The LUX approach to the photon PDF

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- Motivations
- The Master Equation
- The LUX PDF set
- Structure functions data
- Elastic data
- Uncertainties
- Some applications
- LUX and Hoppet resources
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Motivations

- Photon distribution small compared to quarks and gluons.
- With the increasing quest of precision, its knowledge has become relevant for key scattering reaction at LHC, as in W/Z fusion higgs production or H production in association with a weak boson.
- Anomalies in high mass W pair production, and the now disappeared $\gamma\gamma$ resonance have underlined the need for a better understanding of the photon density in the proton.
- "Agnostic" (model independent) fits to photon PDF's (NNPDF) showed a worrisome range of uncertainties.

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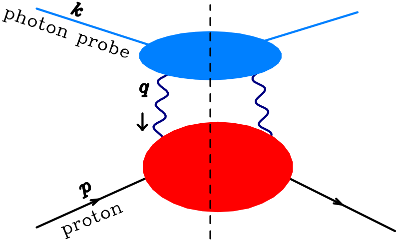
The Master Equation



$$\sigma = \int \frac{d^4 q}{(2\pi)^4} \frac{e^4_{\rm phys}(q^2)}{q^4}$$
$$\times \langle k | \tilde{J}_p^{\mu}(-q) J_p^{\nu}(0) | k \rangle$$
$$\times \langle p | \tilde{J}_{\mu}(q) J_{\nu}(0) | p \rangle$$

Kinematics constraints:

$$Q^2 = -q^2 > 0,$$
$$0 < x_{\rm bj} = Q^2/(2p\cdot q) \leq 1.$$

- Same kinematic restrictions as in DIS.
- $\frac{1}{4\pi}\langle p | \tilde{J}_{\mu}(q) J_{\nu}(0) | p \rangle = -g_{\mu\nu} F_1(Q^2, x_{\rm bj}) + \frac{p^{\mu}p^{\nu}}{p\cdot q} F_2(Q^2, x_{\rm bj}) + \ldots$ (Notice: full F_1 and F_2, not only inelastic)
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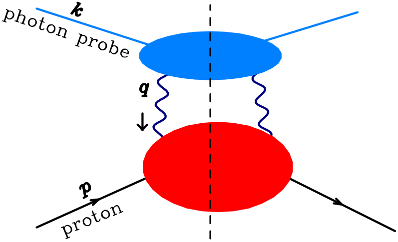

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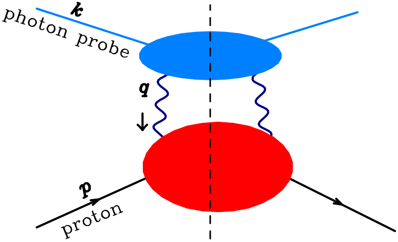

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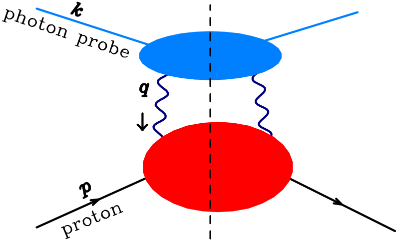

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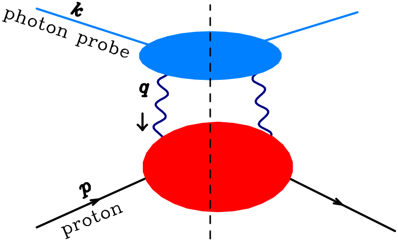

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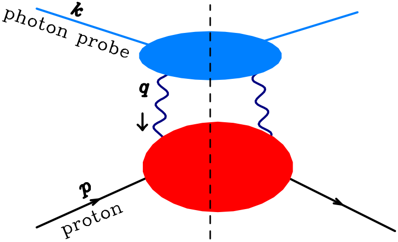

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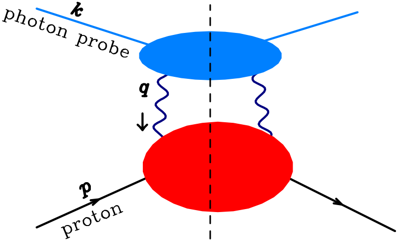

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- Take a BSM interaction of the form $\frac{e}{\Lambda}\bar{l}[\gamma^\mu, \gamma^\nu] L F_{\mu\nu} + \mathrm{cc}$, l massless, L massive with mass M, both neutral. With this choice there are no QED corrections to the probe process. All higher order QED effects are lumped into the physical electromagnetic coupling and in the hadronic tensor.
- Compute the cross section with the Master Formula
- Compute the cross section with the Parton Model formula
- Extract f_γ by identifying the two cross sections.

