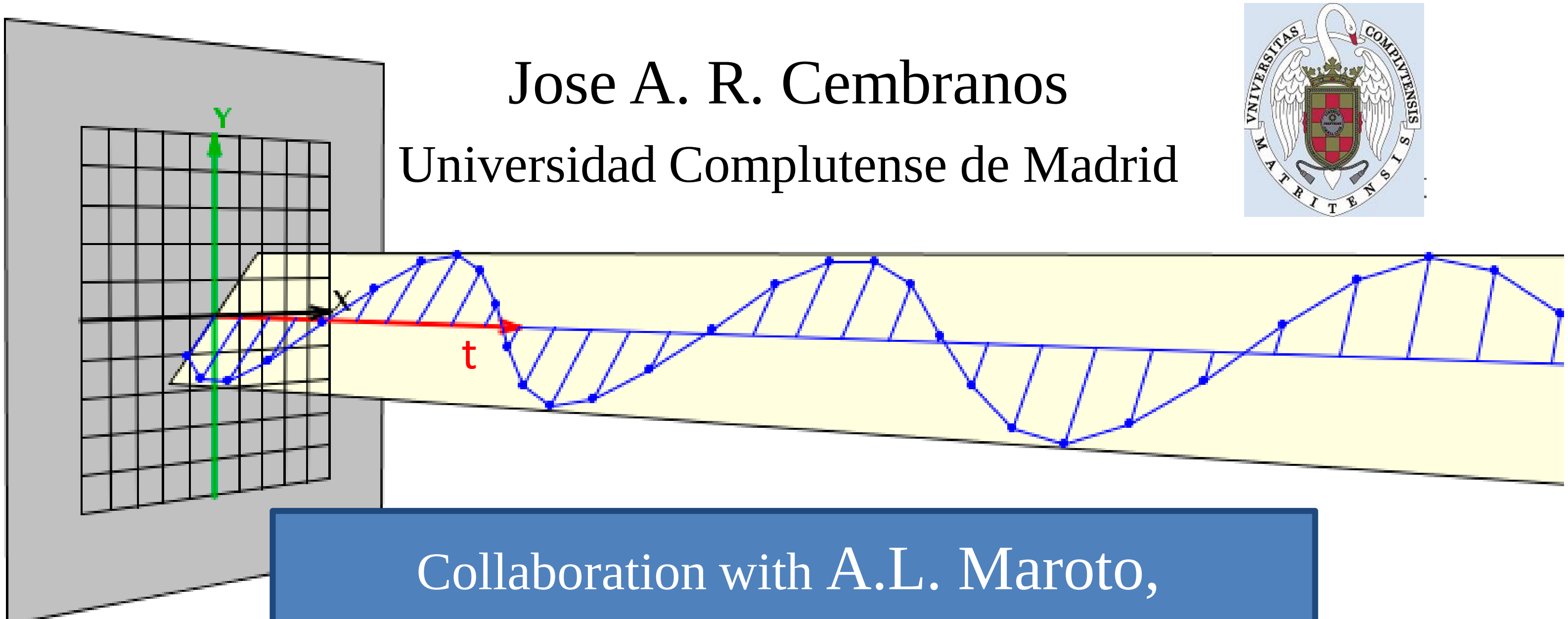
Ultralight bosonic dark matter with arbitrary spin



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Collaboration with A.L. Maroto, S.J. Núñez Jareño and H. Villarrubia

Dark Matter from aeV to ZeV: 3rd IBS-MultiDark-IPPP Workshop

21-25 November 2016
Lumley Castle

Outline

- Introduction
  - Coherent bosons in cosmology:
    Dark Matter Candidates
- Background evolution
  - Scalar fields
  - Isotropy theorem for
    Abelian vector fields
    Non-Abelian vector fields
    General case: Spin 2 fields
- Perturbations:
  - Scalar fields
  - Vector fields
- Conclusions

Introduction

- Most of the DM models based on a **particle** description of the DM candidates

- Small-scale "issues" :

  a) missing satellite, b) too-big-to-fail, c) cusp-core

- Solutions proposed: a) baryonic physics effects,

  b) alternative DM models: warm, self-interacting, decaying DM,…

- Another solution **wave dark matter ΨDM** (Sin, PRD 50, 3650 (1994), Guzmán-Matos, CQG 17, L9 (2000)) also known as **fuzzy DM** (Hu et al, PRL85, 1158 (2000))

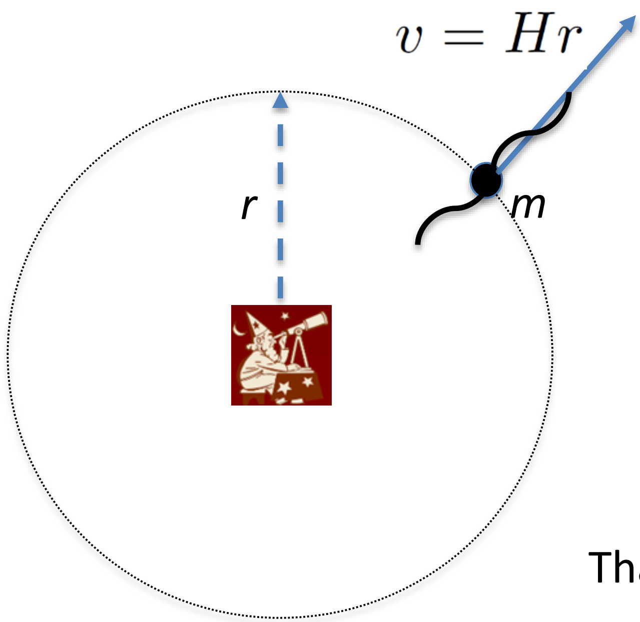
- Existing wave DM models based on ultralight axions or axion-like particles ($m_a \sim 10^{-22}$ eV). What about higher-spin wave DM?

Particle DM vs. Wave DM

Heuristic interpretation (Hu et al, PRL85, 1158 (2000), Hlozek et al, PRD 91 103512 (2015))

Consider a particle of mass *m* << 1 eV moving with the Hubble flow *H*

$\lambda_{\text{Compton}} >> d_{\text{inter part.}}$ DM described by a classical field



The corresponding de Broglie wavelength:

$$\lambda_{\rm dB} = \frac{1}{mv} = \frac{1}{mHr}$$

Thus, the particle can be localized only in a sphere with radius:

$$r \geq \lambda_{\rm dB} \quad \Longrightarrow \quad \boxed{r \geq \frac{1}{\sqrt{Hm}}}$$

