Effective Potentials and Radiative Mass Generation

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Outline

1. The Effective Action
   - Thermodynamics Background
   - QFT equivalent
   - Computing Γ_{eff}
2. Coleman Weinberg mechanism
   - Simple massless scalar model
   - SSB induced via loop corrections
3. Incorporating the CW mechanism
   - The SM and fine tuning
   - CW in the SM
   - CW in extended models

The Partition Function

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- The generating field functional is the QFT equivalent to the partition function of a thermal system
- Thermal fluctuations are replaced by quantum fluctuations

Magnetic system in an external field H

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For a magnetic system one defines the Helmholtz free energy $F(H)$:

$$Z(H) = \exp\big(-\beta F(H)\big) = \int \mathscr{D}s \exp\left(-\beta \int dx(\mathscr{H}(s) - Hs(x))\right)$$

where H is the exterior magnetic field, $\mathscr{H}(s)$ is the spin energy density, and $\beta = 1/k_B T$.

Magnetisation M

The magnetisation M of a system is given by the 1st moment of the spins across the defined spatial region, and can be found via differentiation of the Helmholtz free energy:

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If $H = 0$, G reaches an extremum at a corresponding value of M, and the thermodynamic stable state is the **minimum of** $G(M)$, i.e. $G(M)$ gives a picture of the favoured thermodynamic state that includes all effects of thermal fluctuations.

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We now construct the analogous in QFT. Considering a real scalar field ϕ in the presence of an external source $J(x)$, we define a vacuum energy function $E(J)$:

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$$\Gamma(\phi_{\text{cl}}) \equiv -E(J) - \int d^4y J(y)\phi_{\text{cl}}$$

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i.e. the solutions, are values for the VEV of $\phi(x)$ in the stable quantum states of the theory. Therefore by extremising the effective action one finds the exact vacuum state of the QFT, including all effects of quantum corrections

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Furthermore, since we're only interested in cases in which the VEV is translational invariant, the minimisation of Γ_{eff} reduces to:

$$\Gamma_{\text{eff}}(\phi_{\text{cl}}) = -(VT) \cdot V_{\text{eff}}(\phi_{\text{cl}}) \qquad \Rightarrow \qquad \frac{\partial}{\partial \phi_{\text{cl}}} V_{\text{eff}}(\phi_{\text{cl}}) = 0$$

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Implicitly, if we know the effective action we know the effective potential

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Let's now try and look at a simple model and actually try and compute the effective potential, consider the following **massless** self interacting scalar theory:

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where, $A = Z_\phi - 1$, $B = Z_m - 1$ and $C = Z_\lambda - 1$ are the usual counterterms.

1 Loop Order - I

It turns out that computing the functional determinant in the effective action $\log\left(\det\left(-\frac{\delta\mathcal{L}_1}{\delta\phi\delta\phi}\right)\right)$, is equivalent with finding the 1-loop approximation to the effective potential.

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This in turn can be calculated for our ϕ^4 theory via the polygon graph series:

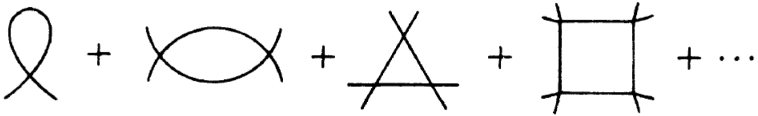

Figure: Polygon Graph Corrections to the potential

1 Loop Order - II

Introducing the counter-terms as well , the 1LO effective potential is:

$$V_{\text{eff}}(\phi_{\text{cl}}) = \frac{\lambda}{4!}\phi_{\text{cl}}^4 - \frac{1}{2}B\phi_{\text{cl}}^2 - \frac{1}{4!}C\phi_{\text{cl}}^4 + i\int \frac{d^dk}{(2\pi)^d}\sum_{n=1}^{\infty}\frac{1}{2n}\left(\frac{\lambda\phi_{\text{cl}}^2/2}{k^2+i\epsilon}\right)^n$$

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Which, after summing the series provides:

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We find:

$$V_{\text{eff}}(\phi_{\text{cl}}) = \frac{\lambda}{4!}\phi_{\text{cl}}^4 + \frac{\lambda^2 \phi_{\text{cl}}^4}{256\pi^2}(\log \frac{\phi_{\text{cl}}^2}{M^2} - \frac{25}{6})$$

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Looking at the potential more closely :

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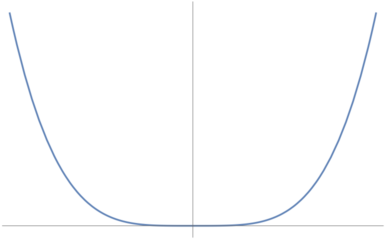

