A map of the non-thermal WIMP

Chang Sub Shin (APCTP)

Based on *arXiv:1611.02287*

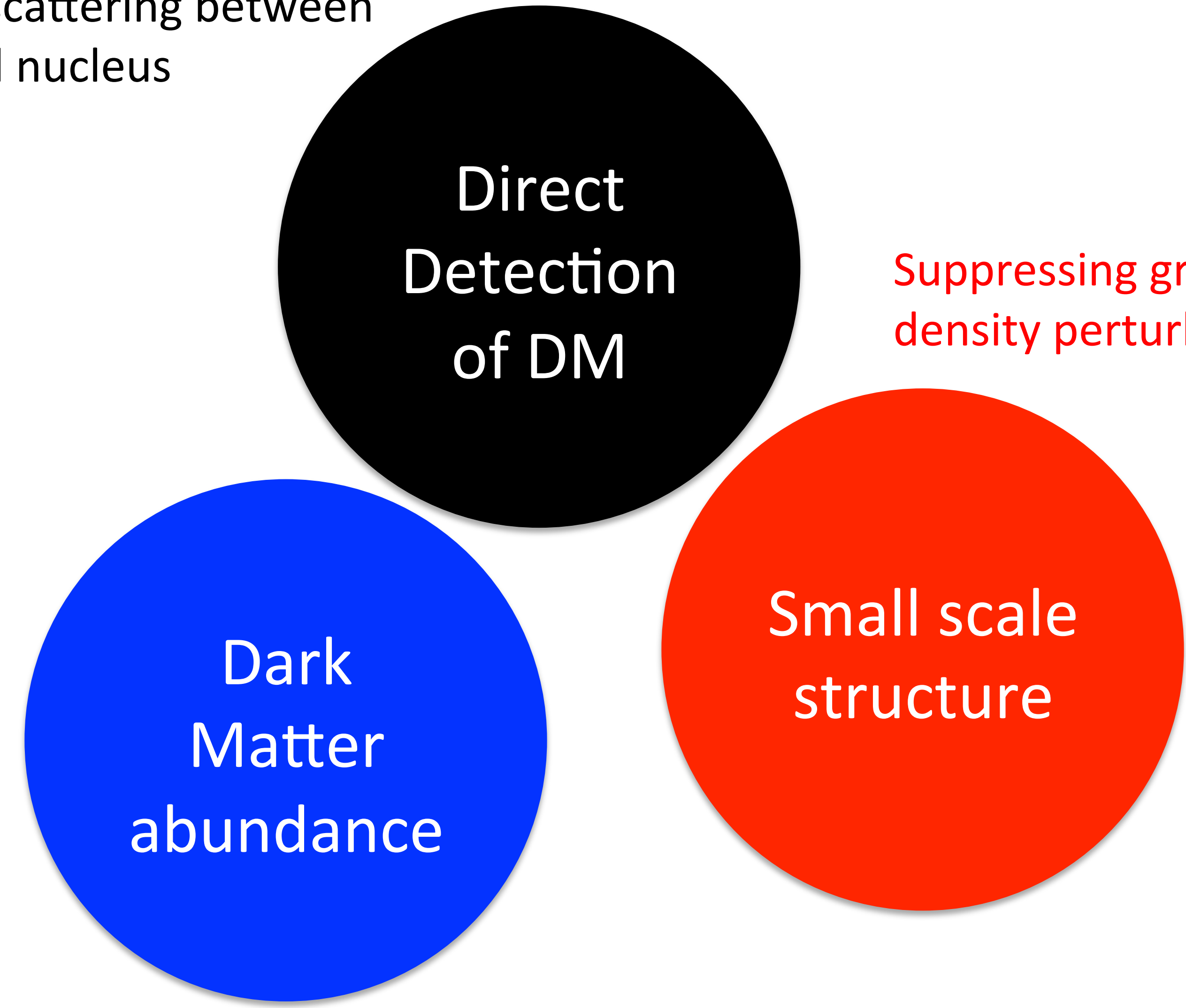
with Hyungjin Kim, Jeong-Pyong Hong

at Lumley Castle, Nov. 21, 2016

Effects of kinetic decoupling

Elastic scattering between
DM and nucleus

Suppressing growth of the
density perturbation of DM



DM annihilation cross section

Effects of kinetic decoupling

Elastic scattering between
DM and nucleus

Direct
Detection
of DM

Suppressing growth of the
density perturbation of DM

Small scale
structure

Dark
Matter
abundance

DM annihilation cross section

Outline

- WIMP in standard thermal history
  - How to determine a DM relic abundance
- Early universe with low reheating temperatures
  - Thermal and Non-thermal production of WIMPs
- Non-thermal WIMP
  - Categorizing mechanisms for the final DM abundance
- Constraints
  - dark matter density, direct/indirect detections
- Summary and Outlook