That corresponds to a (physical) wavenumber k=π/r

$$k_\star = \pi\sqrt{mH}$$

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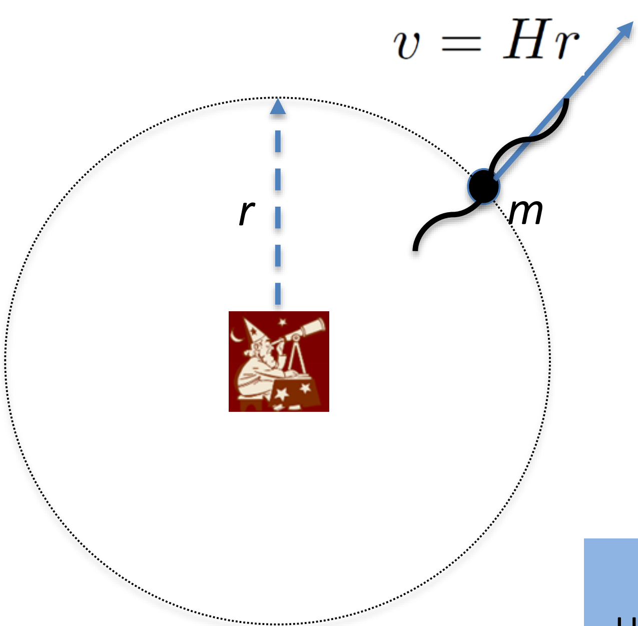


Thus, we have:

$k < \pi\sqrt{Hm}$ particle-like behaviour

$k > \pi\sqrt{Hm}$ wave-like behaviour

Jeans scale = de Broglie wavelength

Uncertainty principle modifies small-scale structure formation

Background evolutions

Scalar fields in cosmology

Coherent scalar fields are the standard candidates for solving cosmological unresolved questions as:

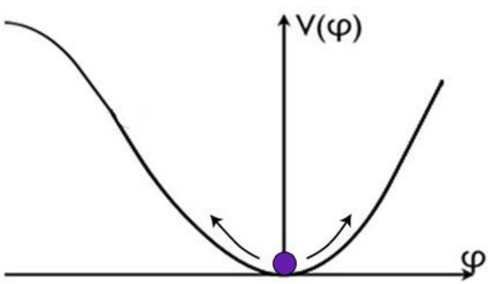
- Inflation: Inflaton.
- Dark matter: Axion and axion-like particles.
- Dark energy: quintessence, scalar-tensor theories of gravity,...

Simalar results can be provided by any bosonic field. Vector fields, and new vectors are maybe the best motivated theoretically.

Background evolutions

Scalar field DM

Homogeneous field (M. S. Turner, Phys. Rev. D 28 (1983) 6.)



$$\boxed{V(\phi) = a\phi^n}$$

$$\omega = \frac{n-2}{n+2}$$

Average eq. of state

$$\begin{cases} n = 2 & \text{matter} \\ n = 4 & \text{radiation} \end{cases}$$

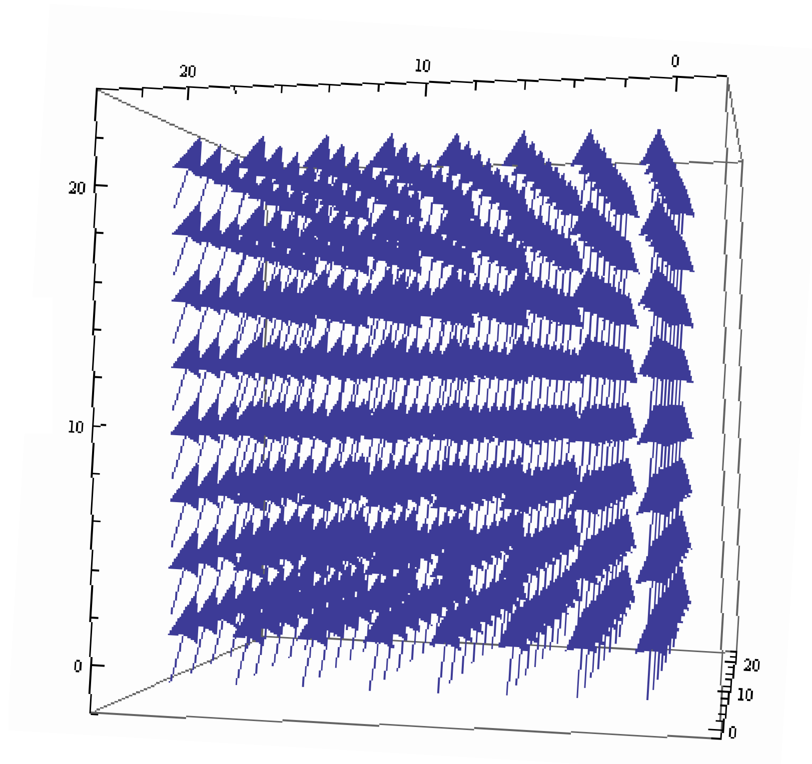
Theoretical frameworks for vector fields

New vector fields appear in many theoretical extensions of the Standard Model:

- **GUT** P. Langacker, Phys. Rep. 72 C, 185 (1981)
- **SUSY** S. Weinberg, Phys. Rev. D 26, 287 (1982); P. Fayet, Nucl. Phys. B 187, 184 (1981)
- **Fith force extensions** E. D. Carlson, Nucl. Phys. B 286, 378 (1987)
- **Paraphoton models** L. B. Okun, Sov. Phys. JETP 56, 502 (1982) [Zh. Eksp. Teor. Fiz. 83, 892 (1982)]
- **Superstring compactifications** J. Ellis et al., Nucl. Phys. B 276, 14 (1986); M. Goodsell, A. Ringwald, Fortsch. Phys. 58, 716 (2010)

The anisotropy problem

However, vector coherent oscillations are generally anisotropic. This fact can be in contradiction with the large isotropy of the universe as shown by the cosmic microwave background (CMB).



The anisotropy problem

There are different solutions in the literature:

- Using the scalar degree of freedom $\mathbf{A_0}$.

Beltran Jimenez, Maroto, Phys. Rev. D78, 063005 (2008)

Beltran Jimenez, Maroto, JCAP 0903, 016 (2009)

- **Particular solutions**: Triads of orthogonal vectors.

H.H. Soleng, Astron. Atrophys. 237, 1 (1990)

Bento, Bertolami, Moniz, Mourao, Sa, Class. Quant. Grav. 10, 285 (1993)

- Large number, N, of **randomly oriented fields**. Reducing anisotropy in $\sqrt{N}$.

Golovnev,, Mukhanov, Vanchurin, JCAP 0806, 009 (2008)

- **Average isotropy** for a linear polarized Abelian vector coherent oscillation with potential $A_\mu A^\mu$.

