UNIVERSITÄT BERN

AEC
ALBERT EINSTEIN CENTER
FOR FUNDAMENTAL PHYSICS

Resummation of global and non-global logarithms

Thomas Becher
University of Bern

PRL 116 (2016) and 1605.02737 (JHEP)
with Matthias Neubert, Dingyu Shao and Lorena Rothen
1610.01608 with Ben Pecjak and Dingyu Shao

Future challenges for precision QCD, Oct. 25-28, 2016, IPPP, Durham

Executive Summary

The main consequence of predictivity from first principles is the existence of the systematic perturbative expansion...

LO is not a model

NLO is not a model

NNLO is not a model

....

Pythia is not QCD

Herwig is not QCD

Sherpa is not QCD

Geneva is not QCD

SCET is not a theory
-- it is a framework !

Kirill Melnikov at QCD@LHC 2016

Corollary

LO is not a model
NLO is not a model
NNLO is not a model
…

LL is not a model
NLL is not a model
NNLL is not a model
…

The question is just how one obtains it!

Both predictions are systematically improvable, but the status is quite different

Fixed order

- *In principle*, we know what to compute at any order, for any IR safe observable.
- *In practice*: general LO and NLO in automated form, NNLO for 2→2 cross sections, N^{3}LO for 2→1 cross sections.

Resummation

- For very simple observables (i.e. global event shapes, q_T spectra), we know *in principle* how to obtain any accuracy.
- Some NLL and NNLL automation. A few selected N^{3}LL results.

Future challenges for precision QCD

For resummation we need both

1. more ``in principle”
   - resummation of more complex observables
2. more ``in practice”
   - automation
   - better observables

In my talk, I will focus on the first point, in particular on higher-log resummation for non-global observables.

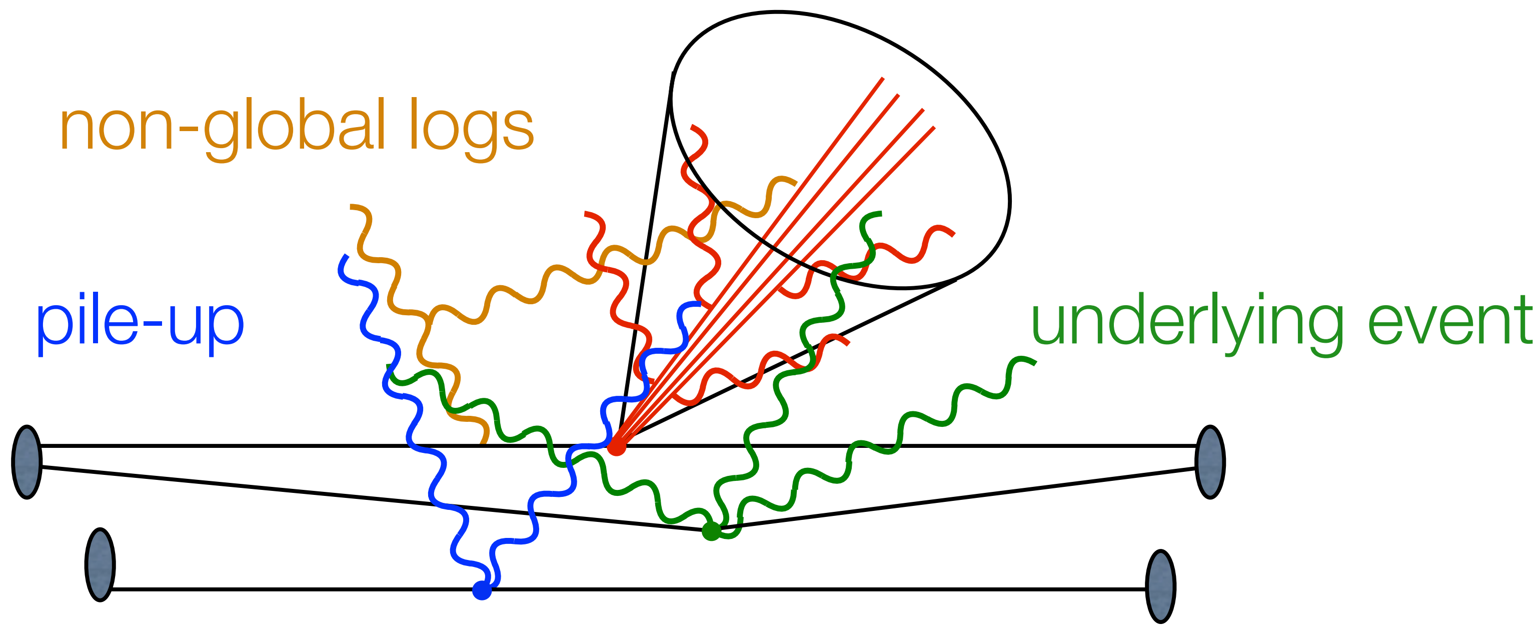
Automated resummation

- Automated computations of 2-loop soft functions Bell, Rahn and Talbert ‘16
- NNLL for jet veto cross sections, TB, Frederix, Neubert and Rothen ‘15
- NLL for *pp*→2 jets Farhi, Feige, Freytsis and Schwartz ’15
- NNLL soft-gluon resummations for arbitrary distributions. *ttH*, Broggio, Ferroglia, Pecjak, Signer and Yang ‘15. *ttW*, Broggio, Ferroglia, Ossola and Pecjak ’16
- ARES: NNLL for 2-jet observables in e^+e^- Banfi, McAslan, Monni and Zanderighi ’15, ‘16
- GENEVA results for Drell-Yan process → talk by Simone Alioli

Note: NNLL resummations use automated one-loop computations of hard functions as input.

Better observables: m_J in $pp \to Z + j$

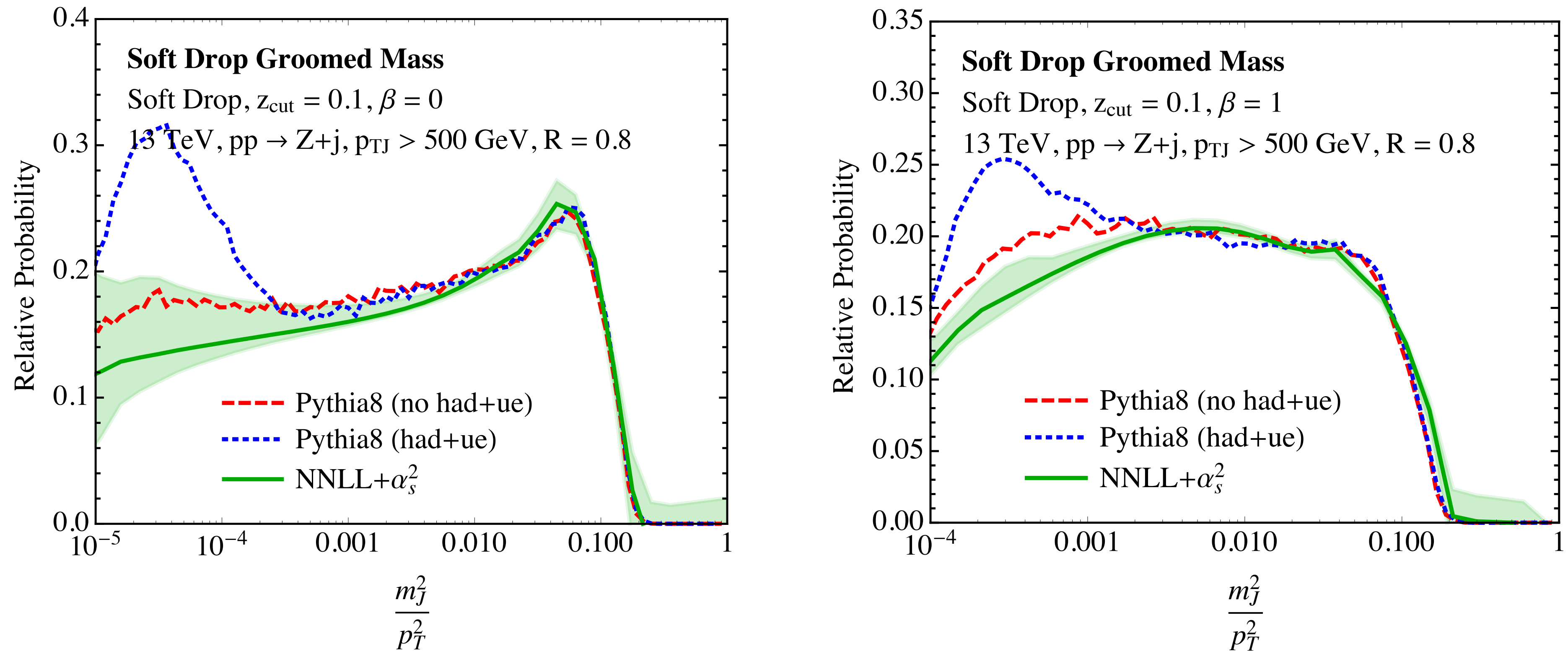
Challenges and contaminations



- Grooming can mitigate these problems
- mMDT also eliminates NGLs in m_J
- Analytical NLL Dasgupta, Fregoso, Marzani, Salam '13, Larkoski, Marzani, Soyez,Thaler '14

NNLL + $O(\alpha_s^2)$ for jet mass

Frye, Larkoski, Schwartz, Yan'16



Based on factorization

$$m_J^2 \ll z_{\rm cut} p_{TJ}^2 \ll p_{TJ}^2$$

$$\frac{d\sigma^{\rm resum}}{dm_J^2} = \sum_{k=q,\bar{q},g} D_k(p_T, z_{\rm cut}, R) S_{C,k}(z_{\rm cut} m_J^2) \otimes J_k(m_J^2)$$

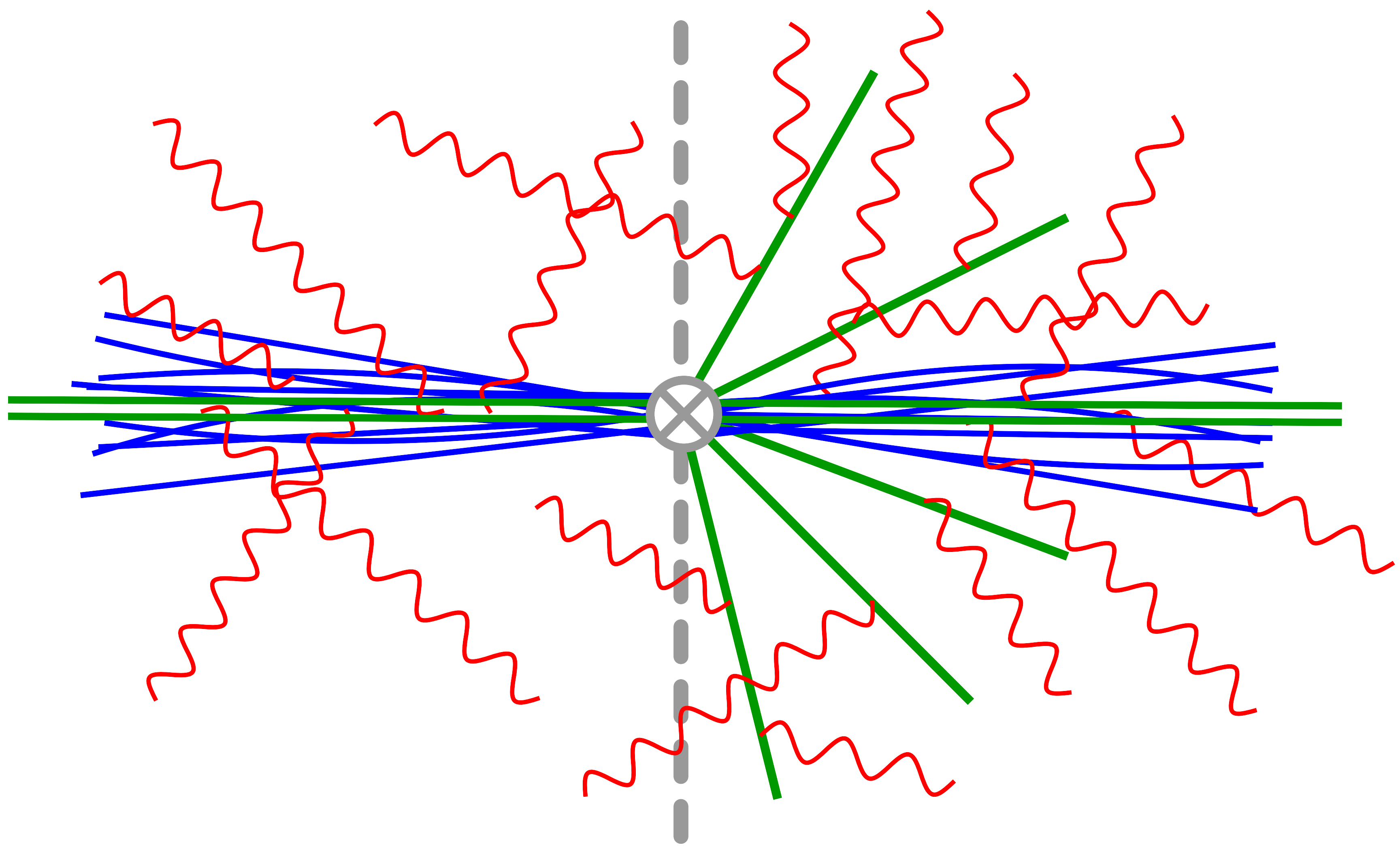
sum over jet flavor

includes pdfs, emissions that were groomed away, out-of-jet radiation,...

collinear-soft radiation

hard collinear radiation

15



Factorization theorems

resummation in principle

A key ingredient to obtain logarithmically enhanced terms is factorization

E.g. q_T spectrum of EW boson for $q_T \ll M$ CSS '84

$$\frac{d\sigma}{dq_T dy} = H\left(M^2,\mu\right) \frac{1}{4\pi} \int d^2x_\perp e^{-iq_\perp \cdot x_\perp} \left(\frac{x_T^2 M^2}{b_0^2}\right)^{-F_{q\bar{q}}\left(x_T^2,\mu\right)}$$
$$\times \sum_q e_q^2 \left[B_{q/N_1}\left(z_1, x_T^2, \mu\right) B_{\bar{q}/N_2}\left(z_2, x_T^2, \mu\right) + (q \leftrightarrow \bar{q})\right] .$$

Scale separation: Only functions of single scale!

(Transverse PDFs $B_{q/N}$ also depend on non-perturbative scale m_p, can again be factorized.)

Factorization and resummation

Once factorization is understood, resummation reduces to

1. Fixed-order computations of ingredients
2. Solution of evolution equations.

In EFTs such as SCET, these are RG evolution equations, driven by anomalous dims.