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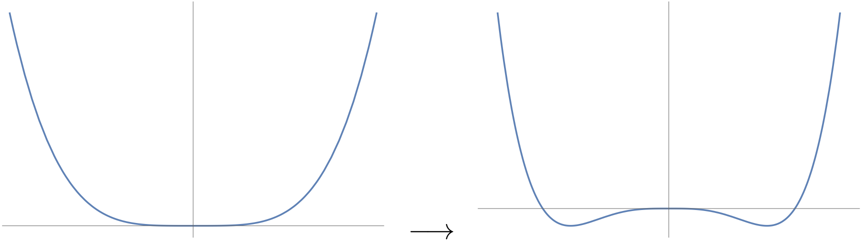
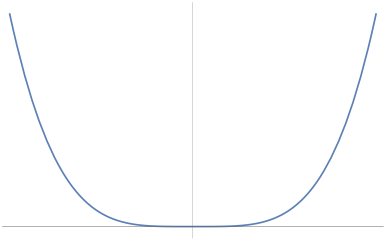
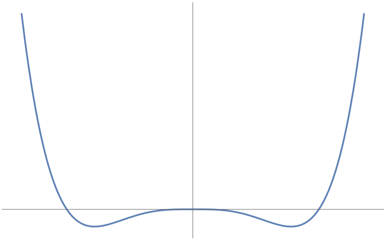


Coleman - Weinberg Mechanism II

 $\longrightarrow$ 

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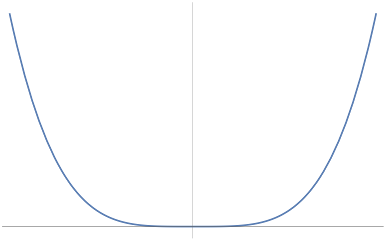

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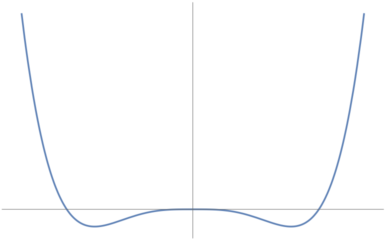

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But why do we care?

The Hierarchy Problem and Fine tuning I

From S.M. Higgs potential:

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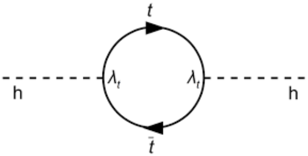

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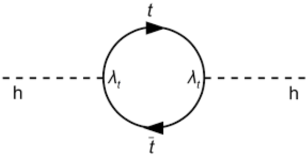

we find the corrected Higgs mass squared (up to one loop):

$$m_H^2 = \mu^2 - \frac{y_t^2}{8\pi^2}\Lambda_{\text{UV}}^2$$

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What can we do?

This is where the CW mechanism comes in. It can provide a naturally small mass scale via radiative generation!

CW mechanism in the Standard Model I

In their original paper, Coleman and Weinberg assumed a Weinberg-Salam theory of leptons, and based on the W, Z masses it predicted a Higgs Mass:

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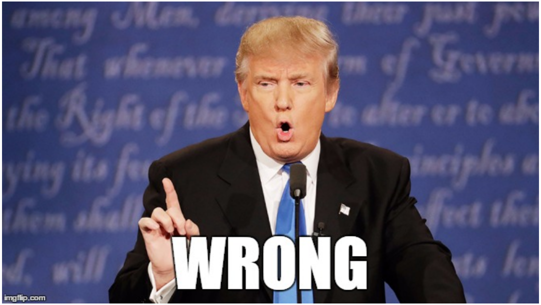

CW mechanism in the Standard Model II

It gets even worse when you add in the top corrections , which make the Higgs mass squared negative!

CW mechanism in the Standard Model II

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So, What can we do?

