Lattice results for spectroscopy

David Wilson

based on work with the Hadron Spectrum Collaboration

arXiv:1708.06667

hadspec

Trinity College Dublin
Coláiste na Tríonóide, Baile Átha Cliath
The University of Dublin

supported by
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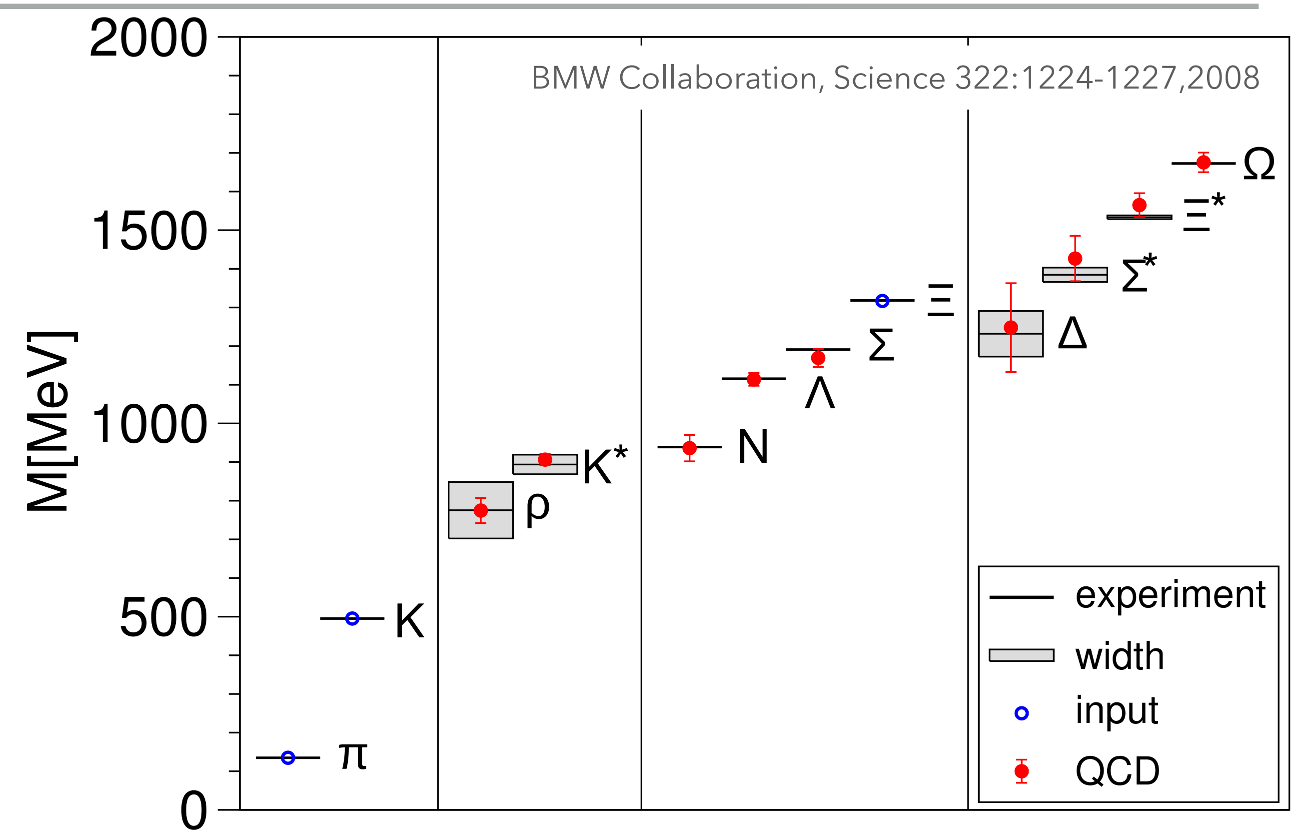
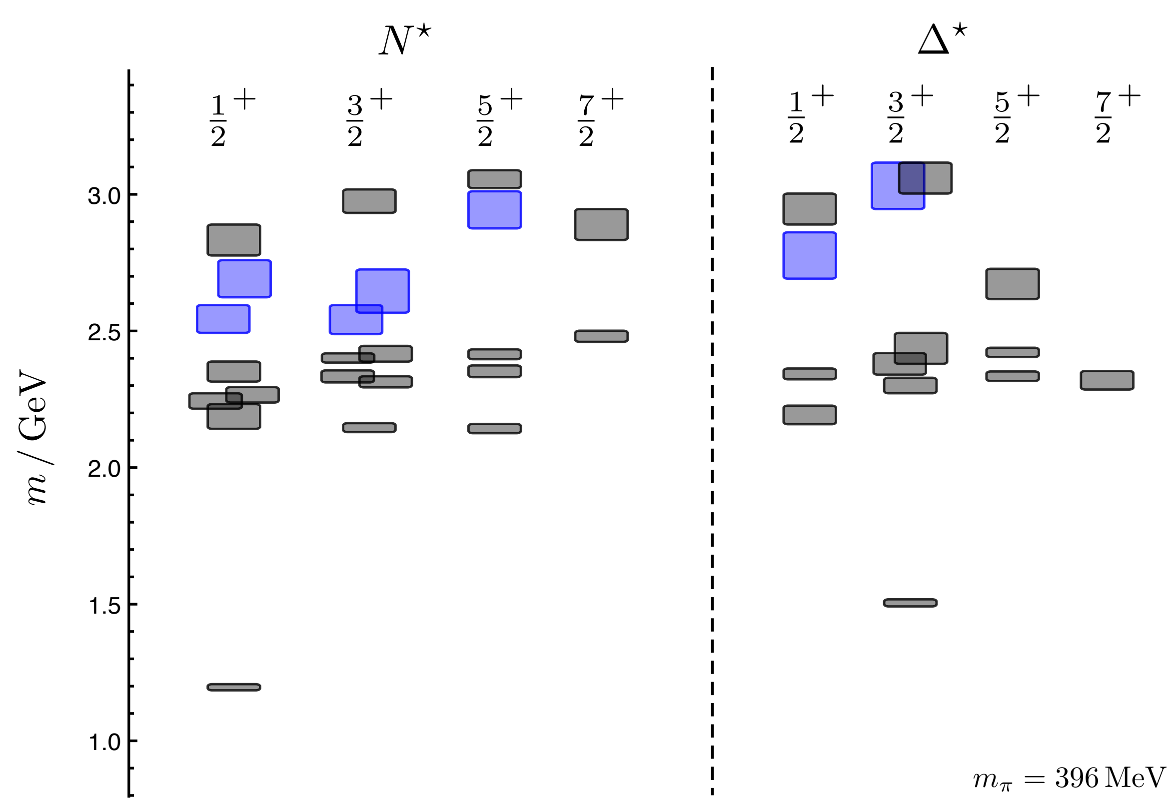
Horizon 2020

Stable states (w.r.t. QCD) are generally well understood - calculations at physical masses in many quantum numbers

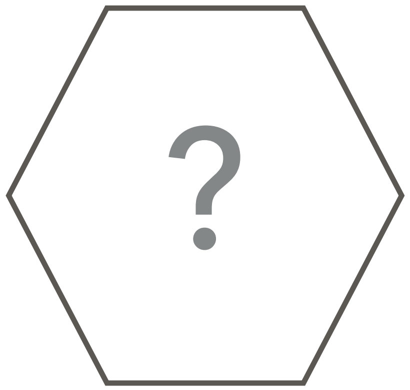


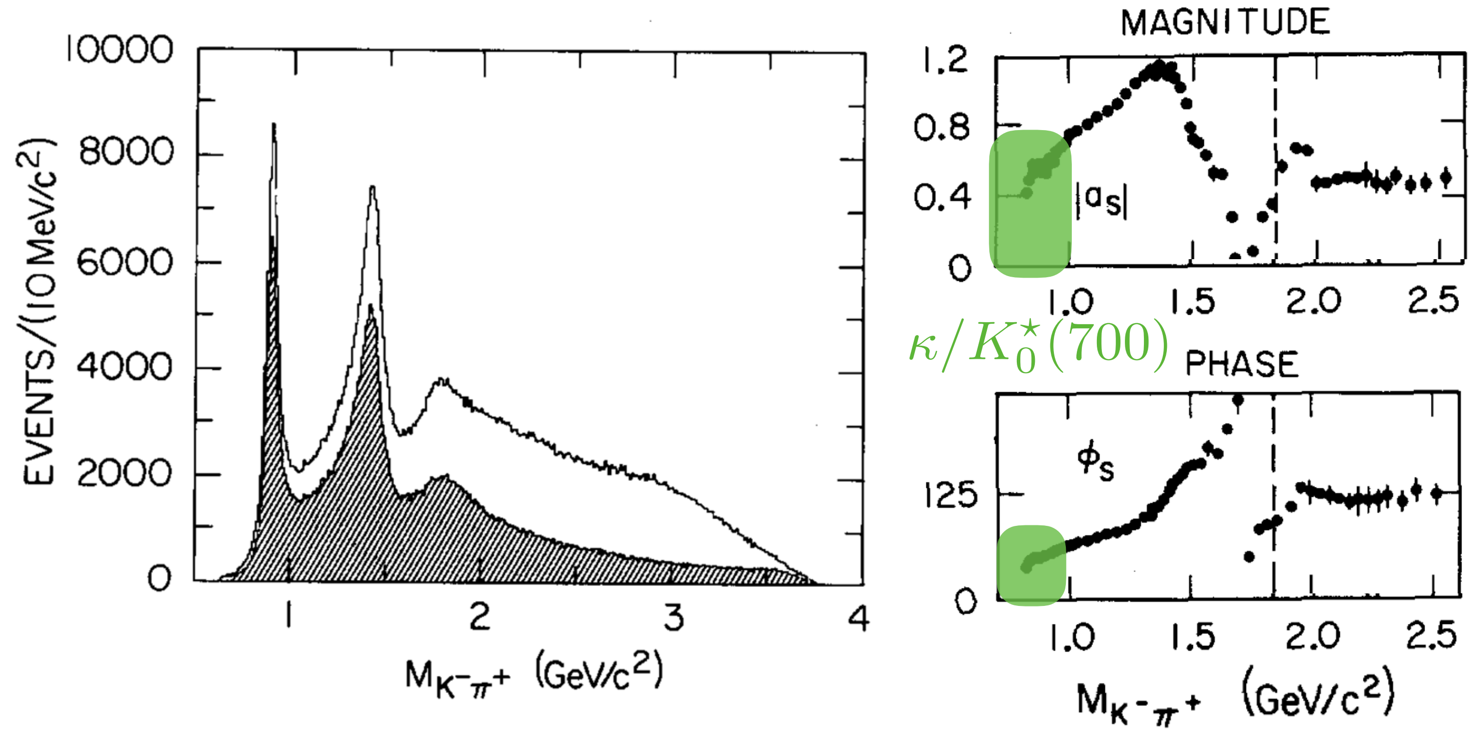
Highly-excited states
- see talk by Gavin Cheung - today 11:00



Dudek & Edwards (for the Hadron Spectrum Collaboration)
Phys. Rev. D85, 054016 (2012)

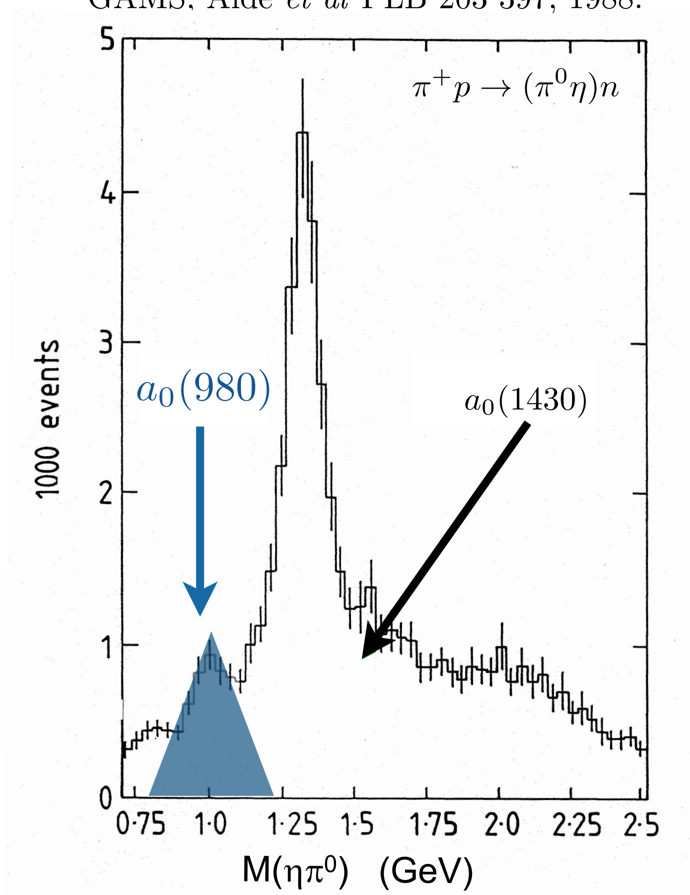
This talk:
- isoscalar resonances
- extracting complex poles & couplings from scattering amplitudes

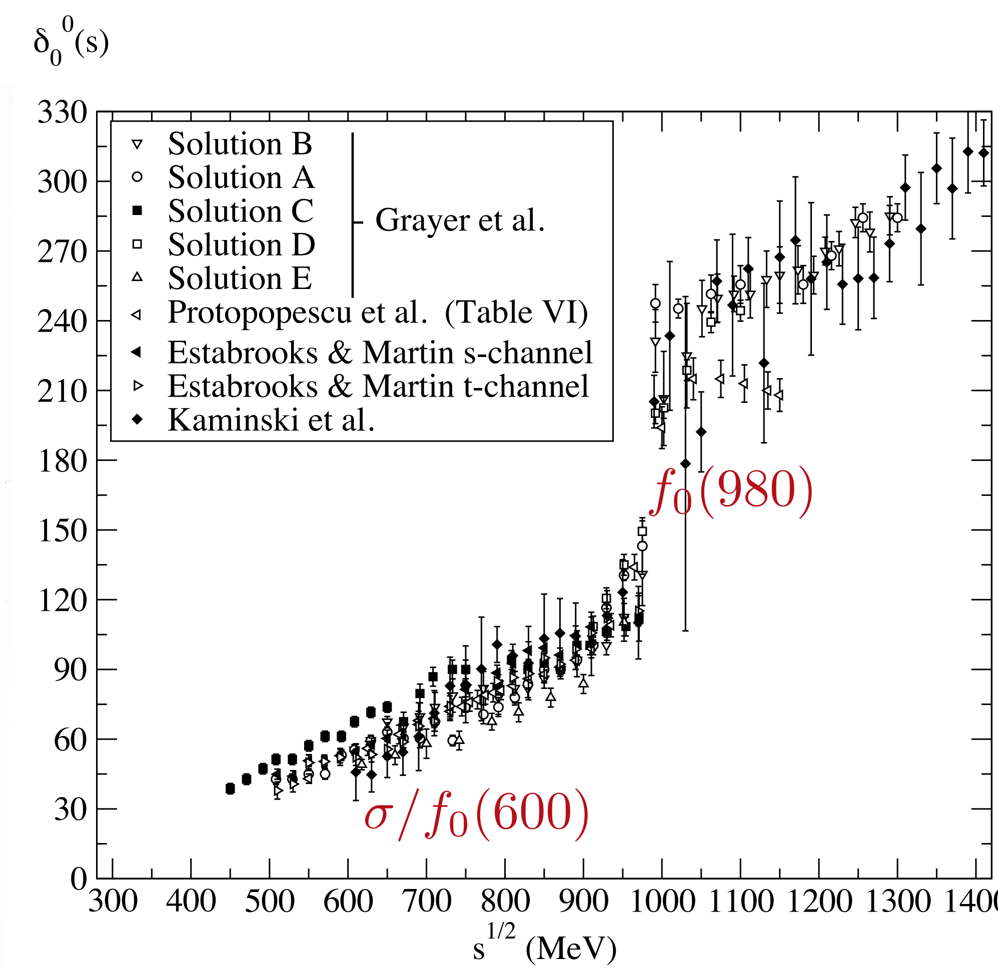


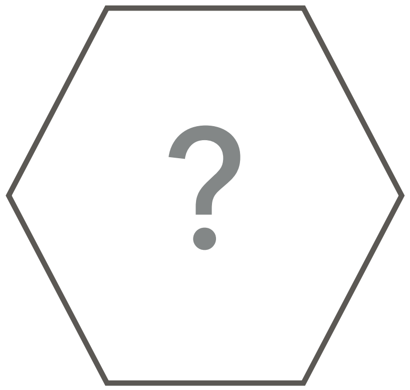


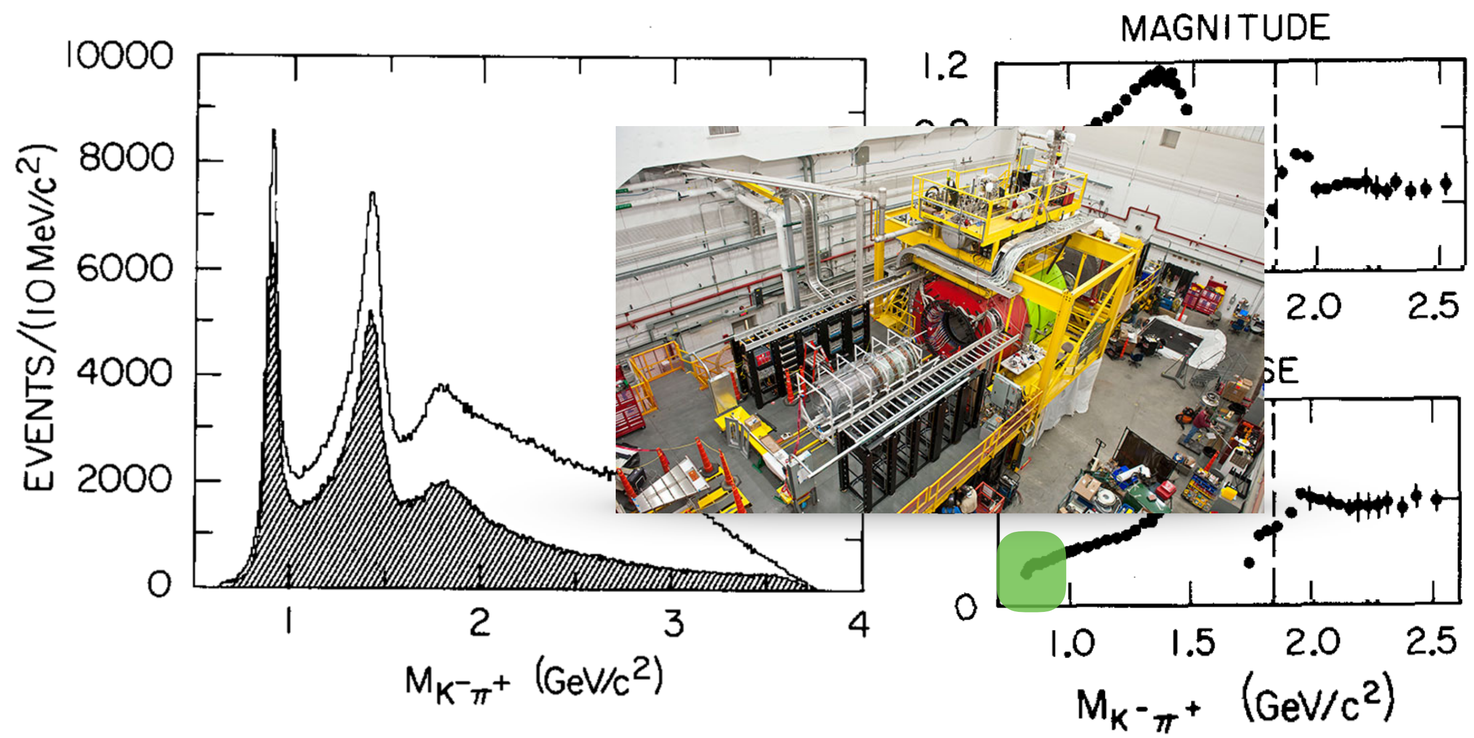
LASS experiment at SLAC $E_K = 11$ GeV

GAMS. Alde *et al* PLB 203 397, 1988.









