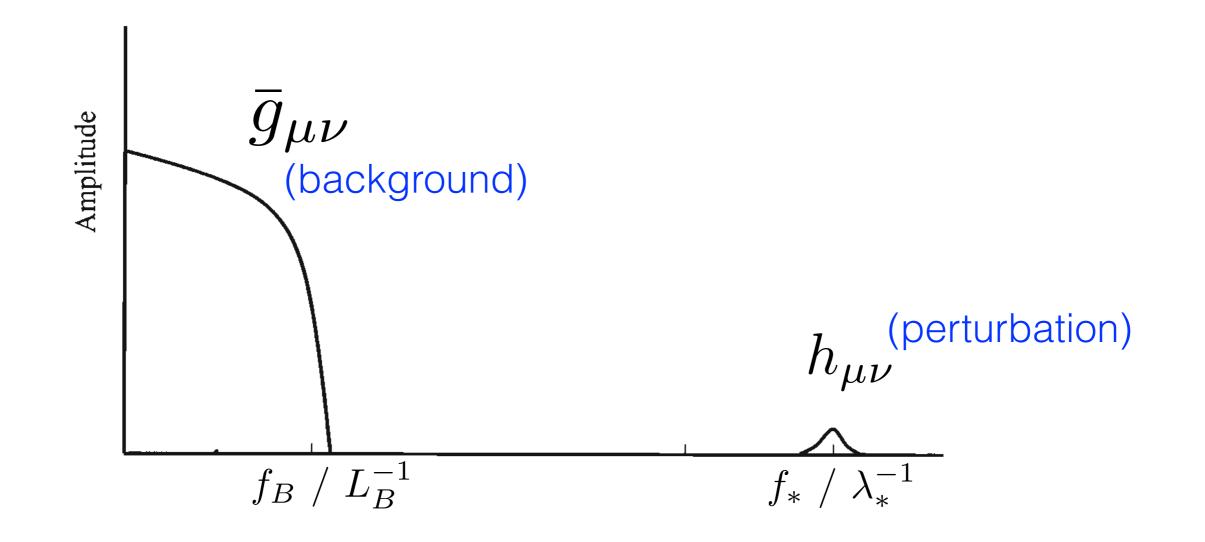
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Low Freq. / Long Scale: $\bar{R}_{\mu\nu} = -\langle R^{(2)}_{\mu\nu} \rangle + \frac{8\pi G}{c^4} \langle T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T \rangle$ average) $t_{\mu\nu} = -\frac{c^4}{8\pi G} \langle R^{(2)}_{\mu\nu} - \frac{1}{2}\bar{g}_{\mu\nu}R^{(2)} \rangle \qquad \langle T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T \rangle = \bar{T}^{\mu\nu} - \frac{1}{2}\bar{g}_{\mu\nu}\bar{T}$



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It can be shown that only TT *dof* contribute to < ... >

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$$t_{\mu\nu} = \frac{c^4}{32\pi G} \left\langle \partial_{\mu} h_{ij}^{\text{TT}} \partial_{\nu} h_{ij}^{\text{TT}} \right\rangle \longrightarrow \frac{dE}{dAdt} = \frac{c^4}{32\pi G} \left\langle \dot{h}_{ij}^{\text{TT}} \dot{h}_{ij}^{\text{TT}} \right\rangle$$

GW energy-momentum tensor GW power/area radiated

What about the High Freq. / Short Scale? $R^{(1)}_{\mu\nu} = -[R^{(2)}_{\mu\nu}]^{\text{High}} + \frac{8\pi G}{c^4} \left(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T\right)^{\text{High}}$

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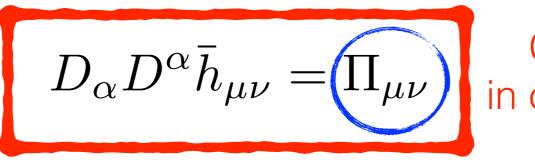
Propagation of GWs in curved space-time

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/ - \

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Creation of GWs in curved space-time

Gravitational Waves as a probe of the early Universe

OUTLINE

0) GW definition 🗸

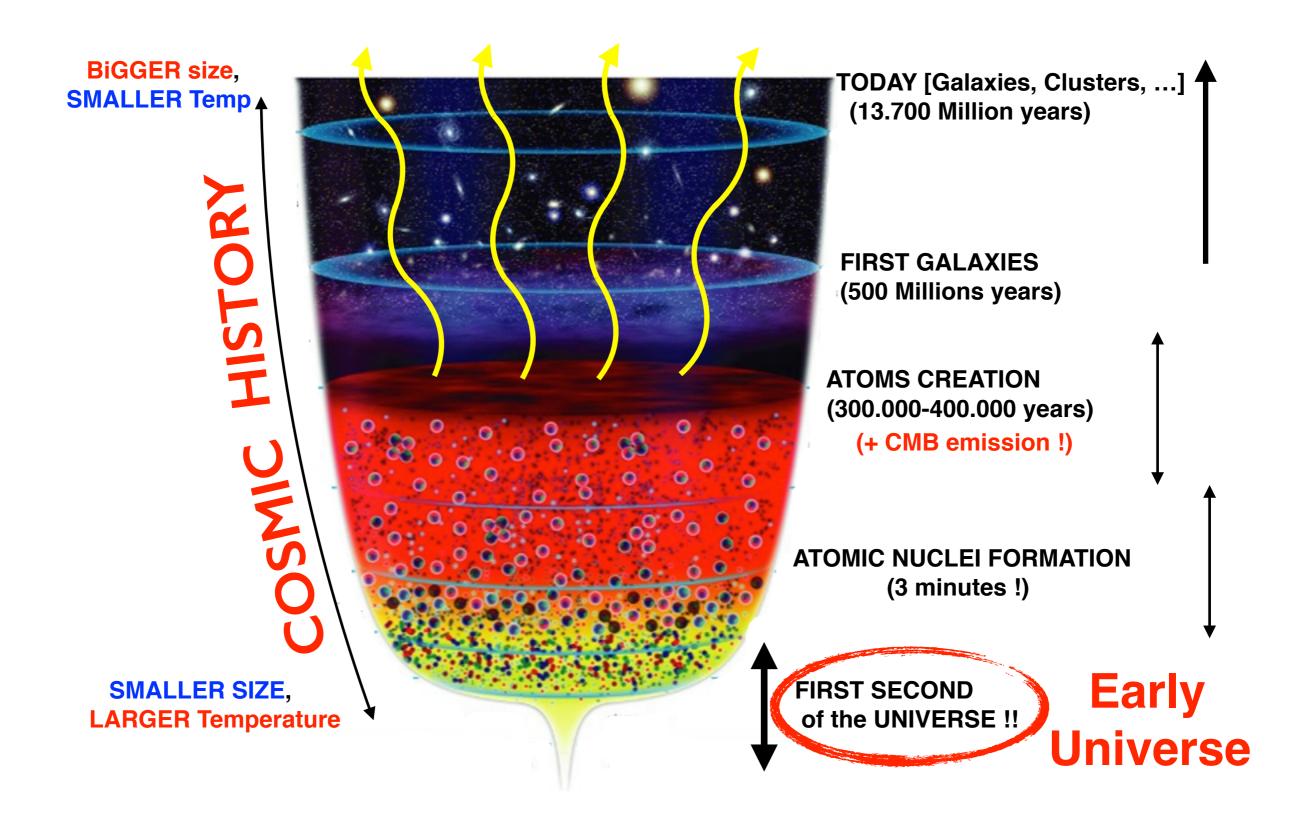
1) GWs from Inflation

Early Universe

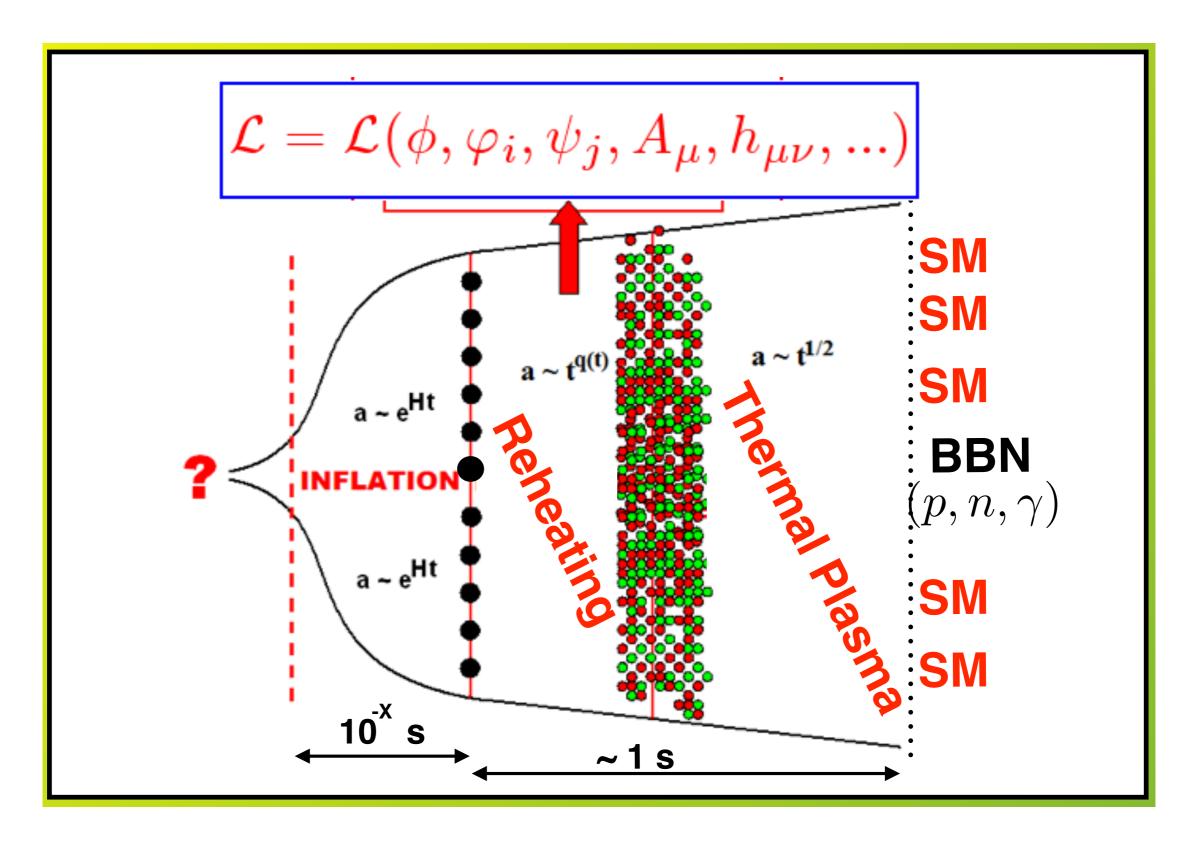
- 2) GWs from Preheating
- 3) GWs from Phase Transitions

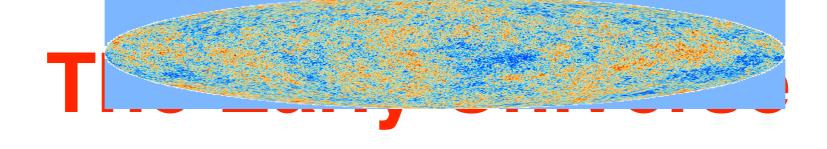
4) GWs from Cosmic Defects

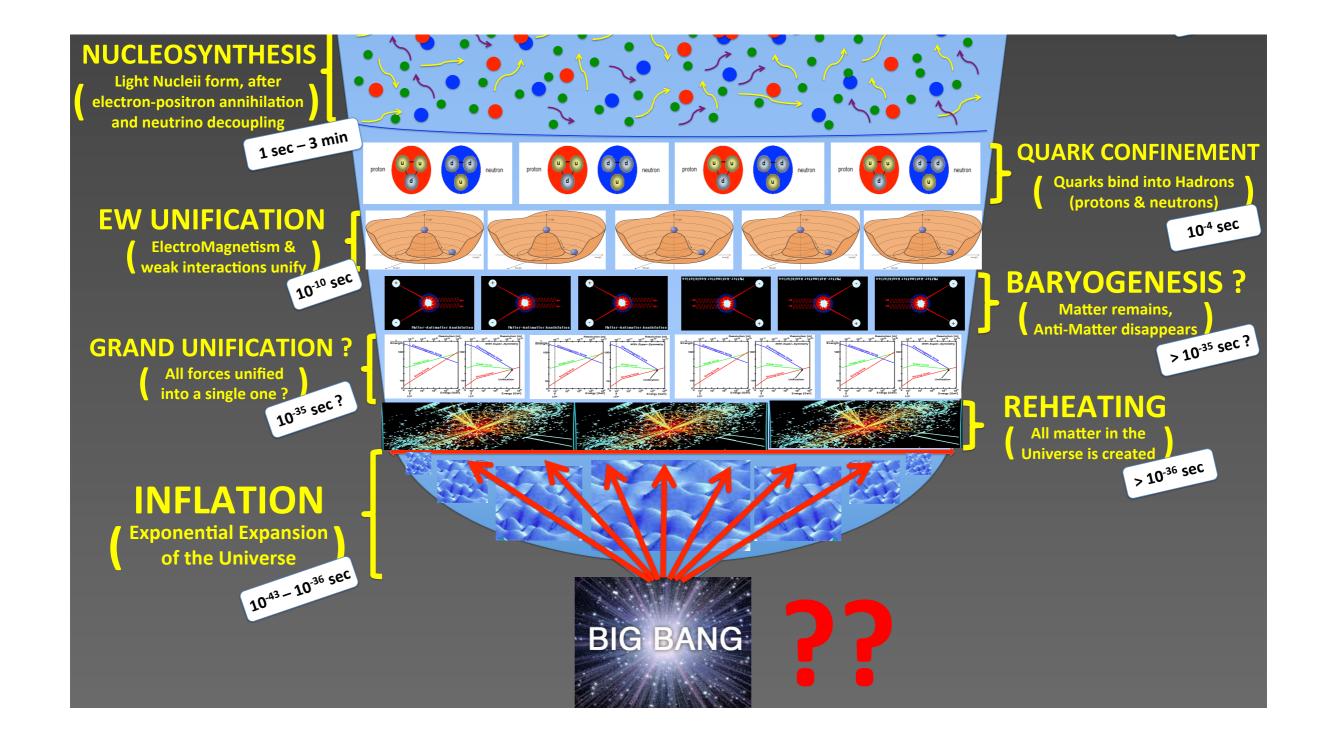
The Early Universe



The Early Universe







GWs as a probe of the early Universe

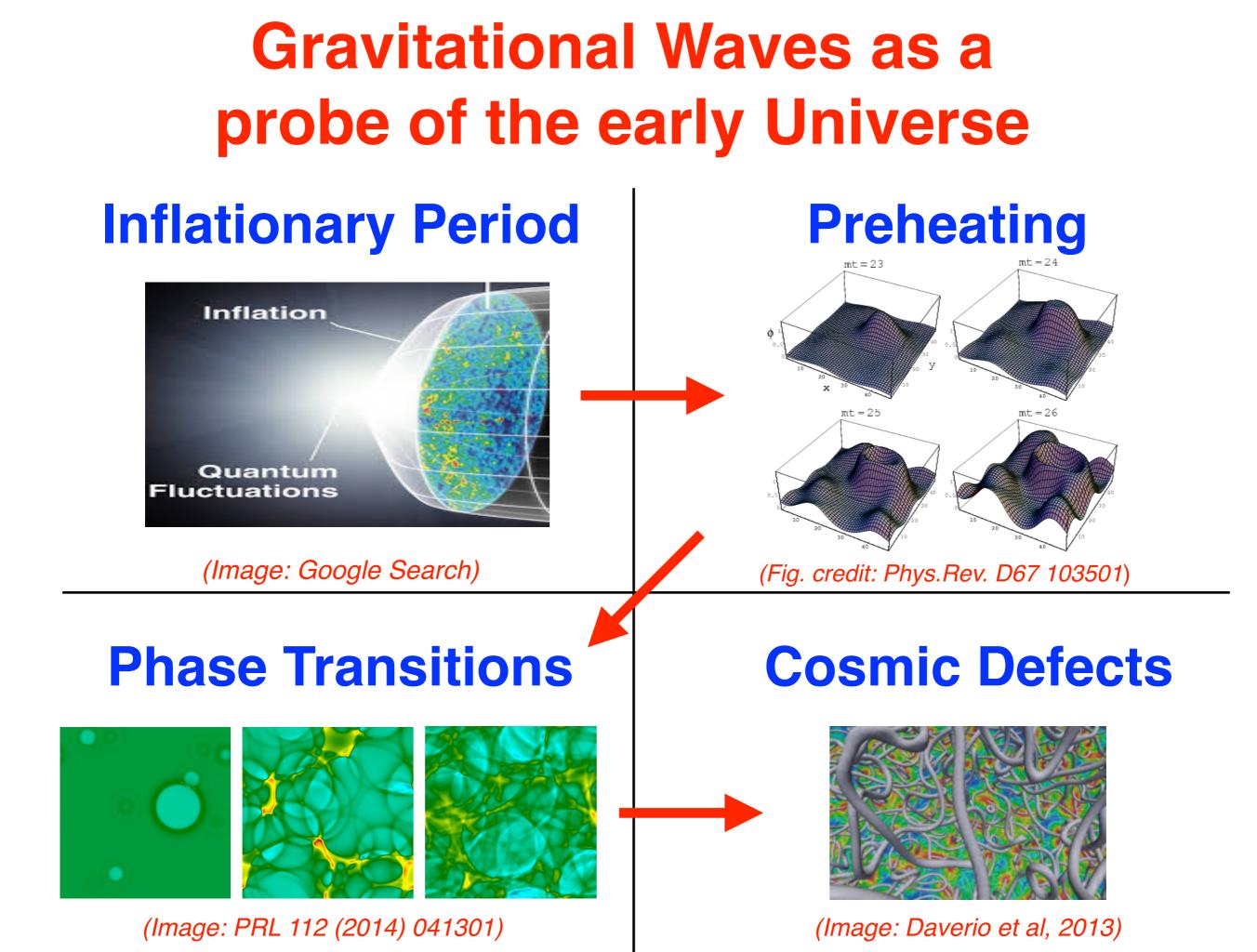
WEAKNESS of GRAVITY:

ADVANTAGE: GW DECOUPLE upon Production **DISADVANTAGE:** DIFFICULT DETECTION

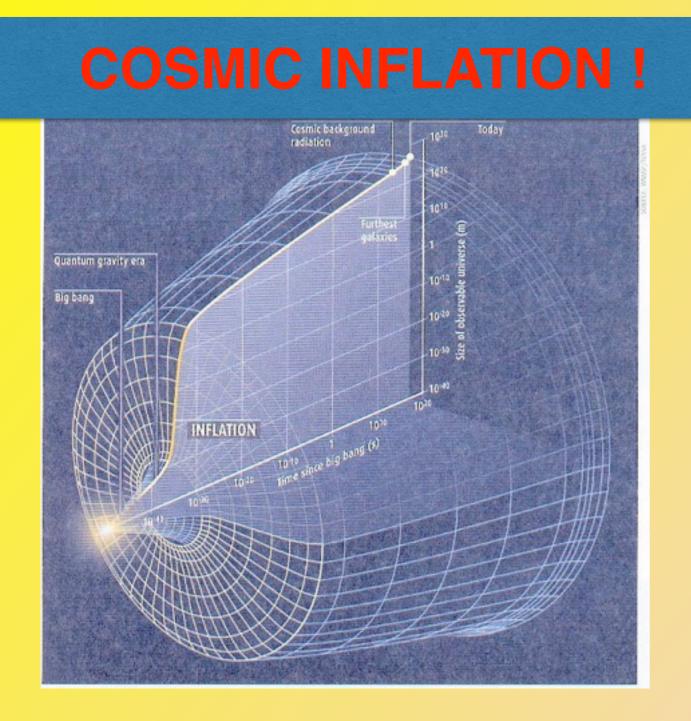
O ADVANTAGE: GW \rightarrow Probe for Early Universe

 $\rightarrow \left\{ \begin{array}{l} \mathbf{Decouple} \rightarrow \mathrm{Spectral} \ \mathrm{Form} \ \mathrm{Retained} \\ \mathbf{Specific} \ \mathbf{HEP} \ \Leftrightarrow \ \mathrm{Specific} \ \mathrm{GW} \end{array} \right.$

Physical Processes:
 Inflation
 Reheating
 Phase Transitions
 Cosmic Defects

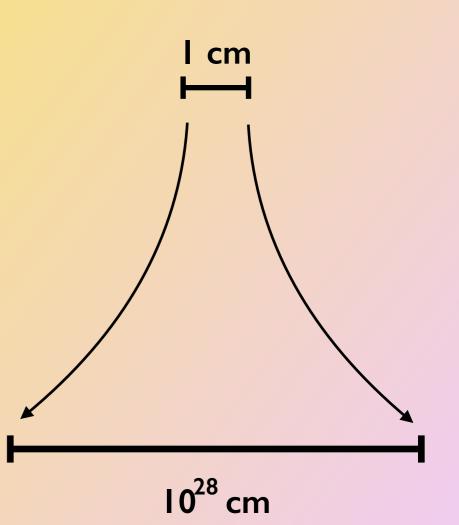


Inflation: Basics



Needed for Consistency of the Big Bang theory !

$$a \sim e^{H_* t} \gtrsim e^{60}$$



Inflation: A generator of Primordial Fluctuations

 $d\tau \equiv dt/a(t)$ $S_{(2)}^{(t)} = \frac{m_p^2}{8} \int dt dx^3 a^3 \left[(\dot{h}_{ij})^2 - a^{-2} (\partial_l h_{ij})^2 \right]$ $h_{ij}(\vec{k}, \tau) = \epsilon_{ij}^{(s)} h_{\vec{k}}^{(s)}$ $v^{(s)} \equiv \frac{a}{2} m_p h_{\vec{k}}^{(s)}$

There is no source!

Inflation: A generator of Primordial Fluctuations

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 (Similarly as with Scalar Pert.)
 Quantization

 Quantize→Bunch-Davies→Power Spectrum
 of Gravity dof!

