A lattice probe of flavour physics: the $B_s \to D_s l\nu$ decay

Euan McLean

Glasgow University

YTF9

Outline

Background & Motivation

The CKM matrix

CKM Element Determination via $B_s \to D_s l \nu$

The need for a lattice calculation in $B_s \to D_s l \nu$

Lattice Calculation

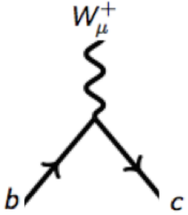
How to lattice

Integration over Gauge Configurations

Extracting Amplitudes from Correlation Functions

Conclusion

The CKM matrix



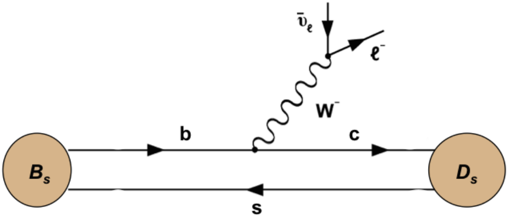
$$\propto \quad V_{cb}$$

$$\mathscr{L}_W = \frac{g}{\sqrt{2}}\left(V_{ij} J_{ij}^{\mu} W_\mu^+ + V_{ij}^\dagger J_{ij}^{\mu\dagger} W_\mu^-\right) \quad , \quad J_{ij}^{\mu} = \bar{u}_i \gamma_\mu P_L d_j \qquad (1)$$

- Our goal is to deduce CKM matrix elements to high precision, in order to uncover new physics.
- V_{cb} currently has the highest uncertainty, so that's a worthwhile element to focus on.

CKM Element Determination

How do you determine V_{cb}? via decays like $B_s \to D_s l\bar{\nu}$:



$$\underbrace{\mathcal{M}(B_s \to D_s l\bar{\nu})}_{\text{get from experiment}} \simeq V_{cb} \underbrace{\langle D_s, l\bar{\nu}|J^{\mu}_{cb} D^{W}_{\mu\nu} J^{\nu}_{l\bar{\nu}}|B_s\rangle}_{\text{get from theoretical calculation}} \tag{2}$$

The need for a lattice calculation in $B_s \to D_s l\nu$

Amplitude we need to compute from SM: integrate out W and factorise

$$\langle D_s, l\bar{\nu}|J^{\mu}_{cb} D^{W}_{\mu\nu} J^{\nu}_{l\bar{\nu}}|B_s\rangle \tag{3}$$

$$= \frac{G_F}{\sqrt{2}} \underbrace{\langle D_s|J^{\mu}_{cb}|B_s\rangle_{QCD}}_{\substack{\text{non-perturbative,} \\ \text{need lattice QCD}}} \underbrace{\langle l, \bar{\nu}|J_{\mu, l\bar{\nu}}|0\rangle_{QED}}_{\substack{\text{easily done in} \\ \text{perturbation theory}}} + \mathcal{O}\left(\frac{q^2}{M_W^2}\right) \tag{4}$$

How to lattice

We want to compute $\langle D_s | J^{\mu}_{cb} | B_s \rangle$. This can be extracted from a correlation function, which can be computed with lattice QCD.

Definition

Lattice QCD = Solving the path integral (correlation function) by throwing computers at it.

$$\langle \Phi_{B_s}(0) J^{\mu}_{cb}(x) \Phi_{D_s}(y) \rangle$$
$$= \int \mathcal{D}A \mathcal{D}q \mathcal{D}\bar{q} \; \Phi_{B_s}(0) J^{\mu}_{cb}(x) \Phi_{D_s}(y) \; e^{-S_G[A] - \sum_i \bar{q}_i M_i[A] q_i}$$

How to lattice

We want to compute $\langle D_s | J^\mu_{cb} | B_s \rangle$. This can be extracted from a correlation function, which can be computed with lattice QCD.

