Cosmological Relaxion Windows and Observational constraints

Sang Hui Im

Dark Matter from aev to ZeV: 3rd IBS-MultiDark-IPPP Workshop
(Nov 21, 2016, Durham)

Kiwoon Choi, SHI : 1610.00680

universitätbonn

bctp Bethe Center for Theoretical Physics

Outline

- Relaxion mechanism
  : Transmutation of the weak scale hierarchy
    i) Large separation of axion scales
    ii) Large number of e-folding

- Cosmological Relaxion Windows

- Observational constraints

- Conclusions

Relaxion mechanism

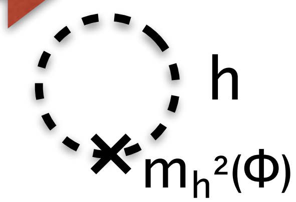
Graham, Kaplan, Rajendran, '15

M : Natural Higgs mass scale

$$m_h^2(\phi) = M^2 \cos\left(\frac{\phi}{f_L}\right) + M_0^2$$

$$V(\phi, h) = m_h^2(\phi)|h|^2 + \cdots$$

Radiative correction

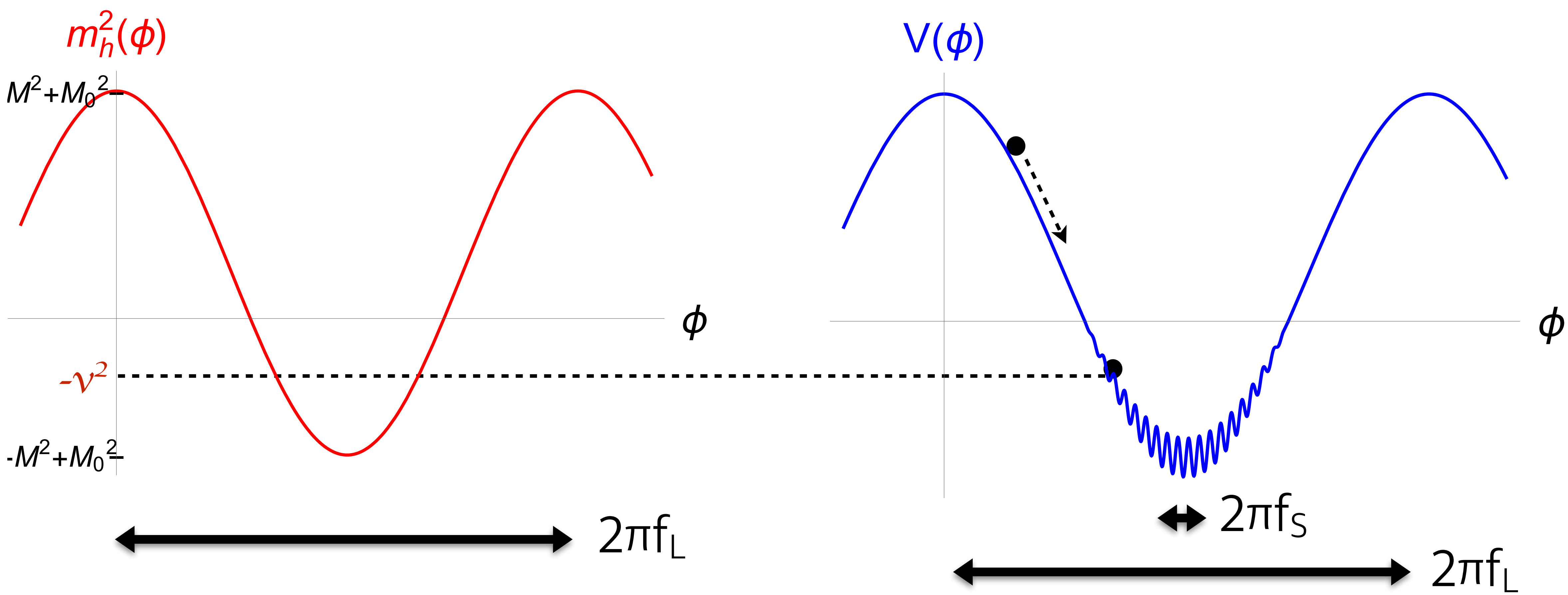


Higgs dependent barrier

$$V(\phi) = \frac{c_0}{16\pi^2} M^2 m_h^2(\phi) + \Lambda_b^4(h)\cos\left(\frac{\phi}{f_S} + \delta_b\right) + V_0$$

$$\Lambda_b^4(h) \sim h^n \neq 0 \quad \text{when} \quad m_h^2(\phi) < 0$$

$$\frac{\partial V}{\partial \phi} = 0 \quad \rightarrow \quad \frac{f_L}{f_S} \sim \frac{c_0}{16\pi^2}\left(\frac{M}{\Lambda_b(h=v)}\right)^4 \frac{1}{\sin(\phi/f_S + \delta_b)}$$



