Exotic Hadrons on the Lattice

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Hadron Spectrum Collaboration

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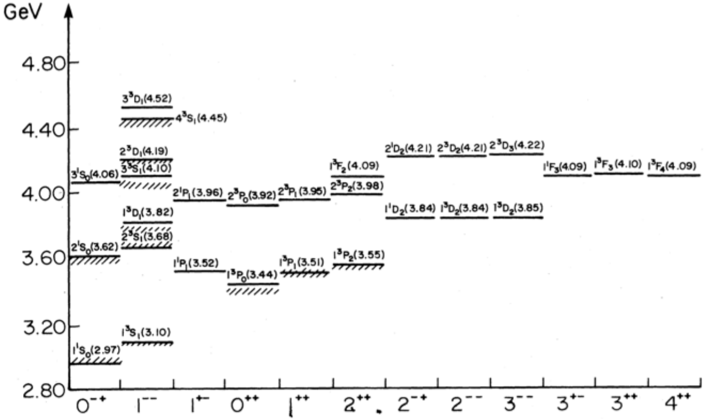
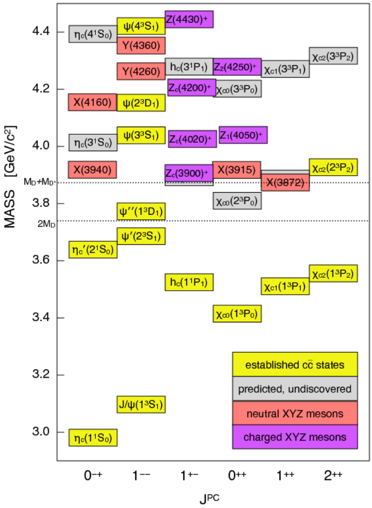
Quark Model and the $c\bar{c}$ Spectrum



No $0^{--}, 0^{+-}, 1^{-+}, 2^{+-}, \ldots$

S. Godfrey, N. Isgur, Phys. Rev. D 32, 189 (1985)

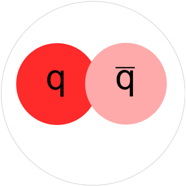
The Situation Today for $c\bar{c}$



Too many states seen compared to what quark models predict.

S. Olsen, arxiv:1511.01589

Exotic Mesons



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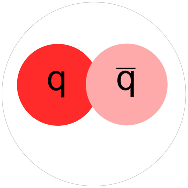
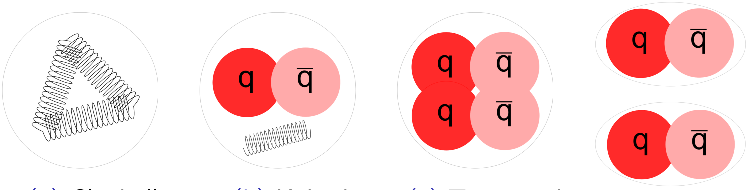


Can these explain the exotic states?



(a) Glueball (b) Hybrid (c) Tetraquark (d) Molecule

Lattice QCD

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- The spectrum is contained in the two-point correlation function and can be extracted. What about $\mathcal{O}$?

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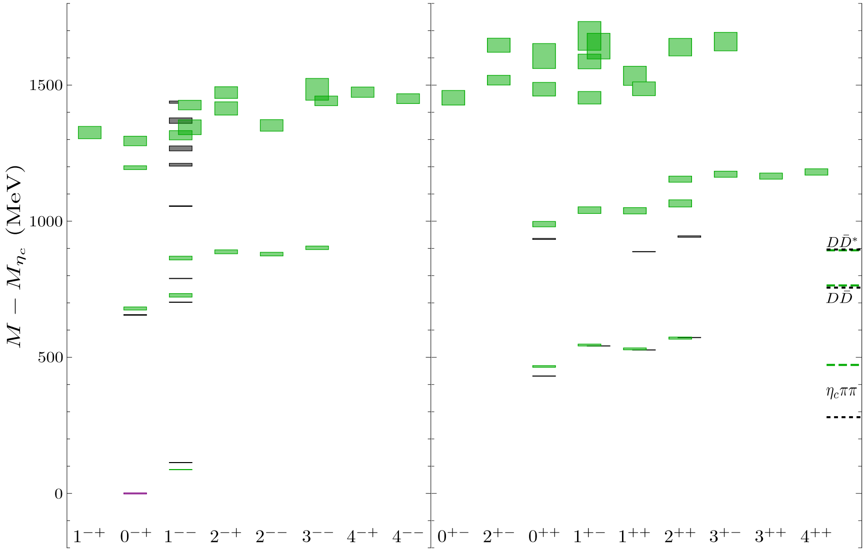
- This construction also gives 'gluey' operators $\mathcal{O}(t) \propto F_{\mu\nu}$ that resemble a hybrid meson structure.

