Precise theory predictions for a threshold scan of top pair production at a lepton collider

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based on work in collaboration with
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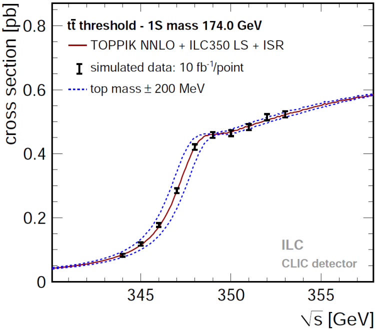
Introduction

- Threshold scan at future linear collider
  - Ultra-precise measurement of top-quark mass: $\delta m_t^{\overline{\text{MS}}} \sim \mathcal{O}(50\ \text{MeV})$
  - High sensitivity to top width and α_s
  - Possibility to measure top Yukawa coupling
- Technically very interesting computation



[Seidel, Simon, Tesar, Poss]

Introduction

- Near threshold tops are nonrelativistic with velocity $v \sim \alpha_s$
  - Multiple scales are relevant:

hard scale	m_t	mass
soft scale	$m_t v$	momentum
ultrasoft scale	$m_t v^2$	energy

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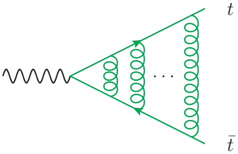
  - Conventional perturbation theory in α_s fails
  - Coulomb singularities $(\alpha_s/v)^n$ from n exchanges of potential gluons $(k^0, \mathbf{k}) \sim (m_t v^2, m_t v)$ have to be summed to all orders

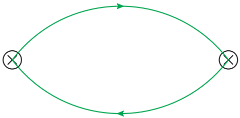


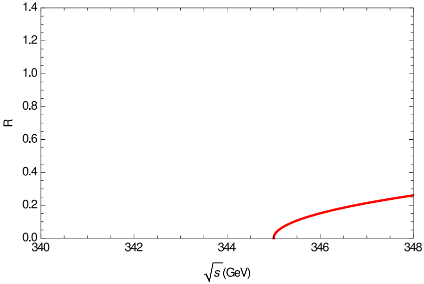
  - This resummation can be organized systematically using the effective theory potential non-relativistic QCD (PNRQCD) [Pineda, Soto; Beneke; Beneke, Signer, Smirnov; Brambilla, Pineda, Soto, Vairo; Beneke, Kiyo, Schuller]

Cross section in pure QCD

- Normalized cross section $R(s) = \frac{\sigma(e^+e^- \to t\bar{t}X)}{\sigma_0(e^+e^- \to \mu^+\mu^-)} = 12\pi e_t^2 f(s)\,\mathrm{Im}\left[\Pi^{(v)}(s)\right]$

Born cross section:





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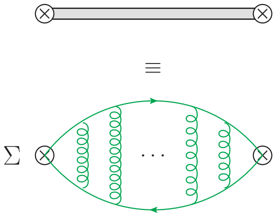
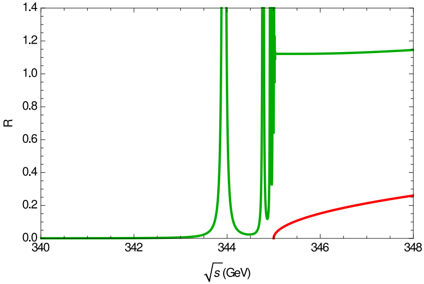
Resummed cross section at LO:



$\Gamma_t = 0$



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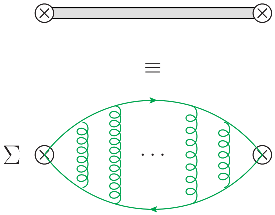
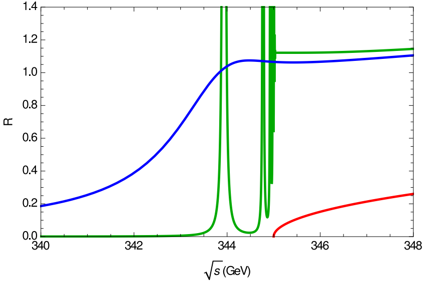


$\Gamma_t \neq 0$



Effective field theory setup

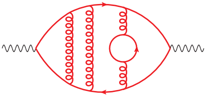
Higher-order corrections require systematic approach.