Large separation of axion scales

$$\frac{f_L}{f_S} \sim \frac{c_0}{16\pi^2}\left(\frac{M}{\Lambda_{\rm b}(h=v)}\right)^4 \frac{1}{\sin(\phi/f_S+\delta_b)} \gtrsim \boxed{\frac{c_0}{(4\pi)^3}\frac{M^4}{v^4}}$$

e.g. $f_L > 10^3 f_S$ for $M > 10\,\mathrm{TeV}$

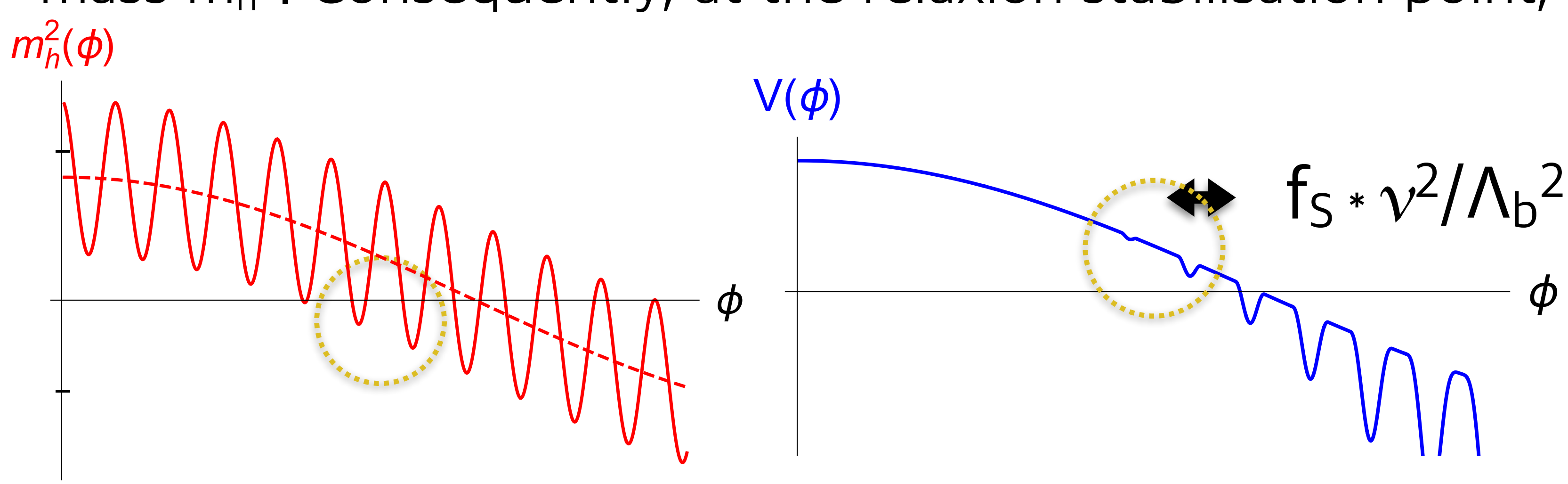
- Naturalness limit on the barrier height

$$V_b(\phi,h) = \Lambda_b^4(h)\cos\left(\frac{\phi}{f_S}+\delta_b\right) = \left(\mu_b^2|h|^2+\mu_0^4\right)\cos\left(\frac{\phi}{f_S}+\delta_b\right)$$

For the barrier to depend on the Higgs VEV, $\mu_b^2 v^2 > \mu_0^4 \gtrsim \frac{1}{16\pi^2}\mu_b^4 \;\to\; \Lambda_b^4(h=v) < \mathcal{O}(16\pi^2 v^4)$

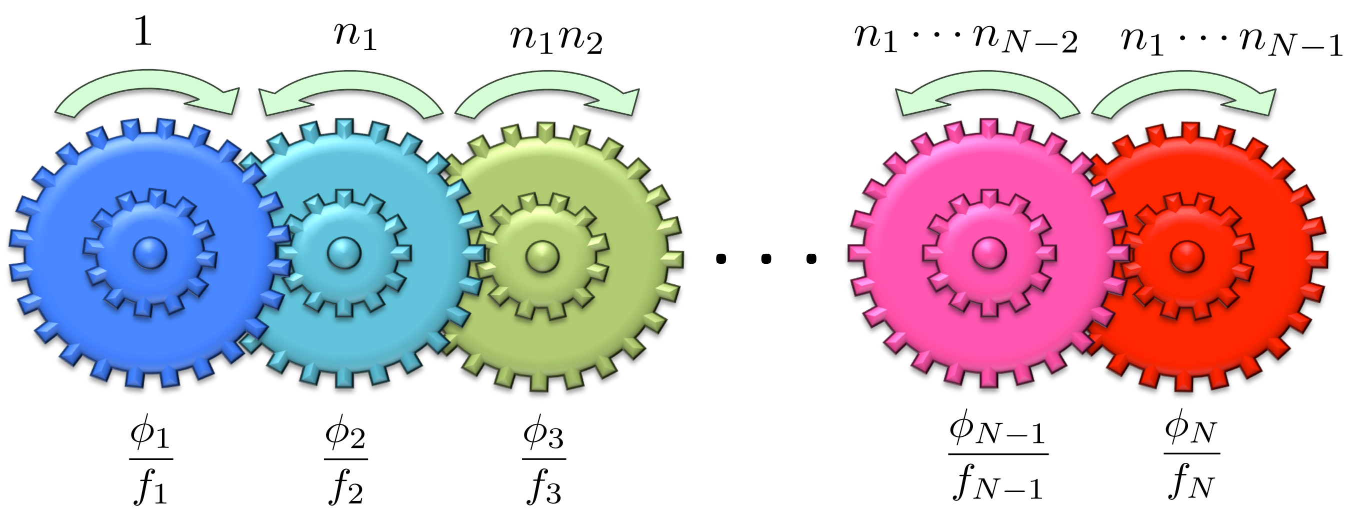
- Barrier shrinking effect when $\Lambda_b > v$: the barrier potential takes part in scanning the Higgs mass m_h^2. Consequently, at the relaxion stabilisation point,

