Higgs pair production with full top-quark mass dependence

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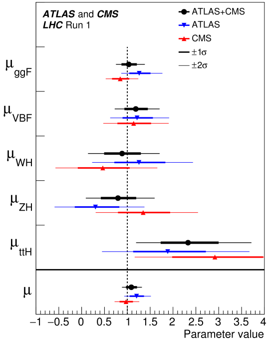
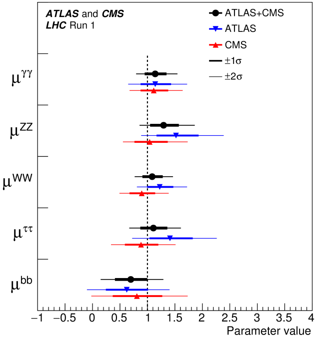
arXiv:1608.04798 [hep-ph] (submitted to JHEP)
Phys. Rev. Lett. 117 (2016) 012001, Erratum 079901

Future challenges for precision QCD, IPPP Durham,
Oct 25^{th} 2016

Discovery of a scalar particle

- Scalar particle discovered which looks like the Higgs boson
  - consistent with EW precision bounds
  - couplings to fermions consistent with SM prediction





ATLAS & CMS Combination Aug '16

Brout-Englert-Higgs potential

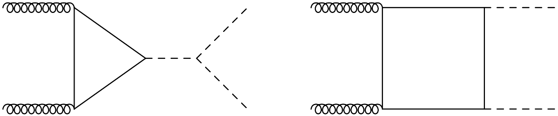
- further verification: does the predicted SM Brout-Englert-Higgs potential

$$V(\phi^\dagger\phi) = -\mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2\,, \quad \lambda, \mu > 0$$

  match what we observe?
- deduction of SM trilinear coupling $\lambda_{HHH} = \frac{m_H^2}{2v}$ directly probes the structure of the potential
- largest Higgs pair production cross section in gluon fusion
- 30% experimental precision expected at high-luminosity LHC (3000fb^{-1}) Barr, Dolan, Englert, Spannowsky '13
  $\rightarrow$ could be a demotivating factor ... BUT

Higgs-boson pair production in gluon fusion

- the leading order is loop induced Glover, van der Bij '88



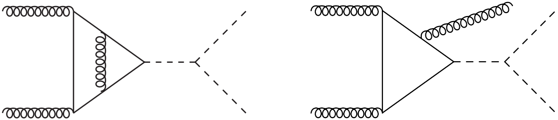
- Higher-order corrections were as of Apr 2016 only known in approximations

Higher-order approximations for $gg \to hh$

- NLO $m_t \to \infty$ limit
  Plehn, Spira, Zerwas '96; Dawson, Dittmaier, Spira '98
- NLO $m_t \to \infty$, supplemented with $1/m_t$ expansion
  Grigo, Hoff, Melnikov, Steinhauser '13; Degrassi, Giardino, Gröber 16
- NNLO $m_t \to \infty$ limit De Florian, Mazzitelli '13
- NNLO $m_t \to \infty$ + all matching coefficients
  Grigo, Melnikov, Steinhauser '14
- Full mass dependence in real radiation part + matching to parton shower Frederix, Hirschi, Mattelaer, Maltoni, Torrielli, Vryonidou, Zaro '14; Maltoni, Vryonidou, Zaro '14
- NNLO $m_t \to \infty$ + all matching coefficients + top quark mass effects Grigo, Hoff, Steinhauser '15
- NNLO $m_t \to \infty$ + NNLL threshold resummation
  De Florian, Mazzitelli '15
- NNLO $m_t \to \infty$ (differential) De Florian, Grazzini, Hanga, Kallweit, Lindert, Maierhöfer, Mazzitelli, Rathlev '16

