$B \to \pi \ell \nu$ form factors from Lattice QCD

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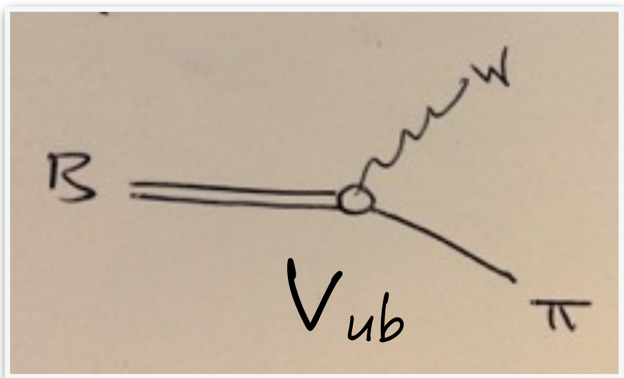
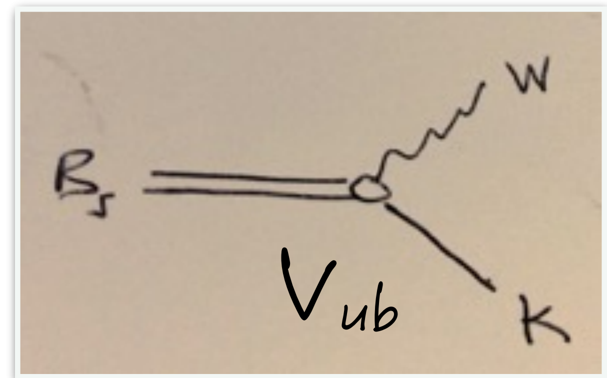
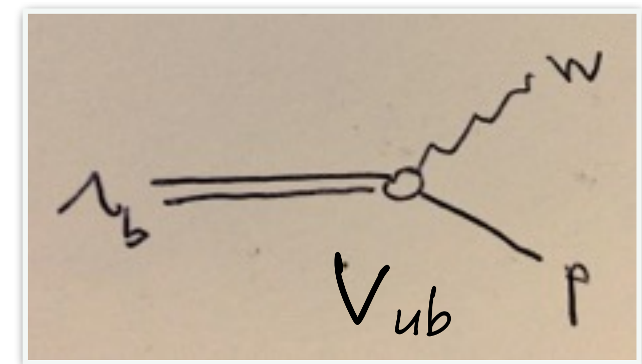
- introduction
- lattice calculation of matrix elements/form factors
  - q^2 coverage on the lattice
- preliminary results
  - form factors
  - $|V_{ub}|$
- summary

introduction

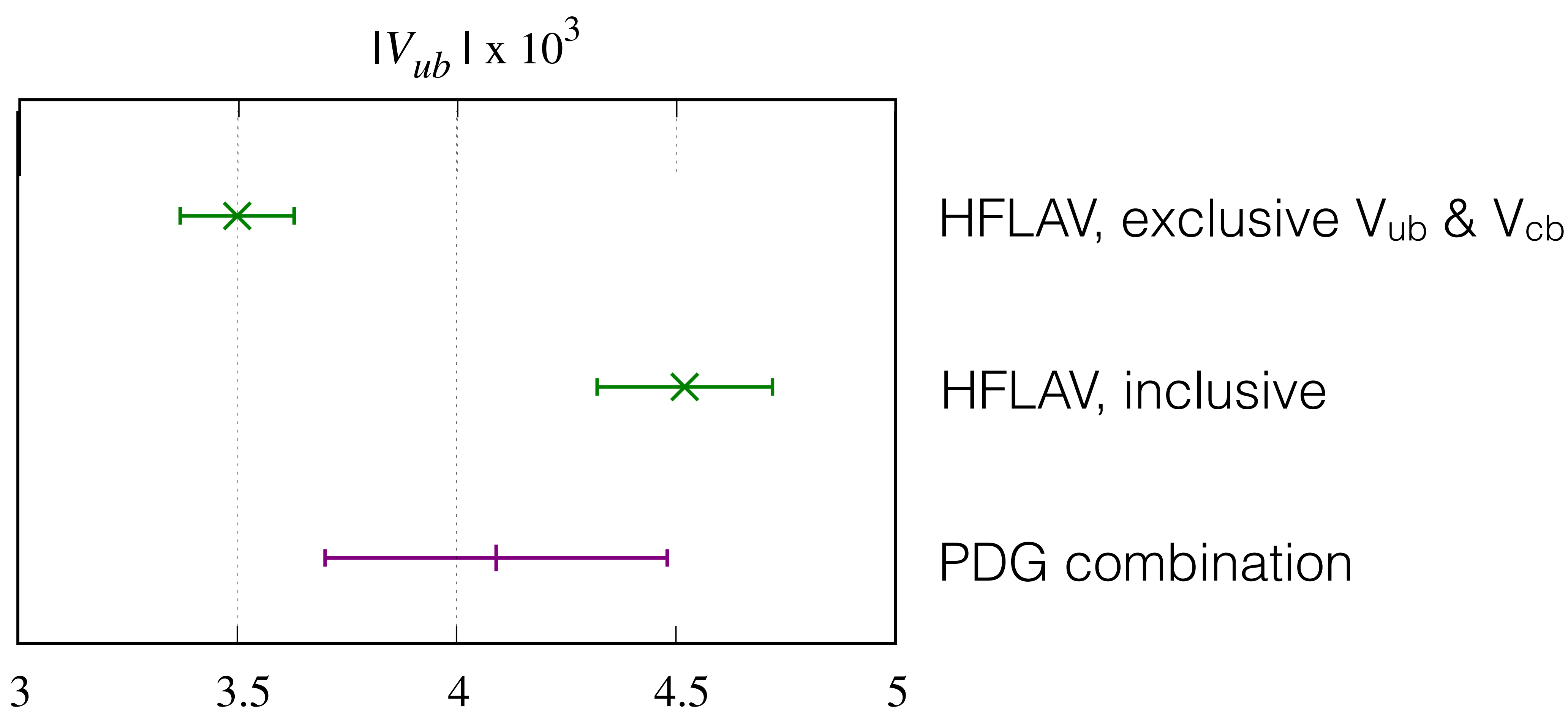
- Standard Model accommodation of flavor changes