Exploit the hierarchy of scales to create an EFT:

QCD	Full theory
↓	Integrated out hard modes $k \sim m_t$ Hard subgraphs become point-like vertices
NRQCD	Contains nonrelativistic modes
↓	Integrated out soft modes $k \sim m_t v$ Soft subgraphs become non-local vertices
PNRQCD	Contains potential heavy quarks and ultrasoft gluons

NRQCD: Integrating out the hard scale

Diagram with hard lines connecting both external current vertices



Imaginary part vanishes, because there are no on-shell cuts

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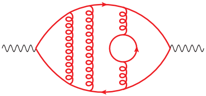

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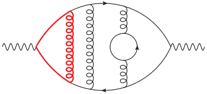
Diagram with hard lines connecting to only one external current vertices



Hard vertex subgraph gets contracted, hard matching correction to external current

$$j^{(v)k} = \bar{u}\gamma^k v = c_v \psi^\dagger \sigma^k \chi + \dots$$

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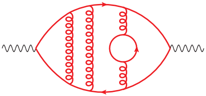

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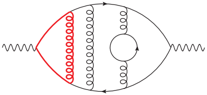

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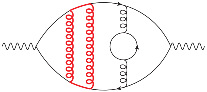
Diagram with hard lines connecting to no external current vertices



Hard subgraph gets contracted to a local four-quark operator in the NRQCD Lagrangian

PNRQCD: Integrating out the soft scale I

- Further matching to potential NRQCD yields a spatially nonlocal Lagrangian

$$\mathcal{L}_{\text{PNRQCD}} = \psi^\dagger \left(i\partial_0 + \frac{\partial^2}{2m_t} + \frac{\partial^4}{8m_t^3} + g_s A_0(t,\mathbf{0}) - g_s \mathbf{x}\cdot\mathbf{E}(t,0) \right) \psi + \left(\text{anti-quark}\right)$$
$$+ \int d^{d-1}\mathbf{r} \left[\psi_a^\dagger \psi_b\right](x+\mathbf{r})\, V_{ab;cd}(\mathbf{r}) \left[\chi_c^\dagger \chi_d\right](x)$$

  - Contains potential (anti)quark fields $\psi(\chi)$ with $p^0 \sim mv^2, \mathbf{p} \sim mv$ and heavy quark potentials $V_{ab;cd}$
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- The colour-singlet projection of the potential has the form

$$V(\mathbf{p},\mathbf{p}') = -\frac{4\pi\alpha_s C_F}{\mathbf{q}^2}\left[1 + \frac{\alpha_s}{4\pi}\mathcal{V}_C^{(1)} + \mathcal{O}(\alpha_s^2)\right] + \ldots,$$

$$\mathcal{V}_C^{(1)} = \left[\left(\frac{\mu^2}{\mathbf{q}^2}\right)^{\epsilon} - 1\right]\frac{\beta_0}{\epsilon} + \left(\frac{\mu^2}{\mathbf{q}^2}\right)^{\epsilon} a_1(\epsilon).$$

PNRQCD: Integrating out the soft scale II

- LO Coulomb potential is of the same order as the leading kinetic terms
  - Must be treated non-perturbatively
  - LO Lagrangian describes propagation of quark-antiquark pairs, where ladder diagrams with exchange of an arbitrary number of potential gluons $(k^0 \sim m_t v^2, \mathbf{k} \sim m_t v)$ have been resummed

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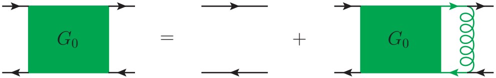
- Green function $\tilde{G}_0(\mathbf{p}, \mathbf{p}'; E)$ satisfies d-dimensional Lippmann-Schwinger equation

$$\left(\frac{\mathbf{p}^2}{m_t} - E\right) \tilde{G}_0(\mathbf{p}, \mathbf{p}'; E) - \tilde{\mu}^{2\epsilon} \int \frac{d^{d-1}\mathbf{k}}{(2\pi)^{d-1}} \frac{4\pi C_F \alpha_s}{\mathbf{k}^2} \tilde{G}_0(\mathbf{p} - \mathbf{k}, \mathbf{p}'; E)$$
$$= (2\pi)^{d-1} \delta^{(d-1)}(\mathbf{p} - \mathbf{p}'),$$