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Isotropy theorem for Abelian vector fields

Abelian vector fields described by the action:

$$S = \int d^4x \sqrt{g} \left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - V\left(A_\mu A^\mu\right) \right)$$

If the **field evolves rapidly** and $\boldsymbol{A_i}$ **,** $\boldsymbol{\dot{A}_i}$ **are bounded** during its evolution:

1.- **The energy momentum tensor is diagonal and isotropic in average.**

JARC, Hallabrin, Maroto, Nunez Jareno, Phys. Rev. D86 (2012)

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Similar argument than for the Virial theorem in classical mechanics:

(for a FLRW background) $\Longrightarrow$

$$0 = \frac{G_{ij}(T) - G_{ij}(0)}{T} = \left\langle 2V'(A^2)\frac{A_iA_j}{a^2}\right\rangle + \left\langle\frac{\dot{A}_i\dot{A}_j}{a^2}\right\rangle$$

$$G_{ij} = \frac{\dot{A}_iA_j}{a^2}, \quad i,j = 1,2,3$$

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If the **field evolves rapidly** and $\boldsymbol{A_i}$ **,** $\boldsymbol{\dot{A}_i}$ **are bounded** during its evolution:

1.- **The energy momentum tensor is diagonal and isotropic in average.**

2.- **Under power law potentials, the equation of state parameter is constant:**

$$V = \lambda (A_\mu A^\mu)^n \quad \rightarrow \quad \omega = \frac{\langle p \rangle}{\langle \rho \rangle} = \frac{n-1}{n+1}$$

JARC, Hallabrin, Maroto, Nunez Jareno, Phys. Rev. D86 (2012)

Isotropy theorem for Yang-Mills theories

Yang-Mills theories associated with semi-simple groups described by the action:

$$S = \int d^4x \sqrt{g} \left(-\frac{1}{4} F^a{}_{\mu\nu} F^{a\,\mu\nu} - V\left(A^a{}_\mu A^{a\,\mu}\right) \right)$$

If the **field evolves rapidly** and $\boldsymbol{A^a{}_i}$ **,** $\boldsymbol{\dot{A}^a{}_i}$ **are bounded** during its evolution,

1.- **The energy momentum tensor is diagonal and isotropic in average.**

2.- **Without potential, the equation of state parameter is w = 1/3 , i.e. it behaves as radiation.**

JARC, Maroto, Nunez Jareno, Phys. Rev. D87 (2013) 043523

Example I: SU(2) theory

The self-interaction for non-Abelian theories changes the average equation of state. For high energy densities or large coupling constants it will behave as radiation, in the opposite limit, the Abelian behavior is recovered.

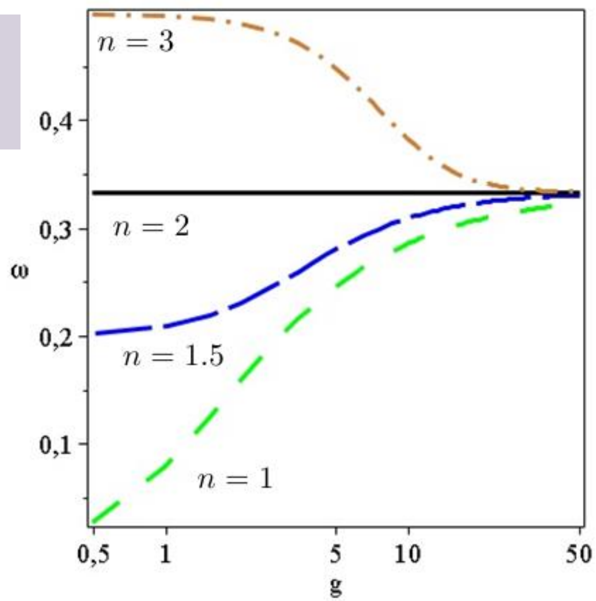
$$V = \frac{1}{2}(-M^2 A^a_\rho A^{a\,\rho})^n$$

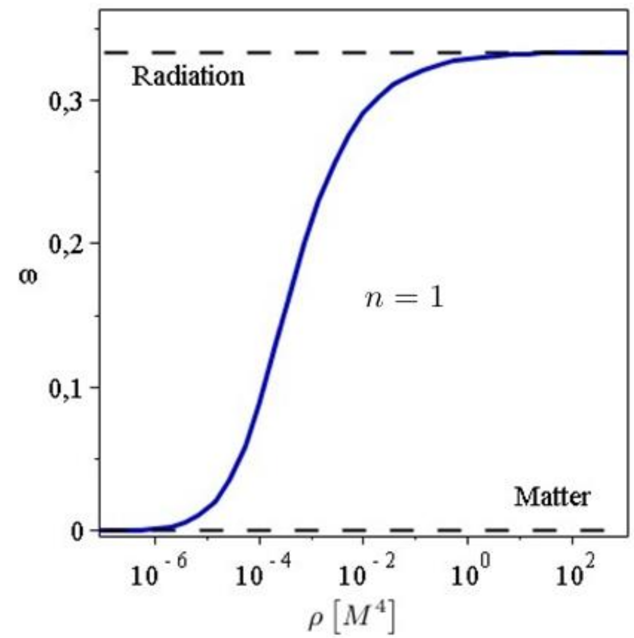
$\boldsymbol{g} \downarrow,\ \boldsymbol{\rho} \downarrow$

$$\boldsymbol{\omega} = \frac{\boldsymbol{n-1}}{\boldsymbol{n+1}}$$

$\boldsymbol{g} \uparrow,\ \boldsymbol{\rho} \uparrow$

$$\boldsymbol{\omega} = \frac{\boldsymbol{1}}{\boldsymbol{3}}$$





JARC, Maroto, Nunez Jareno, Phys. Rev. D87 (2013) 043523

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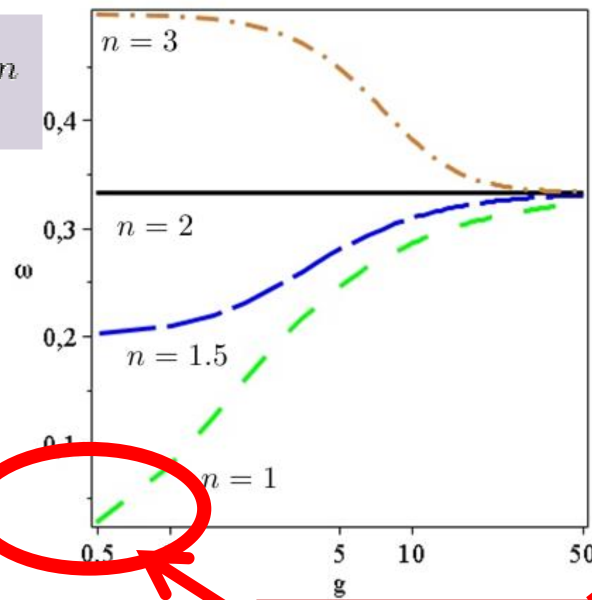
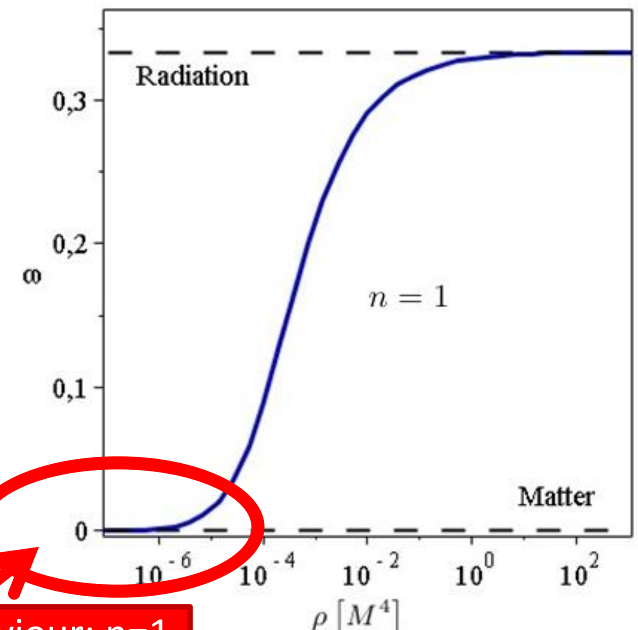