LASS experiment at SLAC E_K = 11 GeV

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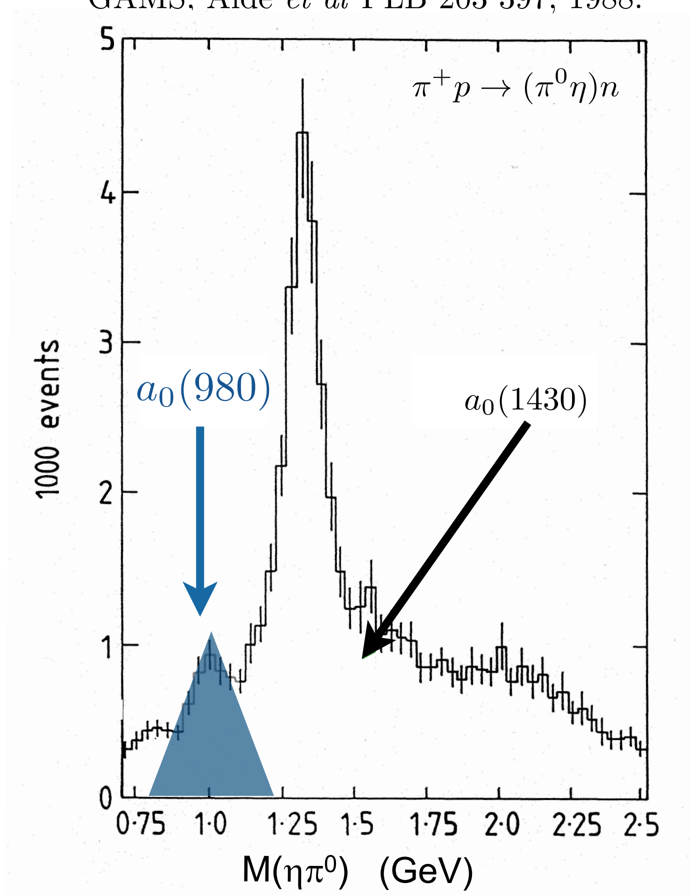
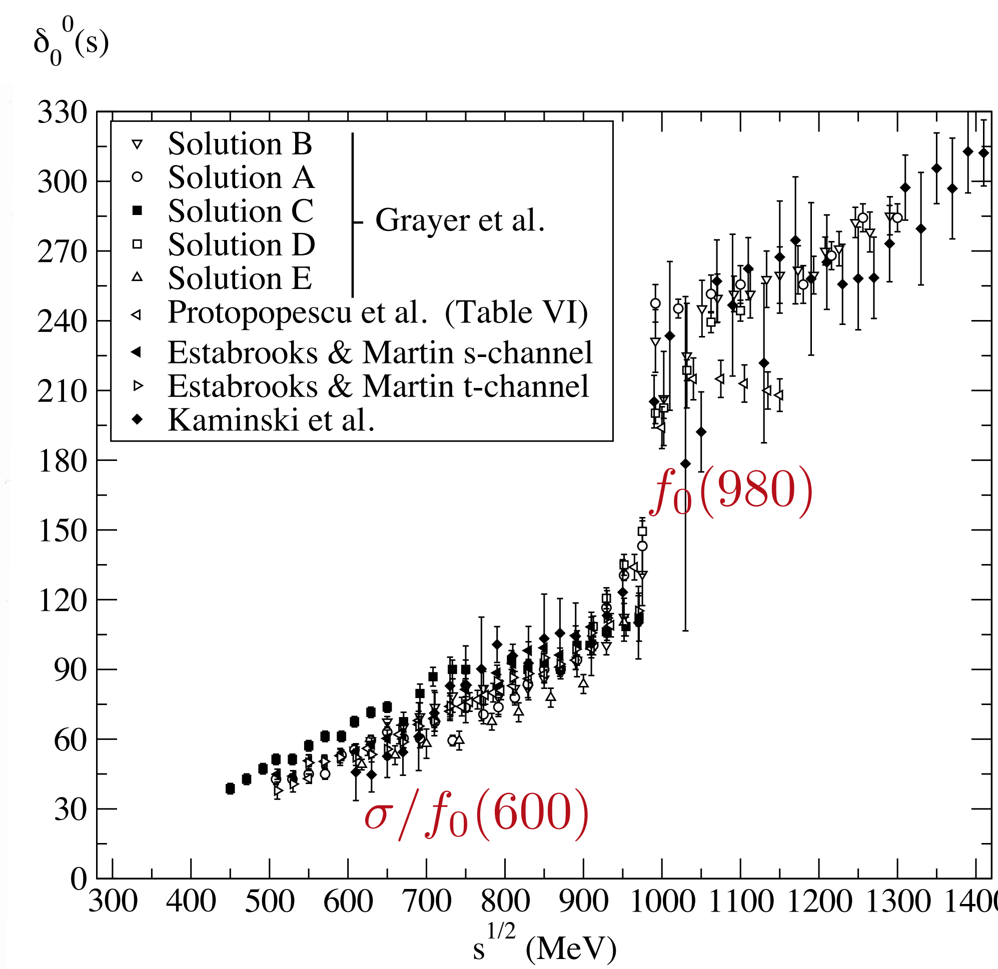
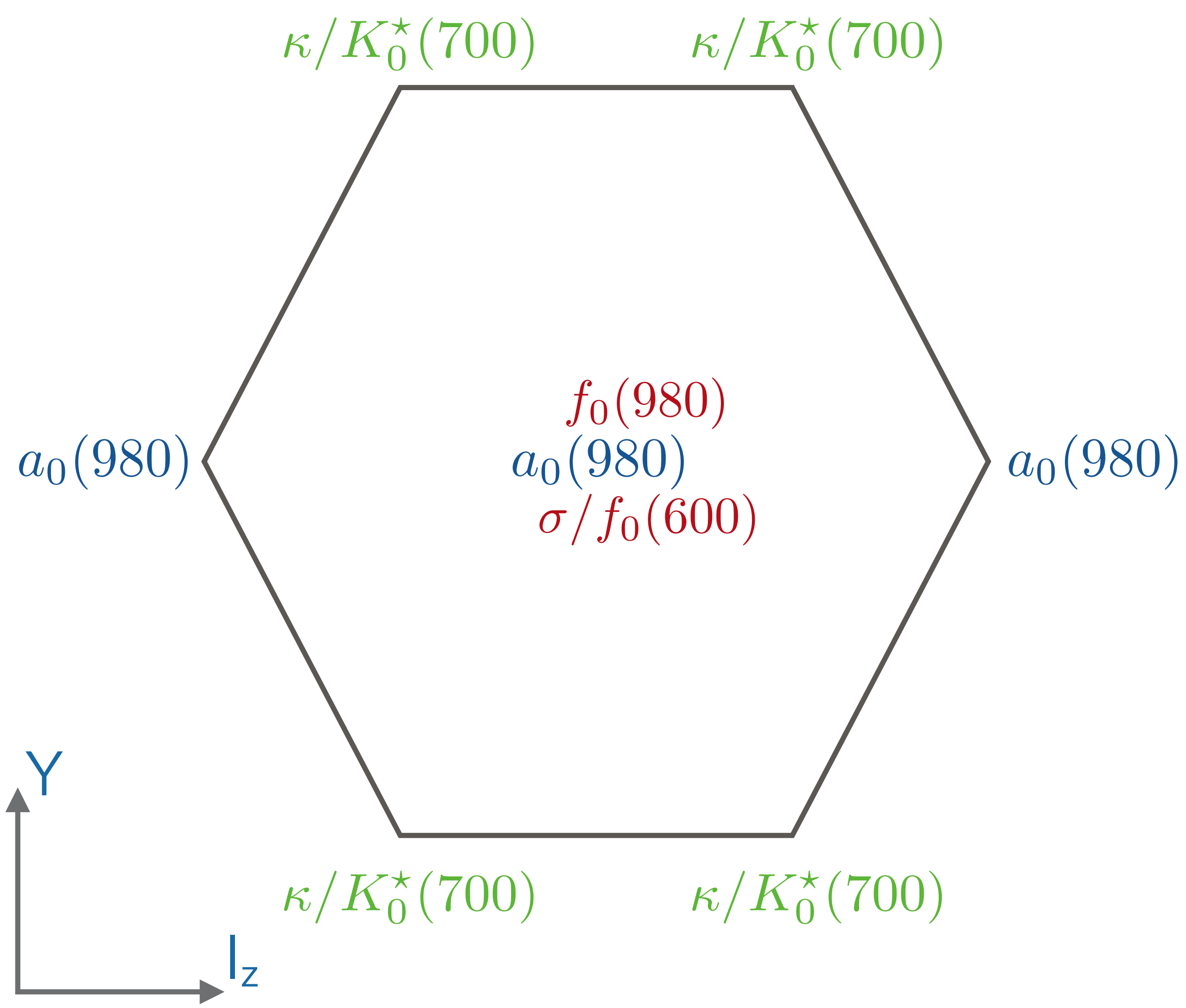
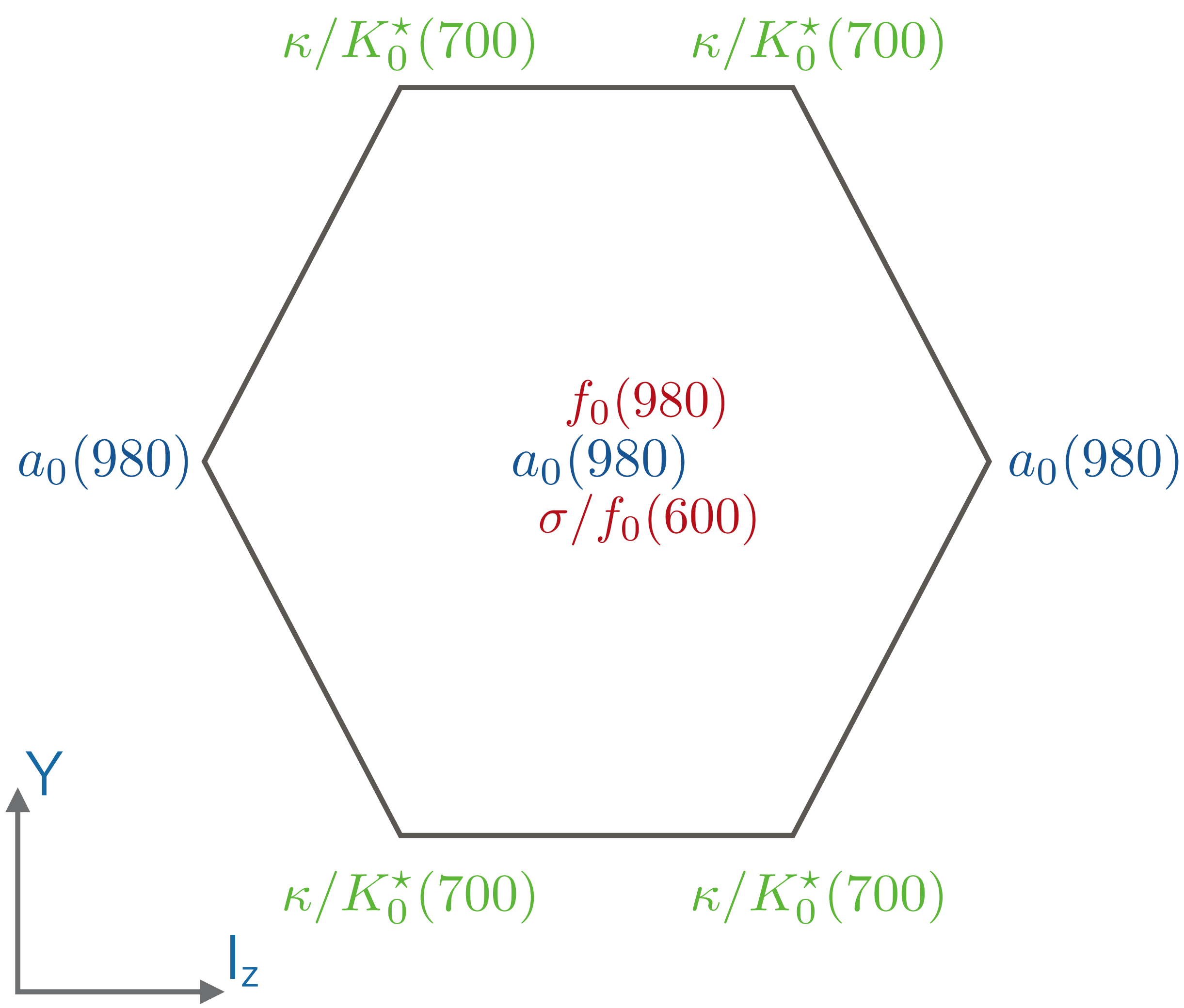
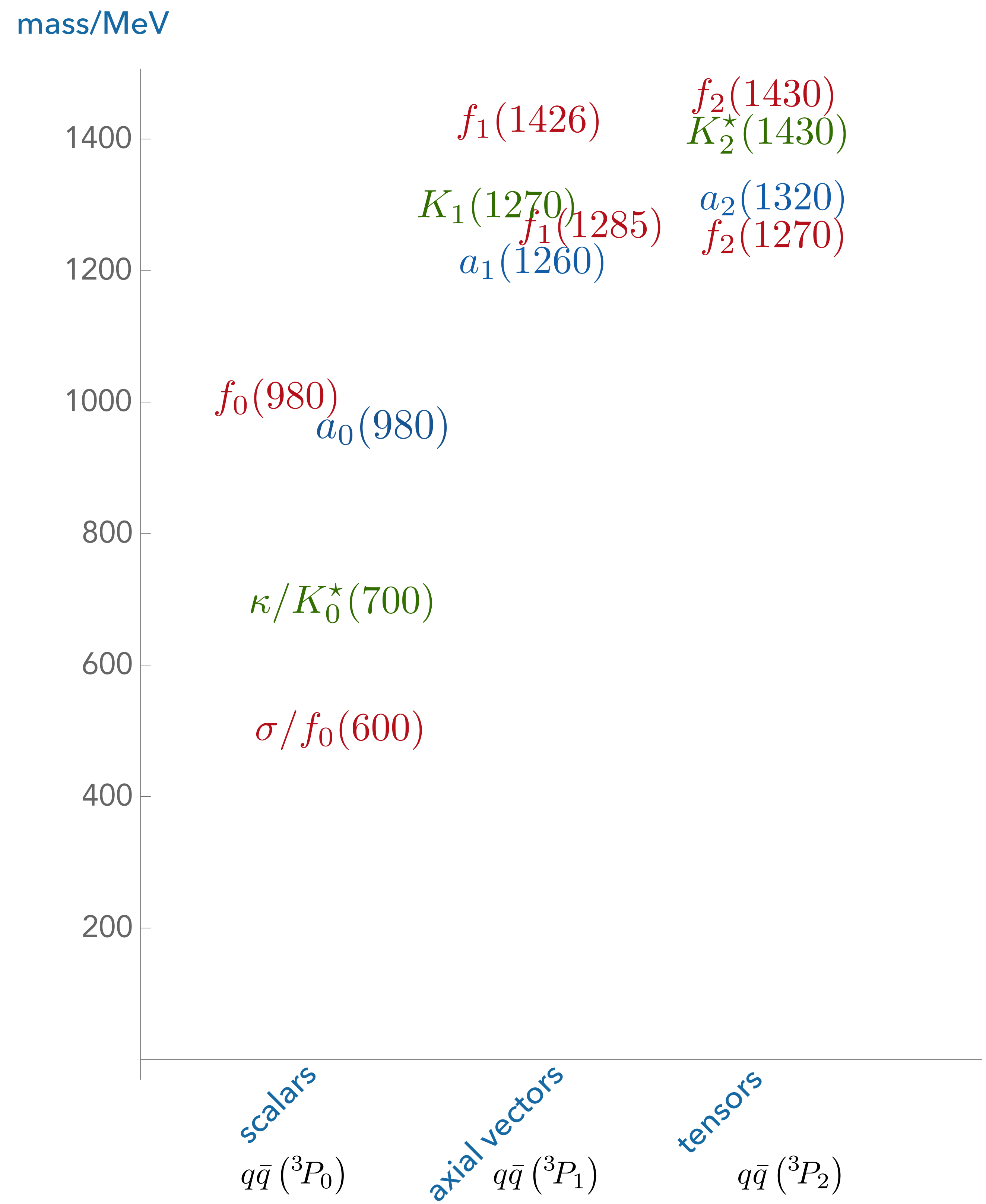




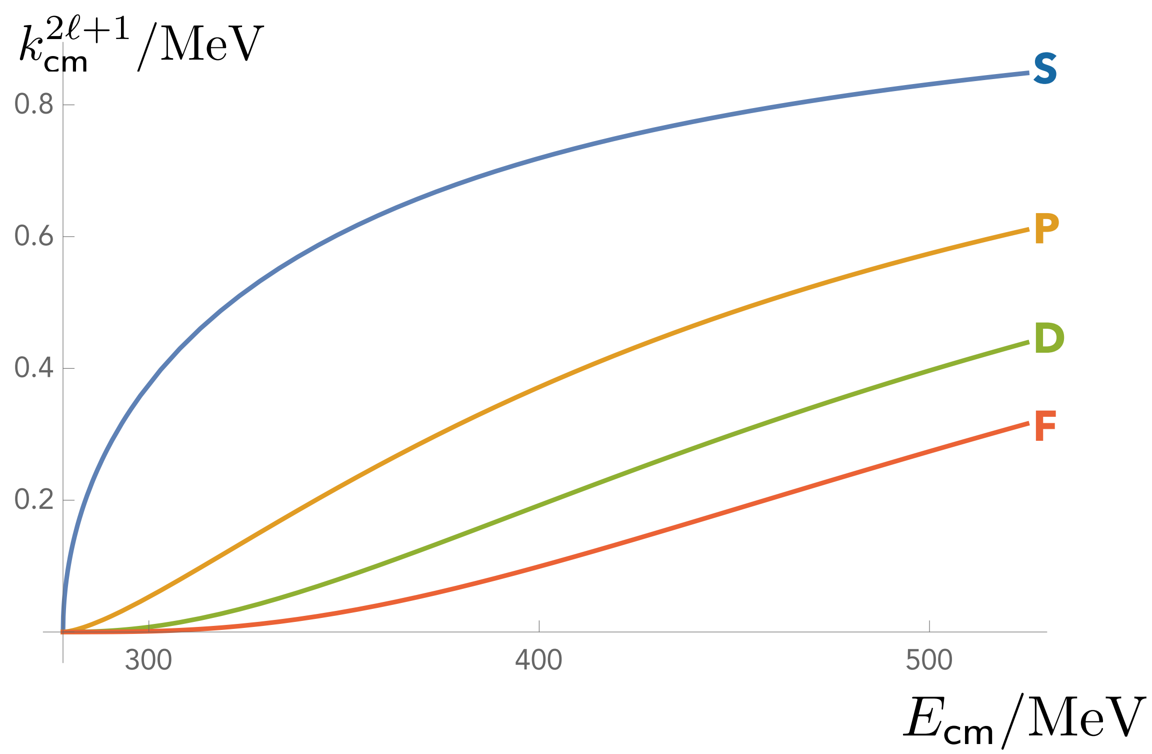


$q\bar{q}\left({}^3P_0\right)$


$\kappa/K_0^\star(700)$
$\kappa/K_0^\star(700)$
$a_0(980)$
$f_0(980)$
$a_0(980)$
$\sigma/f_0(600)$
$a_0(980)$
$\kappa/K_0^\star(700)$
$\kappa/K_0^\star(700)$
Y
I_z


mass/MeV
1400
1200
1000
800
600
400
200
$f_1(1426)$
$f_2(1430)$
$K_2^\star(1430)$
$K_1(1270)$
$f_1(1285)$
$a_1(1260)$
$a_2(1320)$
$f_2(1270)$
$f_0(980)$
$a_0(980)$
$\kappa/K_0^\star(700)$
$\sigma/f_0(600)$
scalars
$q\bar{q}\ (^3P_0)$
axial vectors
$q\bar{q}\ (^3P_1)$
tensors
$q\bar{q}\ (^3P_2)$

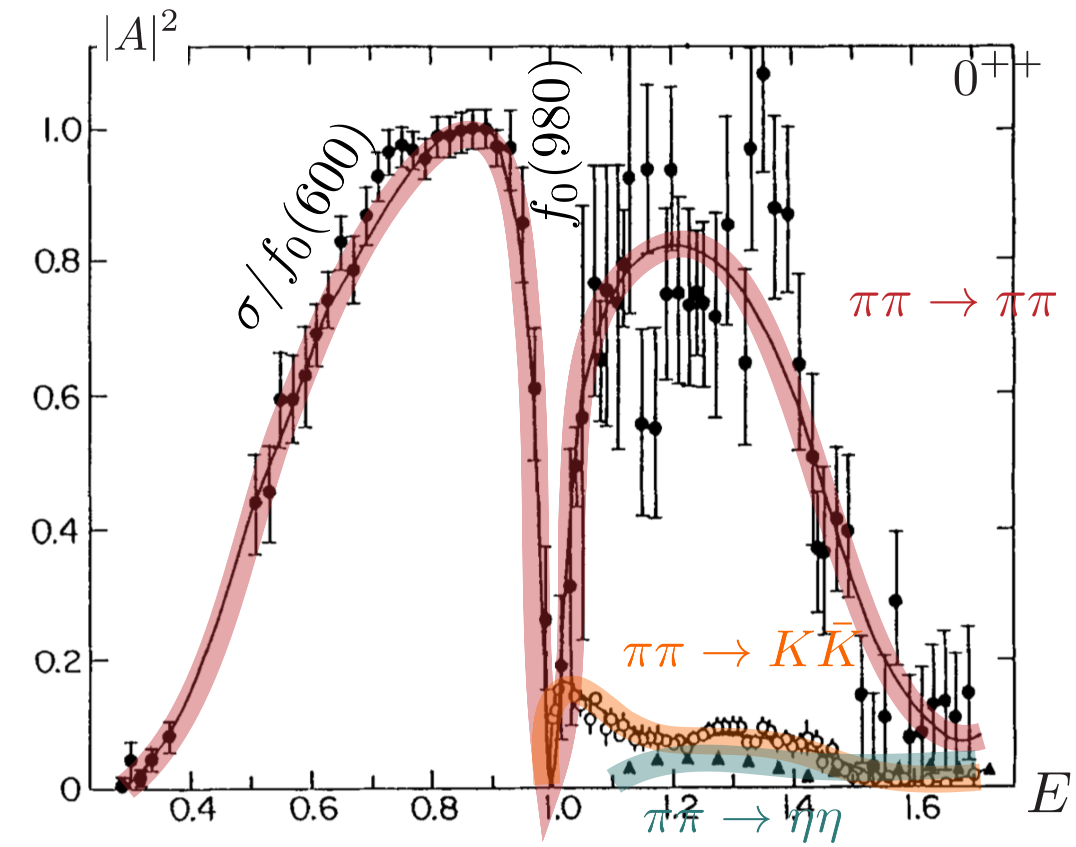
In the scalar sector, amplitudes grow rapidly from threshold:



σ and κ are broad (width ~ mass)

$f_0(980)$ and $a_0(980)$ lie very close to $K\bar{K}$ threshold

CERN-Munich, ANL, BNL,NA48



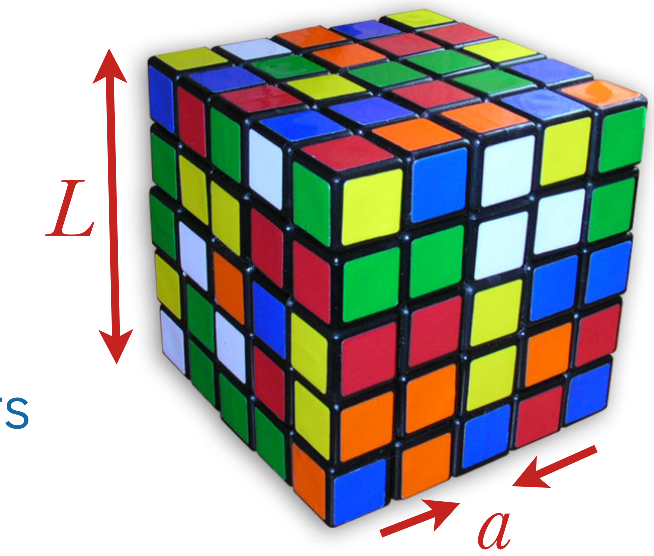
Threshold effects are also essential to understand puzzling states:

$$\begin{array}{llll} \Lambda(1405) & \to & \Sigma\pi & \Delta_E \sim 70\text{MeV} \\ & & N\bar{K} & \Delta_E \sim 30\text{MeV} \\ \\ \text{X}(3872) & \to & D\bar{D}^\star & \Delta_E \sim 1\text{MeV} \\ & \to & \pi\pi J/\psi & \\ & \to & \pi\pi\pi J/\psi & \end{array}$$

What about state composition?

Lattice QCD provides a first principles method to access these resonances

Work in a finite Euclidean volume *L* with a lattice spacing *a*



Compute matrices of correlation functions with many operators

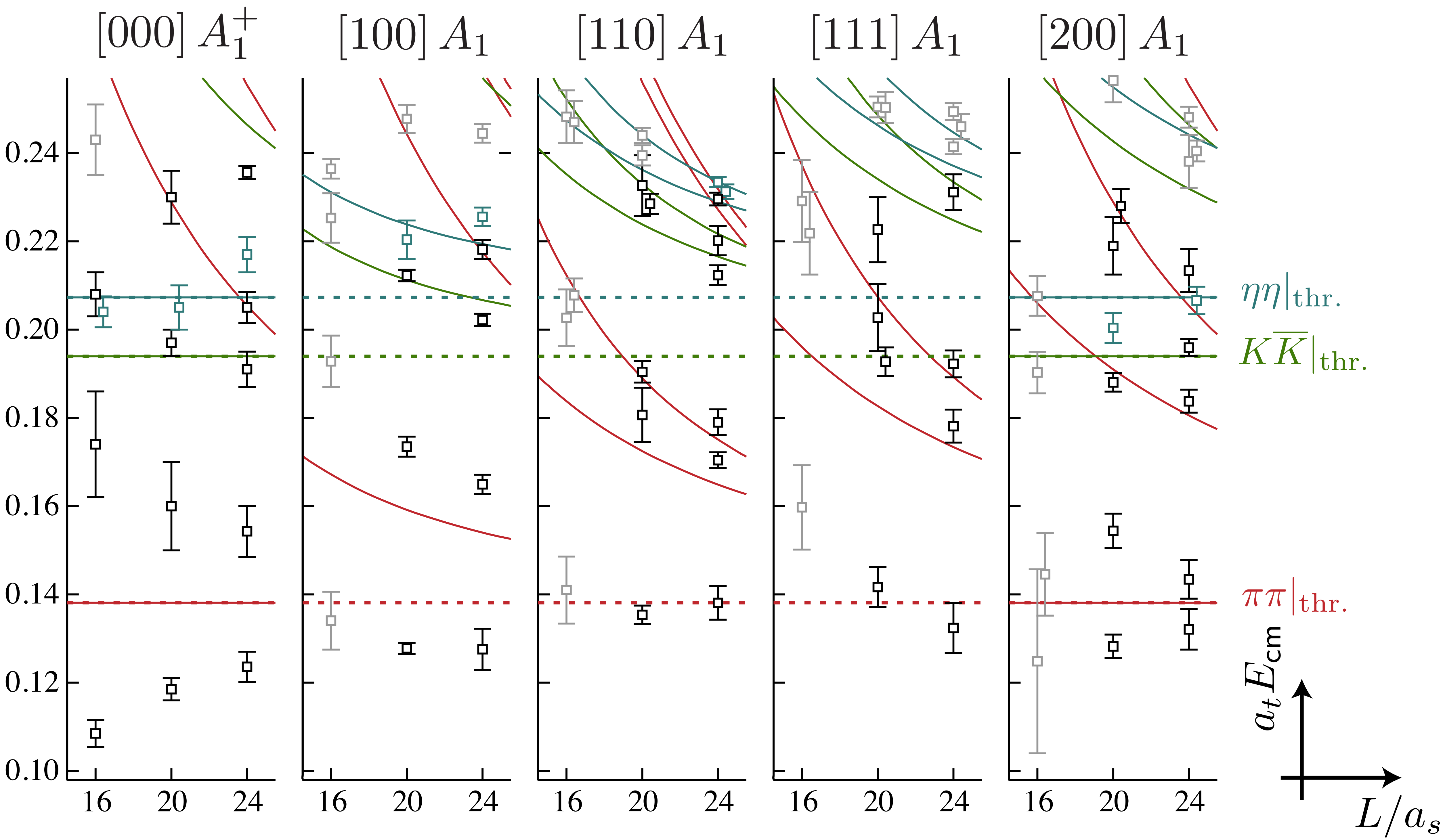
$$C_{ij} = \langle 0|\, \mathcal{O}_i(t)\mathcal{O}_j^\dagger(0)\, |0\rangle$$

Obtain the finite volume spectrum using a variational method

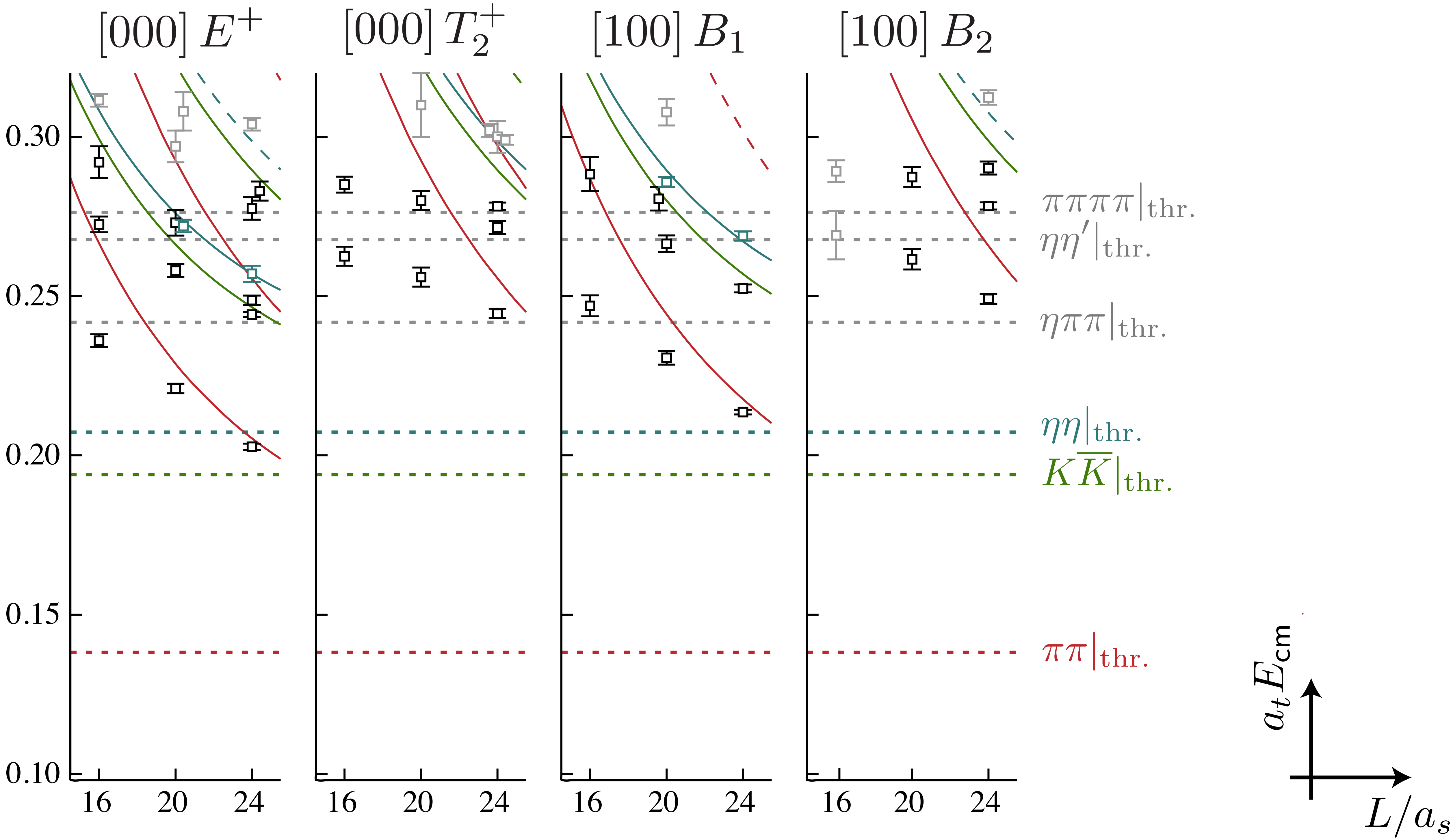
$$C_{ij}(t)v_j^{\mathfrak{n}} = \lambda(t)_{\mathfrak{n}} C_{ij}(t_0)v_j^{\mathfrak{n}} \quad \rightarrow \quad \lambda_{\mathfrak{n}} \sim \exp(-E_{\mathfrak{n}}t)$$