$gg \to hh$ @ NLO with full top mass dependence

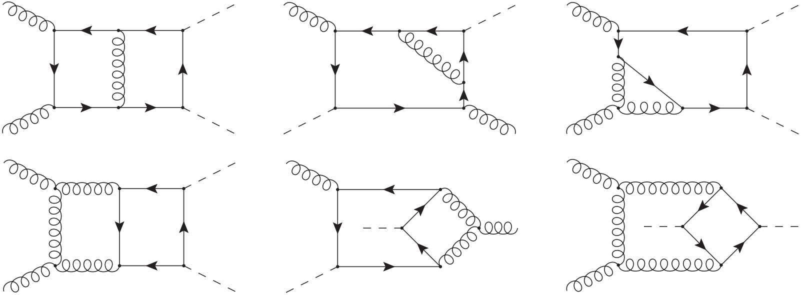
- NLO with full top mass dependence involves the computation of two-loop triangles and one-loop box diagrams (known from single Higgs production), e.g.



$gg \to hh$ @ NLO with full top mass dependence

Higher order correction to box-type LO diagram also needed:

- one-loop pentagon integrals
- unknown two-loop integrals (with 4 independent mass scales: $\hat{s}$, $\hat{t}$, m_t^2, m_h^2 or 3 ratios), e.g.



with up to 4 scalar products $\to$ reduction of virtual 2-loop amplitude to master integrals is highly non-trivial

Analytic vs. numerical approach

Analytical:

- Analytical techniques for calculations at the two-loop level are powerful, but reach their limits when elliptic functions are involved (appear e.g. in two-loop integrals involving multiple mass scales)

Numerical:

- Numerical methods to compute multi-loop multi-scale integrals are in general easier to automate and multiple scales do not pose a problem but precision reached with Monte Carlo integrators is limited

$\rightarrow$ New technical developments needed! Push boundaries!

Prerequisites for $gg \to hh$ @ NLO /w exact top-mass

Important new steps:

- evaluate integrals numerically with Quasi Monte Carlo integrator Dick, Kuo, Sloan '13; Li, Wang, Zan, Zhao '15 and using GPUs
- only integrate up to the necessary accuracy
  - set number of sampling points dynamically for each integral
  - target accuracy is set at amplitude level (3% for one form factor, ≈10% for the other, depending on the ratio of the two)

Also:

- be pragmatic: leave some Feynman integrals unreduced and compute them individually

Virtual two-loop amplitude

- virtual amplitude generated with GoSam-XL
  - diagram generation with Qgraf Nogueira '93 (≈10000 different integrals before accounting for symmetries)
  - further processed with FORM Vermaseren '00; Kuipers, Ueda, Vermaseren '12
  - python interface to Reduze von Manteuffel, Studerus '12
- reduction up to non-planar 6- and 7-propagator topologies with Reduze, partially transformed into finite basis Panzer '14; von Manteuffel, Panzer, Schabinger '14
  $\rightarrow$ 228 planar master integrals
- 99 non-planar integrals
- integrals calculated numerically with SecDec 3 SB, Heinrich, Jones, Kerner, Schlenk, Zirke '15 using a dedicated integration setup Jones '16, Kerner '16

Public codes using the sector decomposition method

SecDec is based on the method of sector decomposition
Hepp '66, Denner & Roth '96, Binoth & Heinrich '00

Public codes:

- sector_decomposition (uses GiNaC) Bogner & Weinzierl '07
  supplemented with `CSectors` Gluza, Kajda, Riemann, Yundin '10
  for construction of integrand in terms of Feynman parameters
- FIESTA* (uses Mathematica, C) A.V. Smirnov, V.A. Smirnov, Tentyukov '08 '09, A.V. Smirnov '13
- SecDec* (uses Mathematica, Fortran/C++)
  Carter & Heinrich '10; SB, Carter, Heinrich '12; SB & Heinrich '13; SB, Heinrich, Jones, Kerner, Schlenk, Zirke '15