We obtain in the $\overline{\mathrm{MS}}$ scheme at NLO:

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$$x f_{\gamma/p}(x,\mu^2) = \frac{1}{2\pi}\int_x^1 \frac{dz}{z}\Bigg\{ -\overbrace{\alpha(\mu^2) z^2 F_2\left(\frac{x}{z},\mu^2\right)}^{\overline{\mathrm{MS}}\ \text{correction}}$$

$$+\underbrace{\int_{\frac{x^2 m_p^2}{1-z}}^{\overbrace{\frac{\mu^2}{1-z}}^{\overline{\mathrm{MS}}}} \frac{dQ^2}{Q^2}}_{\mathcal{O}(\log\frac{\mu^2}{m_p^2})} \frac{\alpha^2(Q^2)}{\alpha(\mu^2)}\left[\left((1+(1-z)^2)+\frac{2x^2 m_p^2}{Q^2}\right)F_2(x/z,Q^2) - z^2 F_L\left(\frac{x}{z},Q^2\right)\right]\Bigg\}.$$

- $f_\gamma \approx \alpha \log\frac{\mu^2}{m_p^2} \approx \alpha/\alpha_s$ $(\alpha_s(\mu^2) \approx 1/\log\frac{\mu^2}{\Lambda^2})$ relative to $f_{u/d}$.
- $Q^2 \approx m_p^2$ region formally of order α, i.e. NLO (as $\overline{\mathrm{MS}}$ term).
- Straightforward to improve at NNLO in α_s (Master Equation is exact, compute the parton model process at NNLO)
- Also accurate at $(\alpha/\alpha_s)^2$, provided that $\alpha(Q^2)$ and F_2 include leading log electromagnetic evolution.
- Valid at all μ's: MUST match evolution accuracy with one extra α_s. Agrees with De FLorian, Sborlini, Rodrigo $\alpha\alpha_s$ splitting functions, arXiv:1512.00612.

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Use:

Ideal use:

- Get $F_{2/L}$ at low Q^2 from available data.
- PDF global fit, including EM evolution, with the photon density constrained by the previous equation, $F_{2/L}$ taken from data at low Q^2 and computed from the PDF's at high Q^2

Much can be done without performing a dedicated global fit. However, if we aim at NLO accuracy:

- Low Q^2 region cannot be neglected.
- $(\alpha/\alpha_s)^2$ terms arising from the evolution of QED coupling cannot be neglected $(\alpha(m_\mu^2))/\alpha(M_Z^2) \approx 0.94)$
- $(\alpha/\alpha_s)^2$ terms arising from the QED evolution of the quarks are small, just do something minimal to account for them.

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- Start from a standard set (e.g. PDF4LHC15_nnlo_100);
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- Evolve down to 10 GeV, including QED evolution only for splitting processes that affect the photon: $P_{\gamma q}$, $P_{\gamma g}$, $P_{\gamma\gamma}$ (with $\alpha\alpha_s$ terms included).
- Fix the momentum sum rule by rescaling the gluon (a factor of 0.99299 is needed).
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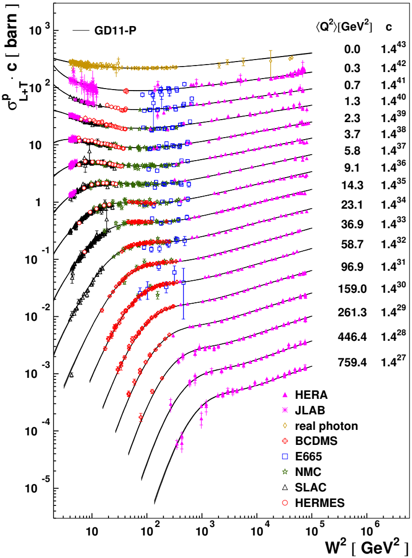
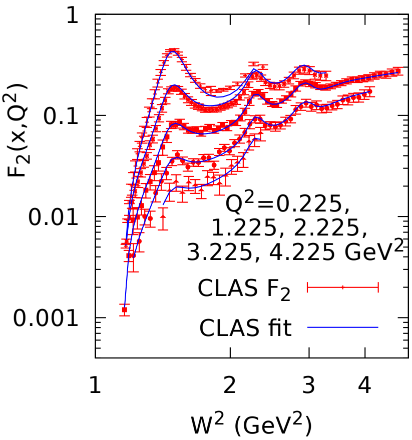
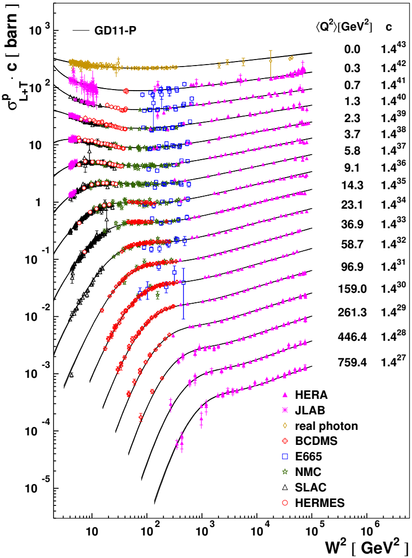


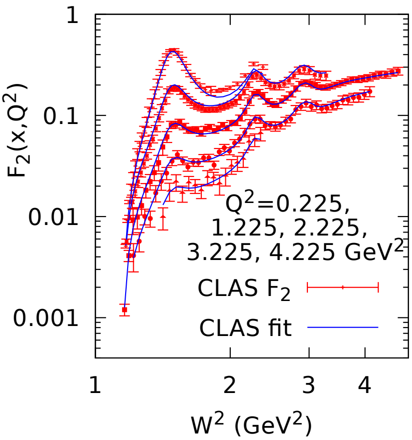


Fitted data from $Q^2 = 0.225$ to 4.725 in steps of $0.05\,\mathrm{GeV}^2$.