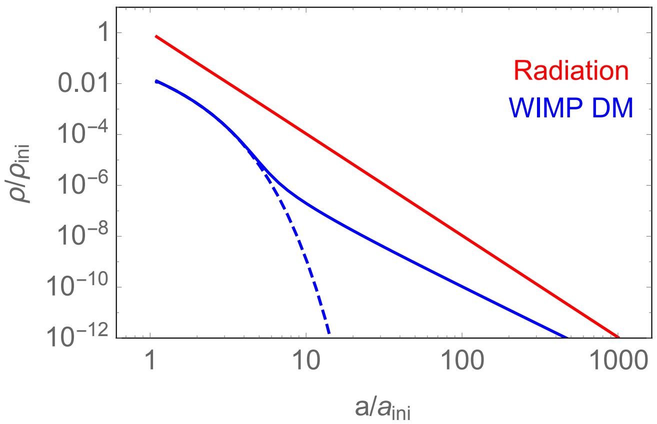
WIMP in
standard thermal history

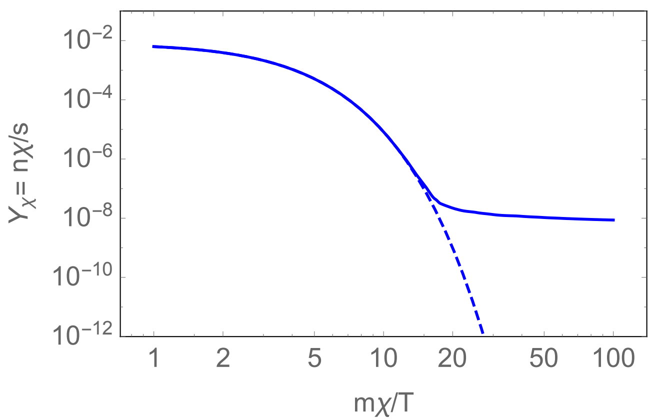
Thermal freeze out

- *At a high temperature, the WIMP DMs are in equilibrium. As the temperature drops below the DM mass, the number changing interactions are frozen so that the total number is preserved.*

$$\dot{n}_\chi + 3Hn_\chi = -\langle\sigma_{\rm ann}v_{\rm rel}\rangle_\chi n_\chi^2 + \langle\sigma_{\rm ann}v_{\rm rel}\rangle_T (n_\chi^{\rm eq})^2$$
$$= -\langle\sigma_{\rm ann}v_{\rm rel}\rangle_T (n_\chi^2 - (n_\chi^{\rm eq})^2)$$

$$\boxed{\begin{aligned}\langle\sigma_{\rm ann}v_{\rm rel}\rangle_\chi &\simeq \frac{\alpha_{\rm ann}^2}{m_\chi^2}\left(\frac{2\langle p_\chi^2\rangle}{m_\chi^2}\right)^{k_{\rm ann}} \\ &= \frac{\alpha_{\rm ann}^2}{m_\chi^2}\left(\frac{6T}{m_\chi}\right)^{k_{\rm ann}}\end{aligned}}$$





$$\left(\Gamma_{\rm ann} = \langle\sigma_{\rm ann}v_{\rm rel}\rangle_\chi n_\chi \simeq H\right)_{\text{at } T=T_{\rm fr}}$$

$$Y_\chi(t_0) = \frac{H_{\rm fr}}{\langle\sigma_{\rm ann}v_{\rm rel}\rangle_{T_{\rm fr}} s_{\rm fr}}$$

- *Why is the annihilation cross-section averaged for a thermal distribution?*

$$\langle\sigma_{\rm ann}v_{\rm rel}\rangle_\chi = \frac{\int d^3p_\chi d^3\tilde{p}_\chi f_\chi^{\rm eq}(p_\chi, T) f_\chi^{\rm eq}(\tilde{p}_\chi, T)\,(\sigma_{\rm ann}v_{\rm rel})}{\int d^3p_\chi d^3\tilde{p}_\chi f_\chi^{\rm eq}(p_\chi, T) f_\chi^{\rm eq}(\tilde{p}_\chi, T)}$$

$$\boxed{f_\chi^{\rm eq} = (e^{E/T} \pm 1)^{-1}}$$

Kinetic decoupling

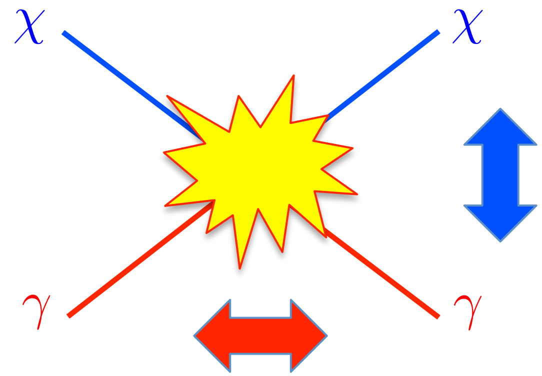
- *Number conserving interactions (elastic scatterings) are decoupled far later than the number changing interactions.*

$$\Gamma_{\rm ann} = \langle \sigma_{\rm ann} v_{\rm rel} \rangle_\chi n_\chi$$

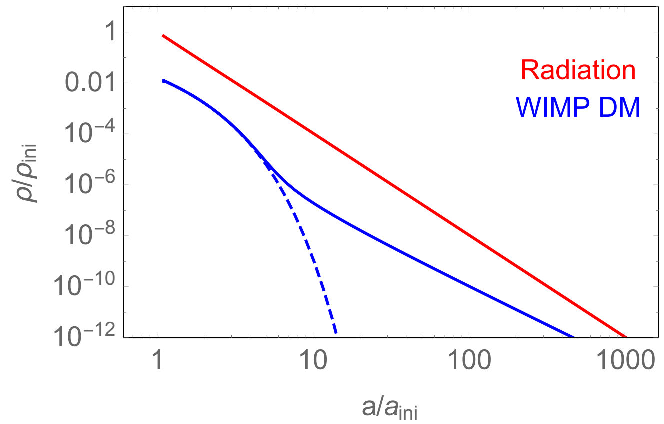
$$\Gamma_{\rm el} = \langle \sigma_{\rm el} v_{\rm rel} \rangle_{\chi T} n_\gamma \frac{T}{m_\chi}$$

$$n_\chi \ll n_\gamma$$

$$x_{\rm fr} \ll x_{\rm kd} = m_\chi / T_{\rm kd}$$



$$\begin{aligned} x_{\rm fr} &\simeq \ln m_\chi M_{\rm pl} \langle \sigma_{\rm ann} v_{\rm rel} \rangle_{T_{\rm fr}} \\ &= 25 + \ln \frac{m_\chi}{100\,{\rm GeV}} + \ln \frac{\langle \sigma_{\rm ann} v_{\rm rel} \rangle_{T_{\rm fr}}}{3 \times 10^{-26} {\rm cm}^3/\,{\rm sec}} \\ x_{\rm kd} &\simeq \sqrt{m_\chi M_{\rm pl} \langle \sigma_{\rm el} v_{\rm rel} \rangle_{\chi T_{\rm kd}}} \\ &= 10^3 \left(\frac{m_\chi}{100\,{\rm GeV}} \frac{\langle \sigma_{\rm el} v_{\rm rel} \rangle_{\chi T_{\rm kd}}}{5 \times 10^{-32} {\rm cm}^3/\,{\rm sec}} \right)^{1/2} \end{aligned}$$