*Multi-scale integrals not limited to the Euclidean region
SB, J. Carter & G. Heinrich '12; A.V. Smirnov '13

Numerical evaluation of Feynman integrals

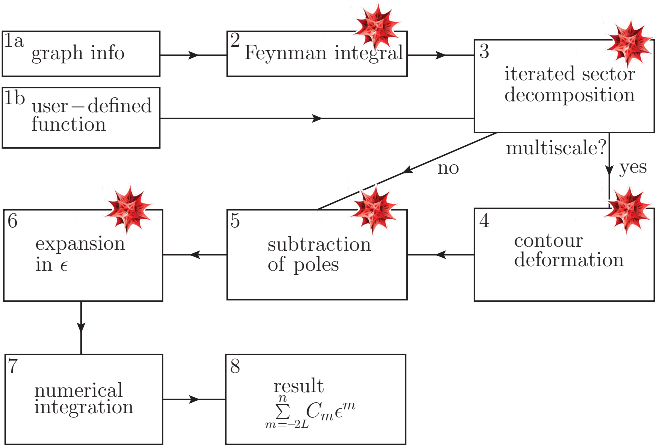
- SecDec is a tool to numerically compute

Feynman graph	or	parametric function

  - General **Feynman** integrals for **arbitrary** kinematics and with numerators
  - Integrals **matching** a Feynman integral **structure**
  - More general **parametric** functions

- Soon to come: PySecDec Stephan Jahn + SecDec3 collab.
  - generate integral libraries
  - instead of Perl, Mathematica → now only Python

Outline of the program SecDec



numerical integration: CUBA library Hahn '04, NIntegrate Wolfram

NEW: QMC Jones '16, Kerner '16

Numerical integration of two-loop amplitude

So far SecDec has been used for...

- fast evaluation of massive bubbles (34 mass topologies, up to 5 scales) to calculate $\mathcal{O}(\alpha_s \alpha_t)$ self-energy contributions to the MSSM Higgs-boson masses

  SB, Hahn, Heinemeyer, Heinrich, Hollik '14
- checks of analytically calculated integrals

NEW:
use SecDec as a library to numerically compute all 327 integrals contributing to the Higgs-pair production amplitude

Real radiation at NLO

- four real radiation channels

$$gg \to hh + g \,, \quad gq \to hh + q \,,$$
$$g\bar{q} \to hh + \bar{q} \,, \quad q\bar{q} \to hh + g$$

- 1-loop amplitudes generated and processed with GoSam
  Cullen, van Deurzen, Greiner, Heinrich, Luisoni, Mastrolia, Mirabella, Ossola, Peraro, Reiter, Schlenk, von Soden-Fraunhofen, Tramontano; '11 '14
- Catani-Seymour dipole formalism used for IR subtraction
  Catani, Seymour '96
- numerical integration using Vegas Lepage '80 of CUBA library Hahn '04

Checks and result

Checks:

- independent calculation of (unreduced) amplitude
- checked invariance under exchange of $\hat{t}$ and $\hat{u}$
- two-loop integrals recomputed with Vegas
- single Higgs production reproduced, comparison to SusHi
  Harlander, Liebler, Mantler '13 '16
- numerical pole cancellation (tested for up to 5 digits accuracy)
- independence of dipole parameter α Nagy '03
- comparison of $1/m_t$ expansion with Jens Hoff
  Grigo, Hoff, Steinhauser '15
- comparison of HEFT result to MG5_aMC@NLO
  Maltoni, Vryonidou, Zaro '14 '15