  - Representation of Green function known in $d = 4$

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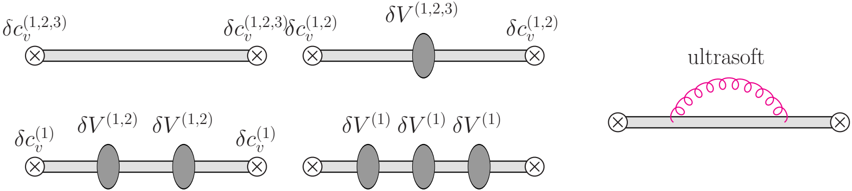
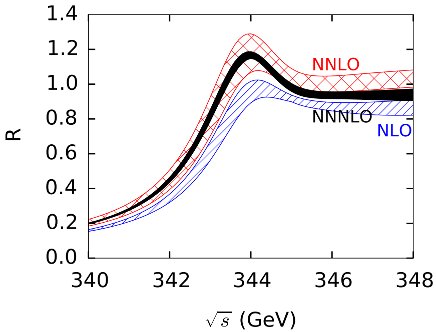
Resummed cross section at NNNLO:



Full third order in QCD

- NNNLO QCD result completed last year (NNLO from late 90's)



Inputs:

$$m_t^{\rm PS}(\mu_f = 20\text{ GeV}) = 171.5\text{ GeV}$$
$$\Gamma_t = 1.33\text{ GeV}$$
$$\alpha_s(m_Z) = 0.1185$$
$$\alpha(m_Z) = 1/128.944$$
$$\sin^2\theta_w = 0.223$$

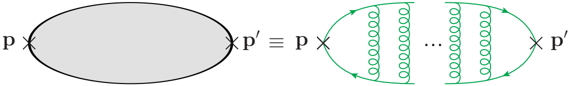
Scale variation: $\mu \in [50\text{ GeV}, 350\text{ GeV}]$ with $\mu^{\rm cent} = 80$ GeV

- Plot from [Beneke, Kiyo, Marquard, Penin, Piclum, Steinhauser]
  NNNLO ingredients: [Anzai, Beneke, Kiyo, Kniehl, Marquard, Penin, Piclum, Schuller, Seidel, Smirnov, Smirnov, Steinhauser, Sumino, Wüster]
  Public code: `QQbar_Threshold` [Beneke, Kiyo, Maier, Piclum]

- NNLL results [Pineda, Signer; Hoang, Stahlhofen]

P wave contribution at NNLO

- Axial-vector component of s-channel Z boson produces top pair in P-wave state
- In non-relativistic region the AV current is suppressed by $v \sim \alpha_s \to$ NNLO effect



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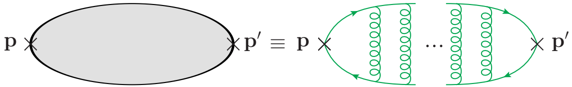
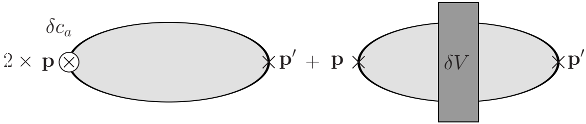


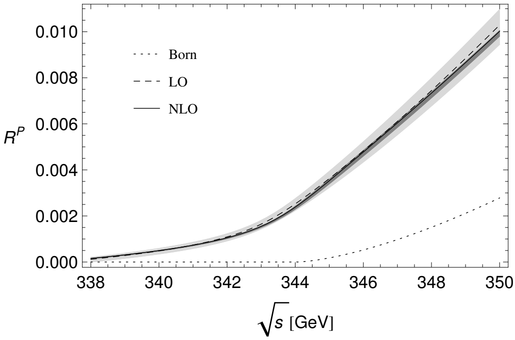
- NNLO correction computed in [Beneke, Piclum, TR]



- Older results exist [Penin, Pivovarov], but are not in dimensional regularization, which is required for a consistent combination with non-resonant effects

P wave contribution at NNNLO

- P wave gives a small effect $\lesssim 1\%$
- Complete NNNLO QCD result (incl. NLO P wave) will be used as reference prediction for the study of subleading effects in the following



