Asymptotic safety and fixed points of gauge theories

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Outline

Renormalisation group

- Couplings λ_i in QFT run with energy scale — described by renormalisation group equations (RGEs)

$$\frac{\partial \lambda_i}{\partial \log \mu} = \beta_i(\{\lambda\})$$

- Beta functions β_i determined by field content and symmetries
- Various approaches available to compute the β_i in some approximation

Fixed points

- Fixed points λ_i^* are points in coupling space that satisfy

$$\beta_i(\{\lambda^*\}) = 0$$

- Infrared means have solutions to RGEs which satisfy $\lim_{\mu\to 0^+} \lambda(\mu) = \lambda^*$
- Ultraviolet means have solutions to RGEs which satisfy $\lim_{\mu\to\infty} \lambda(\mu) = \lambda^*$
- Ultraviolet fixed points allow us to define QFTs up to arbitrarily large energies

Perturbation theory

- Can compute β-functions perturbatively — power series expansion in coupling constants

$$\beta(\lambda) = c_1\lambda^2 + c_2\lambda^3 + \dots$$

- Extensive set of tools available, structure of β-functions is known in general for first few loop orders
- Useful starting point to understand non-perturbatively

Ultraviolet fixed points in perturbation theory

- Two possible fixed point scenarios:
  - Gaussian fixed point $\lambda^* = 0$ — asymptotic freedom
  - Interacting fixed point $\lambda^* \neq 0$ — asymptotic safety
- Perturbation theory $\implies$ need couplings to be small
  - For asymptotic safety need $0 < |\lambda^*| \ll 1$
  - Small corrections to anomalous dimensions — classical mass dimension still governs relevance
- What are the necessary ingredients for perturbative asymptotic safety to be realised?

Gauge theory one-loop beta function

$$\beta(\alpha) \quad = \quad -B\alpha^2 \quad + \quad \mathcal{O}(\alpha^3)$$

B is determined by gauge group and matter content

$$B = \frac{2}{3}\left(11C_2^{\mathcal{G}} - 2S_2^F - \tfrac{1}{2}S_2^S\right)$$

Gauge theory one-loop beta function

$$\beta^{(1)} = -B\alpha^2$$

- No other couplings affect the running of the gauge at this order
- B can take either sign
- Have only the Gaussian (free) fixed point $\alpha^* = 0$
  - $B > 0$ this is UV (asymptotic freedom)
  - $B < 0$ this is IR — Landau pole in UV. Signals that we need to study further — go to higher order!

Two-loop RGE

$$\beta(\alpha) = \alpha^2(-B + C\alpha) \quad + \quad \mathcal{O}(\alpha^4)$$

- Have potential interacting fixed point from cancellation of one- and two-loop contributions

$$\alpha^* = \frac{B}{C} \,.$$

- Physical $\implies$ $BC > 0$
- Perturbative $\implies$ $|B| \ll |C|$

One-loop vs. two-loop contributions

Gauge β-function coefficients are

$$C = 2\left[\left(\frac{10}{3}C_2^{\mathcal{G}} + 2C_2^F\right)S_2^F + \left(\frac{1}{3}C_2^{\mathcal{G}} + 2C_2^S\right)S_2^S - \frac{34}{3}(C_2^{\mathcal{G}})^2\right],$$

$$B = \frac{2}{3}\left(11C_2^{\mathcal{G}} - 2S_2^F - \tfrac{1}{2}S_2^S\right)$$

- Extreme cases offer no fixed point:
  - Not much matter, $B > 0$ and $C < 0$
  - Lots of matter, $B < 0$ and $C > 0$
- In between we can have $B, C > 0$: Banks-Zaks infrared fixed point, e.g. QCD with $N_f = 16$
- $B, C < 0$ not possible! $B < 0 \implies C > 0$. No UV fixed point.

Gauge only UV fixed point?

Is it possible to have $B, C < 0$?

$$C = \frac{2}{11}\left[2S_2^F\left(11C_2^F + 7C_2^{\mathcal{G}}\right) + 2S_2^S\left(11C_2^S - C_2^{\mathcal{G}}\right) - 17B\,C_2^{\mathcal{G}}\right].$$

- Manifestly impossible with only fermions
- In fact for any irrep of a simple gauge group $C_2^R \geq \frac{3}{8}C_2^{\mathcal{G}}$
- No UV fixed points with only a gauge coupling!