Total cross section @ 14 TeV by courtesy of Stephen Jones

	$\sigma_{\rm LO}$ (fb)	$\sigma_{\rm NLO}$ (fb)	$\sigma_{\rm NNLO}$ (fb)
HEFT	$17.07^{+30.9\%}_{-22.2\%}$	$31.93^{+17.6\%}_{-15.2\%}$	$37.52^{+5.2\%}_{-7.6\%}$
B.I. HEFT	$19.85^{+27.6\%}_{-20.5\%}$	$38.32^{+18.1\%}_{-14.9\%}$	$43.63^{+5.2\%}_{-7.6\%}*$
FTapprox	$19.85^{+27.6\%}_{-20.5\%}$	$34.26^{+14.7\%}_{-13.2\%}$	–
Full Theory	$19.85^{+27.6\%}_{-20.5\%}$	$32.91^{+13.6\%}_{-12.6\%}$	–
N.I. HEFT	–	$32.91^{+13.6\%}_{-12.6\%}$	$38.67^{+5.2\%}_{-7.6\%}*$

PDF4LHC15_nlo_30_pdfas
$m_H = 125$ GeV
$m_T = 173$ GeV
Uncertainty:
$\mu_R = \mu_F = \frac{m_{HH}}{2}$
$\mu \in \left[\frac{\mu_0}{2}, 2\mu_0\right]$ $(7-\text{point})$

* re-weighted on total cross-section level

de Florian, Grazzini, Hanga, Kallweit, Lindert, Maierhöfer, Mazzitelli, Rathlev 16;
Maltoni, Vryonidou, Zaro 14 (recalculated by us); Borowka, Greiner, Heinrich, Kerner, Schlenk, Schubert, Zirke 16;
Dawson, Dittmaier, Spira 98 (recalculated by us); Glover, van der Bij 88 (recalculated by us)

Comparison to Full Theory

	$\Delta\sigma^{\rm Full}_{\rm LO}$	$\Delta\sigma^{\rm Full}_{\rm NLO}$
HEFT	-14%	-3.0%
B.I. HEFT	0%	$+16\%$
FTapprox	0%	$+4.1\%$

Can do a similar exercise @ 100 TeV, differences typically larger

Comparison I: virtual amplitude vs. $1/m_t^{2\rho}$ approximations

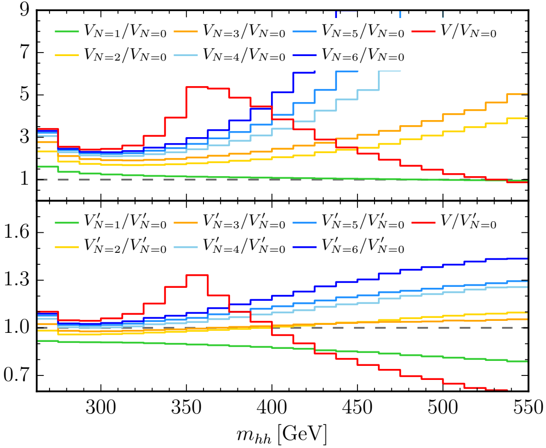
- Expansion of the virtual NLO matrix element

$$\mathrm{d}\hat{\sigma}^V_{\mathrm{exp},N} = \sum_{\rho=0}^{N} \mathrm{d}\hat{\sigma}^{V(\rho)} \left(\frac{\Lambda}{m_t}\right)^{2\rho}, \quad \Lambda \in \left\{\sqrt{\hat{s}}, \sqrt{\hat{t}}, \sqrt{\hat{u}}, m_H\right\}$$

- Born-improved finite part of virtual amplitude

$$\begin{aligned} &\mathrm{d}\sigma^V_{\mathrm{exp},N} \frac{d\sigma^{LO}(\varepsilon)}{d\sigma^{LO}_{\mathrm{exp},N}(\varepsilon)} + \mathrm{d}\sigma^{LO}(\varepsilon) \otimes \mathbf{I} \\ &= \underbrace{\underbrace{\left(\mathrm{d}\sigma^V_{\mathrm{exp},N} + \mathrm{d}\sigma^{LO}_{\mathrm{exp},N}(\varepsilon) \otimes \mathbf{I}\right)}_{\equiv V_N} \frac{d\sigma^{LO}}{d\sigma^{LO}_{\mathrm{exp},N}}}_{\equiv V'_N} + \mathcal{O}(\varepsilon)\,. \end{aligned}$$