Definition

Lattice QCD = Solving the path integral (correlation function) by throwing computers at it.

$$
\begin{aligned}
&\langle \Phi_{B_s}(0) J^\mu_{cb}(x) \Phi_{D_s}(y) \rangle \\
&= \int \mathcal{D}A \mathcal{D}q \mathcal{D}\bar{q} \; \Phi_{B_s}(0) J^\mu_{cb}(x) \Phi_{D_s}(y) \; e^{-S_G[A] - \sum_i \bar{q}_i M_i[A] q_i} \\
&= \int \mathcal{D}A \mathcal{D}q \mathcal{D}\bar{q} (\bar{b}(0)\gamma_5 s(0))(\bar{c}(x)\gamma^\mu b(x))(\bar{s}(y)\gamma_5 c(y)) e^{-S_G[A] - \sum_i \bar{q}_i M_i[A] q_i}
\end{aligned}
$$

How to lattice

We want to compute $\langle D_s | J^{\mu}_{cb} | B_s \rangle$. This can be extracted from a correlation function, which can be computed with lattice QCD.

Definition

Lattice QCD = Solving the path integral (correlation function) by throwing computers at it.

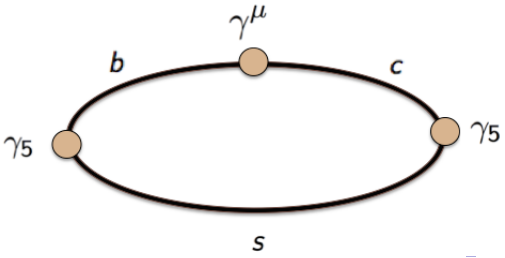
$$
\begin{aligned}
&\langle \Phi_{B_s}(0) J^{\mu}_{cb}(x) \Phi_{D_s}(y) \rangle \\
&= \int \mathcal{D}A \mathcal{D}q \mathcal{D}\bar{q} \; \Phi_{B_s}(0) J^{\mu}_{cb}(x) \Phi_{D_s}(y) \; e^{-S_G[A] - \sum_i \bar{q}_i M_i[A] q_i} \\
&= \int \mathcal{D}A \mathcal{D}q \mathcal{D}\bar{q} (\bar{b}(0)\gamma_5 s(0))(\bar{c}(x)\gamma^{\mu} b(x))(\bar{s}(y)\gamma_5 c(y)) e^{-S_G[A] - \sum_i \bar{q}_i M_i[A] q_i} \\
&= \int \mathcal{D}A \; \mathrm{Tr}\left[\gamma_5 \; M_b^{-1}(0,x)[A] \; \gamma^{\mu} \; M_c^{-1}(x,y)[A] \; \gamma_5 \; M_s^{-1}(y,0)[A]\right] e^{-S_G[A]}
\end{aligned}
$$

Integration over Gauge Configurations

Can perform integral over Gauge field by generating an "ensemble" of gauge configurations $\{A(x)\}$, with probability distribution $\exp(-S_G[A])$. Then-

$$\langle \Phi_{B_s}(0) J^{\mu}_{cb}(x) \Phi_{D_s}(y) \rangle$$
$$\simeq \frac{1}{N} \sum_{\{A(x)\}} \mathrm{Tr}\left[\gamma_5 \; M_b^{-1}(0,x)[A] \; \gamma^{\mu} \; M_c^{-1}(x,y)[A] \; \gamma_5 \; M_s^{-1}(y,0)[A]\right]$$

Problem boils down to computing 3 quark propagators (by inverting Dirac operators) on many of these gauge backgrounds $\{A(x)\}$:



Extracting Amplitudes from Correlation Functions

We compute $\langle \Phi_{B_s}(0) J^{\mu}_{cb}(x) \Phi_{D_s}(y) \rangle$ for many different x_0 and y_0. Then we can perform a fit to extract $\langle D_s | J^{\mu}_{cb} | B_s \rangle$ as a fit parameter, using the below as a fit function:

$$
\begin{aligned}
&\langle 0 | \Phi_{B_s}(0) J^{\mu}_{cb}(x) \Phi_{D_s}(y) | 0 \rangle \\
&= \sum_{n,m} \langle 0 | \Phi_{B_s}(\underline{0}) | n \rangle \langle n | J^{\mu}_{cb}(\underline{x}) | m \rangle \langle m | \Phi_{D_s}(\underline{y}) | 0 \rangle e^{-E_n x_0} e^{-E_m (y_0 - x_0)} \\
&\simeq \langle 0 | \Phi_{B_s}(\underline{0}) | B_s \rangle \underbrace{\langle B_s | J^{\mu}_{cb}(\underline{x}) | D_s \rangle}_{\text{yay physics}} \langle D_s | \Phi_{D_s}(\underline{y}) | 0 \rangle e^{-M_{B_s} x_0} e^{-M_{D_s} (y_0 - x_0)} \\
&\equiv f(x_o, y_0)
\end{aligned}
$$

Conclusion

- We need V_{cb} to higher precision to get us some new physics.
- V_{cb} can be deduced from a lattice calculation of $B_s \to D_s l \bar{\nu}$.
- The lattice calculation boils down to generating an ensemble of gauge configurations, then inverting the Dirac operator on each of those configurations.

Backup slides!

Other motivations for doing the thing

- $\sim 3\sigma$ tensions between theory and experiment in both $R(D)$ and $R(D^*)$, where

$$R(X) = \frac{\mathcal{B}(B \to X\tau\nu_\tau)}{\mathcal{B}(B \to Xe\nu_e)} \tag{5}$$

Could this effect also be in the $B_s \to D_s$ decay?

- There are up to $\sim 3\sigma$ tensions between different determinations of V_{cb}

$$|V_{cb}|^{B\to D^*}_{excl} \neq |V_{cb}|^{B\to D}_{excl} \neq |V_{cb}|_{incl} \tag{6}$$

Something is afoot...

Dirac Operator Inversion

Just need to invert the Dirac operator $M(x, y)[A]$ a bunch of times, plugging in a new gauge configuration $A(x)$ each time.

This typically achieved via *conjugate gradient*.

There are many choices for $M(x, y)[A]$ on the lattice that reduce to the Dirac operator in the continuum limit. In other words, corresponding to a choice of action.

Which action is best to use?

Choice of Action

For c and s quarks, we can use the HISQ (**H**ighly **I**mproved **S**traggered **Q**uark) action:

$$\mathscr{L}_{\text{HISQ}} = \bar{\psi}(\gamma \cdot \mathcal{D} - m + \overbrace{\mathcal{O}(a)}^{\substack{\text{where} \\ \text{"improvement"} \\ \text{happens}}})\psi$$

The b quark is too heavy for the lattice: instead use the NRQCD (**N**on-**R**elativistic **QCD**) action:

$$\mathscr{L}_{\text{NRQCD}} = \bar{\psi}\left(\mathcal{D}_t - \frac{\mathcal{D}^2}{2m} + \text{ relativistic corrections}\right)\psi$$

Continuum Extrapolation

We compute $\langle D_s | J^{\mu}_{cb} | B_s \rangle$ on lattices with many different lattice spacings a, then can extrapolate to $a = 0$.

The a-dependance of any observable $\mathcal{A}$ can be modelled by something like:

$$\mathcal{A}(a) = \mathcal{A}(a_0) \times (1 + Ba^2 + Ca^4)$$

Extrapolation to Physical Quark Masses

It is computationally difficult to use physical quark masses...

We compute $\langle D_s | J^{\mu}_{cb} | B_s \rangle$ using a range "easier" m_s's, then extrapolate to the physical m_s using:

$$\mathcal{A}(a, M, f) = \mathcal{A}(a) + c_1 \left(\frac{M}{f}\right)^2 + c_2 \left(\frac{M}{f}\right)^2 \log\left(\frac{M}{f}\right)^2$$

Inspiration for $\mathcal{A}$'s mass dependance comes from *chiral perturbation theory.*