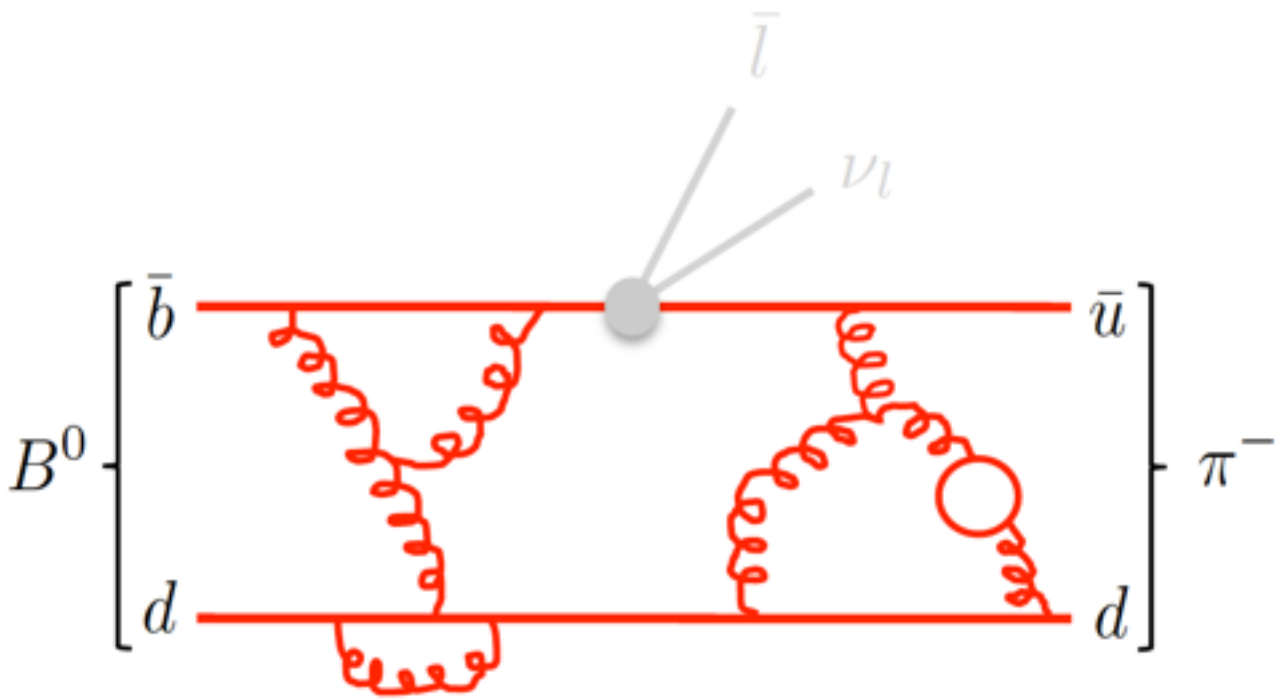




- inclusive vs exclusive

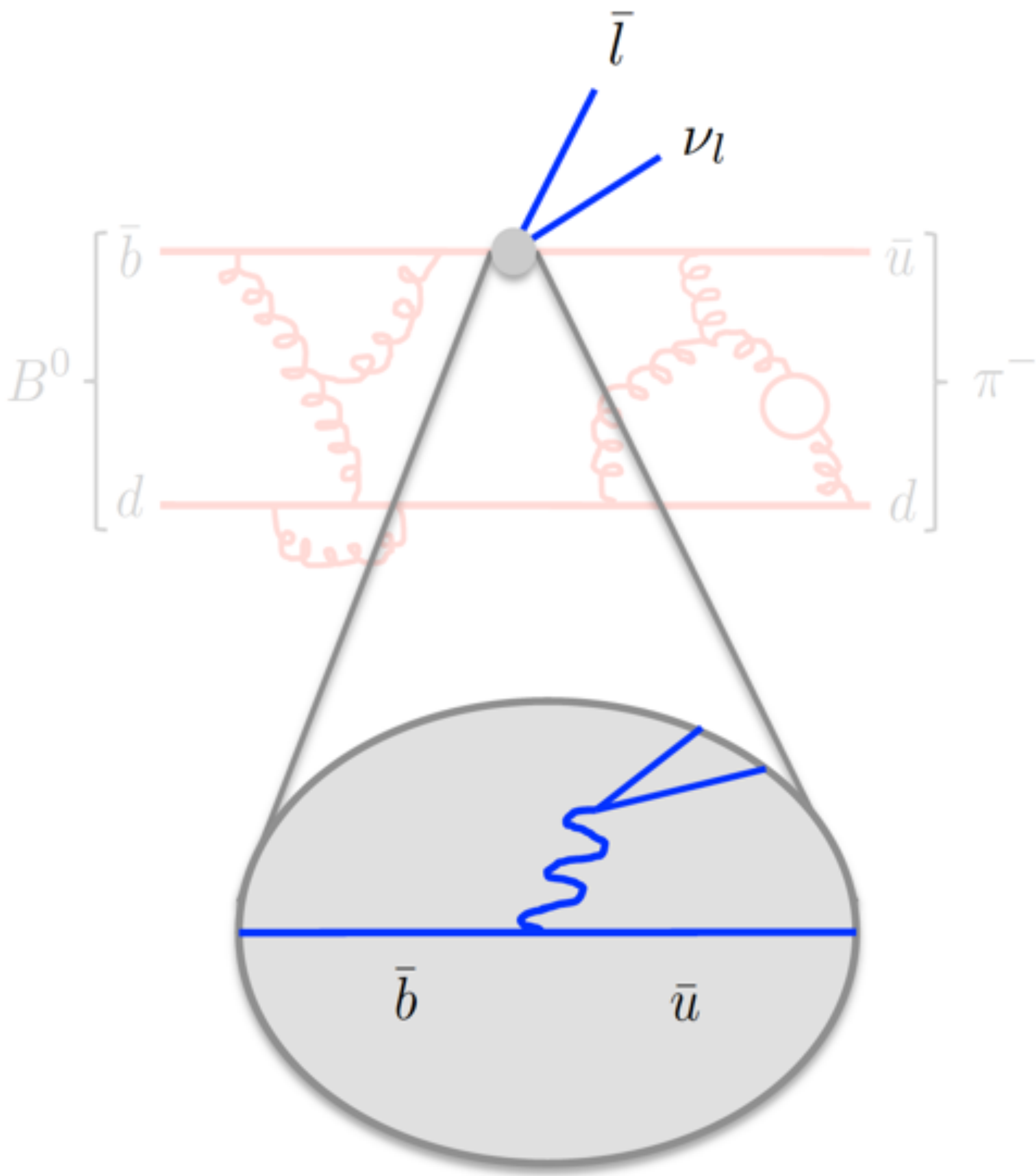


- role of LQCD



Λ_{QCD} ~ few x 100 MeV

- role of LQCD



Λ_{QCD} ~ few x 100 MeV

Λ_{EW} ~ 100 GeV

- role of LQCD

$$\frac{d\mathcal{B}}{dq^2} = \left| V_{ub} \sum_\mu C_\mu \langle \pi | V^\mu | B \rangle \right|^2 + \cdots$$

$\langle \pi | V^\mu | B \rangle$: lattice QCD

C_μ : known or perturbatively calculable

V_{ub} : extract from combination with expt

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lattice calculation of matrix elements/form factors

1) $\langle \pi(T) V^\mu(t) B^\dagger(0) \rangle$: generate data via Monte Carlo evaluation of path integral.

2) fit known time-dependence to get matrix element

$$\langle \pi(T) V^\mu(t) B^\dagger(0) \rangle = \sum_{n,m} \frac{\langle \pi | E_n^\pi \rangle}{\sqrt{2E_n^\pi}} \langle E_n^\pi | V^\mu | E_m^B \rangle \frac{\langle E_m^B | B^\dagger \rangle}{\sqrt{2E_m^B}} \times (-1)^{mt+n(T-t)} e^{-E_n^\pi (T-t)} e^{-E_m^B t}$$