Resummed computations = fixed order in EFT + RG evolution of Wilson coefficients

- RG improved perturbation theory: LO = NLL …

q_T resummation at N^{3}LL

Unknown ingredients at achieve N^{3}LL accuracy

1. Four-loop $\Gamma_{\rm cusp}$ aka A_4
2. Three-loop anomaly $F_{qq}^{(3)}$ *aka* rapidity anomalous dimension γ_r, directly related to B_3 of CSS.
3. Two-loop H functions and beam functions $B_{q/N}$

NEW

$\Gamma_{\rm cusp}$

$$\frac{d\sigma}{dq_T dy} = H\left(M^2,\mu\right) \frac{1}{4\pi} \int d^2 x_\perp e^{-iq_\perp \cdot x_\perp} \left(\frac{x_T^2 M^2}{b_0^2}\right)^{-F_{q\bar{q}}\left(x_T^2,\mu\right)}$$

$$\times \sum_q e_q^2 \left[B_{q/N_1}\left(z_1, x_T^2, \mu\right) B_{\bar{q}/N_2}\left(z_2, x_T^2, \mu\right) + (q \leftrightarrow \bar{q})\right]$$

Catani, Grazzini et al. ‘12

Gehrmann, Luebbert, Yang ’12 ‘14

Full three-loop double differential soft function in QCD

$$-\frac{8}{3} C_A^2 C_F \left(H_{1,1}[x] - \frac{H_{1,1}[x]}{x} \right) + \frac{8}{3} C_A C_F n_f \left(H_{1,1}[x] - \frac{H_{1,1}[x]}{x} \right) +$$

Cancel in N=1 SYM

$C_F^2 n_f \left(-\frac{110}{3} H_{0,1}[x \ldots \right.$

$C_F n_f^2 \left(\frac{400}{81} H_{0,1}[x] \ldots \right.$

$C_A C_F n_f \left(-\frac{7988}{81} H_0 \ldots \right.$

$\frac{2312}{27} H_{0,1,1}[x] \ldots$

$\frac{32}{9} H_{0,1,1,1}[x] \ldots$

$\frac{16}{3} H_{0,1,0,0,1}[x \ldots$

$C_A^2 C_F \left(\frac{30790}{81} H_{0,1} \ldots \right.$

$\frac{88}{3} \zeta_2 H_{0,0,1}[x] \ldots$

$\frac{3112}{9} H_{0,0,1,1}[x \ldots$

$96 \zeta_2 H_{0,1,1,1}[x \ldots$

$\frac{88}{3} H_{0,1,0,0,1}[x \ldots$

$128 H_{0,0,0,0,1,1} \ldots$

3-loop coefficient Li and Zhu '16

$$\gamma_0^r = 0$$

$$\gamma_1^r = C_a C_A \left(28\zeta_3 - \frac{808}{27} \right) + \frac{112 C_a n_f}{27}$$

$$\begin{aligned}\gamma_2^r = {} & C_a C_A^2 \left(-\frac{176}{3}\zeta_3\zeta_2 + \frac{6392\zeta_2}{81} + \frac{12328\zeta_3}{27} + \frac{154\zeta_4}{3} \right. \\ & \left. - 192\zeta_5 - \frac{297029}{729} \right) + C_a C_A n_f \left(-\frac{824\zeta_2}{81} - \frac{904\zeta_3}{27} \right. \\ & \left. + \frac{20\zeta_4}{3} + \frac{62626}{729} \right) + C_a n_f^2 \left(-\frac{32\zeta_3}{9} - \frac{1856}{729} \right) \\ & + C_a C_F N_f \left(-\frac{304\zeta_3}{9} - 16\zeta_4 + \frac{1711}{27} \right)\end{aligned}$$

$$80 H_{0,0,1,1,0,1}[x] + 256 H_{0,0,1,1,1,1}[x] + 24 H_{0,1,0,0,0,1}[x] + 80 H_{0,1,0,0,1,1}[x] + 56 H_{0,1,0,1,0,1}[x] +$$

$$160 H_{0,1,0,1,1,1}[x] + 16 H_{0,1,1,0,0,1}[x] + 96 H_{0,1,1,0,1,1}[x] + 64 H_{0,1,1,1,0,1}[x] + 192 H_{0,1,1,1,1,1}[x] \Big)$$

Hard functions

Three-loop Γ for an any number of legs is known!

Almelid, Duhr, Gardi ’16; confirmed by Henn, Mistlberger ‘16

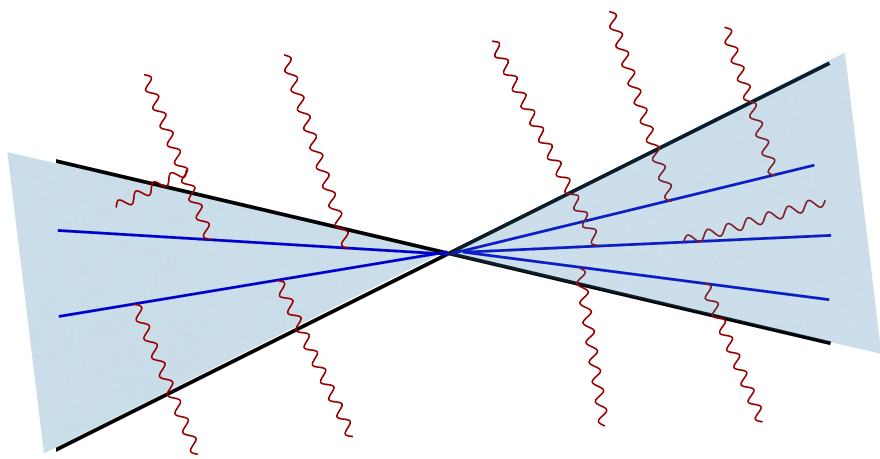
$$\boldsymbol{\Gamma}(\{\underline{p}\},\mu) = \sum_{(i,j)} \frac{\boldsymbol{T}_i \cdot \boldsymbol{T}_j}{2}\, \gamma_{\text{cusp}}(\alpha_s)\, \ln\frac{\mu^2}{-s_{ij}} + \sum_i \gamma^i(\alpha_s) + \Delta_n\left(\{\rho_{ijkl}\}\right)$$

$$\begin{aligned}\Delta_n^{(3)}\left(\{\rho_{ijkl}\}\right) = 16\, f_{abe} f_{cde} \Big\{ \sum_{1\le i<j<k<l\le n} \Big[&\mathbf{T}_i^a \mathbf{T}_j^b \mathbf{T}_k^c \mathbf{T}_l^d\, \mathcal{F}(\rho_{ikjl}, \rho_{iljk}) \\ &+ \mathbf{T}_i^a \mathbf{T}_k^b \mathbf{T}_j^c \mathbf{T}_l^d\, \mathcal{F}(\rho_{ijkl}, \rho_{ilkj}) \\ &+ \mathbf{T}_i^a \mathbf{T}_l^b \mathbf{T}_j^c \mathbf{T}_k^d\, \mathcal{F}(\rho_{ijlk}, \rho_{iklj}) \Big] \\ - C \sum_{i=1}^{n} \sum_{\substack{1\le j<k\le n \\ j,k\neq i}} &\left\{ \mathbf{T}_i^a, \mathbf{T}_i^d \right\} \mathbf{T}_j^b \mathbf{T}_k^c \Big\},\end{aligned}$$

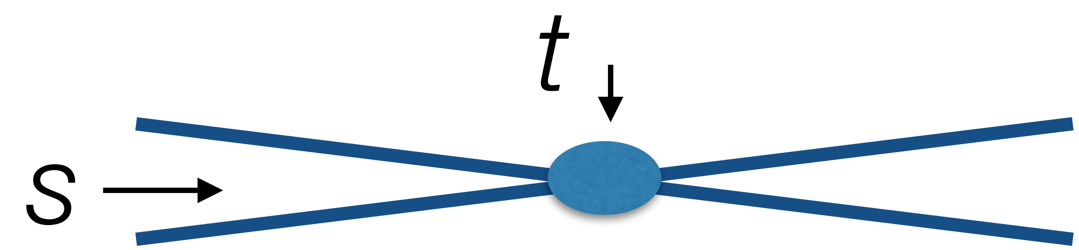
$$\rho_{ijkl} \equiv \frac{(-s_{ij})(-s_{kl})}{(-s_{ik})(-s_{jl})}$$

Δ_n is strongly constrained by factorization TB, Neubert ’09; Gardi, Magnea ’09 + Dixon ’09, + Del Duca, Duhr, and White ‘11

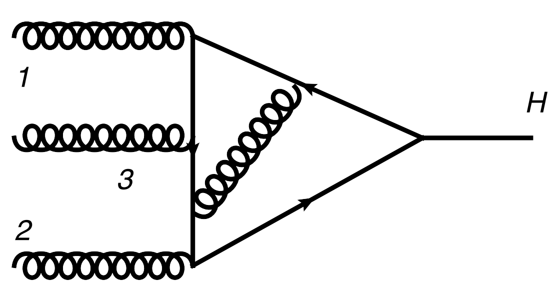
Missing factorization theorems



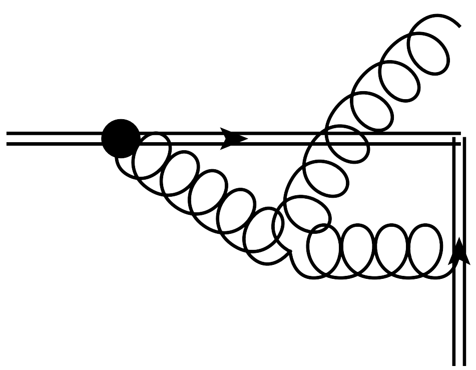
Non-global observables
(e.g. phase-space cuts, jets, …)



forward scattering, Glauber gluons
(*pp* scattering contains forward part)



Small masses
(e.g. *b*-quarks in *H* production,
EW effects at large q_T, ...)



Power corrections
(e.g. corrections to threshold limit,
next-to-eikonal corrections)

A lot of progress during the past year

Caron-Huot ’15
Larkoski, Moult, Neill ’15 '16
TB, Neubert, Shao, Rothen ’15 +
Pecjak ‘16

Non-global observables
(e.g. phase-space cuts, jets, …)

Del Duca,Falcioni, Magnea, Vernazza ’14
Fleming ’14,
Rothstein, Stewart ‘16

forward scattering
(*pp* scattering contains forward part)

Melnikov, Penin ’16 + Tancredi,
Wever ’16
Caola, Forte, Marzani, Muselli, Vita
‘16

Small masses
(e.g. EW effects at large q_T,
b-quarks in *H* production)

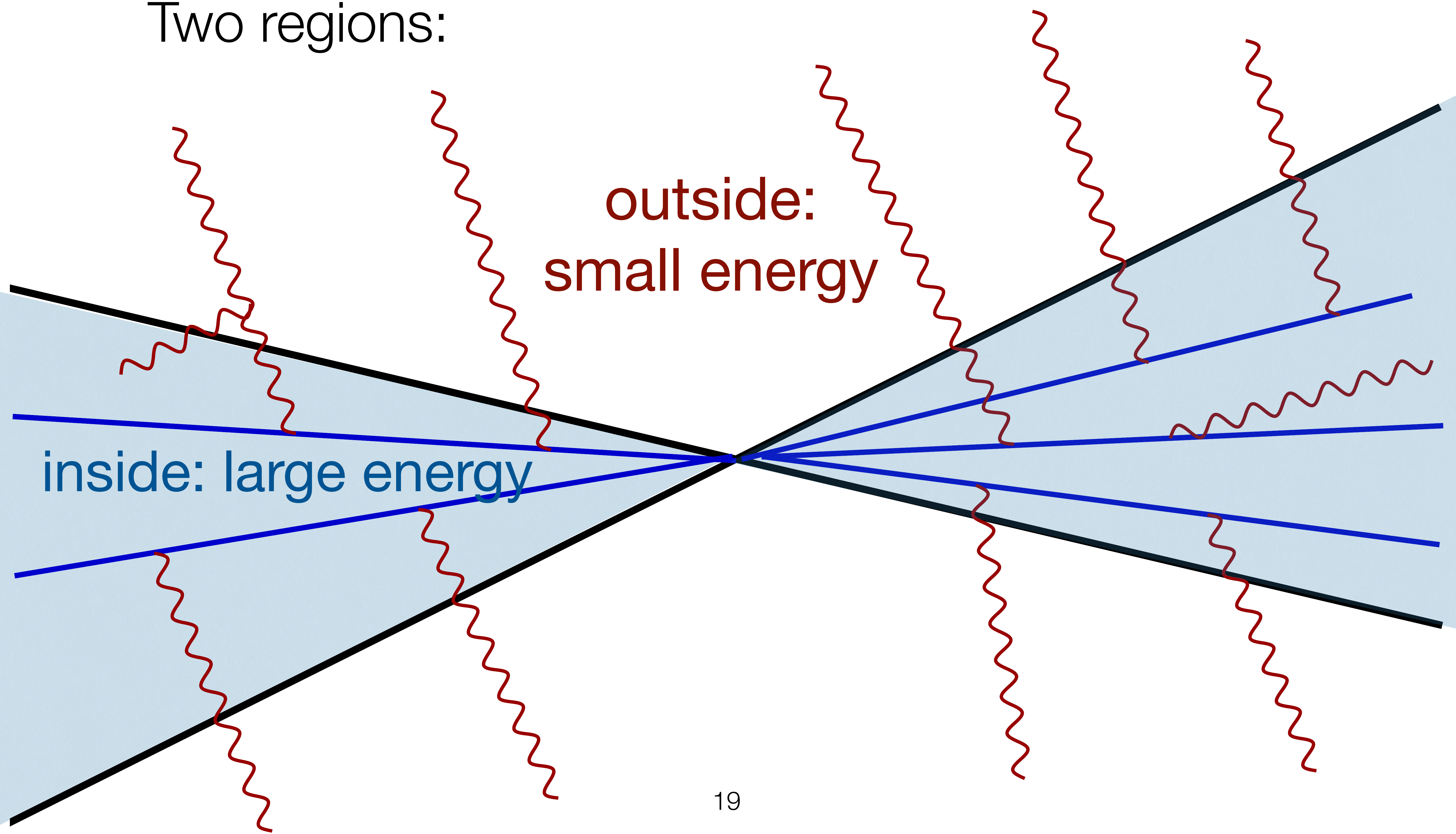
Larkoski, Neill, Stewart ‘14
Bonocore, Laenen, Melville,
Magnea, Vernazza and White
’14,’15,’16

Power corrections
(e.g. corrections to threshold limit,
next-to-eikonal corrections)

