Rare and CP violating Kaon Decays

Based on work in collaboration with:

Andrzej Buras, Sebastian Jäger & Matthias Jamin [1507.06345]

Maria Cerda-Sevilla, Sebastian Jäger & Ahmet Kokulu [1611.08276]

[And based on older calculations with

Joachim Brod, Emanuel Stamou and Ulrich Haisch]

UK Flavour Meeting

Durham, 3 September 2017

Martin Gorbahn

UNIVERSITY OF LIVERPOOL

Content

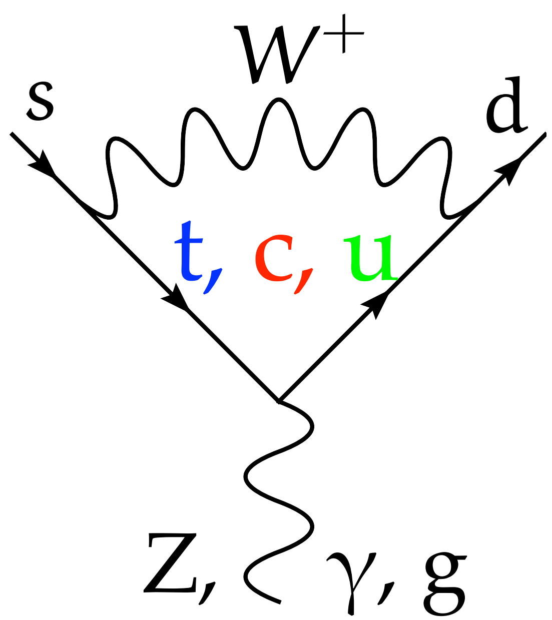
Introduction to rare Kaon decays

$K \to \pi \bar{\upsilon} \upsilon$

$\varepsilon'_K / \varepsilon_K$

For Lattice News on Rare decays, e.g. $K \to \pi l^+ l^-$
$\to$ Talk by A. Portelli

CKM Factors in Kaon physics



Semi-leptonic decays (V_{us}): $\lambda = \mathcal{O}(0.2)$

$$V_{ij} = \mathcal{O}\begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$

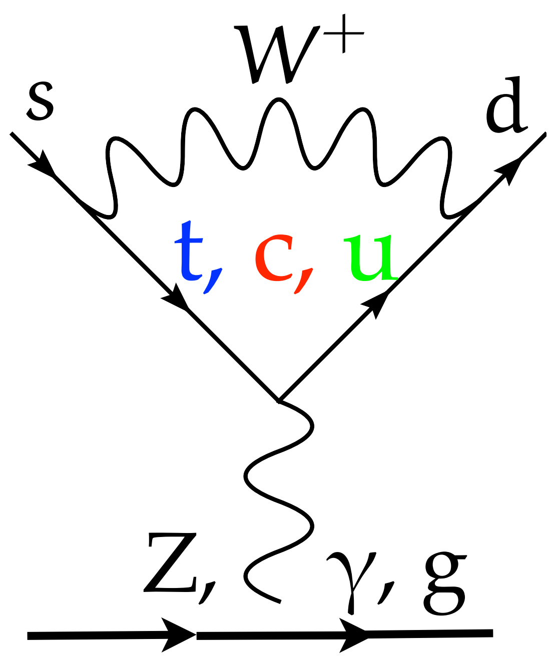
$$\mathrm{Im}V_{ts}^* V_{td} = -\mathrm{Im}V_{cs}^* V_{cd} = \mathcal{O}(\lambda^5) \qquad \mathrm{Im}V_{us}^* V_{ud} = 0$$

$$\mathrm{Re}V_{us}^* V_{ud} = -\mathrm{Re}V_{cs}^* V_{cd} = \mathcal{O}(\lambda^1) \qquad \mathrm{Re}V_{ts}^* V_{td} = \mathcal{O}(\lambda^5)$$

Kaon observables $\propto V_{ts}^* V_{td} \rightarrow$ suppressed in SM sensitive to flavour violating NP

Kaon observables $\propto V_{us}^* V_{ud}$ or $V_{cs}^* V_{cd} \rightarrow$ dominated by QCD, useful for extracting low energy constants

CKM Factors in Kaon physics

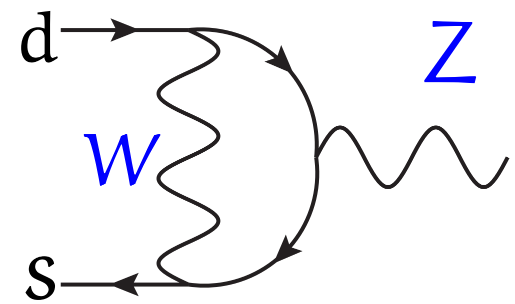
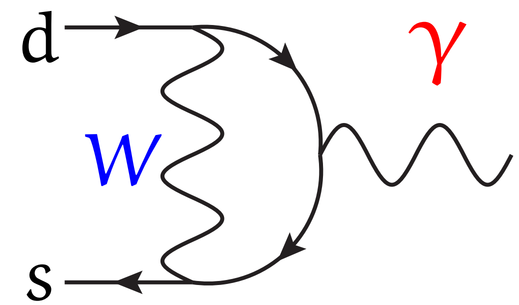
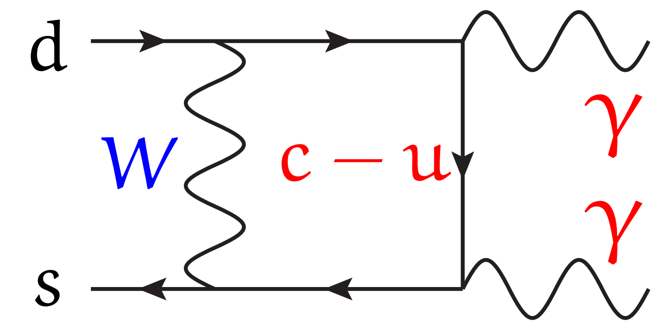


Using the GIM mechanism,
we can eliminate either $V_{cs}^* V_{cd}$ or
$V_{us}^* V_{ud} \rightarrow - V_{cs}^* V_{cd} - V_{ts}^* V_{td}$

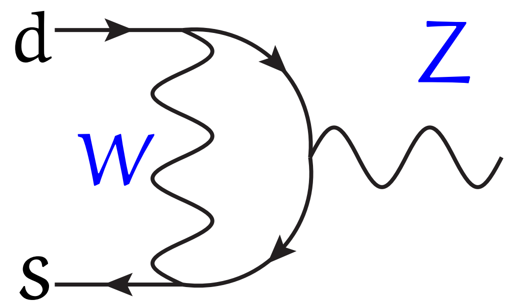
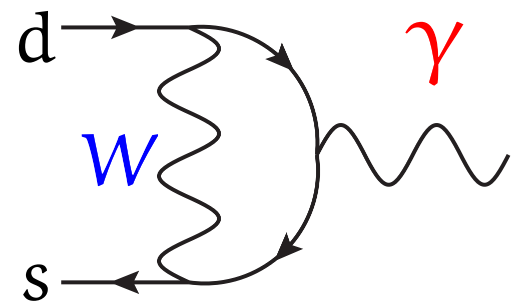
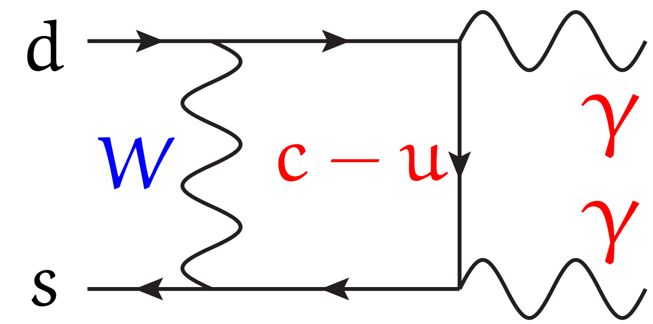
Z-Penguin and Boxes (high virtuality):
power expansion in: $A_c - A_u \propto 0 + O(m_c^2/M_W^2)$

γ/g-Penguin (momentum expansion + e.o.m.):
power expansion in: $A_c - A_u \propto O(\mathrm{Log}(m_c^2/m_u^2))$