The infinite volume scattering t-matrix is determined from the finite volume spectrum by extensions of Lüscher's method

In the calculations that follow m_π=391 MeV
- suppresses some of the difficult many (≥3) hadron channels

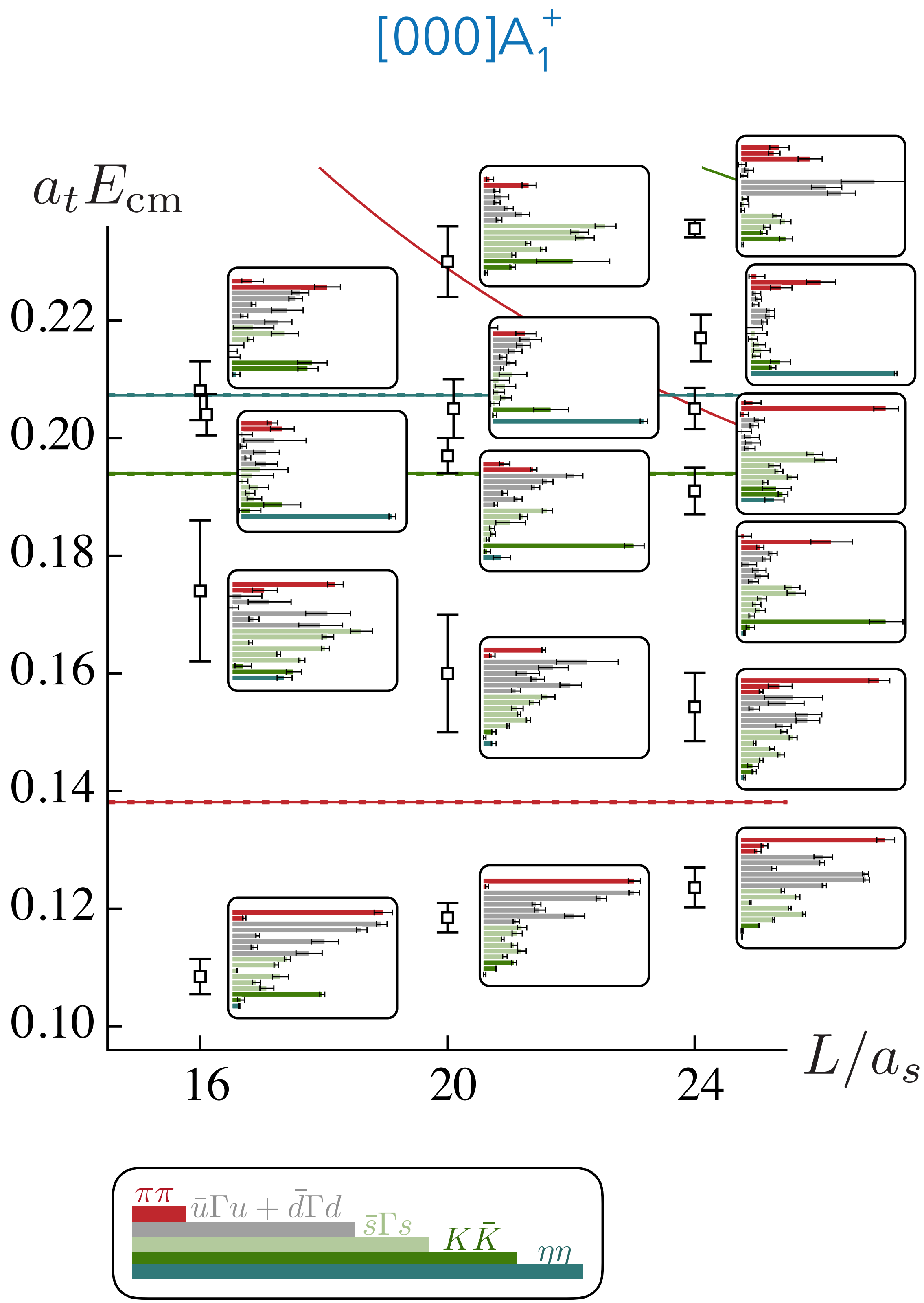


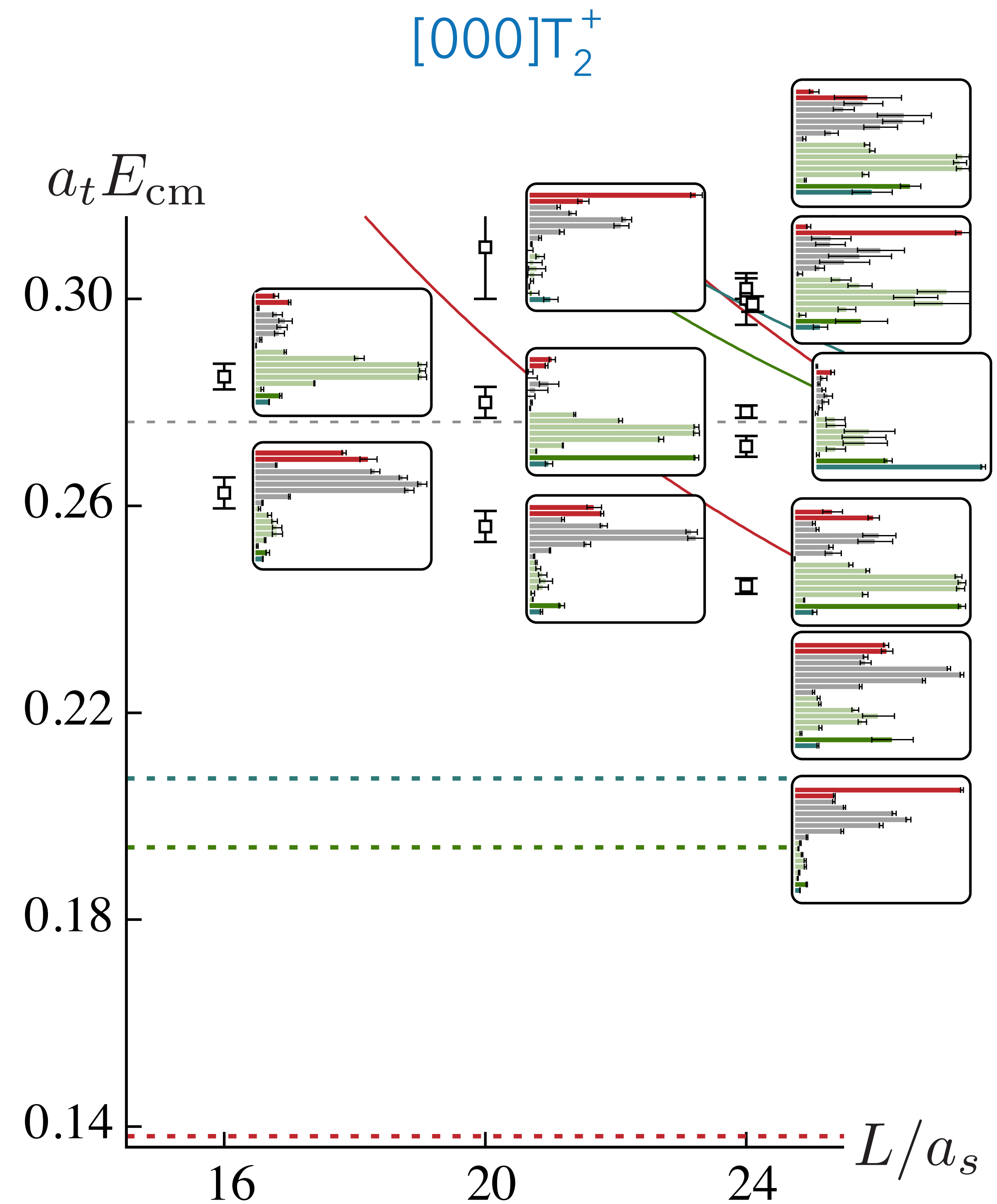
conservatively 57 energy levels
dominated by S-wave interactions



conservatively 34 energy levels
dominated by D-wave interactions

Spectra and overlaps





The amplitudes

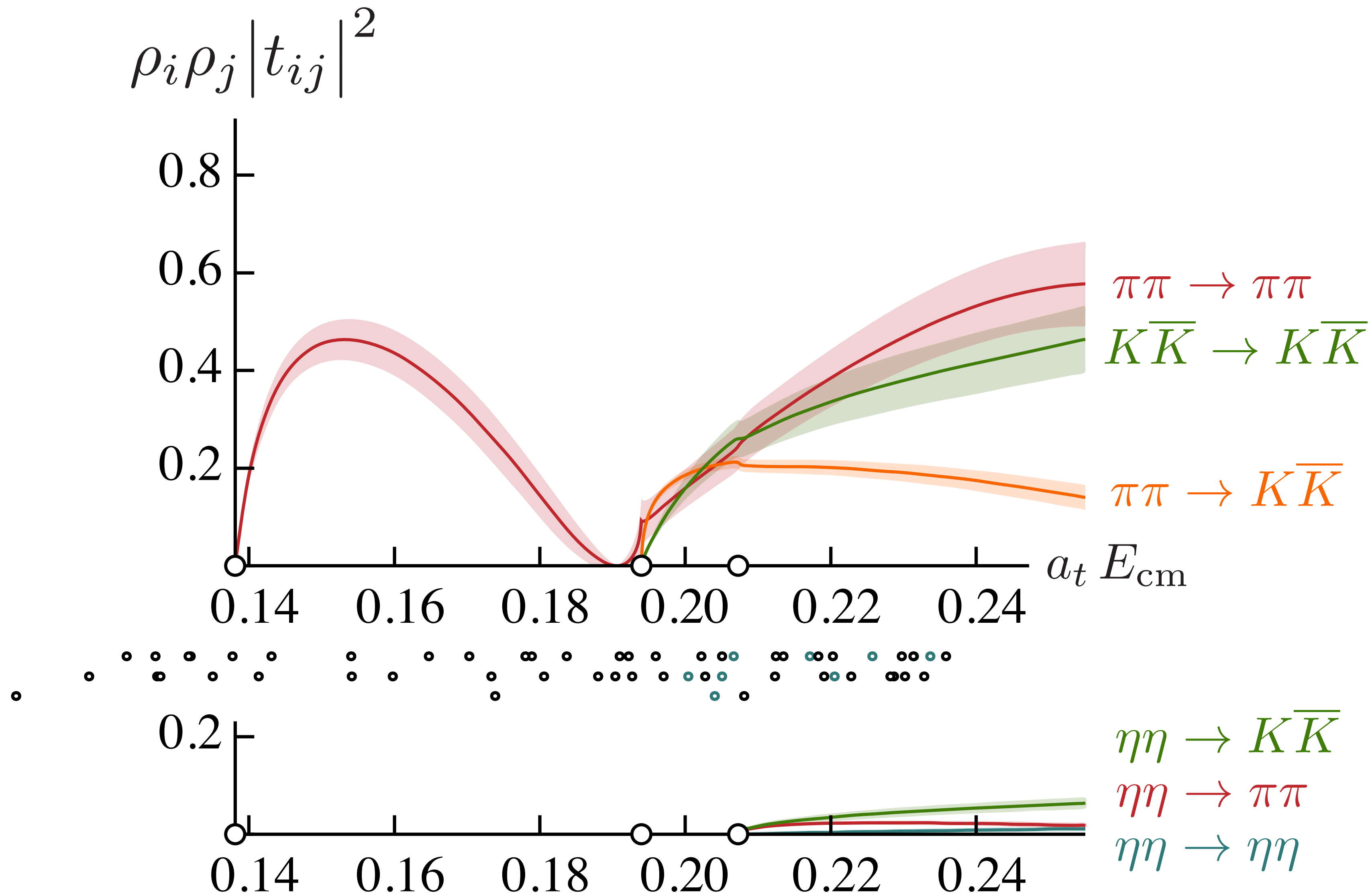
An example S-wave spectrum fit

$$\mathbf{t}^{-1} = \mathbf{K}^{-1} + \mathbf{I}$$

$$\mathbf{K}(s) = \begin{pmatrix} a+bs & c+ds & e \\ c+ds & f & g \\ e & g & h \end{pmatrix}$$

$$\chi^2/N_{\mathrm{dof}} = \frac{44.0}{57-8} = 0.90$$

57 energy levels



Using the coupled-channel extensions of Lüscher's method:

$$\det\left[\mathbf{1} + i\boldsymbol{\rho}(E)\cdot \boldsymbol{t}(E)\cdot\left(\mathbf{1} + i\boldsymbol{\mathcal{M}}(E,L)\right)\right] = 0$$

An example S-wave spectrum fit

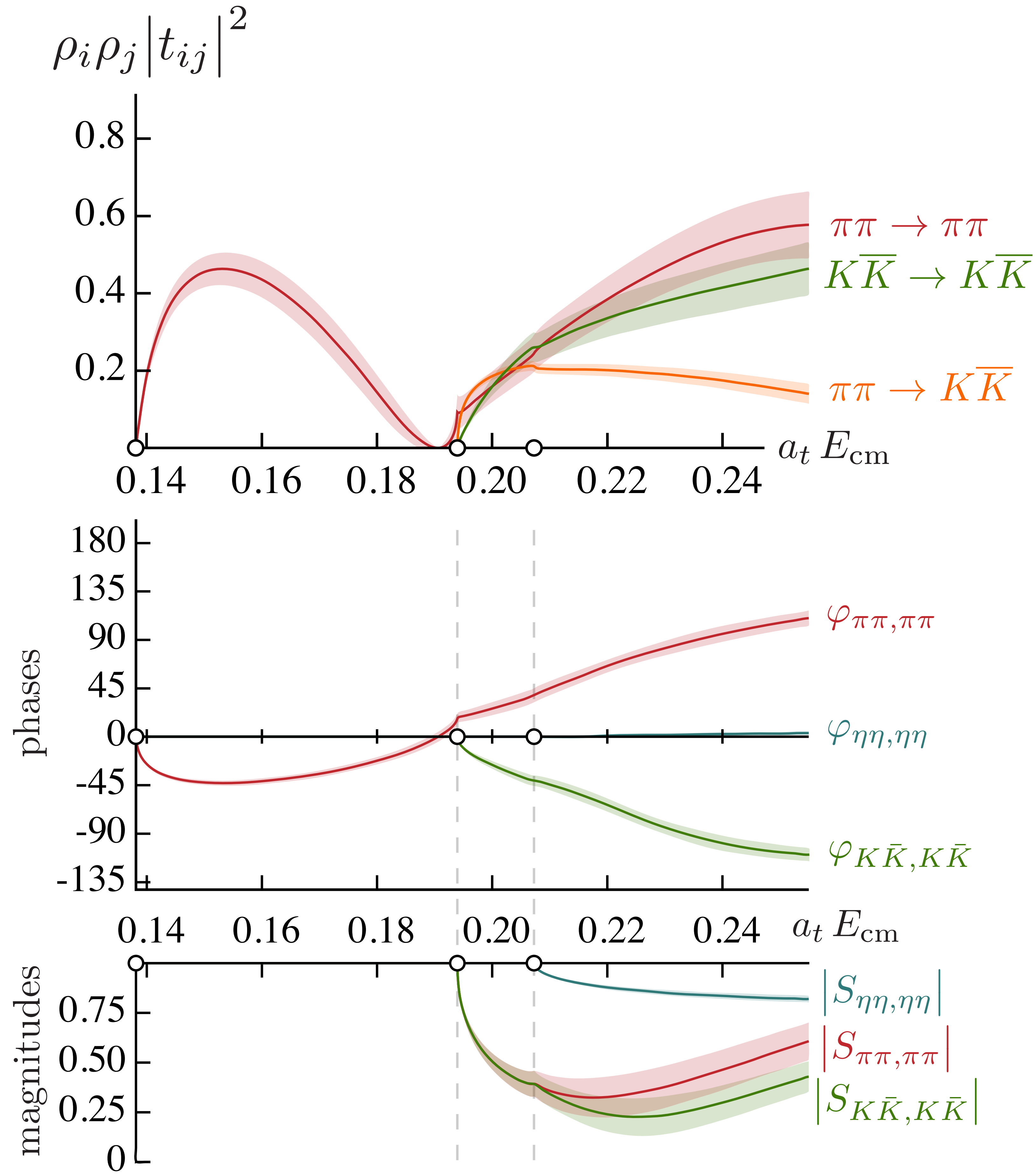
$$\mathbf{t}^{-1} = \mathbf{K}^{-1} + \mathbf{I}$$

$$\mathbf{K}(s) = \begin{pmatrix} a+bs & c+ds & e \\ c+ds & f & g \\ e & g & h \end{pmatrix}$$

$$\chi^2/N_{\mathrm{dof}} = \frac{44.0}{57-8} = 0.90$$

57 energy levels

$$S_{ii}(E_{\mathsf{cm}}) = |S_{ii}(E_{\mathsf{cm}})|\, e^{2i\phi_{ii}(E_{\mathsf{cm}})}$$



An example D-wave spectrum fit

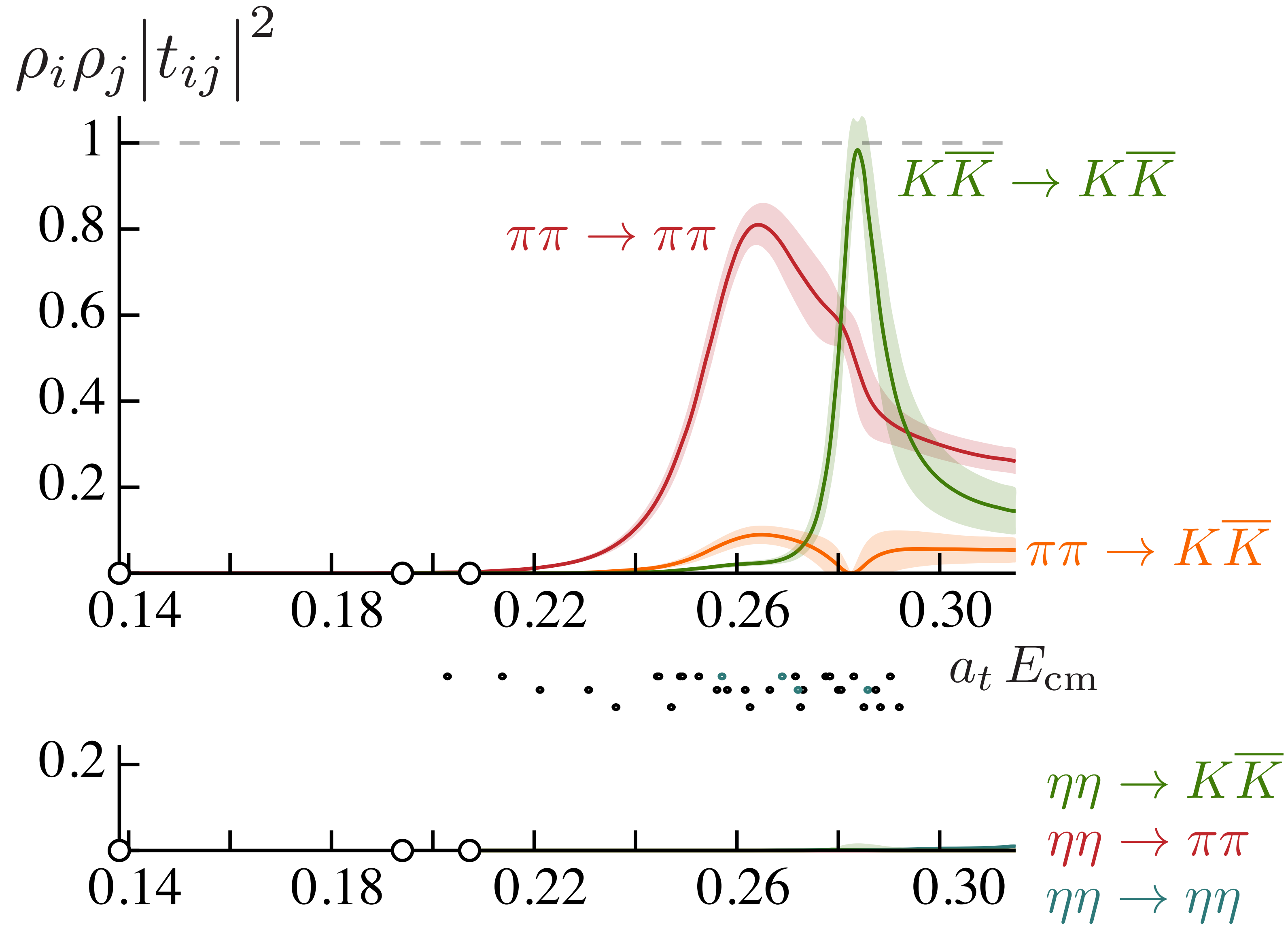
$$\mathbf{t}^{-1} = \mathbf{K}^{-1} + \mathbf{I}$$

$$K_{ij}(s) = \frac{g_i^{(1)} g_j^{(1)}}{m_1^2 - s} + \frac{g_i^{(2)} g_j^{(2)}}{m_2^2 - s} + \gamma_{ij}$$

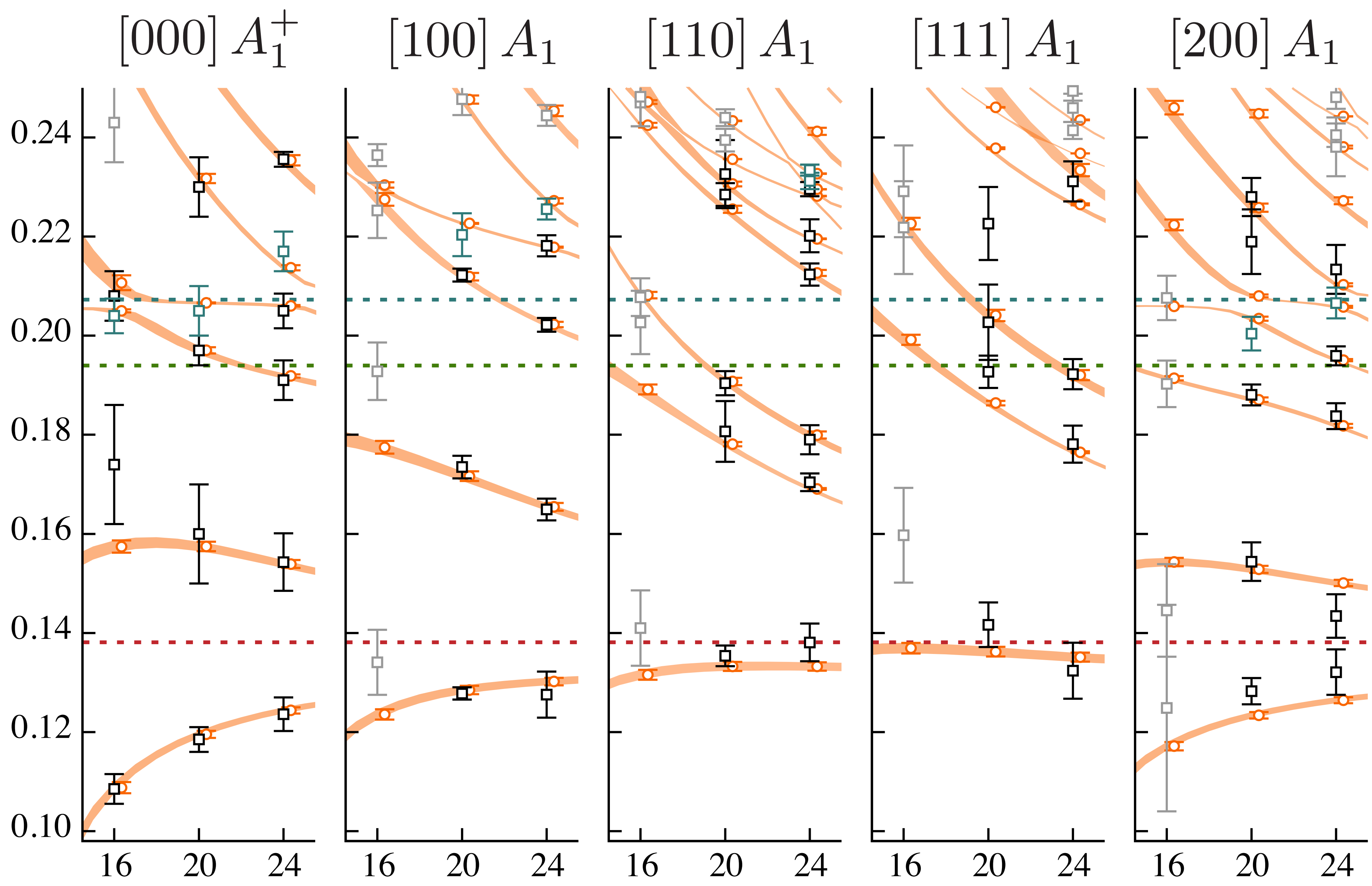
$$\gamma_{\eta\eta} \neq 0$$
$$\gamma_{ij} = 0 \quad \text{otherwise}$$

$$\chi^2/N_{\rm dof} = \frac{28.9}{34-9} = 1.15$$

34 energy levels



An example S-wave spectrum fit



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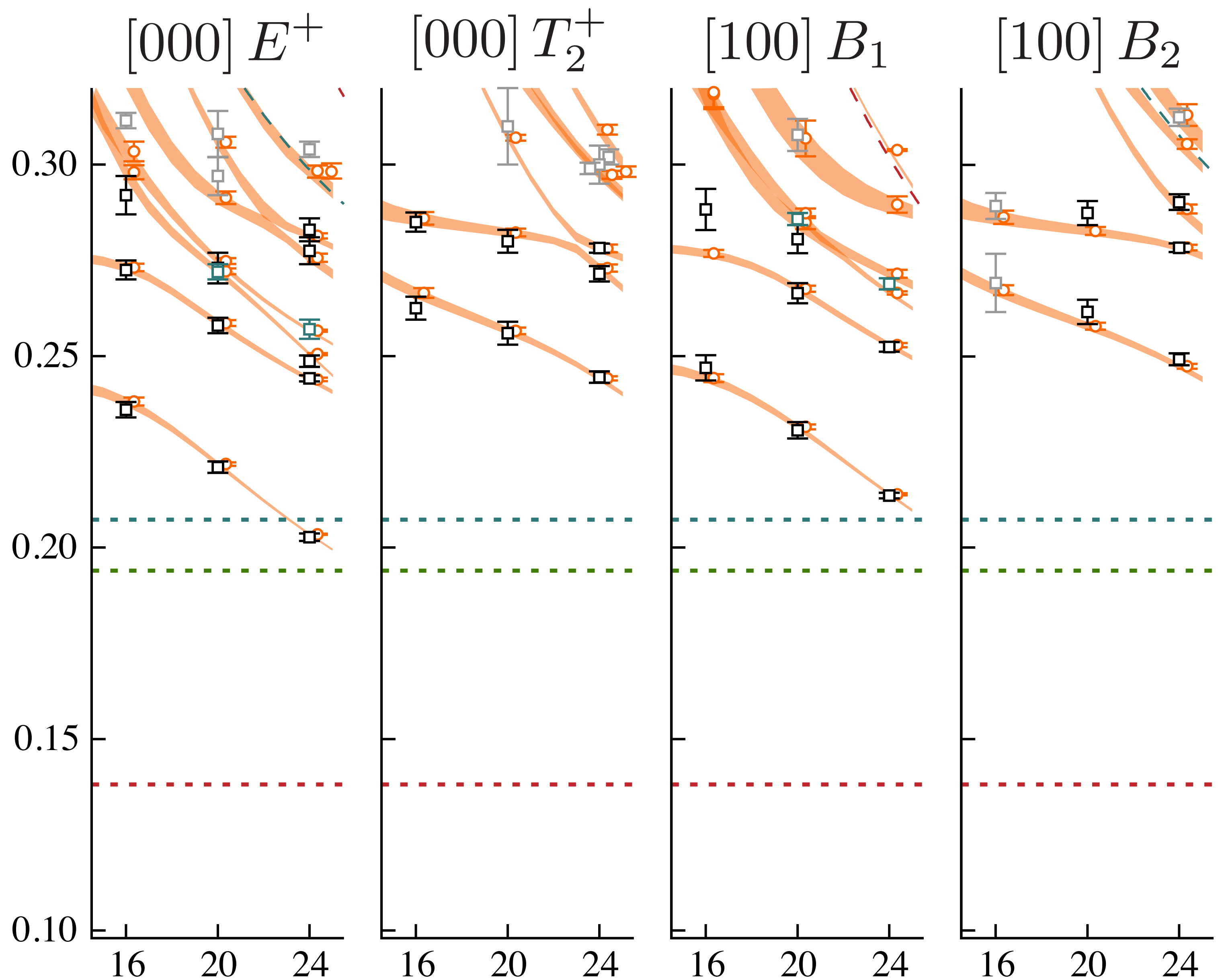
An example D-wave spectrum fit