- *Since the freeze-out happens during kinetic equilibrium, the relic abundance does not explicitly depend on the elastic scattering rate.*

Early Universe with low reheating temperatures

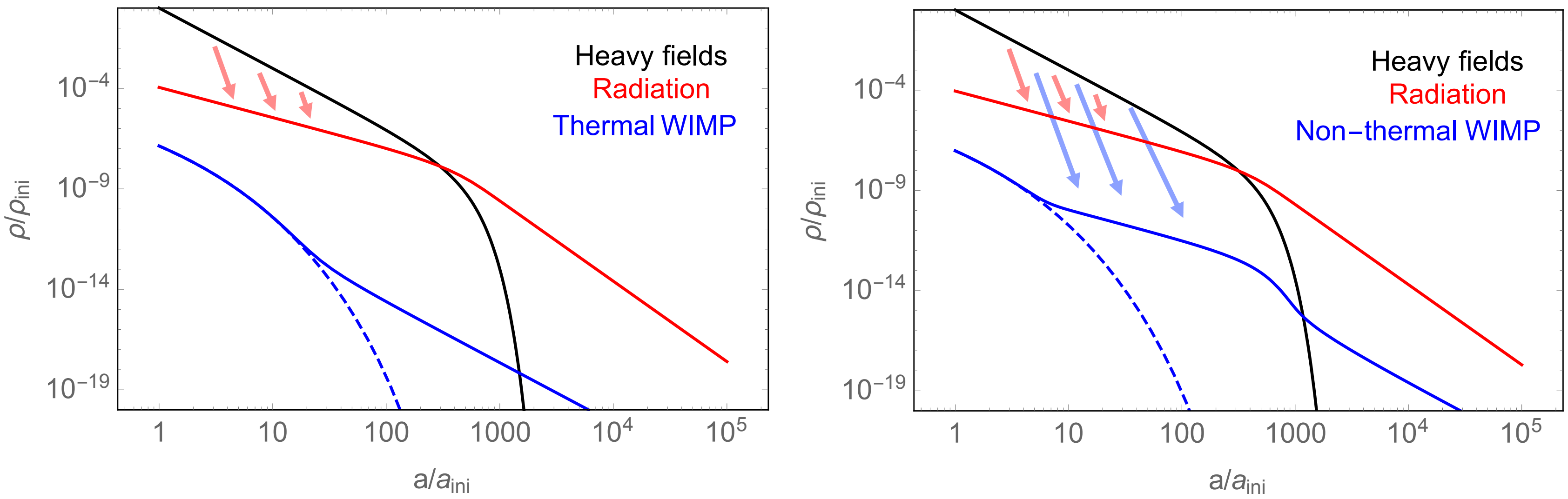
Long lived heavy particles

- *There are many candidates for long lived particles in beyond SM such as string moduli, saxion/axino, Q-ball, etc.* ➡ ***Early Matter domination***

$$\boxed{e.g.\ \Gamma_\phi = \frac{1}{16\pi}\frac{m_\phi^3}{M_{\rm GUT}^2} \simeq 3.4\times 10^{-20}\left(\frac{m_\phi}{100\,{\rm TeV}}\right)^3}$$

$$\Gamma_\phi = H \ \Rightarrow T_{\rm reh} \simeq \sqrt{\Gamma_\phi M_{\rm pl}} = 150\,{\rm MeV}\sqrt{\frac{\Gamma_\phi}{10^{-20}\,{\rm GeV}}}$$

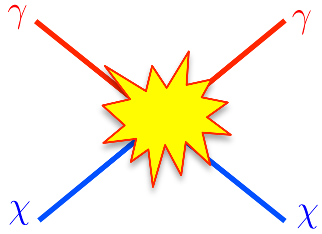
- *The "reheating temperature" could be lower than the DM freeze-out temperature, and also lower than the kinetic decoupling temperature.*

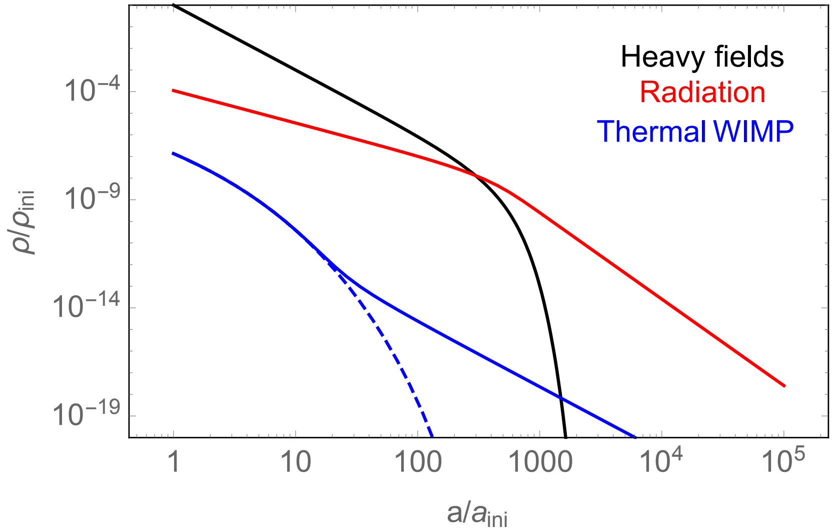


Probing the effect of kinetic decoupling 1

- *Depending on the branching fractions, histories of the dark matter abundances are different.*