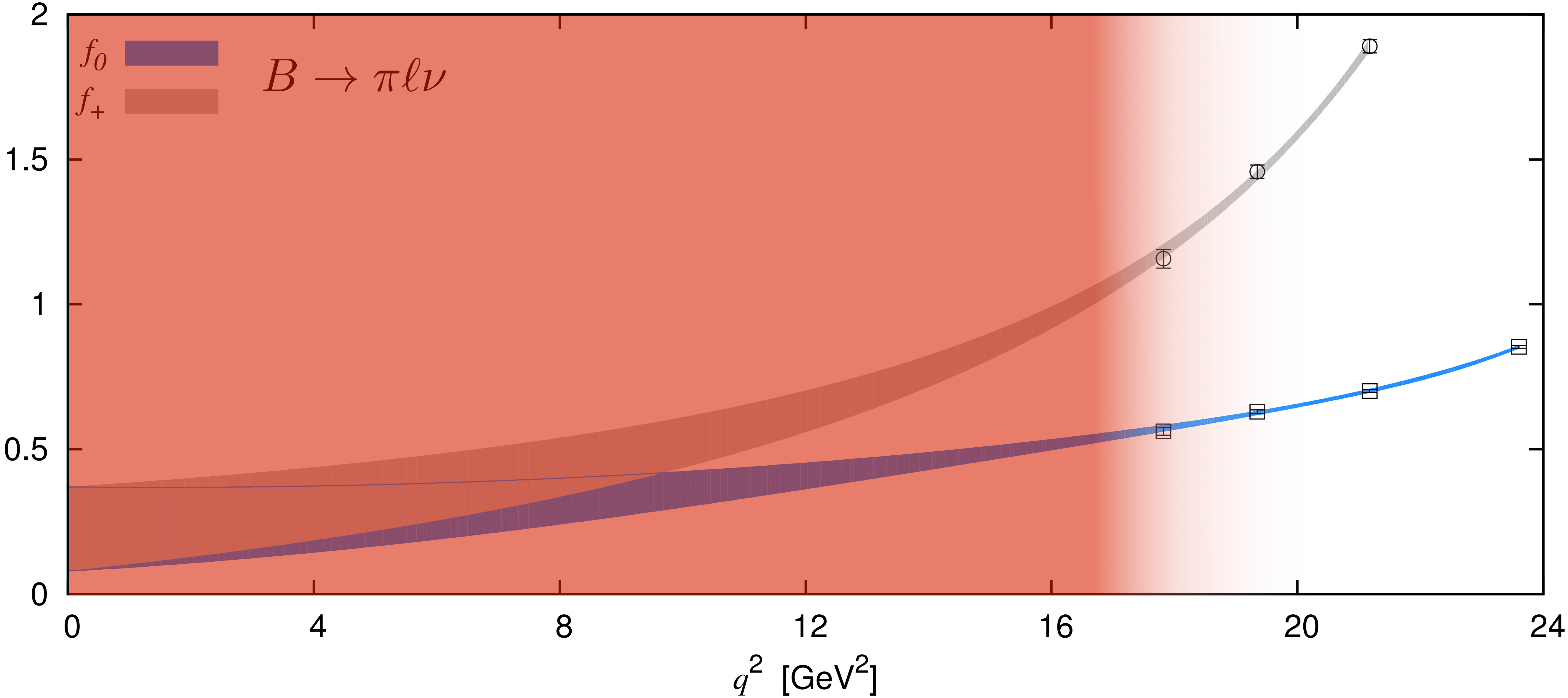
lattice calculation of matrix elements/form factors

- form factors parametrize matrix elements

$$\begin{aligned}\langle \pi | V^\mu | B \rangle &= f_+(q^2)\left(p_B^\mu + p_\pi^\mu - \frac{M_B^2 - M_\pi^2}{q^2} q^\mu\right) \\ &+ f_0(q^2)\frac{M_B^2 - M_\pi^2}{q^2} q^\mu\end{aligned}$$

Problem:

- Chiral Perturbation Theory valid only for $q^2 \gtrsim 17\ \mathrm{GeV}^2$
- kinematics not a problem (z-expansion)



Problem:

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A solution:

- *Hard Pion* Chiral Perturbation Theory Bijnens and Jemos, NPB 846 (2011) 145; 840 (2010) 54; 844 (2010) 182(E)
- chiral physics and kinematics factorize
- HPChPT with z-expansion HPQCD, PRD 90 (2014) 054506

HPChPT modified z expansion

- HPChPT suggests factorization of kinematic and chiral logarithmic effects

$$f(E_\pi) = (1 + [\mathrm{logs}])\,\mathcal{K}(E_\pi)\ .$$

  - may be violated at higher order in ChPT   Colangelo et al, 1208.0498

- Kinematics conveniently described by z expansion

$$P(q^2)f(z) = (1 + [\mathrm{logs}])\sum_k b_k z^k\ .$$

HPChPT modified z expansion

- Add known discretization and chiral analytic effects

$$P(q^2)f(z;a,M_\pi) = (1+[\text{logs}])\sum_k \beta_k\, D_k(a,M_\pi,\dots)\, z(M_\pi)^k$$

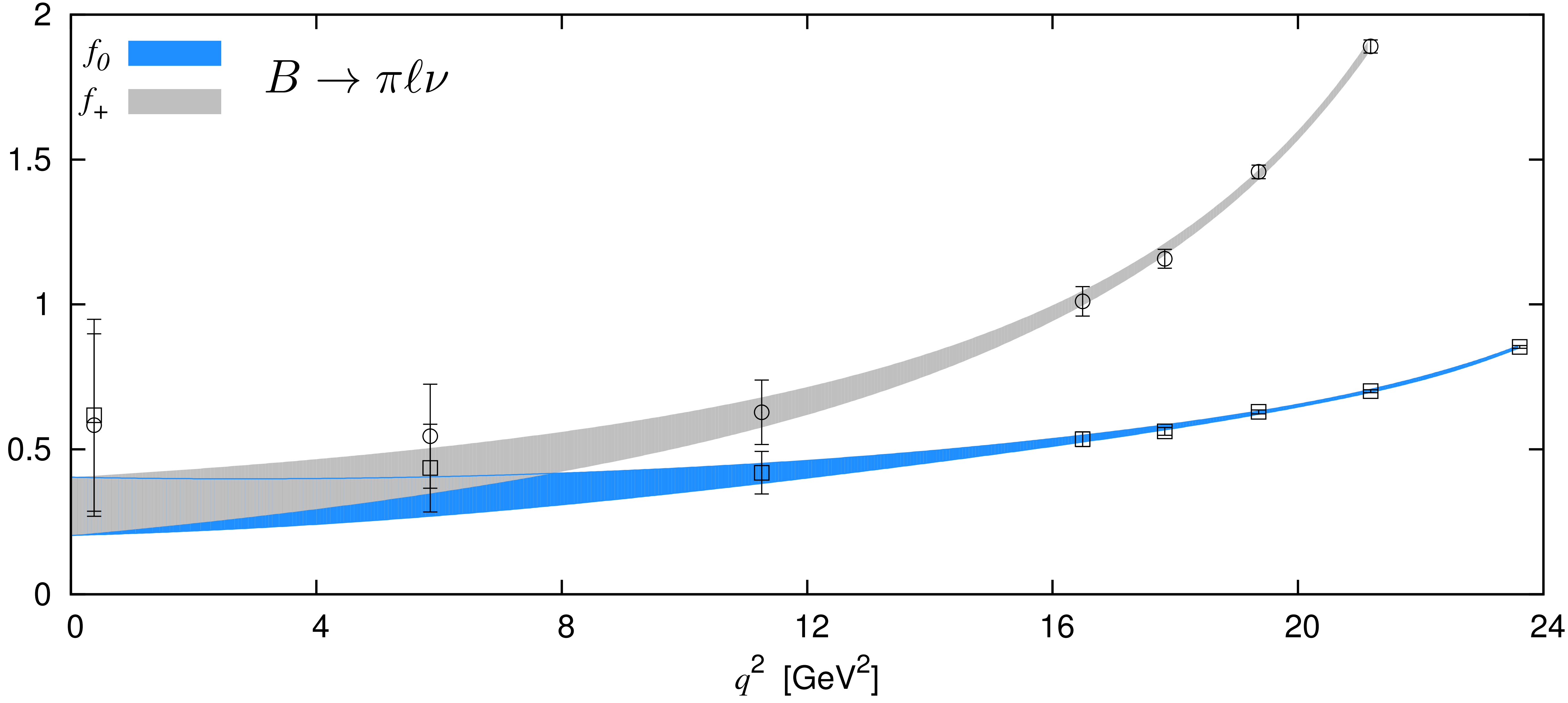
where $D_k(a,M_\pi,\dots) = 1+\dots$.