Non-QCD effects

- Due to top instability the physical final state is $W^+W^-b\bar{b}$
- Top width is of the order of the ultrasoft scale, $\Gamma_t \sim m_t\alpha \sim m_t v^2$
  - Narrow width approximation unphysical!
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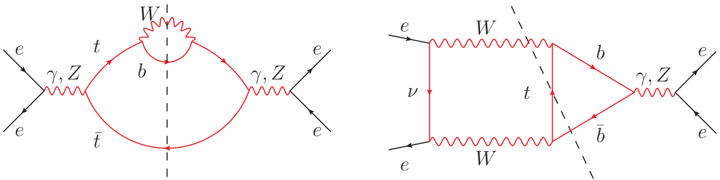
  - Cross section modified in non-perturbative way (smearing)
- $W^+W^-b\bar{b}$ final state: Hard subgraphs connecting to both sides kinematically allowed



Generalization of the EFT framework

- Contributions can be organized systematically within Unstable Particle Effective Theory [Beneke, Chapovsky, Signer, Zanderighi]

$$R \sim \mathrm{Im}\Bigg\{ \sum_{k,l} C_p^{(k)} C_p^{(l)} \int d^4x \left\langle e^+e^- \middle| \mathrm{T}[i\mathcal{O}_p^{(k)\dagger}(0) i\mathcal{O}_p^{(l)}(x)] \middle| e^+e^- \right\rangle + \sum_k C_{4e}^{(k)} \left\langle e^+e^- \middle| i\mathcal{O}_{4e}^{(k)}(0) \middle| e^+e^- \right\rangle \Bigg\}$$

- Dominantly produced through resonant (i.e. on-shell) top pair
- At higher orders: Production with just one or no resonant top
- Both contributions are separately divergent, only the sum is physical

Electroweak contributions

- Full NNLO corrections [Beneke, Maier, TR, Ruiz-Femenía: in preparation]
  - QED Coulomb potential
  - EW corrections to hard matching coefficients [Grzadkowski, Kühn, Krawczyk, Stuart; Guth, Kühn; Hoang, Reißer; Beneke, Maier, TR, Ruiz-Femenía: in preparation]
  - Time dilation effects on top decay
  - Initial state radiation

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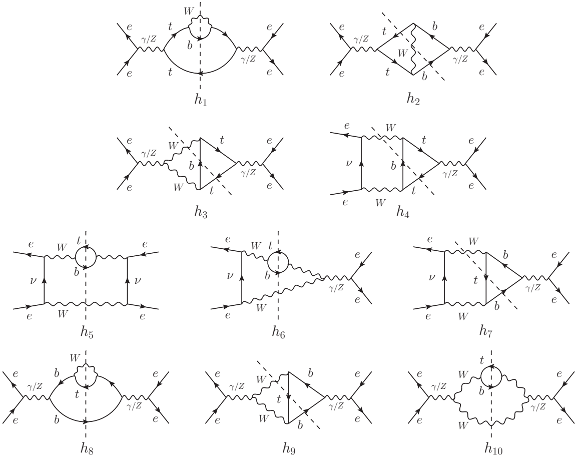
  - Time dilation effects on top decay
  - Initial state radiation
- Higgs effects at NNNLO [Beneke, Maier, Piclum, TR]
  - Mixed Higgs-QCD corrections to hard matching coefficients [Eiras, Steinhauser]
  - Higgs potential

Non-resonant contribution at NLO

- Non-resonant part previously known at NLO [Beneke, Jantzen, Ruiz-Femenía]