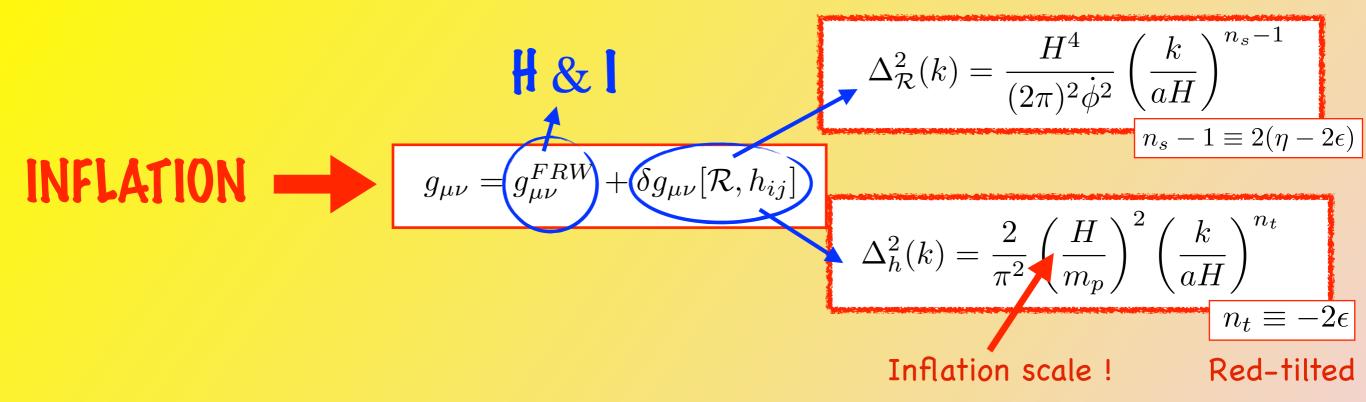
Inflation: A generator of Primordial Fluctuations

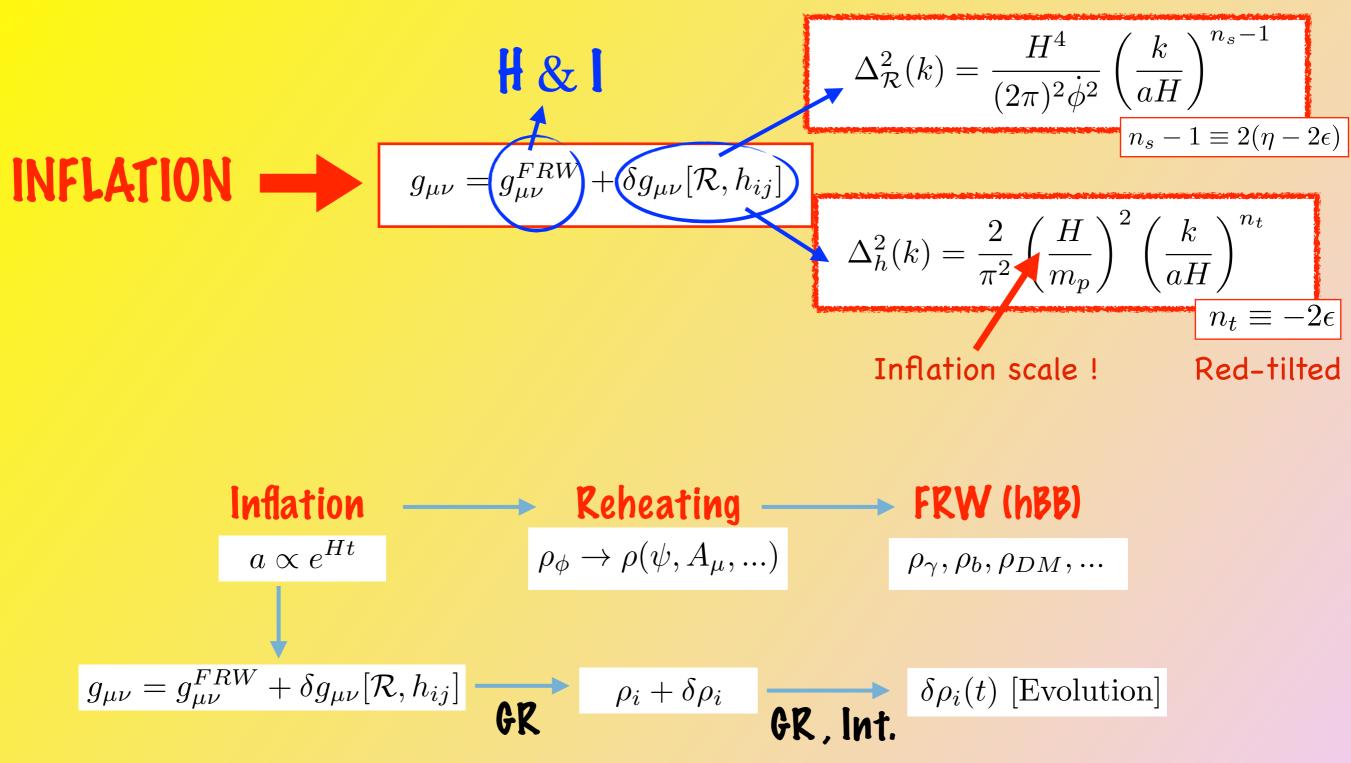
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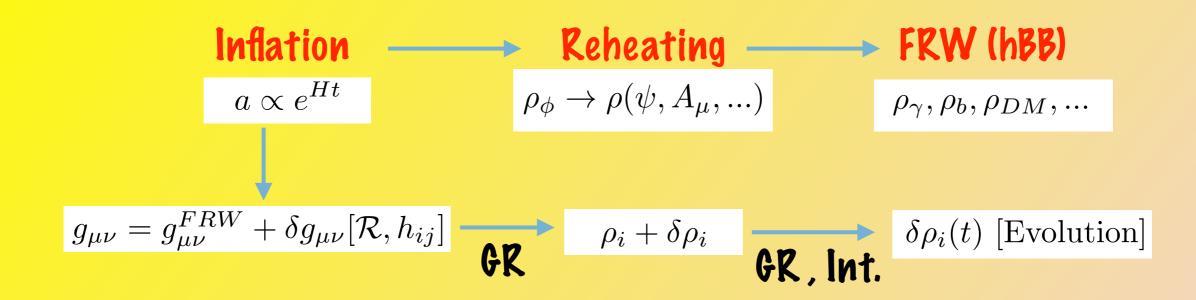
$$\langle h(\mathbf{k})h^*(\mathbf{k}')\rangle = \frac{2\pi^2}{k^3}\delta^{(3)}(\mathbf{k} + \mathbf{k}')P_h(\mathbf{k})$$

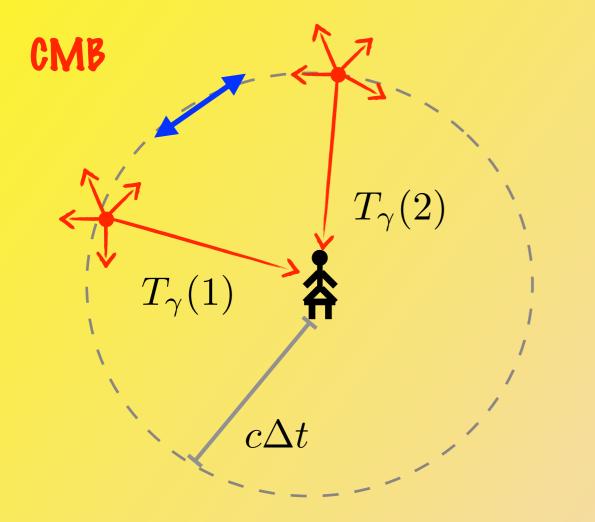
$$\Delta_h^2(k,\tau) \equiv \frac{k^3}{2\pi^2} P_h(k,\tau) \qquad \frac{k\tau \ll 1}{\text{(Super-Horizon)}} \qquad \Delta_h^2(k) = \frac{2}{\pi^2} \left(\frac{H}{m_p}\right)^2 \left(\frac{k}{aH}\right)^{-2\epsilon}$$

Red-tilted !

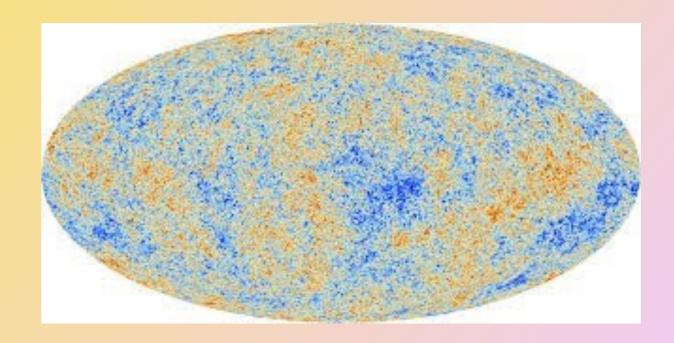


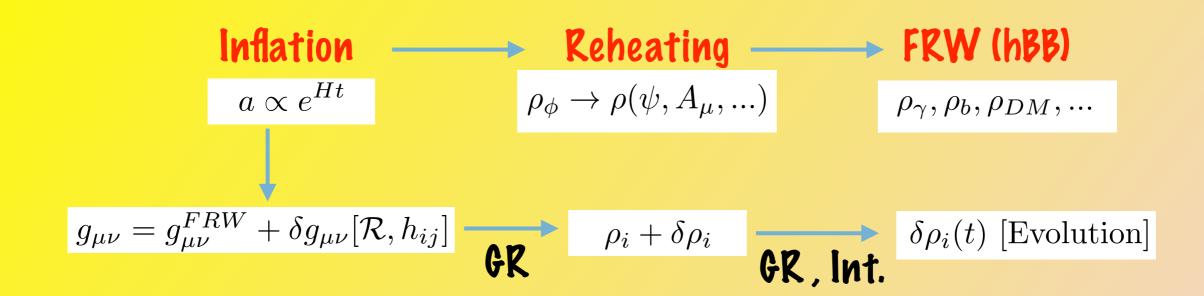


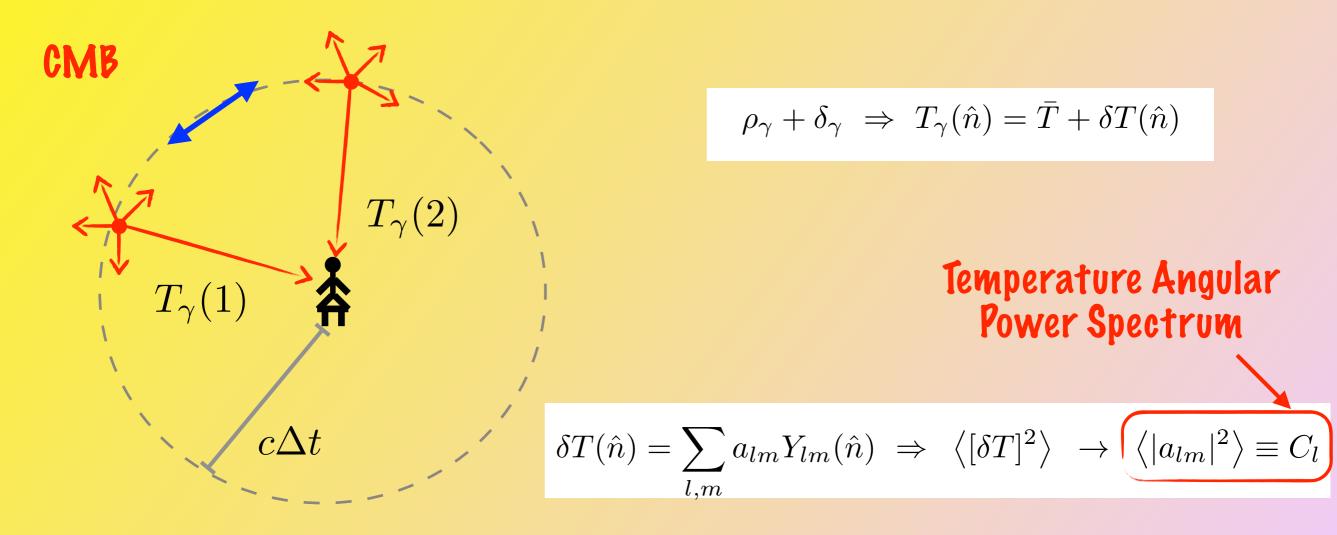




$$\rho_{\gamma} + \delta_{\gamma} \Rightarrow T_{\gamma}(\hat{n}) = \bar{T} + \delta T(\hat{n})$$







$$\delta \rho_{\gamma}, \delta \rho_e \Rightarrow [\text{Thomson Scattering}] \Rightarrow \text{Linear Polarization}$$

Linear Polarization
$$\rightarrow Q, U$$
 (Stokes Parameters)

$$(Q \pm iU)(\hat{n}) = \sum_{l,m} a_{lm}^{(\pm 2)} Y_{lm}^{(\pm 2)} = \sum_{l,m} (e_{lm} \pm ib_{lm}) Y_{lm}^{(\pm 2)}(\hat{n})$$

$$\mathcal{E}(\hat{n}) = \sum_{l,m} e_{lm} Y_{lm}(\hat{n}) , \ \mathcal{B}(\hat{n}) = \sum_{l,m} b_{lm} Y_{lm}(\hat{n})$$

$$\langle \mathcal{E}^2 \rangle, \ \langle \mathcal{B}^2 \rangle \rightarrow \langle |e_{lm}|^2 \rangle \equiv C_l^E, \ \langle |b_{lm}|^2 \rangle \equiv C_l^B$$

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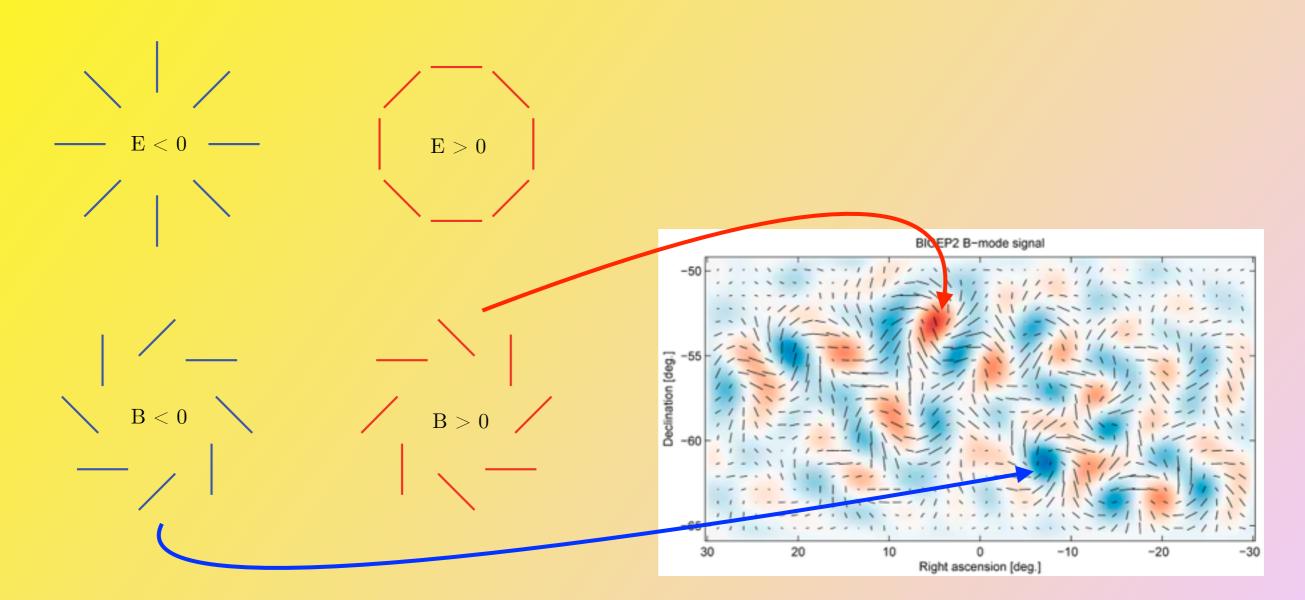
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Polarization Angular Power Spectrum

Depends on Scalar (also tensor) Perturbations

$$\left\langle \mathcal{E}^2 \right\rangle, \ \left\langle \mathcal{B}^2 \right\rangle \ \to \ \left\langle |e_{lm}|^2 \right\rangle \equiv C_l^E, \ \left\langle |b_{lm}|^2 \right\rangle \equiv C_l^B$$

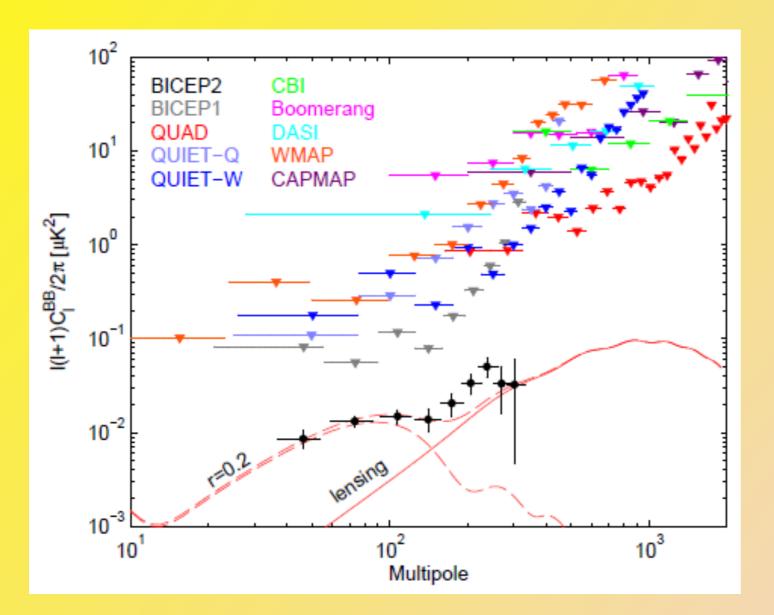
Pepends on Scalar (also tensor) Perturbations Polarization Angular Power Spectrum



$$\left\langle \mathcal{E}^2 \right\rangle, \ \left\langle \mathcal{B}^2 \right\rangle \ \to \ \left\langle |e_{lm}|^2 \right\rangle \equiv C_l^E, \ \left\langle |b_{lm}|^2 \right\rangle \equiv C_l^B$$

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Polarization Angular Power Spectrum



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Depends on Scalar (also tensor) Perturbations

Polarization Angular Power Spectrum

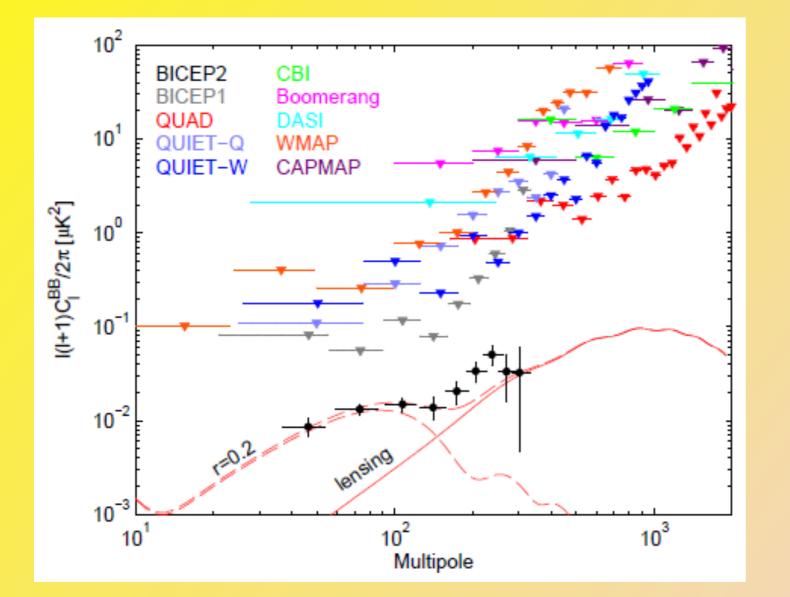
Dashed Line Theoretical

Expectation from

Inflation

 $r \equiv \Delta_t / \Delta_s < 0.07 \ (2\sigma)$

 $r \sim 10^{-2} - 10^{-3} \Rightarrow E_* \sim 5 \cdot 10^{15} \text{GeV}$



Inflation: Summary

Inflation: Solves Causality Problem. Bonus: Universe Flat

$a \propto e^{Ht}$	$ \qquad \qquad$	$+ \delta g_{\mu\nu}[\mathcal{R}, h_{ij}]$	$\rightarrow \Delta T, \mathcal{E}, \mathcal{B}$	$[\text{also } \delta \rho]$
Exponetial Expansion	Quantum Origin of Fluctuations		Angular Temperature/ Polarization Anisotropies	
Observations:	$ \Omega_k \ll 1$	$n_s - 1 \sim -0.04$	$\left< \mathcal{R}^3 \right> \approx 0$	$4\delta_m = 3\delta_\gamma$
	Locally Flat	Almost Scale-Inv	Gaussian	Adiabatic

But CMB polarization B-modes due to GWs not yet found !

Inflation: What else?



Many more things George!

Inflation: What else?

Many more things!

Inflationary period

- * GWs from Particle production during inflation
- * GWs from Spectator fields
 - * GWs in the EFT of space-reparam

Post-Inflationary period

- * GWs from merging of primordial BHs
 - * Kination-domination
- * GWs from (p)reheating

Alternatives to Inflation

Inflation: What else?

Many more things!

Inflationary period

GWs from Particle production during inflation

* GWs from Spectator fields

* GWs in the EFT of space-reparam

Post-Inflationary period

* GWs from merging of primordial BHs

* Kination–domination

* GWs from (p)reheating

Alternatives to Inflation

Particle production during inflation

Axion-inflation model

[J. Cook, L. Sorbo (arXiv:1109.0022)] [N. Barnaby, E. Pajer, M. Peloso (arXiv:1110.3327)]



inflaton φ = pseudo-scalar axion

The rolling inflaton excites the gauge field(s)

$$\boldsymbol{\xi} \equiv \frac{\dot{\varphi}}{2\Lambda H} \qquad \longrightarrow \quad \ddot{A}_{\pm}(k,t) + \left[k^2 \pm 2\xi \frac{k}{t}\right] A_{\pm}(k,t) = 0$$

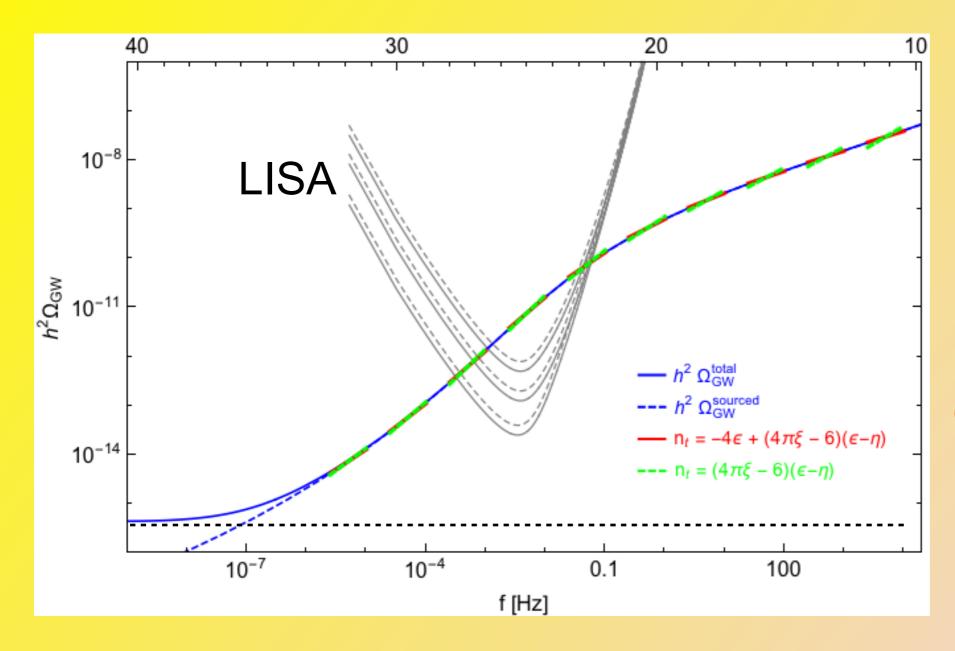
 $A_+ \propto e^{\pi\xi}, \ |A_-| \ll |A_+|$

A+ exponentially amplified, A- has no amplification

Gauge field excitation creates chiral GWs !