Hermes fit: we are interested in the region $Q^2 < 10\,\mathrm{GeV}^2$.

Continuum data region: $4\ \mathrm{GeV}^2 < W^2 \lesssim 10^5\mathrm{GeV}^2$ $(x \to 10^{-4})$.





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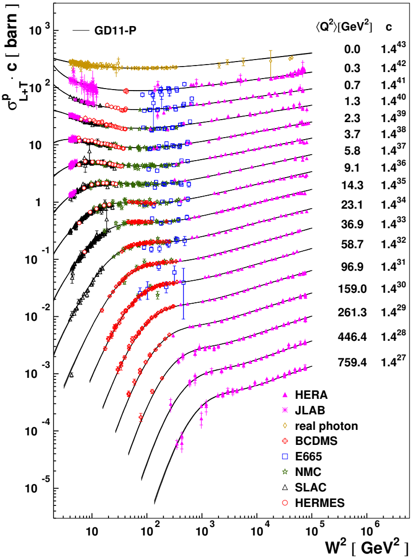

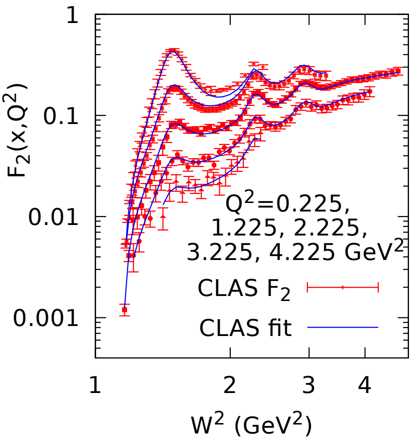

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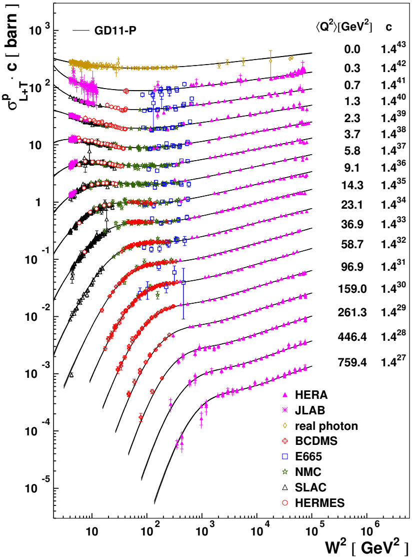

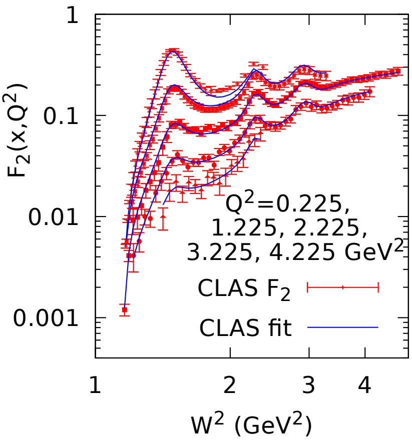

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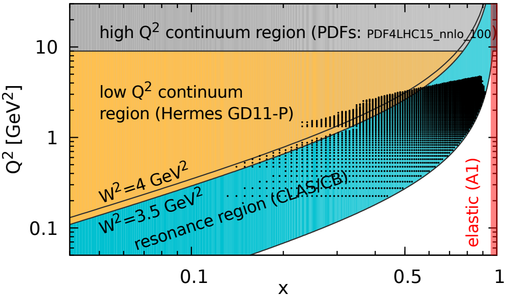

Also:

$$F_2(x, Q^2) = \frac{1}{4\pi^2\alpha}\frac{Q^2(1-x)}{1+\frac{4x^2m_p^2}{Q^2}}(\sigma_T(x, Q^2)+\sigma_L(x, Q^2)) \underset{Q^2\to 0}{\Longrightarrow} \frac{Q^2\sigma_{\gamma p}(W)}{4\pi^2\alpha^2}.$$

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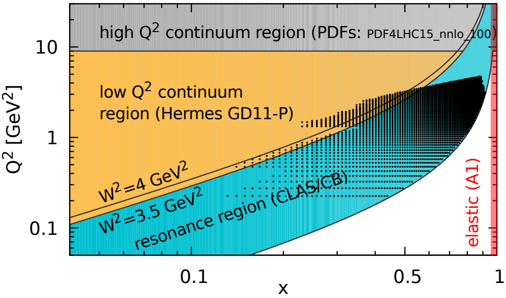

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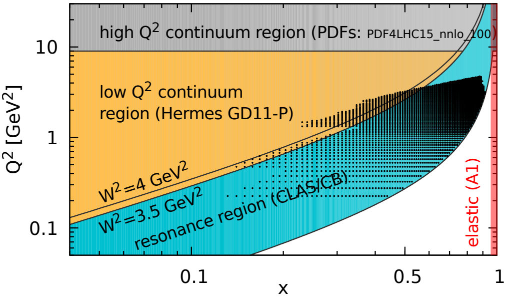