Rare Kaon Decays

			
$K_L \to \mu^+\mu^-$	SD	–	α_e LD

Rare Kaon Decays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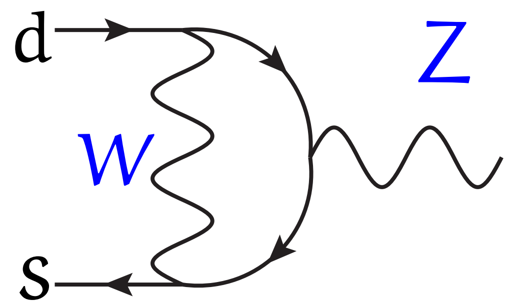
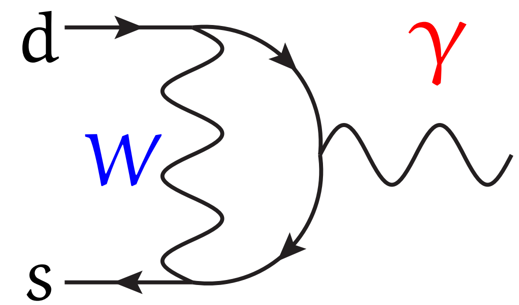
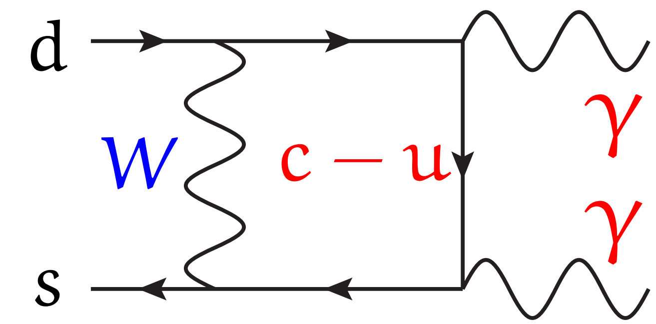
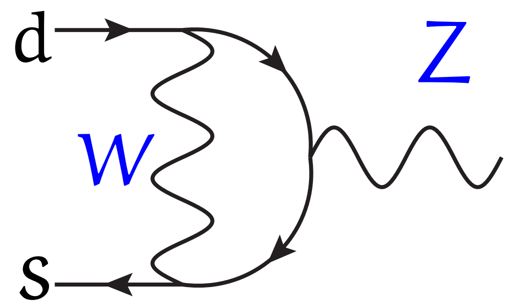
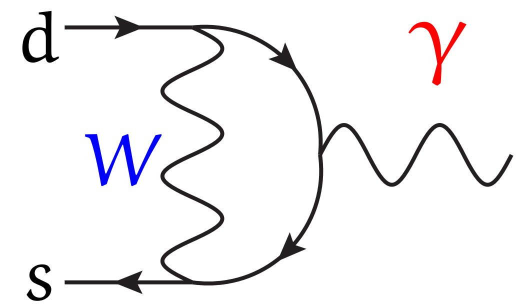
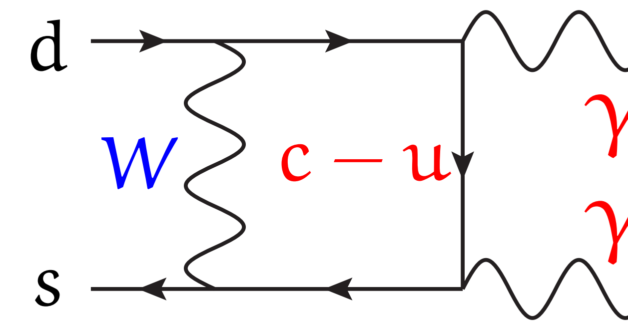
$K \to \pi\nu\bar{\nu}$	SD	–	–

Rare Kaon Decays

			
$K_L \to \mu^+\mu^-$	SD	–	α_e LD
$K \to \pi\nu\bar{\nu}$	SD	–	–
$K_S \to \pi l^+l^-$	–	LD	–

Rare Kaon Decays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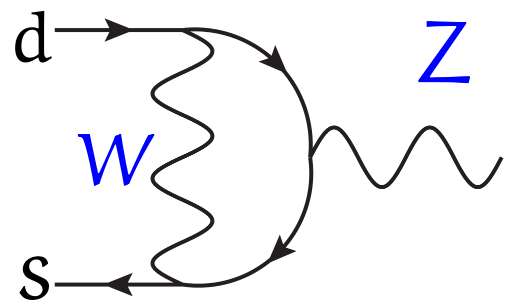
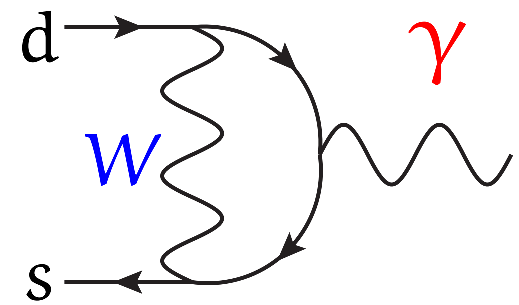
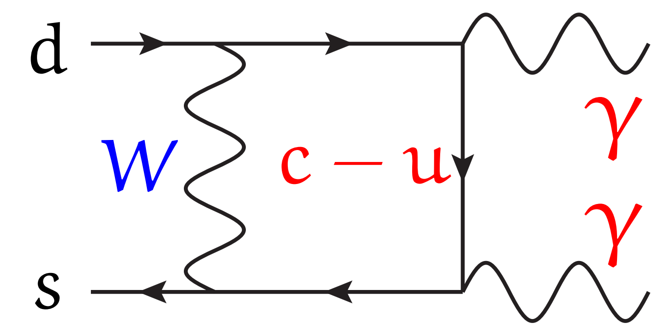
Rare Kaon Decays

			
$K_L \to \mu^+\mu^-$	SD	–	α_e LD
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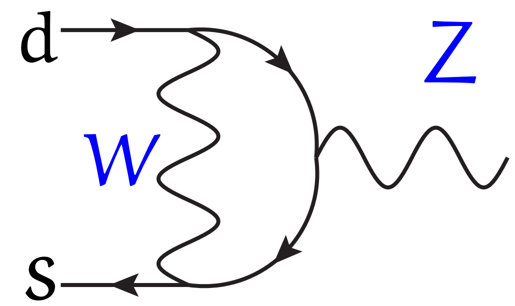
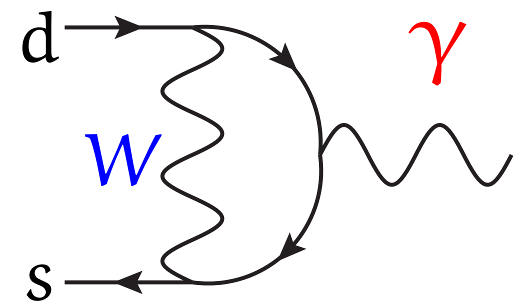
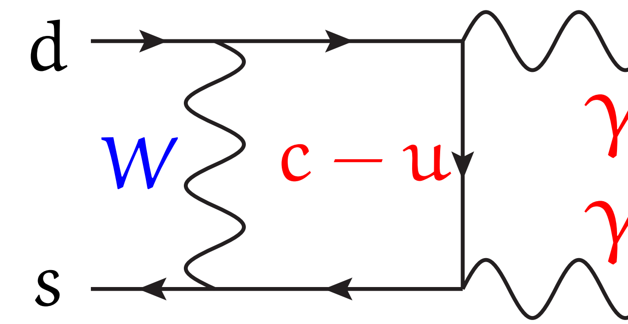
$\cancel{K_S \to \pi l^+ l^-}$	~~–~~	~~LD~~	~~–~~
$K_L \to \pi l^+ l^-$	SD	SD $+\epsilon_K$ LD	α_e LD

CP violating

Rare Kaon Decays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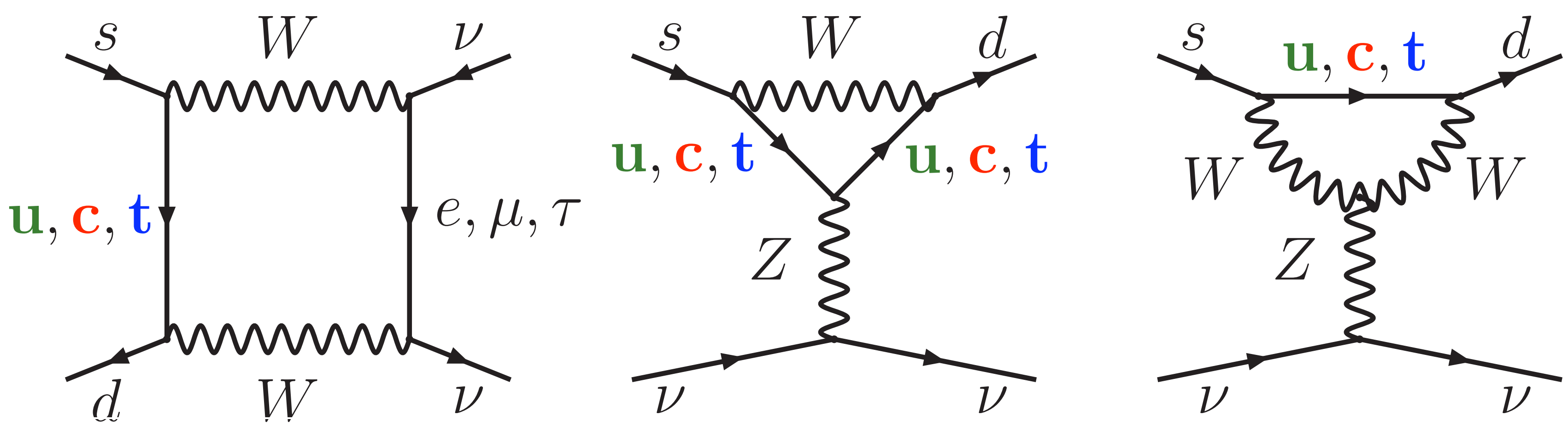
$K_L \to \pi l^+ l^-$	SD	SD $+\epsilon_K$ LD	α_e LD
CP violating	most NNLO QCD known	χPT, Large N [1603.09721 + Ref] Now: Accessible to Lattice Talk by Portelli	