DARK MATTER behaviour: n=1

JARC, Maroto, Nunez Jareno, Phys. Rev. D87 (2013) 043523

Gauge fixing term

The previous results can be extended to actions completed with the gauge fixing term:

$$\mathcal{L} = -\frac{1}{4}F^a_{\mu\nu}F^{a\ \mu\nu} + \frac{\xi}{2}(\nabla_\rho A^{a\ \rho})^2 - V(M_{ab}A^a_\rho A^{b\rho})$$

The result is the same, if the **field evolves rapidly** and $A^a{}_i$, $\dot{A}^a{}_i$ **are bounded** during its evolution,

1.- **The energy momentum tensor is diagonal and isotropic in average.**

2.- **Without potential, the equation of state parameter is w = 1/3 , i.e. it behaves as radiation.**

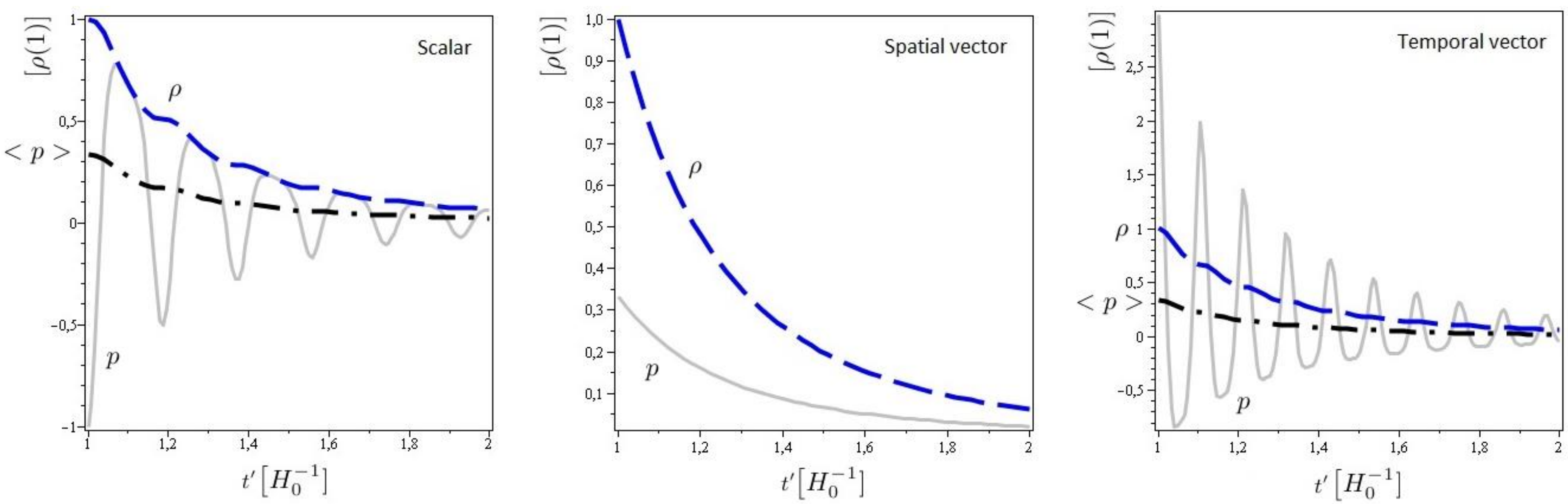
JARC, Maroto, Nunez Jareno, Phys. Rev. D87 (2013) 043523

Example II: n=2

For a power law potential, the equation of state of the average energy is the same for: scalar, Abelian vectors, spatial and temporal Non-Abelian vector components (by assuming a negligible self-interactions).

$$V = \frac{1}{2}(-M^2 A_\rho A^\rho)^n \longrightarrow \omega = \frac{n-1}{n+1}$$

Although their evolutions are very different:



JARC, Maroto, Nunez Jareno, Phys. Rev. D87 (2013) 043523

Isotropy theorem

Rapid evolving coherent vector fields do not suffer from constraining requirements from cosmological isotropy.

Isotropy Theorem: The average Energy-Momentum tensor of a vector field is diagonal and isometric if

1.- its action is

$$\mathcal{L} = -\frac{1}{4}F^a_{\mu\nu}F^{a\ \mu\nu} + \frac{\xi}{2}(\nabla_\rho A^{a\ \rho})^2 - V(M_{ab}A^a_\rho A^{b\rho})$$

2.- The vector field evolves rapidly:

with respect to the background metric evolution.

with respect to spatial variations.

3.- $A^a{}_i$ and $\dot{A}^a{}_i$ remain bounded during its evolution

JARC, Hallabrin, Maroto, Nunez Jareno, Phys. Rev. D86 (2012)

JARC, Maroto, Nunez Jareno, Phys. Rev. D87 (2013) 043523

Isotropy theorem for higher-spin fields

Homogeneous field (J.A.R. Cembranos, A.L.M., S.J. Núñez Jareño, JCAP 1403 (2014) 042)

Homogeneous fields with non-zero spin break isotropy, but:

$$\mathcal{L} \equiv \mathcal{L}\left[\phi^A, \partial_\mu \phi^A\right]$$

ϕ_A and $\dot{\phi}_A$ bounded

$$\omega_A^{-1} \ll T \ll H^{-1}$$

For **rapidly oscillating fields**, virial theorem ensures diagonal and isotropic energy-momentum tensor in average

Power-law theories:

$$\mathcal{H} = \left(\lambda^{AB} g_{00} \Pi_A^0 \Pi_B^0\right)^{n_T} + \left(M_{AB}\phi^A\phi^B\right)^{n_V}$$

Average equation of state:

$$\omega = \frac{2\,n_V}{1+\frac{n_V}{n_T}} - 1$$

Higher-spin DM

Example: **Spin 2 DM**

Spin 2. Massive gravitons as wave DM (Cembranos, A.L.M., Núñez Jareño, JCAP 1403 (2014) 042)