  - *Thermal WIMP* *(* $\mathrm{Br}(\phi \to \chi + \cdots) \simeq 0$ *)*





✓ *No significant effect of kinetic decoupling on the relic abundance*

[J. McDonald, 1991]
[G. F. Giudice, E. W Kolb, A. Riotto 2000]

$$Y_\chi(t_0) \simeq Y_\chi(t_0)|_{\rm std}\left(\frac{T_{\rm reh}}{T_{\rm fr}}\right)^3 \qquad \boxed{x_{\rm kd} > x_{\rm fr}}$$

- *However…*

  - *small scale isocurvature perturbation for thermal WIMP dark matter*
  - *thermal Goldstino production*

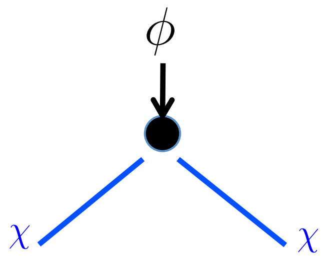
[K.-Y. Choi, J.-O. Gong, CSS 2015]

[A. Monteux, CSS 2015]

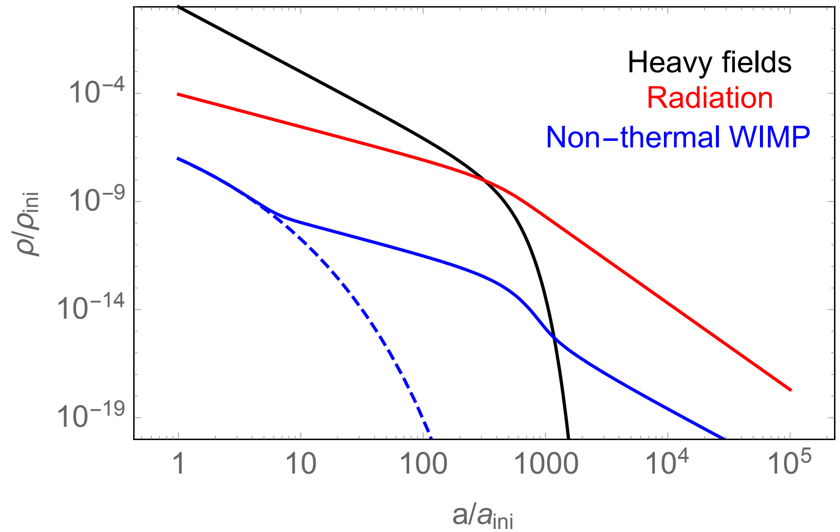
Probing the effect of kinetic decoupling 2

- *Depending on the branching fractions, histories of the dark matter abundances are different.*

  - Non-thermal WIMP ($\mathrm{Br}(\phi \to \chi + \cdots) > 0$ *)*



$p_\chi^{\rm ini} \sim m_\phi \gg m_\chi$



In this plot, we assume instantaneous (kinetic) thermalization of DM :

$$p_\chi \to p_\chi^{\rm eq} = \sqrt{3 m_\chi T} \ll m_\chi$$

$$\boxed{\Gamma_{\rm el} > H_{\rm reh}}$$

✓ *The abundance generated at the end of reheating is important.*

[M. Fujii, K. Hamaguchi 2002] $\boxed{\Gamma^{\rm ini}_{\rm ann} > H_{\rm reh} \;\; \Rightarrow \Gamma_{\rm ann} \to H_{\rm reh}}$ $\quad Y_\chi(t_0) \simeq \dfrac{H_{\rm reh}}{\langle \sigma_{\rm ann} v_{\rm rel} \rangle_{T_{\rm reh}} s_{\rm reh}}$

- *However…*

- complete thermalization could not happen for a low reheating temp.
- cross-sections depend on the momentum evolution
- relative size between $\Gamma_{\rm ann}$ *and* $\Gamma_{\rm el}$ *could be important for p-wave ann. DM*

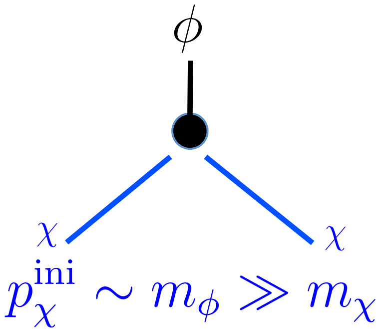
Non-thermal WIMP

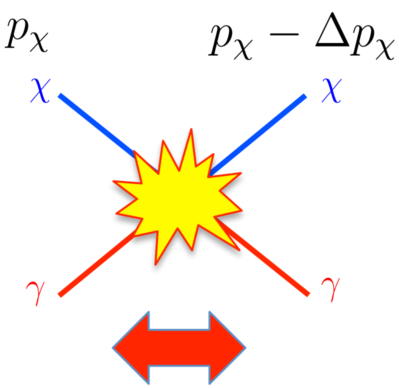
[H. Kim, J-P. Hong, CSS 1611.02287]