Particle production during inflation Axion-inflation model

[J. Cook, L. Sorbo (arXiv:1109.0022)] [N. Barnaby, E. Pajer, M. Peloso (arXiv:1110.3327)]



Gauge fields source a blue tilted & chiral GW background

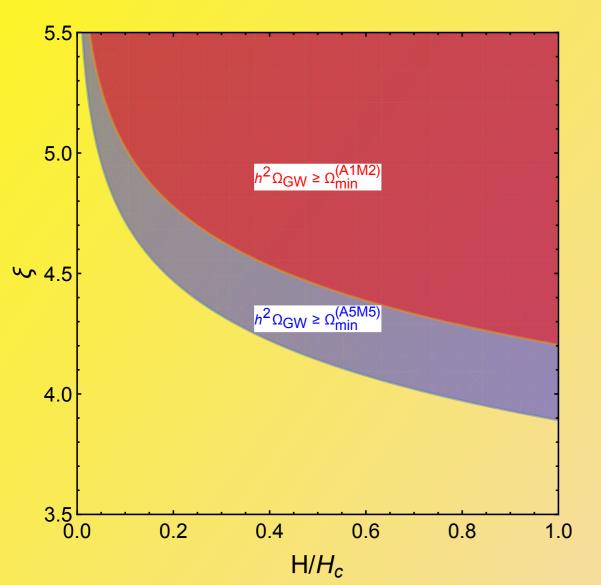
Particle production during inflation Axion-inflation model $V(\varphi) + \frac{\varphi}{\Lambda}F_{\mu\nu}\tilde{F}^{\mu\nu}$

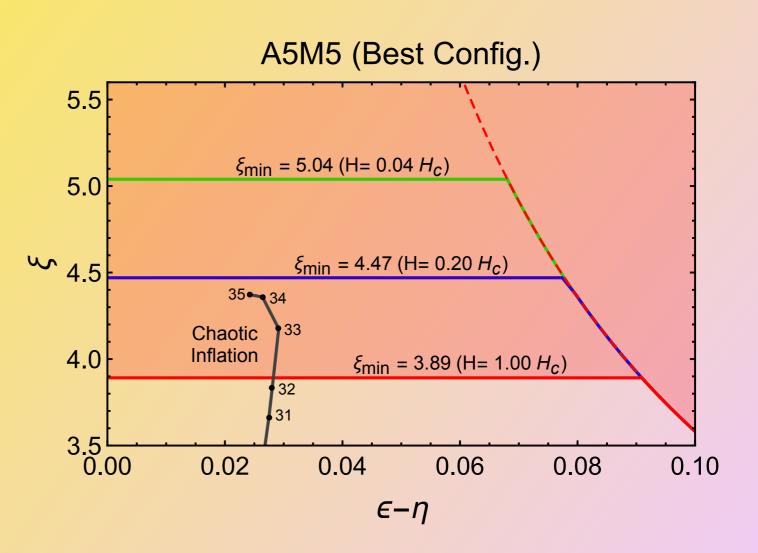
$$h^2 \Omega_{\rm gw} = A_* \left(\frac{f}{f_*}\right)^{n_7}$$

Bartolo et al '16

$$\Omega_{\rm GW} h^2 \simeq 1.5 \cdot 10^{-13} \frac{H^4}{M_{Pl}^4} \frac{e^{4\pi\xi}}{\xi^6} \quad , \quad \xi \gg 1$$
$$n_T \simeq (4\pi\xi - 6)(\epsilon_H - \eta)$$

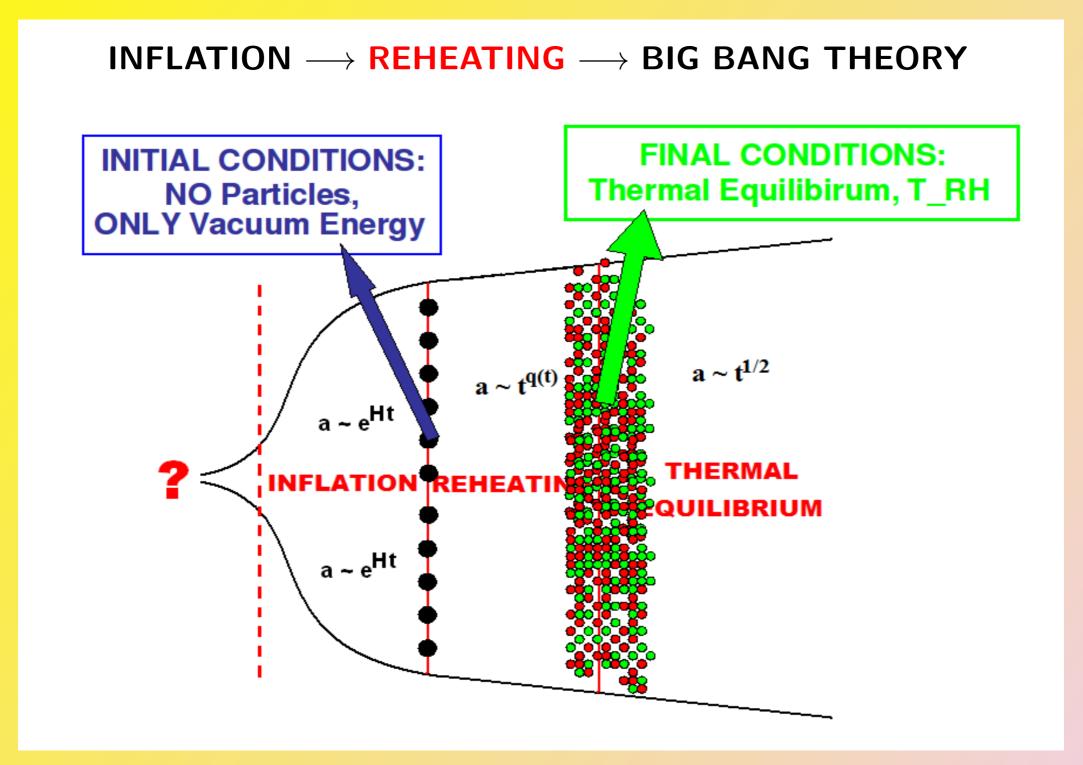
3 parameters
$$H,\,\xi,\,\epsilon_H-\eta$$

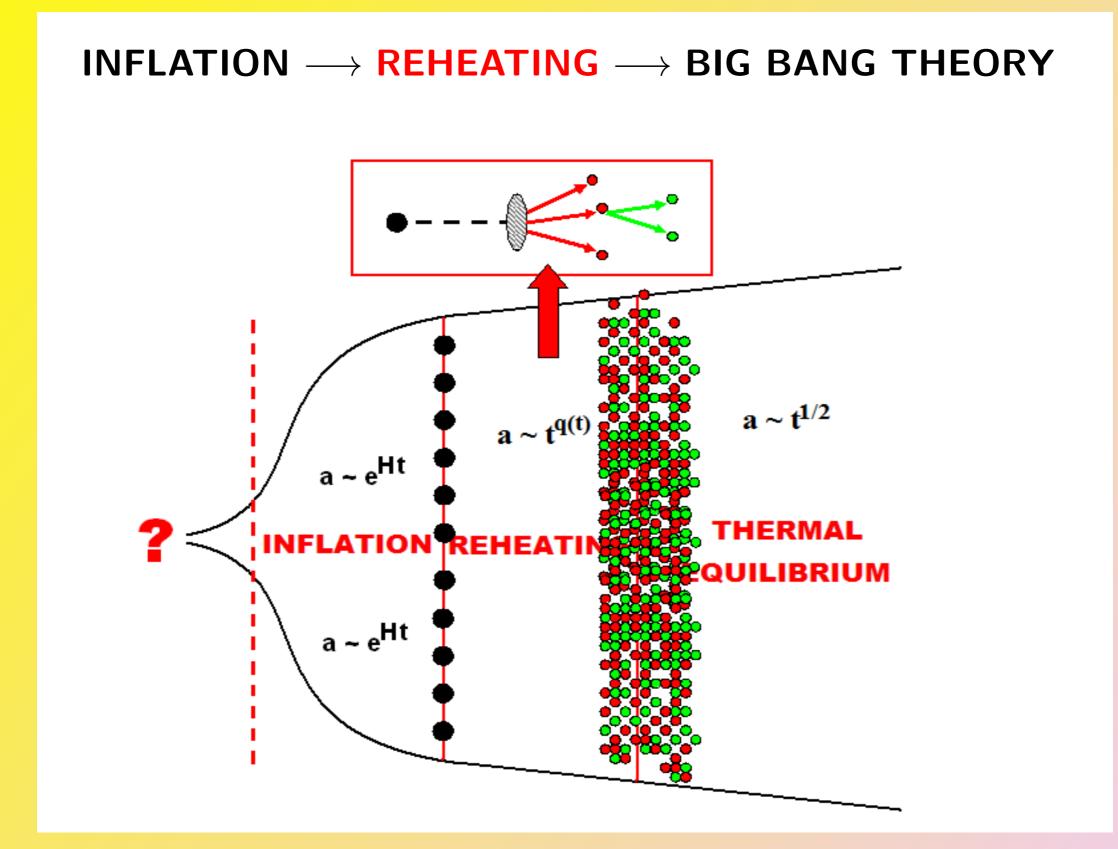




Alright, Inflation ends so what follows afterwards ?

(p)Reheating !

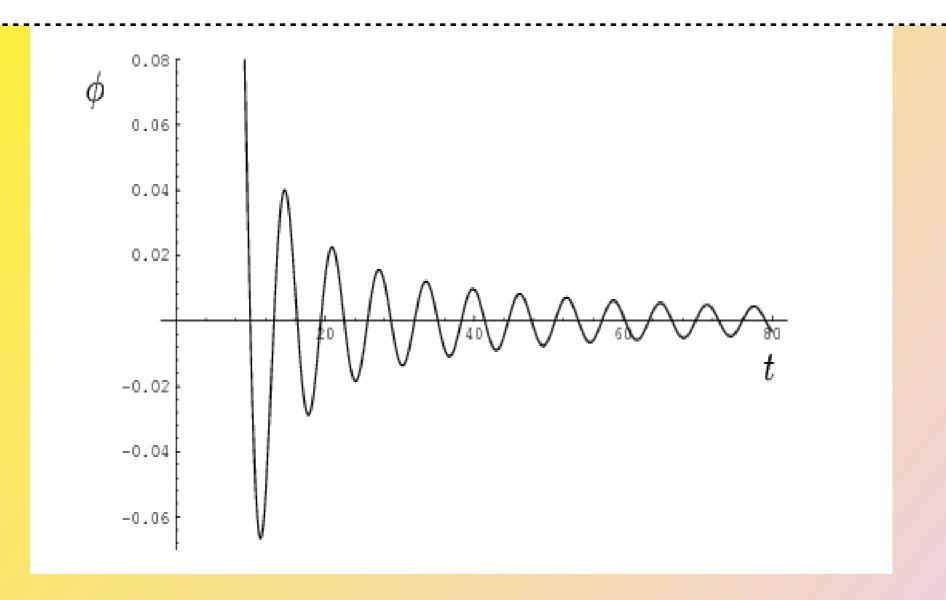


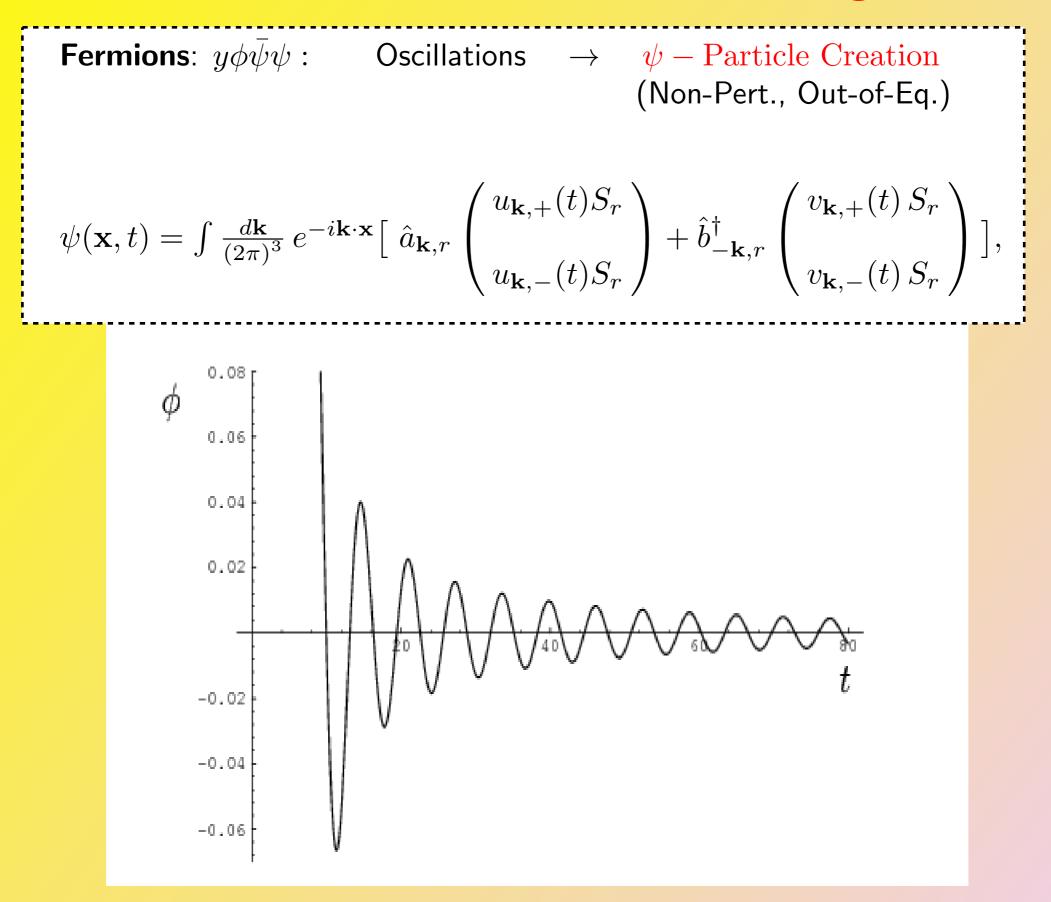


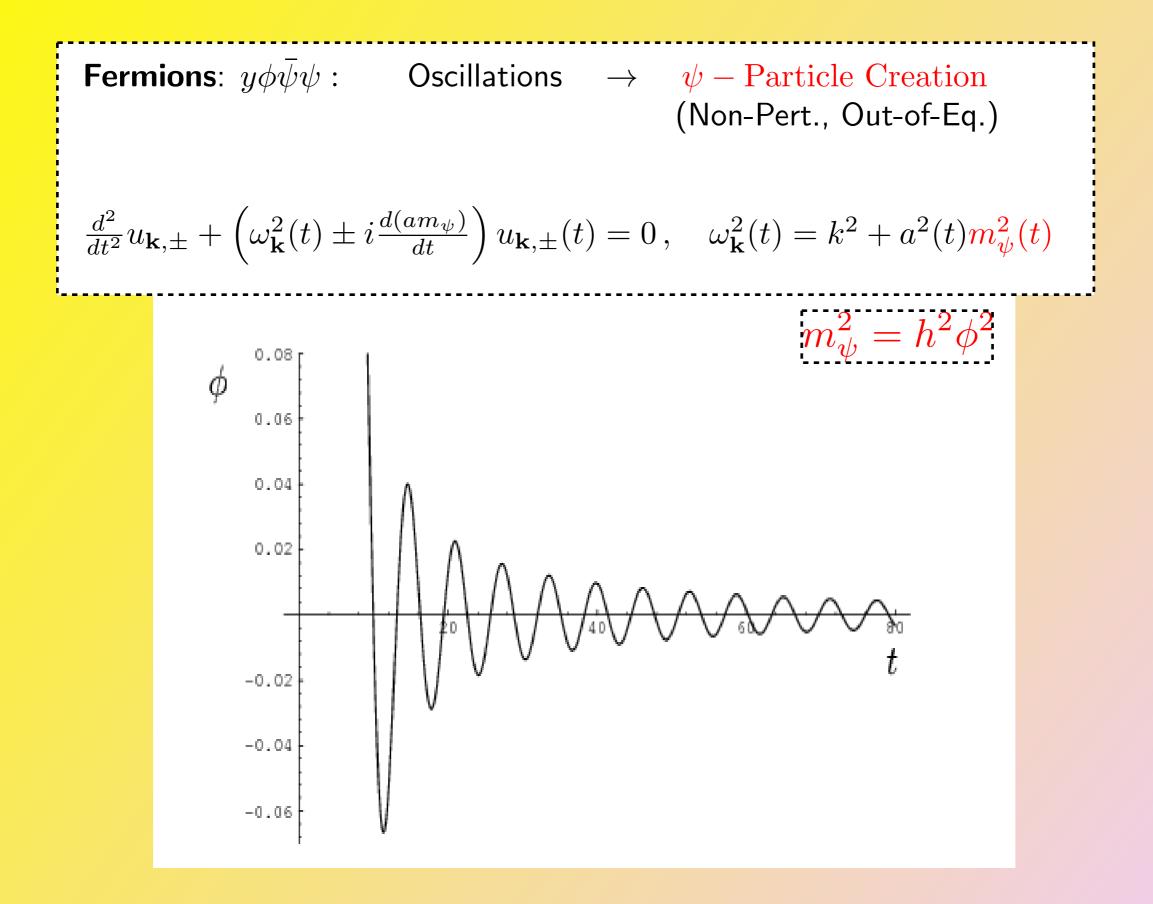
Inflaton: $V(\phi) \propto \phi^n$

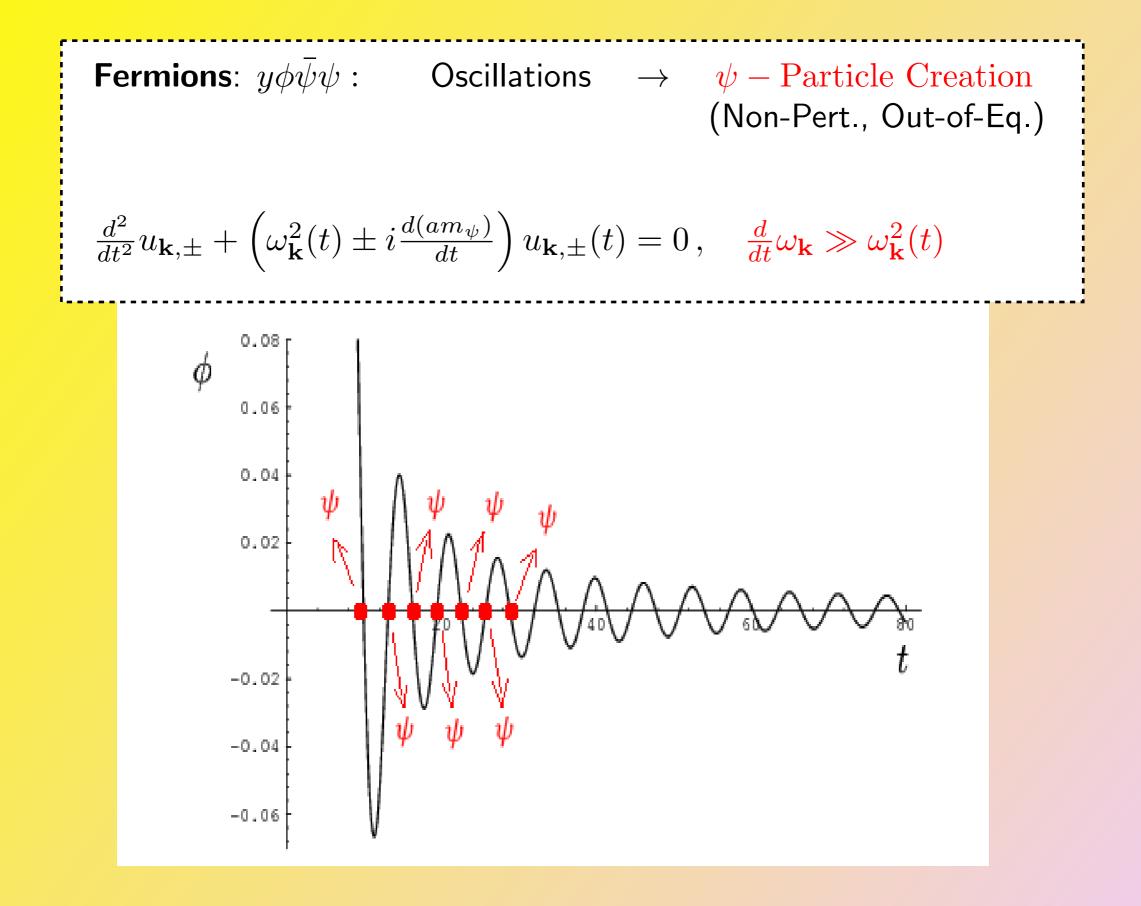
Scalar field (condensate) after Inflation:

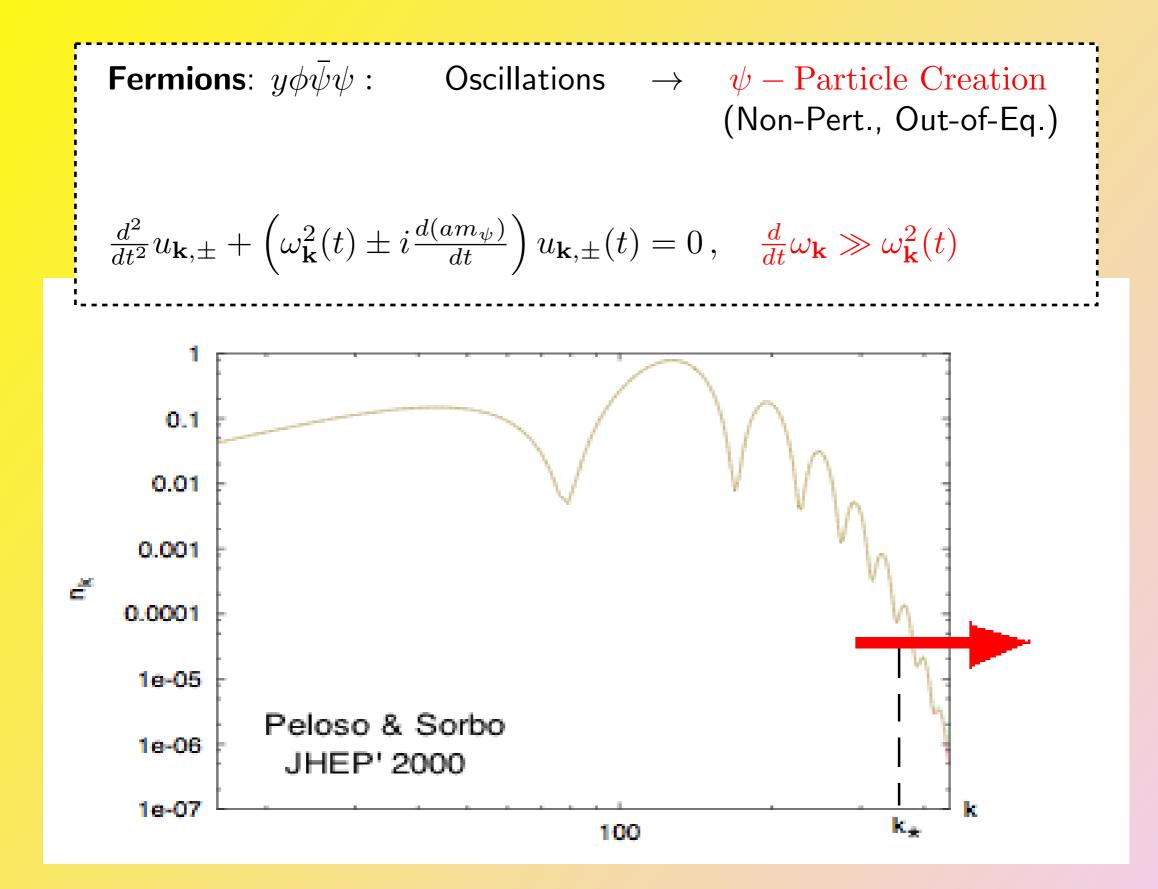
Coherent Oscillations: $\phi(t) \approx \Phi(t)f(t)$, f(t+T) = f(t)





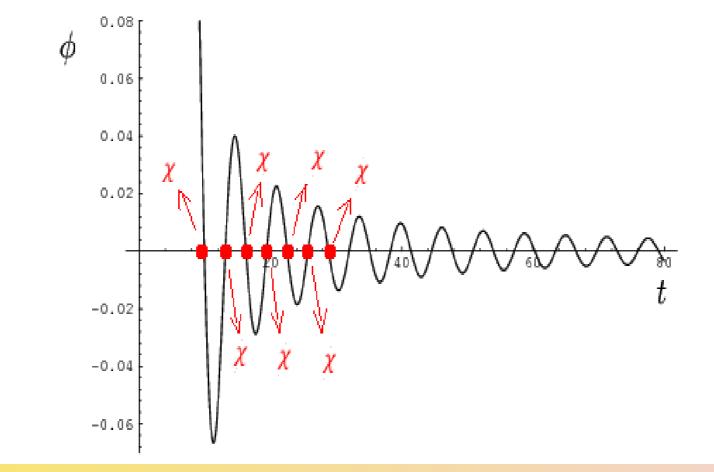


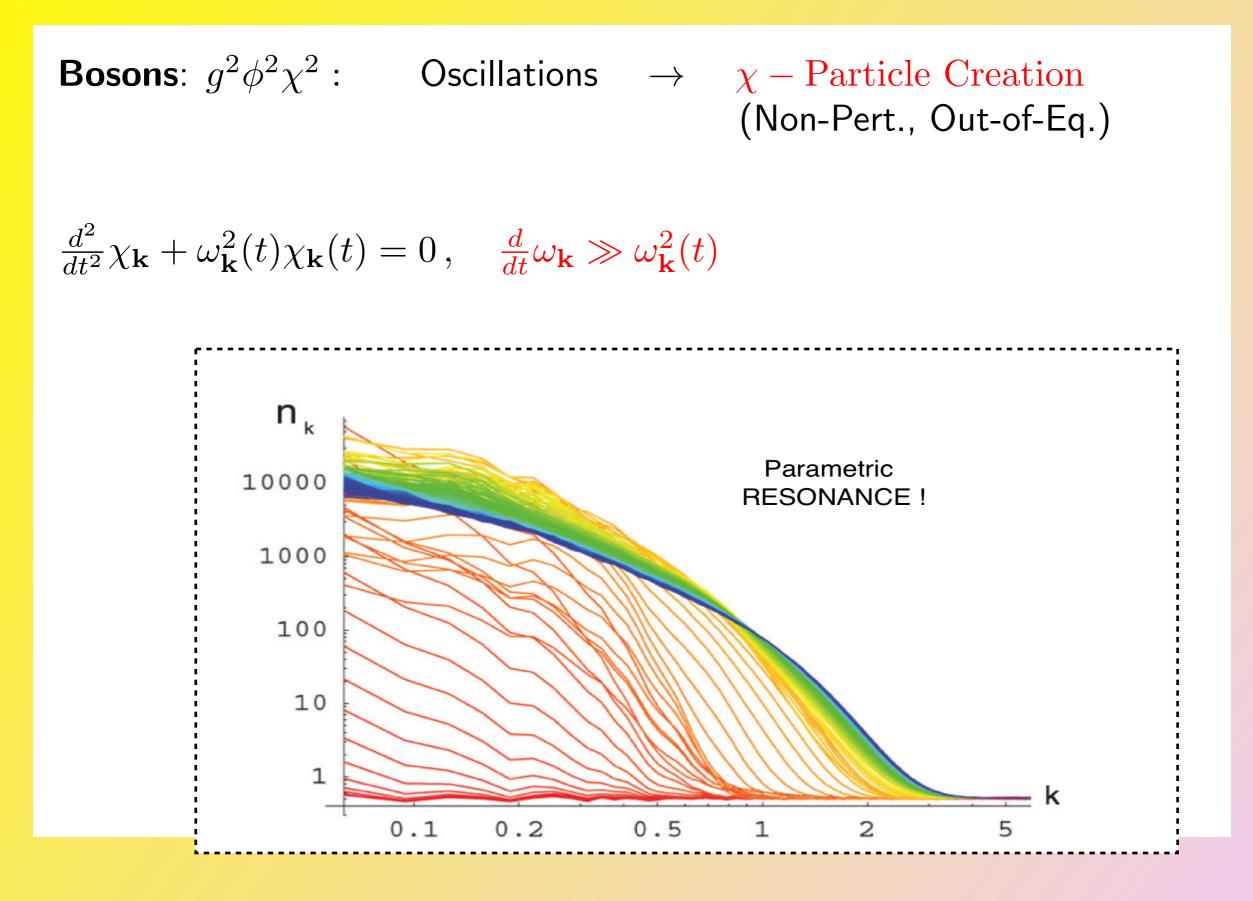


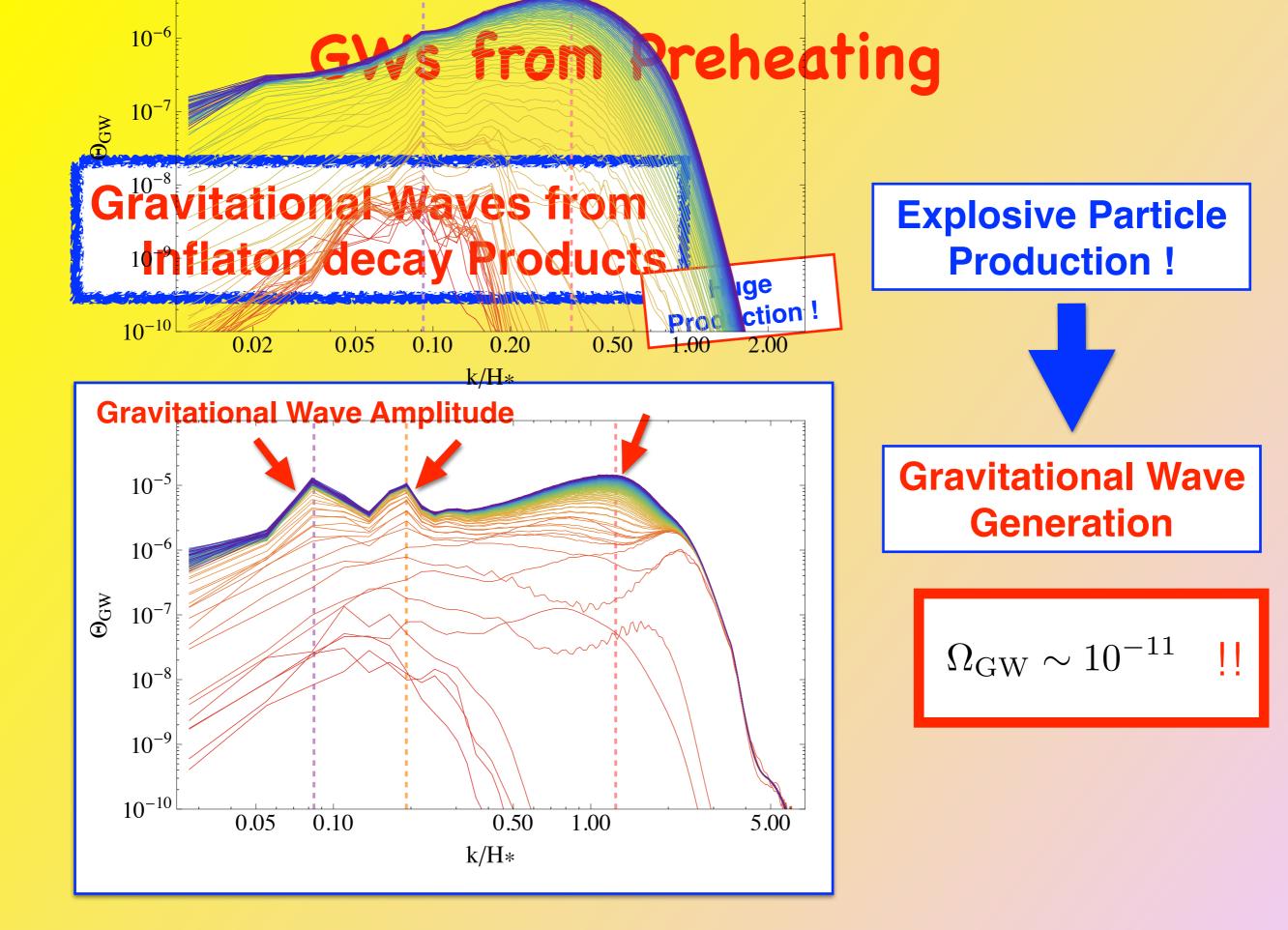


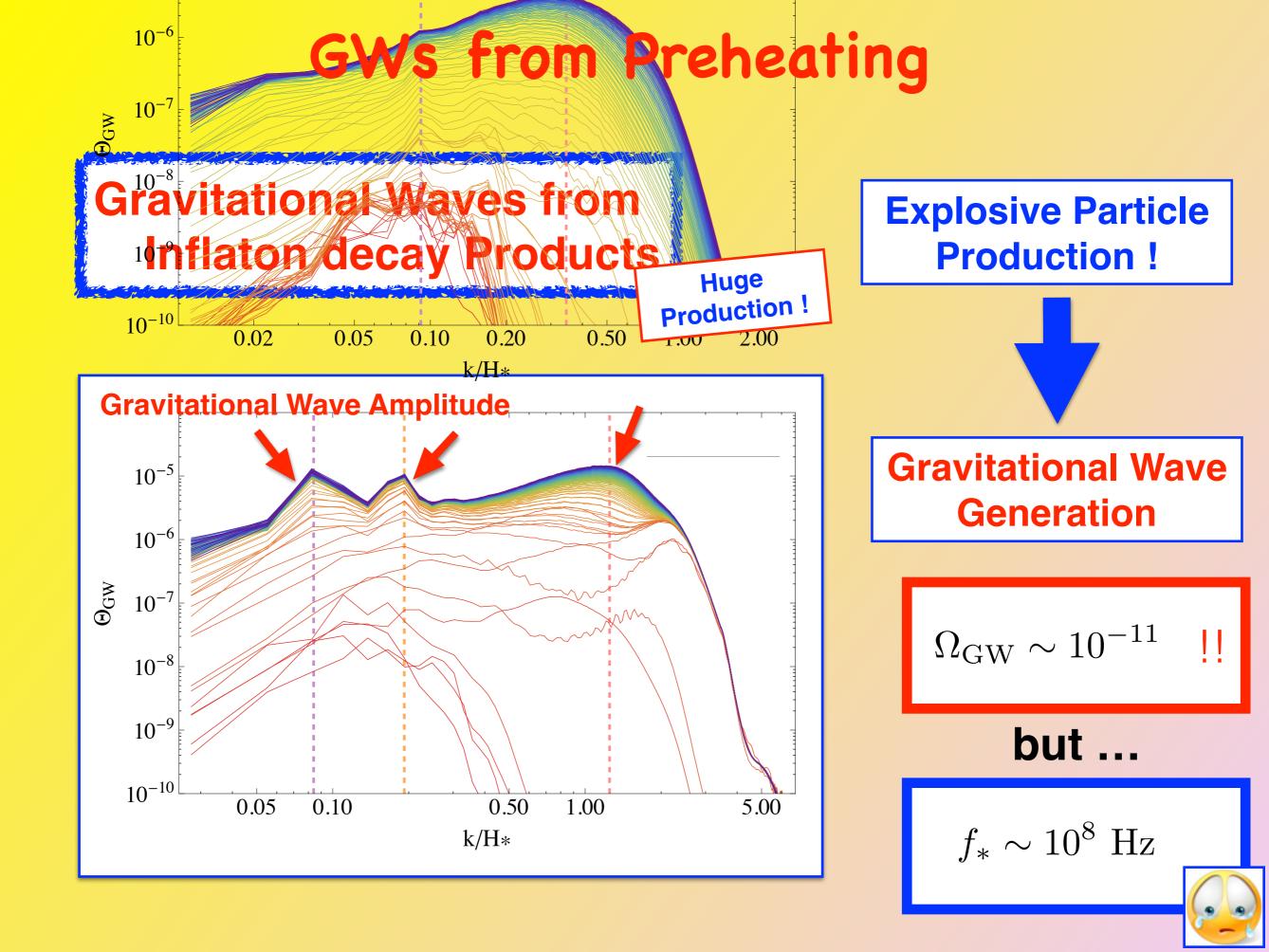
Bosons: $g^2 \phi^2 \chi^2$: Oscillations $\rightarrow \chi - \text{Particle Creation}$ (Non-Pert., Out-of-Eq.)

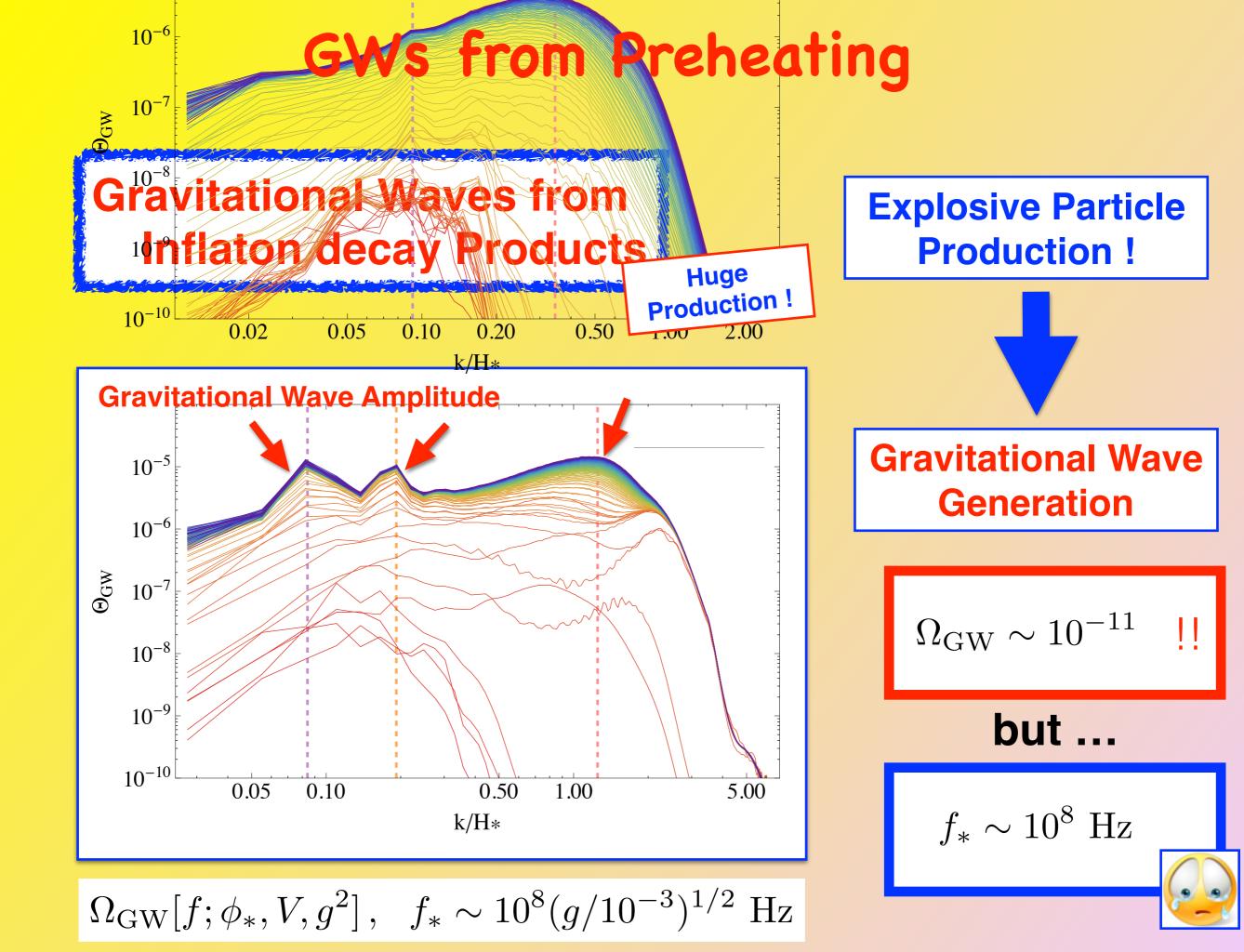
$$\frac{d^2}{dt^2}\chi_{\mathbf{k}} + \omega_{\mathbf{k}}^2(t)\chi_{\mathbf{k}}(t) = 0, \quad \frac{d}{dt}\omega_{\mathbf{k}} \gg \omega_{\mathbf{k}}^2(t)$$

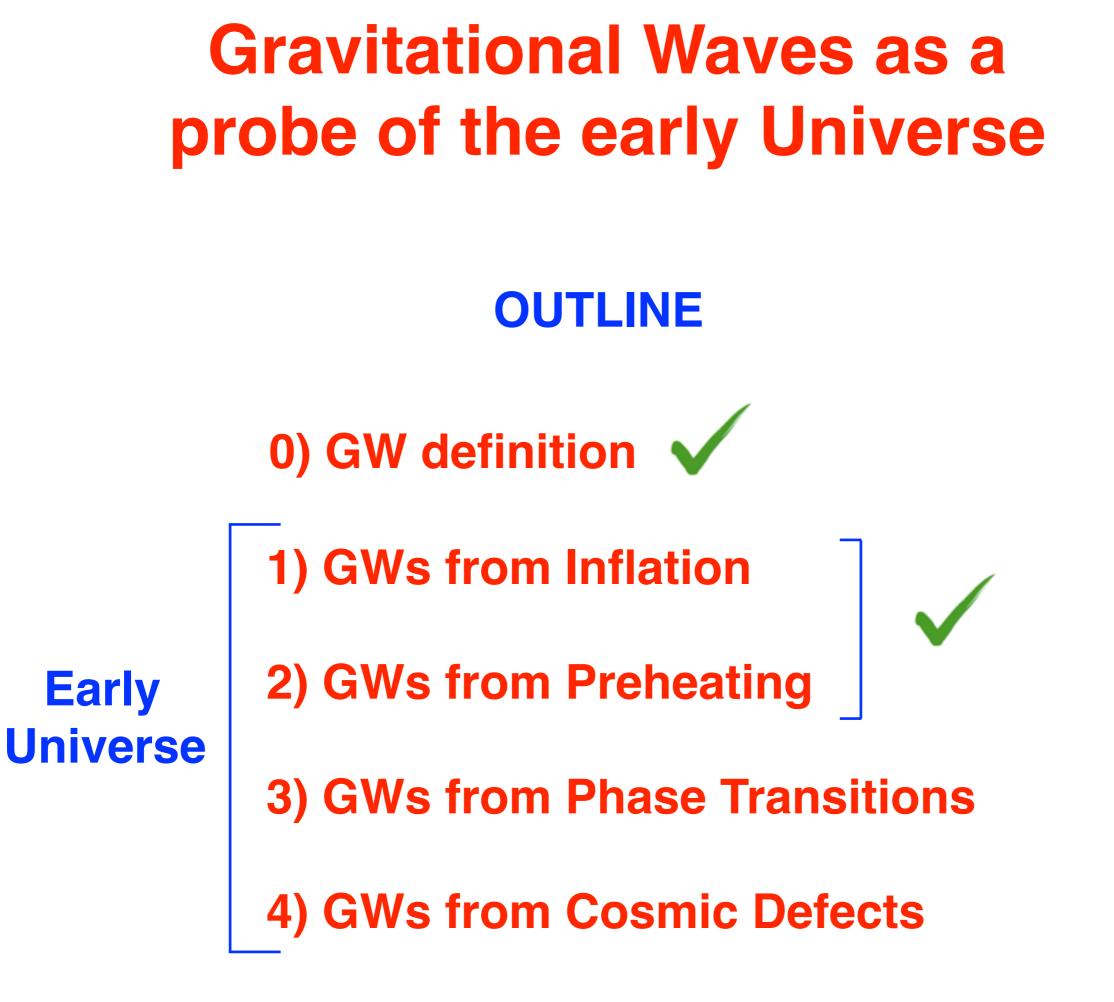












GW background from first order phase transitions

* GW causal source: cannot 'operate' beyond the horizon (Hubble scale)

$$f_* = \frac{H(T_*)}{\epsilon_*} \qquad \qquad \epsilon_* \le 1 \qquad \begin{array}{c} \text{parameter characteristic} \\ \text{of source dynamics} \end{array}$$

Hubble rate \longleftrightarrow temperature in the universe : (assuming standard thermal history)

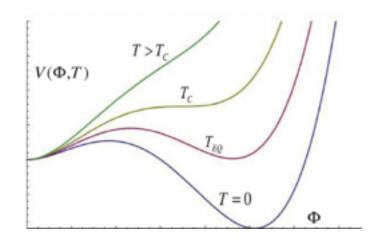
$$f_c = f_* \frac{a_*}{a_0} = \frac{2 \cdot 10^{-5}}{\epsilon_*} \frac{T_*}{1 \text{ TeV}} \text{ Hz} \simeq \text{mHz}$$

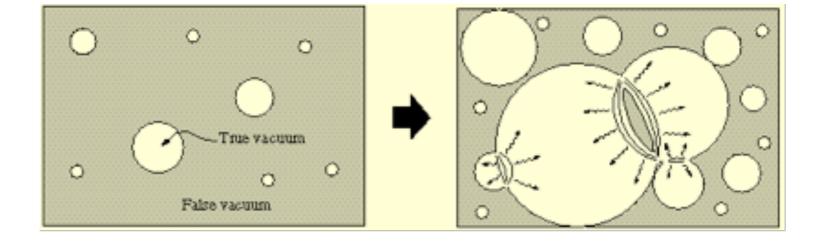
for
$$\epsilon_* \simeq 10^{-2}$$
 $T_* \simeq 1 \,\mathrm{TeV}$

GW background from first order phase transitions

Universe expands, temperature decreases: phase transition triggered !