Fierz-Pauli Lagrangian

$$\mathcal{L} = \frac{M_{Pl}^2}{8}\Big[\nabla_\alpha h^{\mu\nu}\nabla^\alpha h_{\mu\nu} - 2\nabla_\alpha h_\mu^\alpha \nabla_\beta h^{\mu\beta} + 2\nabla_\alpha h_\mu^\alpha \nabla^\mu h_\beta^\beta - \nabla_\alpha h_\mu^\mu \nabla^\alpha h_\nu^\nu - m_g^2\left(h_{\mu\nu}h^{\mu\nu} - \left(h_\mu^\mu\right)^2\right)\Big] .$$

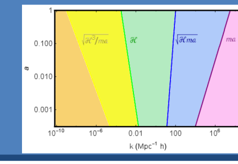
Average equation of state:

$$\omega = \frac{2n_V}{1+\frac{n_V}{n_T}} - 1 = 0$$

Perturbations

$$ds^2 \;=\; a(\eta)^2\left[(1+2\Phi(\eta,\vec{x}))\,d\eta^2-\left((1-2\Psi(\eta,\vec{x}))\,\delta_{ij}+h_{ij}(\eta,\vec{x})\right)dx^i dx^j-2Q_i(\eta,\vec{x})d\eta dx^i\right]$$

Perturbations



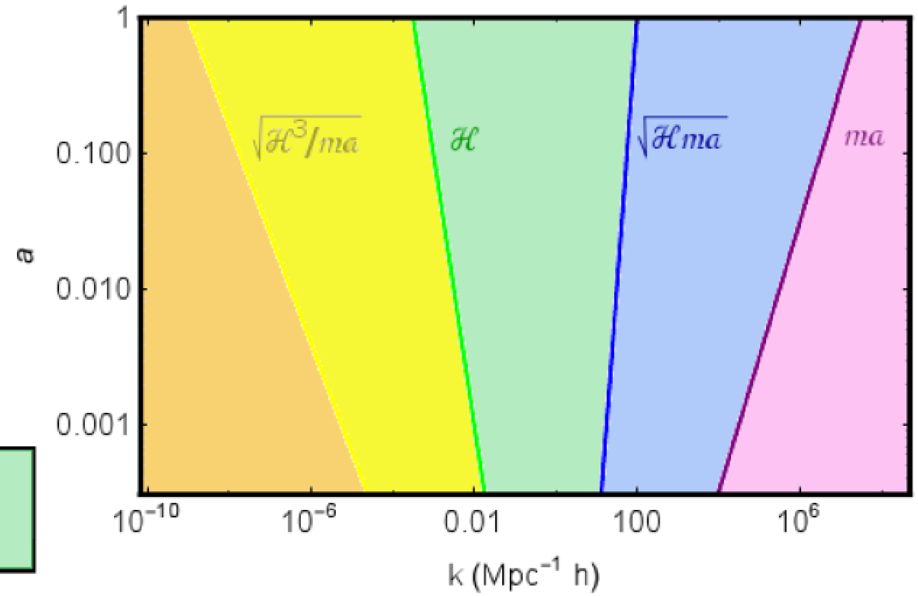
CDM	$\Psi = \Phi \sim$ const. $\delta\rho \sim a^{-3}$ $Q \sim a^{-2}$		$\Psi = \Phi \sim$ const. $\delta\rho \sim a^{-2}$ $Q \sim a^{-2}$		

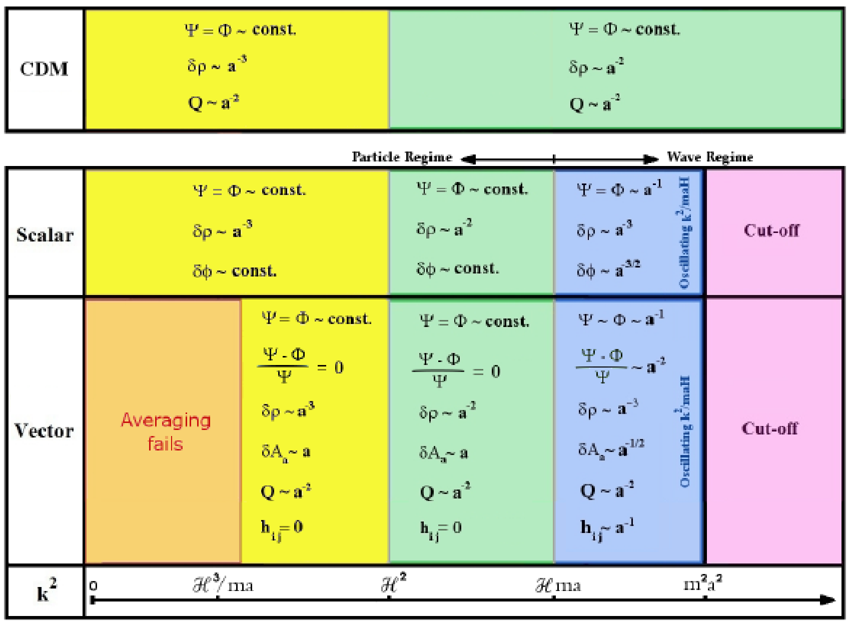
Particle Regime ⟵ | ⟶ Wave Regime

Scalar	$\Psi = \Phi \sim$ const. $\delta\rho \sim a^{-3}$ $\delta\phi \sim$ const.		$\Psi = \Phi \sim$ const. $\delta\rho \sim a^{-2}$ $\delta\phi \sim$ const.	$\Psi = \Phi \sim a^{-1}$ $\delta\rho \sim a^{-3}$ $\delta\phi \sim a^{-3/2}$ Oscillating k^2/maH	**Cut-off**
Vector	Averaging fails	$\Psi = \Phi \sim$ const. $\frac{\Psi - \Phi}{\Psi} = 0$ $\delta\rho \sim a^{-3}$ $\delta A_a \sim a$ $Q \sim a^{-2}$ $h_{ij} = 0$	$\Psi = \Phi \sim$ const. $\frac{\Psi - \Phi}{\Psi} = 0$ $\delta\rho \sim a^{-2}$ $\delta A_a \sim a$ $Q \sim a^{-2}$ $h_{ij} = 0$	$\Psi \sim \Phi \sim a^{-1}$ $\frac{\Psi - \Phi}{\Psi} \sim a^{-2}$ $\delta\rho \sim a^{-3}$ $\delta A_a \sim a^{-1/2}$ $Q \sim a^{-2}$ $h_{ij} \sim a^{-1}$ Oscillating k^2/maH	**Cut-off**
k^2	0	$\mathcal{H}^3/ma$	$\mathcal{H}^2$	$\mathcal{H}ma$	m^2a^2

JARC, A.L.Maroto, Núñez Jareño, arXiv: 1611.03793

Perturbations





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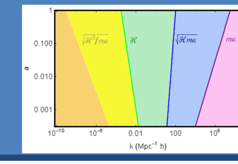