$$\sin\left(\frac{\phi}{f_S}+\delta_b\right) \sim \frac{v^2}{\Lambda_{\rm b}^2+v^2}$$



Clockwork for the axion scale hierarchy

K. Choi, H.J. Kim, S.H. Yun '14; K. Choi, SHI '15; Kaplan, Rattazzi '15



$f_1 \sim f_2 \sim \cdots \sim f_N \sim f$

$$V_{\rm clock} = \Lambda_1^4 \cos\left(n_1 \frac{\phi_1}{f_1} + \frac{\phi_2}{f_2}\right) + \Lambda_2^4 \cos\left(n_2 \frac{\phi_2}{f_2} + \frac{\phi_3}{f_3}\right) + \cdots + \Lambda_{N-1}^4 \cos\left(n_{N-1} \frac{\phi_{N-1}}{f_{N-1}} + \frac{\phi_N}{f_N}\right)$$

N axions with N-1 potentials
: 1 flat direction

Along the flat direction

$$\left(\frac{\Delta\phi_1}{f_1}, \frac{\Delta\phi_2}{f_2}, \frac{\Delta\phi_3}{f_3}, \cdots, \frac{\Delta\phi_N}{f_N}\right) = \left(1, -n_1, n_1 n_2, \cdots, (-1)^{N-1} n_1 \cdots n_{N-1}\right)$$

$$\phi_{\rm flat} \sim \frac{1}{n_1 \cdots n_{N-1}}\phi_1 - \frac{1}{n_2 \cdots n_{N-1}}\phi_2 + \cdots + (-1)^{N-2}\frac{1}{n_{N-1}}\phi_{N-1} + (-1)^{N-1}\phi_N$$

Imposing small perturbing potentials

$$V_{\rm flat} = \epsilon_1 \Lambda_1^4 \cos\left(\frac{\phi_1}{f_1}\right) + \epsilon_N \Lambda_N^4 \cos\left(\frac{\phi_N}{f_N}\right)$$

$$= \epsilon_1 \Lambda_1^4 \cos\left(\frac{\phi_{\rm flat}}{f_L}\right) + \epsilon_N \Lambda_N^4 \cos\left(\frac{\phi_{\rm flat}}{f_S}\right) + \text{heavy modes}$$

$f_L \sim n_1 \cdots n_{N-1} f \sim e^{N \log n} f$

$f_S \sim f$

Large number of e-folding

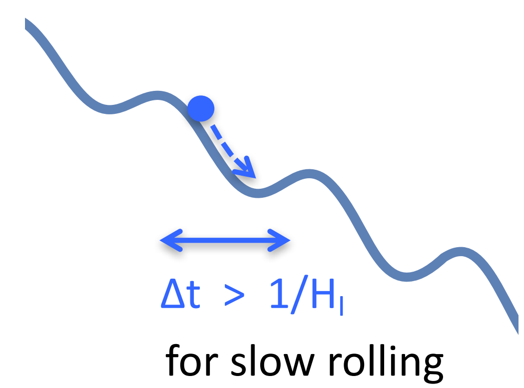
$$\mathcal{N} \sim \frac{f_L}{\dot{\phi}/H_I} \sim \left(\frac{c_0}{16\pi^2}\right)^{-1} \frac{f_L^2 H_I^2}{M^4} \sim \frac{c_0}{16\pi^2} \frac{f^2 M^4}{\Lambda_{\rm b}^8 \sin^2(\phi_0/f)} H_I^2$$

* Lower bound on H_I

i) slow roll

barrier shrinking for $\Lambda_{\rm b} > v$

$$\Delta t \sim \frac{\text{barrier width}}{\dot{\phi}} \sim \frac{f \times v^2/(\Lambda_{\rm b}^2 + v^2)}{\Lambda_{\rm b}^2 \times v^2/(\Lambda_{\rm b}^2 + v^2)} > \frac{1}{H_I} \quad \rightarrow \quad H_I > \frac{\Lambda_{\rm b}^2}{f} \sim m_\phi$$



ii) $\rho_{\rm inflaton} > \rho_\phi$

$$H_I^2 > \frac{\rho_\phi}{3M_{\rm Pl}^2} \sim \frac{c_0}{16\pi^2} \frac{M^4}{M_{\rm Pl}^2}$$

$$\mathcal{N} \gtrsim \max\left[\frac{1}{16\pi^2}\frac{M^4}{\Lambda_{\rm b}^4}, \frac{1}{(16\pi^2)^2}\frac{f^2}{M_{\rm Pl}^2}\frac{M^8}{\Lambda_{\rm b}^8}\right]\left(1 + \frac{\Lambda_{\rm b}^2}{v^2}\right)^2 > \frac{1}{16\pi^2}\frac{M^4}{v^4} \sim \left(\frac{M}{\rm TeV}\right)^4$$