* Potential barrier separates **true** and **false** vacua **quantum tunneling** across the barrier : nucleation of bubbles of true vacuum





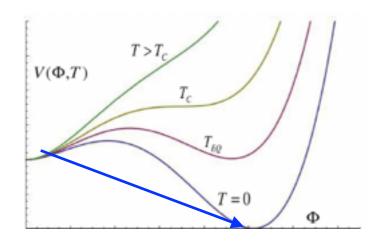
source: \prod_{ij} tensor anisotropic stress

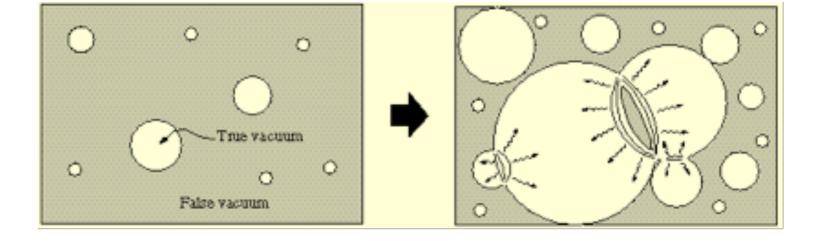
- collisions of bubble walls
- sound waves and turbulence in the fluid
- primordial magnetic fields (MHD turbulence)

GW background from first order phase transitions

Universe expands, temperature decreases: phase transition triggered !

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source: \prod_{ij} tensor anisotropic stress

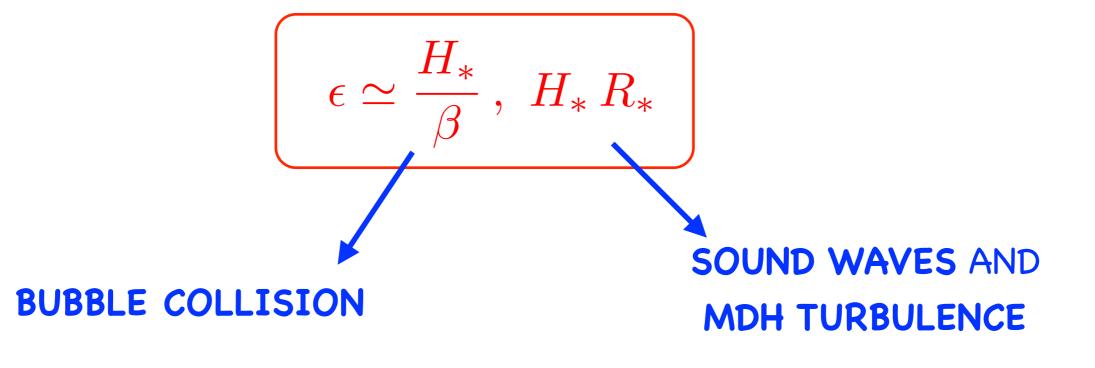
 $\Pi_{ij} \sim \partial_i \phi \, \partial_j \phi$ $\Pi_{ij} \sim \gamma^2 (\rho + p) \, v_i v_j$ $\Pi_{ij} \sim \frac{(E^2 + B^2)}{3} - E^i E^j - B^i B^j$

what is ε in 1st Order PhT's?

$$f_c = f_* \frac{a_*}{a_0} = \frac{2 \cdot 10^{-5}}{\epsilon_*} \frac{T_*}{1 \text{ TeV}} \text{ Hz}$$

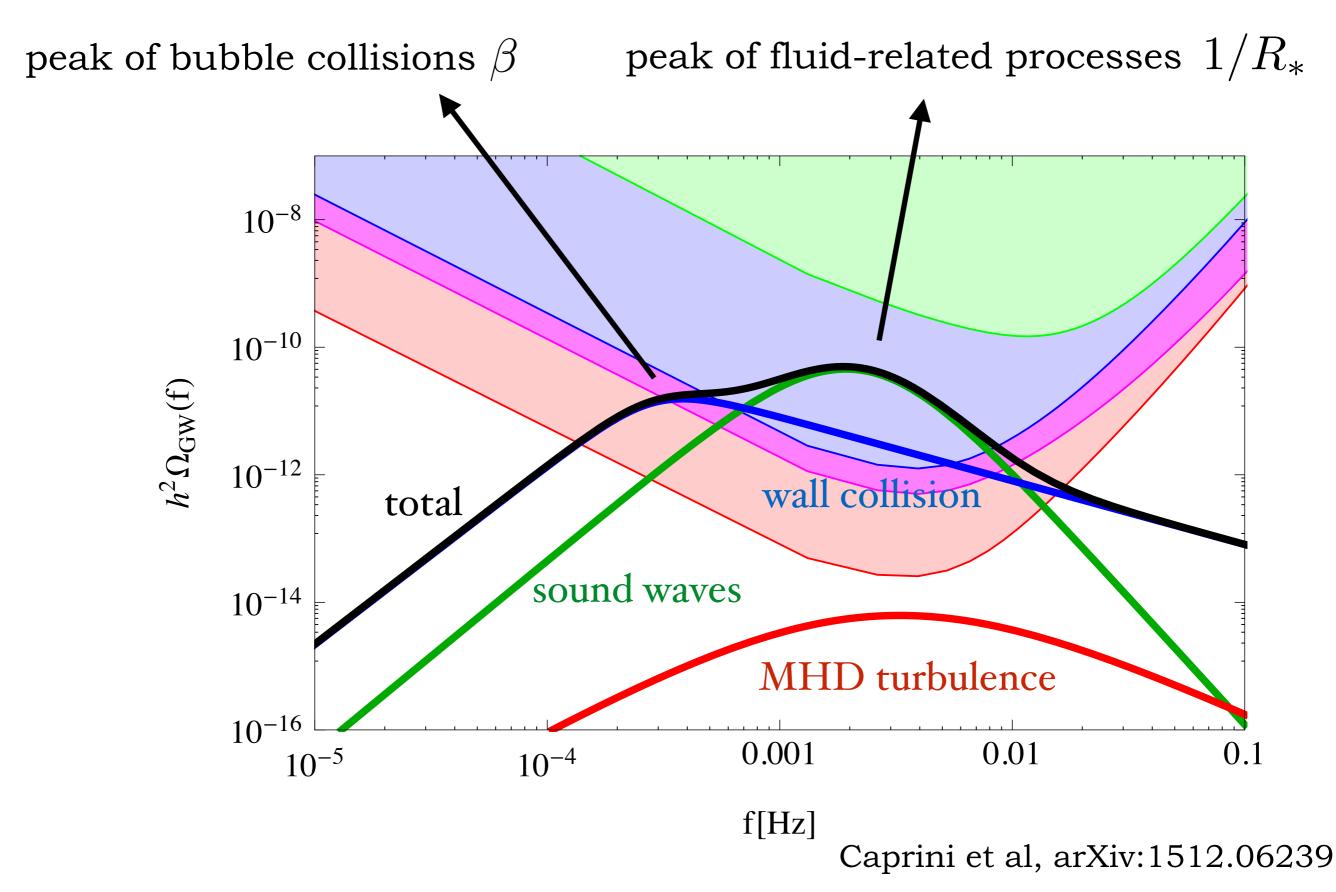
GW generation <--> bubbles properties

$$\beta^{-1}$$
: duration of PhT
 $v_b \leq 1$: speed of bubble walls $\rightarrow R_* = v_b \beta^{-1}$ size of bubbles at collision



Parameters determining the GW spectrum

Example of spectrum ('runaway' solution)



Evaluation of the signal

• bubble collisions: analytical and numerical simulations (Huber and Konstandin arXiv:0806.1828)

• sound waves: numerical simulations with both scalar field and fluid (Hindmarsh et al arXiv:1504.03291)

• MDH turbulence: analytical evaluation

(Caprini et al arXiv:0909.0622)

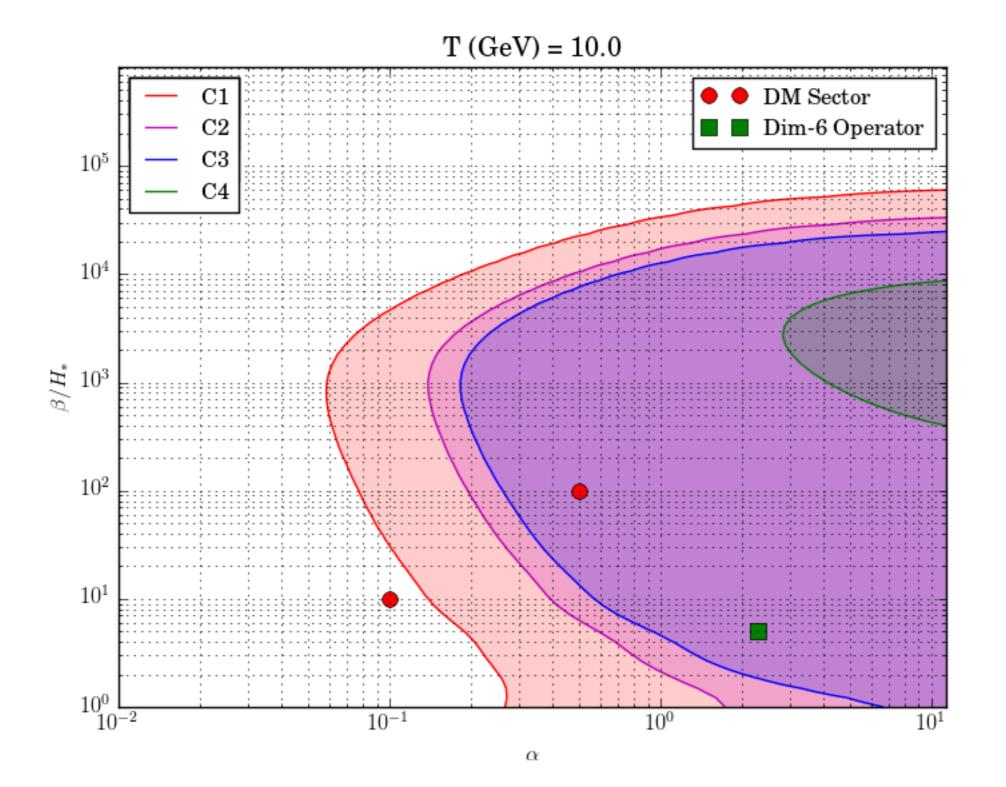
Models for EWPT and beyond

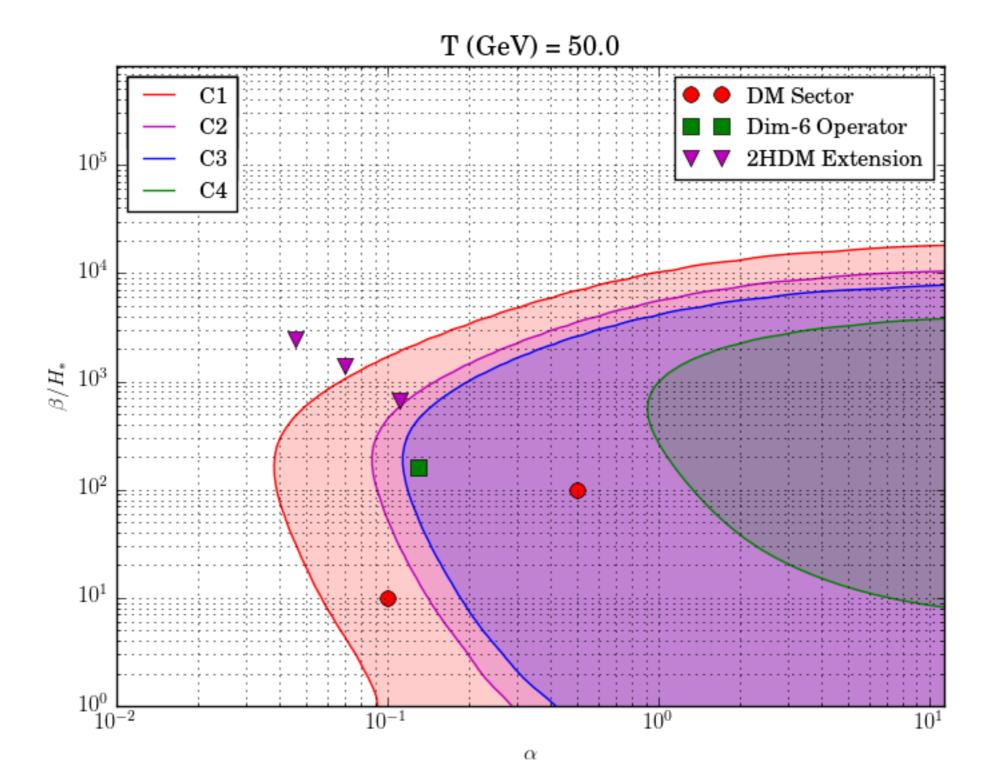
• LISA sensitive to energy scale 10 GeV - 100 TeV !

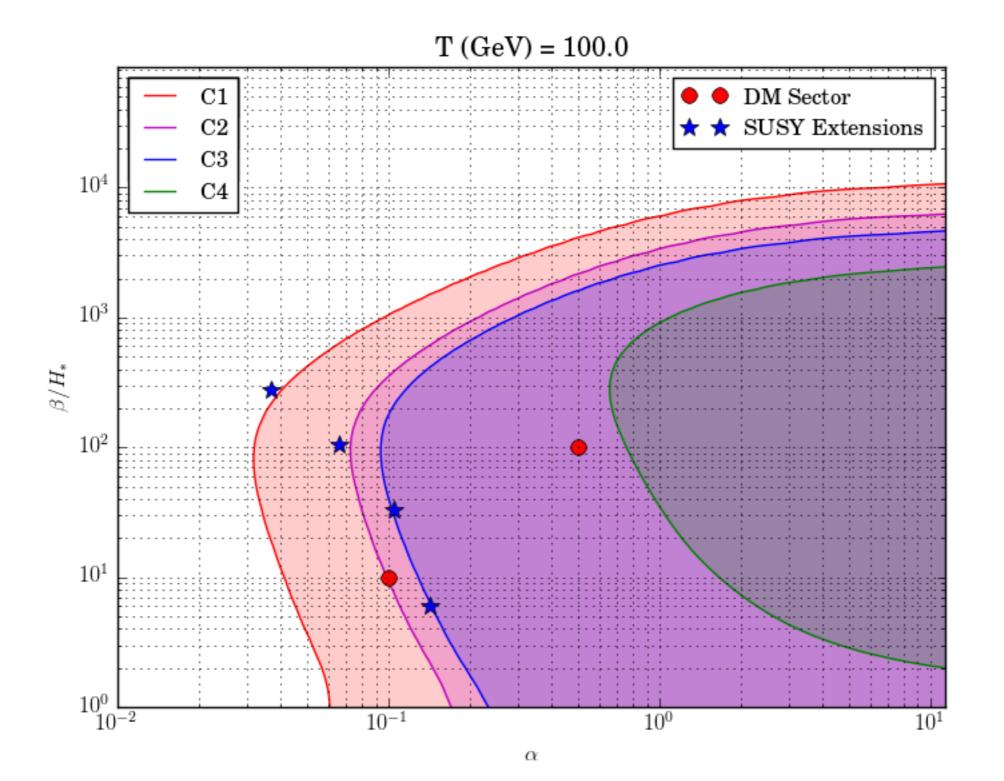
- LISA can probe the EWPT in BSM models ...
 - singlet extensions of MSSM (Huber et al 2015)
 - direct coupling of Higgs to scalars (Kozackuz et al 2013)
 - SM + dimension six operator (Grojean et al 2004)
- ... and beyond the EWPT
 - Dark sector: provides DM candidate and confining PT (Schwaller 2015)
 - Warped extra dimensions : PT from the dilaton/radion stabilisation in RS-like models (Randall and Servant 2015)

Models for EWPT and beyond

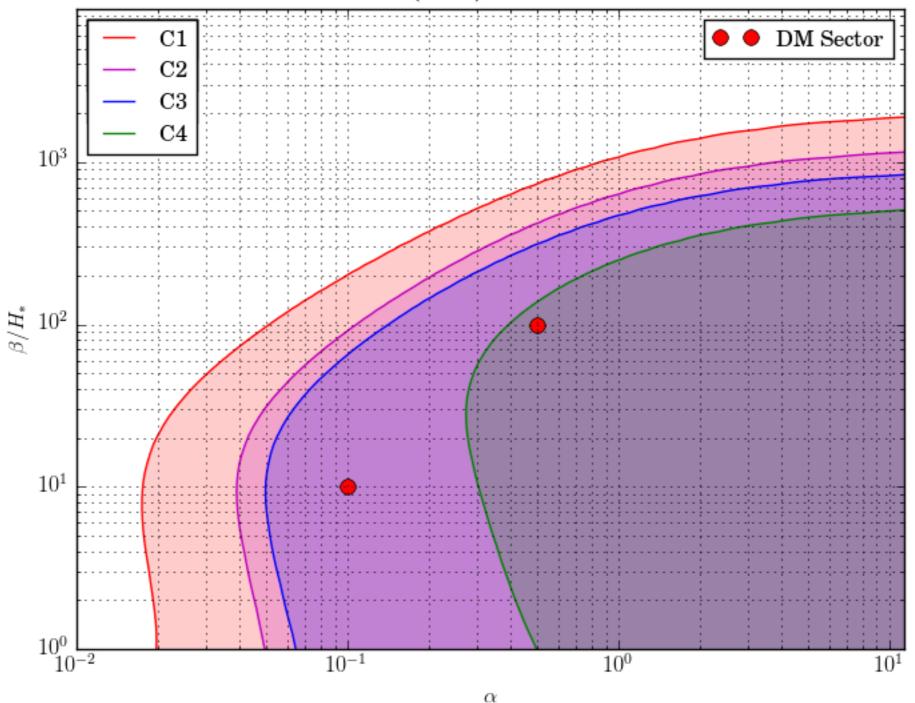
- LISA sensitive to energy scale 10 GeV 100 TeV !
- LISA can probe the EWD - sincle Sincle Physics interplay! - Cosmology and Particle Physics interplay. - Co
- ... and beyond the EWPT - Dark sector: provides DV (Schwell LISA —> new probe of BSM physics! LISA —> new probe of BSM physics! Complementary to particle colliders) (complementary to particle me dilaton/radion (complementary to me models (Randall and Servant 2015)