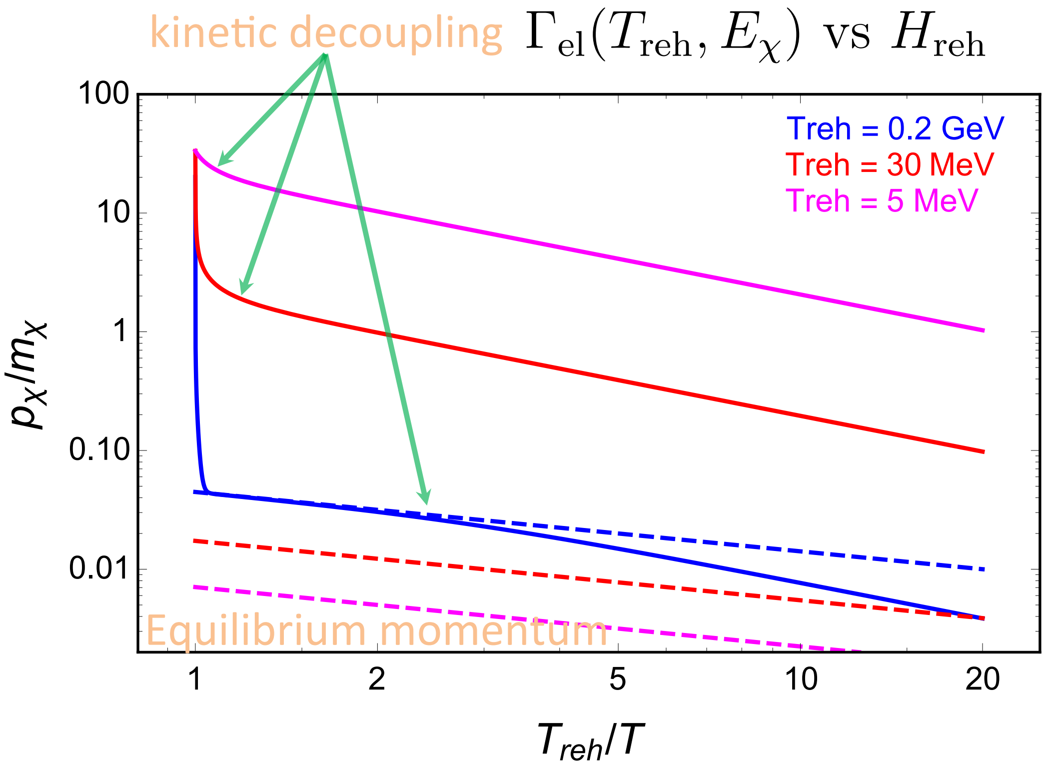
Evolution of the DM momentum

$$\boxed{\frac{dp_\chi}{dt} + Hp_\chi = -\langle \sigma_{\rm el} v_{\rm rel} \Delta p_\chi \rangle_{\chi,T}\, n_\gamma}$$

- *After the DMs are produced by decay of heavy particles at the end of the reheating (at T = Treh), Elastic scatterings between DM and radiations determine the evolution of the DM momentum.*







Categorizing mechanisms

- *(N.A.) No Annihilation :*

$$\Gamma_{\rm ann}(T, E_\chi) < H(T) \quad \text{for } T \leq T_{\rm reh}$$

- *(I.A.) Instantaneous Annihilation :*

$$\Gamma_{\rm el}(T_{\rm reh}, E_\chi) > H_{\rm reh} \ : E_\chi \to E_\chi^{\rm eq}$$

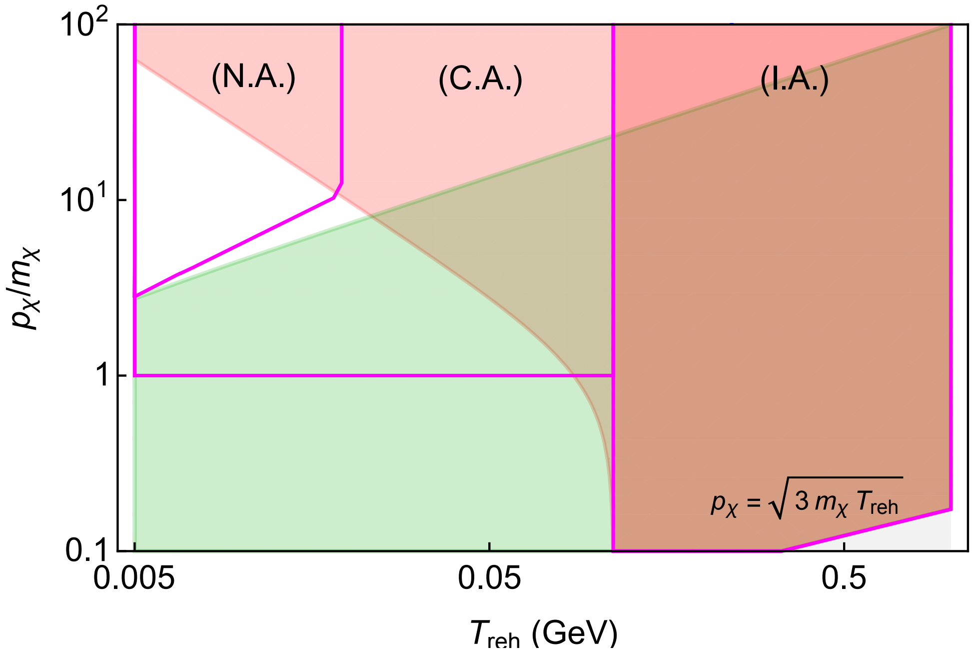
$$\Gamma_{\rm ann}(T_{\rm reh}, E_\chi) \to H_{\rm reh} \quad \text{at } T \simeq T_{\rm reh}$$

- *(C.A.) Continuous Annihilation :*

$$\Gamma_{\rm el}(T_{\rm reh}, E_\chi^{\rm dec}) = H_{\rm reh} \quad \text{for } m_\chi \ll E_\chi^{\rm dec} \leq E_\chi^{\rm ini}$$