Cosmological requirements

- **Upper bound on H_I** : classical rolling > de Sitter quantum fluctuation

$$(\Delta\phi)_{\Delta\mathcal{N}=1} \simeq \frac{\dot{\phi}}{H_I} \sim \frac{V'(\phi)}{H_I^2} > H_I$$

$$\Lambda_{\rm b}(h=v) < \mathcal{O}(\sqrt{4\pi}v)$$

$$H_I < (V'(\phi))^{1/3} \sim \left(\frac{c_0}{16\pi^2}\frac{M^4}{f_L}\right)^{1/3} \sim \left(\frac{\Lambda_{\rm b}}{f}\frac{v^2}{\Lambda_{\rm b}^2+v^2}\right)^{1/3}\Lambda_{\rm b}$$

➡ $H_I \lesssim \mathcal{O}(v)$

: Low scale inflation

- Density perturbation

$$H_* < \mathcal{O}(v)$$

$$\frac{H_*^2}{8\pi^2 M_{\rm pl}^2\epsilon_*} \simeq 2.2\times 10^{-9}$$

➡ $\epsilon \leq \epsilon_* \lesssim 10^{-26}$

: Small Field Inflation (SFI)

* evaluated at the time of horizon exit of the CMB scale $k = 2 \times 10^{-4}\ \mathrm{Mpc}^{-1}$

- Number of e-folding

$$\mathcal{N} \gtrsim \max\left[\frac{1}{16\pi^2}\frac{M^4}{\Lambda_b^4}, \frac{1}{(16\pi^2)^2}\frac{f^2}{M_{Pl}^2}\frac{M^8}{\Lambda_b^8}\right]\left(1+\frac{\Lambda_b^2}{v^2}\right)^2$$

Barrier from QCD

$$\Lambda_b^4 \simeq f_\pi^2 m_\pi^2 \sim (0.1\,\mathrm{GeV})^4, \quad f \gtrsim 4\times 10^8\mathrm{GeV}$$

$$\mathcal{N}_{QCD} \sim \frac{1}{\theta_{QCD}}\frac{c_0}{16\pi^2}\frac{f^2M^4H_I^2}{\Lambda_b^8} \gtrsim \frac{1}{\theta_{QCD}} \times \max\left[\frac{1}{16\pi^2}\frac{M^4}{f_\pi^2 m_\pi^2}, \frac{1}{(16\pi^2)^2}\frac{f^2}{M_{Pl}^2}\frac{M^8}{f_\pi^4 m_\pi^4}\right]$$

$$\gtrsim \max\left[10^{24}\left(\frac{M}{\mathrm{TeV}}\right)^4, 10^{19}\left(\frac{f}{10^9\,\mathrm{GeV}}\right)^2\left(\frac{M}{\mathrm{TeV}}\right)^8\right] \qquad |\theta_{QCD}| \lesssim 10^{-10}$$

SFI with such a large e-folding would imply a fine-tuning on the inflation sector.

- For a natural scenario, we need a new physics for the relaxion barrier.
- A plausible parameter space in terms of $\mathcal{N}$ can be examined.

Cosmological relaxion windows

$$\mathcal{N}_{\rm NP} \sim \frac{c_0}{16\pi^2}\frac{f^2M^4}{\Lambda_{\rm b}^8\sin^2(\phi_0/f)}H_I^2 \gtrsim \max\left[\frac{1}{16\pi^2}\frac{M^4}{\Lambda_{\rm b}^4},\ \frac{1}{(16\pi^2)^2}\frac{f^2}{M_{\rm Pl}^2}\frac{M^8}{\Lambda_{\rm b}^8}\right]\left(1+\frac{\Lambda_{\rm b}^2}{v^2}\right)^2$$

where $\Lambda_{\rm b} < \mathcal{O}(\sqrt{4\pi}v),\ f > M$

Requiring $\mathcal{N}_{\rm NP} < \mathcal{N}_{\rm e}$ (a certain acceptable number of e-folding) & from the upper bound on $H_I < O(v)$,

$$M < \min\left[9\,{\rm TeV}\left(\frac{\mathcal{N}_e}{10^4}\right)^{1/4},\ 10^{11}\,{\rm GeV}\left(\frac{\rm TeV}{f}\right)^{1/6}\right]$$