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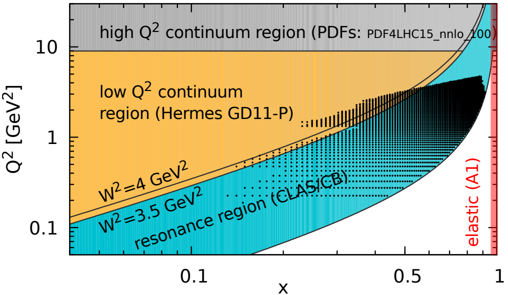

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F_2 and F_L receive an elastic contribution that we must include:

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with $\tau = Q^2/(4m_p^2)$. In the dipole approximation

$$G_E(Q^2) = \frac{1}{(1+Q^2/m_{\mathrm{dip}}^2)^2},\; G_M(Q^2) = \mu_p G_E(Q^2), \quad \begin{array}{l} m_{\mathrm{dip}}^2 = 0.71\,\mathrm{GeV}^2 \\ \mu_p = 2.793 \end{array}$$

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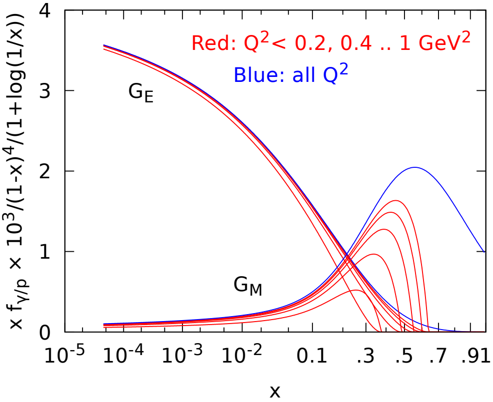

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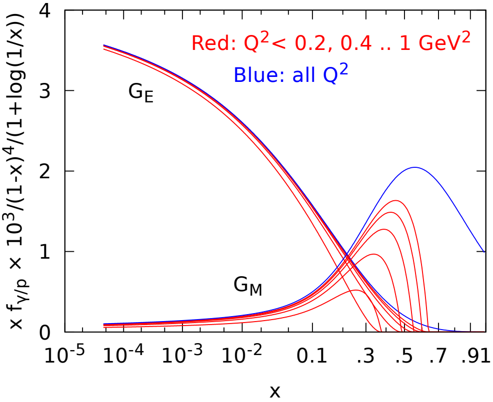

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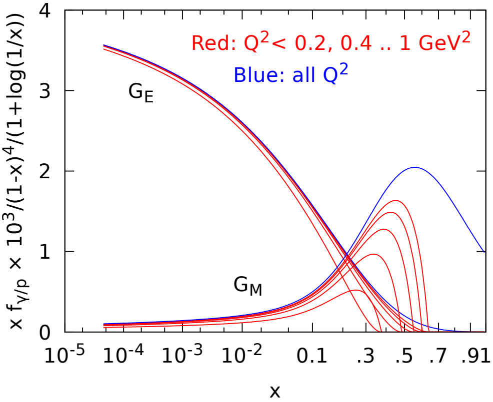

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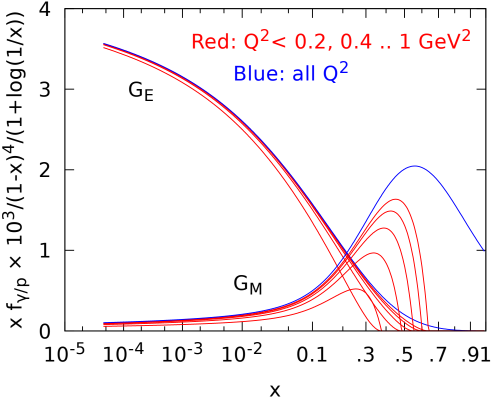

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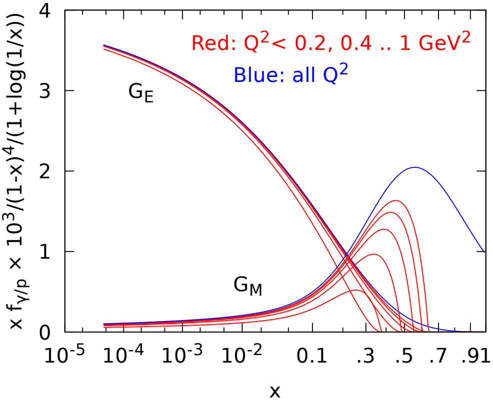

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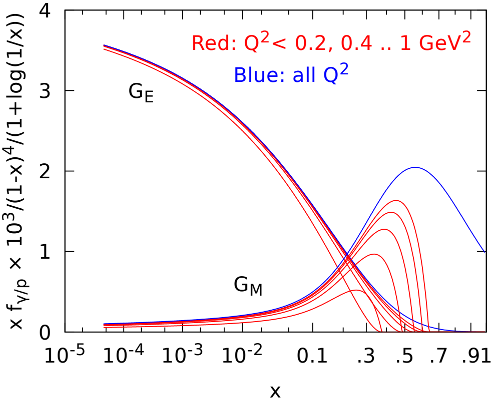
$$x f_\gamma^{\rm el}(x,\mu^2) = \frac{1}{2\pi}\int_{\frac{x^2 m_p^2}{1-x}}^{\frac{\mu^2}{1-x}} \frac{\mathrm{d}Q^2}{Q^2}\frac{\alpha^2(Q^2)}{\alpha(\mu^2)}\left\{\left(1-\frac{x^2 m_p^2}{Q^2(1-x)}\right)\frac{2(1-x)G_E^2(Q^2)}{1+\tau} + \left(2-2x+x^2+\frac{2x^2 m_p^2}{Q^2}\right)\frac{G_M^2(Q^2)\tau}{1+\tau}\right\}.$$

Dipole approximation,
($\mu \to \infty$ in figure.)