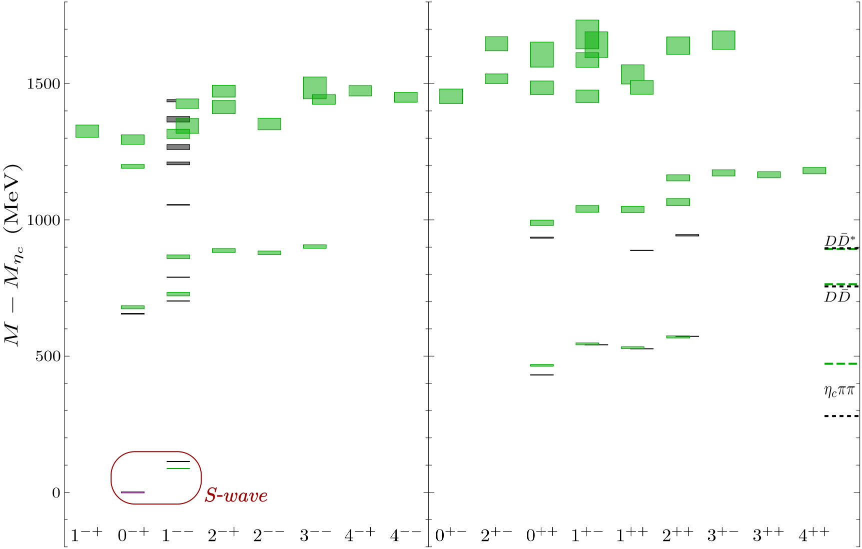
Results

$c\bar{c}$ Spectrum at $m_\pi \sim 240$ MeV



GC et al., arXiv:1610.01073

$c\bar{c}$ Spectrum at $m_\pi \sim 240$ MeV



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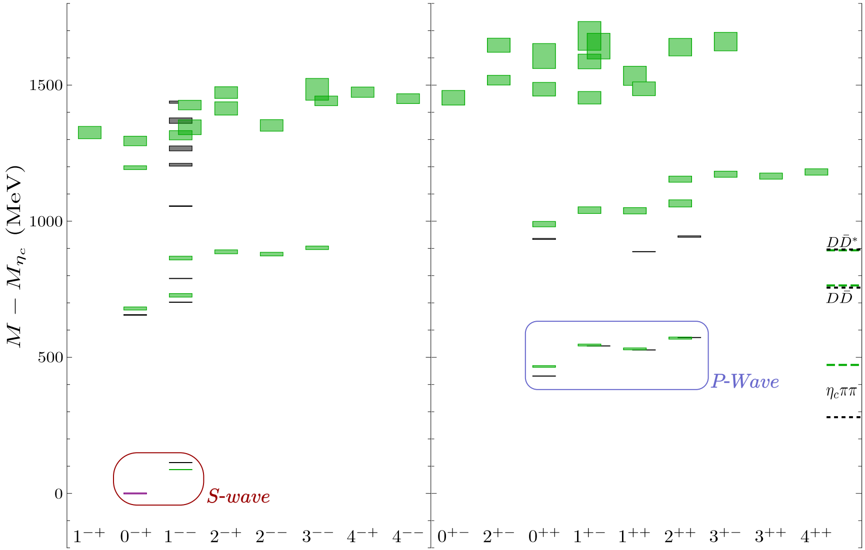

