Disformally Self-Tuning Gravity

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Outline

1. What is the Cosmological Constant Problem (CCP)?

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2. Self-tuning Horndeski theory & a disformal coupling to matter
3. Analysis & Summary

Issues with vacuum energy

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- QFT predicts a non-trivial vacuum energy density for each particle species $\rightarrow$ not a problem in "ordinary" QFT as only energy differences are observable.
- *However*, it becomes relevant when one introduces gravity $\rightarrow$ gravity is sensitive to absolute energy densities!
- Requiring that the equivalence principle holds implies that vacuum energy should gravitate $\rightarrow$ identify this with cosmological constant.

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- Current data requires $\Lambda_{ren} \sim meV^4$ [1]
$\rightarrow$ Significant fine-tuning required. Problematic, but not disastrous!

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- Consequently, the cosmological constant does not have a stable value even in the regime of the Standard Model $\rightarrow$ CCP!

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[where $\hat{G} = R^2 - 4R^{\mu\nu}R_{\mu\nu} + R^{\mu\nu\alpha\beta}R_{\mu\nu\alpha\beta}$ is the "*Gauss-Bonnet*" combination, and $P^{\mu\nu\alpha\beta} := (**R)^{\mu\nu\alpha\beta} = -\frac{1}{4}\varepsilon^{\mu\nu\rho\sigma}R_{\rho\sigma\lambda\gamma}\,\varepsilon^{\lambda\gamma\alpha\beta}$ is the double-dual of the Riemann tensor.]

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- Importantly, Weinberg's famous no-go theorem is avoided by breaking Poincaré invariance at the level of the self-adjusting scalar field $\longrightarrow$ ϕ is allowed to evolve in time.

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$$\bar{g}_{\mu\nu}(x) = A^2(\phi, X)\left[g_{\mu\nu}(x) + B^2(\phi, X)\partial_\mu\phi\partial_\nu\phi\right] \tag{3}$$

[where $X = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$.]

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- Simplifies analysis and corresponds to the physical frame $\longrightarrow$ matter follows the geodesics defined by the physical metric $\bar{g}_{\mu\nu}$, and the energy-momentum tensor is covariantly conserved, $\bar{\nabla}_\mu \bar{T}^{\mu\nu} = 0$..

What is *self-tuning*?

- We are yet to define what "self-tuning" means in the context of theories of gravity. By *self -tuning* it is meant that the theory satisfies the following set of constraints (a so-called *self-tuning filter*):

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  2. This should remain true before and after any phase transition in which the cosmological constant "jumps" (instantaneously) by a finite amount;
  3. The theory should permit a non-trivial cosmology (a vital requirement in order for the theory to match observational data).

Cosmological set-up of the theory

- The background geometries in the Horndeski (HF) and Jordan (JF) frames are taken to be FLRW. In particular, the geometry in the JF should be asymptotically Minkowski. Hence,

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$$ds^2 = g_{\mu\nu}(x)dx^\mu dx^\nu = -N^2(t)dt^2 + a^2(t)\gamma_{ij}(\mathbf{x})dx^i dx^j \tag{5}$$

$$d\bar{s}^2 = \bar{g}_{\mu\nu}(x)dx^\mu dx^\nu = -dt^2 + \bar{a}^2(t)\gamma_{ij}(\mathbf{x})dx^i dx^j \tag{6}$$

  - $g_{\mu\nu}(x)$: *Horndeski-frame metric* (describes geometry defined by gravitation).
  - $\bar{g}_{\mu\nu}(x)$: *Jordan-frame metric* (describes physical geometry on which matter propagates).

Constructing the theory on an FRW background

- We evaluate the Horndeski Lagrangian in the HF, subsequently transforming the relevant dynamical variables into the JF, such that it is of the form

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$$\mathcal{L}_{FRW} = \bar{a}^3 \sum_{i=0}^{3} Z_i(\bar{a}, \phi, \dot{\phi}, \ddot{\phi}) \bar{H}^i \tag{7}$$

Applying the self-tuning filter

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- We assume that $\phi(t)$ is continuous, but that $\dot{\phi}$, $\ddot{\phi}$ and $\dddot{\phi}$ can be discontinuous, compensating for changes in Λ due to phase transitions in matter sector.

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$$\tilde{\mathcal{L}} = \bar{a}^3 \sum_{i=0}^{3} \tilde{Z}_i \bar{H}^i \equiv \bar{a}^3 \sum_{i=1}^{3} \tilde{Z}_i \left[\bar{H}^i - \left(\frac{s}{\bar{a}} \right)^i \right] \tag{10}$$

General form of self-tuning Lagrangian

- *A priori*, $\tilde{\mathcal{L}}$ not a *necessary* condition $\rightarrow$ possibly other equivalent Lagrangians, with $Z_i = \tilde{Z}_i + \Delta Z_i$, that admit same set of self-tuning solutions.

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- It turns out that for the theory to be self-tuning it must be that $\mathcal{H} = \tilde{\mathcal{H}}$ and $\varepsilon^\phi = \tilde{\varepsilon}^\phi$. Hence, $Z_i = \tilde{Z}_i$.

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- These equations can be used to determine the self-tuning form of each of the Horndeski functions K, G_i $(i = 3, 4, 5)$.

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- In the *conformal* case, $\bar{A} = \bar{A}(\phi)$, the theory reproduces a Lagrangian in the same class of self-tuning theories as the Fab Four.

- In the most general case, where $\bar{A} = \bar{A}(\phi, \bar{X})$ and $\bar{B} = \bar{B}(\phi, \bar{X})$ (in principle), requiring self-tuning enforces the constraint:

Analysis - Most general disformal case

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- As a consequence, the Horndeski Lagrangian (evaluated on an FRW background) can be expressed in a self-tuning form:

$$\begin{aligned} \mathcal{L}_{FRW} =& \bar{a}^3 \left[N\sqrt{2X}V_1' - 2V_1 \frac{s}{\bar{a}} + \left(3\bar{G}_5 - 3\sqrt{2X}G_5 - V_2 \right) \left(\frac{s}{\bar{a}} \right)^2 \right] \left[\bar{H} - \frac{s}{\bar{a}} \right] \\ &+ \bar{a}^3 \left[N\sqrt{2X}V_2' + 6NG_4 - 3N\sqrt{2X}\bar{G}_{5,\phi} + 2V_1 \right] \left[\bar{H}^2 - \left(\frac{s}{\bar{a}} \right)^2 \right] \\ &+ 2\bar{a}^3 \frac{X\sqrt{2X}}{N^2} G_{5,X} \left[\bar{H}^3 - \left(\frac{s}{\bar{a}} \right)^3 \right] \end{aligned} \tag{14}$$

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- *Caveat*: the system of differential equations for K, G_i $(i = 3, 4, 5)$ cannot be solved in general and must be done so on a *case-by-case* basis.

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Outlook

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- Possible future research: use of present analysis as a starting point in the construction of a "*beyond Horndeski*" theory.

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Thank you for your time.
Any questions?