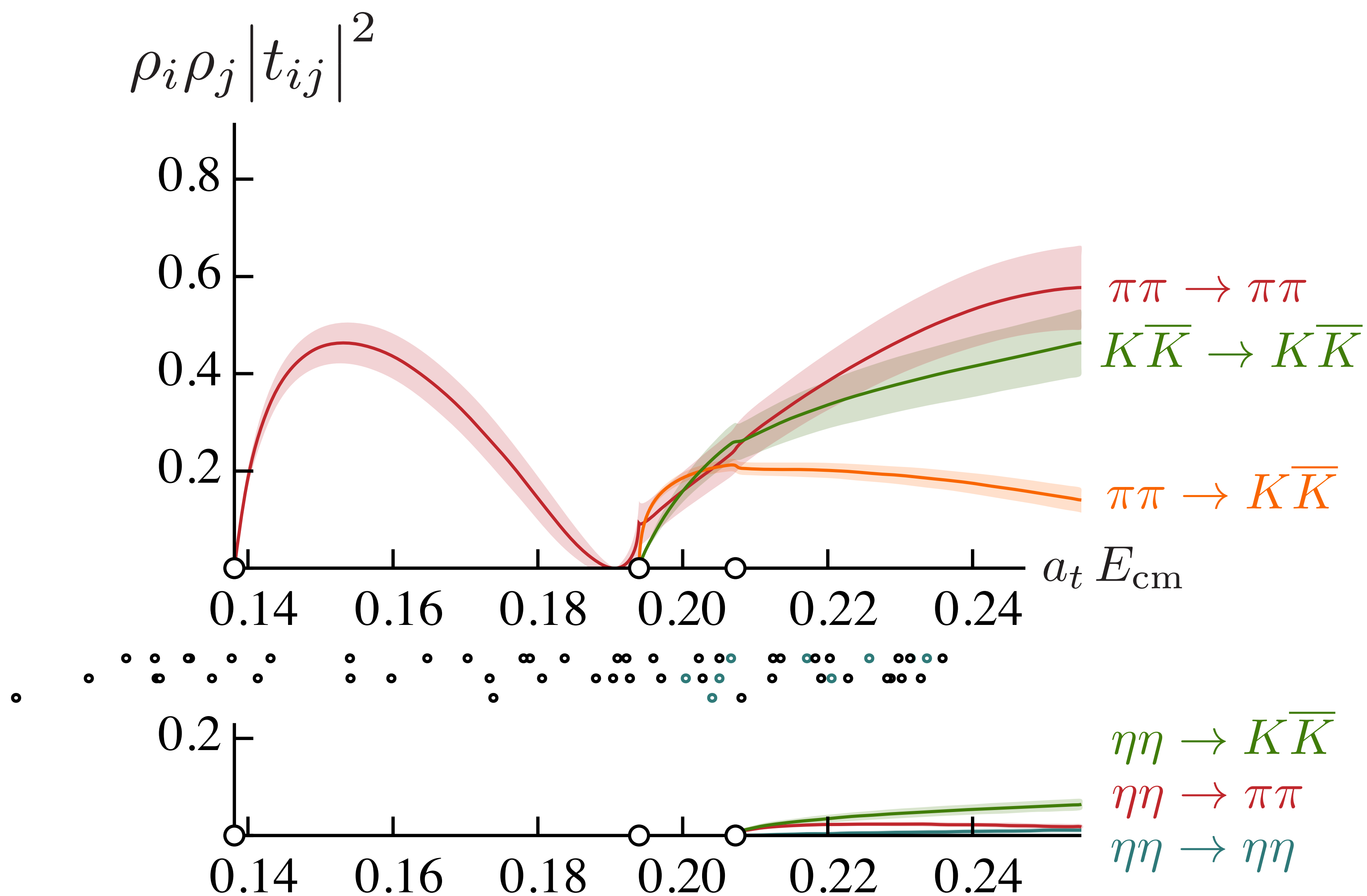
$$\chi^2/N_{\text{dof}} = \frac{28.9}{34-9} = 1.15$$

An example S-wave spectrum fit

$$\mathbf{K}(s) = \begin{pmatrix} a+bs & c+ds & e \\ c+ds & f & g \\ e & g & h \end{pmatrix}$$

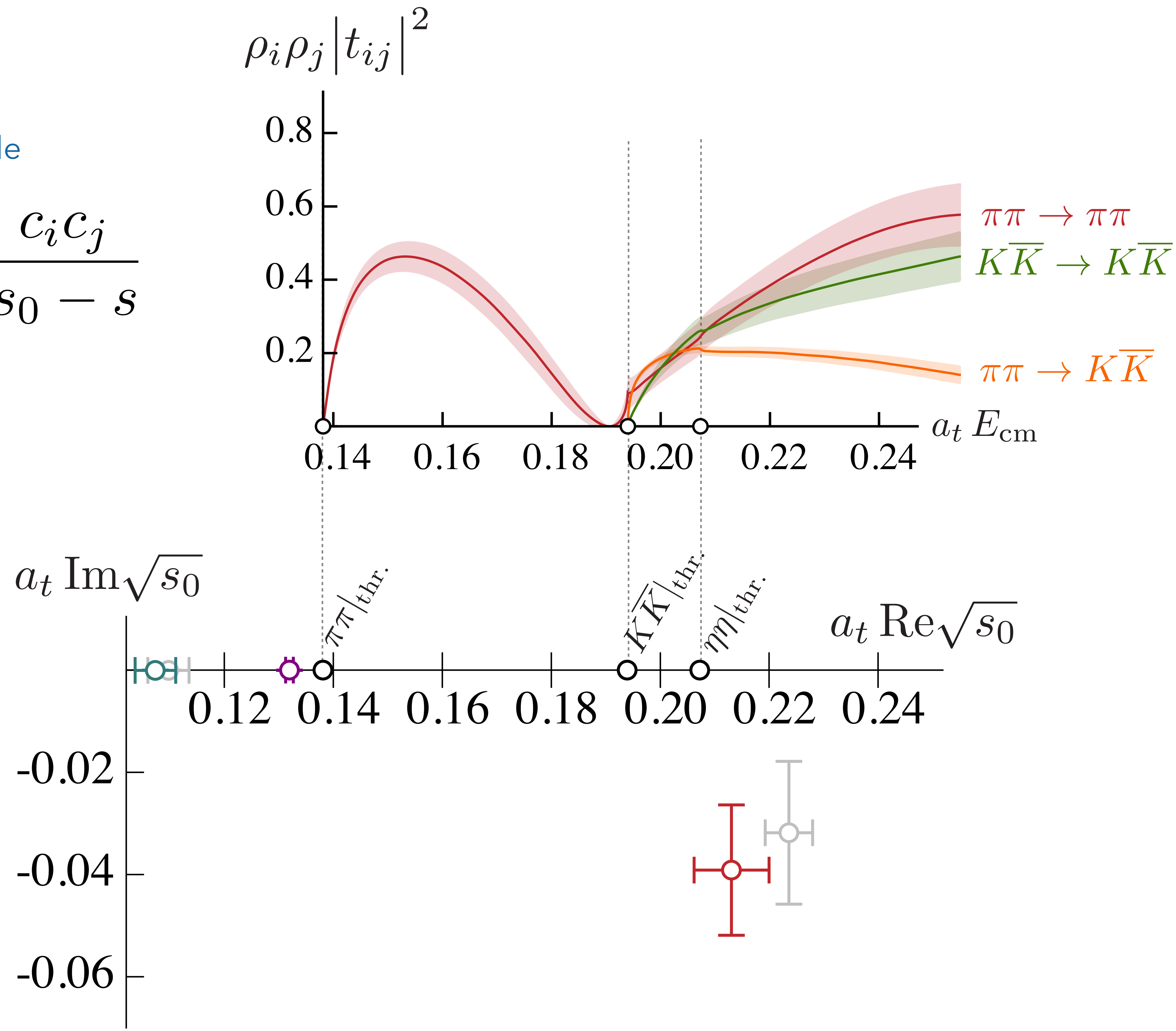
$$\chi^2/N_{\mathrm{dof}} = \frac{44.0}{57-8} = 0.90$$

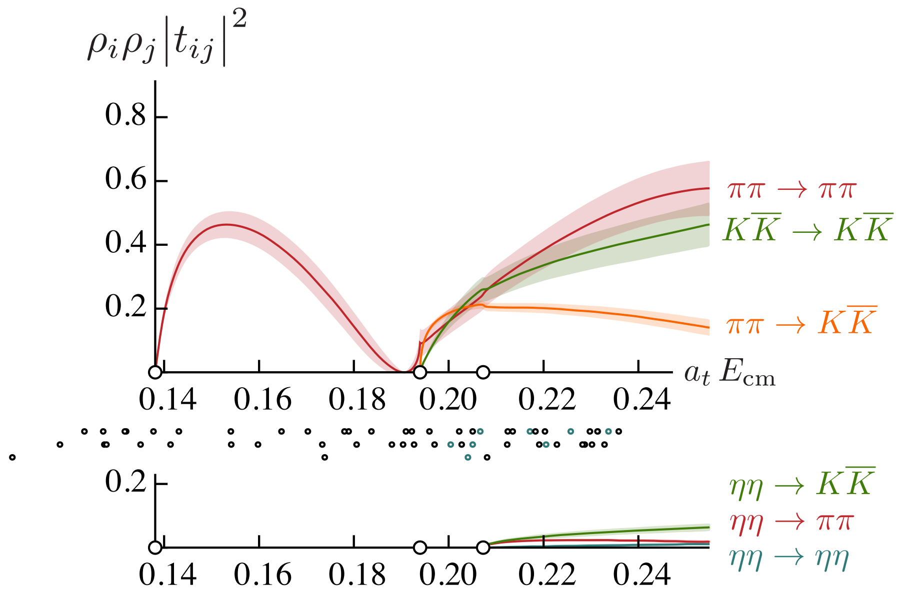
57 energy levels

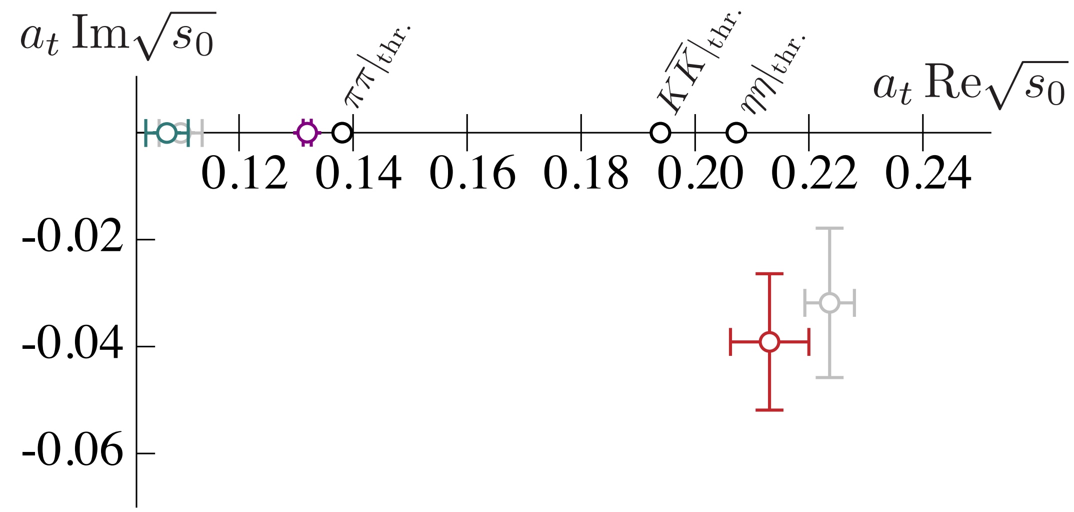


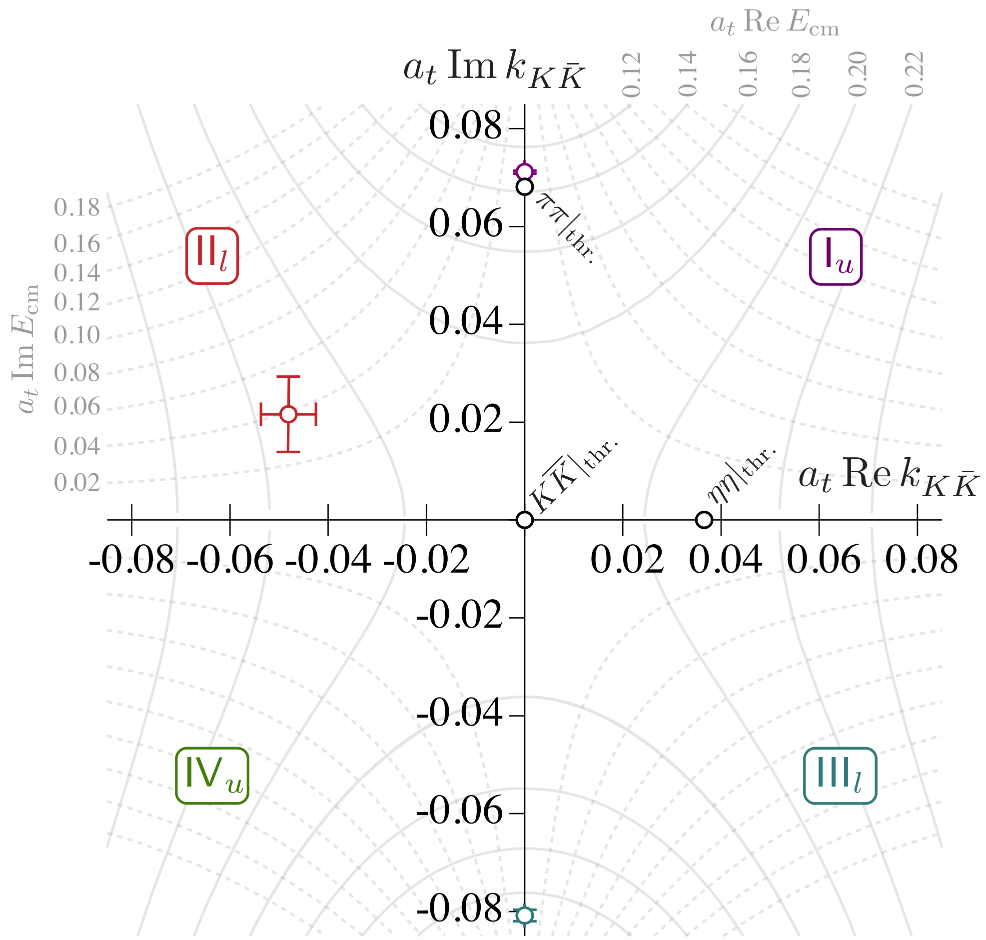
Near a t-matrix pole

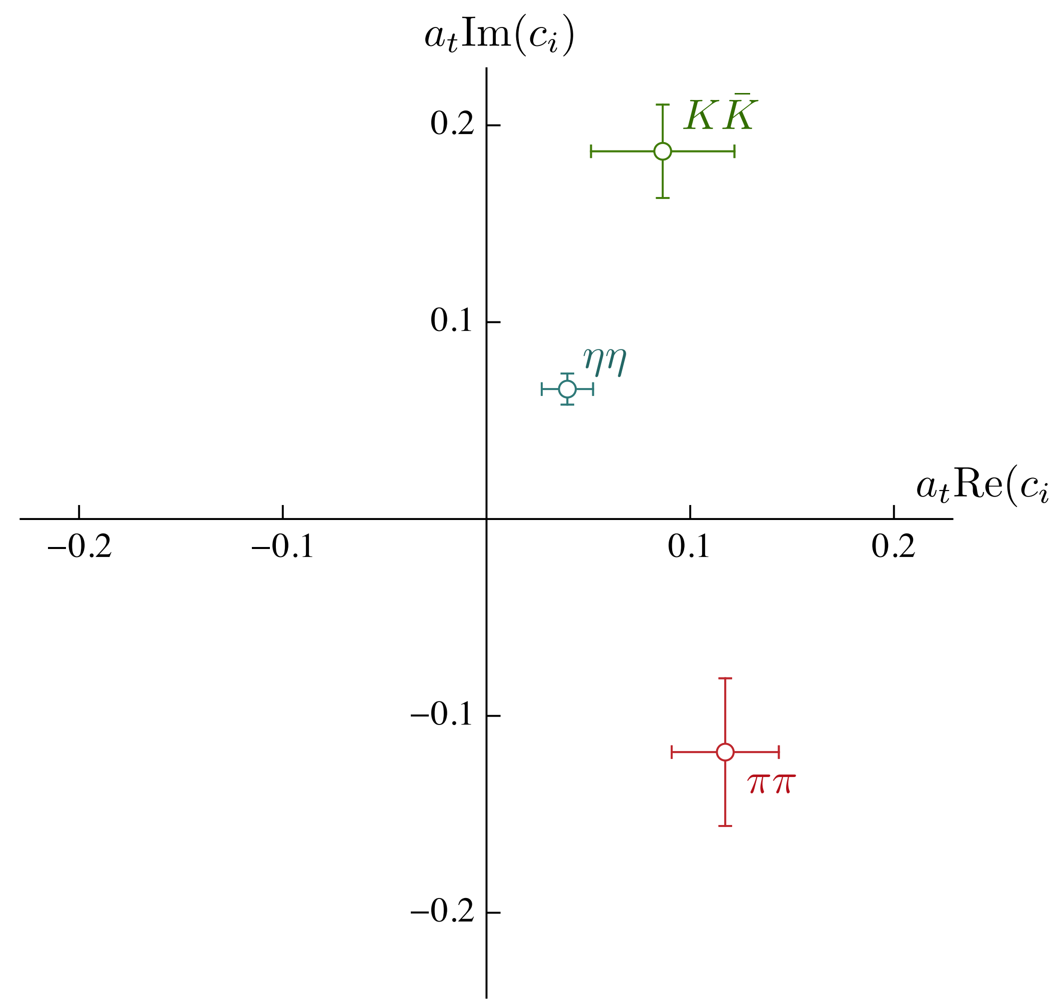
$$t_{ij} \sim \frac{c_i c_j}{s_0 - s}$$

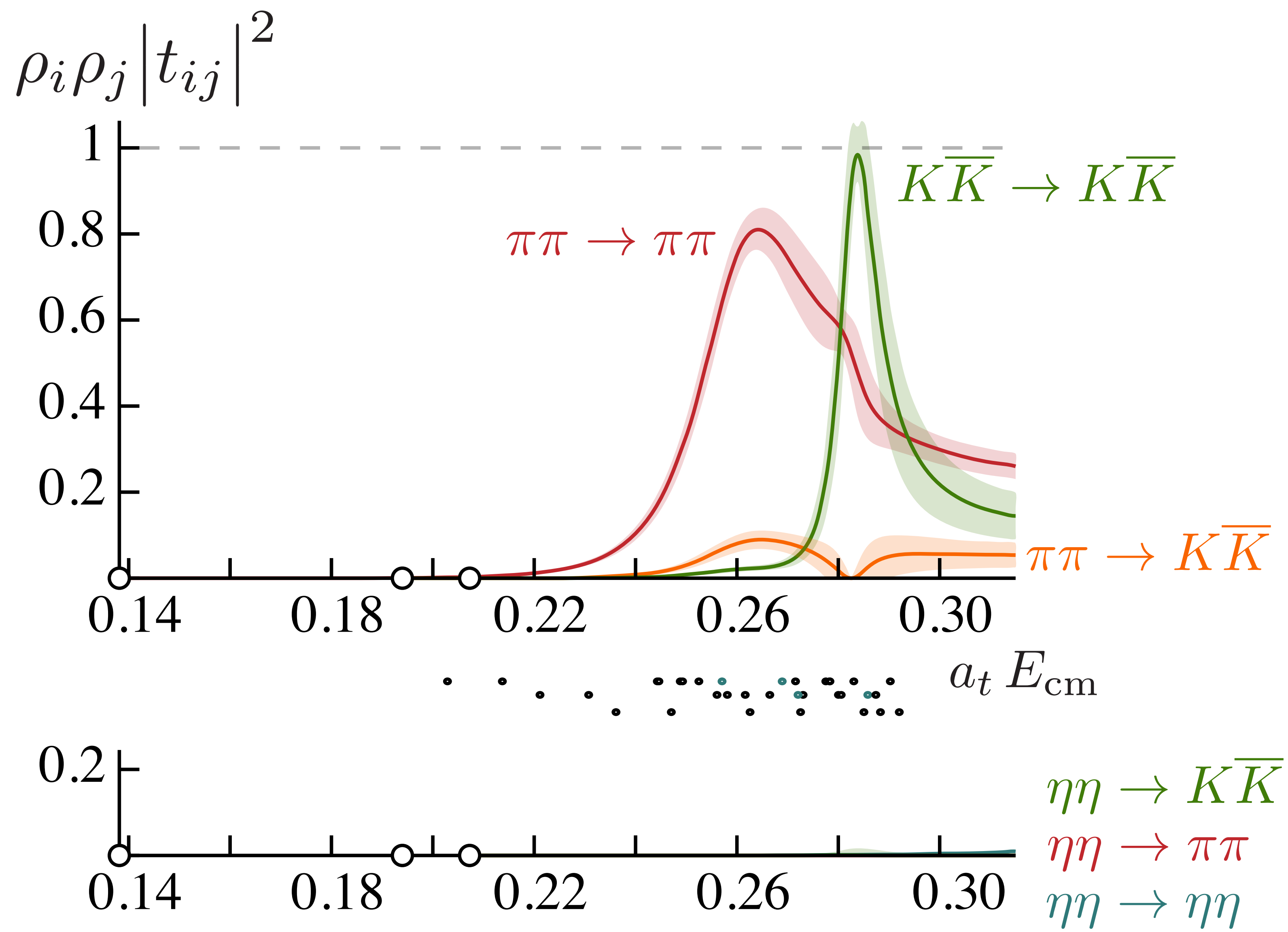


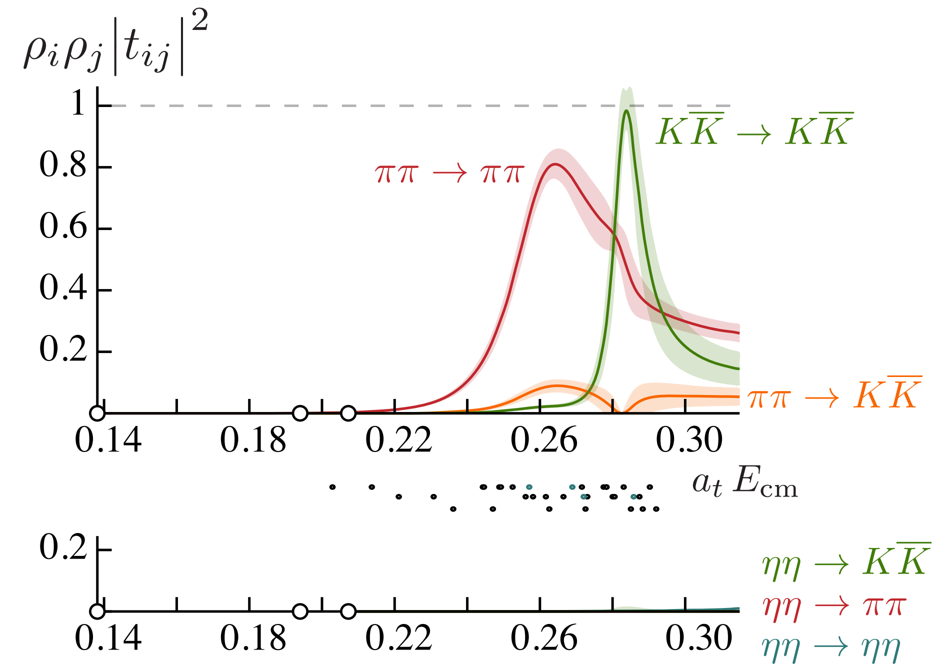

$\rho_i \rho_j |t_{ij}|^2$
0.8
0.6
0.4
0.2
0.14 0.16 0.18 0.20 0.22 0.24
$a_t E_{\rm cm}$
$\pi\pi \to \pi\pi$
$K\bar{K} \to K\bar{K}$
$\pi\pi \to K\bar{K}$
$\eta\eta \to K\bar{K}$
$\eta\eta \to \pi\pi$
$\eta\eta \to \eta\eta$


$a_t \,\mathrm{Im}\sqrt{s_0}$
$a_t \,\mathrm{Re}\sqrt{s_0}$
$\pi\pi|_{\rm thr.}$
$K\bar{K}|_{\rm thr.}$
$\eta\eta|_{\rm thr.}$
0.12 0.14 0.16 0.18 0.20 0.22 0.24
-0.02
-0.04
-0.06