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			Particle Regime ←	→ Wave Regime	
Scalar	$\Psi = \Phi \sim$ const. $\delta\rho \sim a^{-3}$ $\delta\phi \sim$ const.		$\Psi = \Phi \sim$ const. $\delta\rho \sim a^{-2}$ $\delta\phi \sim$ const.	$\Psi = \Phi \sim a^{-1}$ $\delta\rho \sim a^{-3}$ $\delta\phi \sim a^{-3/2}$ Oscillating k^2/maH	**Cut-off**
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k^2	0	$\mathcal{H}^3/ma$	$\mathcal{H}^2$	$\mathcal{H}ma$	m^2a^2

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Perturbations: scalar field

$$V(\phi)=a\phi^n$$

$$c_{\rm eff}^2(k) \equiv \frac{\langle\delta p_k\rangle}{\langle\delta\rho_k\rangle} \begin{cases} \dfrac{n-2}{n+2} & \boxed{n = 2} \;\; = 0 \\[2ex] = \dfrac{k^2}{4m^2a^2} & \end{cases}$$

$$V(\phi)= m^2\phi^2/2$$

$$c_{\rm eff}^2(k) \equiv \frac{\langle\delta p_k\rangle}{\langle\delta\rho_k\rangle} \begin{cases} = \dfrac{k^2}{4m^2a^2} \\ \end{cases}$$

J.A.R. Cembranos, A.L.M., S.J. Núñez Jareño, JHEP 1603 (2016) 013

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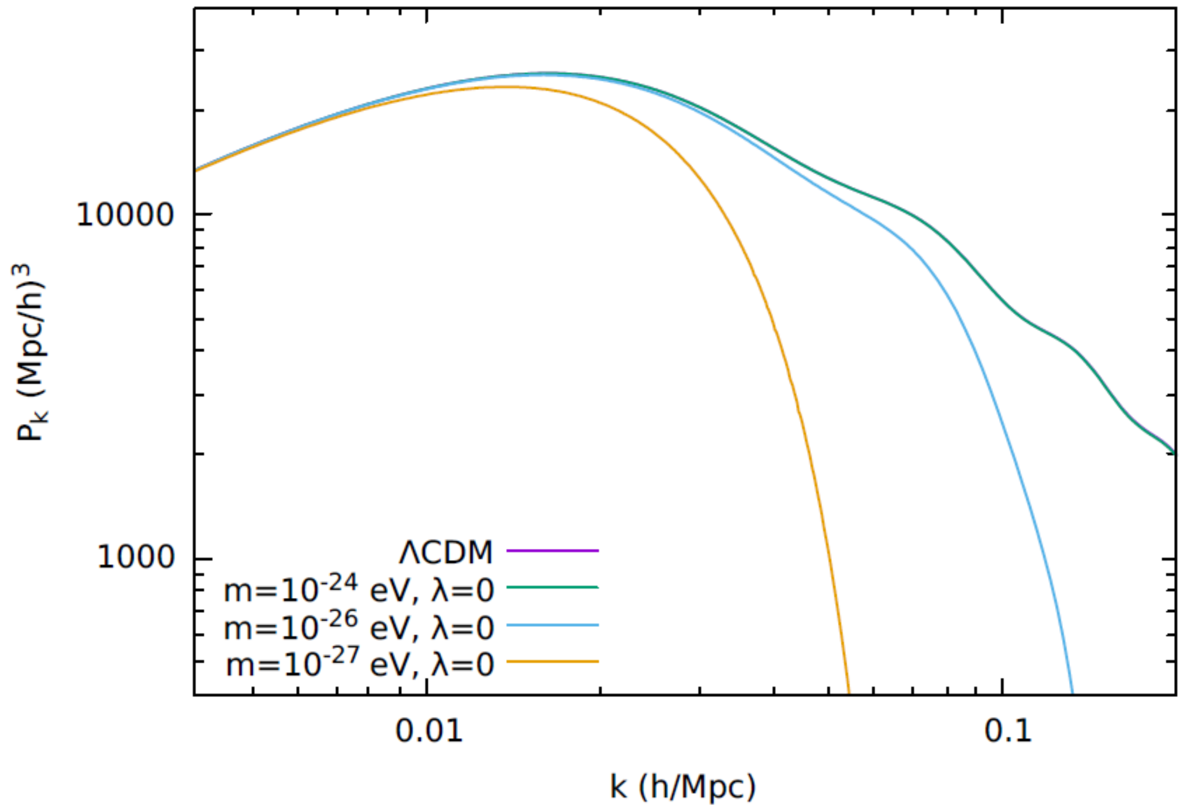
$$V(\phi)= m^2\phi^2/2+\lambda\phi^l/l$$

$$c_{\text{eff}}^2(k) \equiv \frac{\langle \delta p_k \rangle}{\langle \delta \rho_k \rangle} \begin{cases} = \dfrac{k^2}{4m^2a^2} \\[2ex] = \dfrac{k^2}{4m^2a^2} + \dfrac{(p-1)}{2^{2p}} \begin{pmatrix} 2p \\ p \end{pmatrix} \dfrac{\lambda\phi_c^{2p-2}}{m^2a^{3(p-1)}} + \mathcal{O}(\epsilon) & \boxed{l = 2p} \end{cases}$$

J.A.R. Cembranos, A.L.M., S.J. Núñez Jareño, JHEP 1603 (2016) 013

Perturbations: $V(\phi) = m^2\phi^2/2 + \lambda\phi^l/l$ $\boxed{l = 2p}$

Simulations (CLASS):