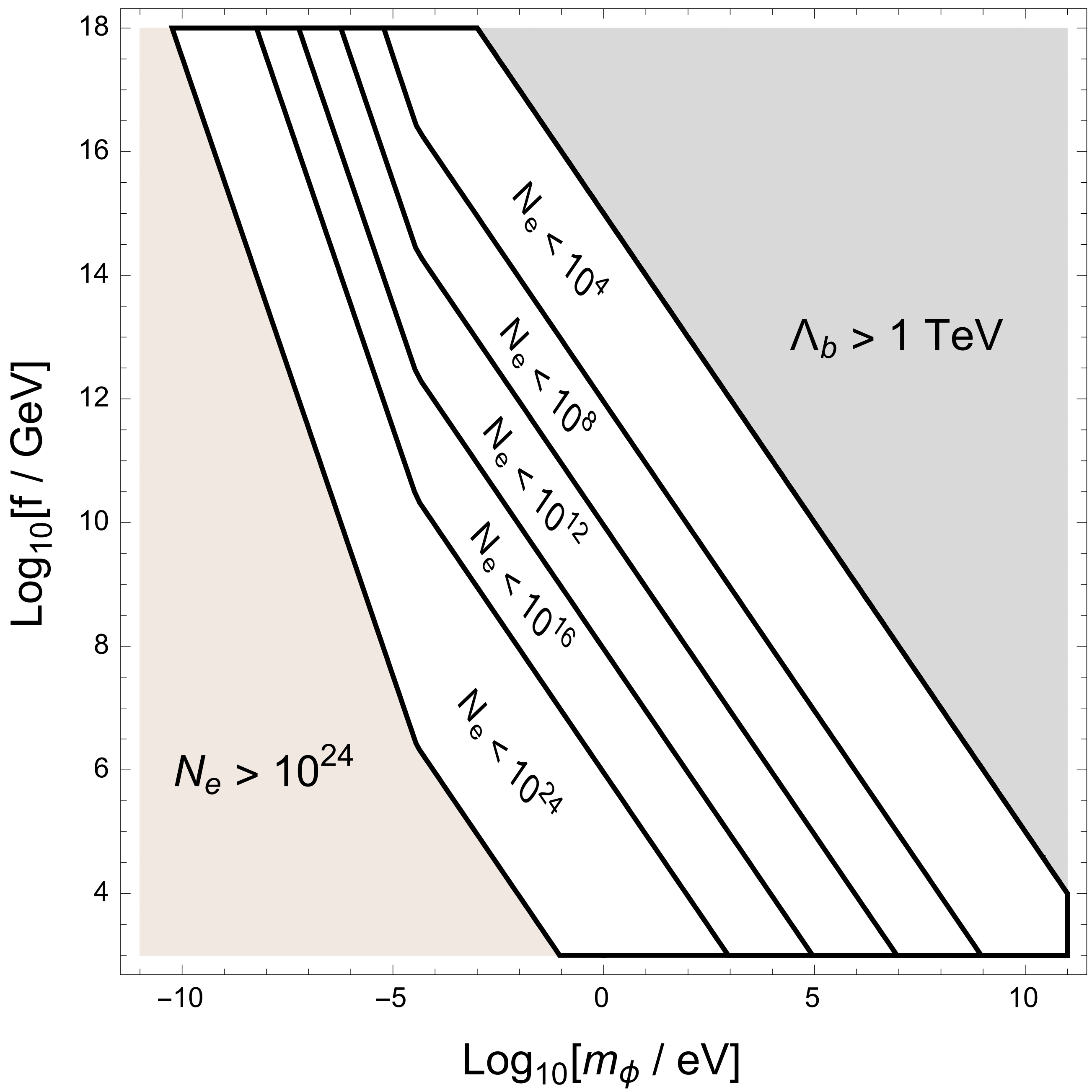
$$30\,{\rm GeV}\left(\frac{M}{1\,{\rm TeV}}\right)\left(\frac{10^4}{\mathcal{N}_e}\right)^{1/4} < \Lambda_{\rm b} \lesssim \mathcal{O}(\sqrt{4\pi}v)$$

$$M < f < 3\times 10^{22}\,{\rm GeV}\left(\frac{1\,{\rm TeV}}{M}\right)^4\left(\frac{\Lambda_{\rm b}}{1\,{\rm TeV}}\right)^4\left(\frac{\mathcal{N}_e}{10^4}\right)^{1/2}\left(1+\frac{\Lambda_{\rm b}^2}{v^2}\right)^{-1}$$

$m_\phi \sim \Lambda_b^2/f$

$$3\times 10^{-8}\,{\rm eV}\left(\frac{M}{\rm TeV}\right)^4\left(\frac{1\,{\rm TeV}}{\Lambda_{\rm b}}\right)^2\left(\frac{10^4}{\mathcal{N}_e}\right)^{1/2}\left(1+\frac{\Lambda_{\rm b}^2}{v^2}\right) < m_\phi < \min\left[v,\ 1\,{\rm TeV}\left(\frac{1\,{\rm TeV}}{M}\right)\left(\frac{\Lambda_{\rm b}}{1\,{\rm TeV}}\right)^2\right]$$

Cosmological relaxion windows



1 TeV < f < M_{Pl}

10^{-10} eV < m_Φ < v_{EW}

For a given m_Φ, the required N_e is smaller as f is larger (due to a higher barrier).