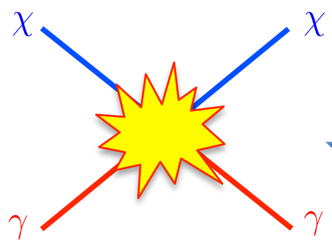
$$\Gamma_{\rm ann}(T_*, E_\chi \sim m_\chi) \to H_* \quad \text{until } T_* < T_{\rm reh}$$

$$\langle \sigma_{\rm ann} v_{\rm rel} \rangle_\chi \propto 1/E_\chi^2 \propto a^2 \quad \text{for } T < T_{\rm reh}$$



$$\Gamma_{\rm ann}(T, E_\chi) = n_\chi \langle \sigma_{\rm ann} v_{\rm rel} \rangle_\chi ,$$
$$\Gamma_{\rm el}(T, E_\chi) = n_\gamma \left\langle \sigma_{\rm el} v_{\rm rel} \frac{\Delta p_\chi}{p_\chi} \right\rangle_{\chi, T}$$

Evolution of the annihilation cross-section



$$\boxed{\dot{n}_\chi + 3Hn_\chi = -\langle\sigma_{\rm ann}v_{\rm rel}\rangle_\chi n_\chi^2}$$

$$Y_\chi(t_0) = Y_\chi(t_{\rm reh})\left(1+\frac{n_\chi^{\rm reh}}{H_{\rm reh}}\int_0^1 du\langle\sigma_{\rm ann}v_{\rm rel}\rangle_{p_\chi(u)}\right)^{-1}$$

$$\boxed{u = T/T_{\rm reh}}$$

- *Evolution of the annihilation cross-section is important to determine the relic density.*

$$\langle\sigma_{\rm ann}v_{\rm rel}\rangle_\chi = \frac{\alpha_{\rm ann}^2}{E_\chi^2}\left(\frac{2p_\chi^2}{E_\chi^2}\right)^{k_{\rm ann}}, \quad \langle\sigma_{\rm el}v_{\rm rel}\rangle_{\chi,T} = \frac{\alpha_{\rm el}^2}{m_\chi^2}\left(\frac{E_\chi^2T^2}{m_\chi^4}\right)^{k_{\rm el}}$$

$$\boxed{\begin{aligned} k_{\rm ann} &= 0 \ (s\text{-wave}) \\ &= 1 \ (p\text{-wave}) \end{aligned}}$$

$$\boxed{\begin{aligned} k_{\rm el} &= 0 \ (e.g.\text{Thomson scattering}) \\ &= 1 \ (e.g. t\text{-channel scalar mediator}) \end{aligned}}$$

- *Introducing momentum independent variables,*

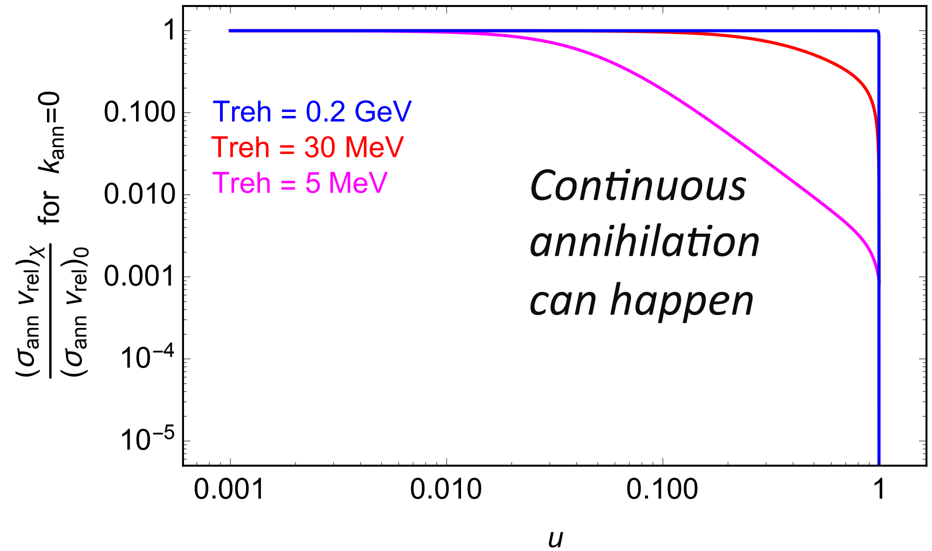
$$\langle\sigma_{\rm ann}v_{\rm rel}\rangle_0 \equiv \frac{\alpha_{\rm ann}^2}{m_\chi^2}, \quad \langle\Gamma_{\rm ann}\rangle_0 \equiv \langle\sigma_{\rm ann}v_{\rm rel}\rangle_0 n_\chi^{\rm reh},$$

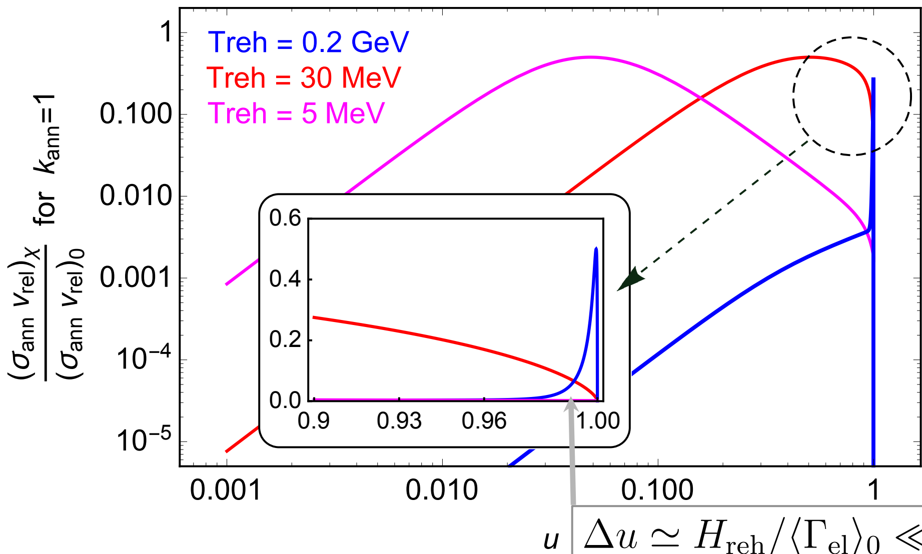
$$\langle\sigma_{\rm el}v_{\rm rel}\rangle_0 \equiv \frac{\alpha_{\rm el}^2T_{\rm reh}^2}{m_\chi^4}, \ \langle\Gamma_{\rm el}\rangle_0 \equiv \langle\sigma_{\rm el}v_{\rm rel}\rangle_0\frac{T_{\rm reh}n_\gamma^{\rm reh}}{m_\chi}.$$