$K \to \pi \bar{\nu} \nu$



$$x_i = \frac{m_i^2}{M_W^2}$$

$$\sum_i V_{is}^* V_{id} F(x_i) = V_{ts}^* V_{td} (F(x_t) - F(x_u)) + V_{cs}^* V_{cd} (F(x_c) - F(x_u))$$

Top (SD),

Charm (Renormalisation Group Improved) &

Light Quarks (Non-Perturbative)

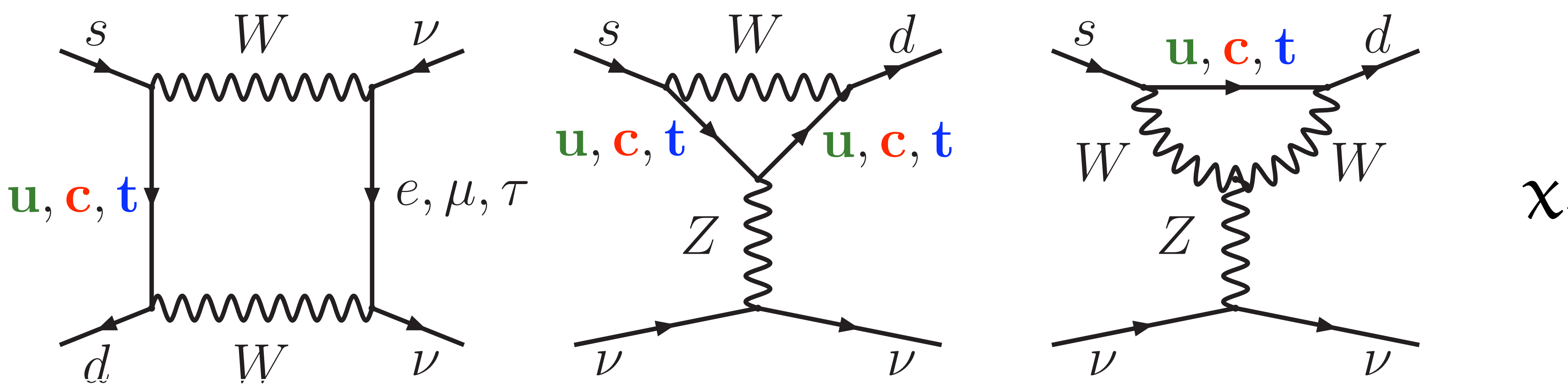
$$\boxed{\lambda^5 \frac{m_t^2}{M_W^2}}$$

$$\boxed{\lambda \frac{m_c^2}{M_W^2} \ln \frac{M_W}{m_c}}$$

$$\boxed{\lambda \frac{\Lambda_{QCD}^2}{M_W^2}}$$

$$Q_\nu = (\bar{s}_L \gamma_\mu d_L)(\bar{\nu}_L \gamma^\mu \nu_L)$$

$K \to \pi \bar{\nu} \nu$



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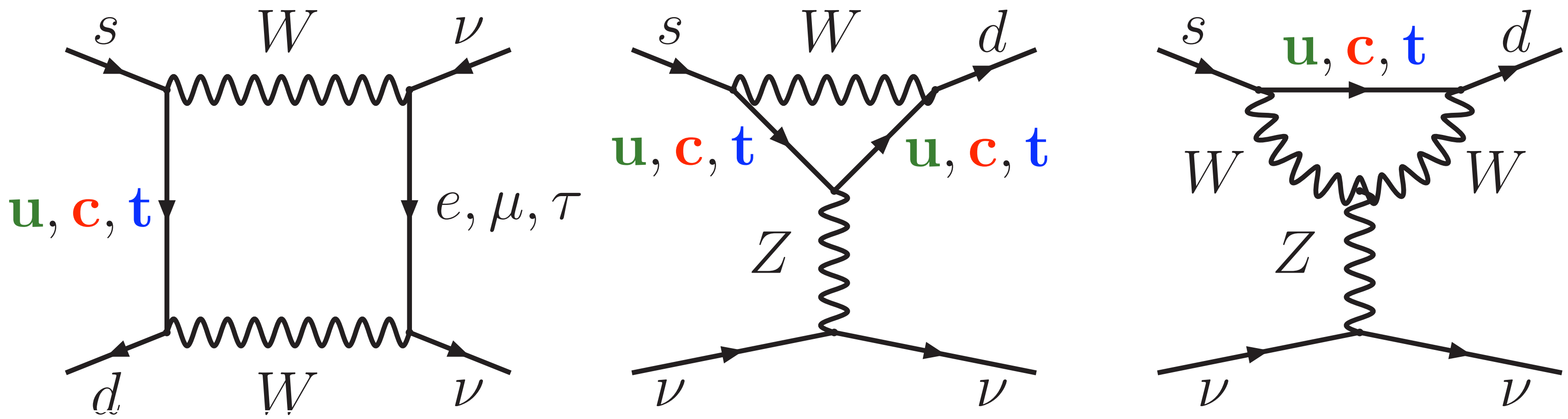
Top quark contribution



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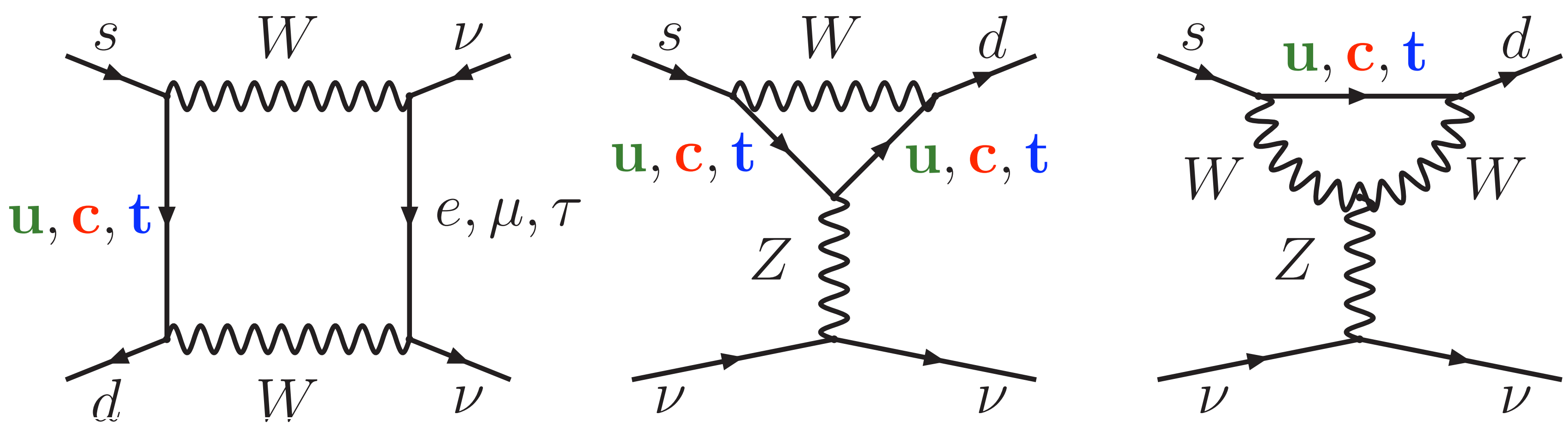

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Quadratic GIM: $\boxed{\lambda^5 \frac{m_t^2}{M_W^2}}$

Matching (NLO +EW):

[Misiak, Urban; Buras, Buchalla;
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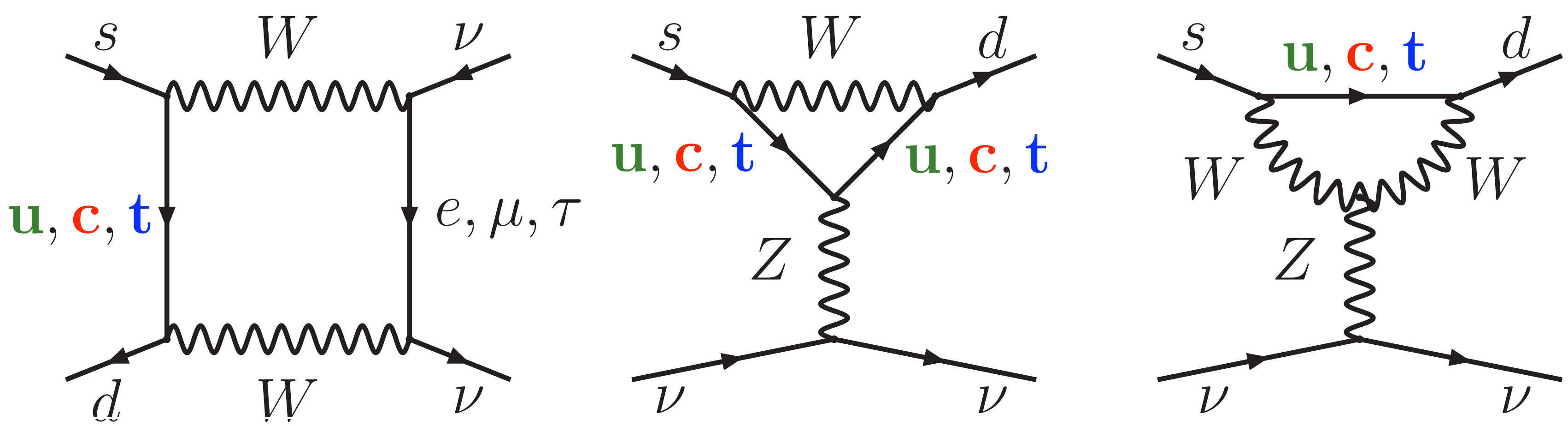