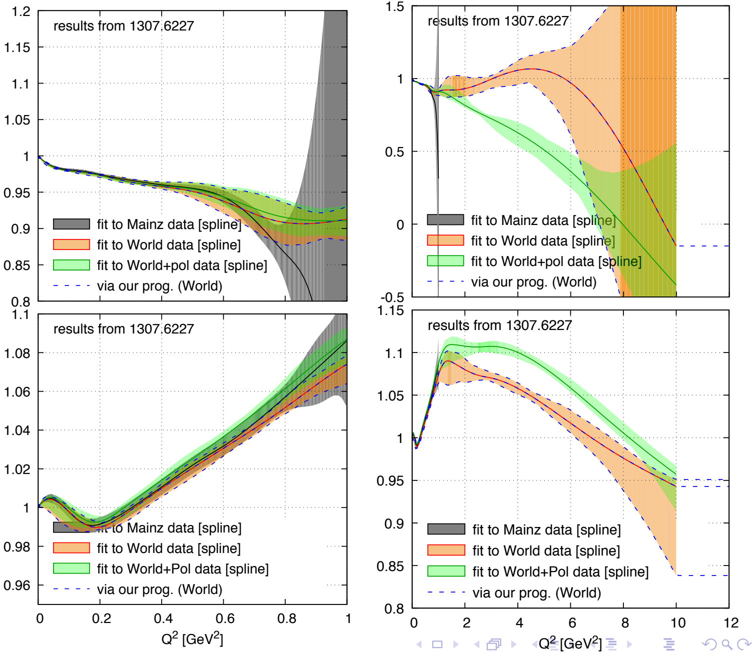
- Mostly G_E at small x.
- Mostly G_M at large x.
- Mostly from $Q^2 < 1 GeV$.



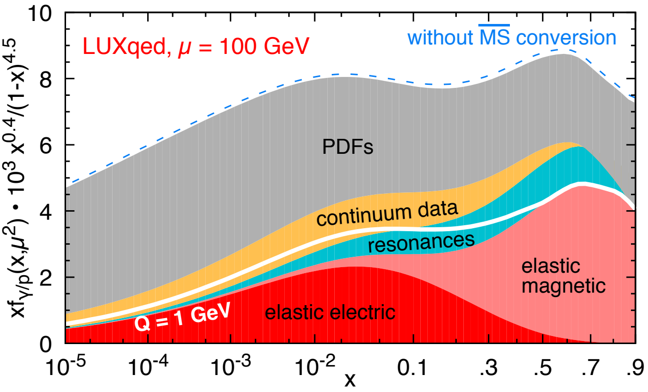
Elastic Data, A1 experiment and World data

$G_E / G_E^{\mathrm{dipole}}$

$G_M / G_M^{\mathrm{dipole}}$



Contributions to f_γ:



- $Q^2 > 9\ GeV^2$, computed from standard PDF sets
- Important elastic component. Magnetic prevails for $x > 0.2$.
- Continuum and resonance contributions not negligible
- Very important contribution from $Q^2 < 1\ GeV^2$.

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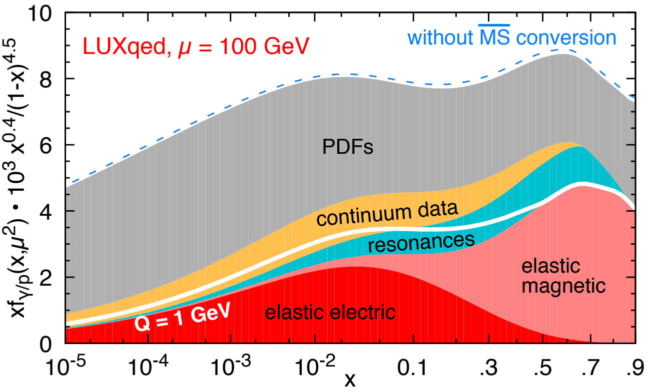

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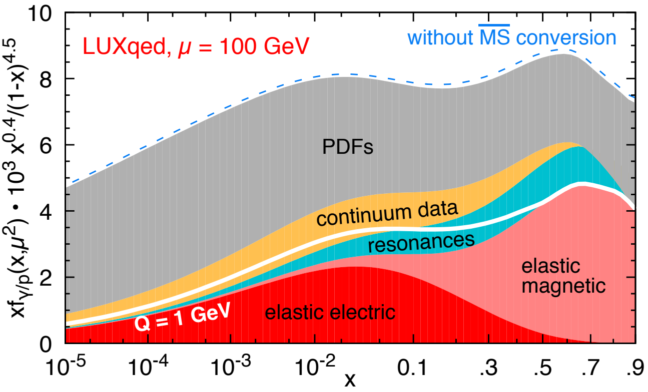