Comparison I: full result to $1/m_t^{2\rho}$ approximations

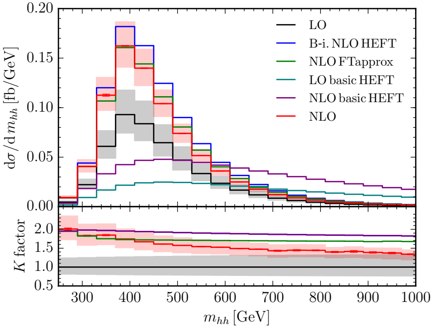


$N < 4$ Tom Zirke,

$N = 4, 5, 6$ thanks to Jens Hoff

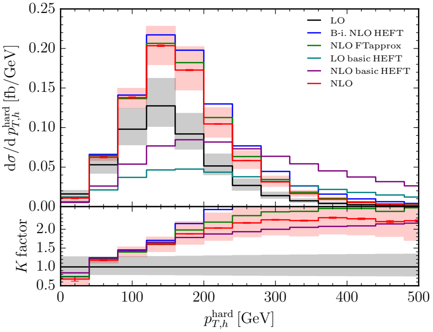
→ shapes very different beyond $m_{hh} \sim 2m_t$

Comparison II: full result to HEFT approximations



- 913 phase-space points with ∼16 dual NVIDIA TESLA K20X GPGPU nodes → total of ∼9 days runtime (median GPU time per PS point: 2 hours)
- NLO HEFT good approximation for $m_{hh} < 2m_t$
- scale uncertainties of HEFT and FT_{approx} do not enclose central value of full result in m_{hh} tail → HEFT breaks down

Comparison II: full result to HEFT approximations



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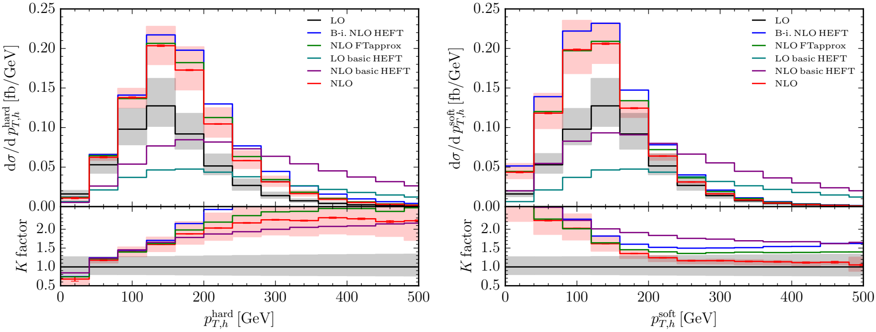
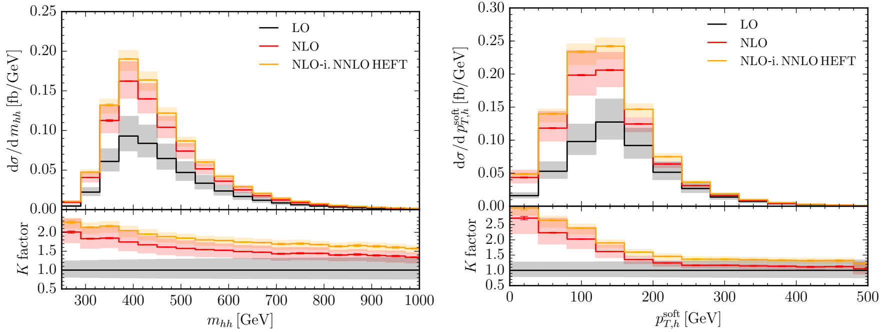