- In the physical limit $\lim_{\substack{m\to m_{\text{physical}}\\ a\to 0}} (1+[\text{logs}])\,\beta_k D_k = b_k$,

this reduces to the standard form

$$P(q^2)f(z) = \sum_k b_k\, z^k\ .$$

constraints:
- large q^2 scaling of f_+
- $f_+(q^2=0) = f_0(q^2=0)$



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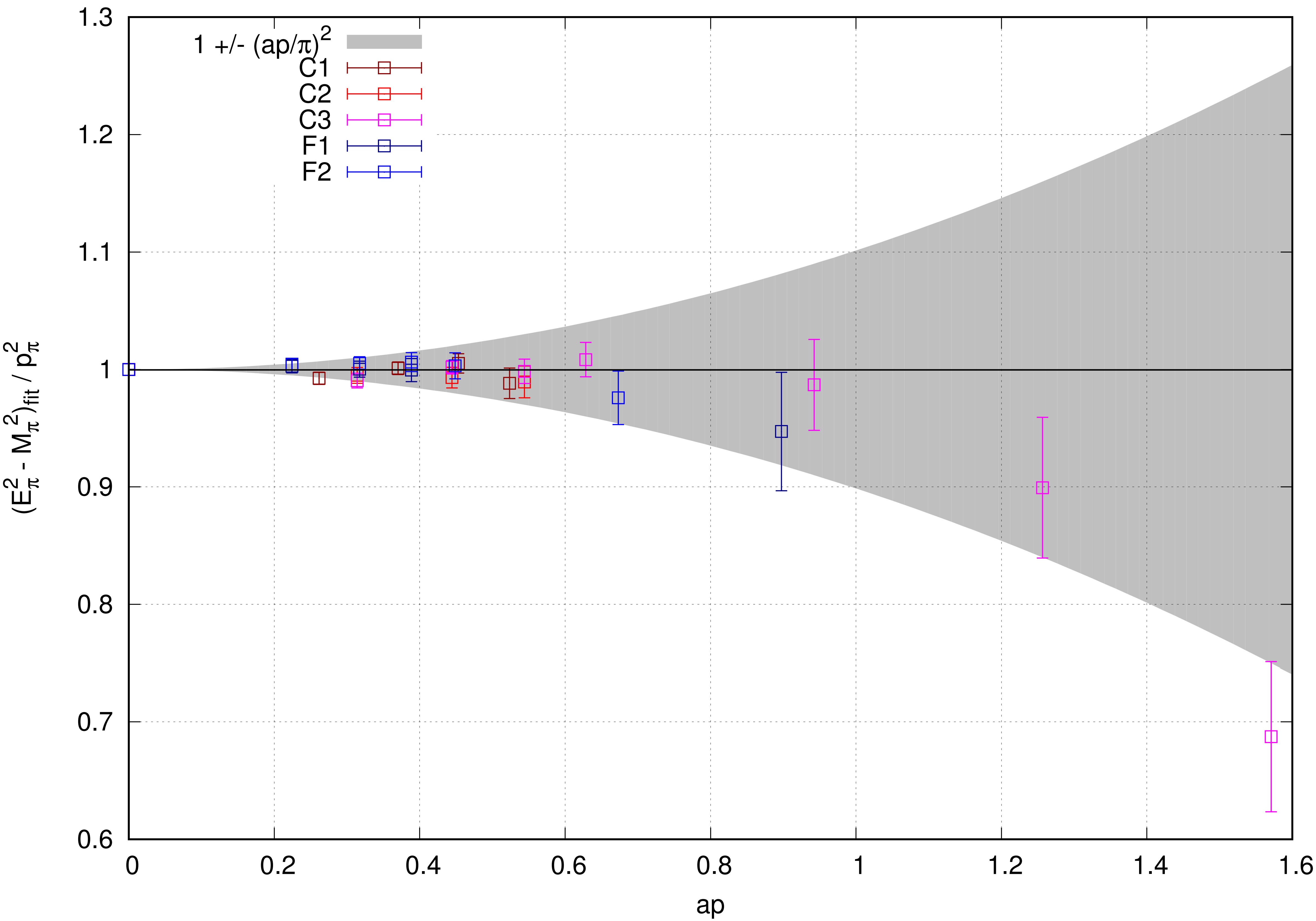
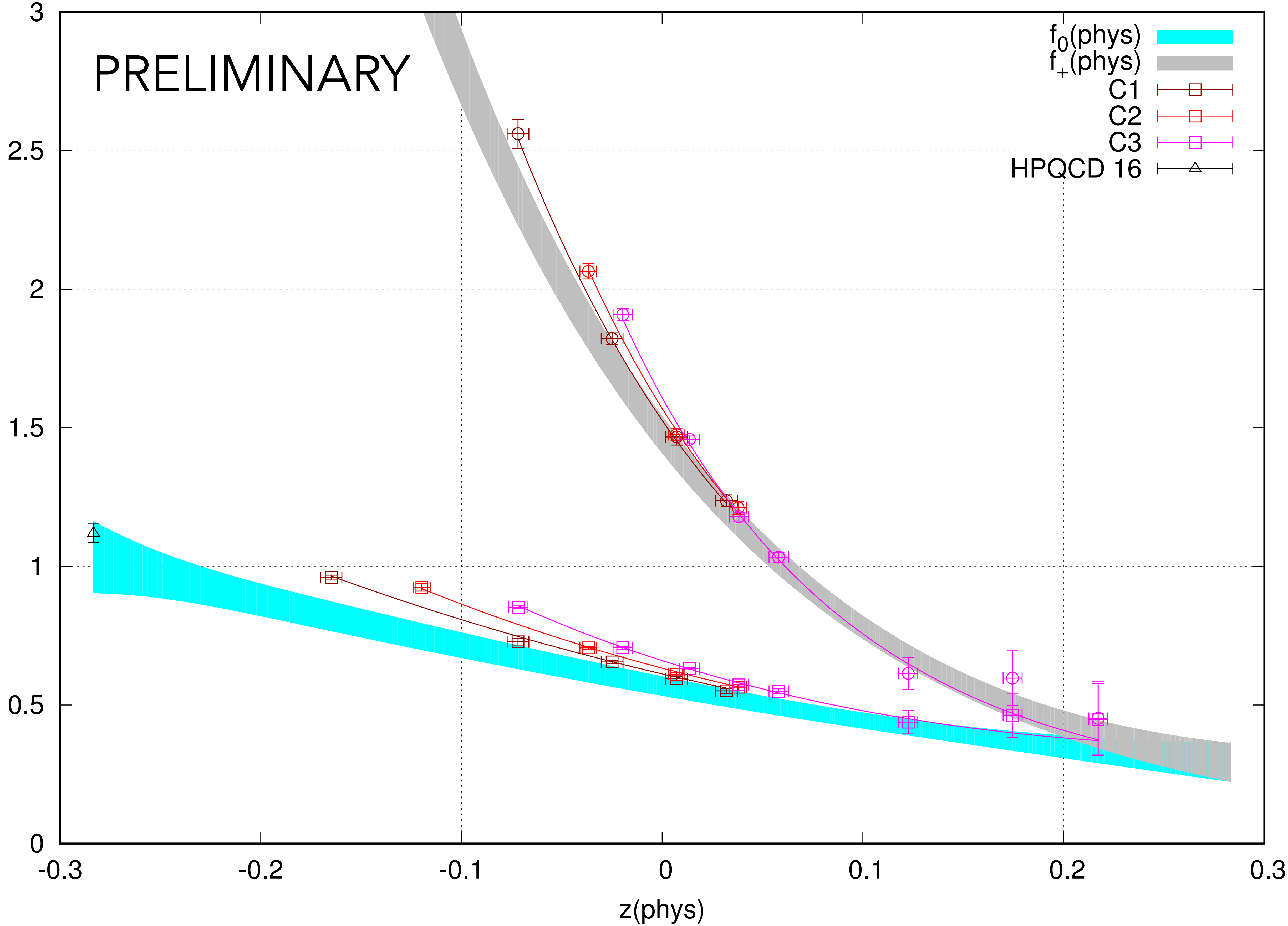
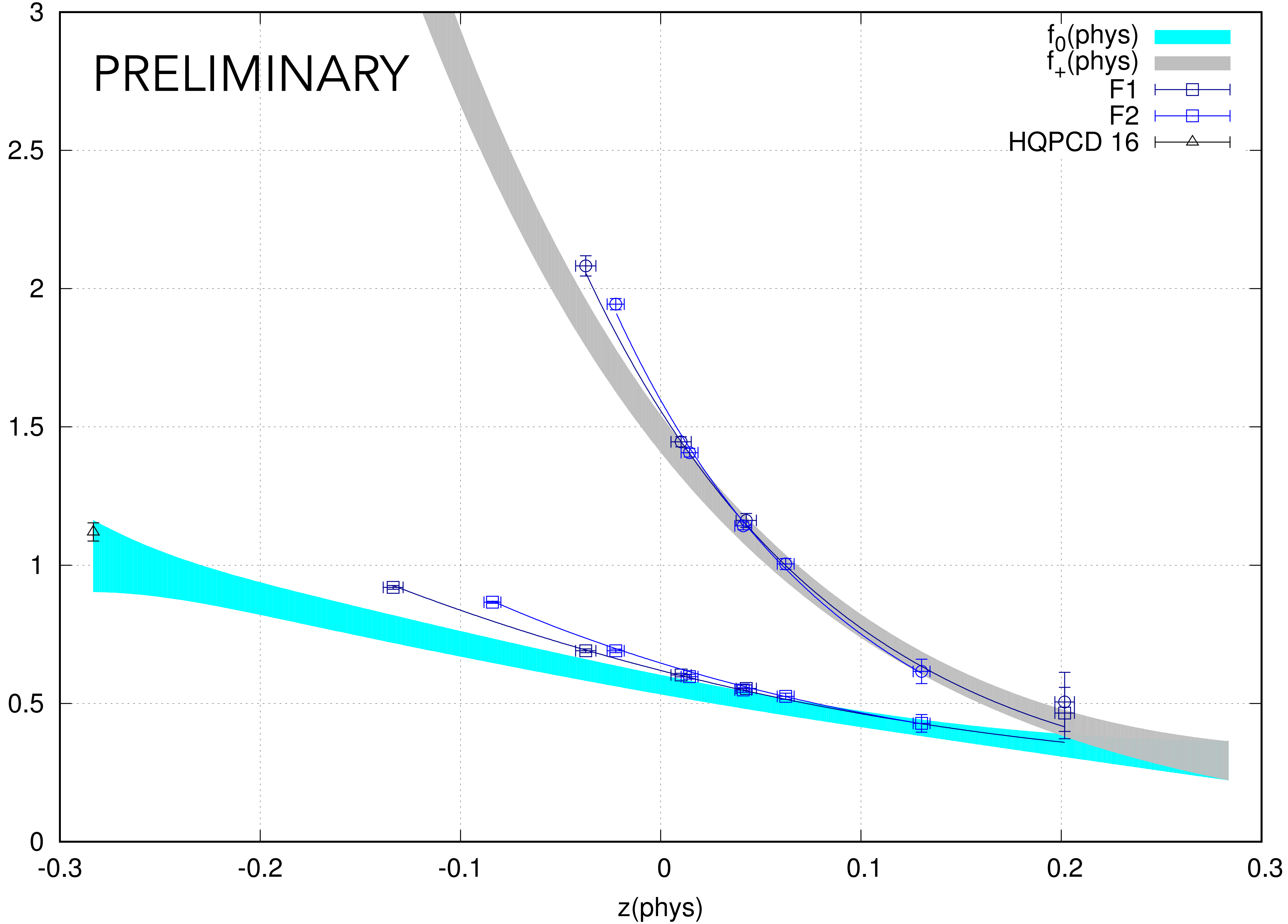
prelimary results with full q^2 coverage