Yukawa couplings

- Yukawa couplings arise naturally when we have fermions and scalars
- They affect the running of the gauge coupling at two-loop via a term

$$\beta_g^{(2,y)} = -\alpha^2 \frac{2}{d_G} \,\mathrm{Tr}[\mathbf{C}_2^F \mathbf{Y}^A (\mathbf{Y}^A)^\dagger] \leq 0$$

- Yukawa running depends on gauge at one-loop

$$\beta^A = \mathbf{E}^A(Y) - \alpha\, \mathbf{F}^A(Y)\,.$$

  - Dimensionally, these vanish on $\mathbf{Y}^A = \frac{g}{4\pi}\mathbf{C}^A$

Yukawas

- Project gauge beta function onto Yukawa nullcline by the replacement

$$C \to C' = C - \frac{2}{d_G}\,\mathrm{Tr}[\mathbf{C}_2^F \mathbf{C}^A (\mathbf{C}^A)^\dagger]$$

- Now *effective* two-loop term C' plays the same role as C did previously
- Necessarily $C > C'$, so may be possible to have $C' < 0$ with $B < 0$
  - Get fixed point $\alpha^* = \frac{B}{C'} > 0$, ultraviolet!
- Get IR fixed point if $B, C' > 0$

Fixed points summary

case	gauge group	Yukawa	parameter	interacting FPs	type
a)	simple	**No**	$B > 0$ and $C > 0$	Banks-Zaks	**IR**
b)	semi-simple, no $U(1)$ factors	**No**	all $B_a > 0$	Banks-Zaks and products thereof	**IR**
c)	simple	**Yes**	$B > 0$ and $C > 0 > C'$	Banks-Zaks	**IR**
	simple	**Yes**	$B > 0$ and $C > C' > 0$	BZ and GYs	**IR**
	simple or abelian	**Yes**	$B < 0$ and $C' < 0$	gauge-Yukawas	**UV/IR**
d)	semi-simple, with or without $U(1)$ factors	**Yes**	all $B'_a > 0$	BZs and GYs and products thereof	**UV/IR**

Scalar self couplings

- Scalar degrees of freedom $\implies$ quartic couplings — not technically natural
- Doesn't affect fixed point, enters gauge (Yukawa) running at three- (two-) loop level
- For consistency, need fixed point for quartics.
  - Solving quadratic equations — not guaranteed to have real solutions!
  - Need quartic tensor to be positive definite for vacuum stability
- Quartics provide independent consistency constraints

$$\lambda^*_{ABCD} = \text{real}\,, \qquad\qquad V_{\text{eff}}(\phi) = \text{stable}\,,$$

Two example theories

- Are interacting IR/UV gauge-Yukawa fixed points achievable in real theories?
- Consider two example theories. Each has:
  - $SU(N_c)$ gauge group
  - N_f fundamental Dirac fermions ψ_i
  - Will consider theories with some 'large' values of N_f, N_c to allow one-loop B to be small, and have control over expansion

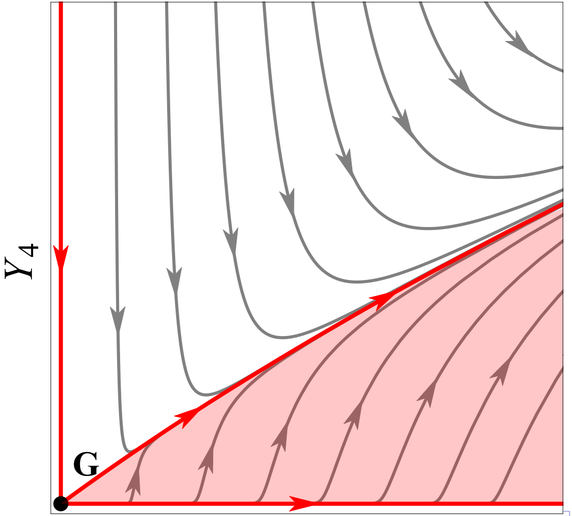
First example theory (a)

- Have a single uncharged scalar field ϕ
- Yukawa term diagonal in flavour $y\psi_{L,i}\phi\psi_{R,i}$
- Yukawa structure means that $C' > 0$
- For small $B > 0$, have Banks-Zaks and interacting IR gauge-Yukawa fixed point
- Theory has no perturbative UV completion with $B < 0$