Comparison II: full result to HEFT approximations



Summary

- We calculated the total cross section and distributions for Higgs pair production in gluon fusion at NLO with full top-quark mass dependence.
- Evaluation of integrals done fully numerically using SecDec in dedicated integration setup
- HEFT does not reproduce shapes beyond top-threshold
- tail of m_{HH} distributions: differences wrt. Born-improved HEFT approximation $\gtrsim 50\%$, and $\sim 20\%$ wrt. FT_{approx} result
- inclusion of the full top-quark mass dependence vital for reliable Higgs-boson pair production predictions over full invariant mass range

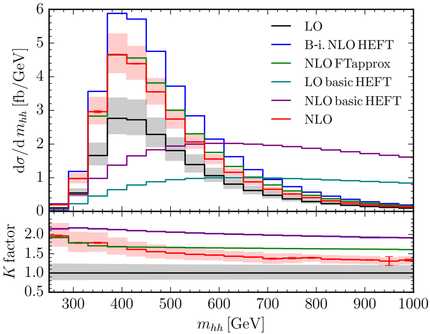
Outlook

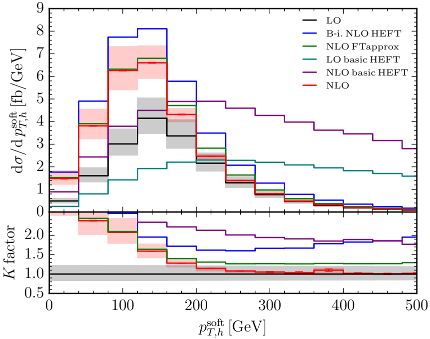
- apply setup to other processes (e.g. gg$\rightarrow$ Hg, gg$\rightarrow$ $\gamma\gamma$, gg$\rightarrow$ WW, gg$\rightarrow$ ZZ)
- make PySecDec publicly available

Thank you.

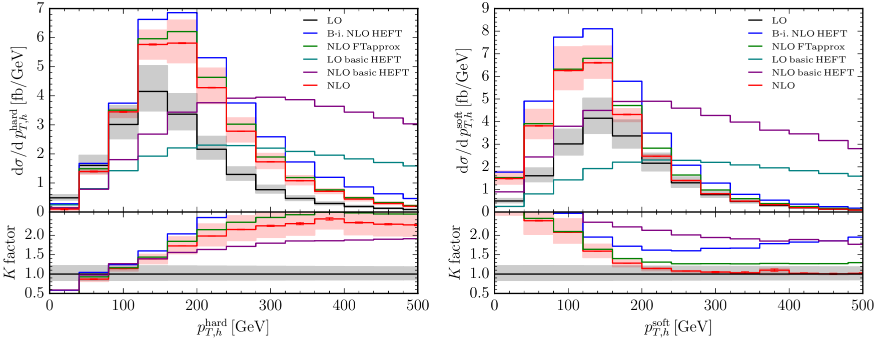
Backup

m_{HH} distributions at 100 TeV

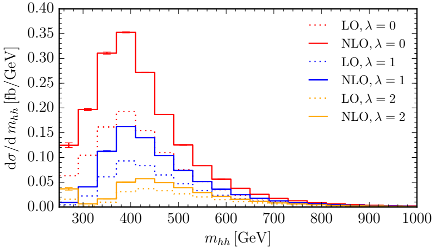


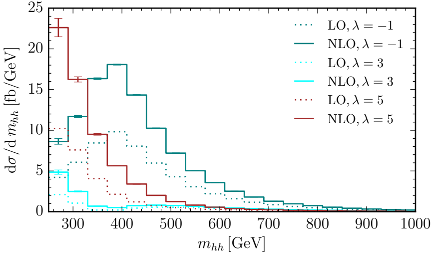


p_T^H distributions at 100 TeV



Variation of $\lambda_{HHH} = \lambda$ at 14 TeV





Variation of $\lambda_{HHH} = \lambda$ at 100 TeV