$a_t \,\mathrm{Re}\, E_{\rm cm}$
$a_t \,\mathrm{Im}\, k_{K\bar{K}}$
$a_t \,\mathrm{Re}\, k_{K\bar{K}}$
$a_t \,\mathrm{Im}\, E_{\rm cm}$
II_l
I_u
IV_u
III_l
$\pi\pi|_{\rm thr.}$
$K\bar{K}|_{\rm thr.}$
$\eta\eta|_{\rm thr.}$
0.12 0.14 0.16 0.18 0.20 0.22
0.08 0.06 0.04 0.02 -0.02 -0.04 -0.06 -0.08

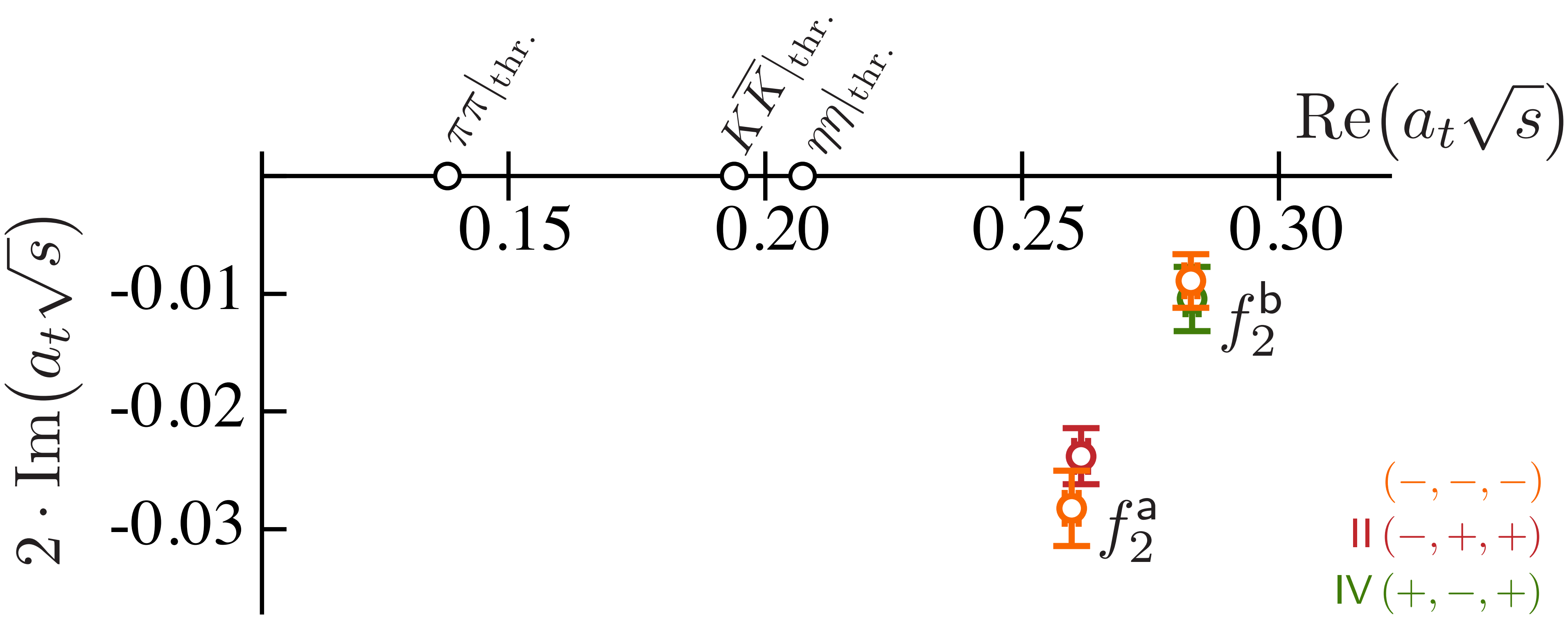

$a_t \mathrm{Im}(c_i)$
$a_t \mathrm{Re}(c_i)$
$K\bar{K}$
$\eta\eta$
$\pi\pi$
−0.2 −0.1 0.1 0.2


$\rho_i \rho_j |t_{ij}|^2$
1
0.8
0.6
0.4
0.2
0.14
0.18
0.22
0.26
0.30
$a_t E_{cm}$
$K\overline{K} \to K\overline{K}$
$\pi\pi \to \pi\pi$
$\pi\pi \to K\overline{K}$
0.2
0.14
0.18
0.22
0.26
0.30
$\eta\eta \to K\overline{K}$
$\eta\eta \to \pi\pi$
$\eta\eta \to \eta\eta$

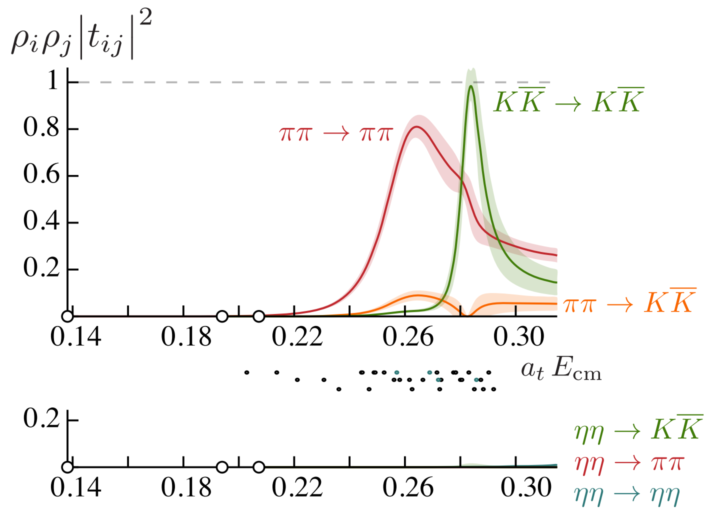


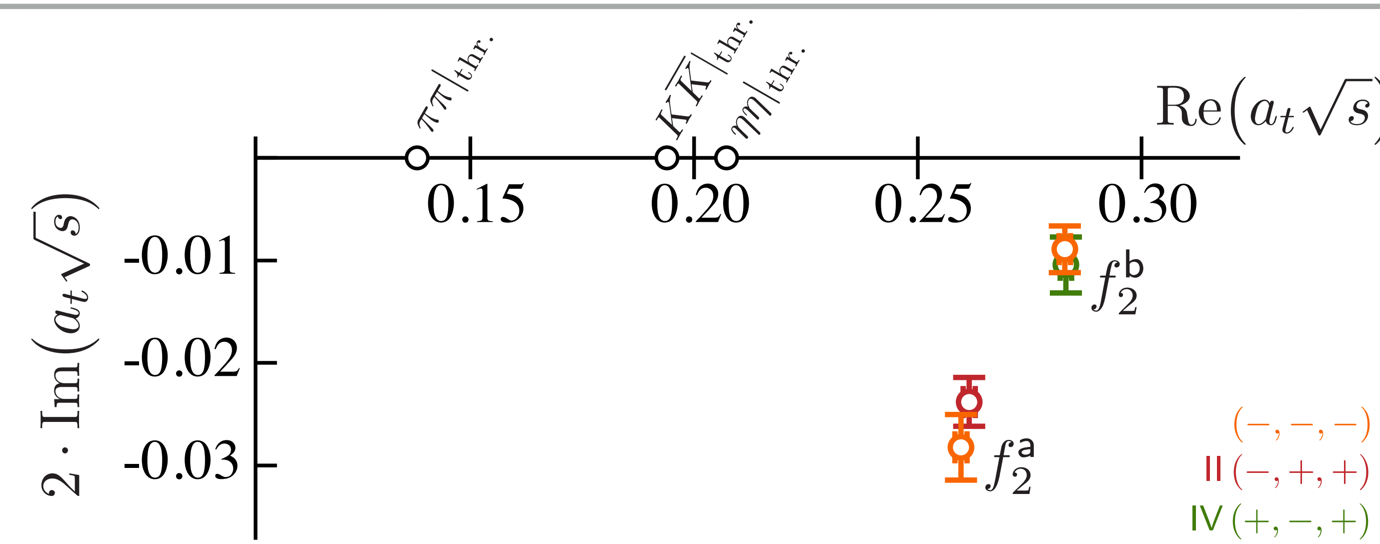
Near a t-matrix pole

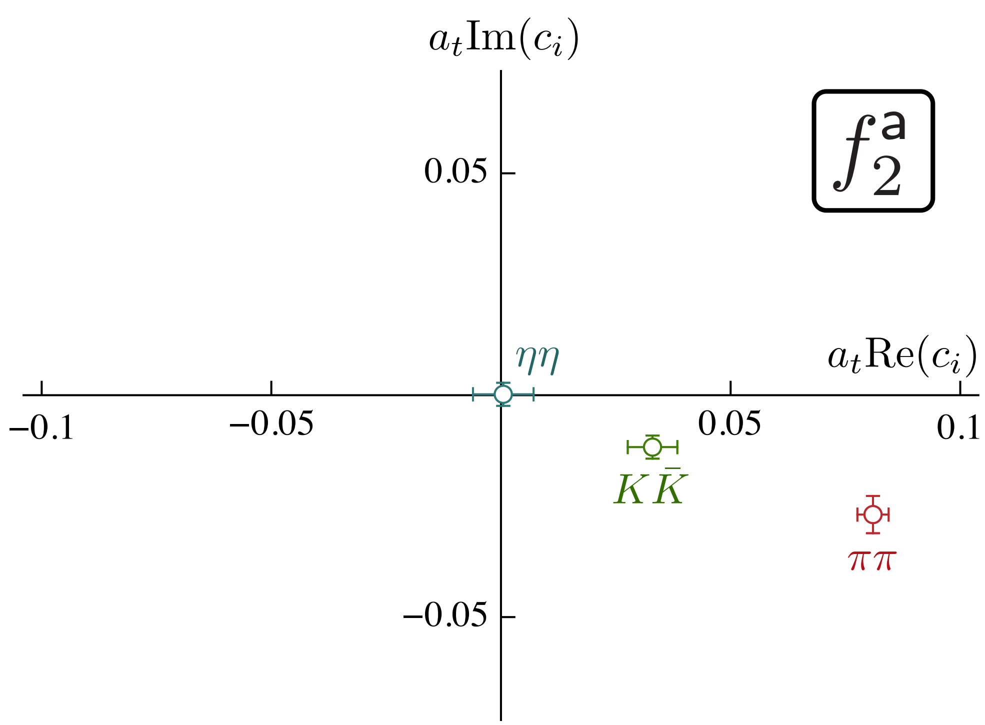
$$t_{ij} \sim \frac{c_i c_j}{s_0 - s}$$

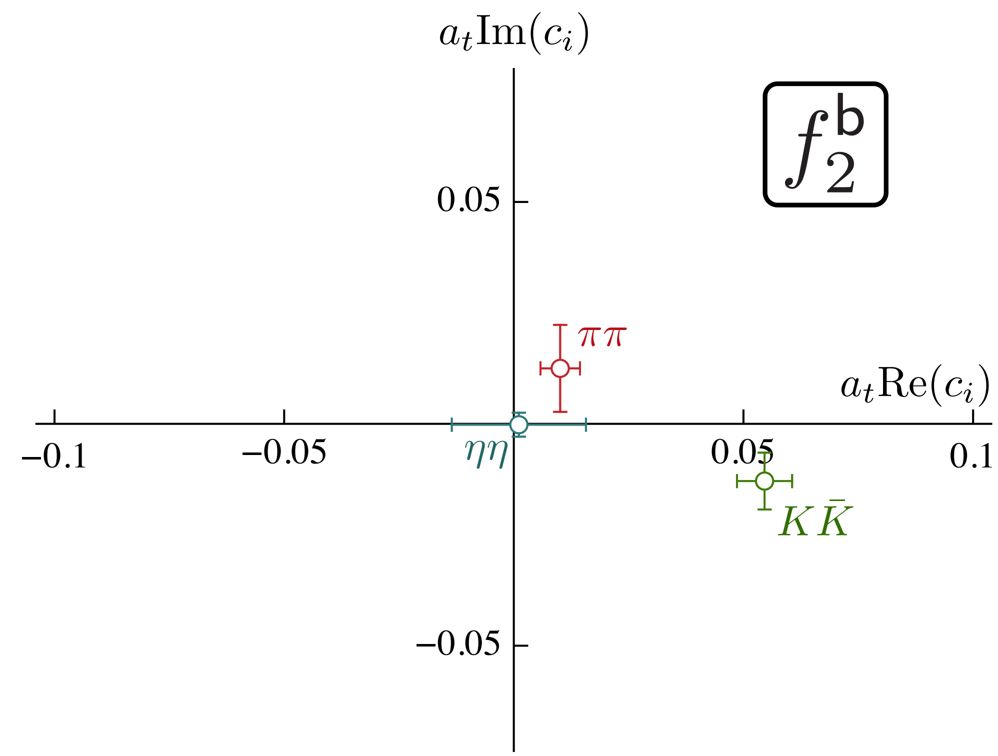


Tensor resonance poles







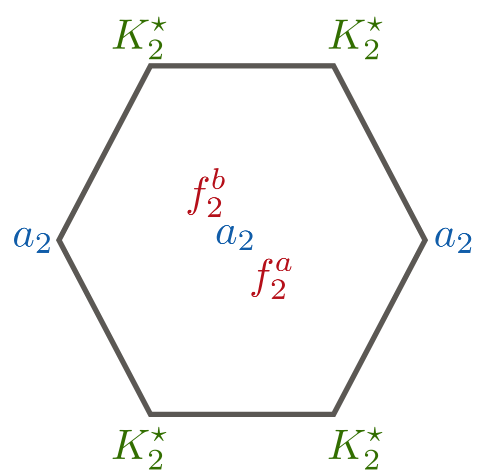


$f_2^{\mathsf{a}}: \quad \sqrt{s_0} = 1470(15) - \tfrac{i}{2}\,160(18)\ \mathrm{MeV}$

$\mathrm{Br}(f_2^{\mathsf{a}} \to \pi\pi) \sim 85\%, \quad \mathrm{Br}(f_2^{\mathsf{a}} \to K\overline{K}) \sim 12\%$

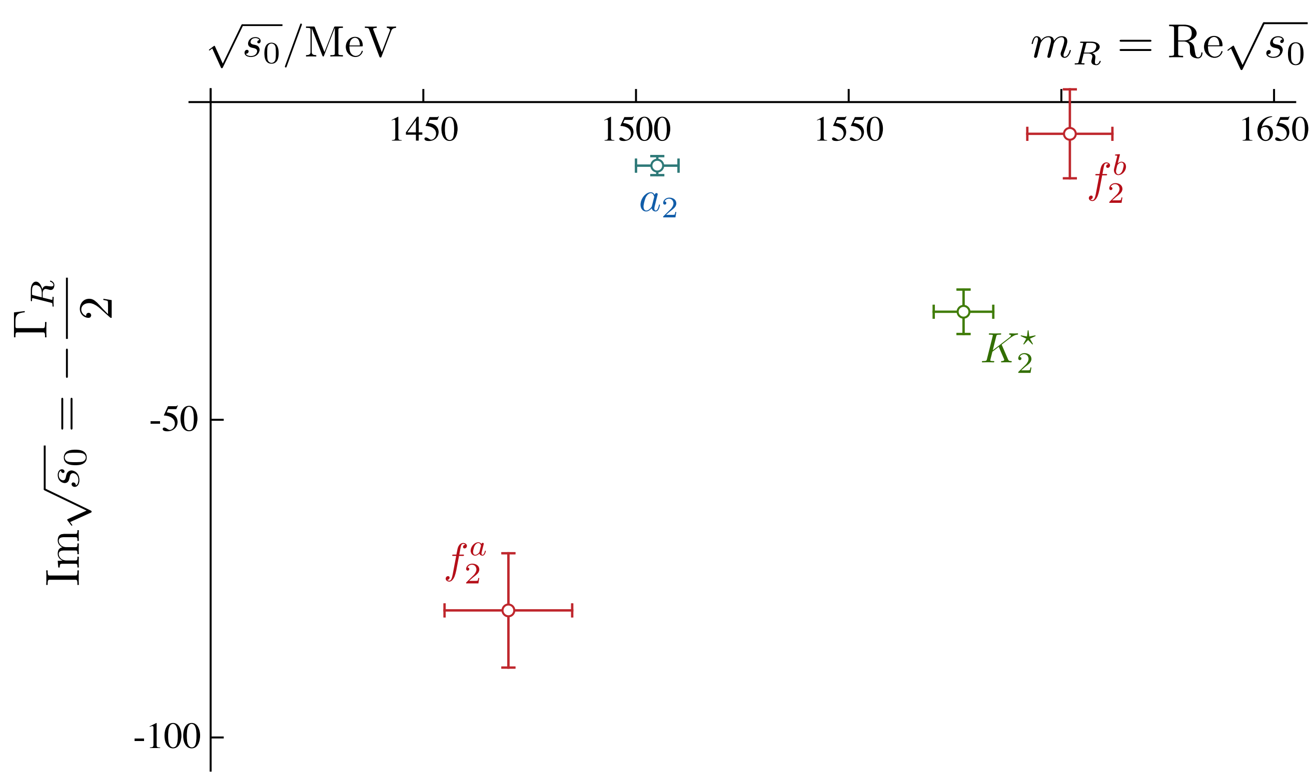
$f_2^{\mathsf{b}}: \quad \sqrt{s_0} = 1602(10) - \tfrac{i}{2}\,54(14)\ \mathrm{MeV}$

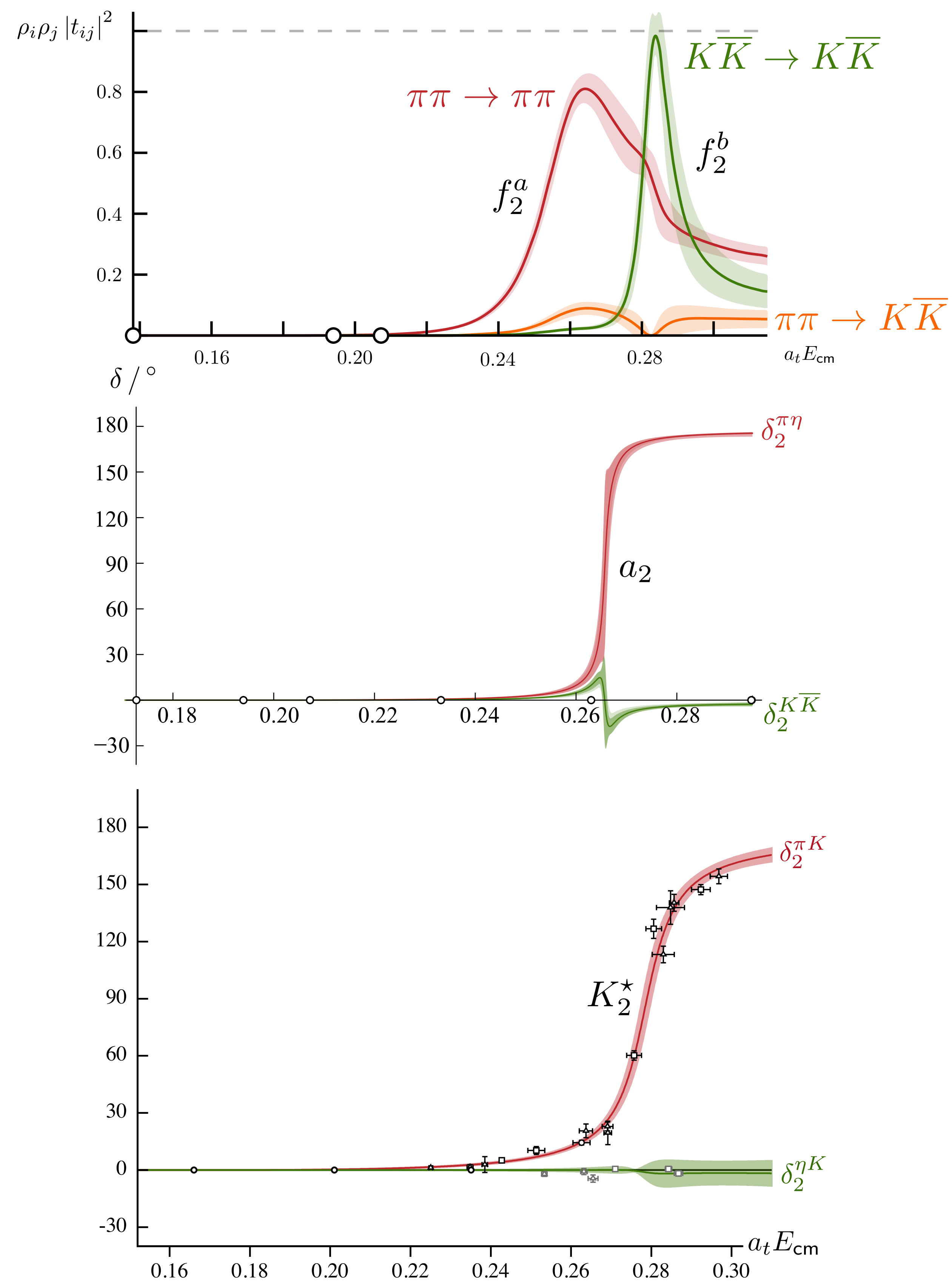
$\mathrm{Br}(f_2^{\mathsf{b}} \to \pi\pi) \sim 8\%, \quad \mathrm{Br}(f_2^{\mathsf{b}} \to K\overline{K}) \sim 92\%$