Analytic approximation

- *Evolution of the annihilation cross-section*

$$\boxed{u = T/T_{\rm reh}}$$





$$\boxed{\Delta u \simeq H_{\rm reh}/\langle\Gamma_{\rm el}\rangle_0 \ll 1}$$

$H_{\rm reh} \gg \langle\Gamma_{\rm el}\rangle_0$

$$Y_\chi(t_0) \simeq \min\left[\underbrace{Y_\chi(t_{\rm reh})}_{\text{(N.A.)}},\ \underbrace{\frac{H_{\rm reh}}{\langle\sigma_{\rm ann}v_{\rm rel}\rangle_0 s_{\rm reh}}\left(\frac{c_0 H_{\rm reh}}{\langle\Gamma_{\rm el}\rangle_0}\right)^{1/3}}_{\text{(C.A.)}}\right]$$

Peak contribution becomes important for
$H_{\rm reh} \ll \langle\Gamma_{\rm el}\rangle_0 < \langle\Gamma_{\rm ann}\rangle_0$

$H_{\rm reh} \ll \langle\Gamma_{\rm el}\rangle_0$

$$Y_\chi(t_0) = \min\left[\underbrace{Y_\chi(t_{\rm reh})}_{\text{(N.A.)}},\ \underbrace{\frac{H_{\rm reh}}{\langle\sigma_{\rm ann}v_{\rm rel}\rangle_0 s_{\rm reh}}}_{\text{(I.A.)}}\right] \quad (k_{\rm ann}=0)$$

$$Y_\chi(t_0) \simeq \min\left[\underbrace{Y_\chi(t_{\rm reh})}_{\text{(N.A.)}},\ \frac{H_{\rm reh}}{\langle\sigma_{\rm ann}v_{\rm rel}\rangle_0 s_{\rm reh}}\underbrace{\left[\frac{c_0 H_{\rm reh}}{\langle\Gamma_{\rm el}\rangle_0} + \left(3 - \frac{T_{\rm kd}^2}{T_{\rm reh}^2}\right)\frac{T_{\rm reh}}{m_\chi}\right]^{-1}}_{\text{(I.A.)}}\right] \quad (k_{\rm ann}=1)$$

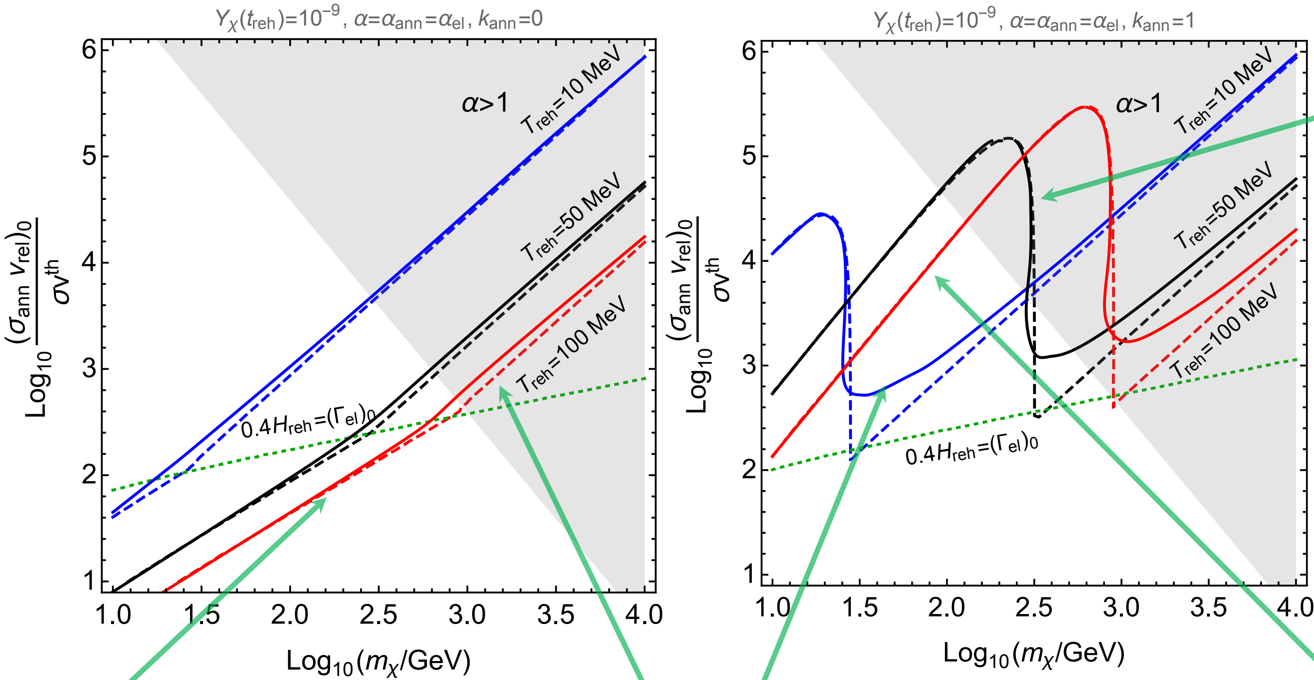
Constraints

Dark matter density

- *Assumption :* $\alpha_{\rm ann} = \alpha_{\rm el} = \alpha \leq 1$
- *Present dark matter density :* $\Omega_\chi h^2 \simeq 0.11$