ensemble	a / fm	M_π / MeV	a**p**L / (2π) q^2 / GeV^2
C1	0.12	267	(0,0,0), (1,0,0), (1,1,0), (1,1,1), (2,0,0) 25.5, 22.9, 21.2, 19.9, 18.7
C2	0.12	348	(0,0,0), (1,0,0), (1,1,0), (1,1,1) 24.8, 21.8, 19.8, 18.2
C3	0.12	489	(0,0,0), (1,0,0), (1,1,0), (1,1,1), (2,0,0), (3,0,0), (4,0,0), (5,0,0) 23.7, 21.2, 19.4, 17.9, 16.5, 11.3, 5.9, 0.4
F1	0.09	313	(0,0,0), (1,0,0), (1,1,0), (1,1,1), (4,0,0) 24.9, 21.8, 19.7, 18.1, 5.8
F2	0.09	438	(0,0,0), (1,0,0), (1,1,0), (1,1,1), (2,0,0), (3,0,0) 23.9, 21.3, 19.4, 17.8, 16.5, 11.2



- dispersion relation satisfied within expected discretization error
- this uncertainty is embedded in the analysis via Bayesian priors

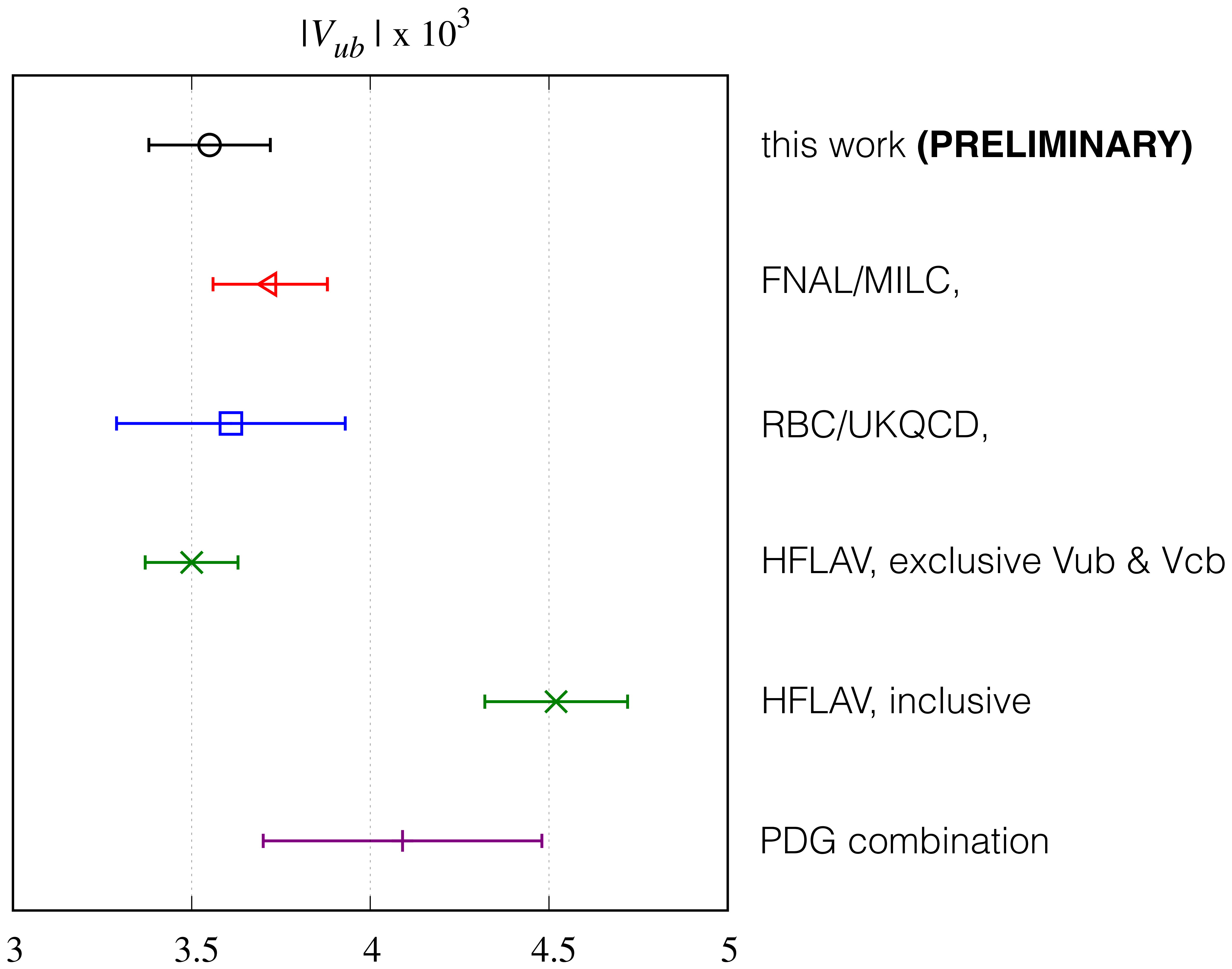

PRELIMINARY
f0(phys)
f+(phys)
C1
C2
C3
HPQCD 16
3
2.5
2
1.5
1
0.5
0
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
z(phys)


PRELIMINARY
f0(phys)
f+(phys)
F1
F2
HQPCD 16
3
2.5
2
1.5