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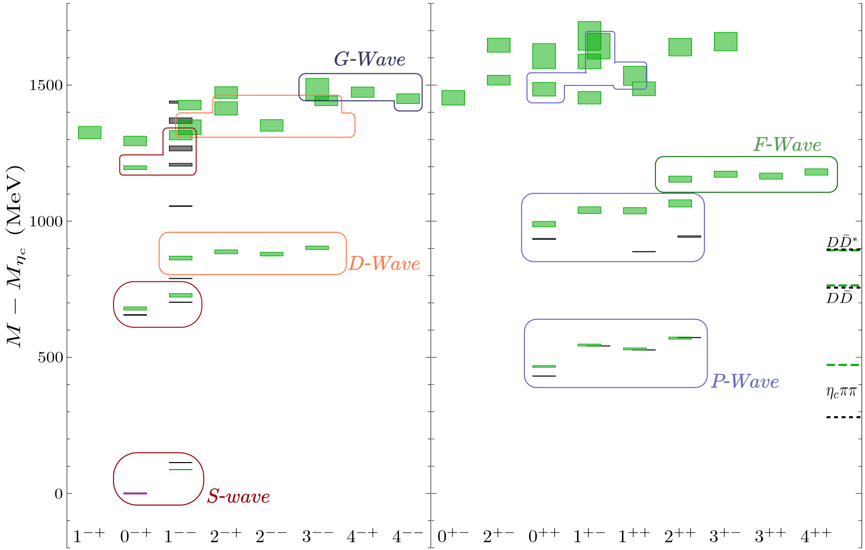

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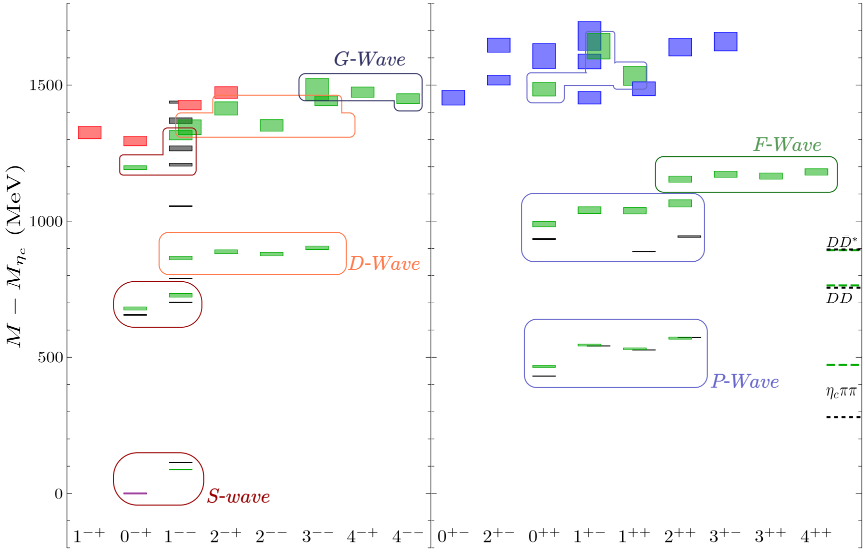

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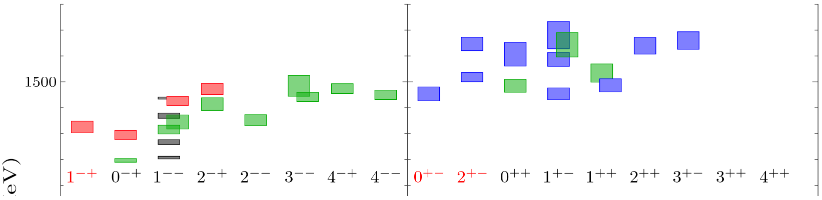
Hybrid Mesons



Consistent with adding an effective gluonic degree of freedom $J^{PC} = 1^{+-}$ to quark model.