$$\langle \sigma_{\rm ann} v_{\rm rel} \rangle_\chi = \frac{\alpha_{\rm ann}^2}{E_\chi^2} \left(\frac{2p_\chi^2}{E_\chi^2} \right)^{k_{\rm ann}} , \quad \langle \sigma_{\rm el} v_{\rm rel} \rangle_{\chi,T} = \frac{\alpha_{\rm el}^2}{m_\chi^2} \left(\frac{E_\chi^2 T^2}{m_\chi^4} \right)^{k_{\rm el}}$$

$$\Omega_\chi h^2 = 0.11 \left(\frac{m_\chi}{100\,{\rm GeV}} \right) \left(\frac{Y_\chi(t_0)}{4 \times 10^{-12}} \right)$$



$\Omega_\chi h^2 \propto T_{\rm reh}^3 m_\chi^{-2}$

(I.A.) that peak contributions are dominant

$\Omega_\chi h^2 \propto \langle \sigma_{\rm ann} v_{\rm rel} \rangle_0^{-1} T_{\rm reh}^{-1} m_\chi$

(I.A.)

$\Omega_\chi h^2 \propto \langle \sigma_{\rm ann} v_{\rm rel} \rangle_0^{-4/3} T_{\rm reh}^{-7/3} m_\chi^2$

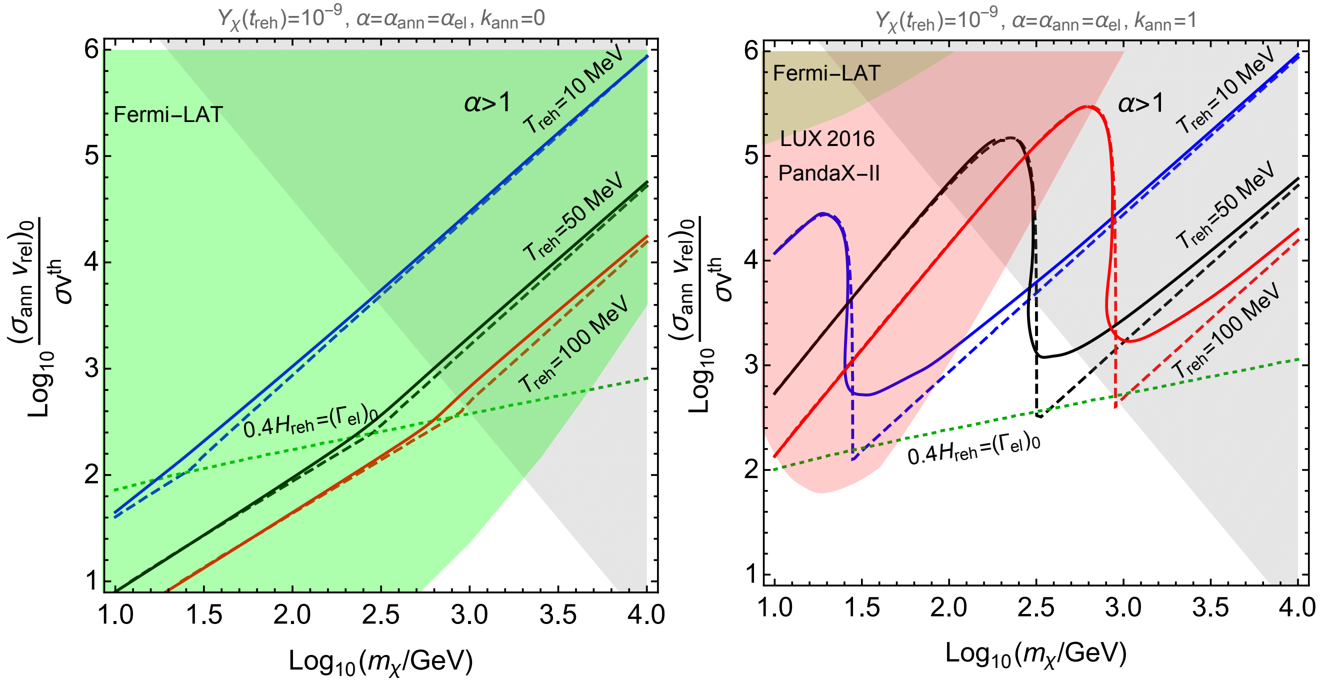
(C.A.)

$\Omega_\chi h^2 \propto \langle \sigma_{\rm ann} v_{\rm rel} \rangle_0^{-1} T_{\rm reh}^{-2} m_\chi^2$

(I.A.)

Direct/indirect detections

- *The s-wave annihilating DM is already ruled out by indirect detection constraints.*
- *The DM should be lepto-philic to be thermalized at a low temperature and quark-phobic by direct detection constraints*



- *The effective operator as a benchmark example :*

$$\mathcal{L}_{\rm eff} = \frac{(\bar{\chi}\chi)(\bar{l}l)}{\Lambda^2}$$

- two loop induced elastic scattering between the DM and nucleus

Summary and Outlook

- *We have studied the effect of kinetic decoupling/elastic scattering on non-thermally produced WIMP dark matter phenomenology.*

- *We categorize each mechanisms and present the approximated analytic formulae.*

- *With low reheating temperatures, a p-wave annihilating DM is still viable and the corresponding relic abundance explicitly depends on the elastic scattering rate.*

- *It is also probable to think "dark wimp" in which dark matters are thermalized by dark radiations. Interesting connections between dark matter property and early history of the universe and late time cosmology can be made and predicted.*