Khronometric Theory, Numerical GR and Blackholes

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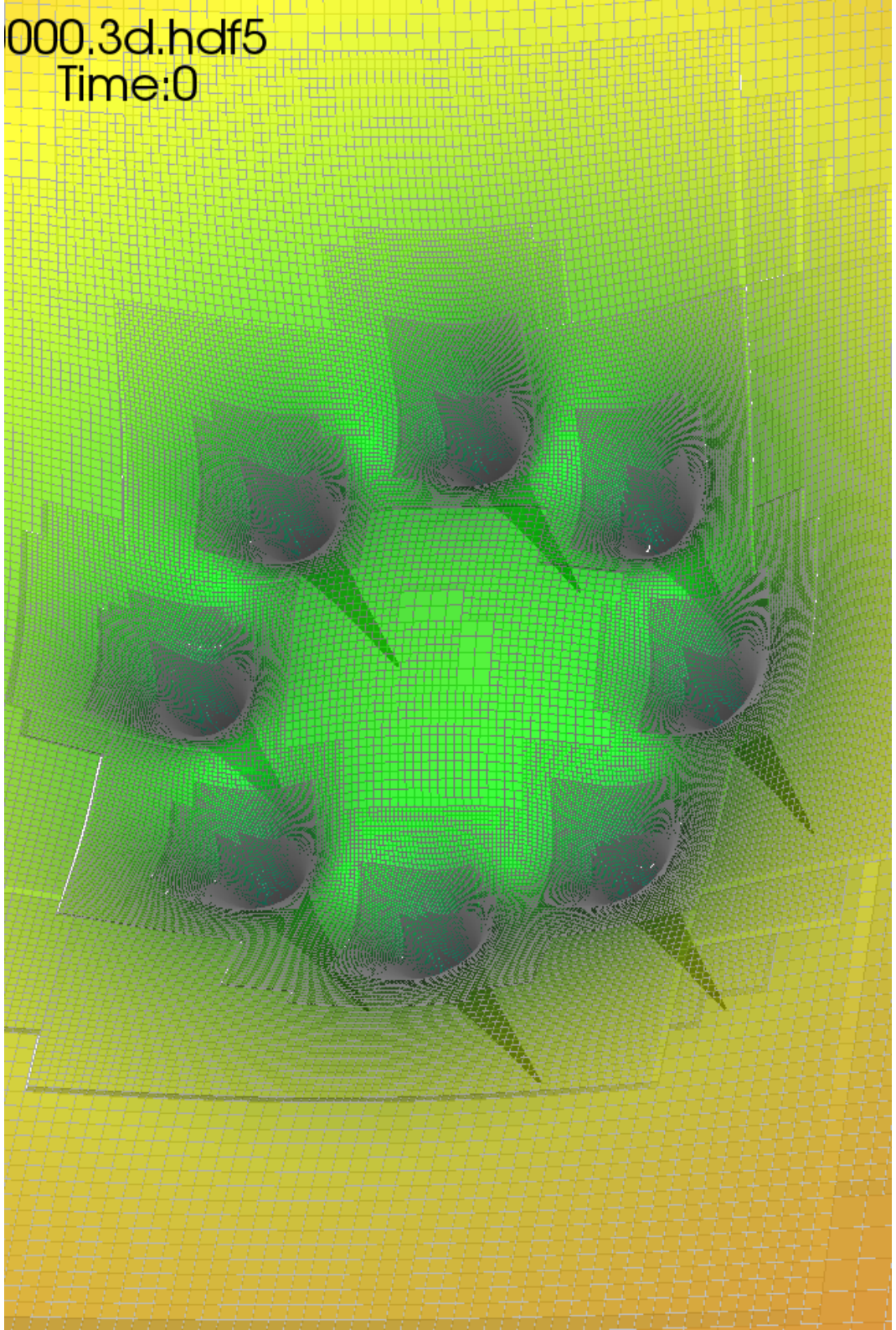
Plan for the talk

- ADM and Numerical GR
- Einstein - Aether Theory
- Khronometric Theory
- Blackholes in Khronometric Theory
- Current and Future Work