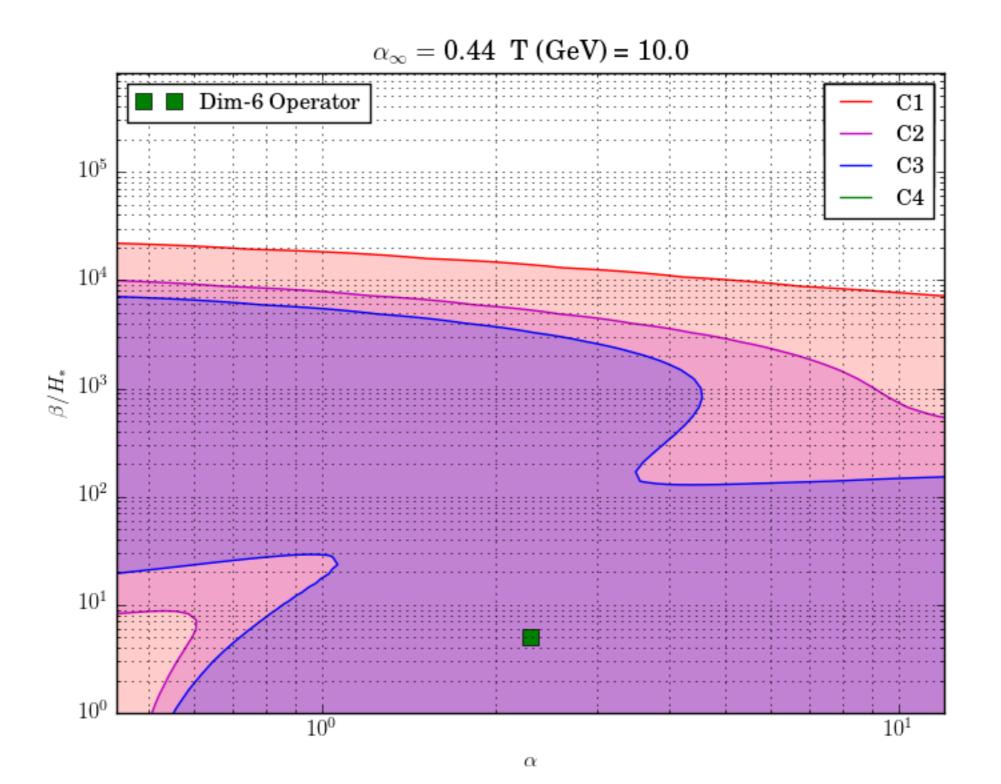


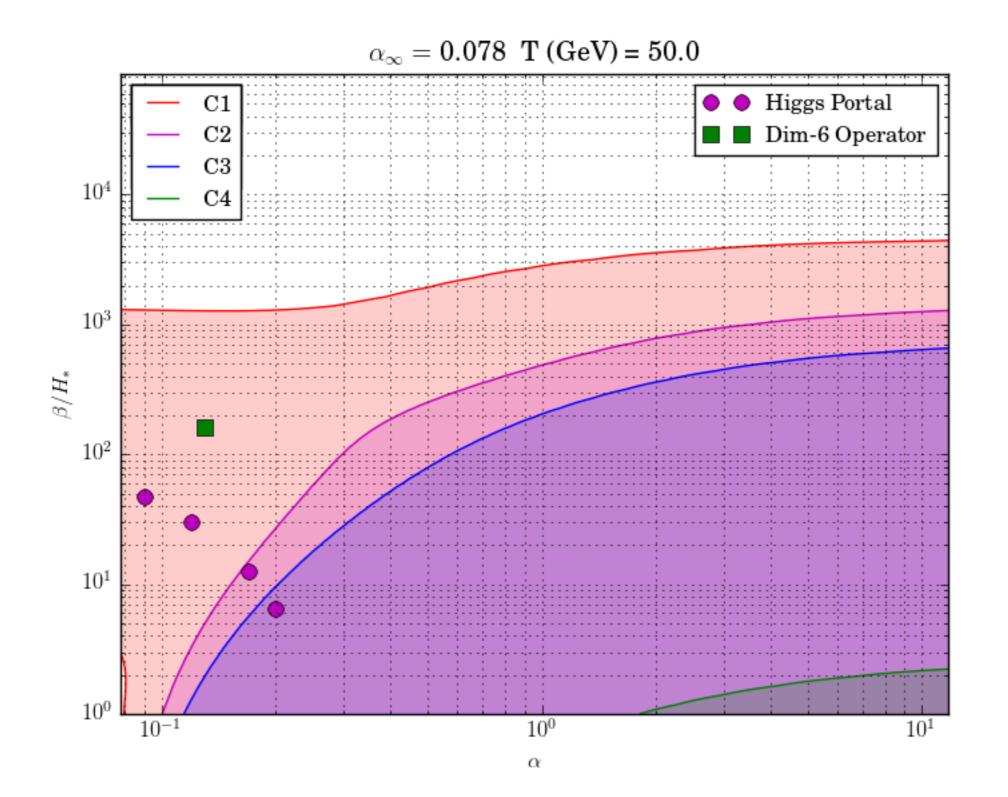


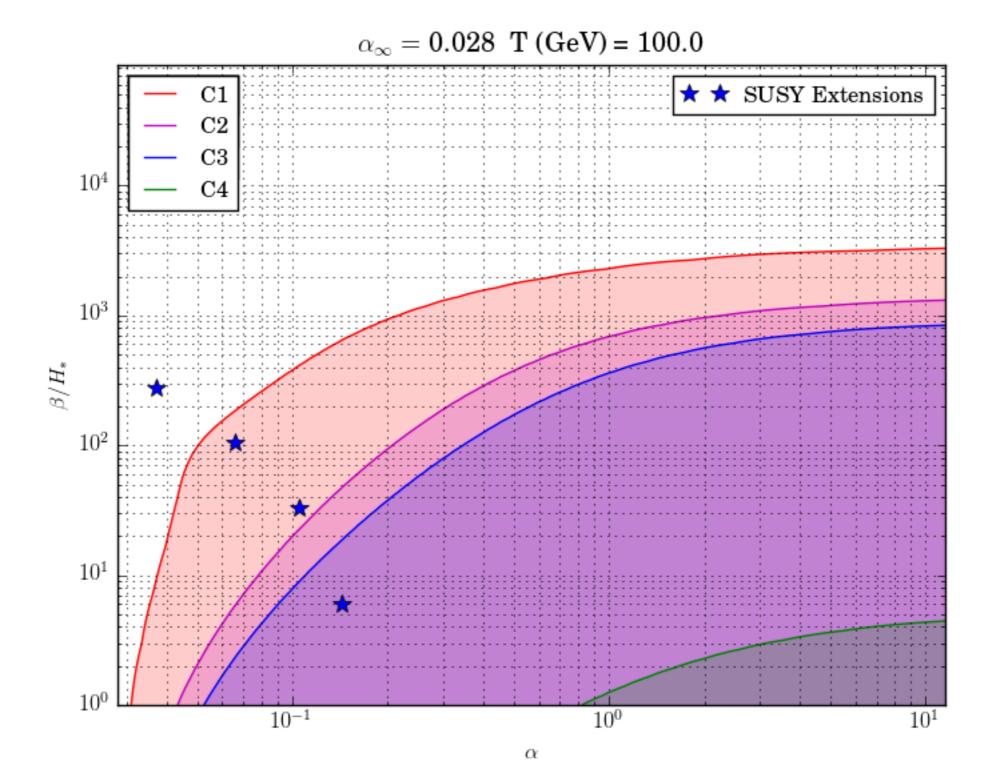


T (GeV) = 1000.0

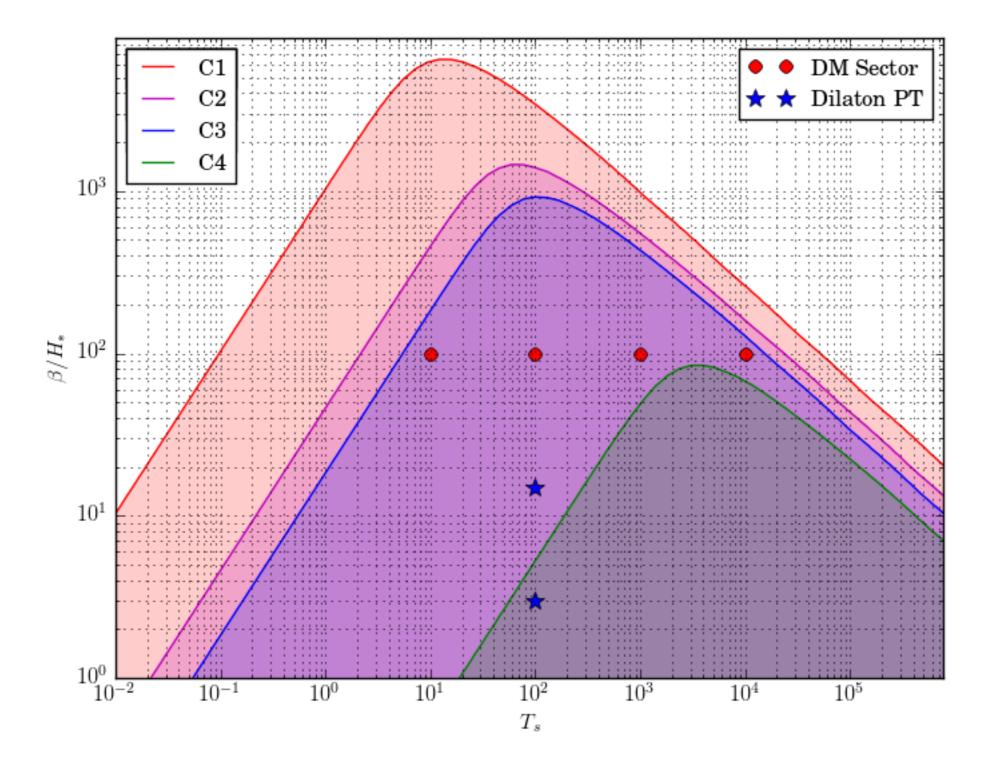






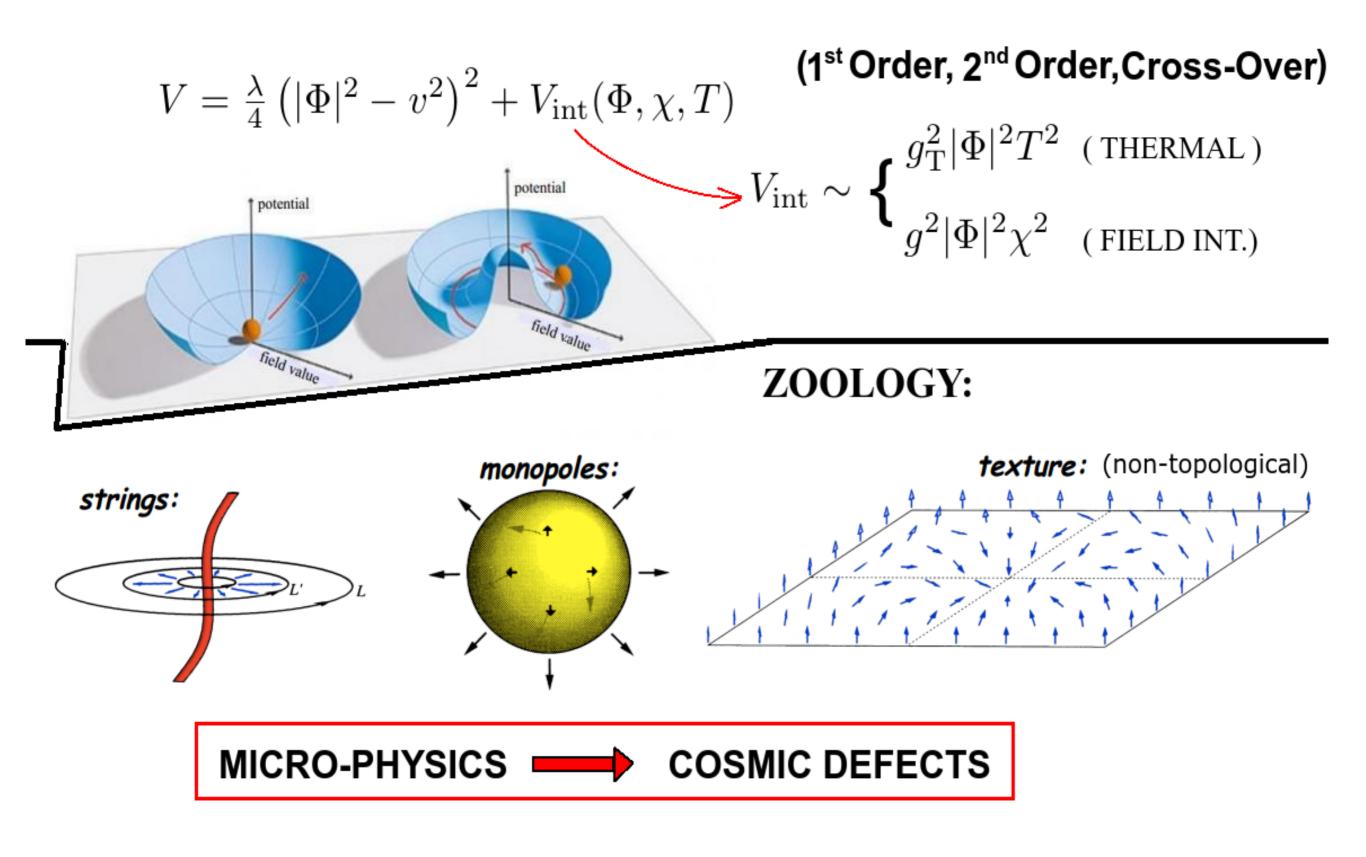


Detection prospects for LISA: runaway in vacuum



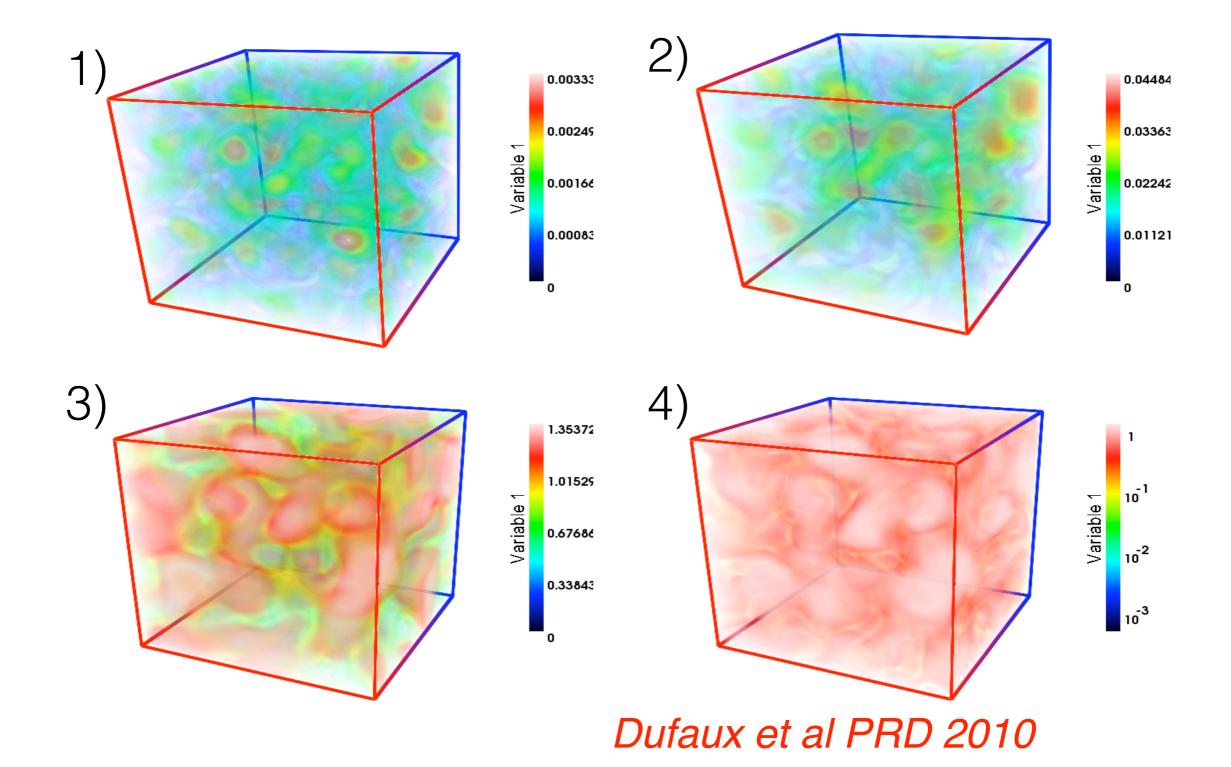
What about Cosmic Defects ? (aftermath products of a PhT)

0) Phase Transition \leftrightarrow Cosmic Defects (if conditions met)



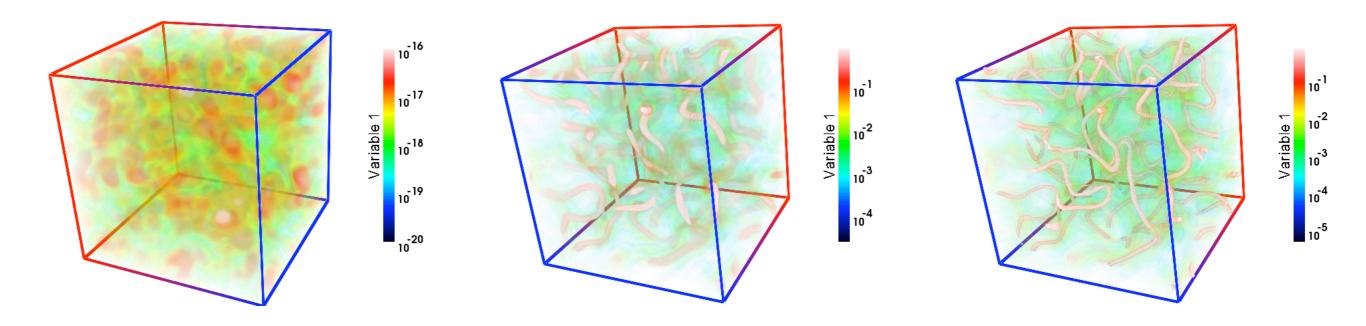
0) Phase Transition \leftrightarrow Cosmic Defects (if conditions met)

U(1) Breaking (e.g. after Hybrid Inflation)



0) Phase Transition \leftrightarrow Cosmic Defects (if conditions met)

U(1) Breaking (after Hybrid Inflation): Mag. Fields



Dufaux et al PRD 2010

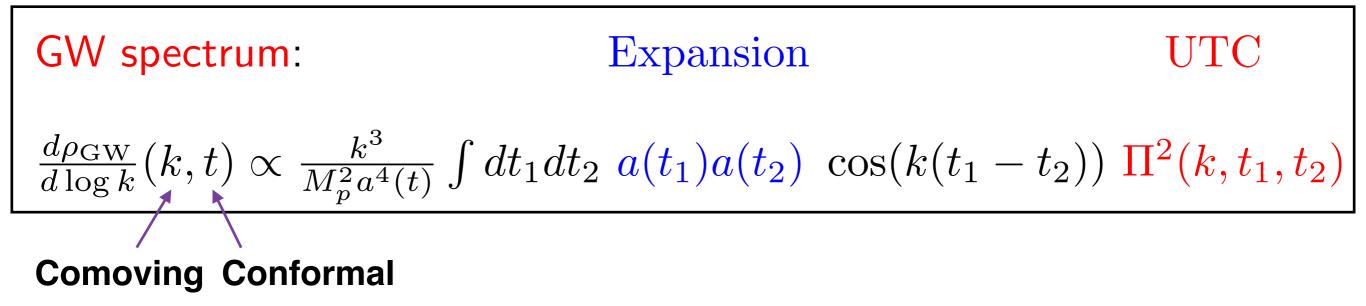
$$\mathsf{DEFECTS:} \ \mathsf{Aftermath} \ \mathsf{of} \ \mathsf{PhT} \to \left\{ \begin{array}{l} \left\{ \begin{array}{l} \mathsf{Domain} \ \mathsf{Walls} \\ \mathsf{Cosmic} \ \mathsf{Strings} \\ \mathsf{Cosmic} \ \mathsf{Monopoles} \\ \mathsf{Non} - \mathsf{Topological} \end{array} \right. \right.$$

CAUSALITY & MICROPHYSICS \Rightarrow Corr. Length: $\xi(t) = \lambda(t) H^{-1}(t)$

(Kibble' 76) SCALING: $\begin{cases} \lambda(t) = \text{const.} \rightarrow \lambda \sim 1 \Rightarrow k/\mathcal{H} = kt \\ \langle T_{ij}^{\text{TT}}(\mathbf{k}, t) T_{ij}^{\text{TT}}(\mathbf{k}', t') \rangle = (2\pi)^3 \frac{V^4}{\sqrt{tt'}} U(kt, kt') \delta^3(\mathbf{k} - \mathbf{k}') \\ \text{Unequal Time Correlator (UTC)} \end{cases}$

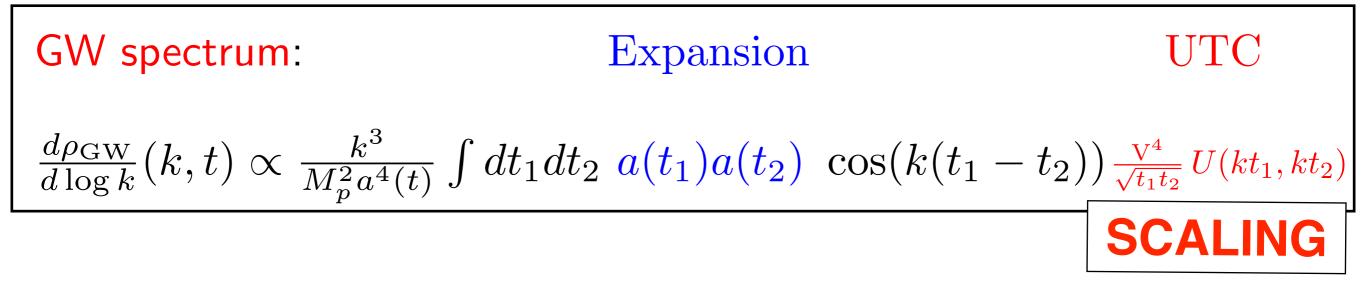
DEFECTS: GW Source $\rightarrow \{T_{ij}\}^{TT} \propto \{\partial_i \phi \partial_j \phi, E_i E_j, B_i B_j\}^{TT}$

UTC: $\langle T_{ij}^{TT}(\mathbf{k},t)T_{ij}^{TT}(\mathbf{k}',t')\rangle = (2\pi)^3 \Pi^2(\mathbf{k},t_1,t_2) \ \delta^3(\mathbf{k}-\mathbf{k}')$ (Unequal Time Correlator)



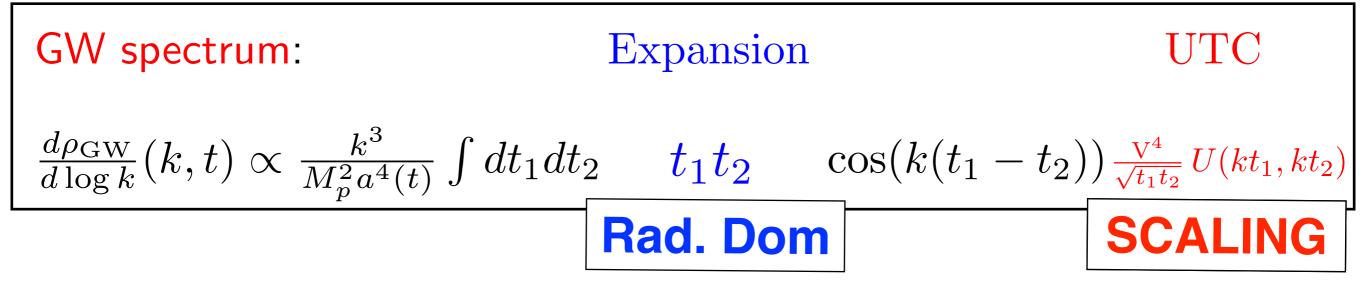
DEFECTS: GW Source $\rightarrow \{T_{ij}\}^{TT} \propto \{\partial_i \phi \partial_j \phi, E_i E_j, B_i B_j\}^{TT}$

SCALING
UTC:
$$\langle T_{ij}^{\text{TT}}(\mathbf{k},t)T_{ij}^{\text{TT}}(\mathbf{k}',t')\rangle = (2\pi)^3 \frac{V^4}{\sqrt{tt'}} U(kt,kt')\delta^3(\mathbf{k}-\mathbf{k}')$$



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SCALING
UTC:
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SCALING