$q\bar{q}$ $L = 0$

$$\{0^{-+}; 1^{--}\} \rightarrow \{1^{--}; 0^{-+}, 1^{-+}, 2^{-+}\}$$

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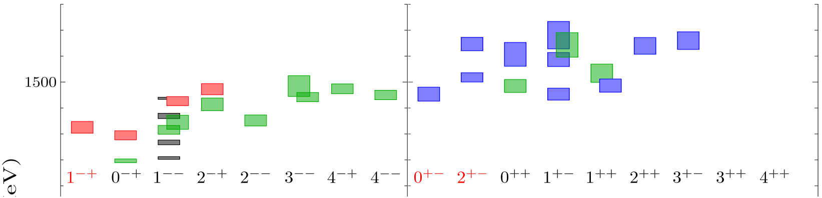

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$$\{0^{-+};1^{--}\} \to \{1^{--};0^{-+},{\color{red}1^{-+}},2^{-+}\}$$

$q\bar{q}\ L = 1$

$$\{1^{+-};0^{++},1^{++},2^{++}\} \to \{0^{++},1^{++},2^{++};{\color{red}0^{+-}},1^{+-}(3),{\color{red}2^{+-}}(2),3^{+-}\}$$

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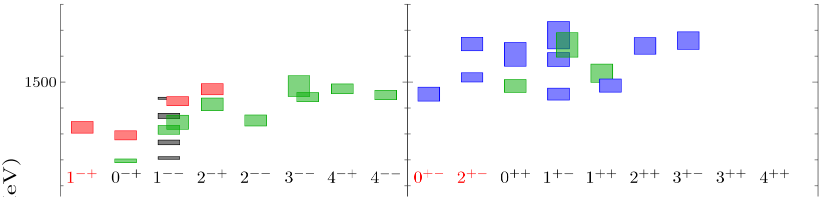

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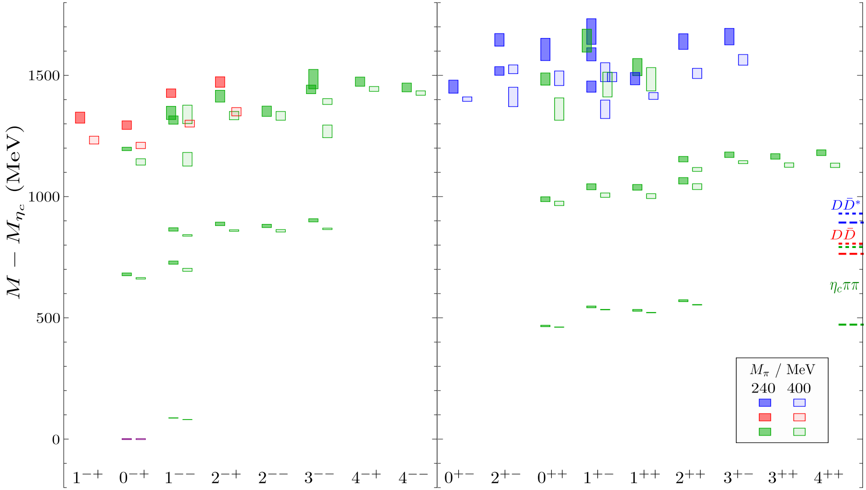
$$\{1^{+-}; 0^{++}, 1^{++}, 2^{++}\} \to \{0^{++}, 1^{++}, 2^{++}; 0^{+-}, 1^{+-}(3), 2^{+-}(2), 3^{+-}\}$$

These states are only seen when we include gluey operators in the calculation.