ADM and Numerical GR

Numerical GR

- There exists many problems that have no analytical solutions
- We want to solve these problems!
- We want to reform GR into a 3+1 system of equations where we provide initial data, solve then evolve
- We use ADM - Formulate as a Cauchy Problem



ADM (Arnowitt-Deser-Misner)

- We foliate spacetime into space like hypersurfaces

$$ds^2 = \left(-\alpha^2 + \beta_i \beta^i\right) dt^2 + 2\,\beta_i\, dt dx^i + \gamma_{ij}\, dx^i dx^j$$

- We calculate Extrinsic Curvature

$$K_{ij} = \frac{1}{2\alpha}\left[-\partial_t \gamma_{ij} + D_i \beta_j + D_j \beta_i\right]$$

- Use projectors to calculate constraint equations and evolution equations

- Spacetime separated in 3+1 so can be solved numerically provided initial K and γ

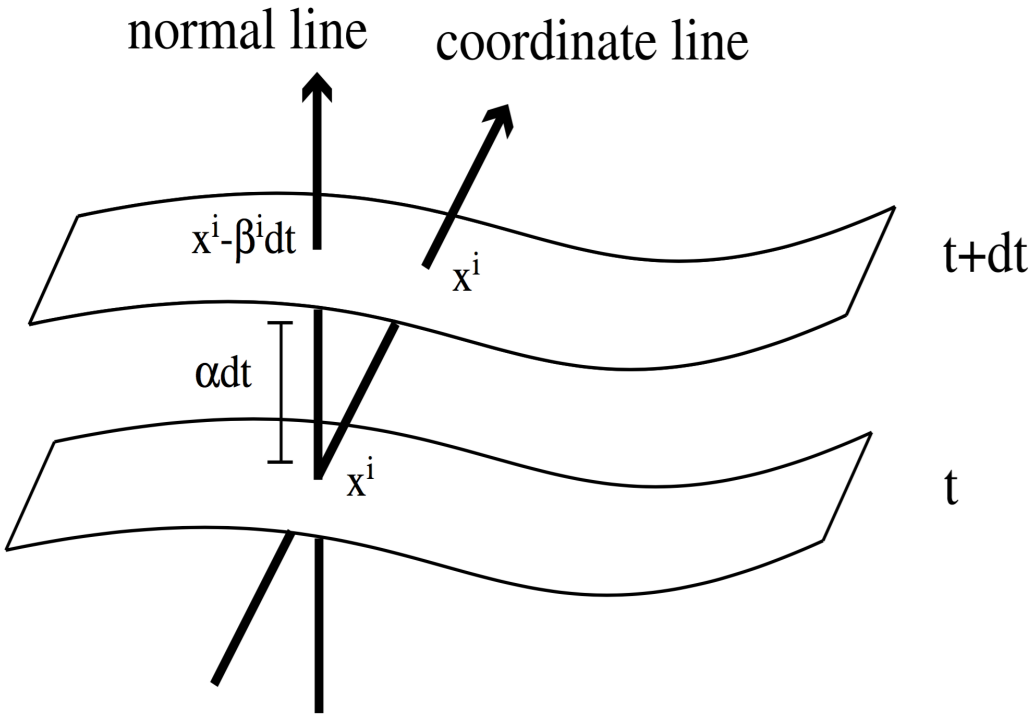


Diagram to show two adjacent spacelike hypersurfaces, and the definitions for α and β