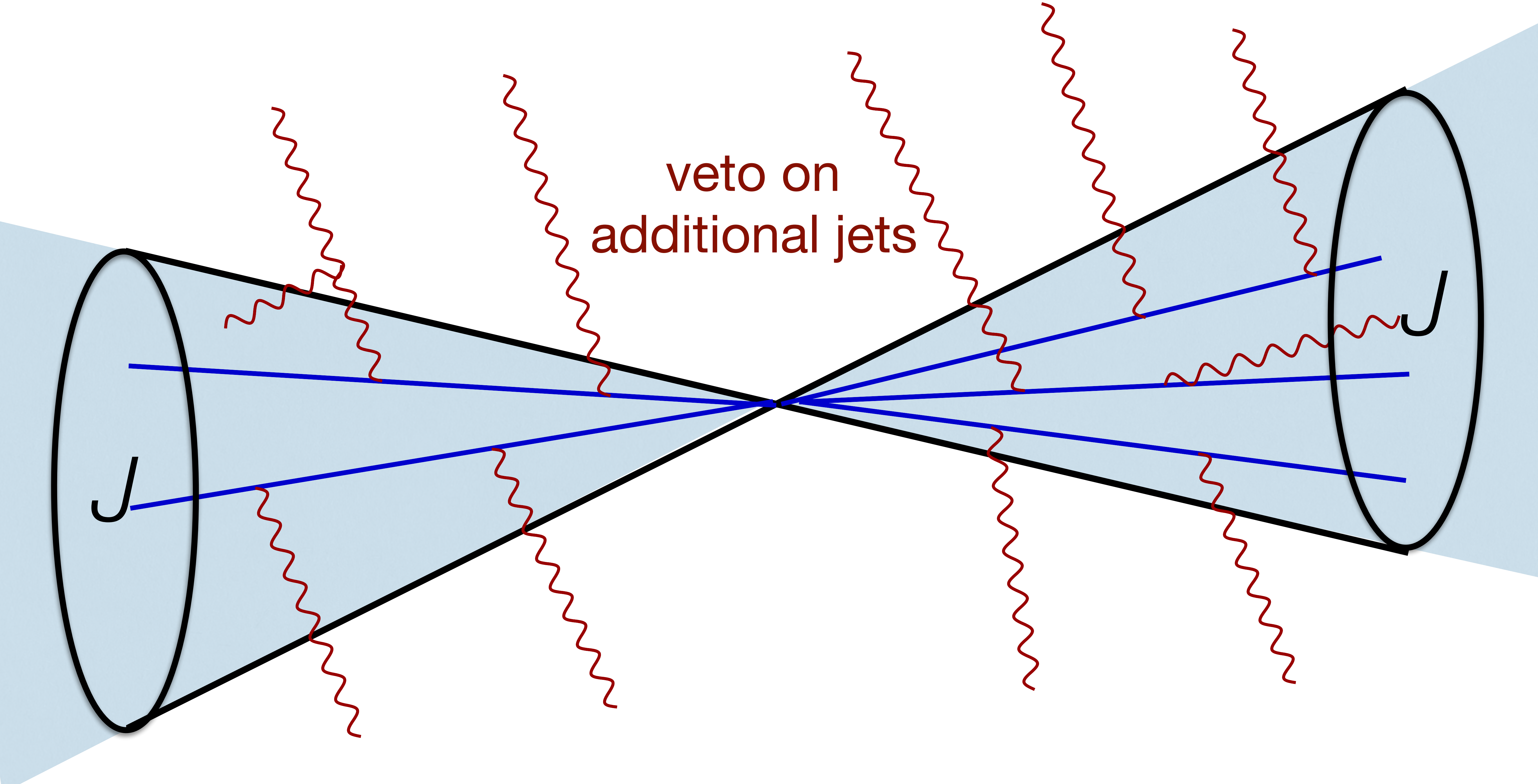
Non-global logarithms

Consider the simplest collider-physics problem involving large logarithms.

Two regions:



Arises in many situation, in particular in all exclusive **jet cross sections**

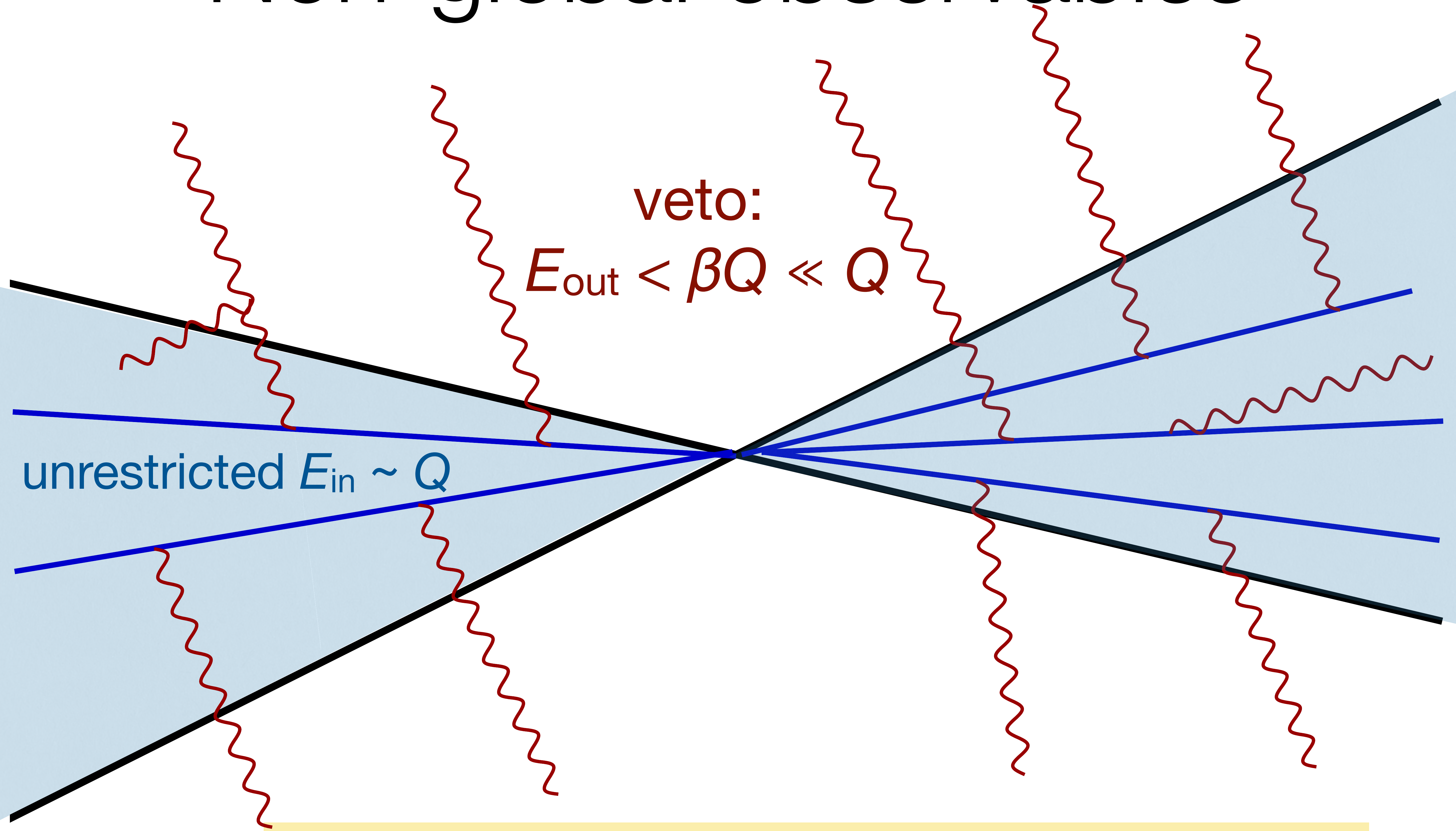


Many more examples

- jet vetoes (includes unrestricted radiation near the beam pipe)
- gaps between jets
- jet substructure
- isolated photons (veto on radiation near photon)
- event shapes such as the light-jet mass and narrow jet broadening
- …

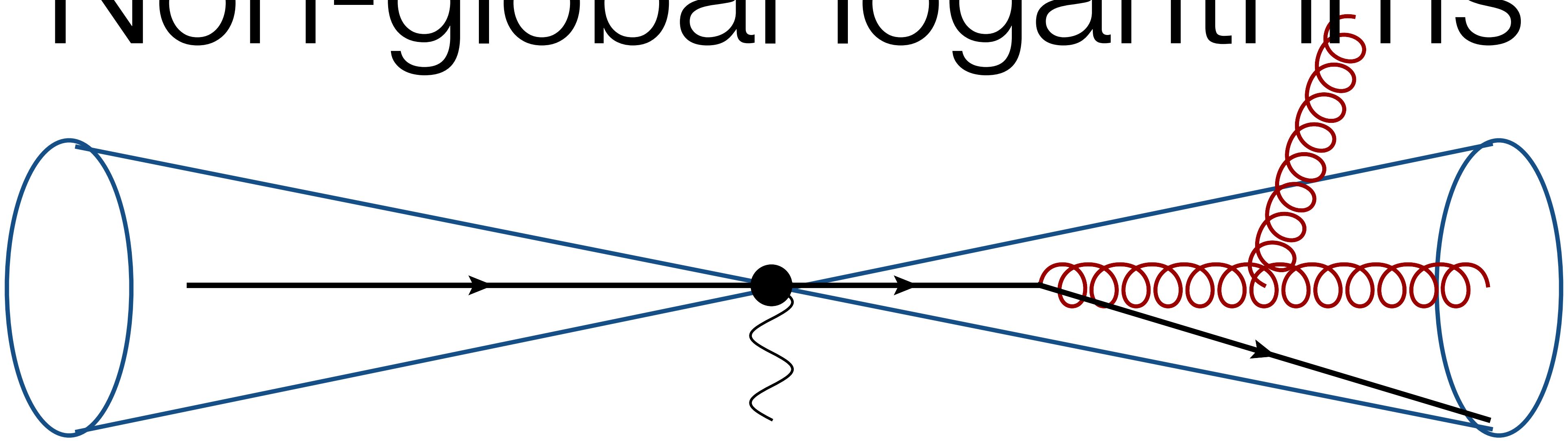
Such observables are called non-global, since they are insensitive to radiation inside certain regions of phase space.

Non-global observables



→ large logs $\alpha_s^n \ln^n(E_{out} / E_{in}) \sim \alpha_s^n \ln^n(\beta)$

Non-global logarithms



Large logarithms $\alpha_s^n \ln^m(\beta)$ in non-global observables do not exponentiate Dasgupta and Salam '02.

Leading logarithms at large N_c can be obtained from non-linear integral equation

$$\partial_{\hat{L}} G_{kl}(\hat{L}) = \int \frac{d\,\Omega(n_j)}{4\pi}\, W_{kl}^{j} \left[\Theta_{\rm in}^{n\bar{n}}(j)\, G_{kj}(\hat{L})\, G_{jl}(\hat{L}) - G_{kl}(\hat{L})\right]$$

Banfi, Marchesini, Smye '02

$\hat{L} \sim N_c\, \alpha_s \ln \beta$

$$W_{kl}^{j} = \frac{n_k \cdot n_l}{n_k \cdot n_j\, n_l \cdot n_j}$$ dipole radiator

LL resummation

- The leading logarithms arise from configurations in which the emitted gluons are strongly ordered

$$E_1 \gg E_2 \gg E_3 \gg \ \ldots \ \gg E_m$$

- Multi-gluon emission amplitudes become extremely simple in this limit, especially at large N_c

$$\left|\mathcal{M}_{ab}^{1\cdots m}\right|^2 = \left|\langle p_1 \cdots p_m \left| Y_a^\dagger Y_b \right| 0 \rangle\right|^2 = N_c^m g^{2m} \sum_{\text{perms of } 1\cdots m} \frac{(p_a \cdot p_b)}{(p_a \cdot p_1)\,(p_1 \cdot p_2) \cdots (p_m \cdot p_b)}$$

- Their simple structure is the basis for the BMS equation.

Non-global logarithms (NGLs)

A lot of recent work on NGLs

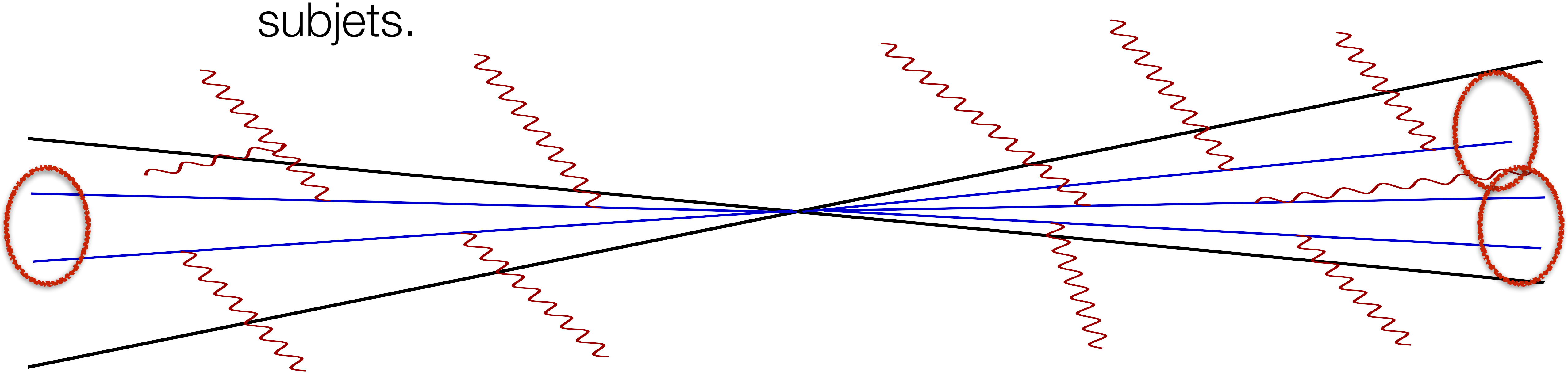
- Resummation of leading logs beyond large N_c Weigert '03, Hatta, Ueda '13 + Hagiwara '15; Caron-Huot '15.
  - Caron-Huot's functional RG has a close relation to our results
- Fixed-order results: 2 loops for $S(\omega_L,\omega_R)$. Kelley, Schwartz, Schabinger and Zhu '11; Hornig, Lee, Stewart, Walsh and Zuberi '11; with jet-cone Kelley, Schwartz, Schabinger and Zhu '11; von Manteuffel, Schabinger and Zhu '13, leading non-global log up to 5 loops by solving BMS equation Schwartz, Zhu '14, up to 12 loops Caron-Huot '16, up to 5 loops and arbitrary N_c Delenda, Khelifa-Kerfa '15
- Approximate resummation of such logs, based on resummation for observables with *n* soft subjets. Larkoski, Moult and Neill '15

A systematic factorization of non-global observables was missing.