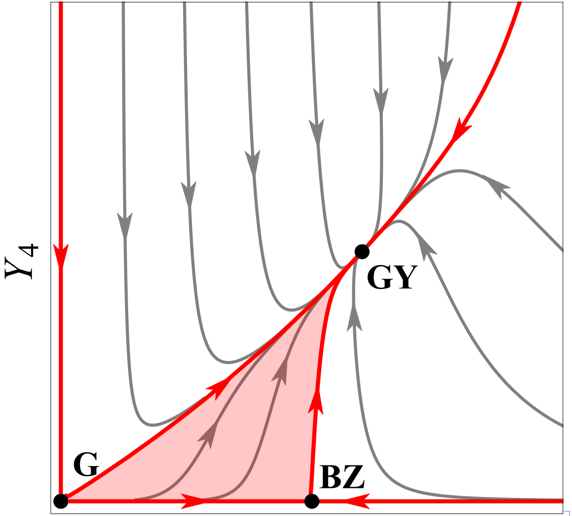
Second example theory (b)

- Have $N_f \times N_f$ matrix of uncharged scalar fields Φ_{ij}
- Yukawa term mixes flavours $y\psi_{L\,i}\Phi_{ij}\psi_{R\,j}$
- Yukawa structure means that $C' < 0$
- For small $B > 0$, have Banks-Zaks fixed point only
- For small $B < 0$ have interacting UV fixed point — asympotic safety

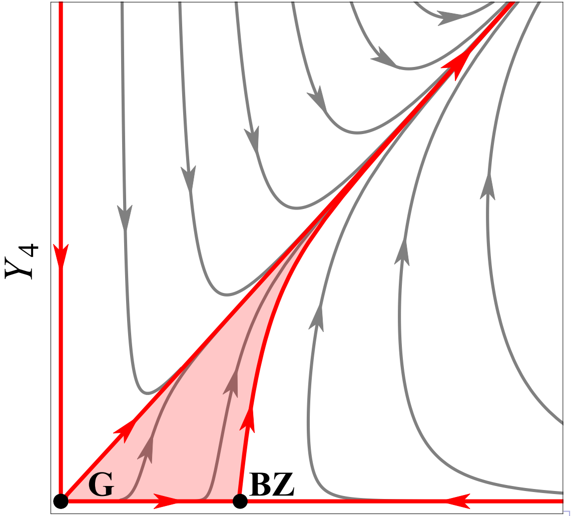
$C\,,\,C' < 0$



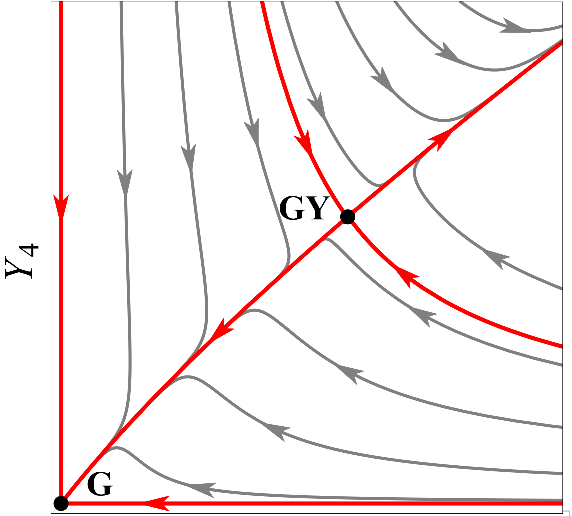
$C > 0 > C'$



$C > C' > 0$



$C' < 0 < C$



Outlook

- Explore further beyond perturbation theory — persistence of fixed points
- Size of UV/IR conformal windows
- Understand better the landscape of asympotically safe gauge theories
- Applications to BSM model building — interacting strong coupling constant fixed point?
- Gain further insight into ultraviolet fixed points in general — quantum gravity?

Conclusion

- Only available perturbative fixed points are Gaussian, gauge-only (IR), or gauge-Yukawa, or products of these
- Yukawa couplings offer a unique mechanism for gauge theories to develop perturbative interacting UV fixed points
- If one-loop term is small $1 \gg |B| > 0$ then we generically expect interacting gauge-Yukawa fixed points
- Gauge-Yukawa fixed points can be either UV or IR depending on Yukawa structure