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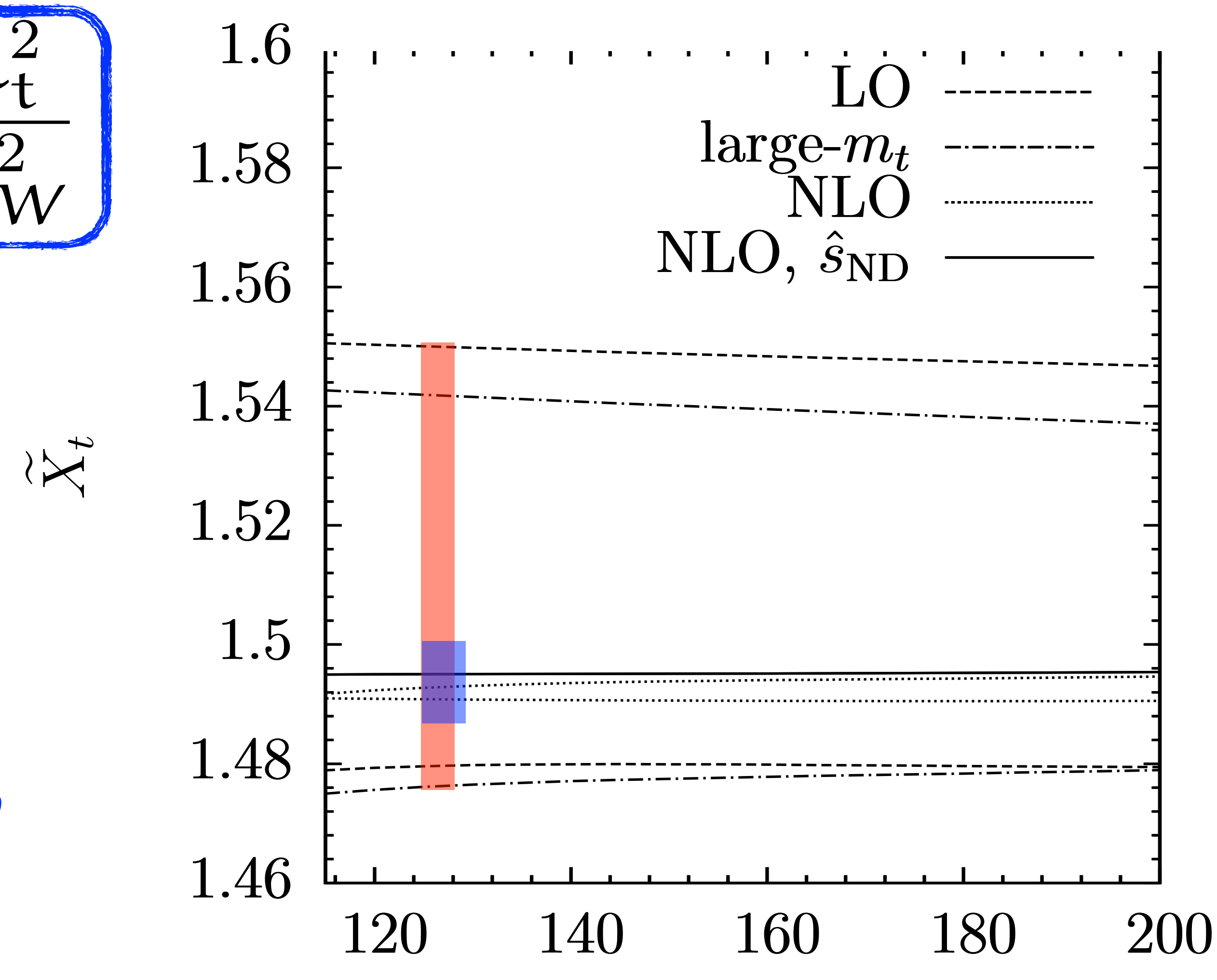
After 2011 uncertainty at 1%



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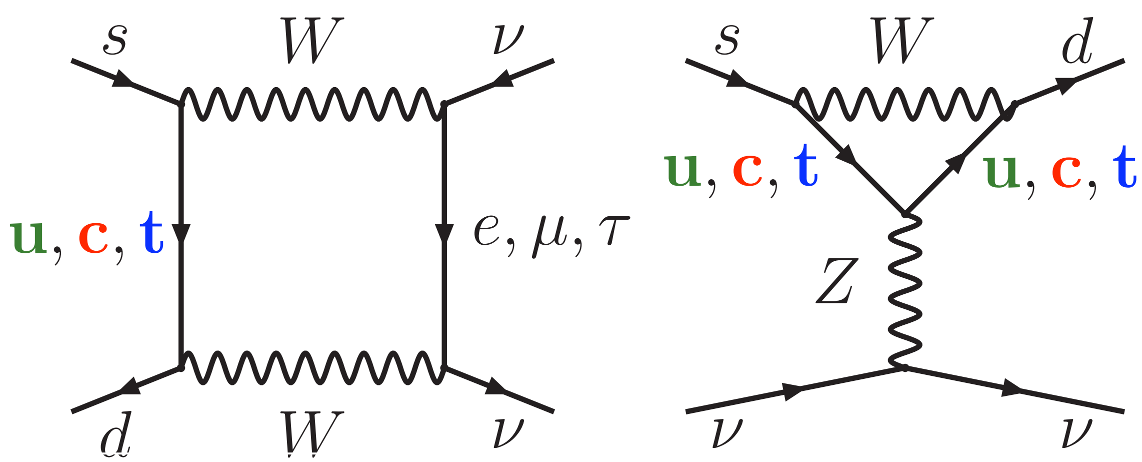

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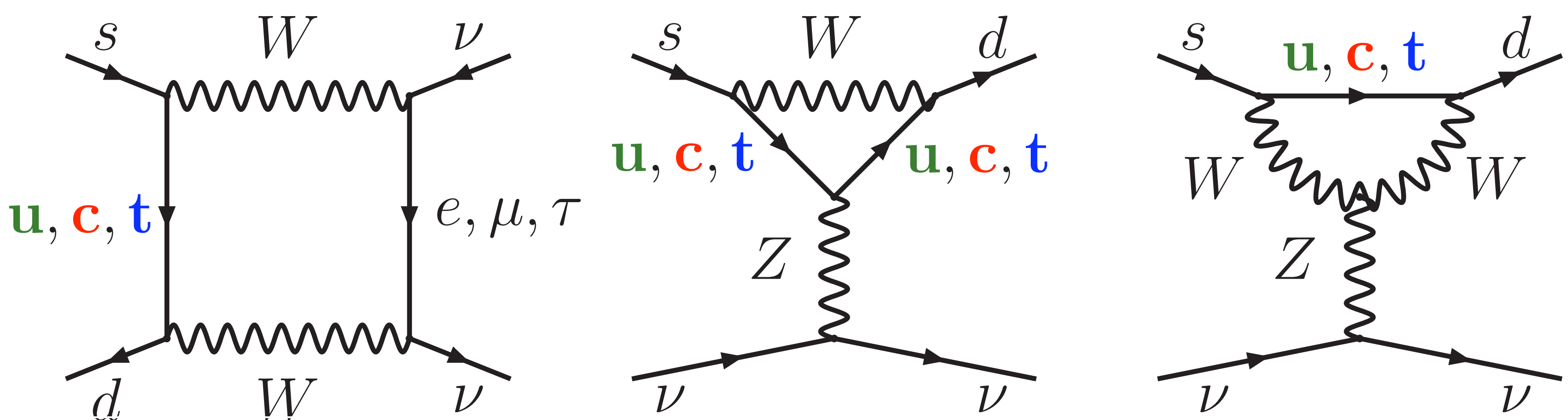
For CP violating $K_L \to \pi^0 \bar{\upsilon}\, \upsilon$ only top contribution relevant.