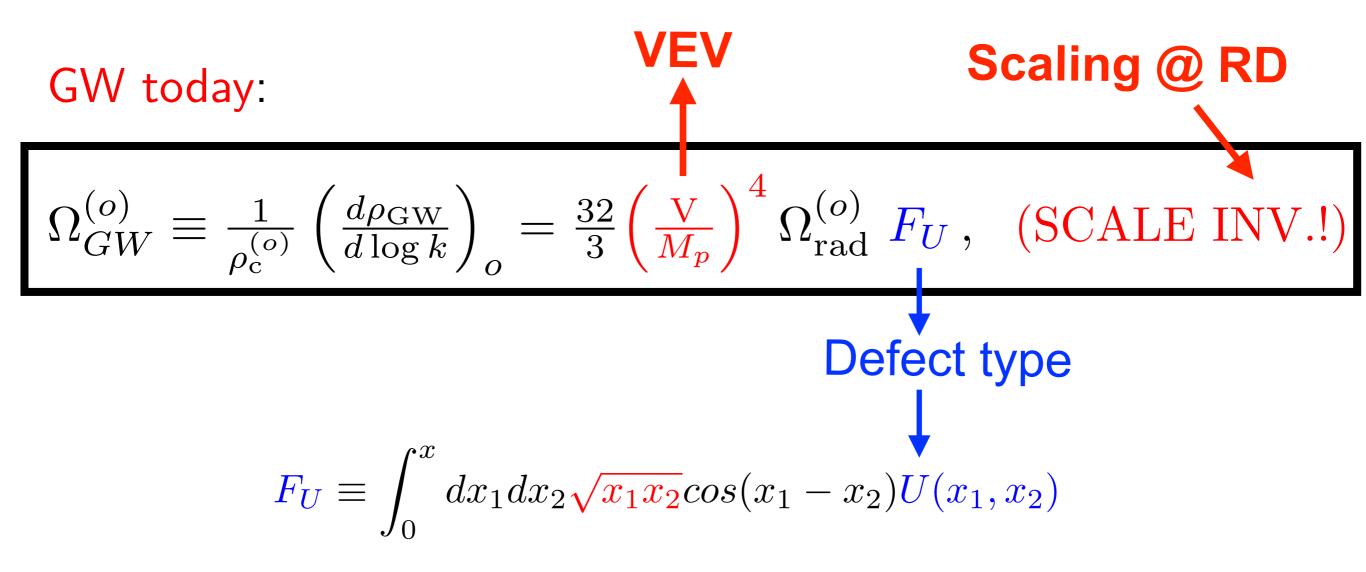
DEFECTS: GW Source $\rightarrow \{T_{ij}\}^{TT} \propto \{\partial_i \phi \partial_j \phi, E_i E_j, B_i B_j\}^{TT}$

$$\langle T_{ij}^{\mathrm{TT}}(\mathbf{k},t)T_{ij}^{\mathrm{TT}}(\mathbf{k}',t')\rangle = (2\pi)^3 \frac{\mathbf{V}^4}{\sqrt{tt'}} U(kt,kt')\delta^3(\mathbf{k}-\mathbf{k}')$$

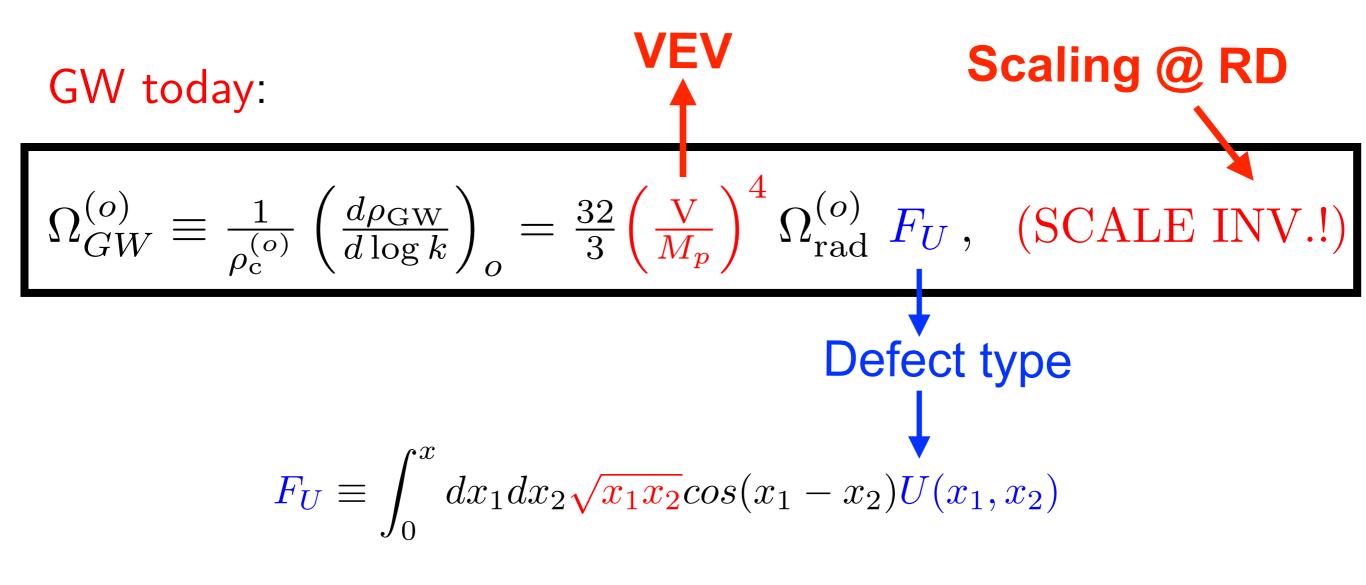
GW spectrum:
$$(x_i \equiv kt_i)$$
ExpansionUTC $\frac{d\rho_{\rm GW}}{d\log k}(k,t) \propto \left(\frac{V}{M_p}\right)^4 \frac{M_p^2}{a^4(t)} \left[\int^x dx_1 dx_2 \sqrt{x_1 x_2} \cos(x_1 - x_2) U(x_1, x_2)\right]$ Rad. DomSCALING

GW spectrum:
$$(x_i \equiv kt_i)$$
 R.D. and SCALING
 $\frac{d\rho_{\text{GW}}}{d\log k}(k,t) \propto \left(\frac{V}{M_p}\right)^4 \frac{M_p^2}{a^4(t)} \left[\int^x dx_1 dx_2 \sqrt{x_1 x_2} \cos(x_1 - x_2) U(x_1, x_2)\right]$

GW spectrum:
$$(x_i \equiv kt_i)$$
 R.D. and SCALING
 $\frac{d\rho_{\text{GW}}}{d\log k}(k,t) \propto \left(\frac{V}{M_p}\right)^4 \frac{M_p^2}{a^4(t)} \int dx_1 dx_2 \sqrt{x_1 x_2} \cos(x_1 - x_2) U(x_1, x_2)$
 $\frac{d\rho_{\text{GW}}}{d\log k}(k,t) \propto \left(\frac{V}{M_p}\right)^4 \frac{M_p^2}{a^4(t)} F_U$ $F_U \sim \text{Const.}$ (Dimensionless)



(Figueroa et al, PRL 2013)



 \forall PhT (1st, 2nd, ...), \forall Defects (top. or non-top.)

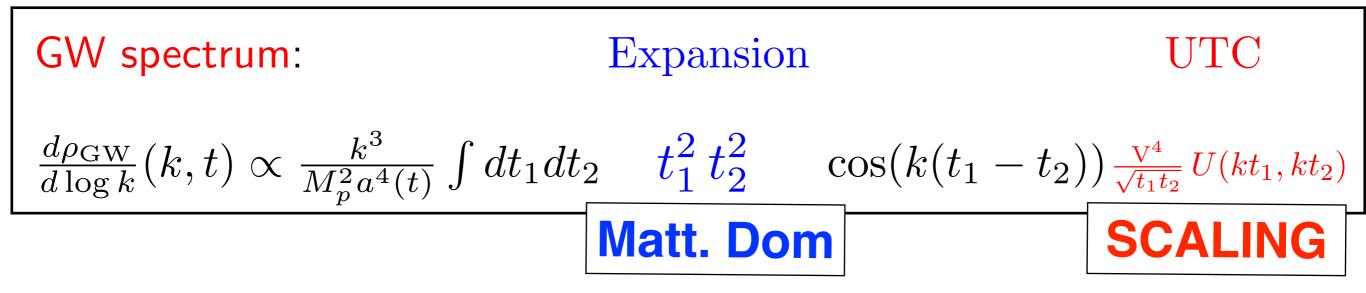
(Figueroa et al, PRL 2013)

$$\Omega_{GW}^{(o)} \equiv \frac{1}{\rho_{c}^{(o)}} \left(\frac{d\rho_{GW}}{d\log k} \right)_{o} = \frac{32}{3} \left(\frac{V}{M_{p}} \right)^{4} \Omega_{rad}^{(o)} F_{U} , \quad \text{(SCALE INV.!)} \\ \downarrow \quad \text{Scaling @ RD} \\ F_{U} \equiv \int_{0}^{x} dx_{1} dx_{2} \sqrt{x_{1}x_{2}} \cos(x_{1} - x_{2}) U(x_{1}, x_{2})$$

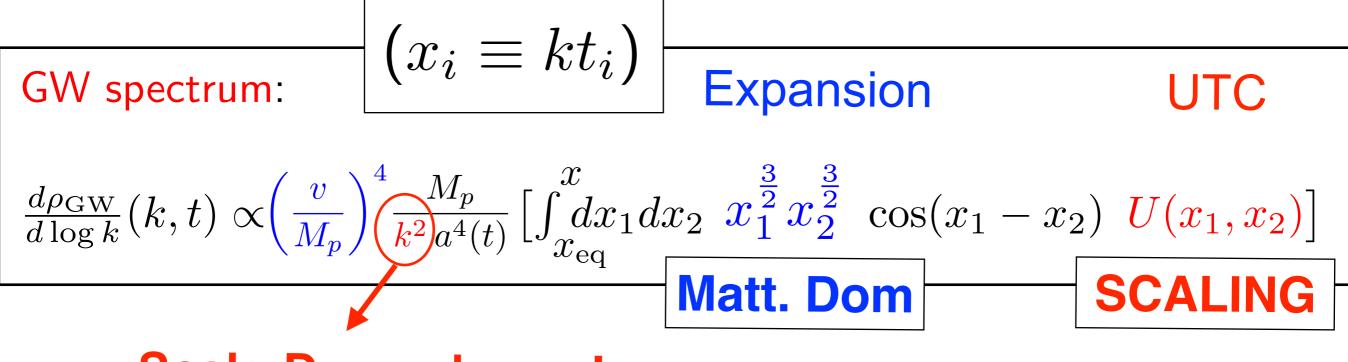
$$\Omega_{GW}^{(o)} \equiv \frac{1}{\rho_{c}^{(o)}} \left(\frac{d\rho_{GW}}{d\log k} \right)_{o} = \frac{32}{3} \left(\frac{V}{M_{p}} \right)^{4} \Omega_{rad}^{(o)} F_{U} , \quad \text{(SCALE INV.!)} \\ \downarrow \quad \text{Scaling @ RD} \\ F_{U} \equiv \int_{0}^{x} dx_{1} dx_{2} \sqrt{x_{1}x_{2}} \cos(x_{1} - x_{2}) U(x_{1}, x_{2})$$

GW spectrum:ExpansionUTC
$$\frac{d\rho_{\rm GW}}{d\log k}(k,t) \propto \frac{k^3}{M_p^2 a^4(t)} \int dt_1 dt_2 \ a(t_1)a(t_2) \ \cos(k(t_1-t_2)) \frac{V^4}{\sqrt{t_1t_2}} U(kt_1,kt_2)$$
SCALING

$$\Omega_{GW}^{(o)} \equiv \frac{1}{\rho_{c}^{(o)}} \left(\frac{d\rho_{GW}}{d\log k} \right)_{o} = \frac{32}{3} \left(\frac{V}{M_{p}} \right)^{4} \Omega_{rad}^{(o)} F_{U} , \quad \text{(SCALE INV.!)} \\ \downarrow \quad \text{Scaling @ RD} \\ F_{U} \equiv \int_{0}^{x} dx_{1} dx_{2} \sqrt{x_{1}x_{2}} \cos(x_{1} - x_{2}) U(x_{1}, x_{2})$$



$$\Omega_{GW}^{(o)} \equiv \frac{1}{\rho_{c}^{(o)}} \left(\frac{d\rho_{GW}}{d\log k} \right)_{o} = \frac{32}{3} \left(\frac{V}{M_{p}} \right)^{4} \Omega_{rad}^{(o)} F_{U}, \quad \text{(SCALE INV.!)} \\ \downarrow \quad \text{Scaling @ RD} \\ F_{U} \equiv \int_{0}^{x} dx_{1} dx_{2} \sqrt{x_{1}x_{2}} \cos(x_{1} - x_{2}) U(x_{1}, x_{2})$$



Scale-Dependence !

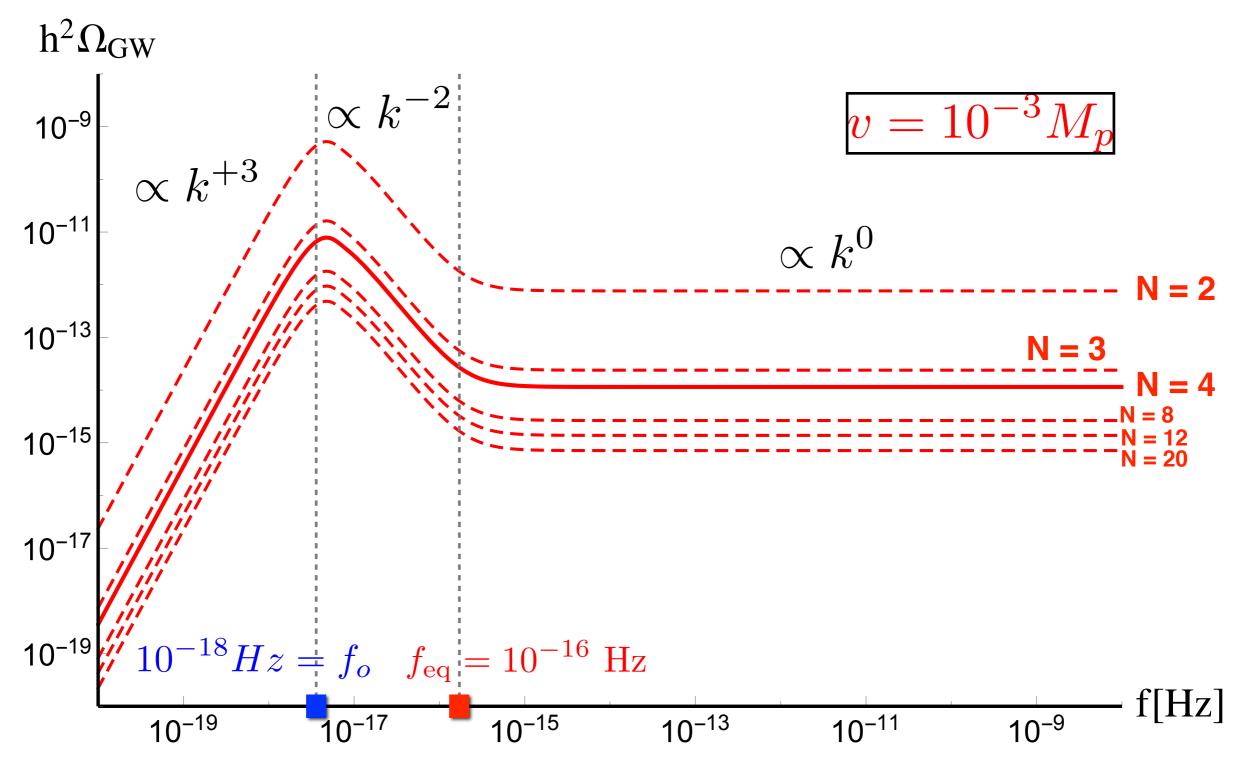
Total GW Spectrum
$$h^{2}\Omega_{\rm GW}^{(\rm o)} = h^{2}\Omega_{\rm rad}^{(\rm o)} \left(\frac{V}{M_{p}}\right)^{4} \left[F_{U}^{(\rm R)} + F_{U}^{(\rm M)} \left(\frac{k_{\rm eq}}{k}\right)^{2}\right]$$

$$F_U^{(R)} \equiv \frac{32}{3} \int_0^x dx_1 dx_2 \, (x_1 x_2)^{1/2} \cos(x_1 - x_2) \, U_{RD}(x_1, x_2)$$

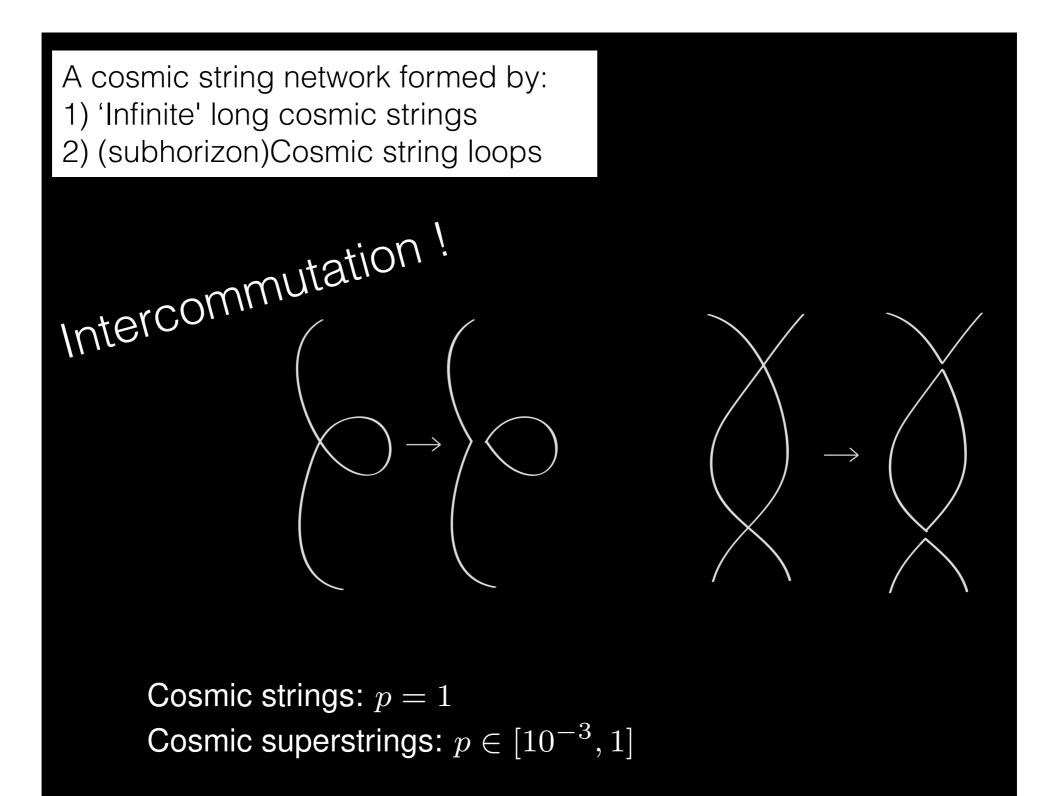
RD Scaling

$$F_U^{(M)} \equiv \frac{32}{3} \frac{(\sqrt{2}-1)^2}{2} \int_{x_{eq}}^x dx_1 dx_2 (x_1 x_2)^{3/2} \cos(x_1 - x_2) U_{MD}(x_1, x_2)$$
MD Scaling

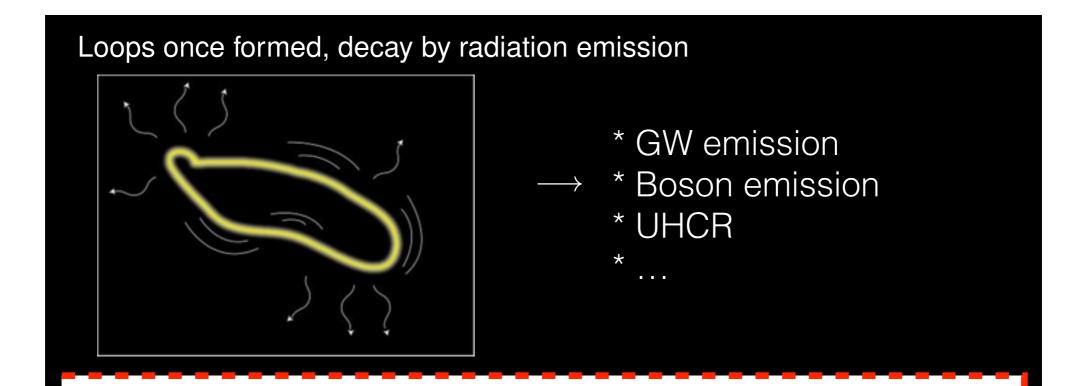
$$h^2 \Omega_{\rm GW}^{\rm (o)} = h^2 \Omega_{\rm rad}^{\rm (o)} \left(\frac{V}{M_p}\right)^4 \left[F_U^{\rm (R)} + F_U^{\rm (M)} \left(\frac{k_{\rm eq}}{k}\right)^2\right]$$



Extra emission of GWs ! (Vilenkin '81)



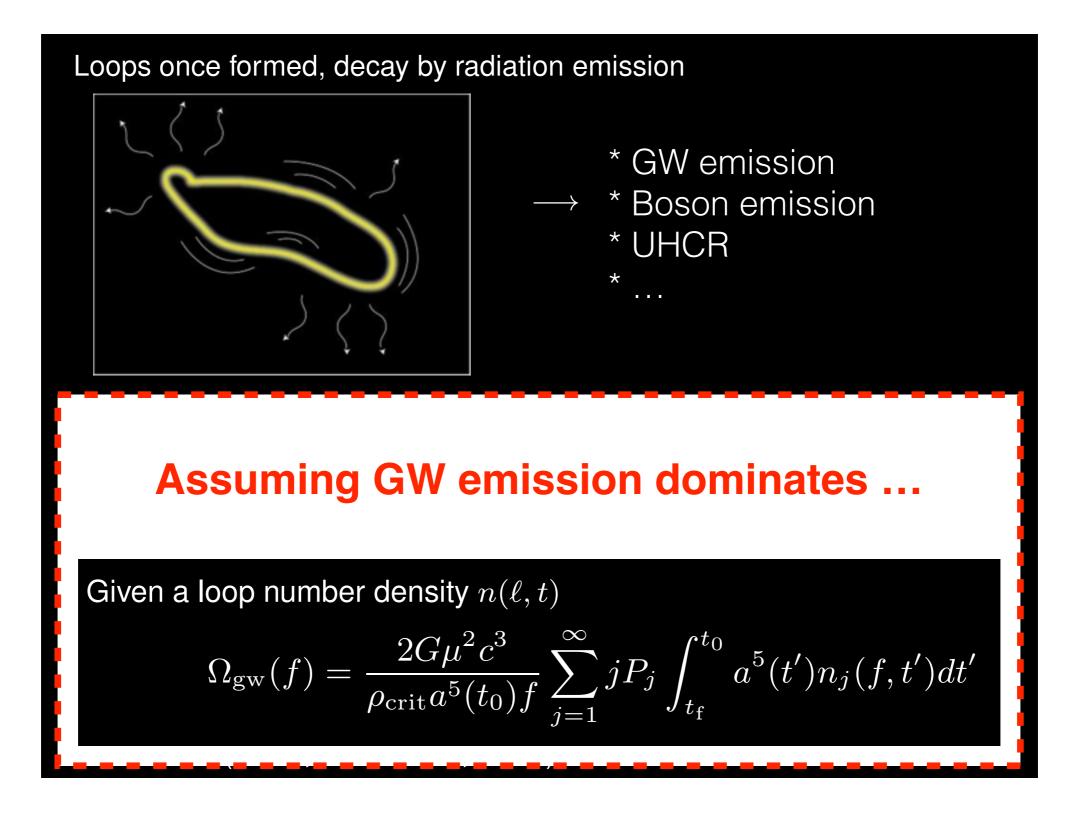
Extra emission of GWs ! (Vilenkin '81)