JARC, A.L.Maroto, H. Villarrubia, in preparation

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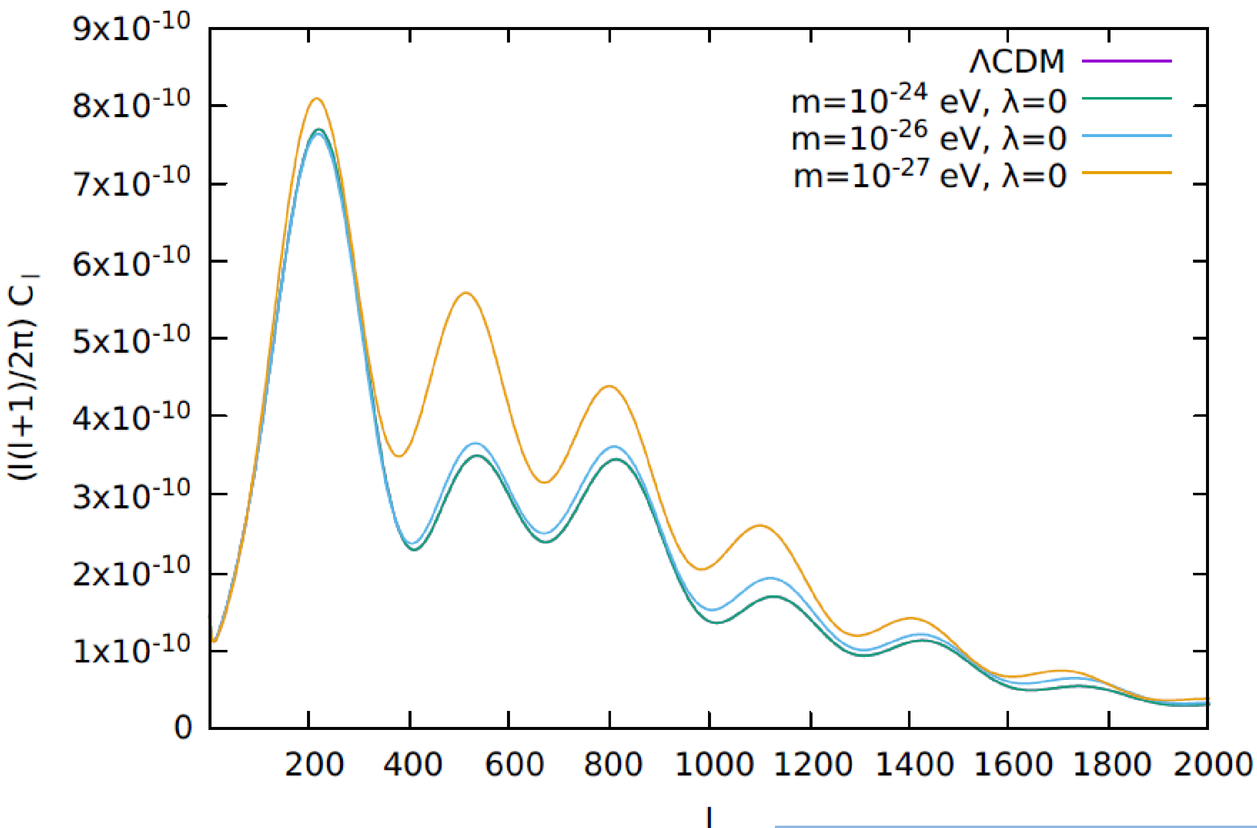

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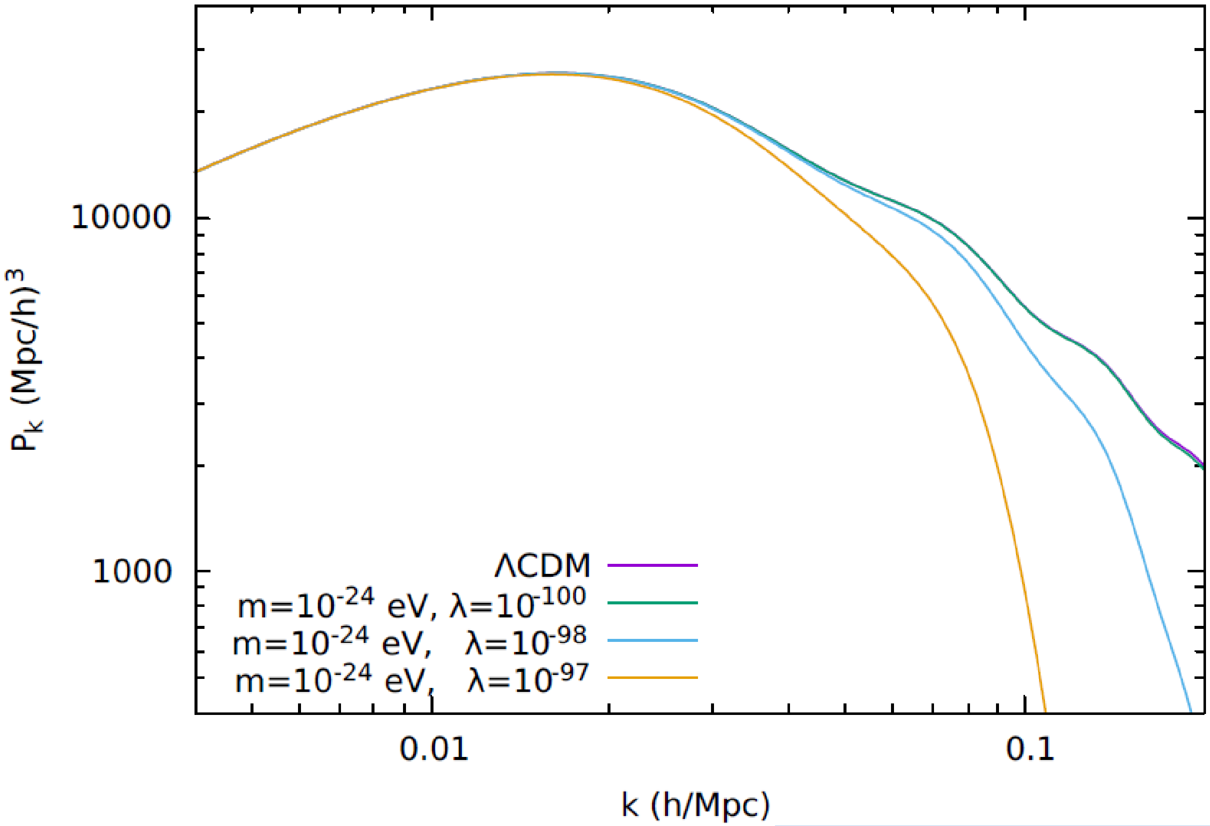

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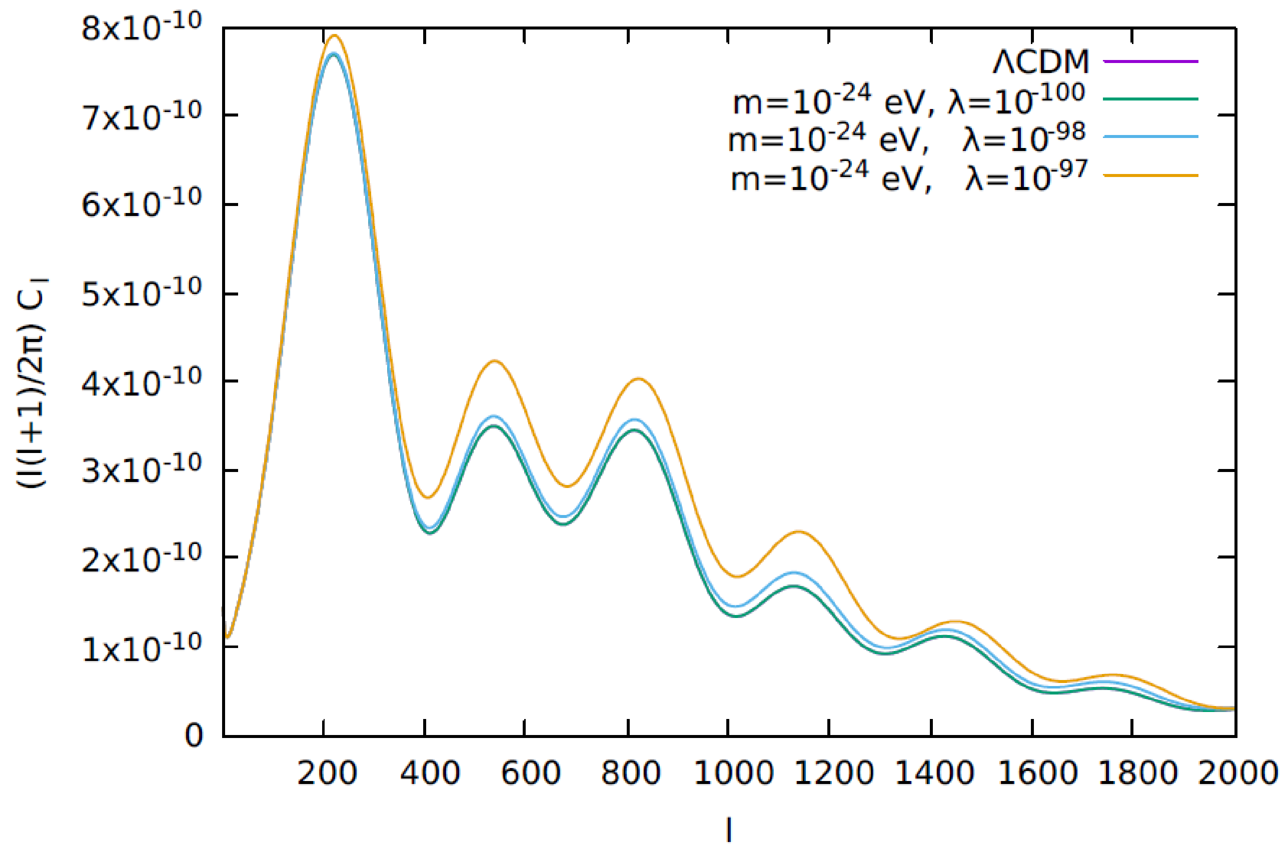