Clean theory and CKM suppression: NP sensitivity

$K^+ \to \pi^+ \bar{\upsilon}\, \upsilon$ at M_W



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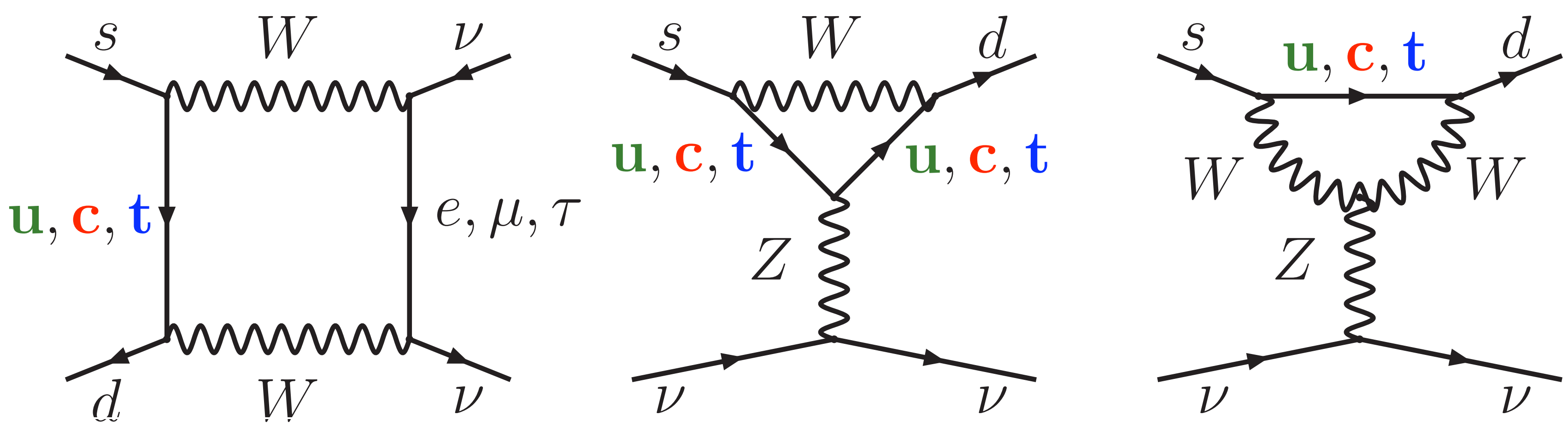

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Operator
Mixing (RGE)

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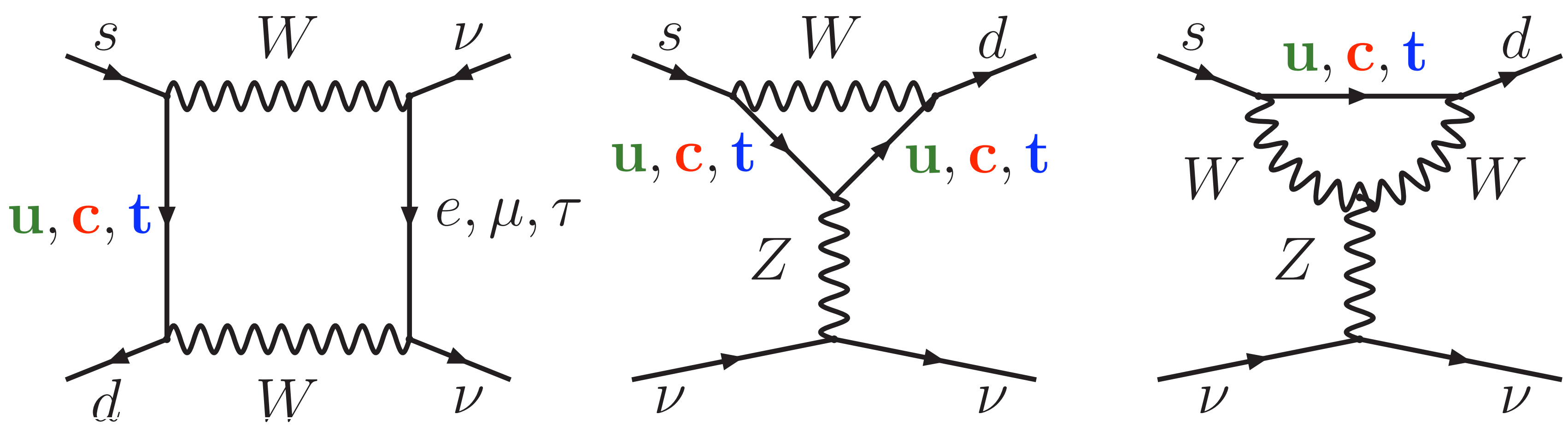

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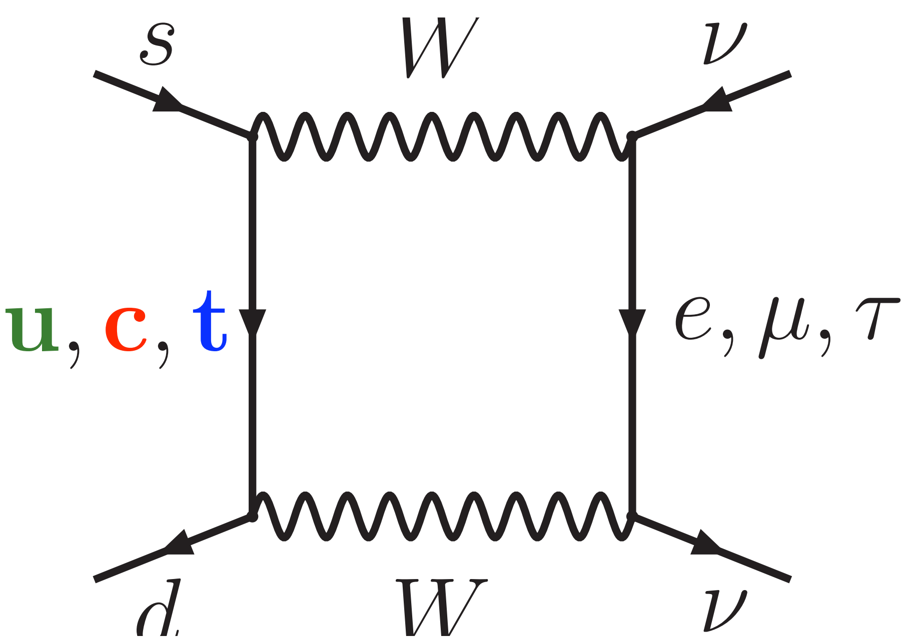
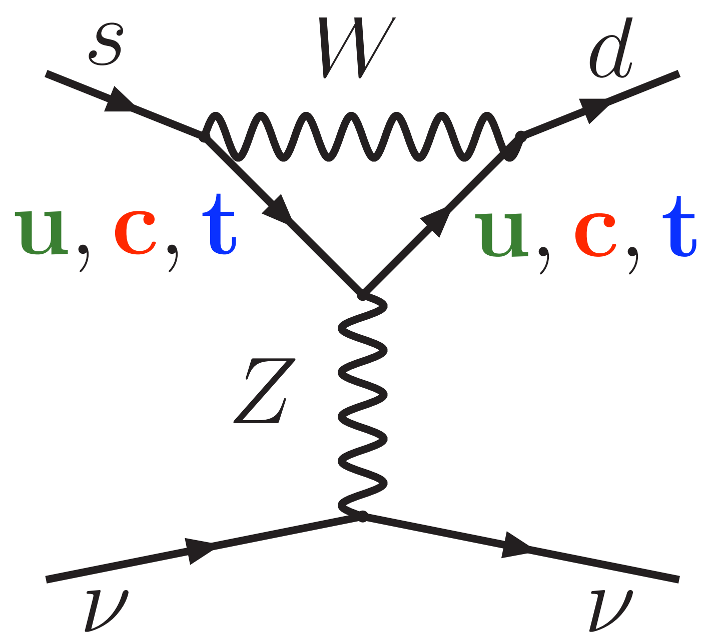
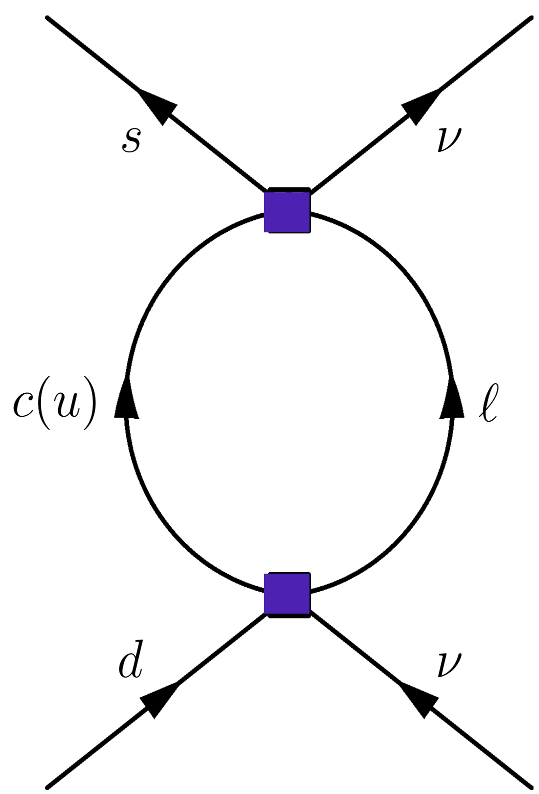
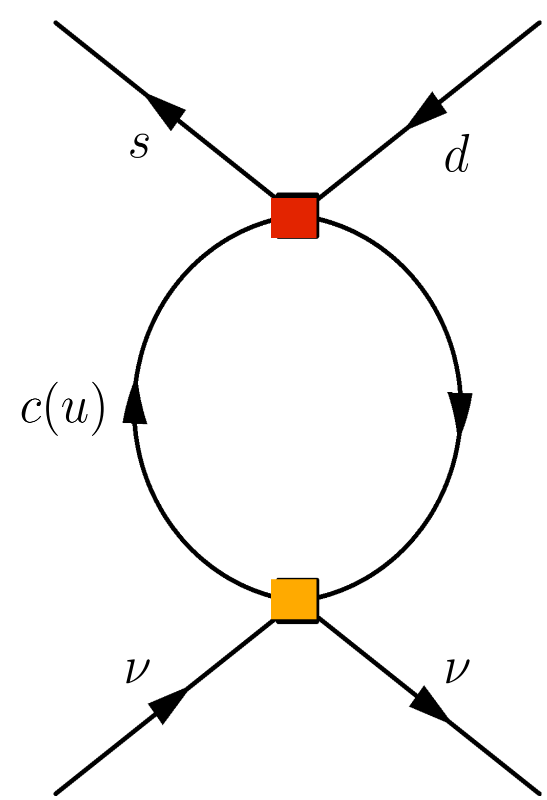
$K^+ \rightarrow \pi^+ \bar{\upsilon}\, \upsilon$ charm contribution