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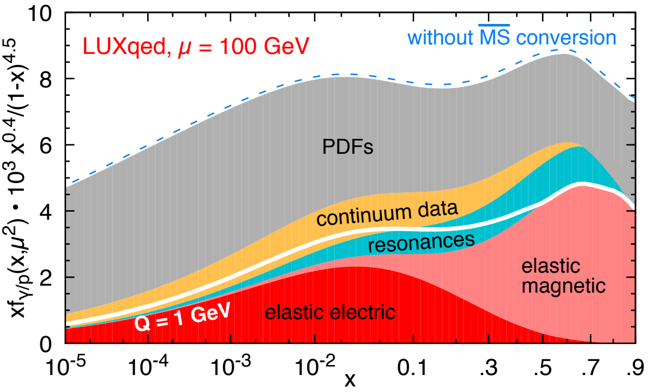

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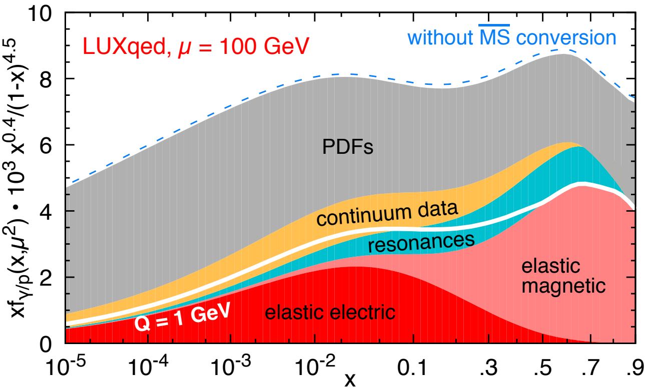

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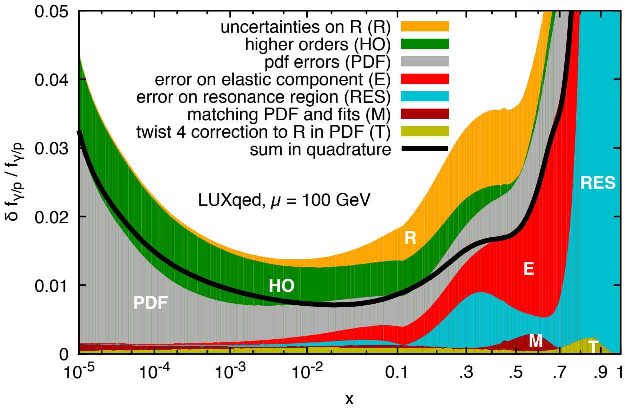

- At small x, higher order effects and PDF's dominate the error.
- At large x, elastic and resonant region dominant.
- Total uncertainty at the percent level.
  Further improvements possible!

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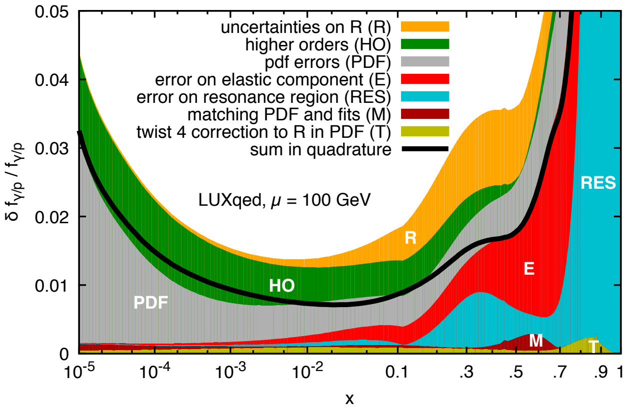

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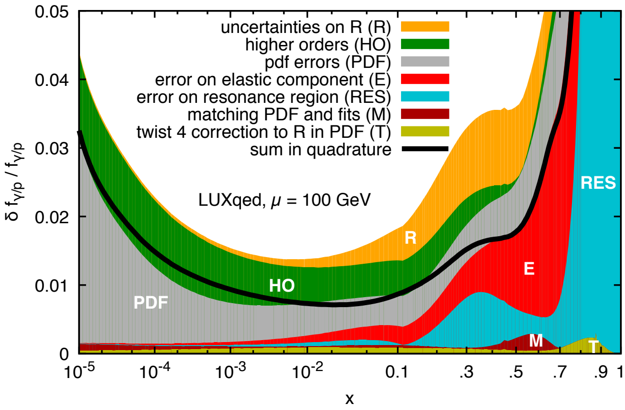

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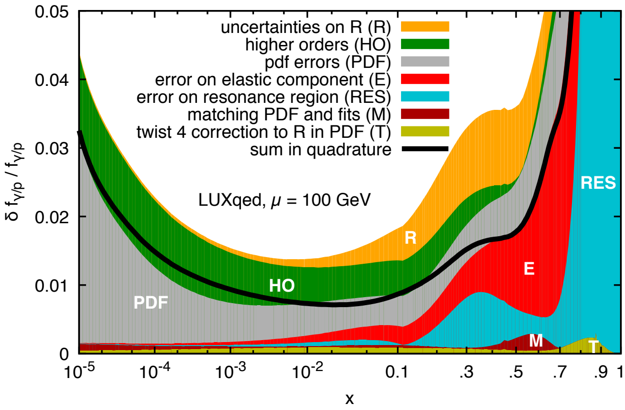

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Uncertainties included in LUX