JARC, A.L.Maroto, H. Villarrubia, in preparation

Higher-spin wave DM

Perturbations: **Spin 1**

Dark photons

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{m^2}{2}A_\mu A^\mu \ .$$

$$A_\mu \ = \ \left(\delta A_0(\eta,\vec{x}),\ \vec{A}(\eta) + \delta\vec{A}(\eta,\vec{x})\right)$$

3 independent perturbations (δA_0 is fixed)

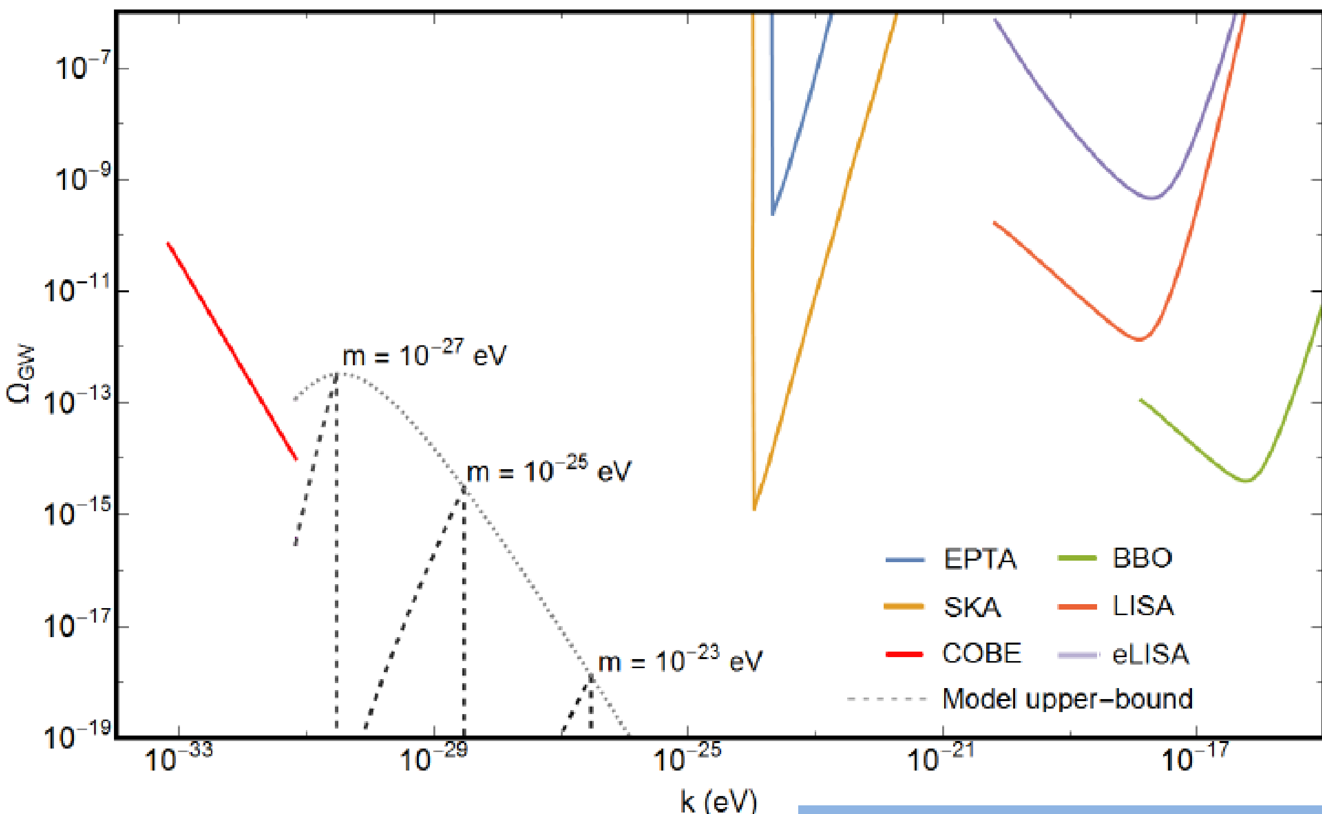
$$ds^2 \ = \ a(\eta)^2\left[(1+2\Phi(\eta,\vec{x}))\,d\eta^2 - ((1-2\Psi(\eta,\vec{x}))\,\delta_{ij} + h_{ij}(\eta,\vec{x}))\,dx^i dx^j - 2Q_i(\eta,\vec{x})d\eta dx^i\right]$$

S-V-T mixing

JARC, A.L.Maroto, Núñez Jareño, arXiv: 1611.03793

Vector DM

Tensor perturbations: Gravitational waves



JARC, A.L.Maroto, Núñez Jareño, arXiv: 1611.03793

Conclusions

- Ultralight scalars are interesting DM candidates with difference cosmological perturbation dynamics than CDM. It modifies the small scale structure formation (wave DM).

- Higher-spin fields can also play the role of wave DM. (No isotropy problem: isotropic average energy-momentum).

- Arbitrary-spin fields with power-law Hamiltonians behave as perfect fluids with average equation of state: $\omega = \frac{2\, n_V}{1+\frac{n_V}{n_T}} - 1$

- Ultralight vectors are indistinguishable from scalars in the particle regime, however in the wave regime they generate scalar-vector-tensor mixing, anisotropic stress and GW.