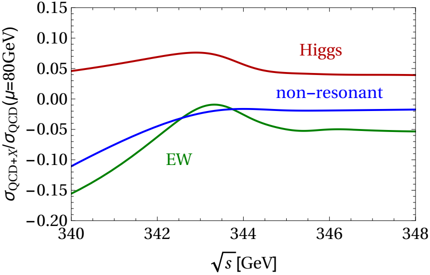


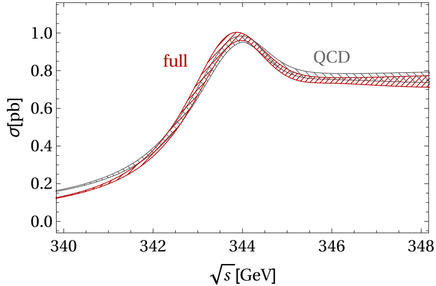
Non-resonant contribution at NNLO

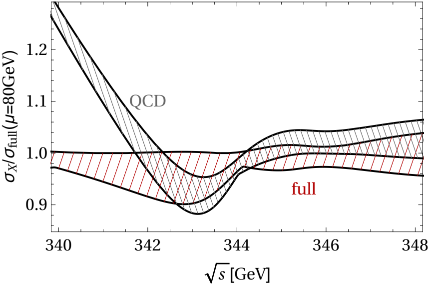
- $\mathcal{O}(\alpha_s)$ corrections to the NLO diagrams
- Contains IR divergences, which cancel with resonant part
- Cancellation demonstrated in [Jantzen, Ruiz-Femenía]
- These divergences occur in only about 15% of the $\mathcal{O}(100)$ diagrams
  $\rightarrow$ computed by hand
- Remainder computed with edited `MadGraph` code
- Full NNLO electroweak and non-resonant corrections in [Beneke, Maier, TR, Ruiz-Femenía: in preparation]

Non-QCD effects (Numerics not final!)

- Relative size of Higgs, QED and non-resonant contributions (down)
- Impact on the cross section (right)
- Effects significantly larger than QCD uncertainty, particularly in the important region below threshold

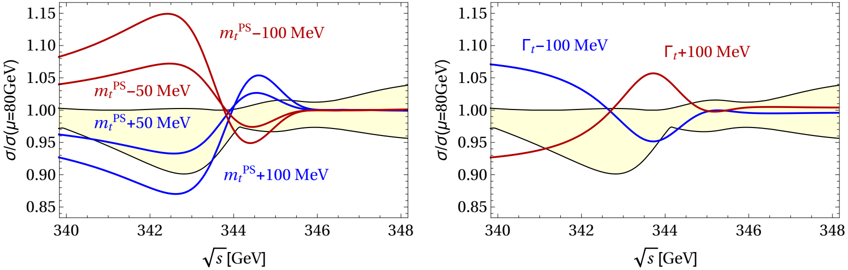






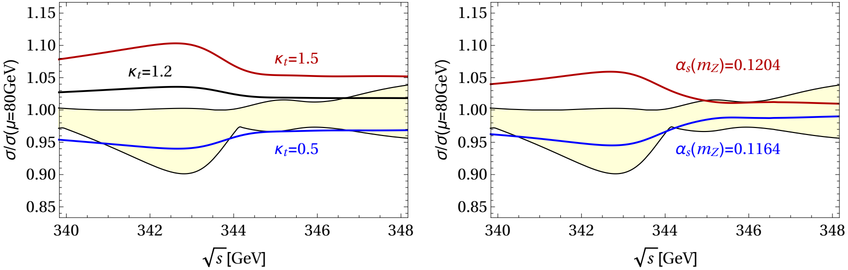
Parameter dependence (Numerics not final!)

- The region at and below the peak is very sensitive to variations of m_t and Γ_t
  - Increase (decrease) of m_t shifts the peak to the right (left)
  - Increase (decrease) of Γ_t makes the peak less (more) pronounced
  - Allows ultra-precise measurements in theoretically well-defined mass schemes (unlike reconstructions of the top mass at LHC)



Yukawa coupling dependence (Numerics not final!)

- Assume that some new physics modify the SM Yukawa coupling, parametrization through $y_t = \kappa_t \frac{\sqrt{2}m_t}{v}$
  - Changes normalization of cross section
  - Variation of α_s has a similar effect
  - Degeneracy possibly restricts measurement of y_t, but α_s should be known sufficiently well by the time a measurement is possible



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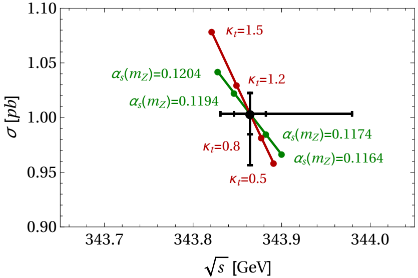

Peak height and position

Summary

- Strong dynamics in $e^+e^- \to t\bar{t}X$ near threshold are under control
- Non-QCD effects are important and (almost) available at NNLO plus partial NNNLO
- Threshold scan at a future linear collider will give an ultra-precise measurement of m_t
- Determination of Γ_t, α_s and y_t