$$\bar{s}_i \gamma^\mu (1-\gamma_5) q_j \;\; \bar{q}_j \gamma_\mu (1-\gamma_5) d_i$$

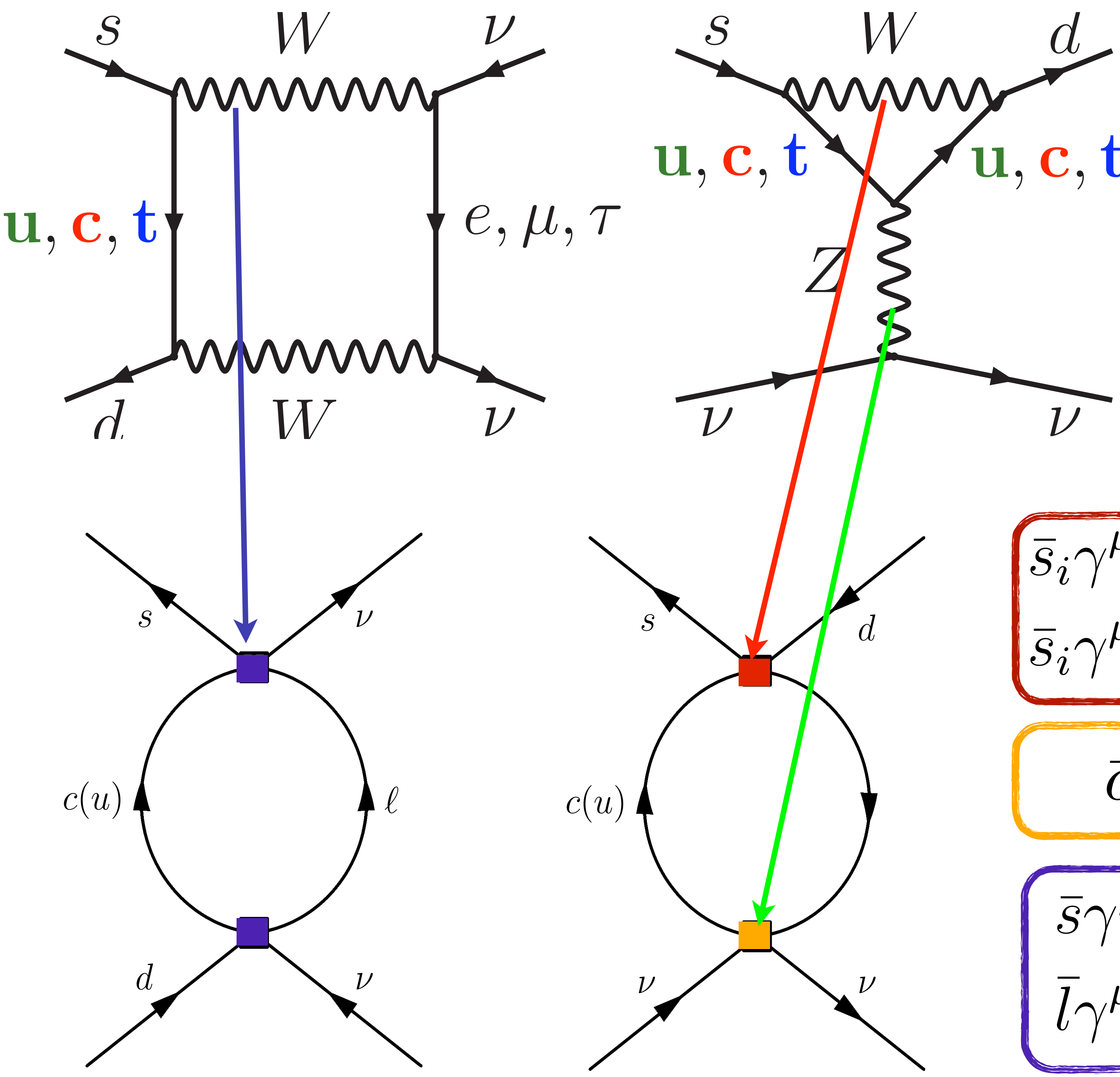
$$\bar{s}_i \gamma^\mu (1-\gamma_5) q_i \;\; \bar{q}_j \gamma_\mu (1-\gamma_5) d_j$$

$$\bar{c}\gamma^\mu \gamma_5 c \;\; \bar{\nu}\gamma^\mu (1-\gamma_5)\nu$$

$$\bar{s}\gamma^\mu (1-\gamma_5) q \;\bar{\nu}_l \gamma_\mu (1-\gamma_5) l$$

$$\bar{l}\gamma^\mu (1-\gamma_5)\nu_l \;\bar{q}\gamma_\mu (1-\gamma_5) d$$

$K^+ \to \pi^+ \bar{\upsilon}\, \upsilon$ charm contribution



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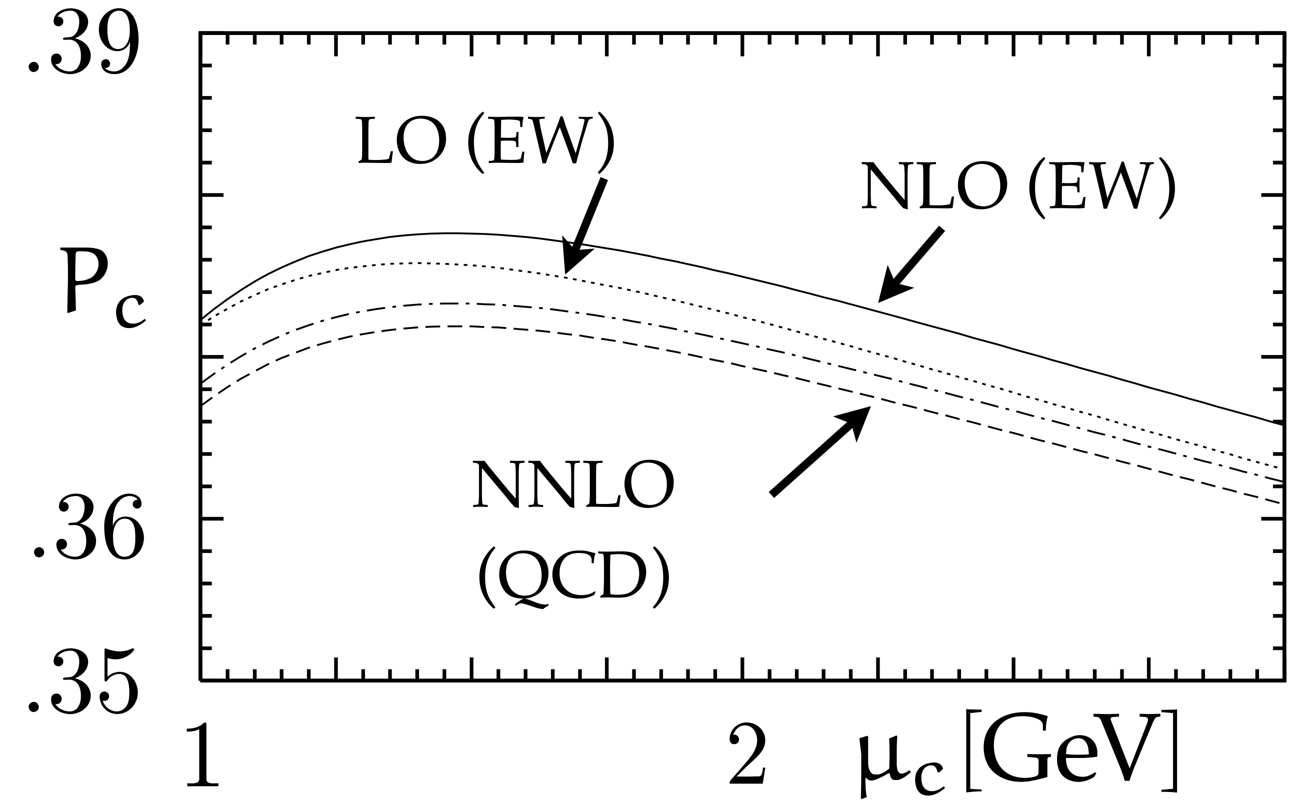
$K^+ \to \pi^+ \bar{\upsilon}\, \upsilon$ from M_W to m_c