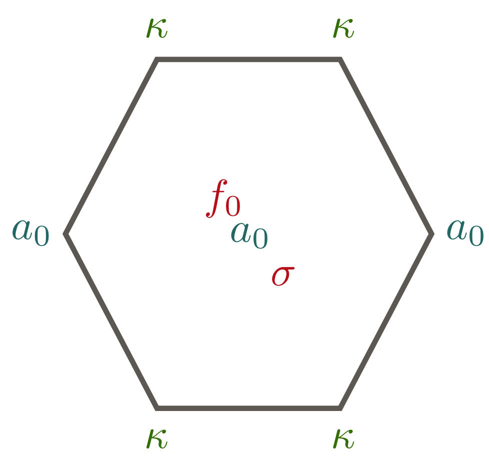


$$t_{ij} \sim \frac{c_i c_j}{s_0 - s}$$

$$\sqrt{s_0} = m_R - i\frac{\Gamma_R}{2}$$







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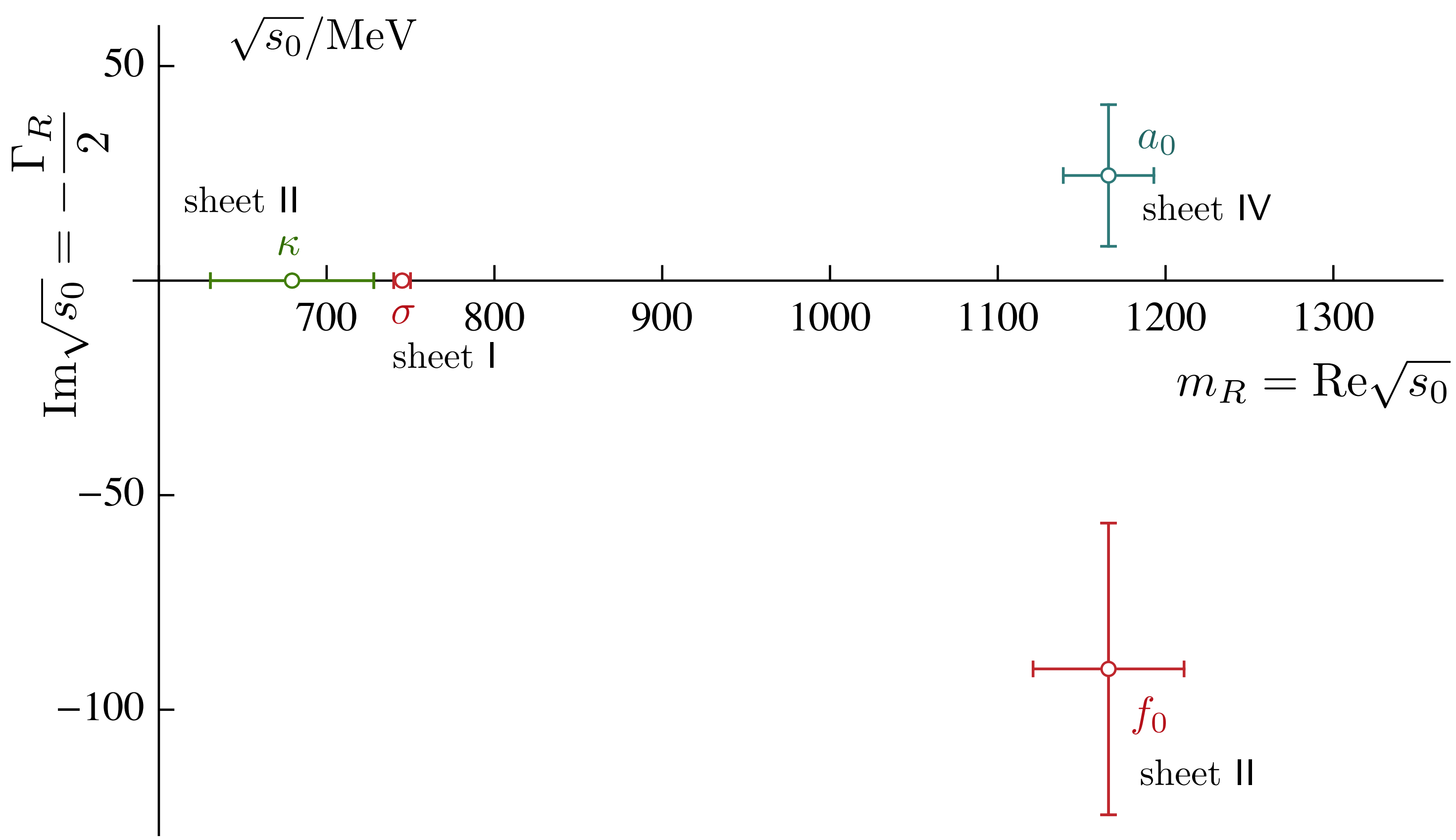
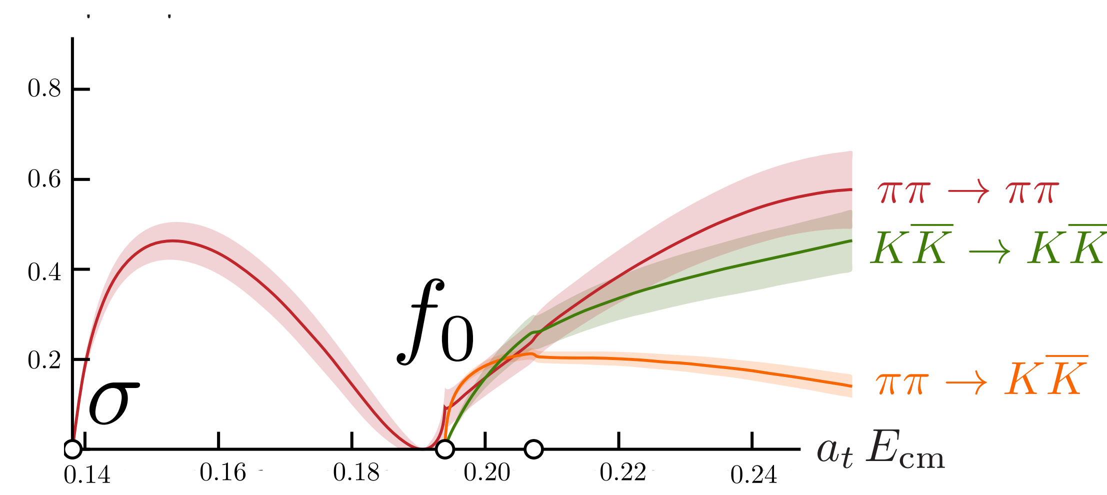
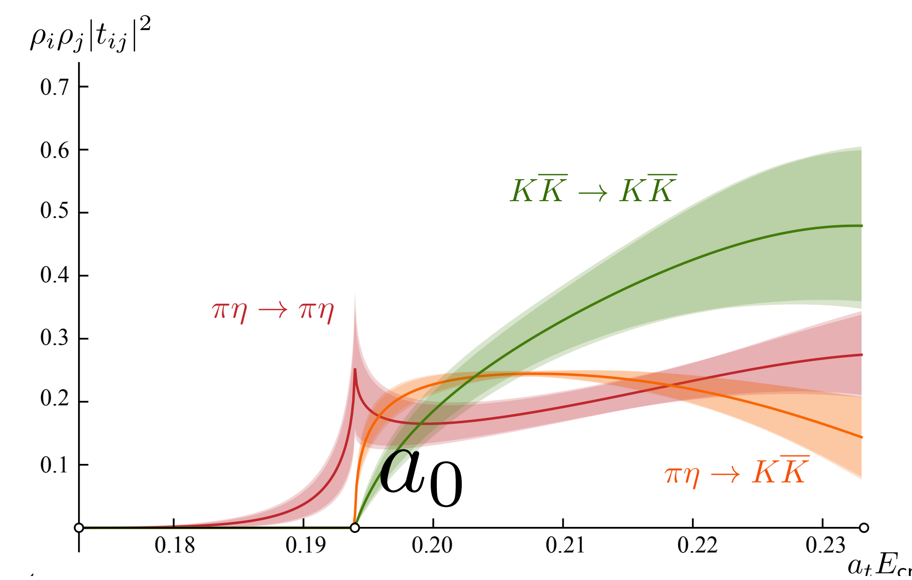
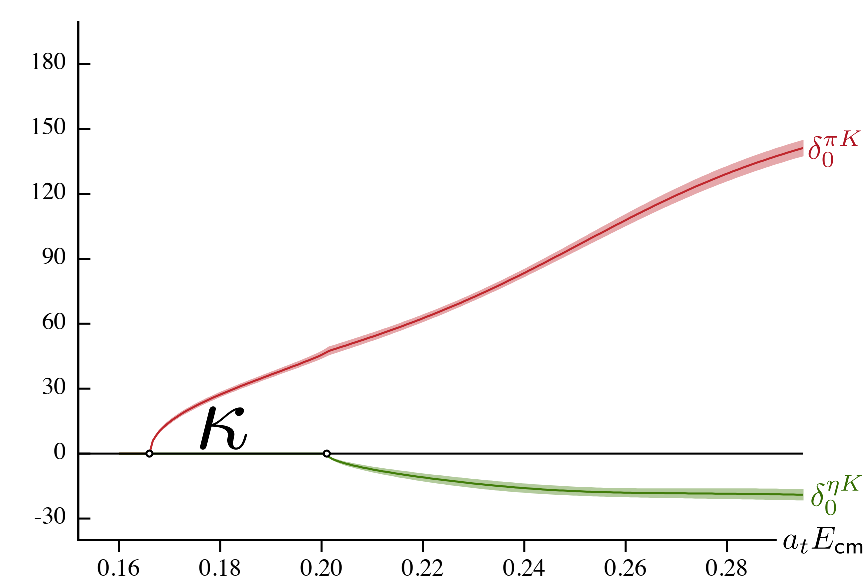
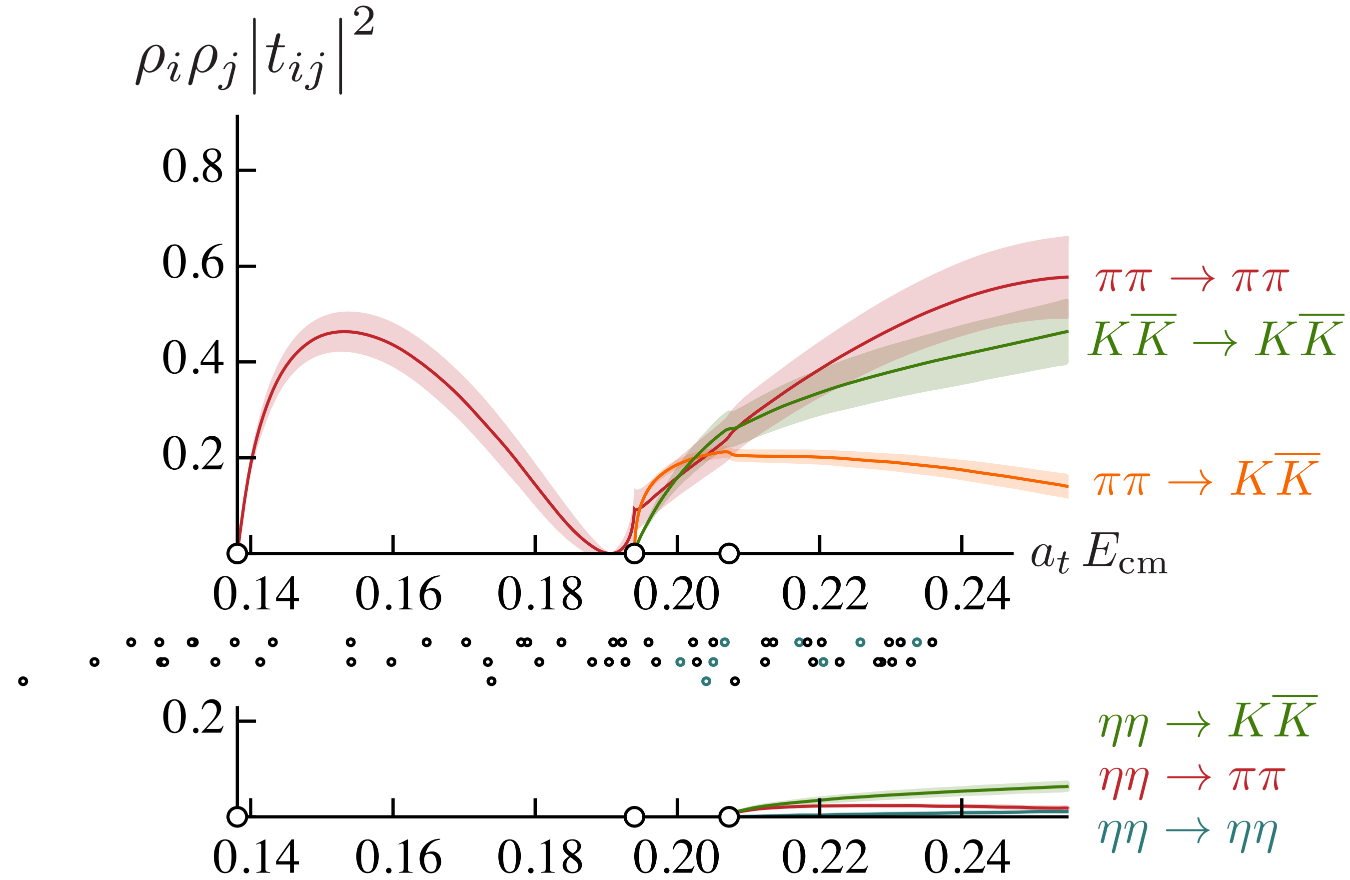






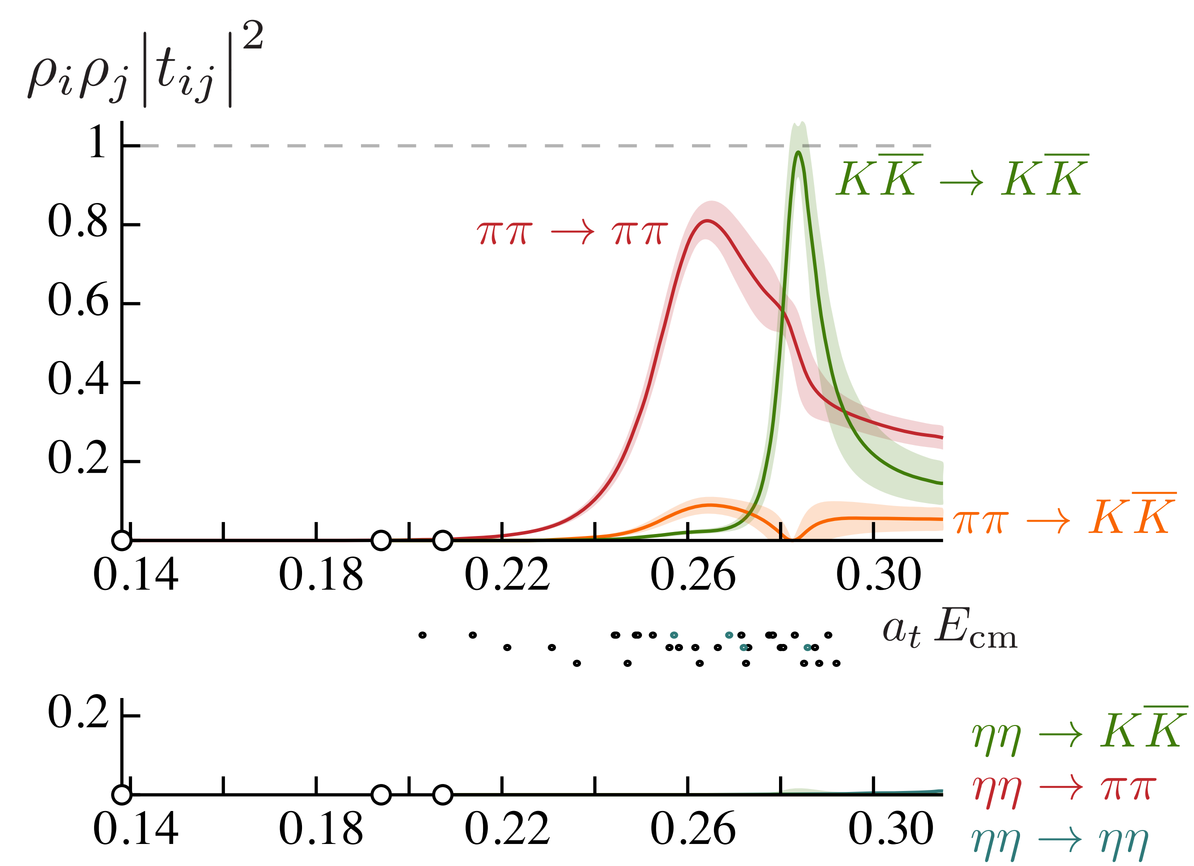


We have extracted S & D wave scattering amplitudes for coupled-channel ππ, KK, ηη scattering.



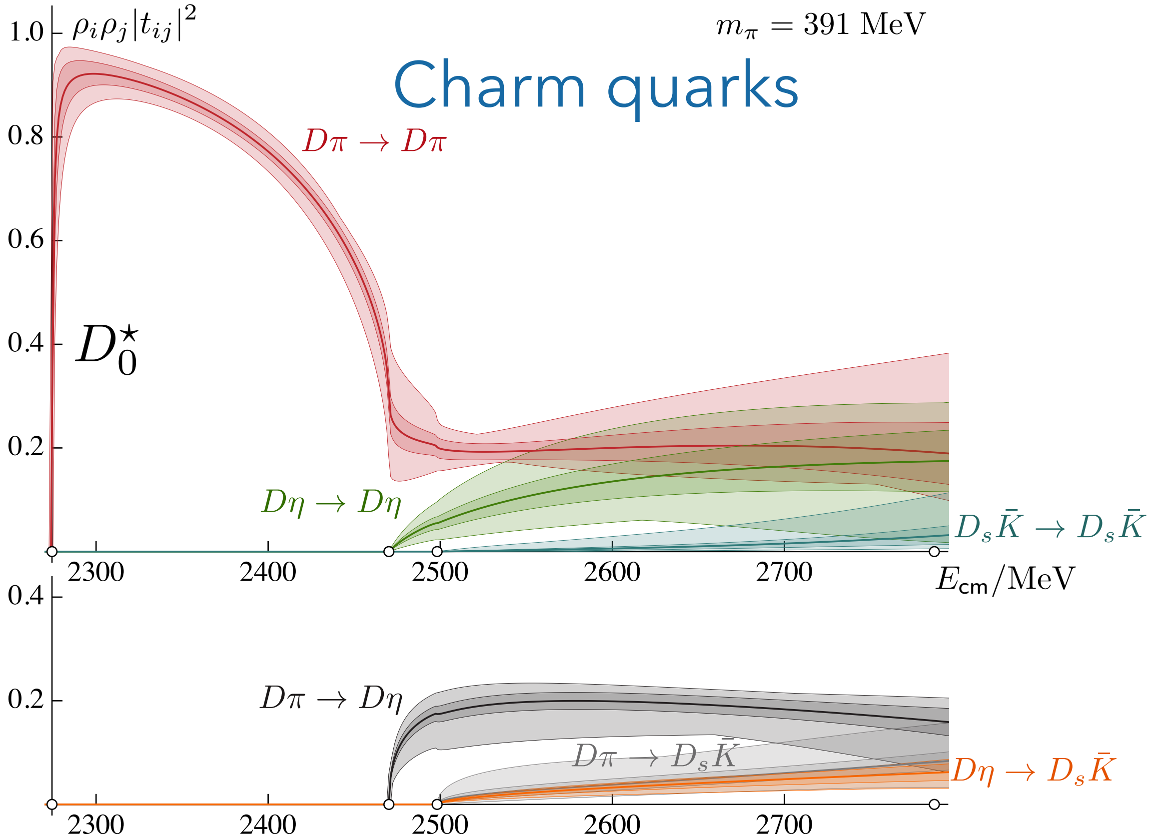
We find analogues of f_0 and f_2 resonances and are able to extract pole positions and residues - giving the coupling to each channel.

The **σ** appears as a bound state which strongly influences ππ channel, and the f_0 as a dip around KK threshold, similar to experiment.

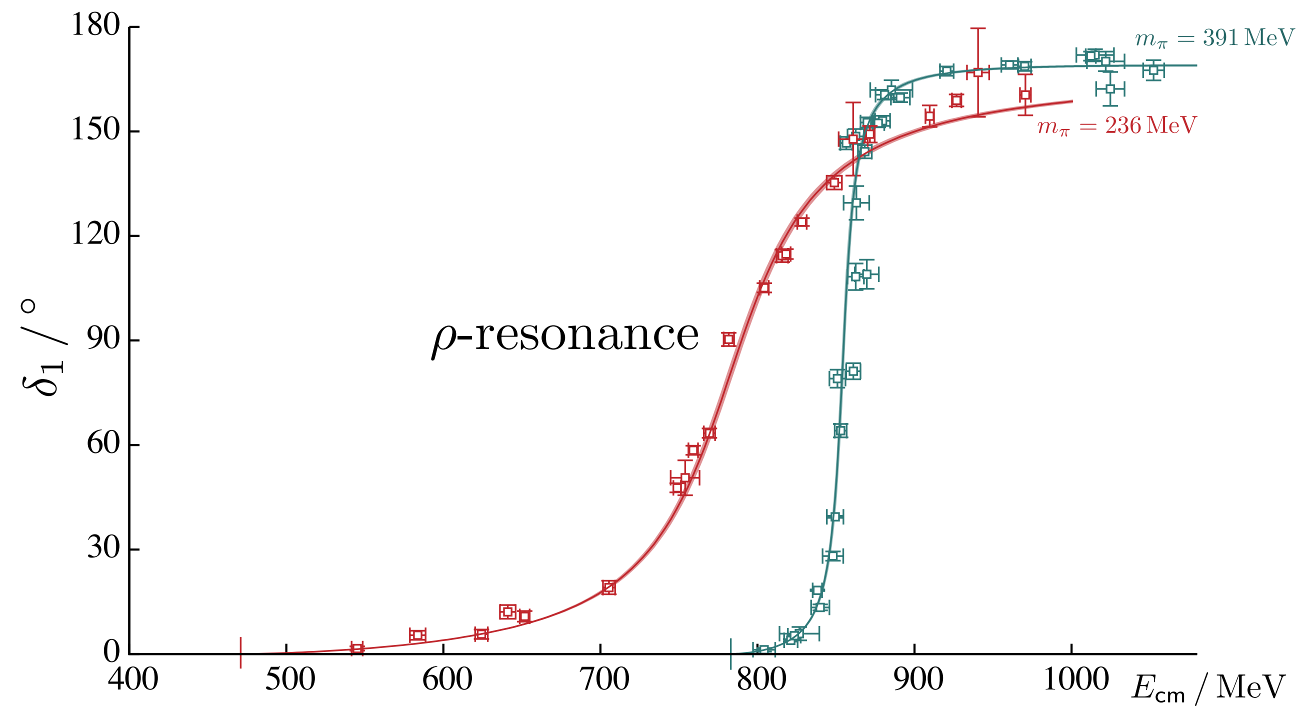


The D-wave features two narrow f_2 resonances, one that couples mostly to ππ and the other to KK.