As per usual in BSM physics, **you add in more stuff!**

Neutrino Scalar extension to the SM - I

For example, following Meissner and Nicolai (arXiv:hep-th/0612165) we can look at a scalar extension to the SM which couples with neutrinos via a Yukawa interaction, and to the Higgs field via a quadratic coupling:

$$\begin{aligned}\mathcal{L} =&(\bar{L}^i \Phi Y^E_{ij} E^j + \bar{Q}^i \epsilon \Phi^* Y^D_{ij} D^j + \bar{Q}^i \epsilon \Phi^* Y^U_{ij} U^j + \\ &+ \bar{L}^i \epsilon \Phi^* Y^\nu_{ij} \nu^j_R + \phi (\nu^i_R)^T \mathcal{C} Y^M_{ij} \nu^j_R + h.c.) - \\ &- \frac{\lambda_1}{4} (\Phi^\dagger \Phi)^2 - \frac{\lambda_2}{2} \phi (\Phi^\dagger \Phi) - \frac{\lambda_3}{4} \phi^4\end{aligned}$$

Neutrino Scalar extension to the SM - II

Assuming small neutrino masses, discarding the $SU(2)_W \times U(1)_Y$ and $SU(3)_C$ and keeping the top quark as the dominant fermion contribution, one can find the effective potential:

$$V_{\text{eff}}(H,\phi) = \frac{\lambda_1 H^4}{4} + \frac{\lambda_2 H^2\phi^2}{2} + \frac{\lambda_3\phi^4}{4} + \\ + \frac{3}{256\pi^2}(\lambda_1 H^2 + \lambda_2\phi^2)^2 \ln\left(\frac{\lambda_1 H^2 + \lambda_2\phi^2}{\nu^2}\right) + \\ + \frac{1}{64\pi^2}F_+^2 \ln\left(\frac{F_+}{\nu^2}\right) + \frac{1}{64\pi^2}F_-^2 \ln\left(\frac{F_-}{\nu^2}\right) - \\ - \frac{3}{16\pi^2}g_t^4 H^4 \ln\left(\frac{H^2}{\nu^2}\right) - \frac{1}{32\pi^2}g_M^4\phi^4 \ln\left(\frac{\phi^2}{\nu^2}\right)$$

Neutrino Scalar extension to the SM - III

Since minimising the potential to obtain the VEV of the fields analytically isn't really feasible we turn to a numeric search. Redefining our fields via the auxiliary dimensionless fields h, φ, we find the numerical minimum of the potential:

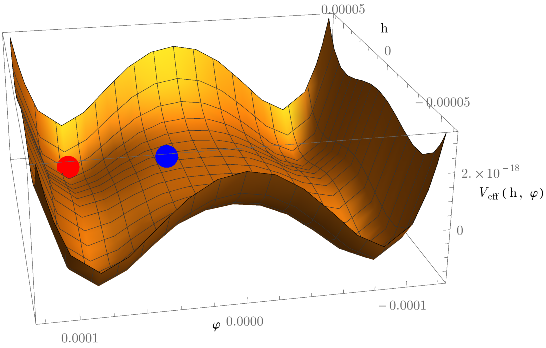

Figure: The dimensionless effective potential as a function of h, φ

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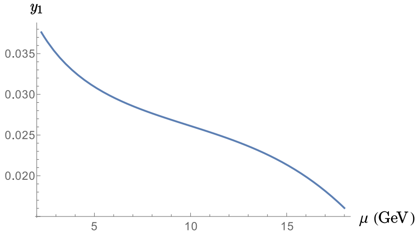
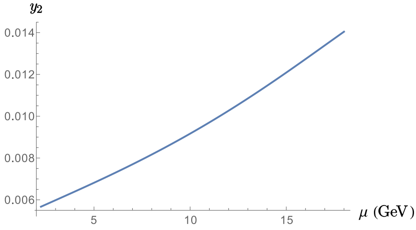
we can scan over the free parameter range and find a set of suitable values!

Stability and Landau poles - I

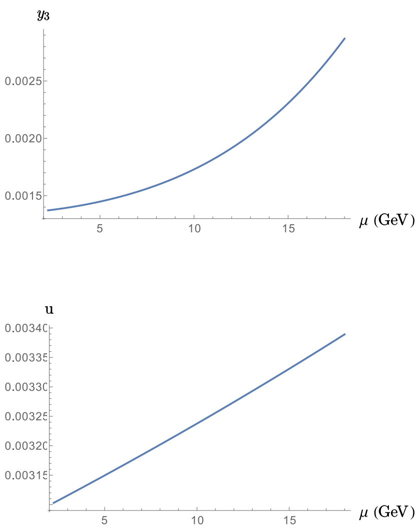
After finding the respective set of suitable bare couplings we still need to check the effective couplings for Landau poles or instabilities up to a high scale.

Based on where we consider the cut off scale (e.g. the Planck Scale) the couplings will behave as :





Stability and Landau poles - II



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If the high scale is dictated by the Planck scale, then we end up with a real scalar with a mass of the order:

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No Fine Tuning!

And note that we didn't have to fine tune anything!

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Future Work will be centred around incorporating the CW mechanism into a $SO(10)$ GUT model.