P_c: charm quark contribution to $K^+ \to \pi^+ \bar{\upsilon}\, \upsilon$ (30% to BR)

Series converges very well (NNLO:10%→2.5% uncertainty)

NNLO+EW [Buras, MG, Haisch, Nierste; Brod MG]



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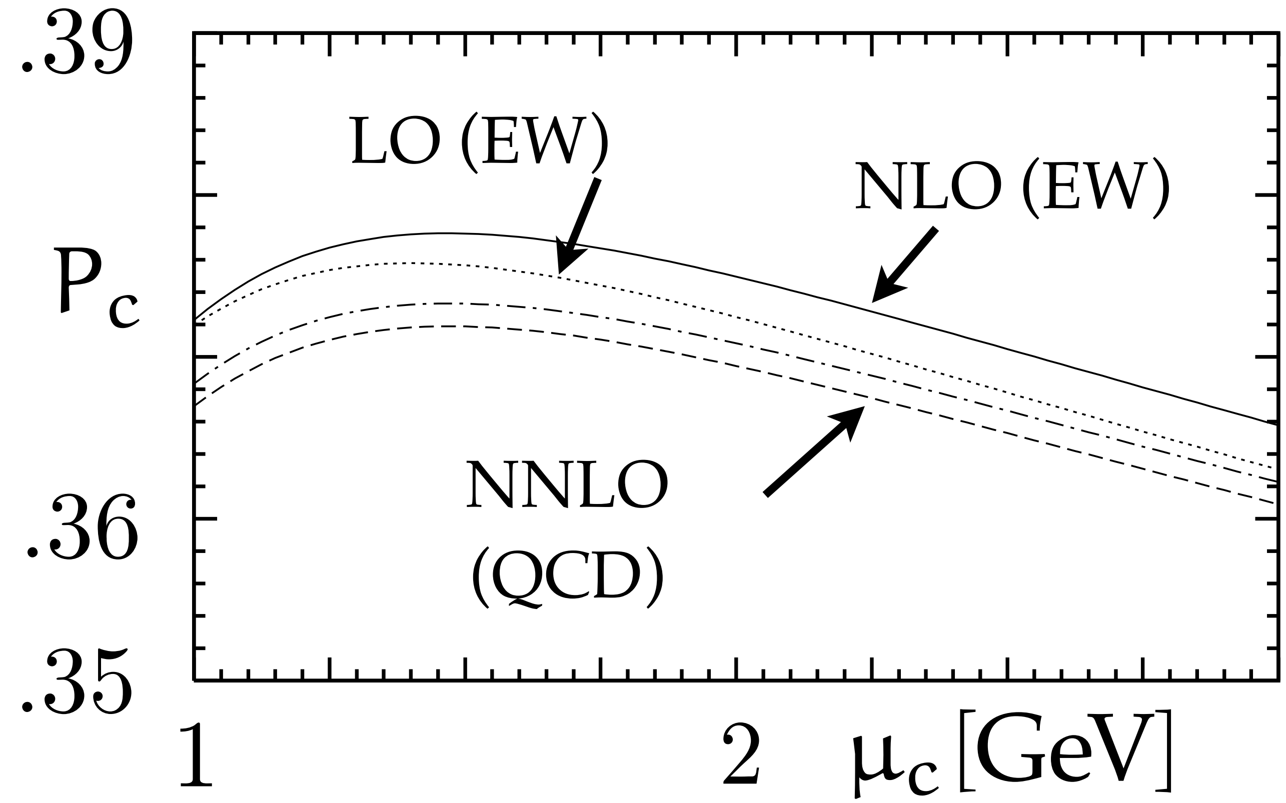

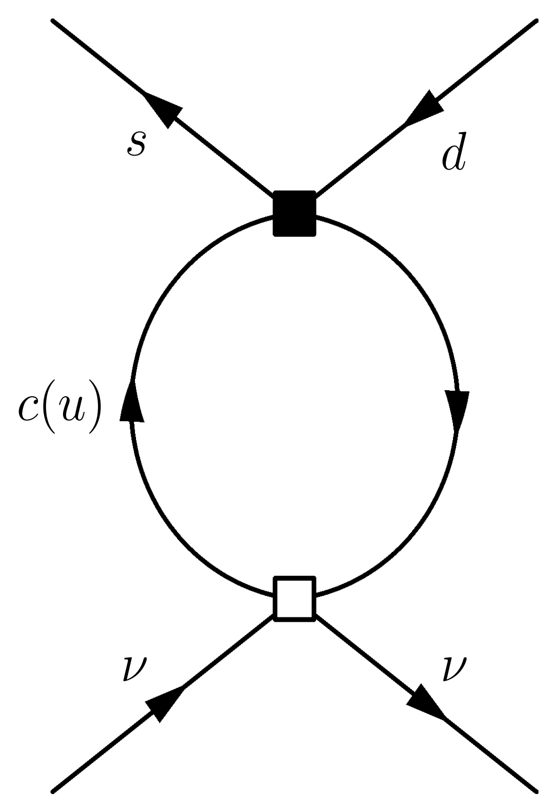

No GIM below the charm quark mass scale

higher dimensional operators UV scale dependent

One loop ChiPT calculation approximately cancels this scale dependence $\delta P_{c,u} = 0.04 \pm 0.02$

[Isidori, Mescia, Smith `05]

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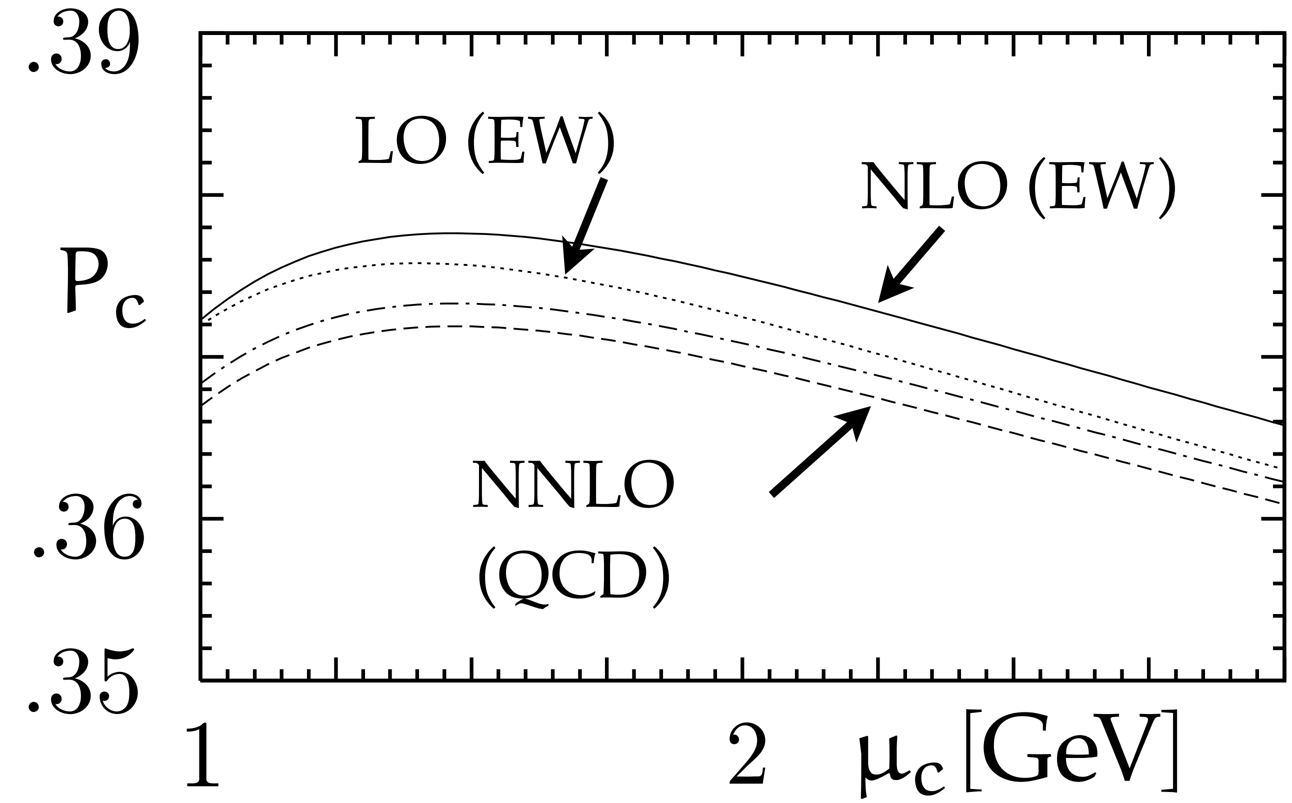