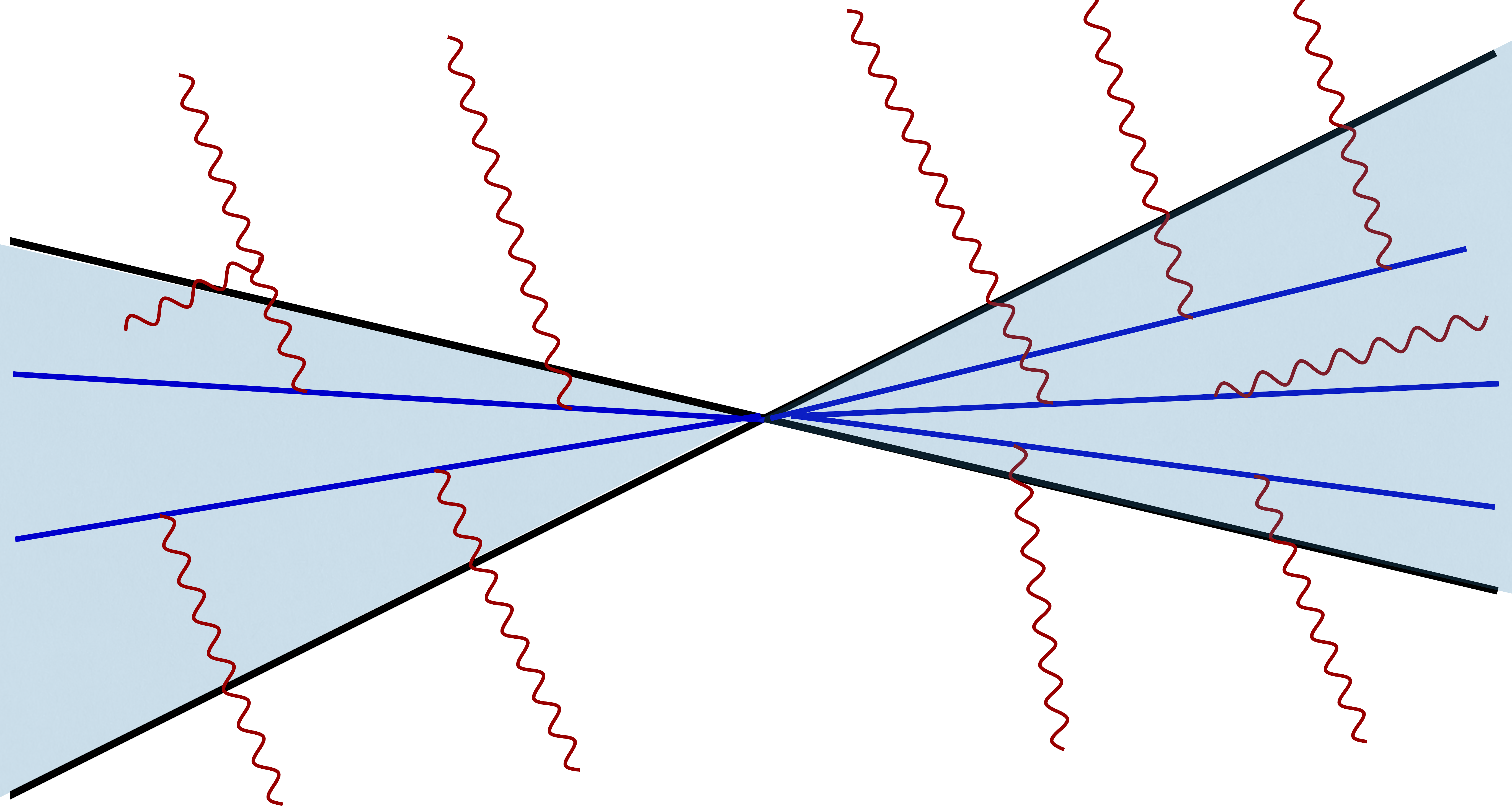
``Globalization"

Alternative SCET approach to observables with NGLs based on resummation for substructure. Larkoski, Moult, Neill '15

- Divide jet cross section into contributions from *n* sub-jets. Idea is to lower the hard scale in the NGLs by resolving the subjets.

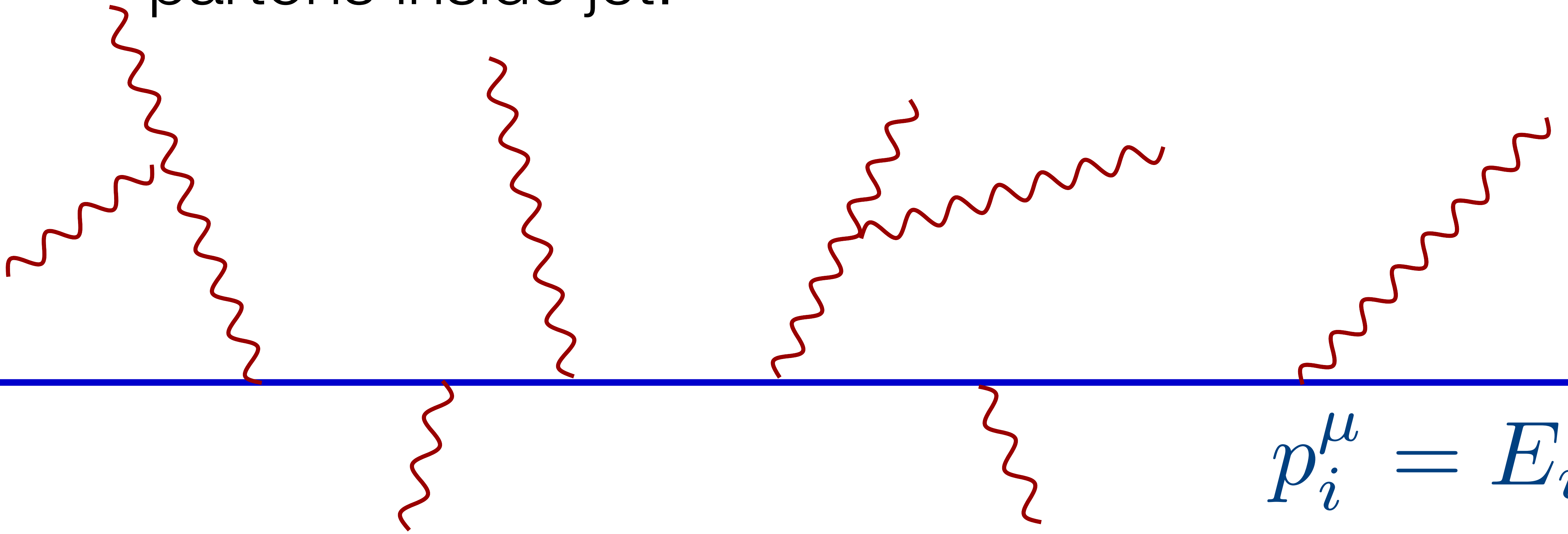


- Resum global logarithms in subjet observables: ``Dressed gluons".
- At leading-log level, this maps into iterative solution of BMS equation Larkoski, Moult, Neill '16



Factorization for NGLs

Basic physics is soft radiation off energetic partons inside jet.



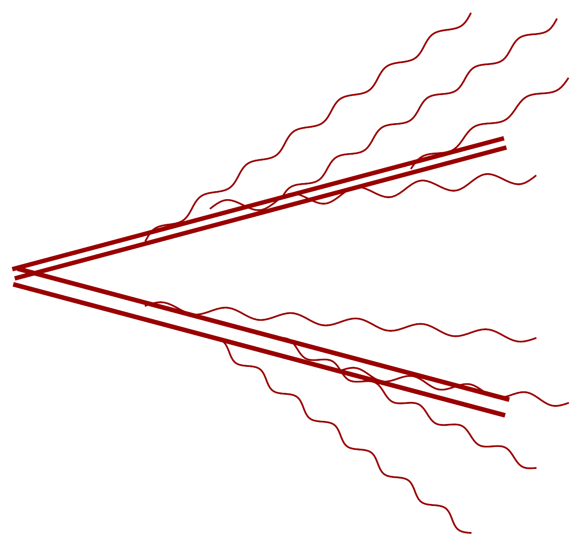
$$p_i^\mu = E_i n_i^\mu$$

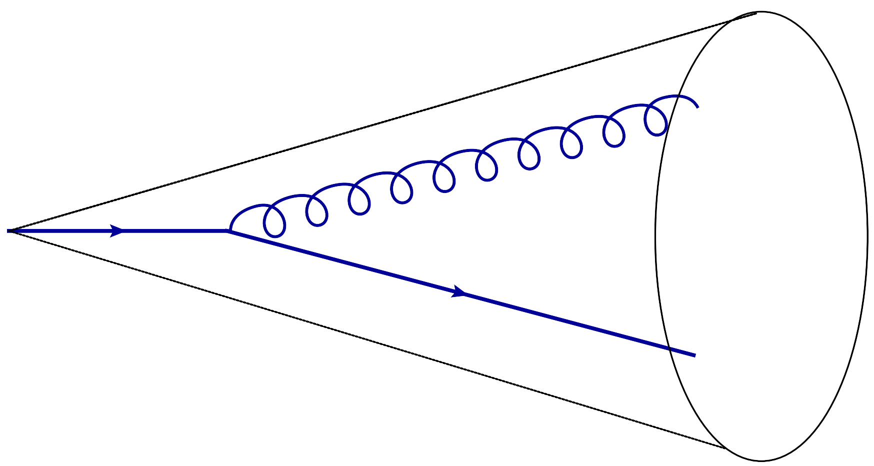
Wilson line along direction of each hard parton inside the jet.

$$\boldsymbol{S}_i(n_i) = \mathbf{P}\exp\left(ig_s \int_0^\infty ds\, n_i \cdot A_s^a(sn_i)\, \boldsymbol{T}_i^a\right)$$

Soft emissions in process with m energetic particles are obtained from the matrix elements of the operator

$$\boldsymbol{S}_1(n_1)\,\boldsymbol{S}_2(n_2)\,\ldots\,\boldsymbol{S}_m(n_m)|\mathcal{M}_m(\{\underline{p}\})\rangle$$





soft Wilson lines along the directions of the energetic particles / jets (color matrices)

soft particles can be inside or outside

hard scattering amplitude with *m* particles (vector in color space)

energetic partons must be inside

For a jet of several (nearly) collinear energetic particles, one can combine

$$\boldsymbol{S}_1(n)\,\boldsymbol{S}_2(n) = \mathbf{P}\exp\left(ig_s\int_0^\infty ds\, n\cdot A_s^a(sn)\,(\boldsymbol{T}_1^a+\boldsymbol{T}_2^a)\right)$$

into a single Wilson line with the total color charge.

For non-global observables one cannot combine the soft Wilson lines → complicated structure of logs!

- For a wide-angle jet, the energetic particles are not collinear.

- For a narrow-angle jets (see later), we find that small-angle soft radiation plays an important role. Resolves directions of individual energetic partons!

Factorization theorem

TB, Neubert, Rothen, Shao ’15 ’16, see also Caron-Huot ‘15

Hard function.
m hard partons along
fixed directions {n₁, …, nₘ}

Soft function
with *m* Wilson lines

$$\sigma(\beta) = \sum_{m=2}^{\infty} \big\langle \boldsymbol{\mathcal{H}}_m(\{\underline{n}\}, Q, \mu) \otimes \boldsymbol{\mathcal{S}}_m(\{\underline{n}\}, Q\beta, \mu) \big\rangle \, ;$$

color trace

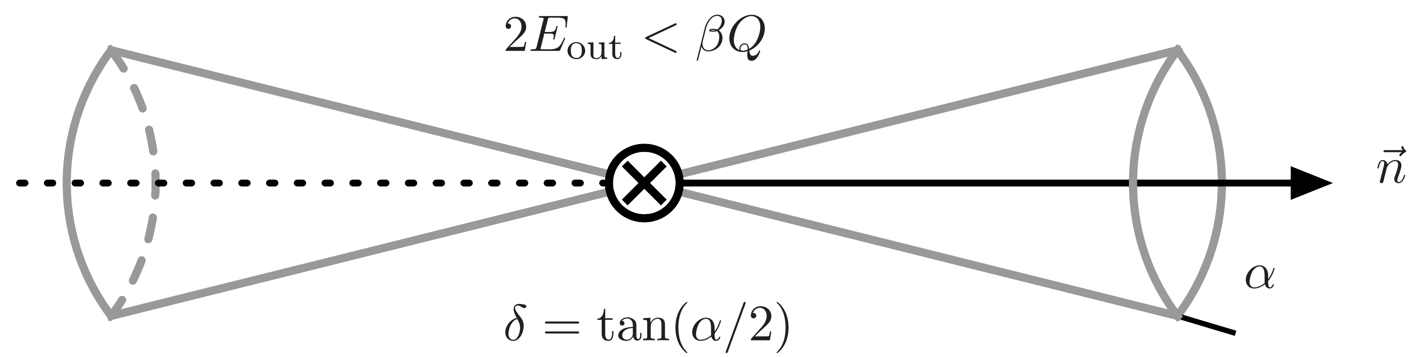
integration over the *m* directions

First all-order factorization theorem for non-global observable. Achieves full scale separation!