$m_\pi \sim 240$ MeV vs $m_\pi \sim 400$ MeV



$m_\pi \sim 400$ MeV from Liu et al., arXiv:1204.5425

Bigger Operators

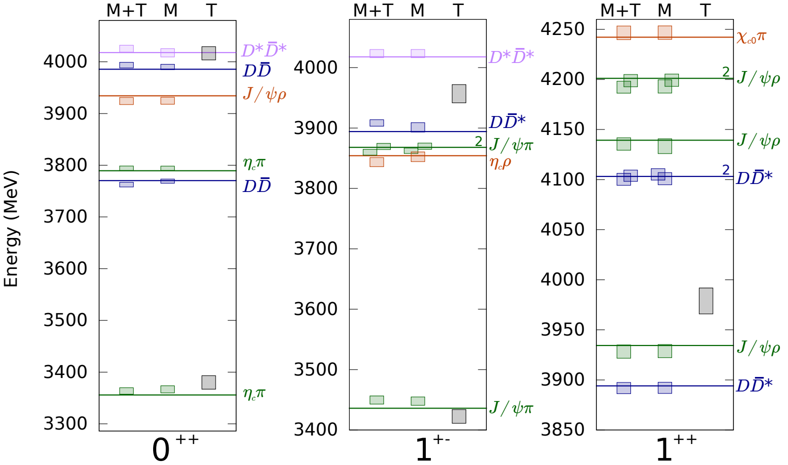
- No multi-meson states seen in the previous spectrum. Need different operators?
- Meson-meson operators (M)

$$\mathcal{O}(t) \sim (\bar{c}\Gamma q')(\bar{q}\Gamma' c).$$

- Tetraquark operators (T)

$$\mathcal{O}(t) \sim G_{ad} \underbrace{\left(g_{abc} c_b (C\Gamma_1) q_c^T\right)}_{\text{Diquark}} \underbrace{\left(g_{def} \bar{c}_e^T (\Gamma_2 C) \bar{q}_f\right)}_{\text{Anti-diquark}}.$$

Isospin-1 hidden charm spectrum ($c\bar{c}q\bar{q}$) for $m_\pi \sim 400$ MeV

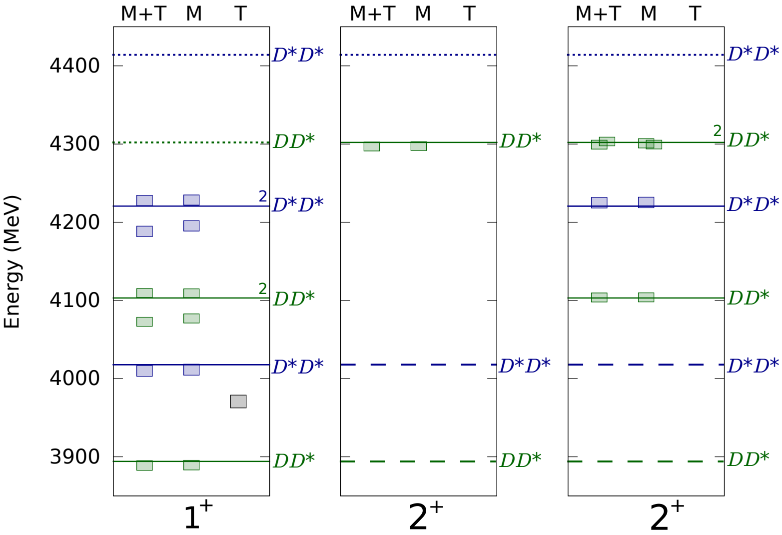


GC, C.E.Thomas, J.J.Dudek, R.G.Edwards, arXiv:1709.01417

Isospin-1 hidden charm spectrum ($c\bar{c}q\bar{q}$) for $m_\pi \sim 400$ MeV

- Tetraquark operators do not seem to have a significant effect on the finite volume spectrum.
- Finite volume spectrum lie close to non-interacting meson-meson levels suggesting there are weak meson-meson interactions.
- Therefore, there is no strong indication for a bound state or narrow resonance in these channels. $Z_C(3900)$?

Isospin-0 doubly charmed spectrum ($cc\bar{q}\bar{q}$)



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Conclusions and outlook

- Quark model does not fully describe all the experimentally observed mesons and lattice QCD provides an attractive way to study exotic mesons.
- In lattice QCD, we find states with exotic J^{PC} quantum numbers and identify states that are consistent with a quark-antiquark combination coupled to a 1^{+-} gluonic excitation.
- Turning to four-quark states, we do not find significant changes to spectrum when including a class of operators resembling tetraquarks in our calculations. The extracted spectrum does not show any clear signs of bound states or narrow resonances.
- Next steps are to relate the discrete finite volume spectrum to scattering phenomena using the Lüscher formalism. This would require more spectra in moving frames and different volumes. [David Wilson, Lattice results for spectroscopy, 8:45am]