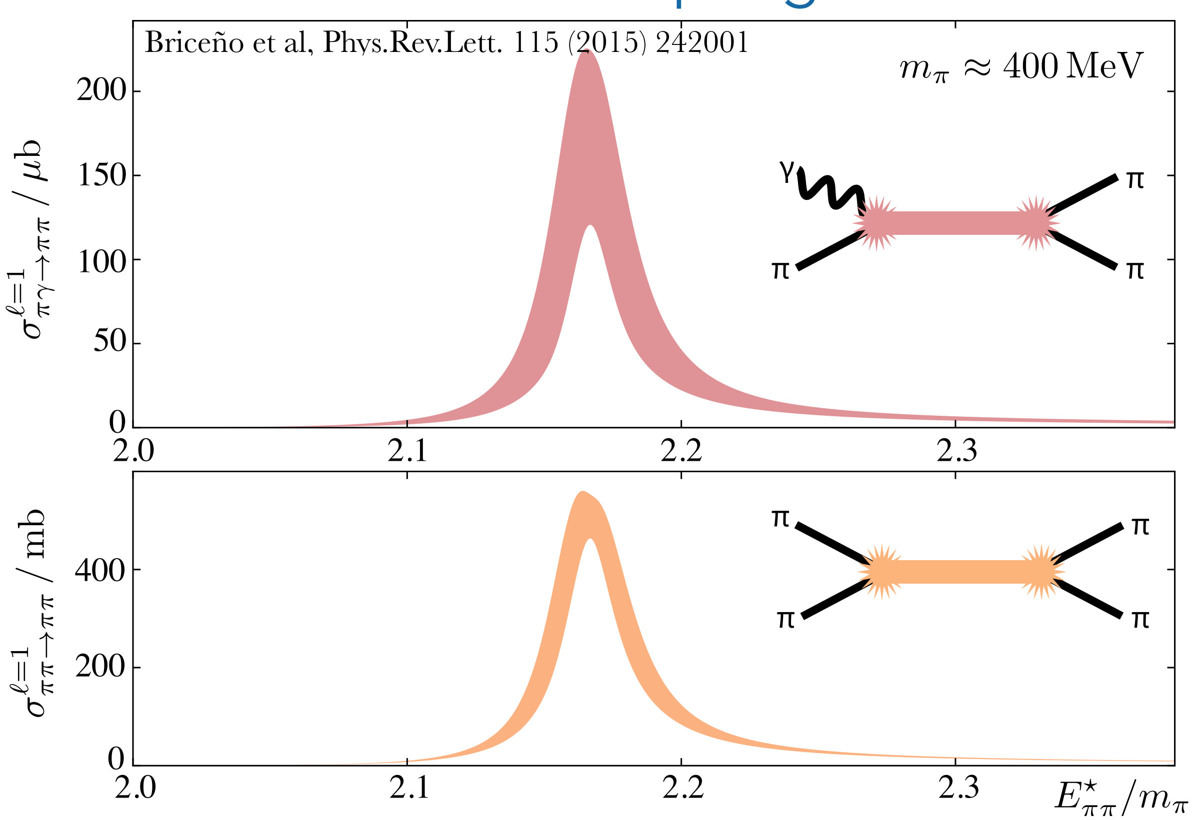
Charm quarks



Lighter masses



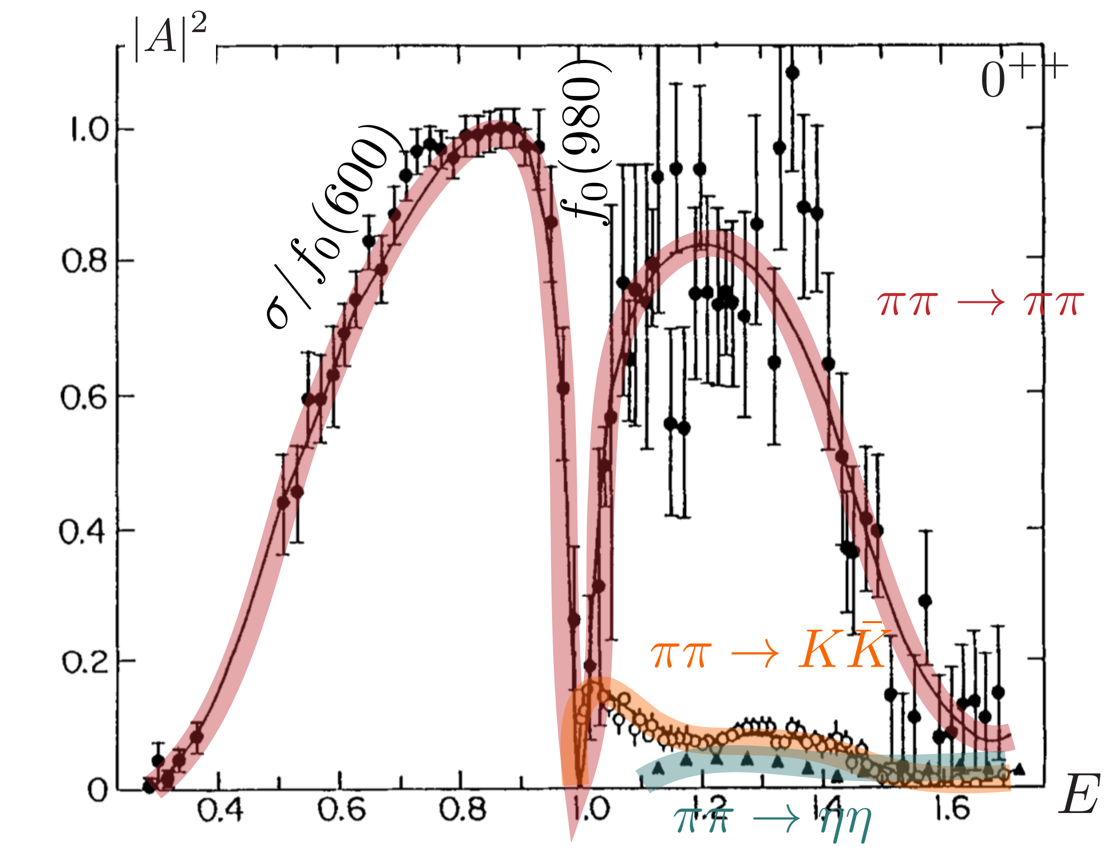
Photocouplings

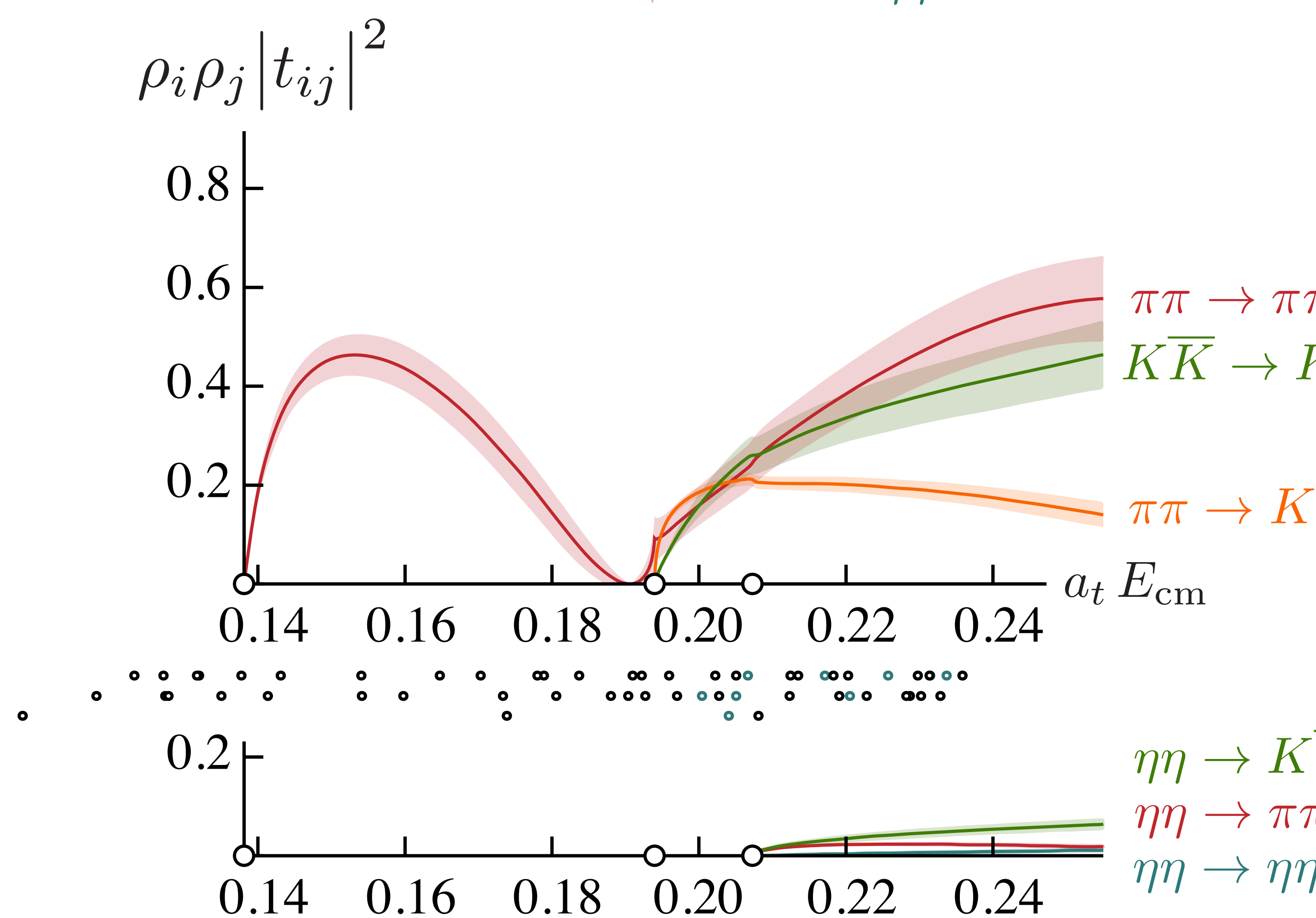


Many exciting possibilities and interesting challenges to overcome

- higher resonances
- charmonium
- many coupled-channels
- three-body formalism

CERN-Munich, ANL, BNL

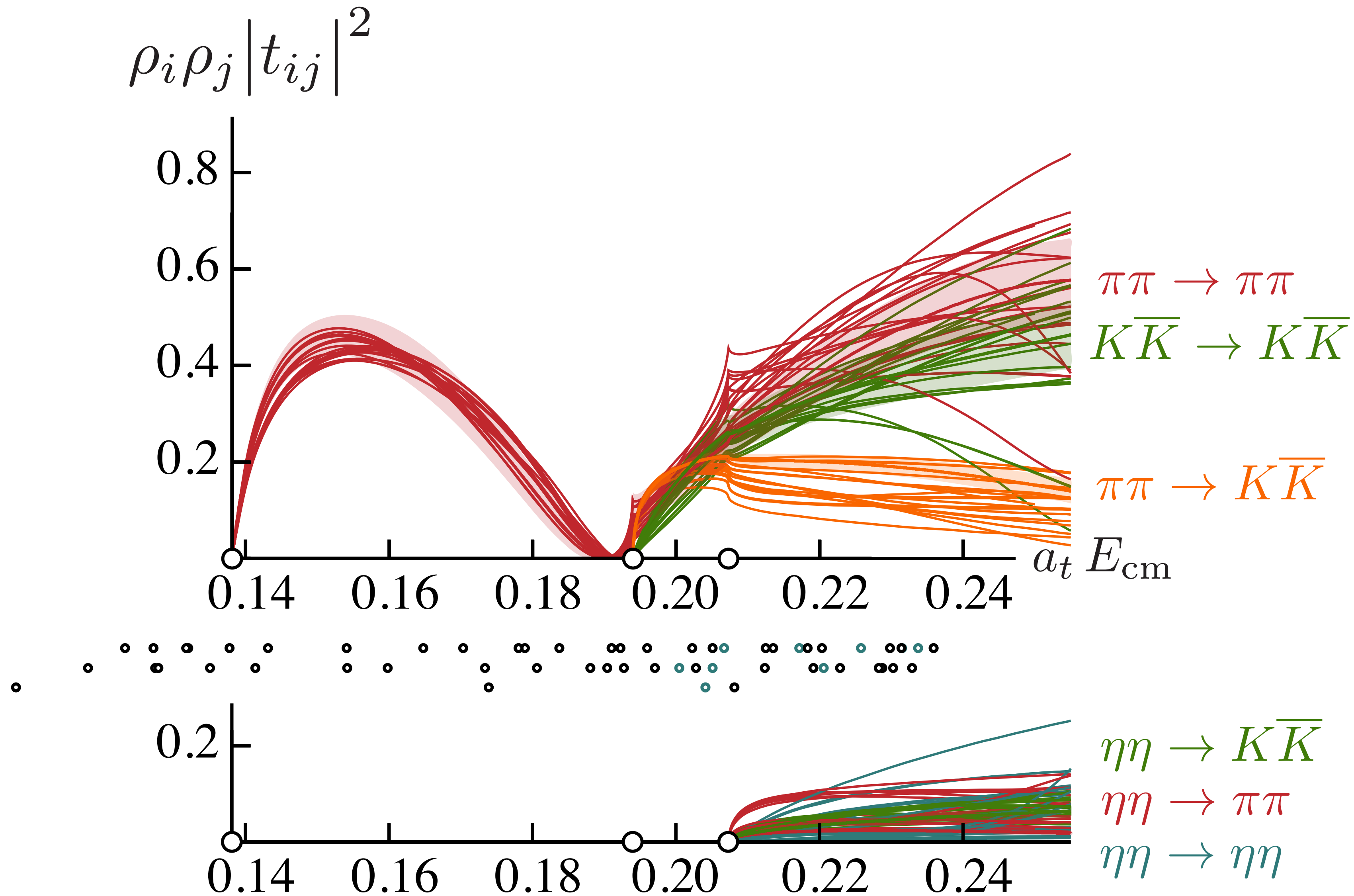




20 amplitudes

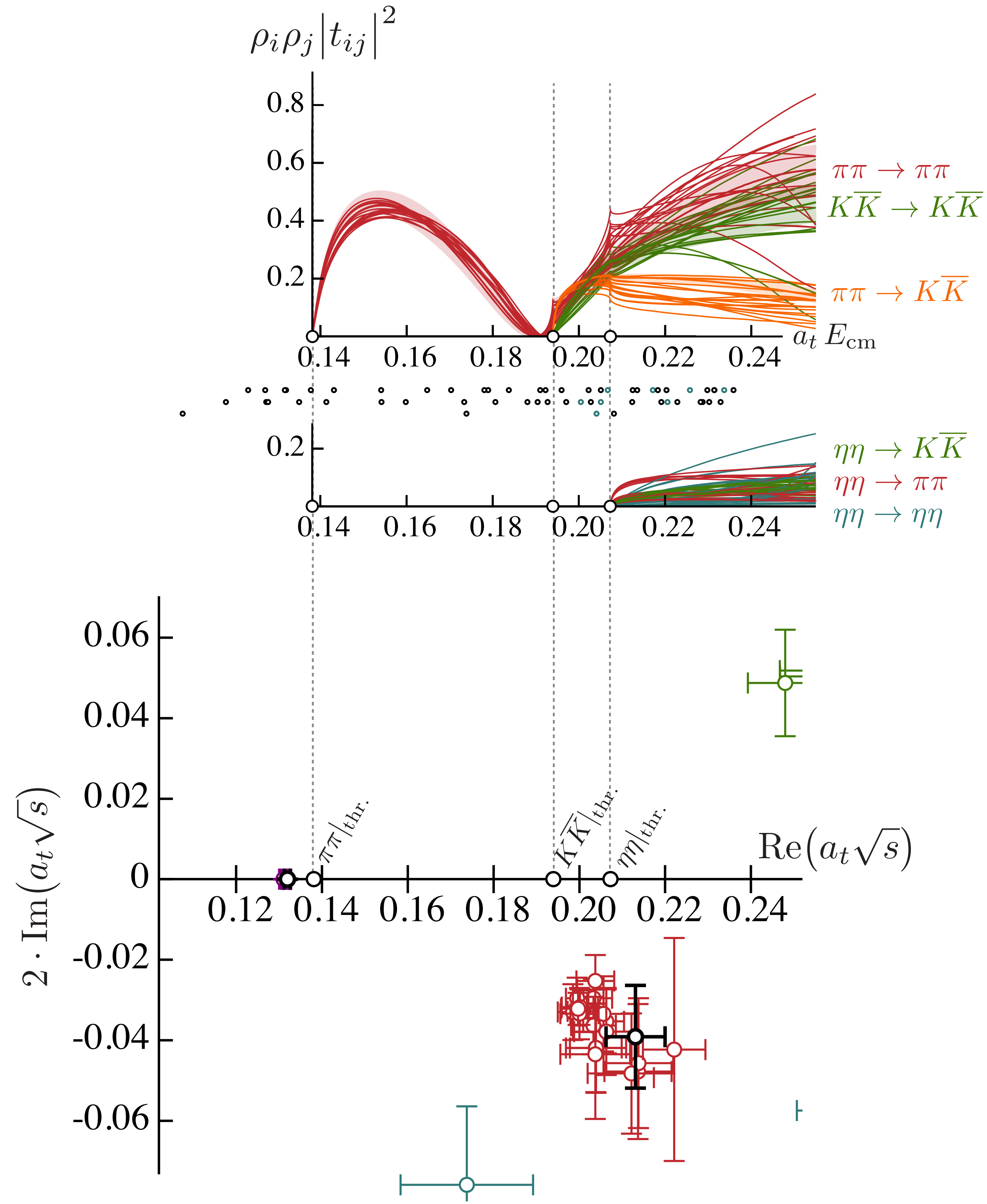
$\chi^2/N_{\mathrm{dof}} < 1.05$

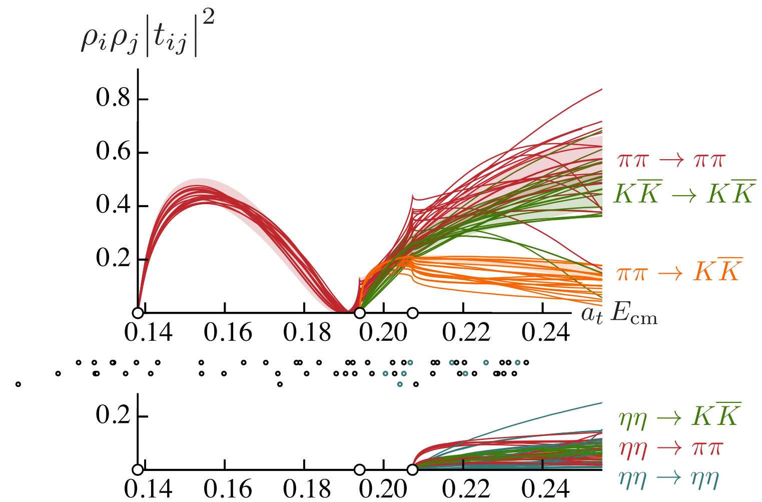
57 energy levels

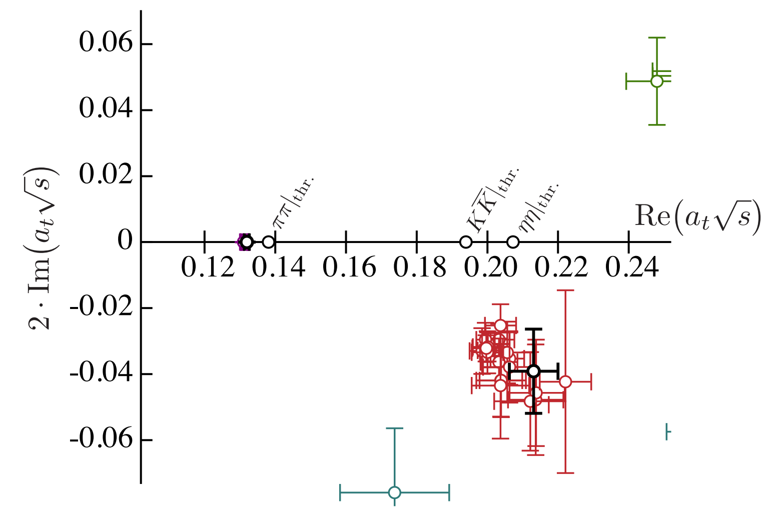


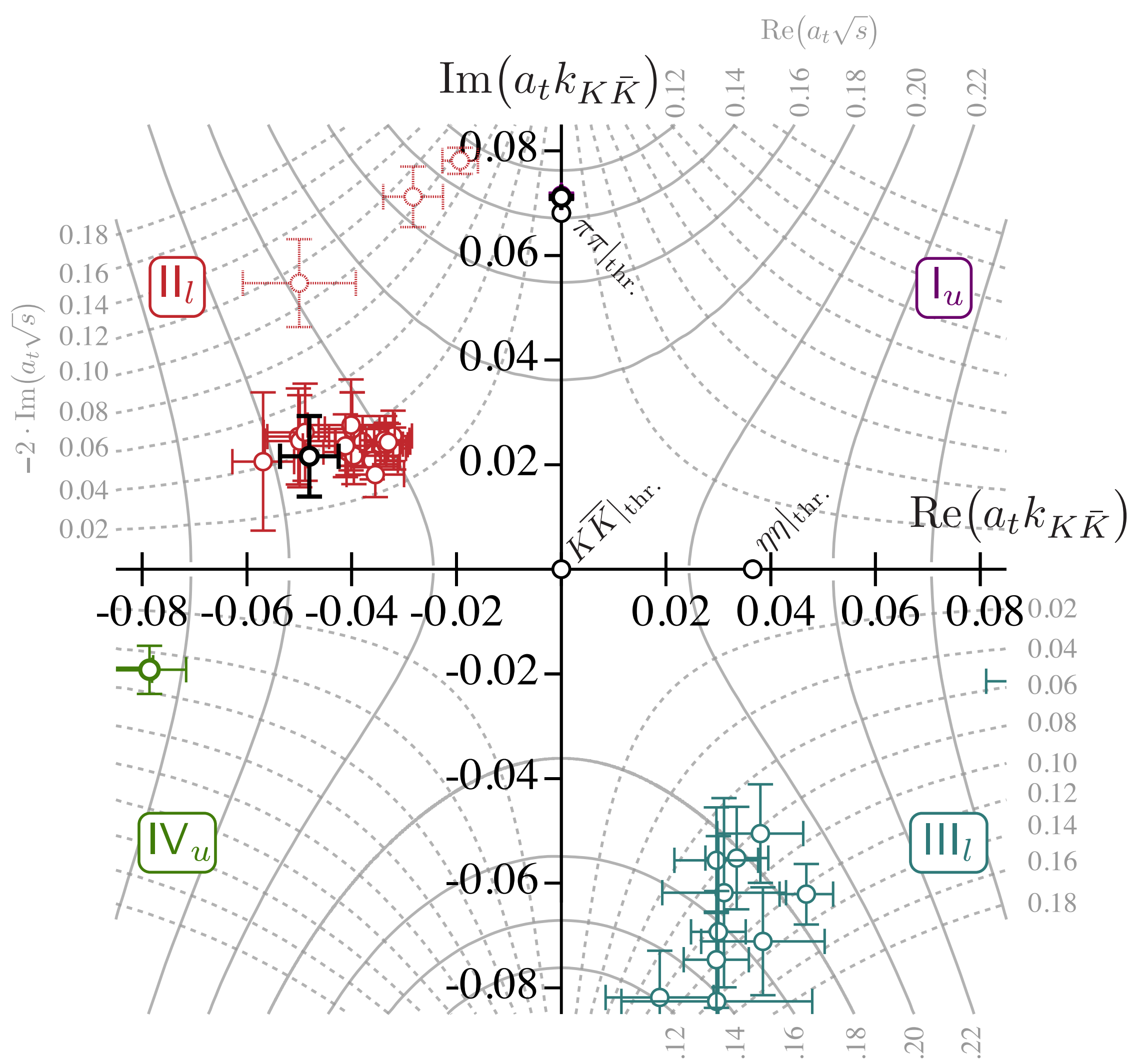
Near a t-matrix pole

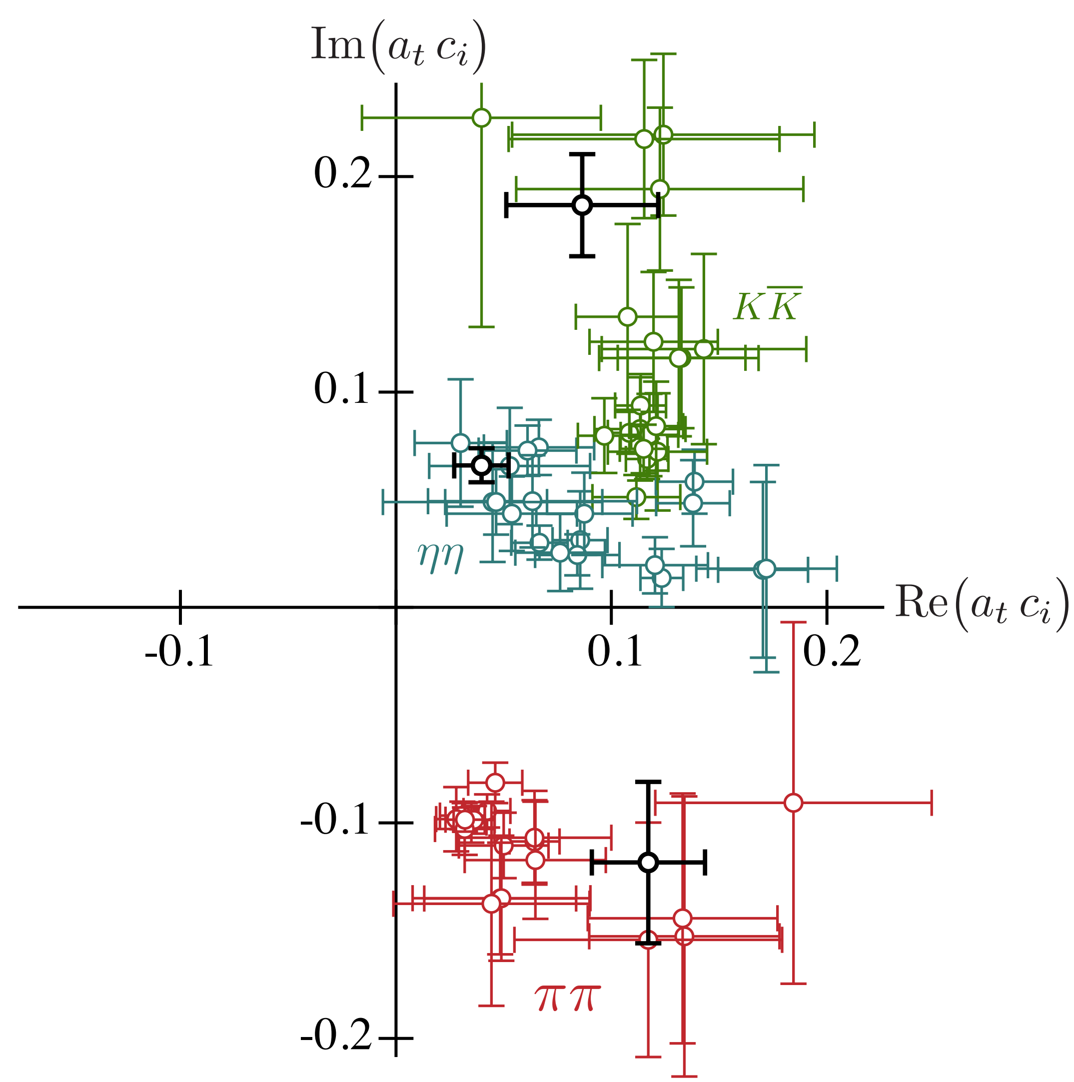
$$t_{ij} \sim \frac{c_i c_j}{s_0 - s}$$

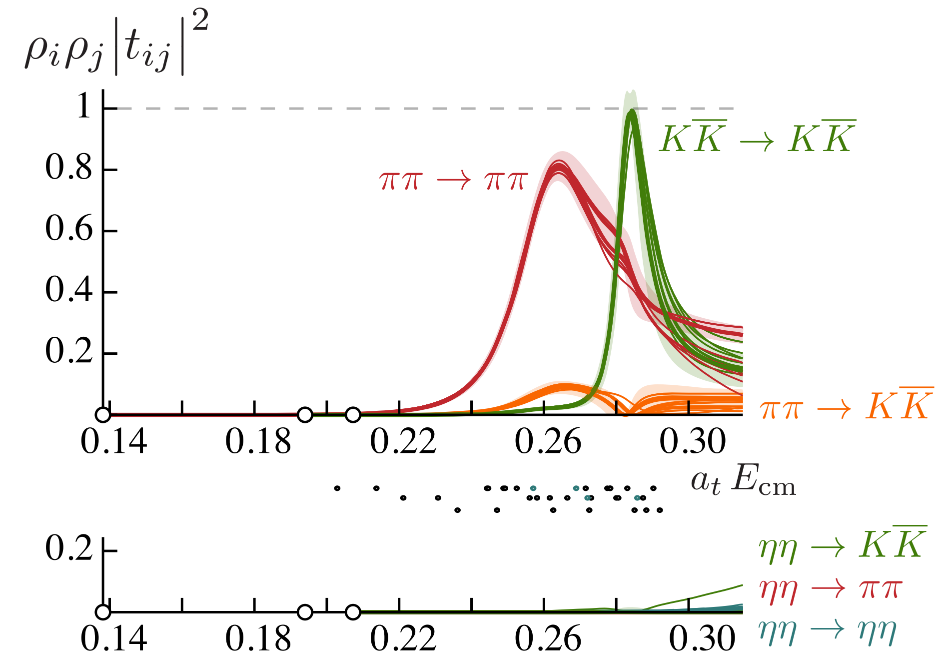



$\rho_i \rho_j |t_{ij}|^2$
$\pi\pi \to \pi\pi$
$K\bar{K} \to K\bar{K}$
$\pi\pi \to K\bar{K}$
$a_t E_{\rm cm}$
$\eta\eta \to K\bar{K}$
$\eta\eta \to \pi\pi$
$\eta\eta \to \eta\eta$


$2 \cdot \mathrm{Im}(a_t\sqrt{s})$
$\mathrm{Re}(a_t\sqrt{s})$
$\pi\pi|_{\rm thr.}$
$K\bar{K}|_{\rm thr.}$
$\eta\eta|_{\rm thr.}$


$\mathrm{Im}(a_t k_{K\bar{K}})$
$\mathrm{Re}(a_t\sqrt{s})$
$-2 \cdot \mathrm{Im}(a_t\sqrt{s})$
$\mathrm{Re}(a_t k_{K\bar{K}})$
II_l
I_u
IV_u
III_l
$\pi\pi|_{\rm thr.}$
$K\bar{K}|_{\rm thr.}$
$\eta\eta|_{\rm thr.}$