1
0.5
0
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
z(phys)

combination with experiment

$$\frac{d\mathcal{B}}{dq^2}(B \to \pi\ell\nu) = \frac{\tau_B G_F^2 |V_{ub}|^2}{24\pi^3 M_B^2}\left(1-\frac{m_\ell^2}{q^2}\right)^2 |\mathbf{p}_\pi| \times \\ \left[\left(1+\frac{m_\ell^2}{2q^2}\right) M_B^2 \mathbf{p}_\pi^2 |f_+(q^2)|^2 \\ + \frac{3m_\ell^2}{8q^2}(M_B^2 - M_\pi^2)^2 |f_0(q^2)|^2\right]$$

- data on LHS: BABAR 11, Belle 11, BABAR 12, Belle 13
- fit parameters on RHS: $|V_{ub}|, b_k^{(+)}, b_k^{(0)}$
  - prior constraints on b's from form factor results
  - $|V_{ub}|$ unconstrained


$|V_{ub}|$ x 10^3
this work **(PRELIMINARY)**
FNAL/MILC,
RBC/UKQCD,
HFLAV, exclusive Vub & Vcb
HFLAV, inclusive
PDG combination
3
3.5
4
4.5
5

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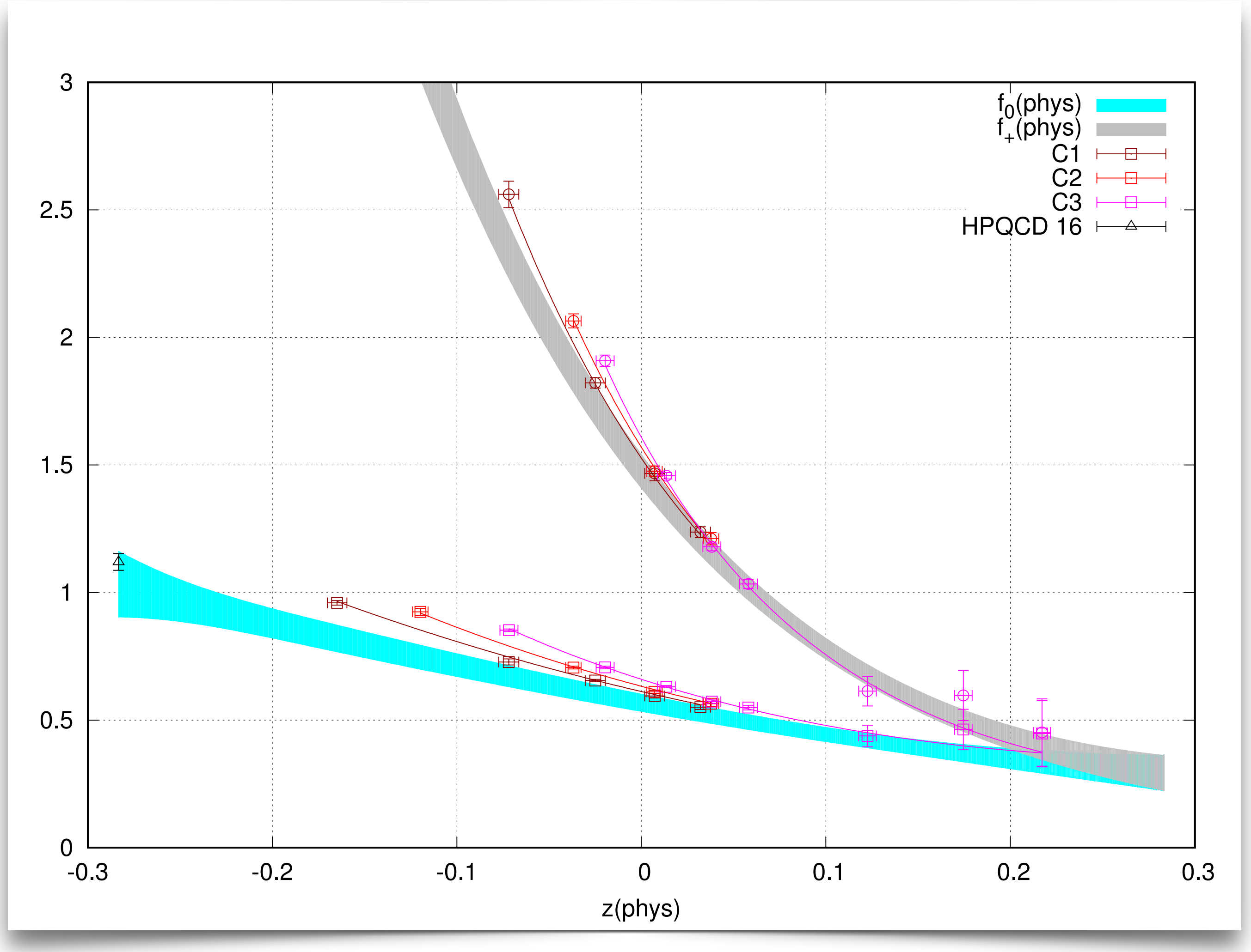
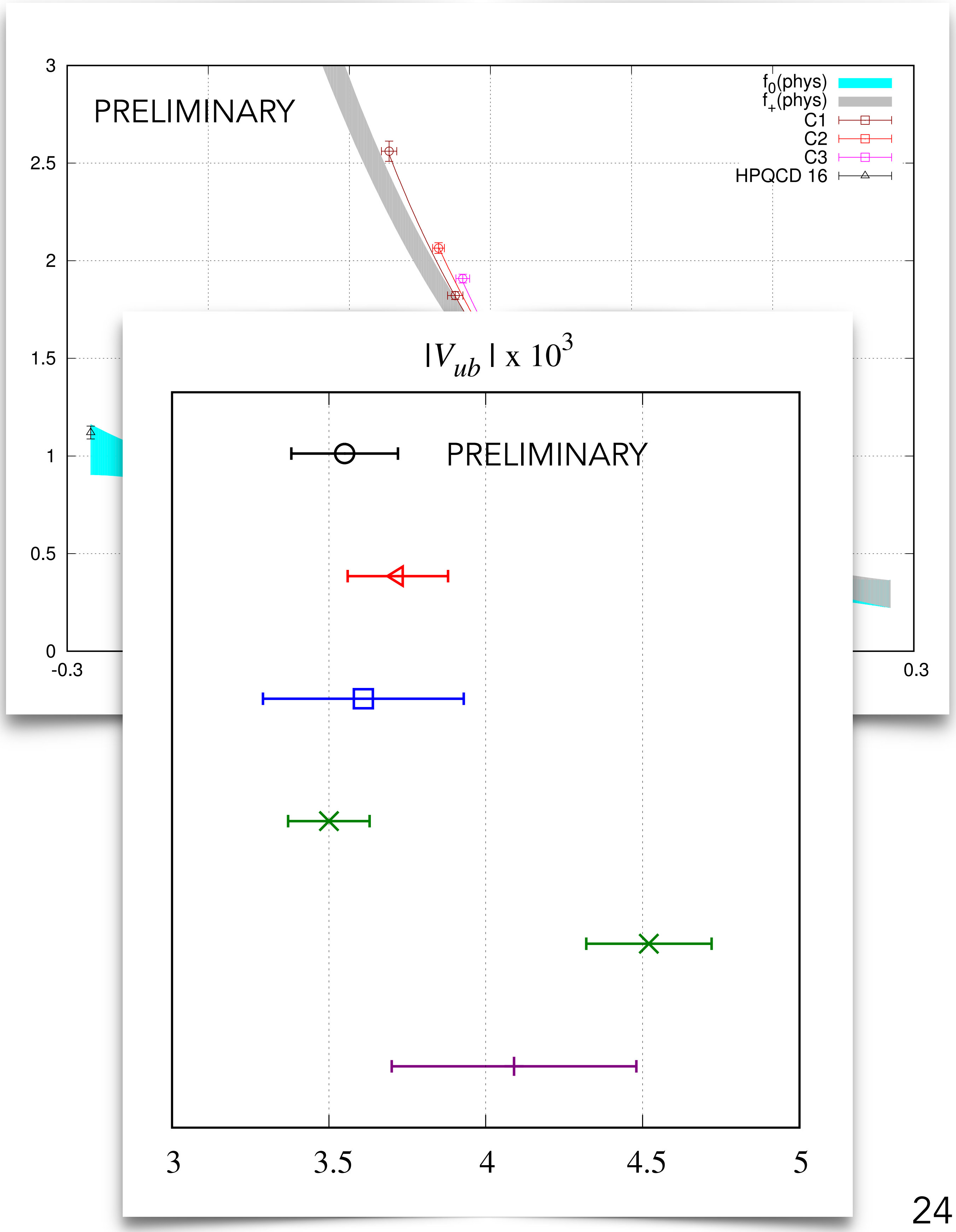
summary