Relaxion effective couplings to SM

i) $$V_{\rm b}(\phi, h) = (\mu_0^4 + \mu_b^2 |h|^2)\cos\left(\frac{\phi}{f}\right) \quad \text{where} \quad \mu_0^4 < \mu_b^2 v^2 \text{ and } \mu_b \lesssim \mathcal{O}(4\pi v)$$

ii) $$c_{\phi\gamma}\frac{\alpha}{4\pi f}\phi F_{\mu\nu}\tilde{F}^{\mu\nu} \quad \text{with} \quad c_{\phi\gamma} = \mathcal{O}(1)$$

CP violation : EDM

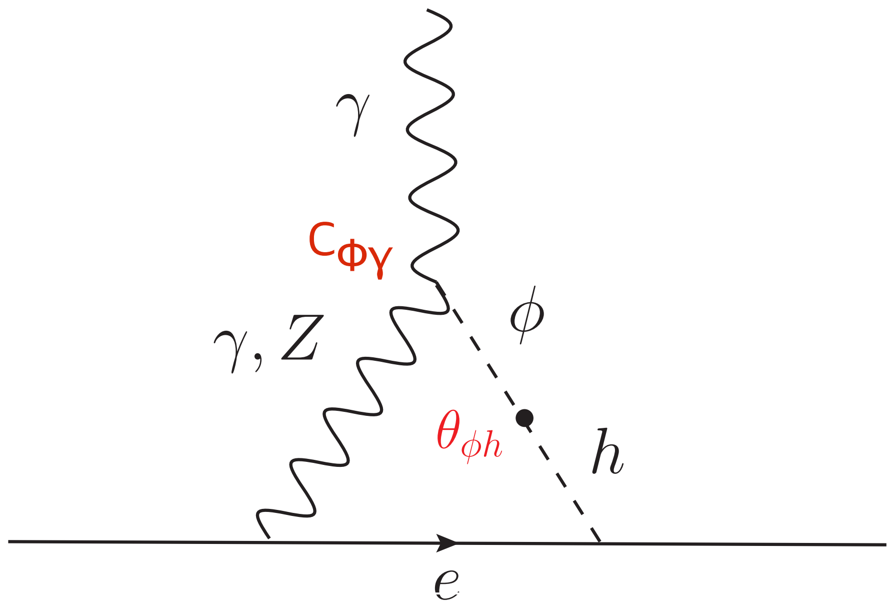
$$m_\phi \sim \frac{\mu_b v}{f}, \quad \theta_{\phi h} \sim \frac{\mu_b^2 v}{f(m_h^2 - m_\phi^2)} \sin\left(\frac{\phi_0}{f}\right) \sim \frac{m_\phi^2}{m_h^2 - m_\phi^2}\frac{f}{v}\left(1 + \frac{f m_\phi}{v^2}\right)^{-1}$$

Relaxion-Higgs mixing

Flacke, Frugiuele, Fuchs, Gupta, Perez '16

- For a given m_ϕ, the mixing angle is sizeable if f is large : large f region will be constrained by the Higgs mixing.
- The relaxion-photon-photon coupling $c_{\phi\gamma}$ constrains a small f region.

EDM and sub-GeV relaxion couplings



$$d_f \simeq 4\frac{e^3}{(4\pi)^4}\frac{m_f}{v}\frac{c_{\phi\gamma}}{f}\sin\theta_{\phi h}\cos\theta_{\phi h}\ln\left(\frac{m_h^2}{m_\phi^2}\right)$$

$$d_e \sim 7\times 10^{-29} c_{\phi\gamma}\left(\frac{m_\phi}{10\,\mathrm{GeV}}\right)^2 \ln\left(\frac{10\,\mathrm{GeV}}{m_\phi}\right)\left(1+\frac{f m_\phi}{v^2}\right)^{-1} e\cdot\mathrm{cm}$$

$d_e < 8.7\times 10^{-29}\, e\cdot\mathrm{cm}$: $m_\Phi > 10/\sqrt{c_{\Phi\gamma}}$ GeV is excluded.