Diagram from Introduction to 3+1 Numerical Relativity - Miguel Alcubierre

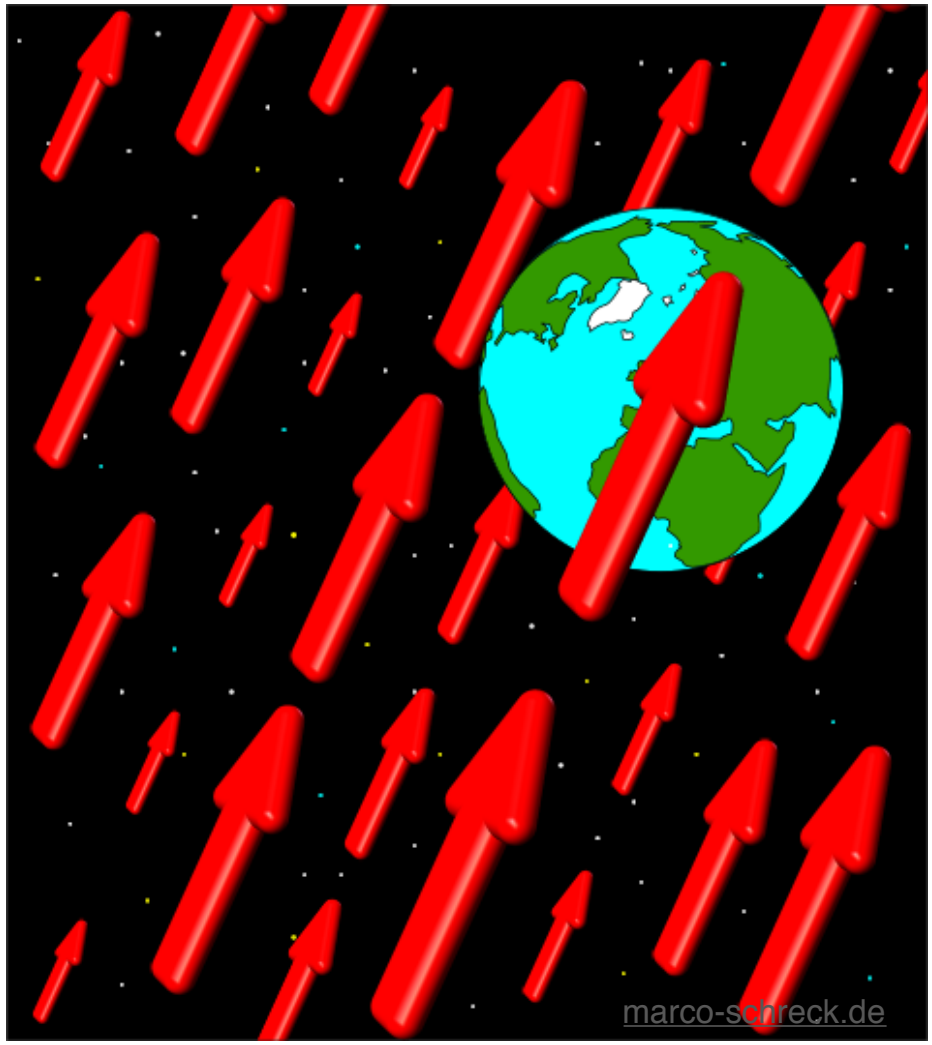
Einstein-Aether Theory

Einstein-Aether Theory - Motivation

- All current tests of GR involve systems that are weakly gravitating - Binary Pulsars and Solar System. (Gravitational waves have only just been discovered, need eLISA to detect Extreme Mass Ratio Inspirals)
- GR incompatible with Quantum Gravity due to mathematical pathologies, such as singularities in blackhole collapse
- Cosmological Constant is not theoretically satisfying - why is it so small?
- Possible Solution - Break Lorentz Invariance

Einstein-Aether Theory - Introduction

- Presume the direction of a preferred direction in spacetime - Violation of boost invariance
- We do this by introducing a timeline unit norm dynamical vector field (Aether Field) U
- Couple to gravity and not matter (No fifth force!)



Einstein-Aether Theory - The Action

- We can write our action in the most general form with two of fewer derivatives as

$$S_{AE} = \frac{1}{16\pi G_{AE}} \int d^4x\sqrt{-g}\left(-R - M^{\alpha\beta}{}_{\mu\nu}\nabla_\alpha U^\mu \nabla_\beta U^\nu\right)$$

$$M^{\alpha\beta}{}_{\mu\nu} = c_1 g^{\alpha\beta}g_{\mu\nu} + c_2\delta^\alpha{}_\mu\delta^\beta{}_\nu + c_3\delta^\alpha{}_\nu\delta^\beta{}_\nu + c_4 U^\alpha U^\beta g_{\mu\nu}$$