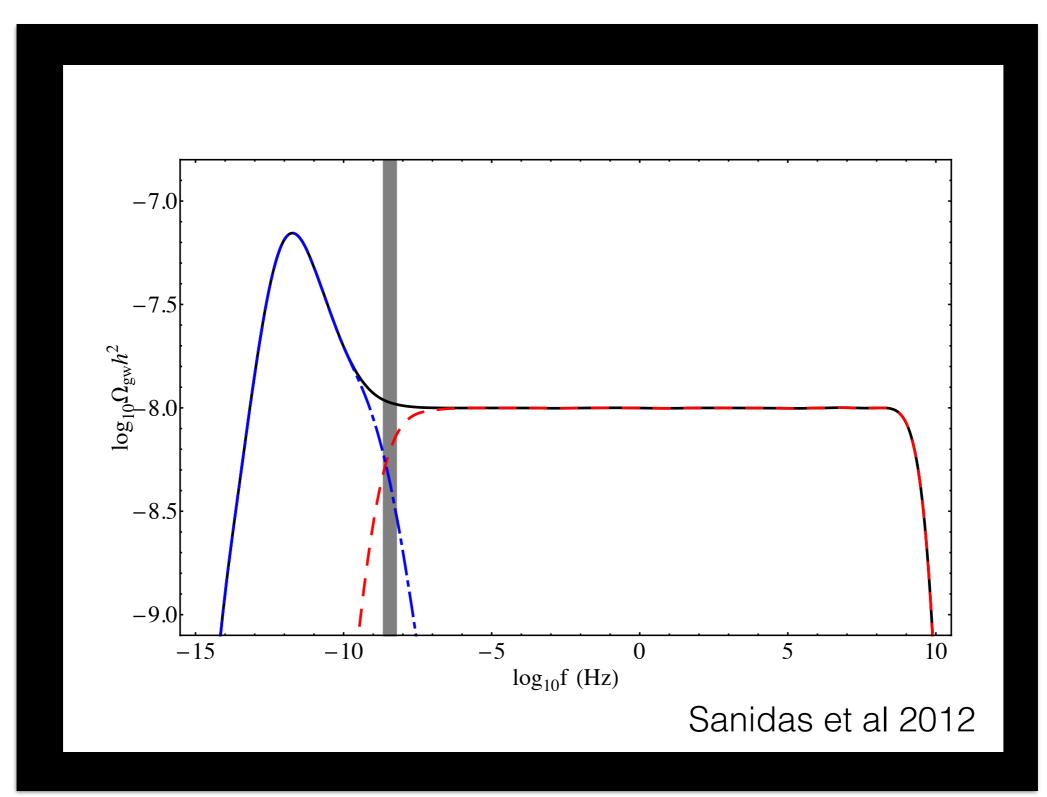
* Widely believed that GW represents dominant emission channel (Nambu-Goto)

* However... Abelian-Higgs field theory simulations show loops decay into bosons

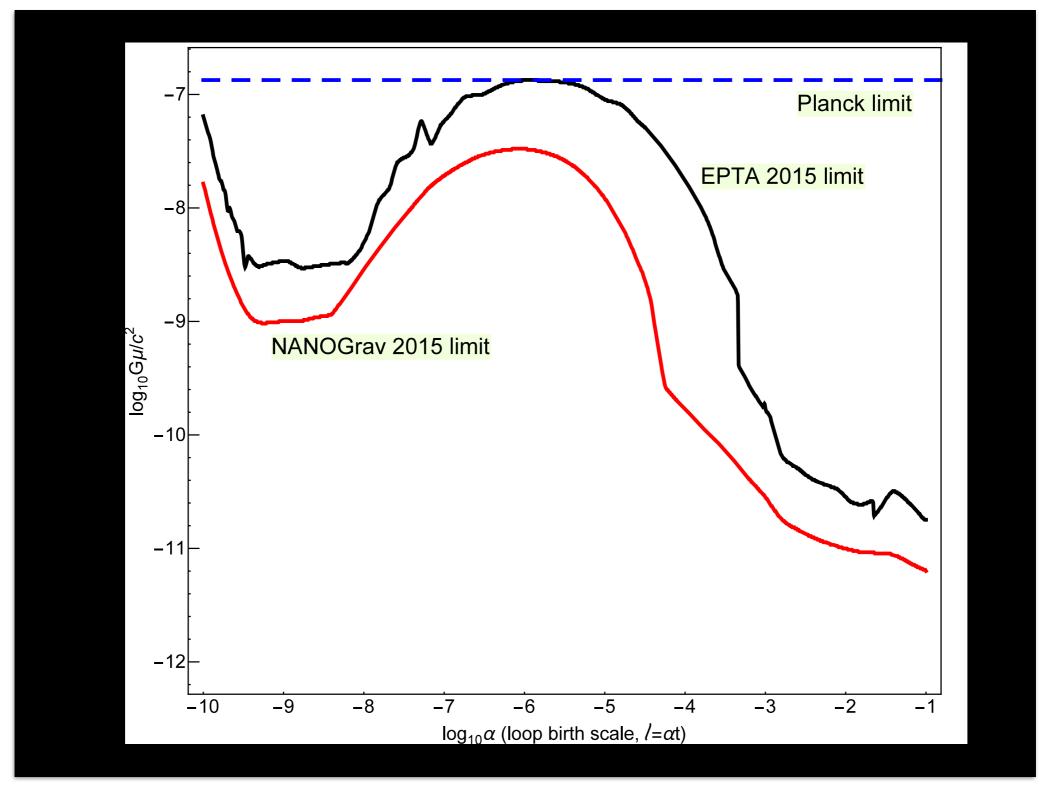
Extra emission of GWs ! (Vilenkin '81)



Extra emission of GWs ! (Vilenkin '81)



Extra emission of GWs !



(From Sanidas et al, LISA GW cosmology 3rd encounter)

Extra emission of GWs !

Results for 6 links, SNR=20

A1M2

LISA Prospects

Conservative limit: $G\mu/c^2 < 4.4 \times 10^{-10}$ Large loops: $G\mu/c^2 < 1.5 \times 10^{-16}$

A2M2

Conservative limit: $G\mu/c^2 < 1.1 \times 10^{-10}$ Large loops: $G\mu/c^2 < 2.1 \times 10^{-17}$

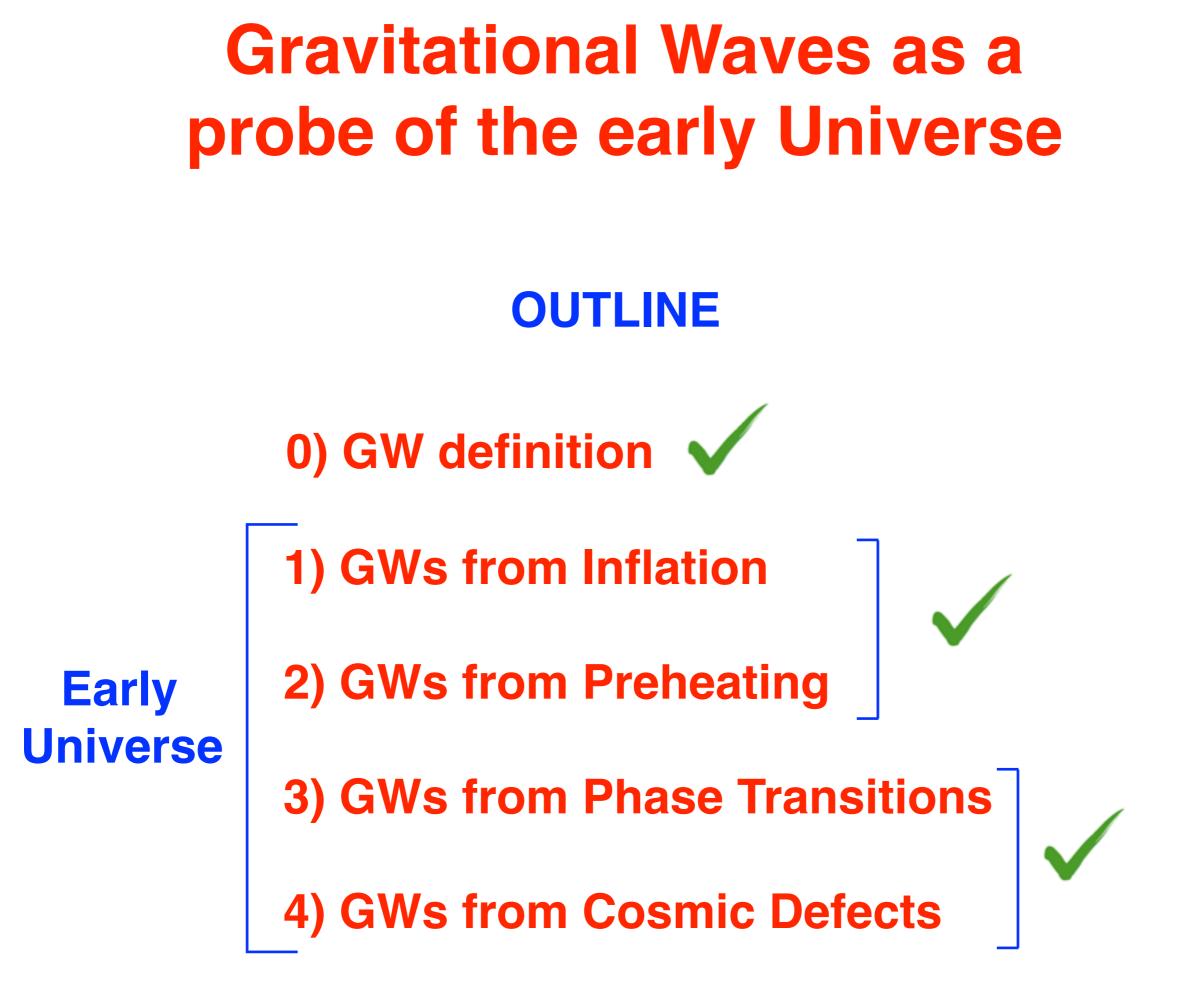
A2M5

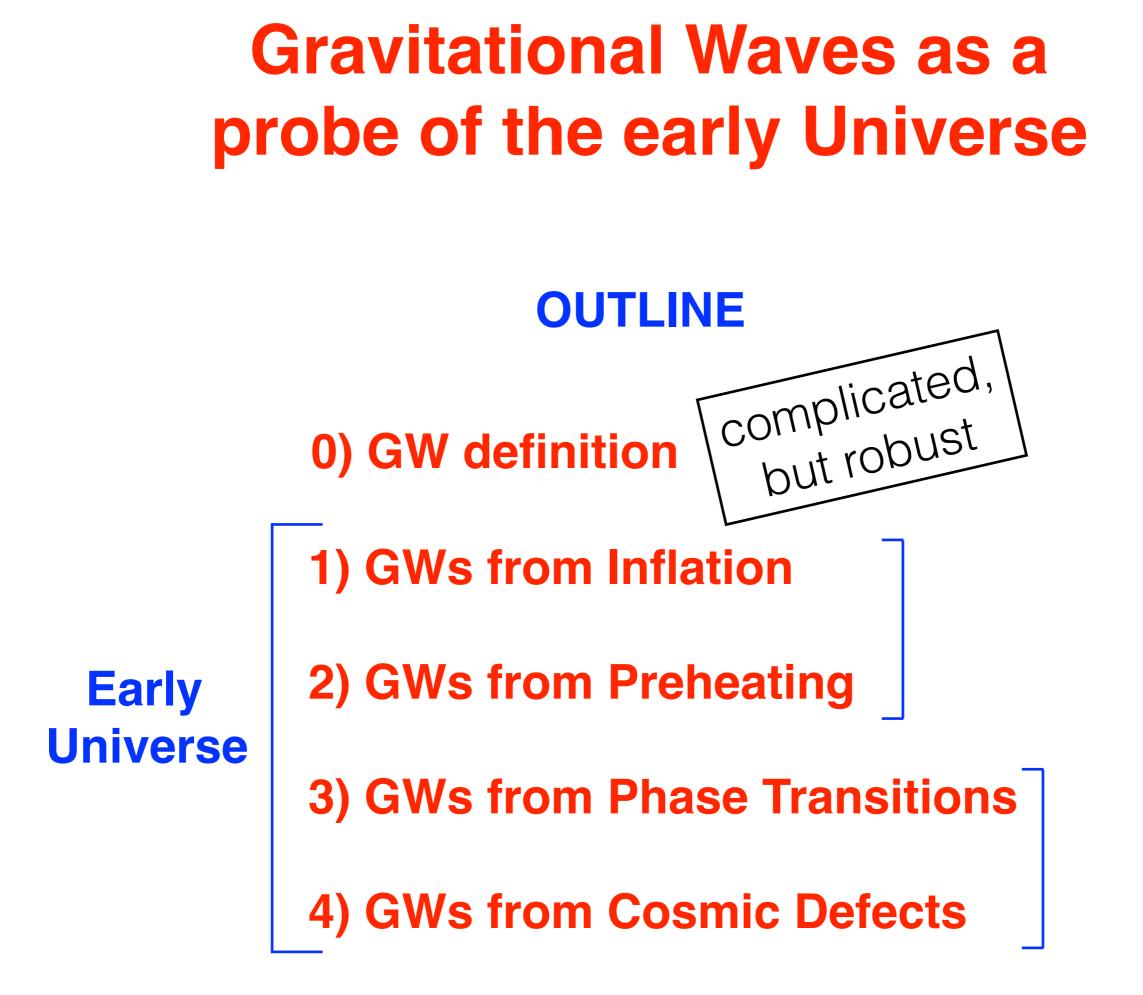
Conservative limit: $G\mu/c^2 < 7.0 \times 10^{-11}$ Large loops: $G\mu/c^2 < 1.3 \times 10^{-17}$

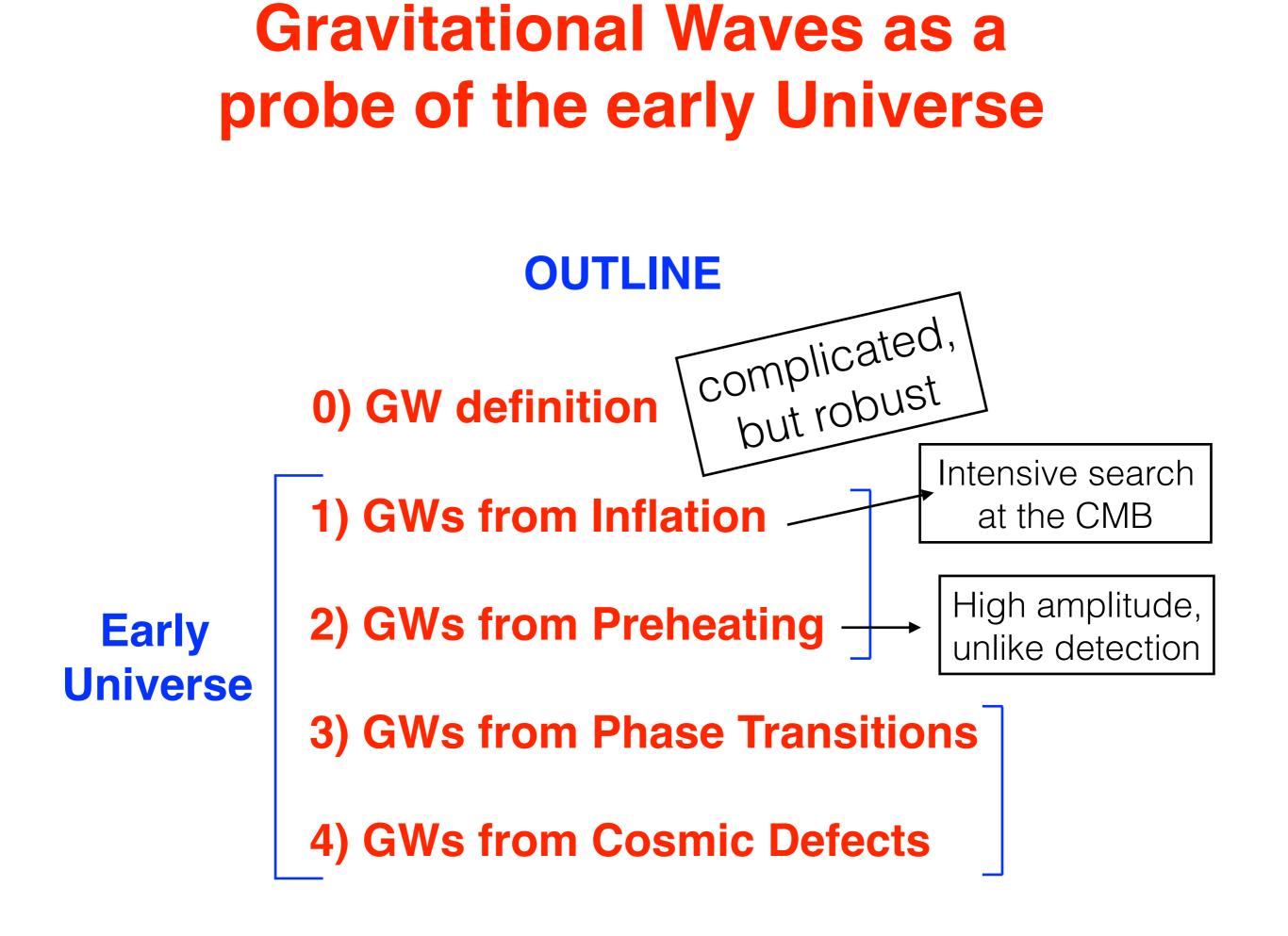
A5M5

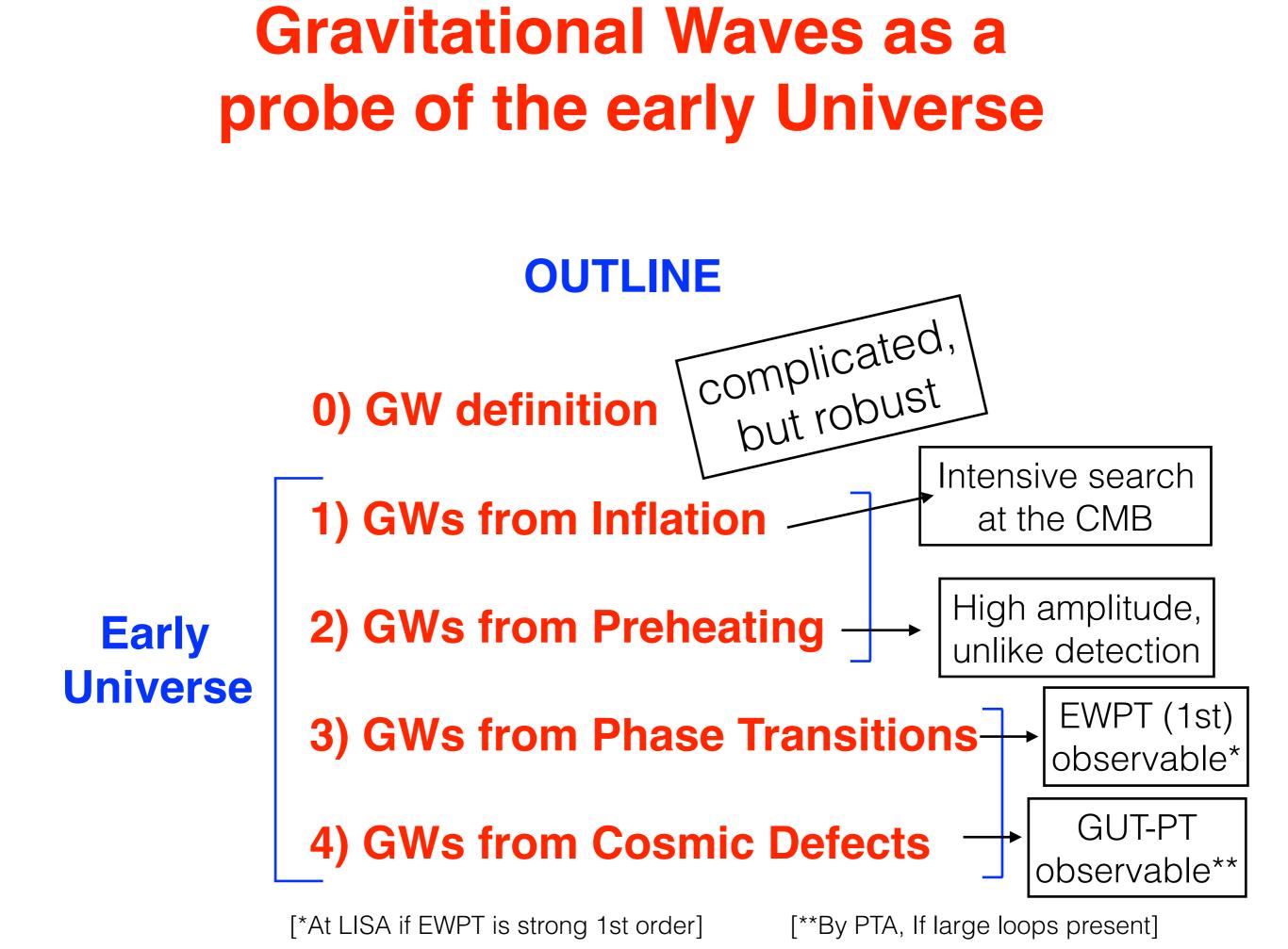
Conservative limit: $G\mu/c^2 < 1.4 \times 10^{-11}$ Large loops: $G\mu/c^2 < 4.4 \times 10^{-18} \rightarrow v \leq 10^{10} GeV$

(From Sanidas et al, LISA GW cosmology 3rd encounter)









Coming Soon ...

* Caprini & Figueroa, REVIEW (expected Feb 2017)

* Benasque GW school (2 weeks): May 28 - June 10

http://benasque.org/2017gw/

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Thanks for your attention !