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- 0-100: original PDF members (PDF4LHC15_nnlo_100)
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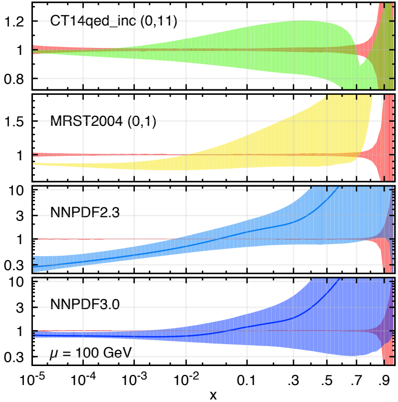
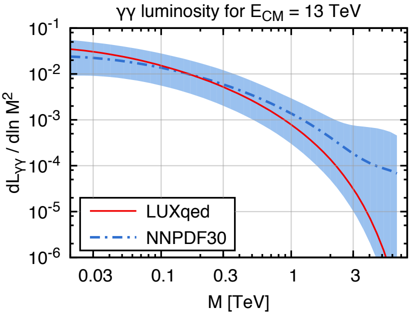




The LUX method achieves by far better precision than other methods.

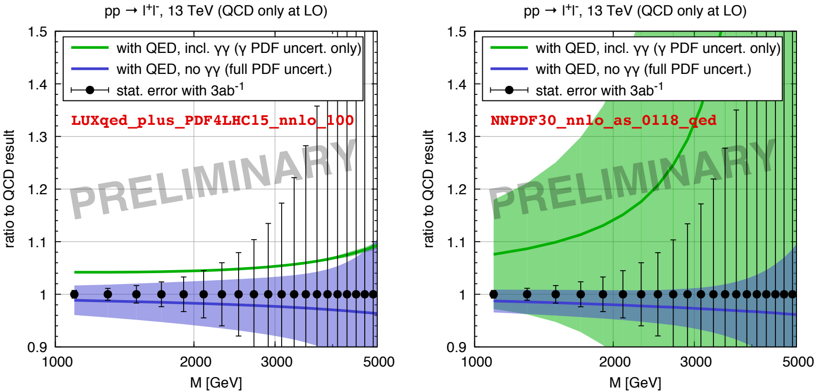
Approaches that use some lepton scattering information (in particular `CT14qed_inc`) achieve better precision than "totally agnostic" approaches (NNPDF) (note different *y* axis in panel).

APPLICATION TO HIGGS PHYSICS

$pp \to H W^+ (\to l^+ \nu) + X$ at 13 TeV	
non-photon induced contributions	91.2 ± 1.8 fb
photon-induced contribs (NNPDF23)	$6.0^{+4.4}_{-2.9}$ fb
photon-induced contribs (LUXqed)	4.4 ± 0.1 fb

non-photon numbers from LHCHXSWG (YR4)

di-lepton spectrum



LUXQED photon has few % effect on di-lepton spectrum and negligible uncertainties

RESOURCES

- `LUXqed_plus_PDF4LHC15_nnlo_100` set available from LHAPDF
- Additional plots and validation info available from http://cern.ch/luxqed
- Preliminary version of HOPPET DGLAP evolution code with QED (order α and $\alpha\alpha_s$) corrections available from hepforge:

```
svn checkout http://hoppet.hepforge.org/svn/branches/qed hoppet-qed
```

(look at `tests/with-lhapdf/test_qed_evol_lhapdf.f90` for an example; interface may change, documentation missing)

Conclusions

- Photon PDF can be extracted with great precision from available knowledge of proton structure function and form factors.
- The needed low Q^2 data is available thanks to extensive low and intermediate energy Nuclear Physics studies.
- Our study aimed at NLO precision including terms suppressed by one power of α_s or by a power of α/α_s relative to the leading term. This leads to precisions at the percent level.
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EXTRA SLIDES

Going NNLO

Going to one extra order in α_s is not difficult. We need to compute our "probe" process in the parton model, at NNLO, subtracting the collinear singularities in the $\overline{MS}$ scheme.

The d-dimensional NLO and NNLO corrections to the probe process are obtained by

- writing our master formula in $d = 4 - 2\epsilon$ dimension, and replacing the $W^{\mu\nu}$ tensor with the partonic $w_i^{\mu\nu}$ tensor.
- Compute $w_i^{\mu\nu}$ along the lines of the Altarelli-Ellis-Martinelli calculation of NLO corrections to DIS of 1979, keeping however one extra power of ϵ in its expansion.

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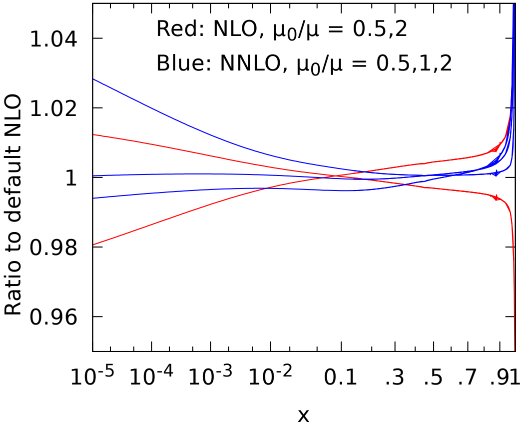

Reduction of the uncertainty band visible from $x \approx 0.3$. The crossing point in scale dependence makes it difficult to appreciate what happens for smaller values of x.