Comments

- Infinitely many operators $\mathcal{S}_m$, mix under RG
- Also for narrow-cone jets, the same type of structure is relevant TB, Neubert, Rothen, Shao ’15 ’16
- **Check:** Have computed all ingredients for cone cross section at NNLO. Obtain full logarithmic structure at this order.

$$B(\beta,\delta) = \frac{1}{4}\langle \boldsymbol{\mathcal{H}}_2^{(2)} \otimes \mathbf{1} + \boldsymbol{\mathcal{H}}_2^{(0)} \otimes \boldsymbol{\mathcal{S}}_2^{(2)} + \boldsymbol{\mathcal{H}}_2^{(1)} \otimes \boldsymbol{\mathcal{S}}_2^{(1)} + \boldsymbol{\mathcal{H}}_3^{(1)} \otimes \boldsymbol{\mathcal{S}}_3^{(1)} + \boldsymbol{\mathcal{H}}_3^{(2)} \otimes \mathbf{1} + \boldsymbol{\mathcal{H}}_4^{(2)} \otimes \mathbf{1}\rangle$$



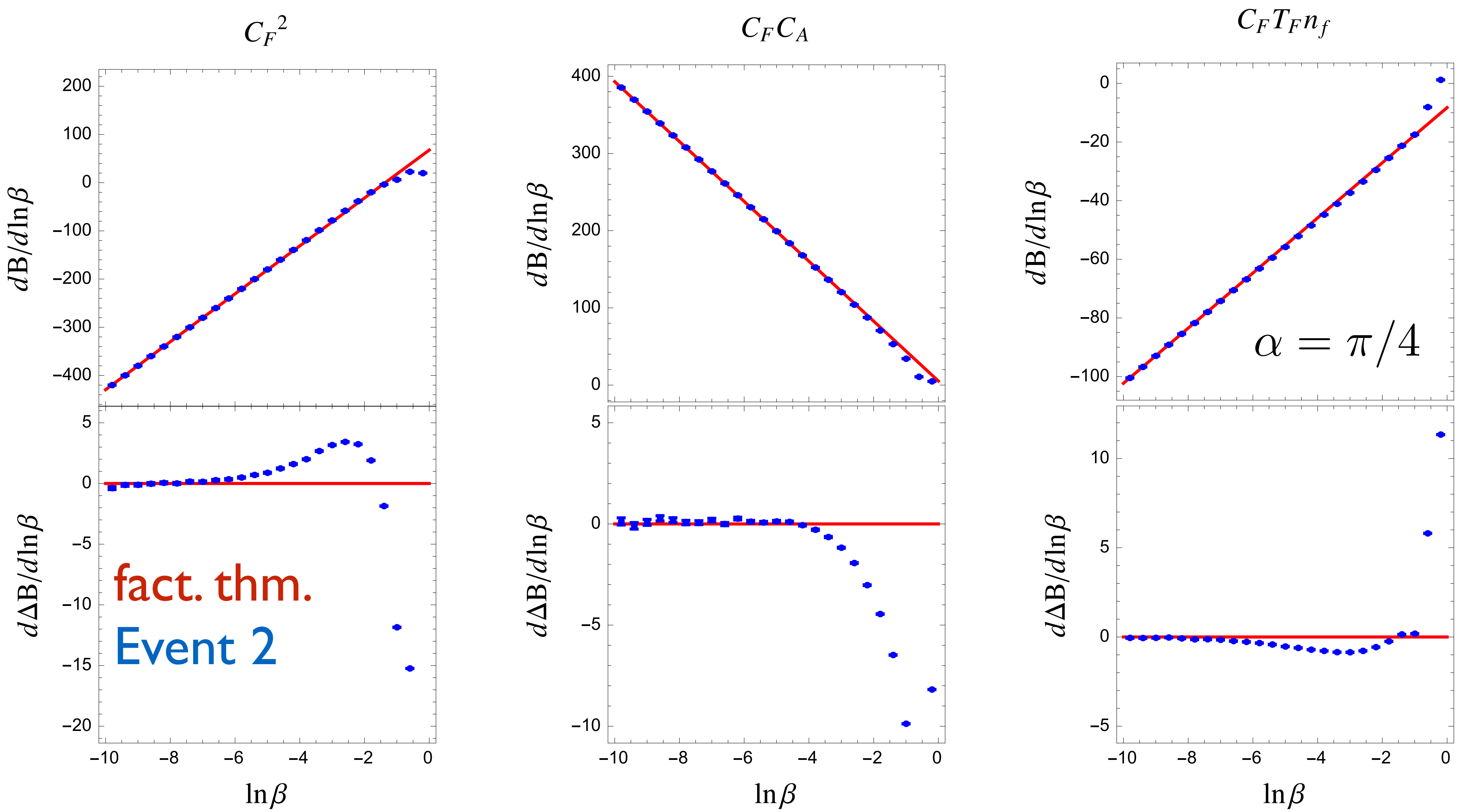
$$\frac{\sigma(\beta,\delta)}{\sigma_0} = 1 + \frac{\alpha_s}{2\pi}\, A(\beta,\delta) + \left(\frac{\alpha_s}{2\pi}\right)^2 B(\beta,\delta) + \dots .$$

$$A(\beta,\delta) = C_F\left[-8\ln\delta\ln\beta - 1 + 6\ln 2 - 6\ln\delta - 6\delta^2 + \left(\frac{9}{2} - 6\ln 2\right)\delta^4 + 4\,\mathrm{Li}_2(\delta^2) - 4\,\mathrm{Li}_2(-\delta^2)\right]$$

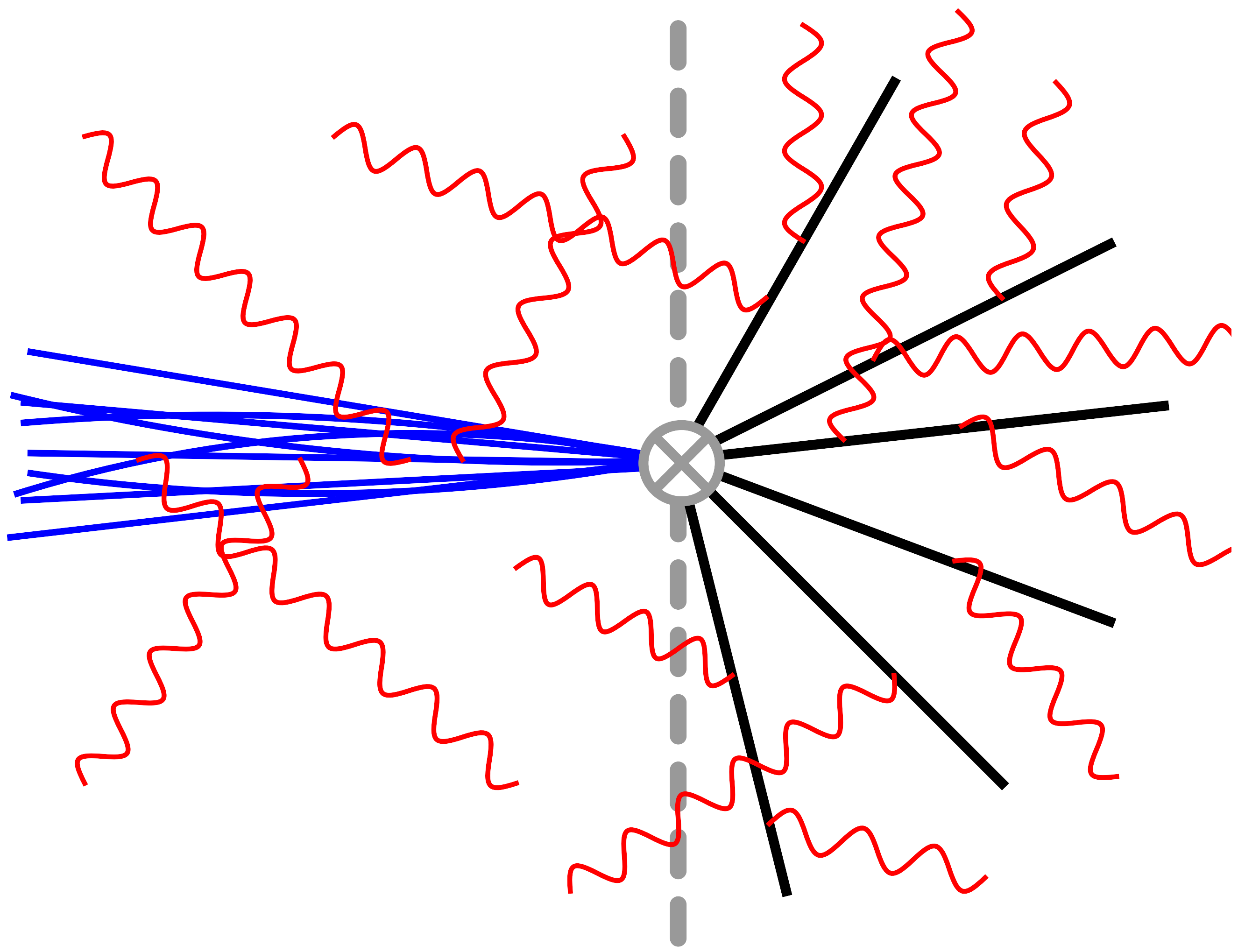
$$B(\beta,\delta) = C_F^2 B_F + C_F C_A B_A + C_F T_F n_f B_f$$

$$\begin{aligned}
B_A = {} & \frac{4}{3}\left[11\ln\delta - \frac{\pi^2}{2} + 3\,\mathrm{Li}_2(\delta^4)\right]\ln^2\beta + \frac{4}{3}\Bigg[11\ln^2\delta - \frac{67\ln\delta}{3} + \frac{4\delta^4\ln\delta}{(1-\delta^4)^2} + \frac{1}{1-\delta^4} \\
& + 36\ln\delta\ln^2\left(1-\delta^2\right) - 12\ln\delta\ln^2\left(1+\delta^2\right) + 22\ln\delta\ln\left(1-\delta^2\right) - 5\pi^2\ln\left(1-\delta^2\right) \\
& + 22\ln\delta\ln\left(1+\delta^2\right) - \pi^2\ln\left(1+\delta^2\right) - 4\ln^3\left(1+\delta^2\right) + 33\,\mathrm{Li}_2\left(-\delta^2\right) + 22\,\mathrm{Li}_2\left(\delta^2\right) \\
& + 48\ln\delta\,\mathrm{Li}_2\left(-\delta^2\right) - 12\ln\left(1-\delta^2\right)\mathrm{Li}_2\left(-\delta^2\right) - 36\ln\left(1+\delta^2\right)\mathrm{Li}_2\left(-\delta^2\right) \\
& + 12\ln 2\,\mathrm{Li}_2\left(-\delta^2\right) + 24\ln\delta\,\mathrm{Li}_2\left(\delta^2\right) + 24\ln\left(1-\delta^2\right)\mathrm{Li}_2\left(\delta^2\right) + 12\ln 2\,\mathrm{Li}_2\left(\delta^2\right) \\
& + 12\ln\left(1-\delta^4\right)\mathrm{Li}_2\left(1-\delta^2\right) - 6\,\mathrm{Li}_3\left(1-\delta^4\right) + 24\,\mathrm{Li}_3\left(1-\delta^2\right) - 36\,\mathrm{Li}_3\left(-\delta^2\right) \\
& - 36\,\mathrm{Li}_3\left(\delta^2\right) + 24\,\mathrm{Li}_3\left(\frac{\delta^2}{1+\delta^2}\right) - 12\,\zeta_3 - \frac{11\pi^2}{12} - \frac{1}{2} - \pi^2\ln 2 - \frac{3}{8}M_A^{[1]}(\delta)\Bigg]\ln\beta \\
& + c_2^A(\delta)\,,
\end{aligned}$$

Numerical check against **Event2**



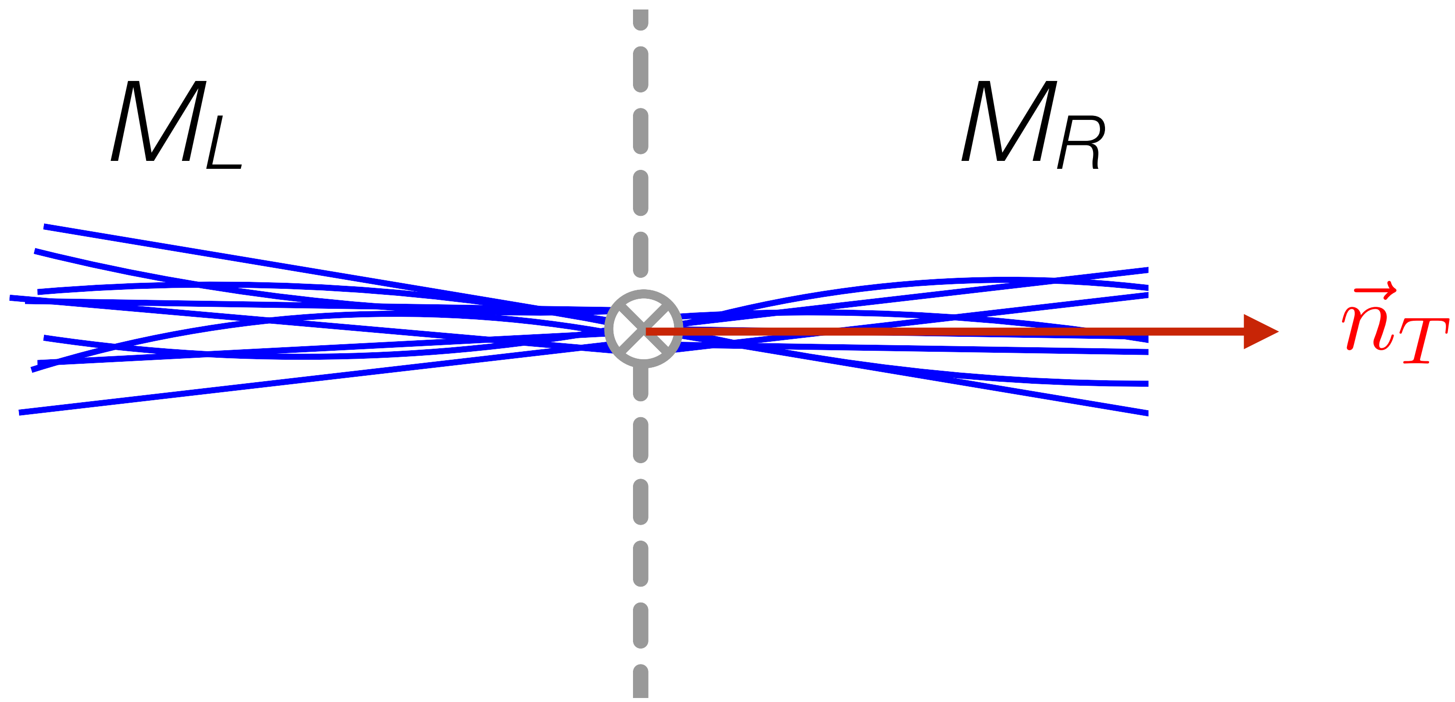
- Works: agreement for small β.
- **Reproduce all logs**, not only the leading ones!



