Exotic Hadrons on the Lattice

Gavin Cheung
Hadron Spectrum Collaboration

DAMTP, University of Cambridge

6 September 2017
Quark Model and the $c\bar{c}$ Spectrum

No $0^{--}, 0^{+-}, 1^{--}, 2^{+-}, \ldots$

The Situation Today for $c\bar{c}$

Too many states seen compared to what quark models predict.

S. Olsen, arxiv:1511.01589
Exotic Mesons

- Glueball
- Hybrid
- Tetraquark
- Molecule
Exotic Mesons

Can these explain the exotic states?

(a) Glueball
(b) Hybrid
(c) Tetraquark
(d) Molecule
Lattice QCD

- Lattice QCD is an attractive framework for performing first principle calculations to understand exotic mesons.

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- The spectrum is contained in the two-point correlation function and can be extracted. What about \( O \)?
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Meson Operators

- We want to build good operators with the correct quantum numbers of the states we’re interested in. Starting with fermion bilinears,

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- This construction also gives ‘gluey’ operators \( \mathcal{O}(t) \propto F_{\mu\nu} \) that resemble a hybrid meson structure.
Results
$c\bar{c}$ Spectrum at $m_\pi \sim 240$ MeV

GC et al., arXiv:1610.01073
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$S$-wave

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$S$-wave

$P$-Wave

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- **S-wave**
- **P-Wave**
- **D-Wave**
- **F-Wave**
- **G-Wave**

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Hybrid Mesons

Consistent with adding an effective gluonic degree of freedom $J^{PC} = 1^{+-}$ to quark model.

$q\bar{q} \ L = 0$

\[
\{0^{-+}; 1^{--}\} \rightarrow \{1^{--}; 0^{-+}, 1^{-+}, 2^{-+}\}
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$q\bar{q} \ L = 1$

$$\{1^{+-}; 0^{++}, 1^{++}, 2^{++}\} \rightarrow \{0^{++}, 1^{++}, 2^{++}; 0^{+-}, 1^{--}(3), 2^{+-}(2), 3^{+-}\}$$
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\{1^{+-}; 0^{++}, 1^{++}, 2^{++}\} \rightarrow \{0^{++}, 1^{++}, 2^{++}; 0^{+-}, 1^{+-}(3), 2^{+-}(2), 3^{+-}\}
\]

These states are only seen when we include gluey operators in the calculation.
$m_\pi \sim 240$ MeV vs $m_\pi \sim 400$ MeV

$m_\pi \sim 400$ MeV from Liu et al., arXiv:1204.5425
Bigger Operators

- No multi-meson states seen in the previous spectrum. Need different operators?
- Meson-meson operators (M)

\[ \mathcal{O}(t) \sim (\bar{c} \Gamma q')(\bar{q}' \Gamma c). \]

- Tetraquark operators (T)

\[ \mathcal{O}(t) \sim G_{ad} \left( g_{abc} c_b \left( C \Gamma_1 \right) q_c^T \right) \left( g_{def} \bar{c}_e^T \left( \Gamma_2 C \right) \bar{q}_f \right). \]

[Diquark] [Anti-diquark]
Isospin-1 hidden charm spectrum \((c\bar{c}q\bar{q})\) for \(m_\pi \sim 400\) MeV

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- Tetraquark operators do not seem to have a significant effect on the finite volume spectrum.
- Finite volume spectrum lie close to non-interacting meson-meson levels suggesting there are weak meson-meson interactions.
- Therefore, there is no strong indication for a bound state or narrow resonance in these channels. \(Z_C(3900)\)?
Isospin-0 doubly charmed spectrum ($cc\bar{q}\bar{q}$)
Conclusions and outlook

▶ Quark model does not fully describe all the experimentally observed mesons and lattice QCD provides an attractive way to study exotic mesons.

▶ In lattice QCD, we find states with exotic $J^{PC}$ quantum numbers and identify states that are consistent with a quark-antiquark combination coupled to a $1^{+-}$ gluonic excitation.

▶ Turning to four-quark states, we do not find significant changes to spectrum when including a class of operators resembling tetraquarks in our calculations. The extracted spectrum does not show any clear signs of bound states or narrow resonances.

▶ Next steps are to relate the discrete finite volume spectrum to scattering phenomena using the Lüscher formalism. This would require more spectra in moving frames and different volumes. [David Wilson, Lattice results for spectroscopy, 8:45am]