$\mathrm{Im}(a_t\, c_i)$
$\mathrm{Re}(a_t\, c_i)$
$K\bar{K}$
$\eta\eta$
$\pi\pi$



Near a t-matrix pole

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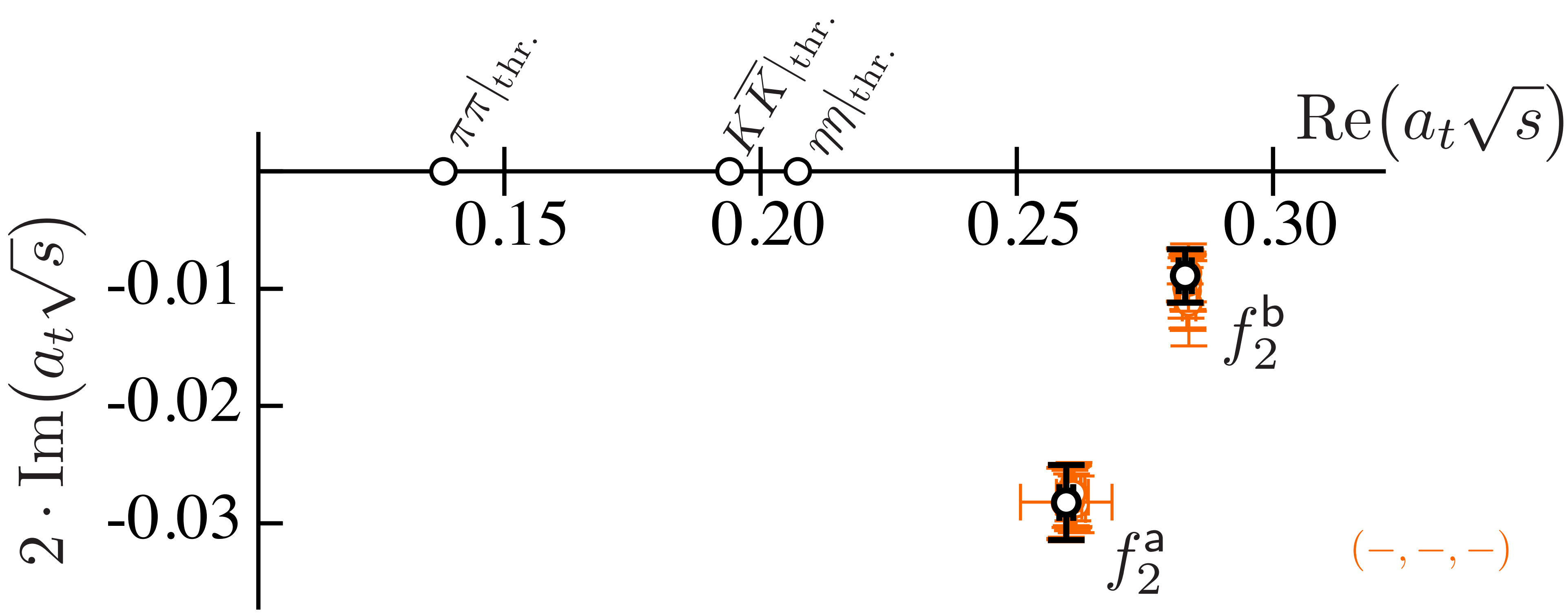
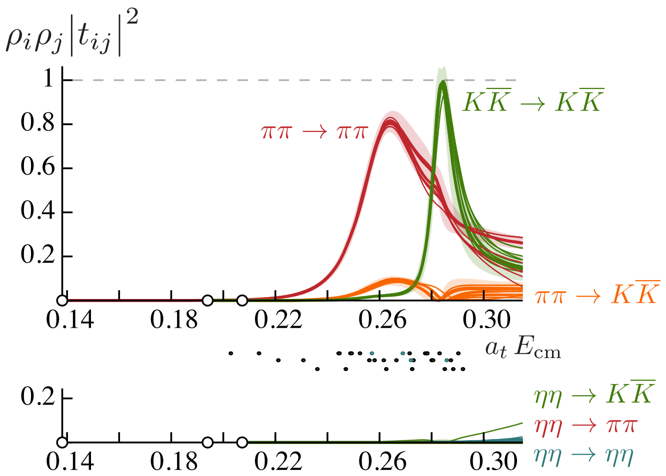
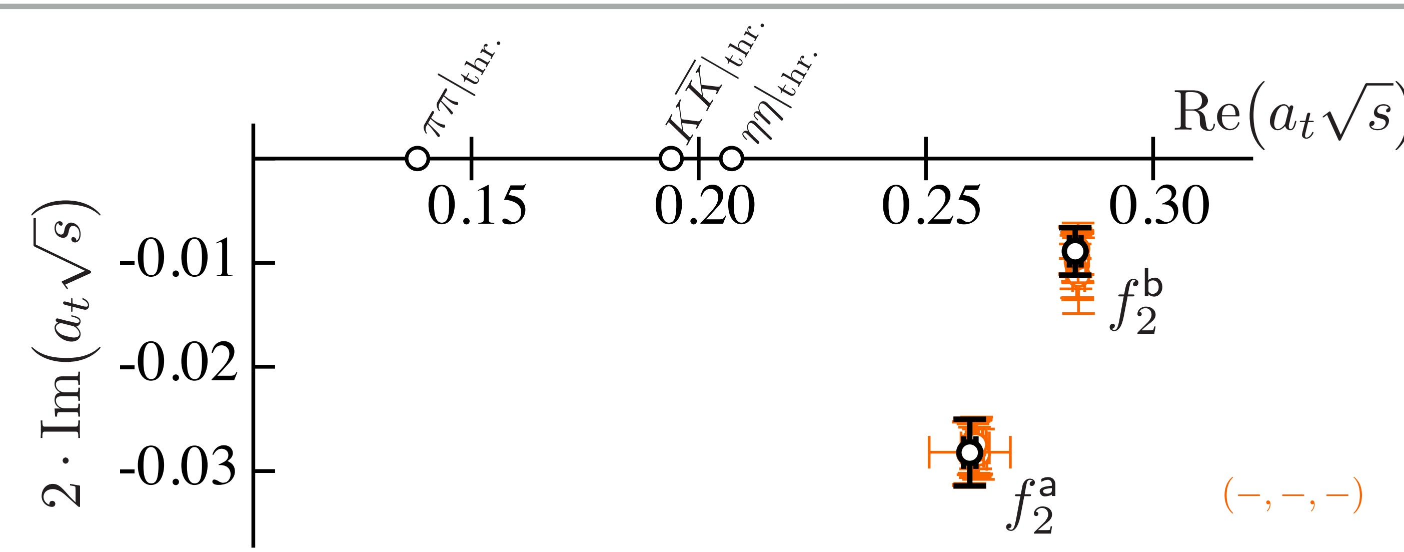
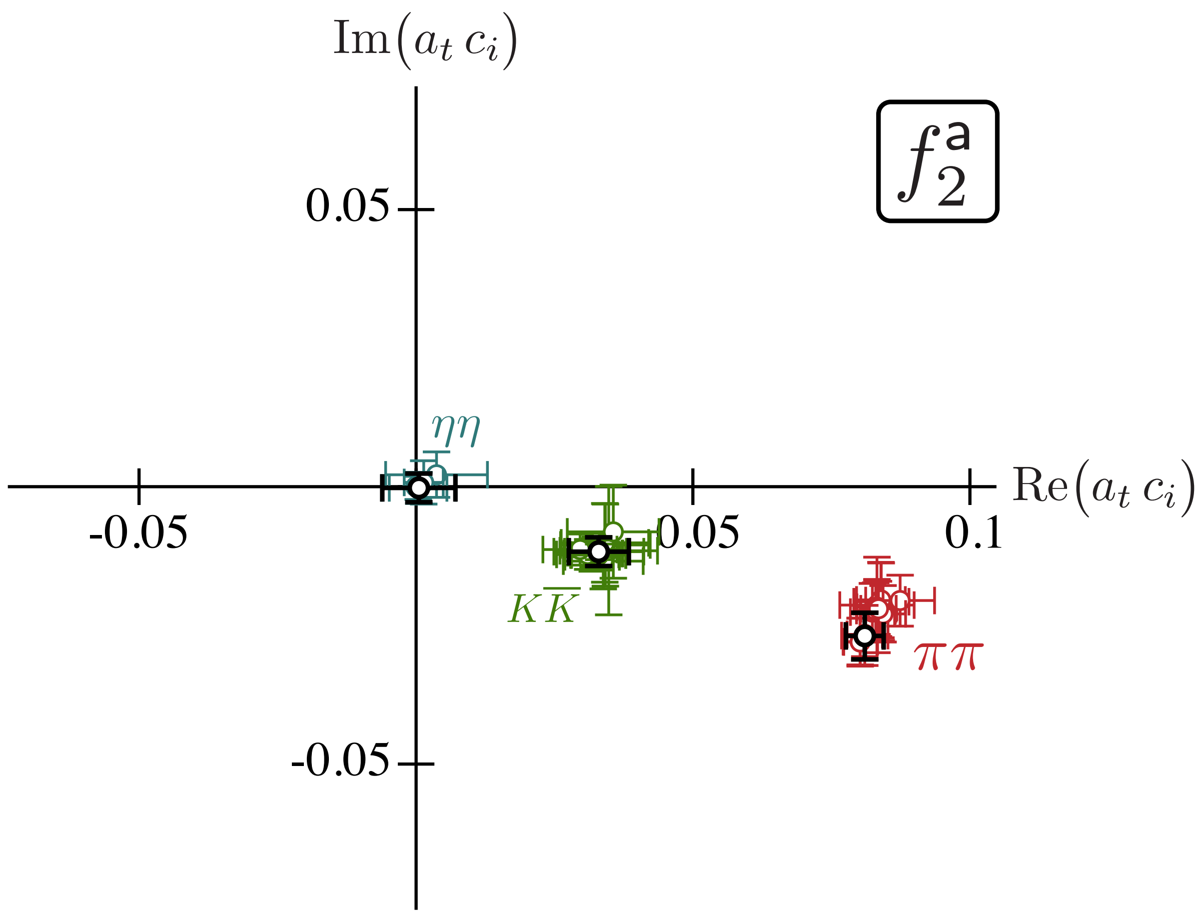
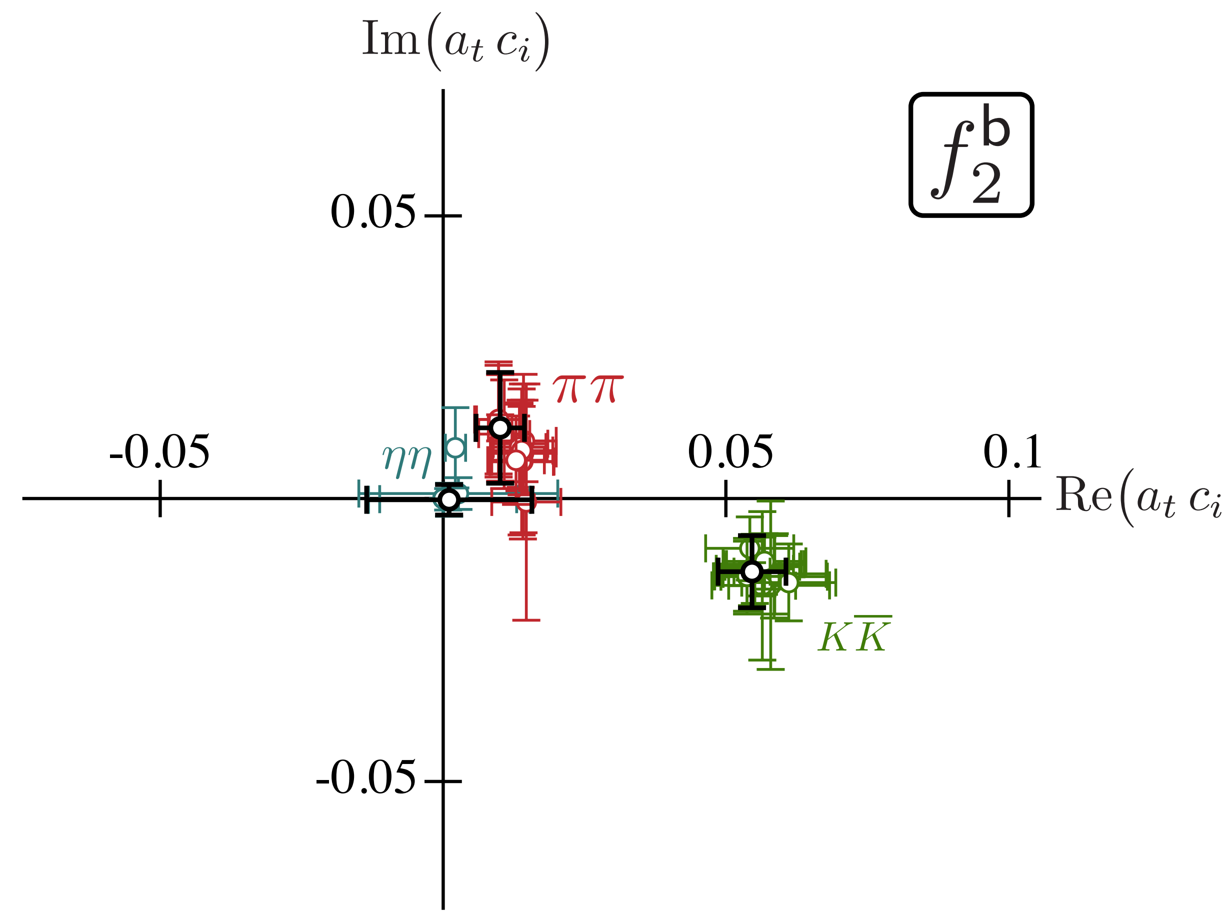










$f_2^{\mathsf{a}}: \quad \sqrt{s_0} = 1470(15) - \tfrac{i}{2}\,160(18)$ MeV

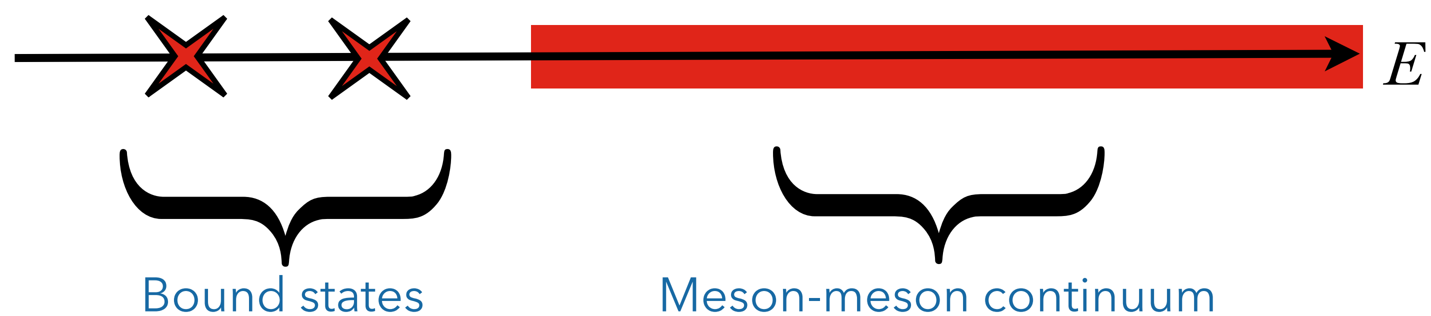
$\mathrm{Br}(f_2^{\mathsf{a}} \to \pi\pi) \sim 85\%, \quad \mathrm{Br}(f_2^{\mathsf{a}} \to K\overline{K}) \sim 12\%$

$f_2^{\mathsf{b}}: \quad \sqrt{s_0} = 1602(10) - \tfrac{i}{2}\,54(14)$ MeV

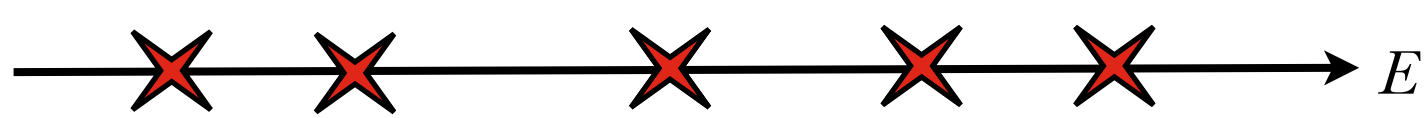
$\mathrm{Br}(f_2^{\mathsf{b}} \to \pi\pi) \sim 8\%, \quad \mathrm{Br}(f_2^{\mathsf{b}} \to K\overline{K}) \sim 92\%$

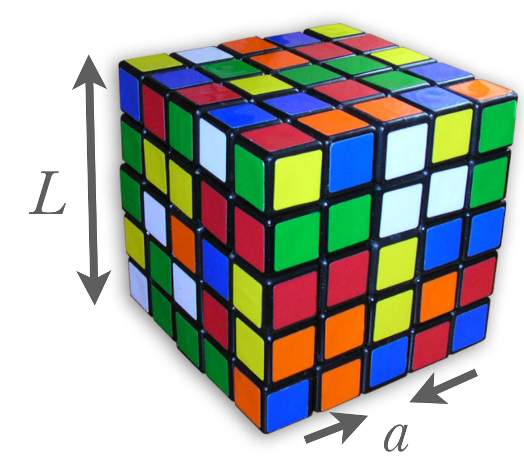
excited states seen as resonant enhancements
in the scattering of lighter stable particles

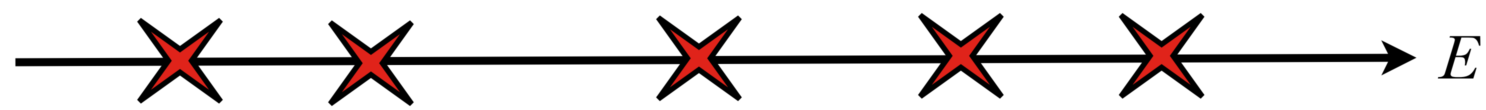
Infinite volume



Finite volume





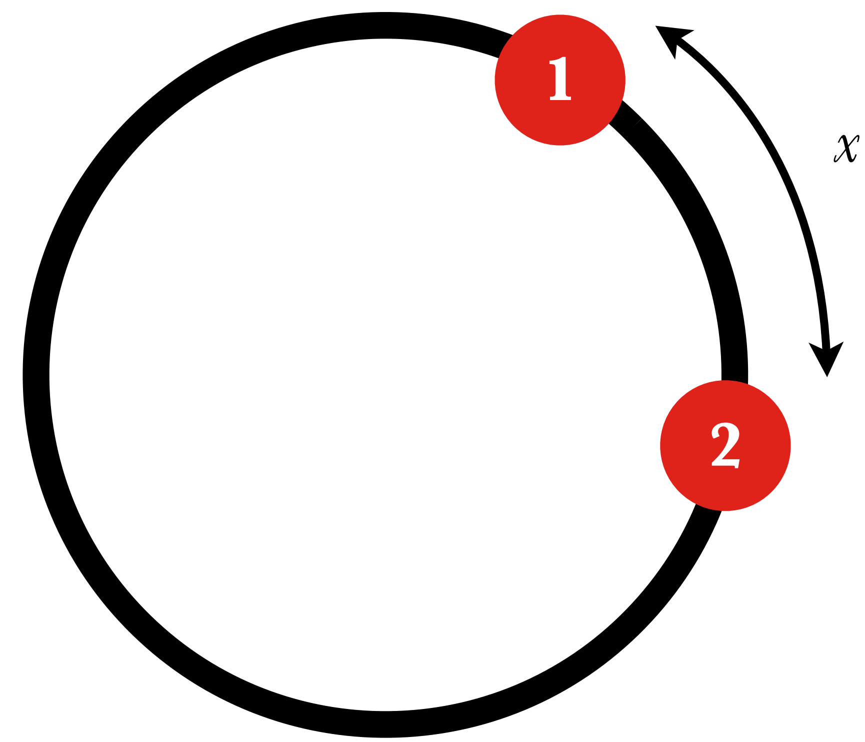


Infinite volume phase shifts from a finite volume

$$\psi(0) = \psi(L), \quad \frac{\partial\psi}{\partial x}\Big|_{x=0} = \frac{\partial\psi}{\partial x}\Big|_{x=L}$$

$$\sin\left(\frac{pL}{2} + \delta(p)\right) = 0$$

$$p = \frac{2\pi n}{L} - \frac{2}{L}\delta(p)$$



Phase shifts via the Lüscher method:

$$\tan\delta_1 = \frac{\pi^{3/2} q}{\mathcal{Z}_{00}(1;q^2)}$$

$$\mathcal{Z}_{00}(1;q^2) = \sum_{n\in\mathbb{Z}^3} \frac{1}{|\vec{n}|^2 - q^2}$$

Multiple channels:

$$\det\left[\mathbf{1} + i\boldsymbol{\rho}(E)\cdot\boldsymbol{t}(E)\cdot\left(\mathbf{1} + i\boldsymbol{\mathcal{M}}(E,L)\right)\right] = 0$$

$$\mathbf{t} = \begin{pmatrix} \pi\pi \to \pi\pi & \pi\pi \to K\bar{K} \\ K\bar{K} \to \pi\pi & K\bar{K} \to K\bar{K} \end{pmatrix}$$

$$\det\left[\mathbf{1} + i\boldsymbol{\rho}(E) \cdot \boldsymbol{t}(E) \cdot (\mathbf{1} + i\boldsymbol{\mathcal{M}}(E, L))\right] = 0$$

determinant condition:
- several unknowns at each value of energy
- energy levels typically do not coincide
- underconstrained problem for a single energy

one solution: use energy dependent parameterizations
- Constrained problem when #(energy levels) > #(parameters)
- Essential amplitudes respect unitarity of the S-matrix

$$\mathbf{S}^\dagger \mathbf{S} = \mathbf{1} \quad \rightarrow \quad \mathrm{Im}\, \mathbf{t}^{-1} = -\boldsymbol{\rho} \qquad \rho_{ij} = \delta_{ij} \frac{2k_i}{E_{\mathsf{cm}}}$$

K-matrix approach:

$$\mathbf{t}^{-1} = \mathbf{K}^{-1} - i\boldsymbol{\rho}$$

Several methods - many challenges similar to experimental analyses

- Weinberg method, uses renormalisation factor Z

  Useful for bound states - distinguishes composite meson-meson vs compact

- Morgan pole counting

  Generalisation of Weinberg - one pole ~ molecular, vs two poles ~ compact

- Photocouplings - determine radial extent

- Decay constants

- N_c dependence

  qq states become stable, meson-meson sink into continuum - Pelaez *et al*

- m_π dependence

  How a near-threshold state reacts to changes in the masses can give clues

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  How a near-threshold state reacts to changes in the masses can give clues

requires significant extra computation

- Weinberg method

Useful for bound states - distinguishes composite meson-meson vs compact

$$a = -2\frac{1-Z}{2-Z}\frac{1}{\sqrt{m_\pi \epsilon}}$$

$$r = -\frac{Z}{1-Z}\frac{1}{\sqrt{m_\pi \epsilon}}$$

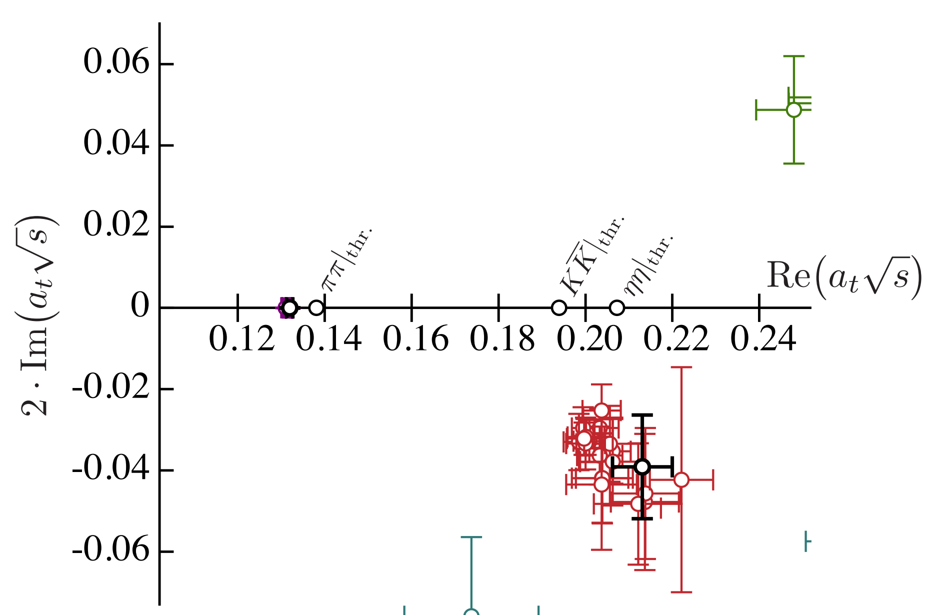
$$Z = 1 \sim \text{compact}$$

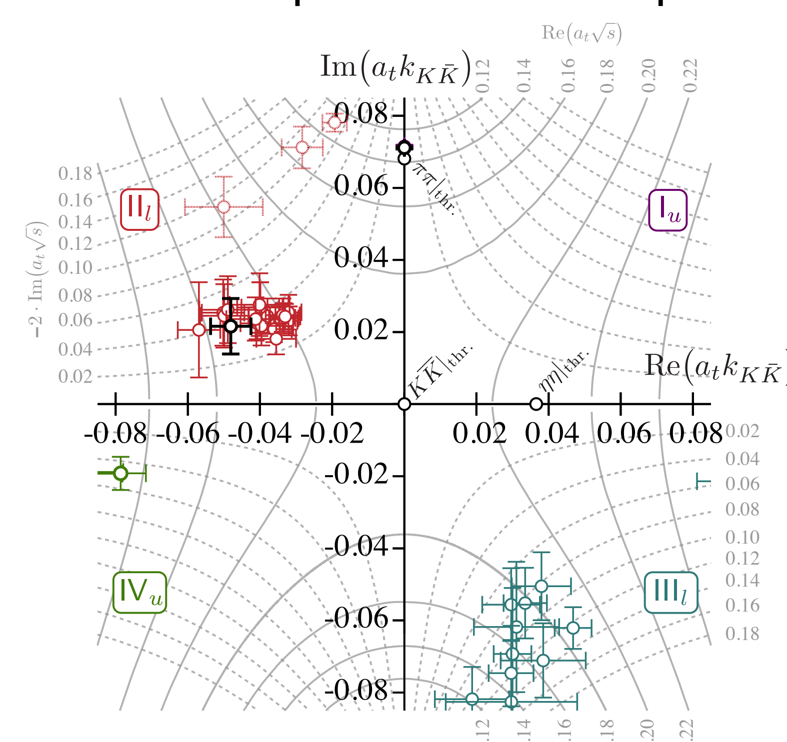
$$Z = 0 \sim \text{molecule}$$

$$Z \sim 0.3(1)$$

- Morgan pole counting

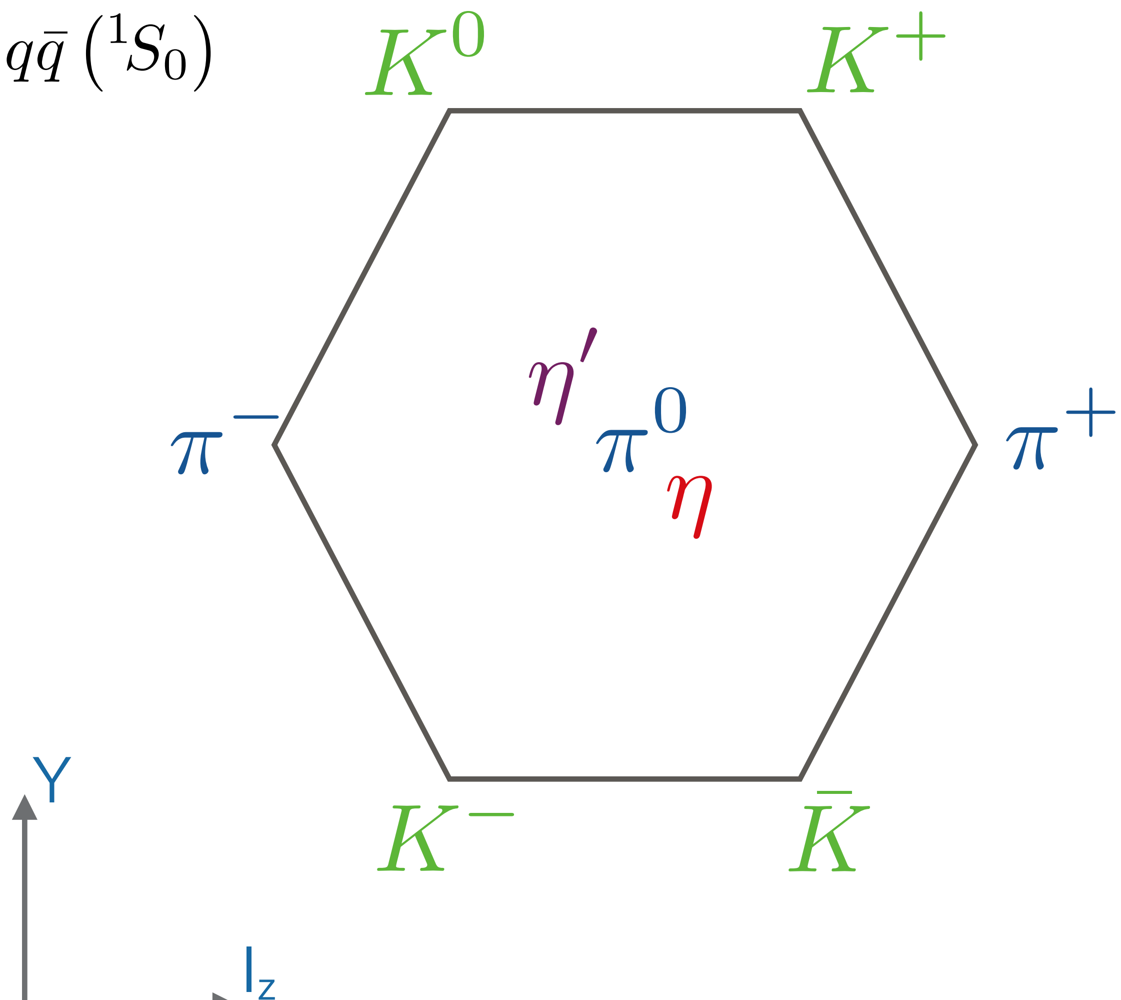
Generalisation of Weinberg - one pole ~ molecular, vs two poles ~ compact





pseudoscalar nonet

$q\bar{q}\ ({}^1S_0)$



vector nonet $q\bar{q}\ ({}^3S_1)$