Light-jet mass & hemisphere soft function

TB, Ben Pecjak and Dingyu Shao 1610.01608

Hemisphere mass observables



Thrust axis splits e^+e^- events in two hemispheres with masses M_L, M_R

$$\text{heavy jet mass:}\ \ \rho_h = \frac{1}{Q^2}\max(M_L^2, M_R^2)$$

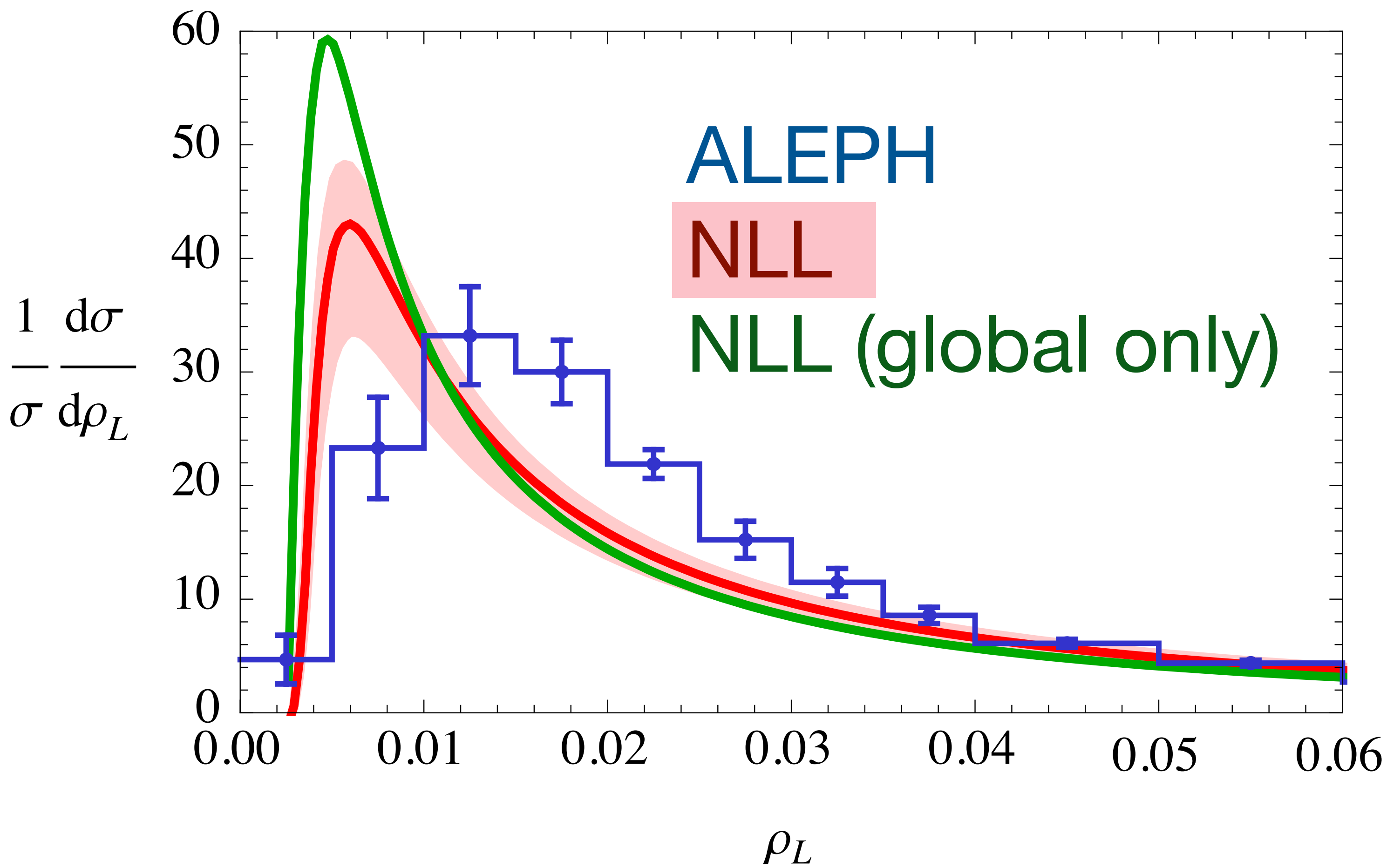
$$\text{light jet mass:}\ \ \rho_\ell = \frac{1}{Q^2}\min(M_L^2, M_R^2)$$

Light-jet versus heavy-jet mass

- Heavy jet mass ρ_h is a global observable, resummed to N^{3}LL Chien Schwartz '10, light-jet mass ρ_l is non-global.
- Burby and Glover '01 computed ρ_l at NLL in coherent branching formalism. Dasgupta and Salam '02 discover additional non-global logarithms
- Can analyze left-jet mass ρ_L instead of light jet mass. Relation

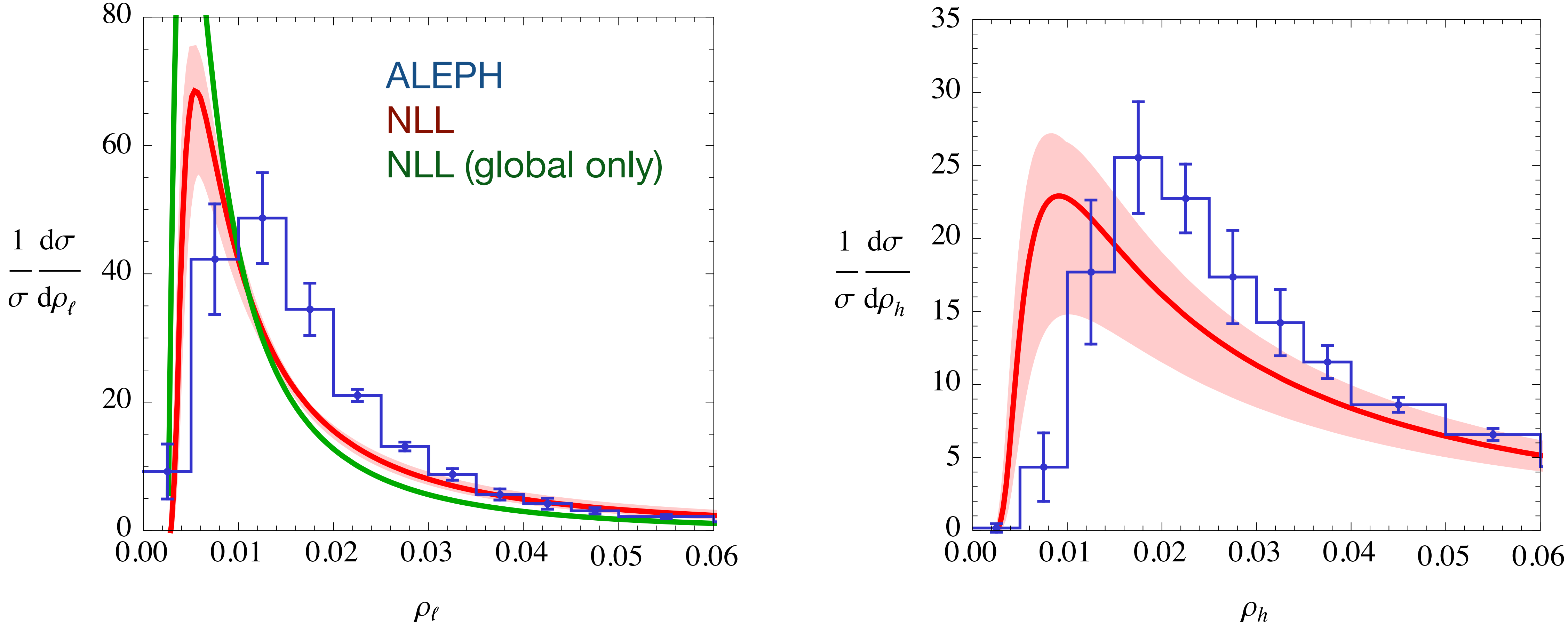
$$\frac{d\sigma}{d\rho_\ell} = 2\frac{d\sigma}{d\rho_L} - \frac{d\sigma}{d\rho_h}\bigg|_{\rho_L=\rho_h=\rho_\ell}$$

- Left-jet mass M_L is manifestly non-global.



$$R(\rho_L) = \int_0^{\rho_L} d\rho'_L \frac{1}{\sigma}\frac{d\sigma}{d\rho'_L} = S_{\rm NG}(\mu_s, \mu_h)\Sigma_q(\rho_L)$$

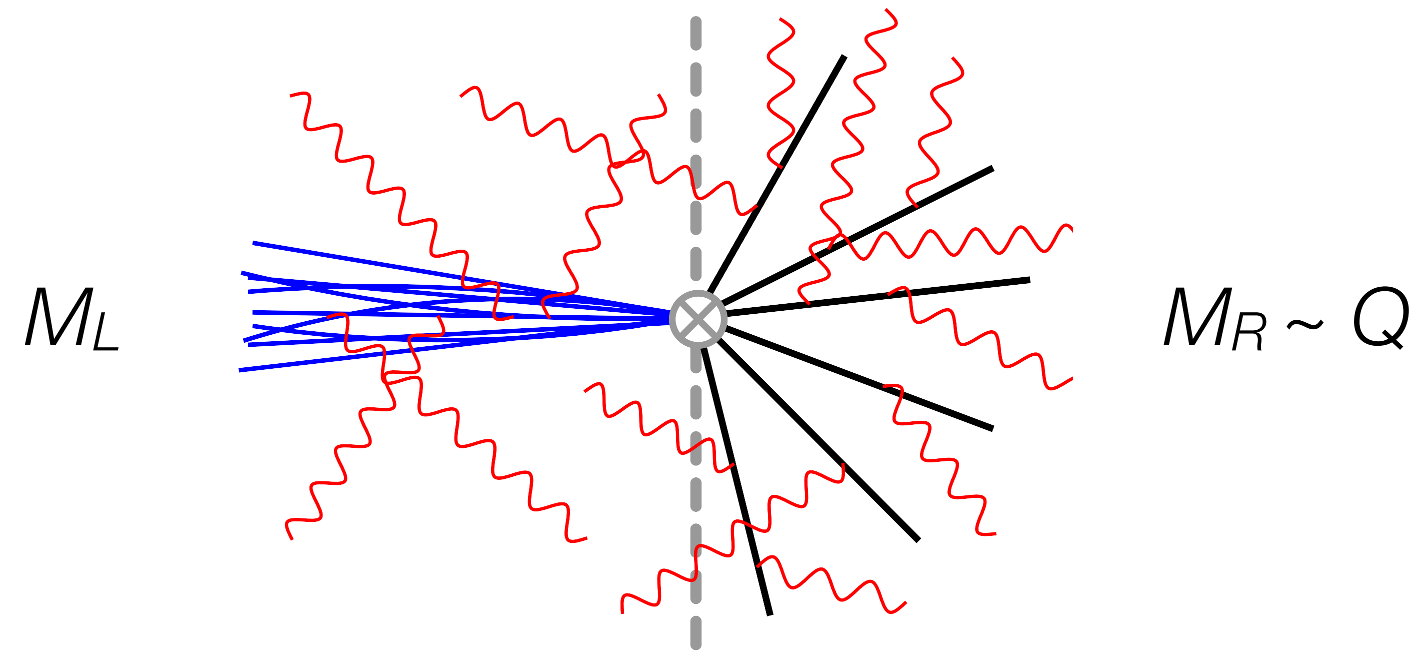
- $S_{\rm NG}$ includes leading nonglobal logs. Taken from MC parameterisation of Dasgupta Salam '02.
- Many SCET papers resum NG observables to NNLL *up to NGLs*. Byers beware…



- Heavy-jet mass $R(\rho_h) = [\Sigma_q(\rho_h)]^2$
- Non-perturbative corrections are important in the peak region and will shift the peak to the right.

Factorization theorem for left-jet mass

TB, Ben Pecjak and Dingyu Shao 1610.01608



$$\frac{d\sigma}{dM_L^2} = \sum_{i=q,\bar{q},g} \int_0^\infty d\omega_L \, J_i(M_L^2 - Q\,\omega_L) \sum_{m=1}^{\infty} \langle \boldsymbol{\mathcal{H}}_m^i(\{\underline{n}\}, Q) \otimes \boldsymbol{\mathcal{S}}_m(\{\underline{n}\}, \omega_L) \rangle$$

- Hard function $\mathcal{H}_m$ has m partons on the right hemisphere and a single parton in the left one (which then branches into a jet J_i).
- Soft functions $\mathcal{S}_m$ are exactly the same as in the narrow jet case (later).

Hemisphere soft function

- Many previous studies of NGLs were performed for hemisphere soft function

$$S(\omega_L,\omega_R) = \frac{1}{N_c}\sum_X \mathrm{Tr}\langle 0|S(\bar{n})S^\dagger(n)|X\rangle\langle X|S(n)S^\dagger(\bar{n})|0\rangle\delta(\omega_R - n\cdot P_R)\,\delta(\omega_L - \bar{n}\cdot P_L)$$

  - Leading logs are related to the ones arising in light-jet mass event shape

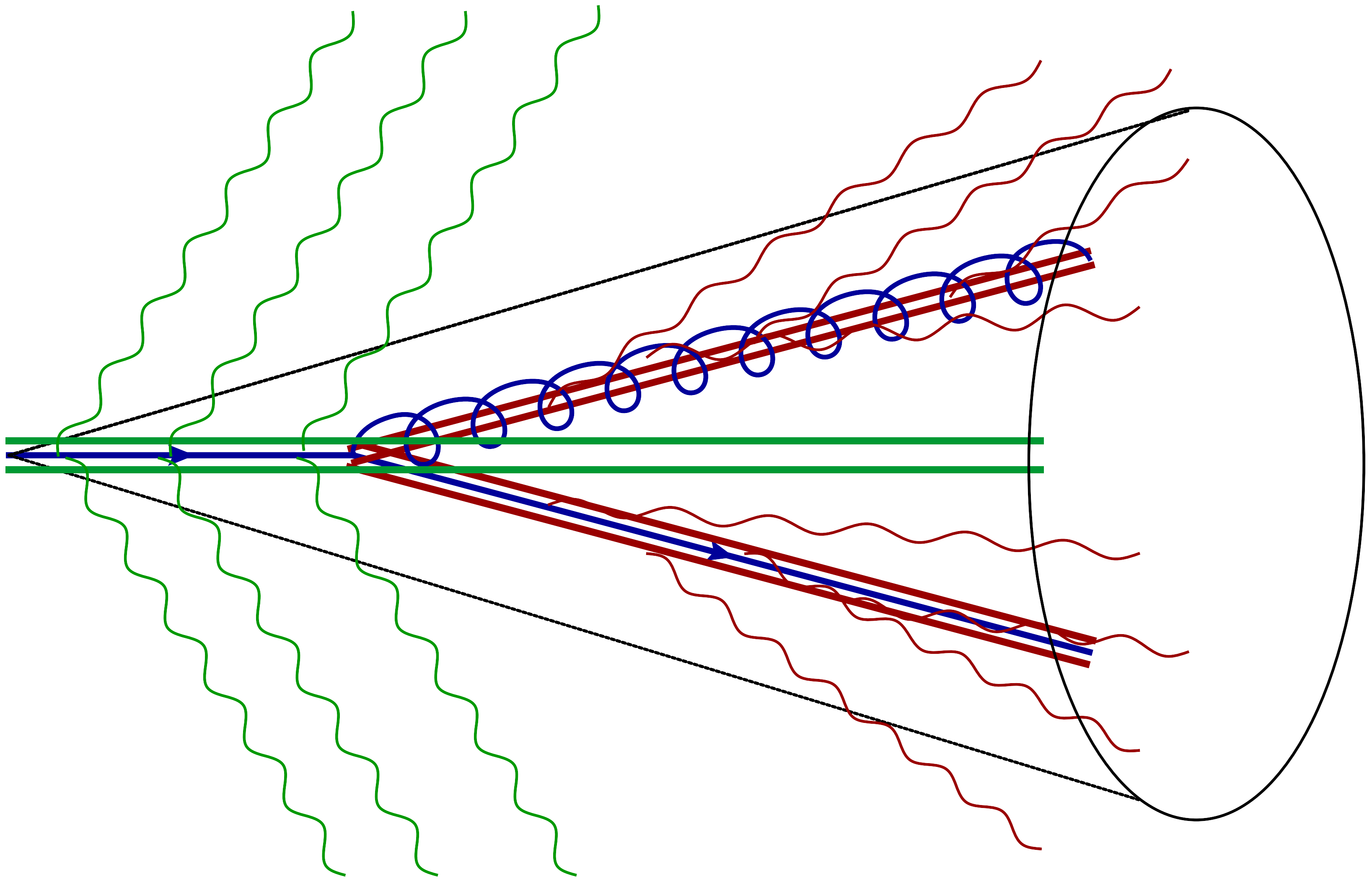
- Factorization formula for $\omega_L \ll \omega_R$

$$S(\omega_L,\omega_R) = \sum_{m=0}^{\infty}\big\langle \boldsymbol{\mathcal{H}}_m^S(\{\underline{n}\},\omega_R)\otimes\boldsymbol{\mathcal{S}}_{m+1}(\{n,\underline{n}\},\omega_L)\big\rangle$$

mode with $p_\mu \sim \omega_R$

mode with $p_\mu \sim \omega_L$

- Structure of theorems is similar in all cases
  - Characteristic feature: Wilson lines along directions of all energetic partons!
- Have computed both observables to NNLO
- Checks
  - Numerical for light-jet mass, using Event2
  - Analytical check for $S(\omega_L,\omega_R)$ using Kelley, Schwartz, Schabinger, Zhu ’11; Hornig, Lee, Stewart, Walsh, Zuberi ’11



Narrow-cone jets

Soft emissions from a narrow jet

For a narrow jet $\delta \to \mathbf{0}$ in direction *n* one would expect that one could combine

$$\boldsymbol{S}_1(n_1)\,\boldsymbol{S}_2(n_2) \approx \mathbf{P}\exp\left(ig_s\int_0^\infty ds\,n\cdot A_s^a(sn)\,(\boldsymbol{T}_1^a+\boldsymbol{T}_2^a)\right)$$

since $n_1 \approx n_2 \approx n$.