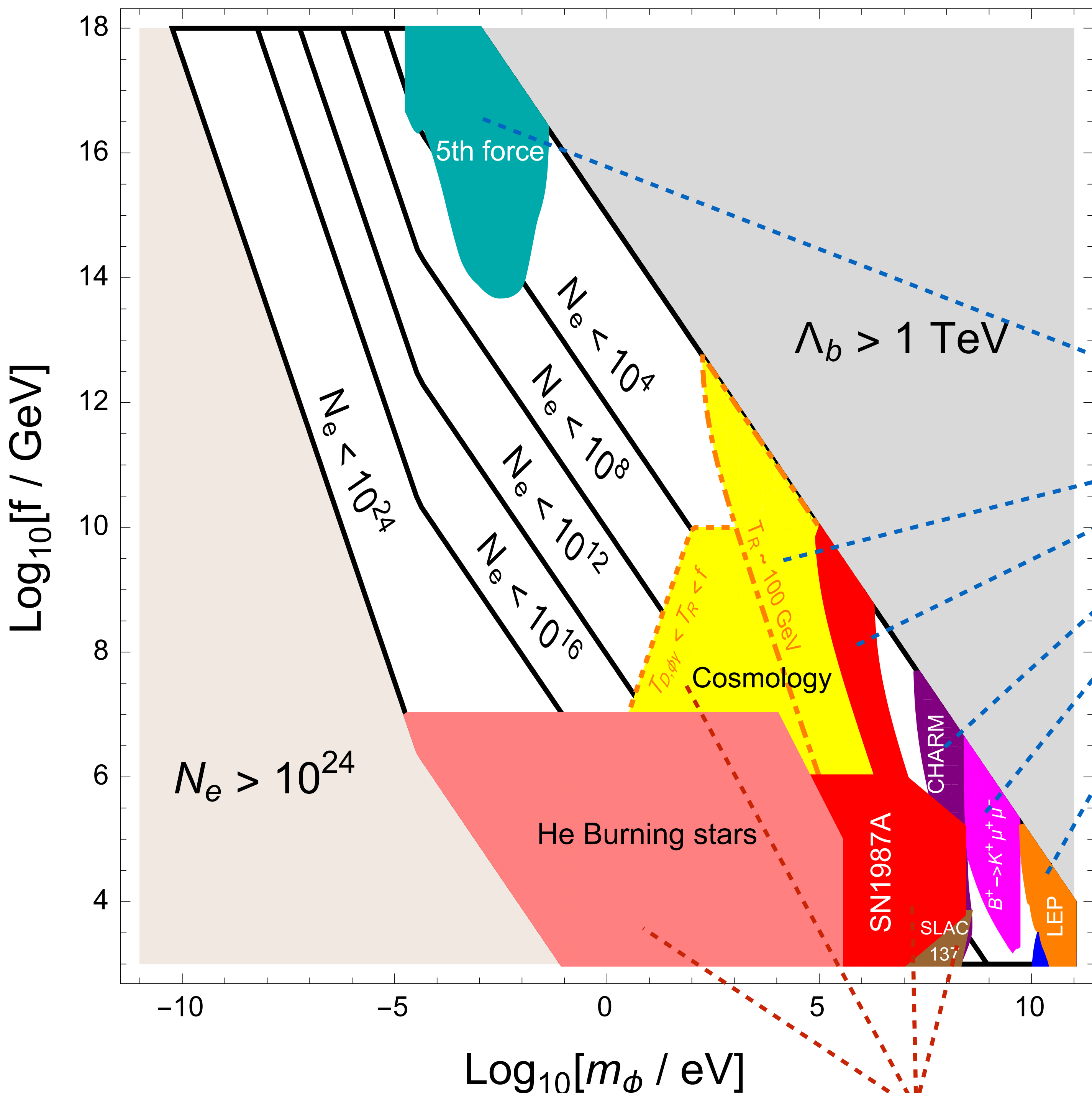
ACME '13

- The future EDM experiment like the proton storage ring experiment can probe the relaxion mass region below about 10 GeV.
- For m_Φ < 1 GeV, the relaxion couplings to pions and nucleons through the relaxion-Higgs mixing should be taken into account.

$m_\phi < 1\,\mathrm{GeV}$

$$\mathcal{L}_{\rm eff} = 2 s_\theta \kappa \frac{\phi}{v}\left(\frac{1}{2}\partial_\mu\pi^0\partial^\mu\pi^0 + \partial_\mu\pi^+\partial^\mu\pi^-\right) - \frac{5 s_\theta}{3}\frac{\phi}{v} m_\pi^2\left(\frac{1}{2}\pi^0\pi^0 + \pi^+\pi^+\right) - \frac{s_\theta}{6}\frac{g_2 m_N}{m_W}\phi\bar{N}N + s_\theta\frac{c_{h\gamma}\alpha}{4\pi v}\phi F^{\mu\nu}F_{\mu\nu} + \frac{c_{\phi\gamma}\alpha}{4\pi f}\phi F_{\mu\nu}\tilde{F}^{\mu\nu} + s_\theta\sum_{l=e,\mu}\frac{m_l}{v}\phi\,\bar{\psi}_l\psi_l$$

Observational constraints on the relaxion windows



Cosmology bounds from BBN, CMB, EBL, DM, etc :
Depends on reheating temperature T_R
(Figure : $T_R \sim f$, 100 GeV)