- Terms not present can are due to them being integrated out by parts and unit vector constraints
- Can obtain equations of motion by varying the action
- We want to specify what the Aether field is before continuing - Khronometric Theory

Khronometric Theory

Khronometric Theory - Introduction

- Make the Aether Field orthogonal to hyper surfaces of constant time i.e T is a constant gauge

$$U_\mu = \frac{\partial_\mu T}{\sqrt{g^{\mu\nu}\partial_\mu T \partial_\nu T}}$$

- T is known as a Khronon

- We can now use our expression for the Aether Field, as well as using ADM to transform our Einstein - Aether action into that of Khronometric Theory

Khronometric Theory - The Action

$$S = \frac{1-\beta}{16\pi G}\int dT d^3x N\sqrt{\gamma}\left(K_{ij}K^{ij} - \frac{1+\lambda}{1-\beta}K^2 + \frac{1}{1-\beta}R^{(3)} + \frac{\alpha}{1-\beta}a_i a^i)\right)$$

- α, β and γ are combinations of c_1 to c_4. Can express without loss of generality.
- a is the acceleration of the aether flow
- N is the lapse, N_i is our shift vector
- We can vary this action by γ_{ij}, N and N_i to produce 3 sets of equations (Not shown here)

Blackholes in Khronometric Theory

Blackholes in Khronometric Theory

- Multiple Horizons
- Horizons are a low energy artefact, and excitations of sufficiently high momenta can escape
- Can we form a blackhole?
- Changes orbits of relativistic binaries

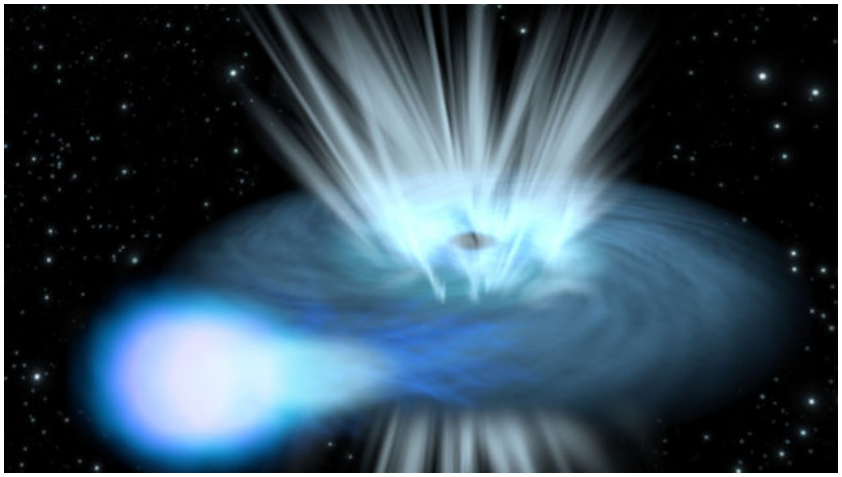


Image From: https://cdn.arstechnica.net

Blackholes in Khronometric Theory - Our Setup

- Spherical Symmetry, Static, Vacuum
- Shift Vector is equal to 0 (Gauge choice)
- Blackhole characterised by 3 parameters, A, B and F

$$N = A(t,r)^2$$

$$\gamma_{ij}dx^i dx^j = F(t,r)^2 \mathrm{d}r^2 + B(t,r)^2 r^2 \mathrm{d}\Omega^2$$

$$\partial_t \boldsymbol{u} + \boldsymbol{M} \cdot \partial_r \boldsymbol{u} = \boldsymbol{S}$$

$$\partial_r^2 A = R$$

Conclusion

- We have looked at blackholes in Khronometric Theory
- Results still to come from simulation
- Khronometric Theory is a low energy limit of Horava Gravity
- Lorentz invariance only an approximate symmetry at low energies

Current and Future Work

- Current - Gravitational Collapse in Spherical Symmetry and solving using PETSc
- Current - Early Universe Bubble Collisions
- Cosmic Strings
- Higher Dimensional Spacetime
- f(R) Modified Gravity