Doing so, one ends up with a single Wilson line per jet and a simple form of the soft radiation.

- Works for global observables such as thrust, broadening, …

Soft emissions from a narrow jet

Consider the emission of single soft a gluon from energetic particles with momenta p_i inside a narrow jet:

$$\sum_i Q_i \frac{p_i \cdot \varepsilon}{p_i \cdot k} = Q_{\text{tot}} \frac{n \cdot \varepsilon}{n \cdot k} + \ldots$$

Approximation: $p_i^\mu \approx E_i \, n^\mu$

This approximation breaks down when the soft emission has a small angle, i.e. when $k^\mu \approx \omega \, n^\mu$!

Small region of phase space, but it turns out that it gives a leading contribution to jet rates!

Momentum modes for jet processes

TB, Neubert, Rothen, Shao,1508.06645; Chien, Hornig and Lee 1509.04287

	Region	Energy	Angle	Inv. Mass
standard SCET	Hard	Q	1	Q
	Collinear	Q	δ	$Q\delta$
	Soft	βQ	1	βQ
new	Coft	βQ	δ	$\beta\delta Q$

Full jet cross section is recovered after adding the contributions from all regions ("method of regions")

- Additional coft mode has very low characteristic scale $\beta\delta Q$! Jets are less perturbative than they seem!
- Effective field theory has additional "coft" degree of freedom.

Factorization for two-jet cross section

TB, Neubert, Rothen, Shao ‘16

Laplace space
$\tau \leftrightarrow \beta$

color trace

integration over angles

$$\widetilde{\sigma}(\tau) = \sigma_0\, H(Q)\, \widetilde{S}(Q\tau) \left[\sum_{m=1}^{\infty} \left\langle \mathcal{J}_m(Q\delta) \otimes \widetilde{\mathcal{U}}_m(Q\delta\tau) \right\rangle \right]^2$$

Coft functions with
m Wilson lines

Soft function

Hard function

Jet functions with *m* partons
at fixed direction

Checks against wide-angle result and fixed-order event generator.

All-order resummation

$$\sigma(\beta) = \sum_{m=2}^{\infty} \big\langle \boldsymbol{\mathcal{H}}_m(\{\underline{n}\}, Q, \mu) \otimes \boldsymbol{\mathcal{S}}_m(\{\underline{n}\}, Q\beta, \mu) \big\rangle \, ;$$

High-*E* physics
Wilson coefficients

Low-*E* physics
EFT Operator

- Renormalization of hard Wilson coefficients

$$\boldsymbol{\mathcal{H}}_m(\{\underline{n}\}, Q, \delta, \epsilon) = \sum_{l=2}^{m} \boldsymbol{\mathcal{H}}_l(\{\underline{n}\}, Q, \delta, \mu) \, \boldsymbol{Z}^H_{lm}(\{\underline{n}\}, Q, \delta, \epsilon, \mu)$$

  - Same *Z*-factor must render $\mathcal{S}_\mathrm{m}$ finite!
  - Associated anomalous dimension $\boldsymbol{\Gamma}^H$

$$\frac{d}{d\ln\mu} \boldsymbol{Z}^H_{km}(\{\underline{n}\}, Q, \delta, \epsilon, \mu) = \sum_{l=k}^{m} \boldsymbol{Z}^H_{kl}(\{\underline{n}\}, Q, \delta, \epsilon, \mu) \, \hat{\otimes} \, \boldsymbol{\Gamma}^H_{lm}(\{\underline{n}\}, Q, \delta, \mu)$$

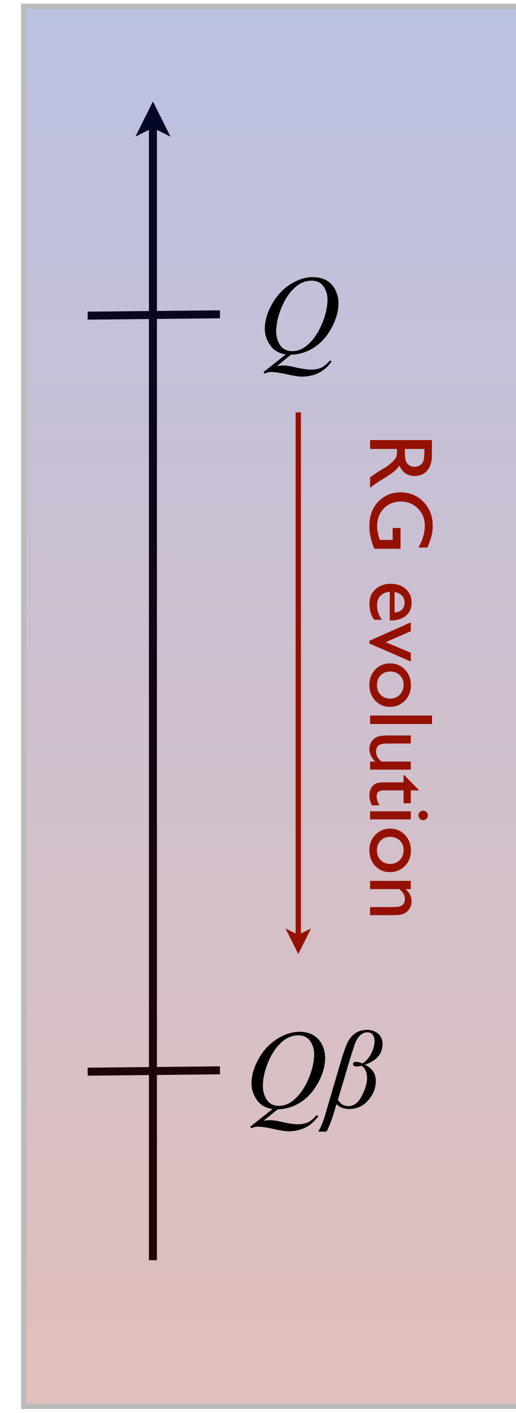
Resummation by RG evolution

Wilson coefficients fulfill renormalization group (RG) equations

$$\frac{d}{d\ln\mu}\boldsymbol{\mathcal{H}}_m(Q,\mu) = -\sum_{l=2}^{m}\boldsymbol{\mathcal{H}}_l(Q,\mu)\,\boldsymbol{\Gamma}^H_{lm}(Q,\mu)$$

1. Compute $\mathcal{H}_m$ at a characteristic high scale $\mu_h \sim Q$
2. Evolve $\mathcal{H}_m$ to the scale of low energy physics $\mu_l \sim Q\beta$

Avoids large logarithms $\alpha_s{}^n \ln^n(\beta)$ of scale ratios which can spoil convergence of perturbation theory.



RG = Parton Shower

- Ingredients for LL

$$\boldsymbol{\mathcal{H}}_2(\mu = Q) = \sigma_0$$
$$\boldsymbol{\mathcal{H}}_m(\mu = Q) = 0 \text{ for } m > 2$$
$$\boldsymbol{\mathcal{S}}_m(\mu = \beta Q) = 1$$

$$\boldsymbol{\Gamma}^{(1)} = \begin{pmatrix} \boldsymbol{V}_2 & \boldsymbol{R}_2 & 0 & 0 & \dots \\ 0 & \boldsymbol{V}_3 & \boldsymbol{R}_3 & 0 & \dots \\ 0 & 0 & \boldsymbol{V}_4 & \boldsymbol{R}_4 & \dots \\ 0 & 0 & 0 & \boldsymbol{V}_5 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

- RG

$$\frac{d}{dt}\boldsymbol{\mathcal{H}}_m(t) = \boldsymbol{\mathcal{H}}_m(t)\boldsymbol{V}_m + \boldsymbol{\mathcal{H}}_{m-1}(t)\boldsymbol{R}_{m-1} \,.$$

$$t = \int_{\alpha(\mu)}^{\alpha(Q)} \frac{d\alpha}{\beta(\alpha)}\,\frac{\alpha}{4\pi}$$

- Equivalent to parton shower equation

$$\boldsymbol{\mathcal{H}}_m(t) = \boldsymbol{\mathcal{H}}_m(t_1)e^{(t-t_1)V_n} + \int_{t_1}^{t} dt'\boldsymbol{\mathcal{H}}_{m-1}(t')\boldsymbol{R}_{m-1}e^{(t-t')\boldsymbol{V}_n}$$

1-loop anomalous dimension

$$\boldsymbol{V}_m = \boldsymbol{\Gamma}^{(1)}_{m,m} = -2\sum_{(ij)} (\boldsymbol{T}_{i,L}\cdot\boldsymbol{T}_{j,L} + \boldsymbol{T}_{i,R}\cdot\boldsymbol{T}_{j,R}) \int \frac{d\Omega(n_k)}{4\pi} W_{ij}^k,$$

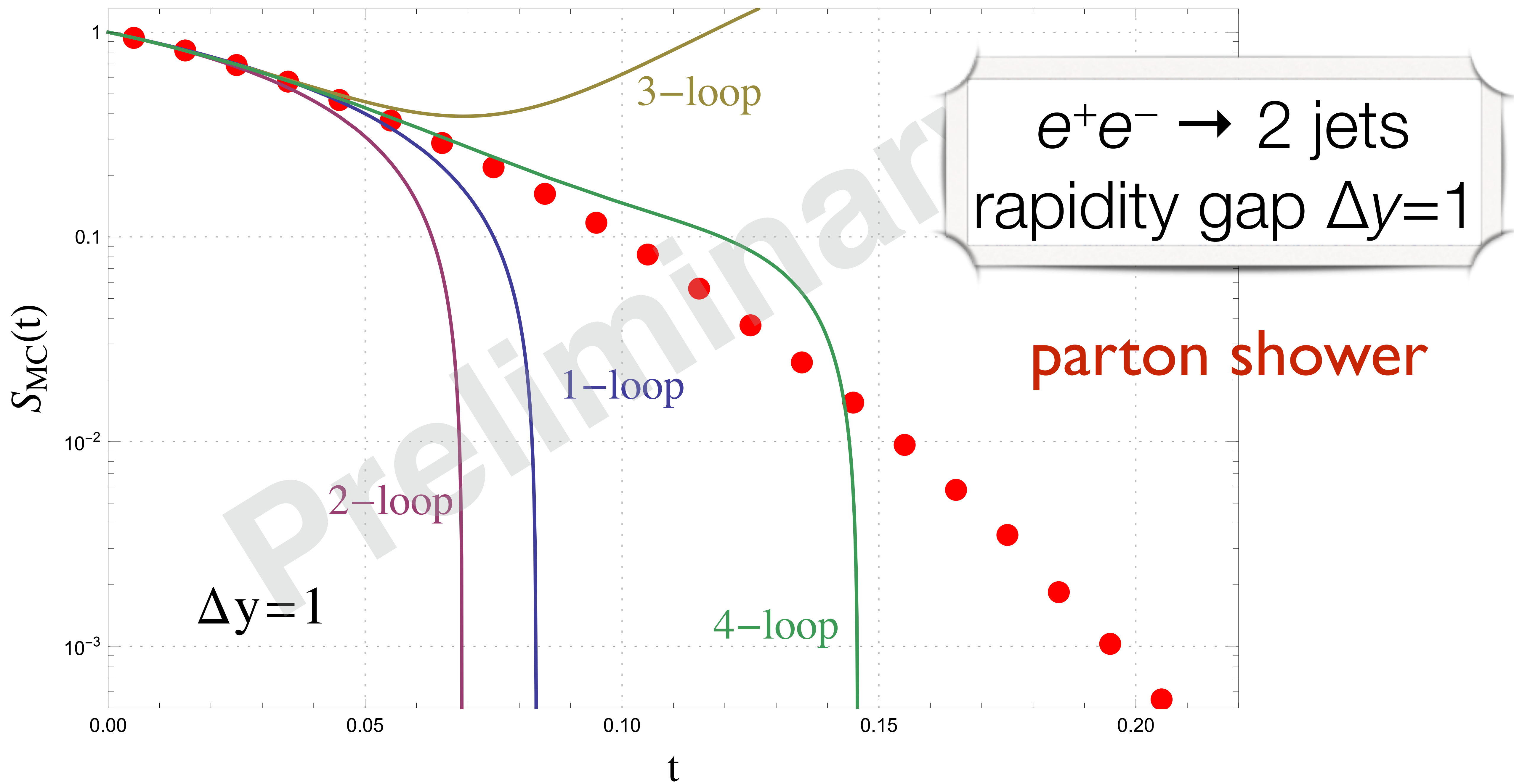
$$\boldsymbol{R}_m = \boldsymbol{\Gamma}^{(1)}_{m,m+1} = 4\sum_{(ij)} \boldsymbol{T}_{i,L}\cdot\boldsymbol{T}_{j,R}\, W_{ij}^{m+1}\, \Theta_{\text{in}}^{n\bar{n}}(n_{m+1})\,.$$

- Contain dipoles → dipole shower

$$W_{ij}^k = \frac{n_i\cdot n_j}{n_i\cdot n_k\, n_j\cdot n_k}$$

- Trivial color structure at large N_c :

$$\boldsymbol{T}_i\cdot\boldsymbol{T}_j \to -\frac{N_c}{2}\,\delta_{j,i\pm 1}$$



- Equivalent to the dipole shower used by Dasgupta and Salam ’02.