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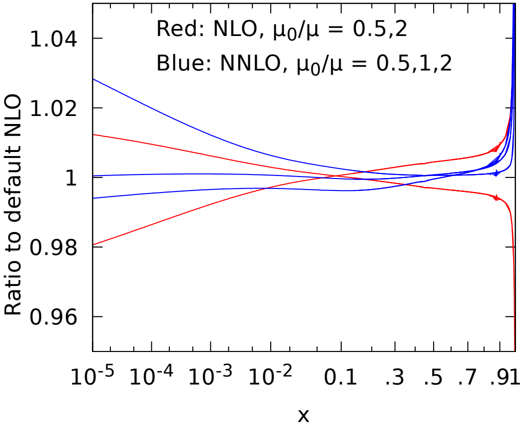

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There is a vast literature touching this topic.

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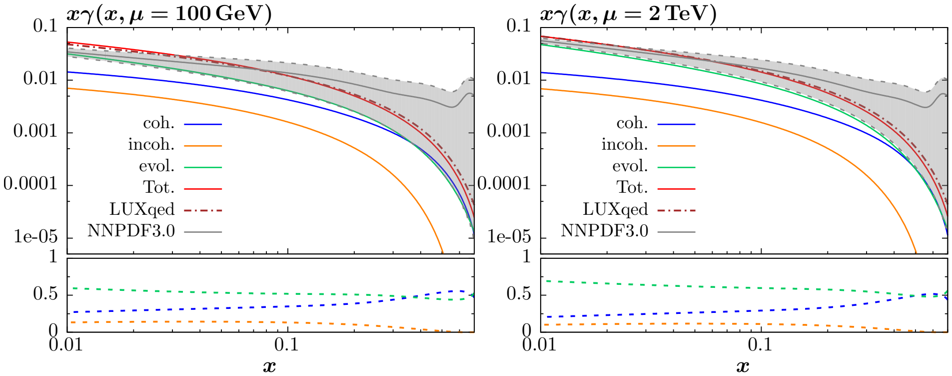
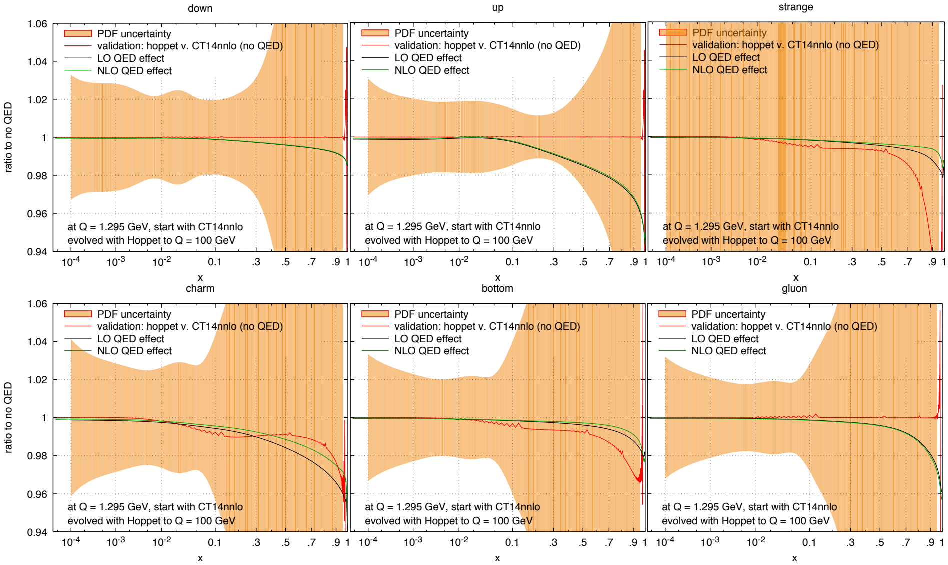
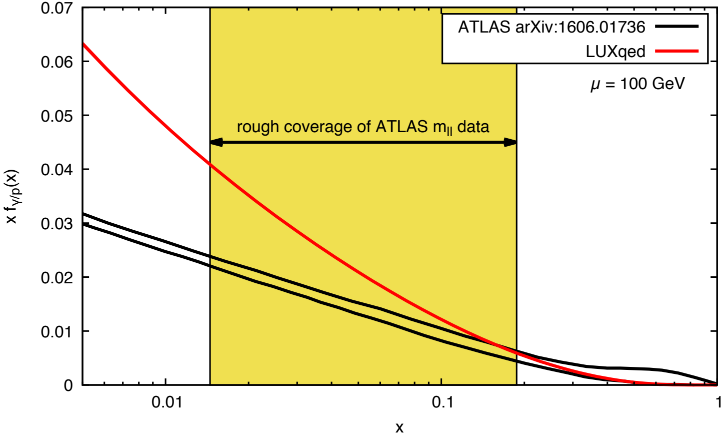
Comparison with Harland-Lang, Khoze and Ryskin, 1607.04635v3 (October 10, 2016).



Impact of QED evolution



ratio of ATLAS photon (1606.01736) to LUXqed



ATLAS result based on reweighting of NNPDF23 with high-mass (M_{ll} > 116 GeV) data