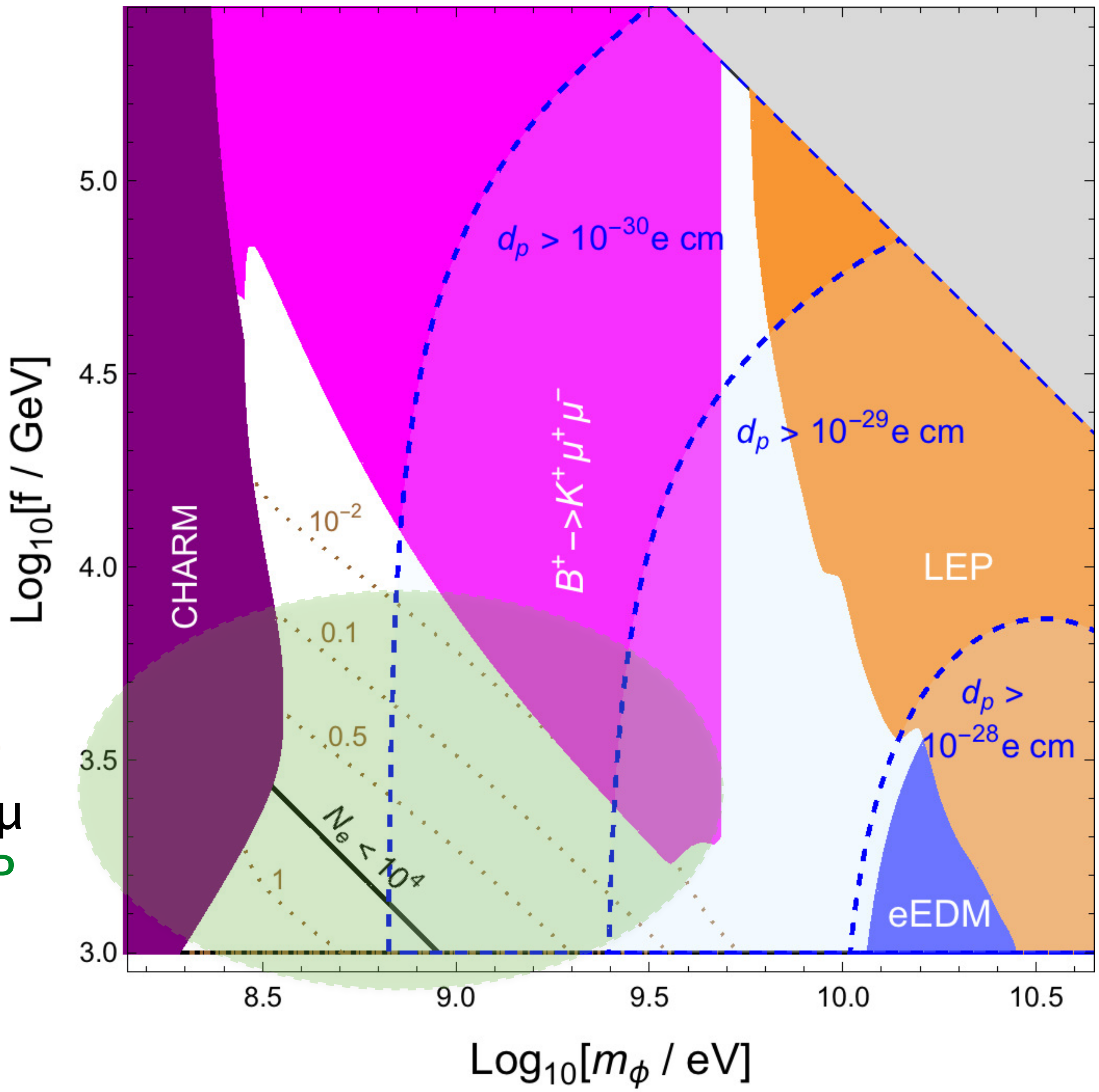
excluded by $\theta_{\Phi h}$

Three distinctive windows

i) $m_\Phi \sim 0.2 - 10$ GeV, $f \sim$ few $- 200$ TeV
ii) $m_\Phi \sim$ few $- 50$ MeV, $f \sim 10^6 - 10^9$ GeV
iii) $m_\Phi < 100$ eV, $f > 10^7$ GeV

excluded by $c_{\Phi\gamma}$ (=1)

Enlarged picture for the first window



- BR(Φ->γγ)
- Φ->ππ, μμ
- CERN SHiP

- EDM with **$c_{\Phi\gamma}$ =1**
- Proton EDM estimated by the QCD sum rule

$$d_p = 0.78\, d_u(\mu_*) - 0.20\, d_d(\mu_*)$$

where $\mu_* = 1\ \mathrm{GeV}$

- NDA with s quark : an order of magnitude larger

- **Storage ring experiment for proton EDM**

Conclusions

- The relaxion mechanism can explain the weak scale in a technically natural way by transmuting the hierarchy to a large separation between two axion scales.
- The separation between two axion scales can be addressed by the clockwork mechanism with multiple axions.
- The large separation also leads to the requirement of a large number of e-folding for the relaxion dynamics, assuming the Hubble friction is the main source of dissipation of relaxion kinetic energy.
- The cosmological relaxion window identifies the favored relaxion parameter space in terms of the required number of e-folding.
- After imposing various observational constraints, three distinctive windows remain viable, all of which include the relatively small number of e-folding region below 10^4.
- The first window ($m_\Phi \sim 0.2$ - 10 GeV, $f \sim$ few - 200 TeV) can be probed by future EDM experiments and CERN SHiP.