- lattice QCD **not** necessarily restricted to large q^2
- more useful for rare decays

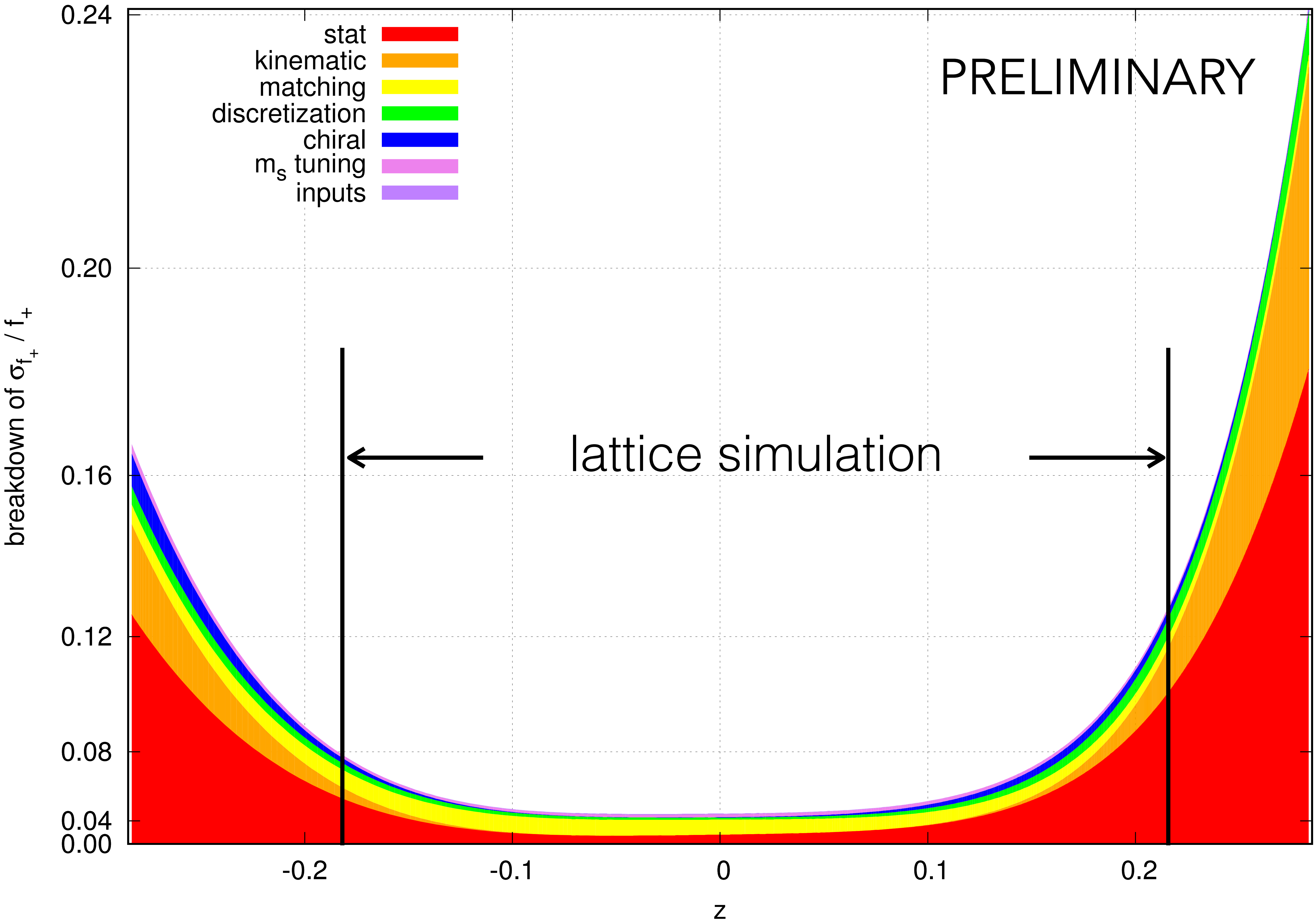


summary



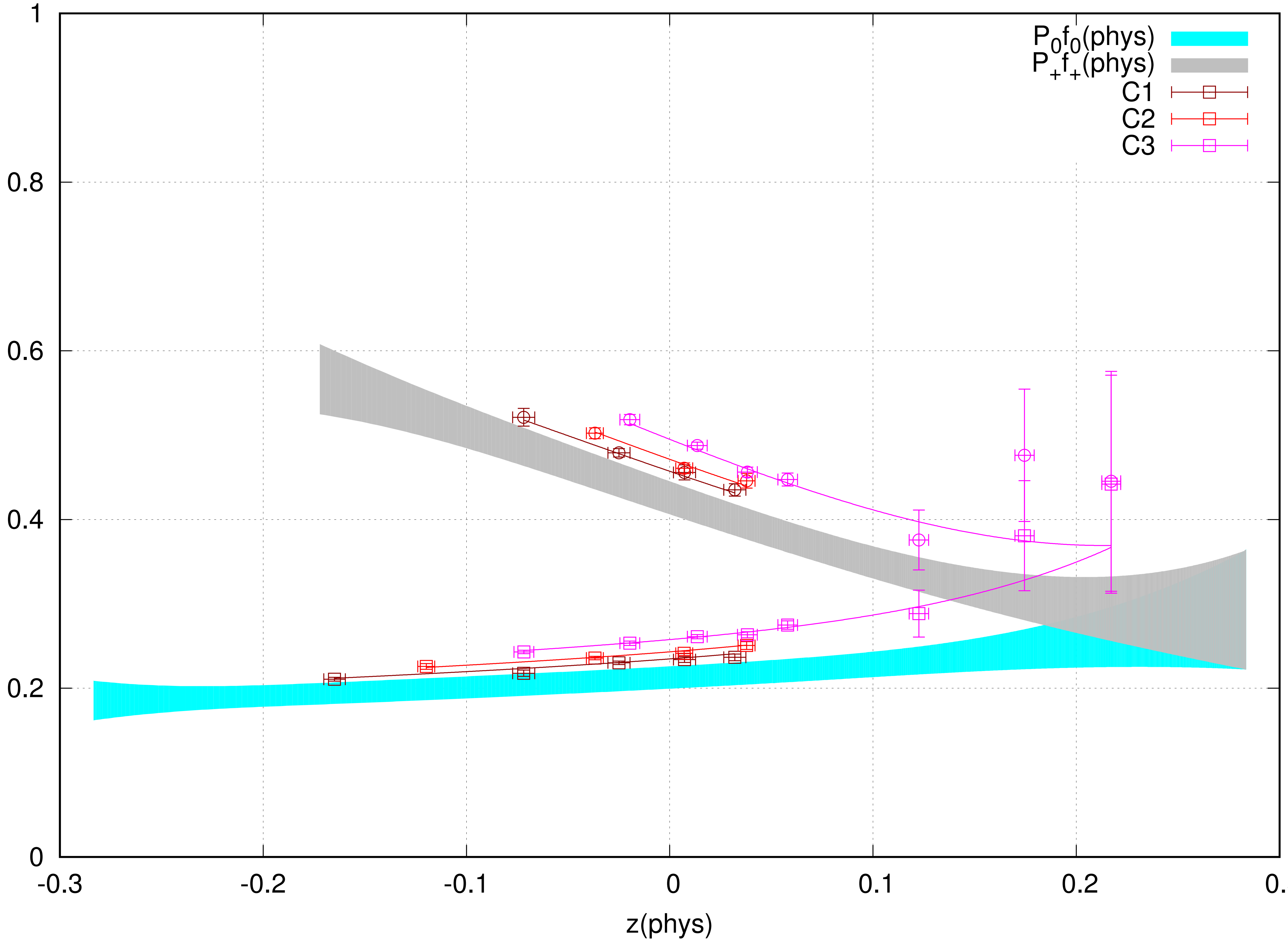
- lattice QCD **not** necessarily restricted to large q^2
- more useful for rare decays
- preliminary result for V_{ub}

the end

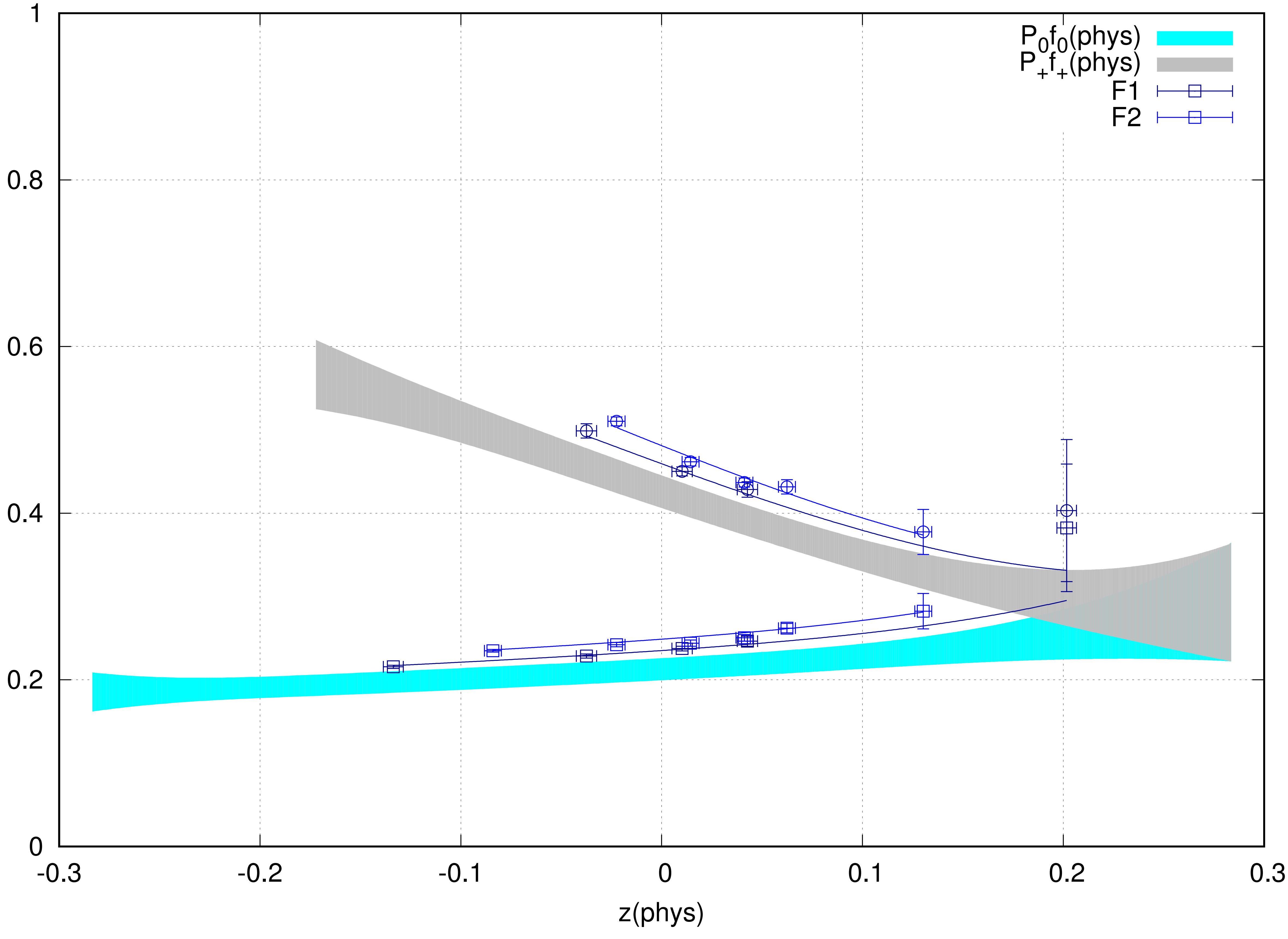


stat
kinematic
matching
discretization
chiral
m_s tuning
inputs
PRELIMINARY
lattice simulation
breakdown of σ_{f_+} / f_+
z
0.24
0.20
0.16
0.12
0.08
0.04
0.00
-0.2
-0.1
0
0.1
0.2

i



P_0f_0(phys)
P_+f_+(phys)
C1
C2
C3
1
0.8
0.6
0.4
0.2
0
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
z(phys)



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