NGL parton shower

- Not a general-purpose parton shower! Produces leading NGLs in this type of observables. Nothing more, nothing less.
- Since it derives from RG, we know exactly what needs to be added to go to NLL

1. Hard functions $\mathcal{H}_2^{(1)}$ and $\mathcal{H}_3^{(1)}$
2. One-loop soft functions $\mathcal{S}_m^{(1)}$
3. Two-loop anomalous dimension $\boldsymbol{\Gamma}_{\mathrm{nm}}^{(2)}$. Can be extracted from Caron-Huot '15.

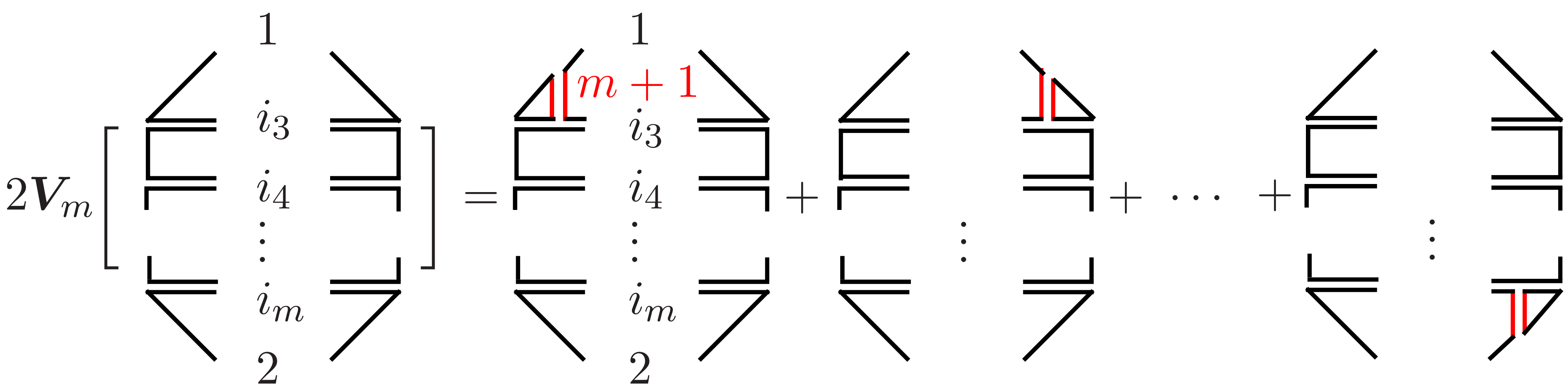
- Need to Monte-Carlo NLL corrections to be efficient.

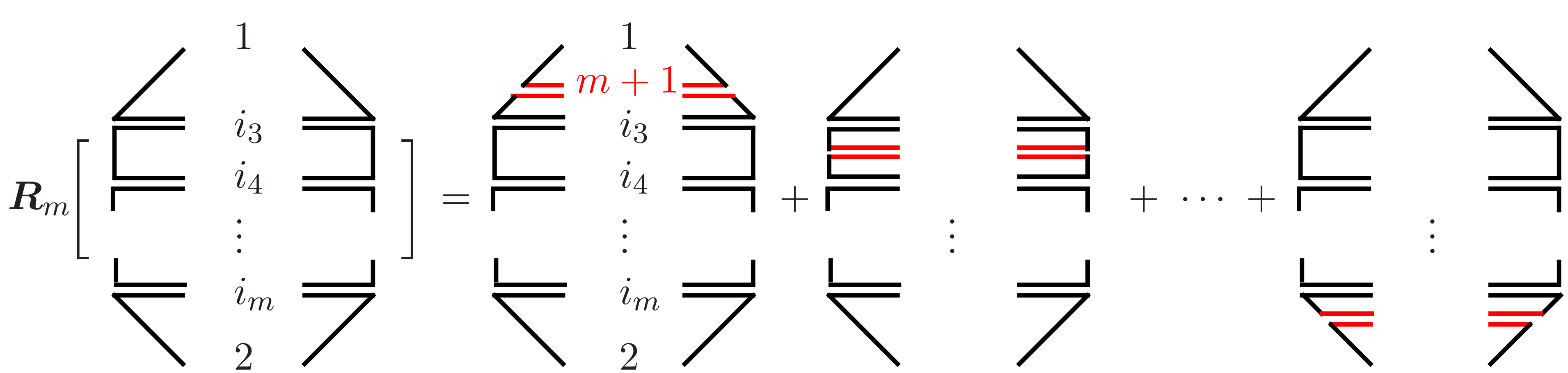
Summary and Outlook

- Resummation = factorization + fixed order + evolution.
- Have obtained factorization formulas for non-global observables
  - wide and narrow cone jets, light-jet mass, hemisphere soft function
  - computed *all* logs at NNLO, verified against fixed order
- Key features
  - Multi-Wilson line structure of soft radiation
  - Resummation of NGLs from RG evolution
- Are developing MC formalism for higher-log resummation
- Applications …
- Interplay with Glauber gluons? Superleading logs?

Extra slides

Action of Γ on H_n





Momentum modes again (for experts)

Split momenta into light-cone components

$$p^\mu = p_+ \frac{n^\mu}{2} + p_- \frac{\bar{n}^\mu}{2} + p_\perp^\mu$$

Scaling of the momentum components ($\beta \sim \delta^2$)

$$\begin{array}{rl} & \quad\ \ (\ p_+\ ,\ p_-\ ,\ p_\perp\ ) \\ \text{collinear:} & p_c \sim Q\,(\ 1\ ,\ \delta^2\ ,\ \delta\ ) \\ \text{soft:} & p_s \sim Q\,(\ \beta\ ,\ \beta\ ,\ \beta\ ) \\ \text{coft:} & p_t \sim \beta Q\,(\ 1\ ,\ \delta^2\ ,\ \delta\ ) \end{array}$$

Note: every component of coft mode is smaller than the corresponding collinear one. Different than SCET_I , SCET_II, $\text{SCET}_{1.5}$, SCET_n , SCET_+, …

Method of regions expansion

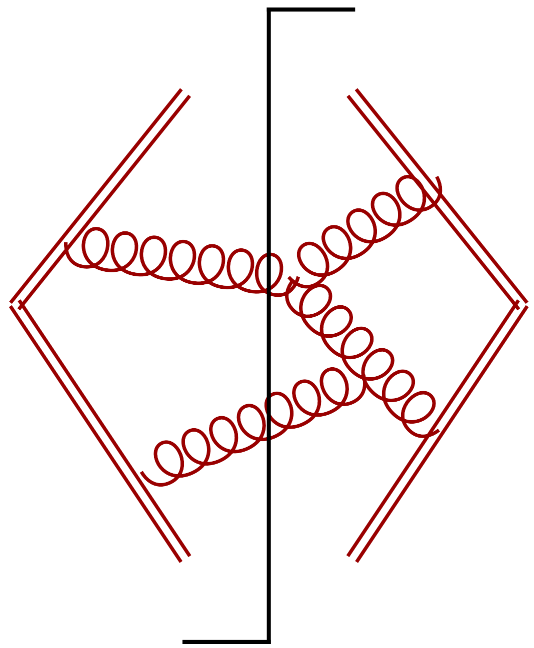
To isolate the different contributions, one expands the amplitudes as well as the phase-space constraints in each momentum region.

- Generic soft mode has O(1) angle: after expansion, it is always outside the jet.
- Collinear mode has large energy $E \gg \beta Q$. Can never go outside the jet.
- Coft mode can be inside or outside, but its contribution to the momentum inside the jet is negligible.

Expansion is performed on the integrand level: the full result is obtained after combining the contributions from the different regions.

Computation of γ_r / d_3 / B_3

Easiest to extract coefficient from a three-loop soft function



Same matrix element as H production near threshold Anastasiou et al., Li et al. '14, but constraint on $p_{X,T}$ instead of E_X.

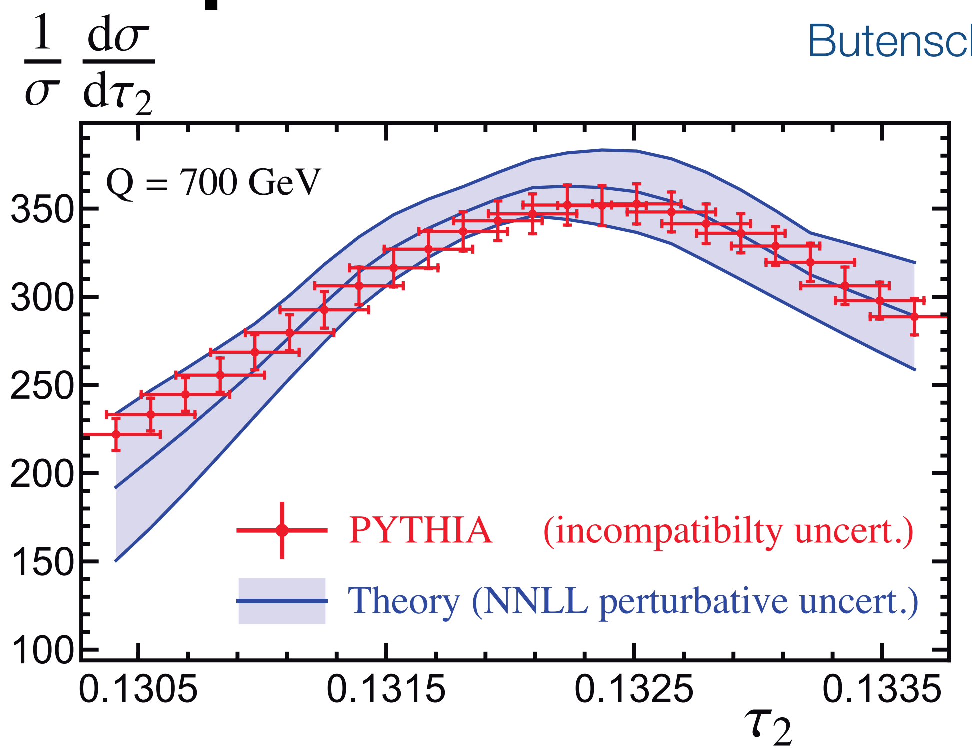
- $p_{X,T}$ function needs additional regulator

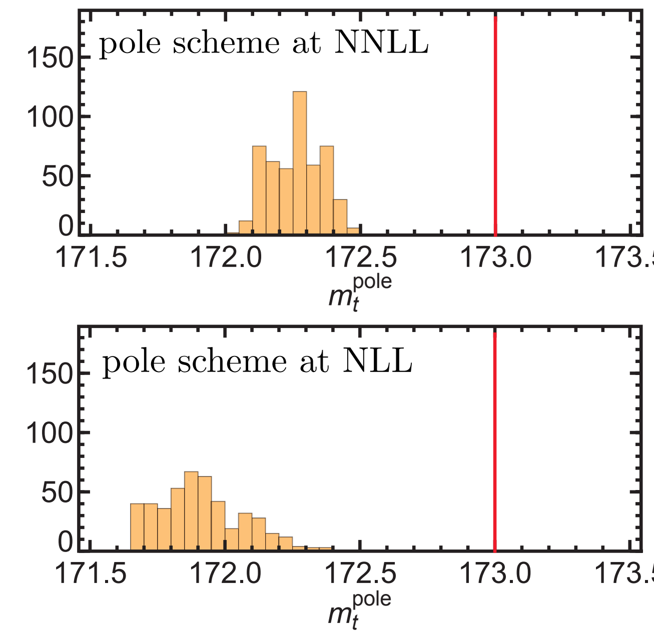
Computation of γ_r / d_3 / B_3

- Interesting to consider *double* differential soft function in E_X and $p_{X,T}$. Li, Neill, Zhu '16
  - E_X regularizes rapidity divergences
  - intriguing relations among threshold and q_T soft functions
- Li and Zhu '16 have computed three-loop double differential soft function
  - make general ansatz, fix coefficients using Taylor expansion

Top mass in Pythia?

Butenschoen, Dehnadi, Hoang, Mateu, Preisser and Stewart '16





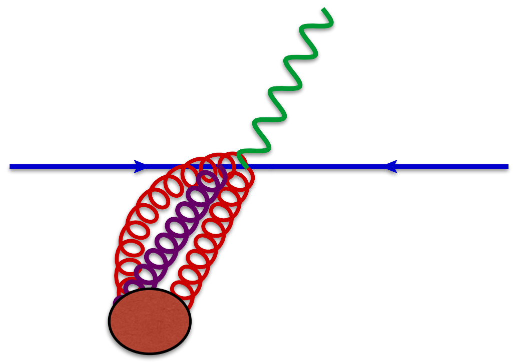
Butenschoen et al. compute NNLL resummed result for thrust event shape in $e^+e^- \to t\bar{t}$

- Exclusive observable, sensitive to m_t
- Compare to MC predictions at different Q and relate Pythia parameter m_t^{MC} to m_t^{pole}
- Universality? Initial state effects at hadron colliders?

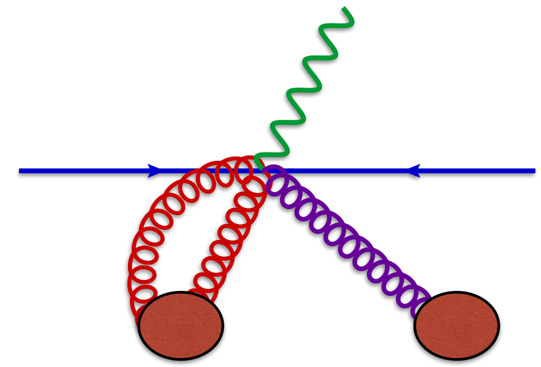
N-jettiness subtraction

Boughezal, Liu, Petreillo 15, Gaunt, Stahlhofen, Tackmann Walsh 15

q_T or τ_N



Two-loop graphs
Soft and collinear
emissions: factorization



NLO

Use event shape τ_N to separate out most singular region of NNLO computations

- Use SCET to compute σ in singular region
- Use existing NLO code away from end-point.

Extension of q_T subtraction Catani, Grazzini ’07 to processes with jets in the final state.

N-jettiness subtraction

- Advantage: can use existing NLO codes to obtain NNLO results
  - Already an impressive list of applications H, Z, W, W+j, H+j, Z+j, HZ, HW, $\gamma\gamma$, …
  - MCFM 8 includes NNLO for color neutral final states Boughezal, Campbell, Ellis, Focke, Giele, Liu, Petriello and Williams '16
- Challenge: independence of slicing parameter q_T or τ_N. Parameter needs to be small, but numerical problems if too small.

Jet radius logarithms

A lot of work during the past year

- Inclusive jet cross section Chien, Kang, Ringer and Vitev ’16; Idilbi, Kim ’16; Dai, Kim, Leibovich ‘16
  - based on jet fragmentation function
- Exclusive jet cross sections Chien, Hornig, Lee ’15, Kolodrubetz, Hornig, Makris and Mehen ’16 Pietrulewicz, Stewart, Tackmann and Waalewijn ‘16
  - non-global logarithms are not resummed