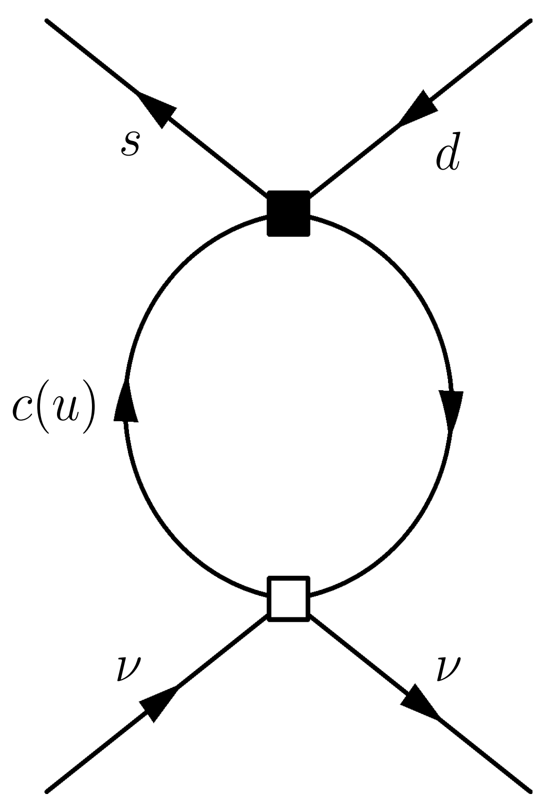

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Explorative (unphysical) Lattice calculation:

$\delta P_{c,u}$ =0.0040(±13)(±32)(-45) [Bai et.al. `17]

$K \to \pi \bar{\upsilon} \upsilon$: Error Budget

$BR^{th}(K^+ \to \pi^+ \bar{\upsilon}\upsilon) = 7.8(8)(3) \cdot 10^{-11}$

$BR^{exp}(K^+ \to \pi^+ \bar{\upsilon}\upsilon) = 17(11) \cdot 10^{-11}$

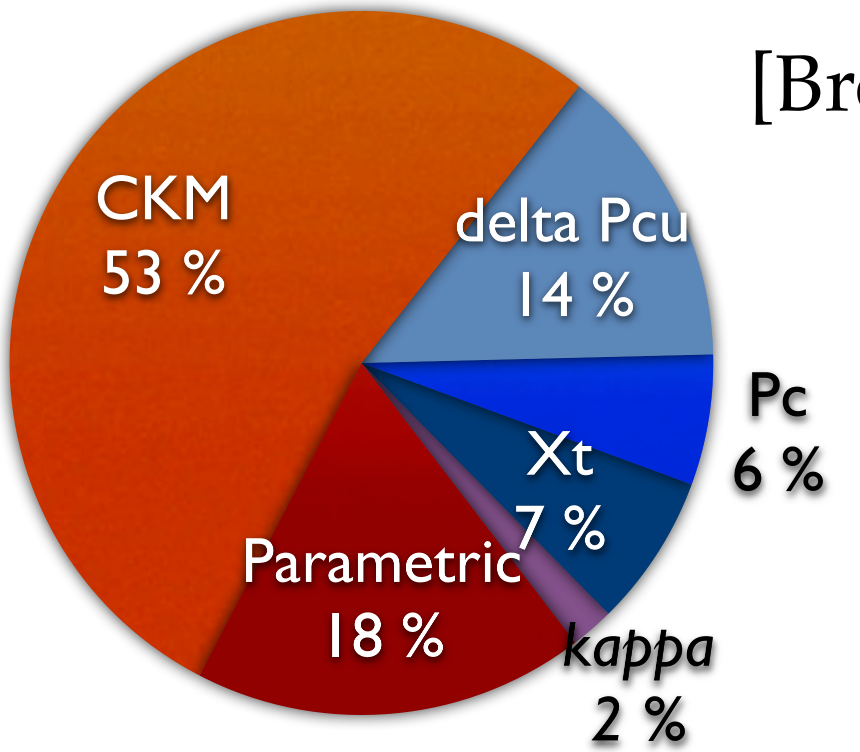
[E787, E949 ´08] NA62 → 10% accuracy

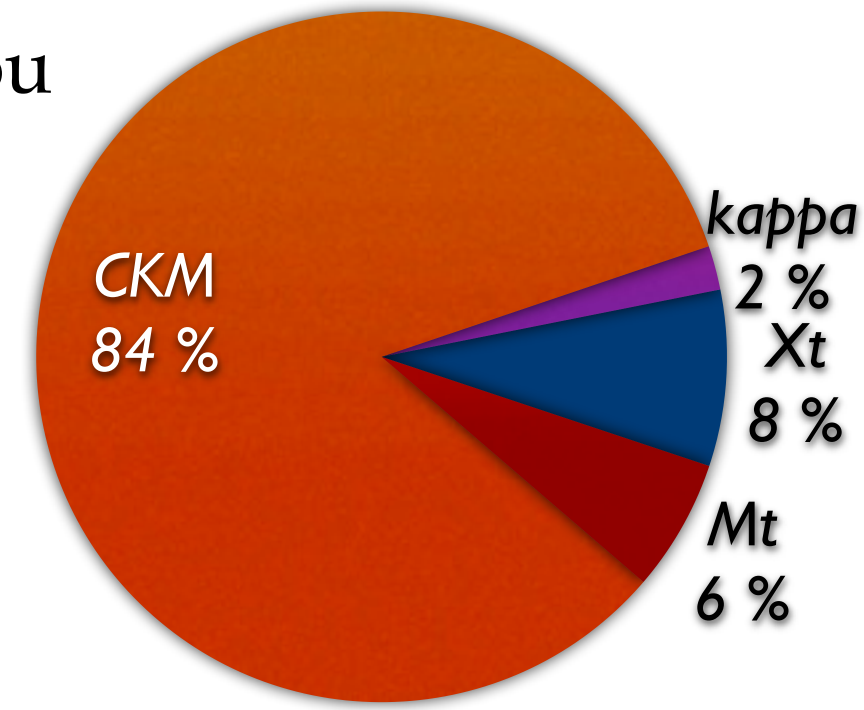
$BR^{th}(K_L \to \pi^0 \bar{\upsilon}\upsilon) = 2.43(39)(6) \cdot 10^{-11}$

$BR^{exp}(K_L \to \pi^0 \bar{\upsilon}\upsilon) < 6.7 \cdot 10^{-8}$

[E391a ´08]

[Brod, MG, Stamou \`2011]





$BR^+ = 8.4(6) \cdot 10^{-11}$ (CKM tree) $BR_L = 3.4(6) \cdot 10^{-11}$ (CKM tree)

Using the same calculations: [Buras et.al. \`15]

CP violation in Kaons

CP violation in mixing, interference & decay → non-zero

$$\eta_{+-} = \frac{\langle \pi^+\pi^- | K_L^0 \rangle}{\langle \pi^+\pi^- | K_S^0 \rangle} \qquad \eta_{00} = \frac{\langle \pi^0\pi^0 | K_L^0 \rangle}{\langle \pi^0\pi^0 | K_S^0 \rangle}$$

Only CP violation in mixing (Re ε), interference of mixing and decay (Im ε, Im ε') and direct CP violation (Re ε')

$$\epsilon_K = (\eta_{00} + 2\eta_{+-})/3 \qquad \epsilon' = (\eta_{+-} - \eta_{00})/3$$

Using: $\lambda_{ij} = \frac{q}{p}\frac{\langle \pi^i\pi^j | \bar{K}^0 \rangle}{\langle \pi^i\pi^j | K^0 \rangle}$ and $|1 - \lambda_{ij}| \ll 1$

$$\boxed{\epsilon' \approx \frac{1}{6}(\lambda_{00} - \lambda_{+-})} + \frac{1}{12}(\lambda_{00} - \lambda_{+-})(2 - \lambda_{00} - \lambda_{+-}) + \ldots$$

Formula for ε‘ / ε

a_0, a_2 & a_2^+ from experiment

[Cirigliano, et.al. `11]

a_0 & a_2: isospin amplitudes for isospin conservation

$$\langle \pi^0 \pi^0 | K^0 \rangle = a_0 e^{i\chi_0} + a_2 e^{i\chi_2} / \sqrt{2}$$

$$\langle \pi^+ \pi^- | K^0 \rangle = a_0 e^{i\chi_0} - a_2 e^{i\chi_2} \sqrt{2}$$

$$\langle \pi^+ \pi^0 | K^+ \rangle = 3 a_2^+ e^{i\chi_2^+} / 2$$

Formula for ε‘ / ε

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$$\langle \pi^+ \pi^0 | K^+ \rangle = 3 a_2^+ e^{i\chi_2^+}/2$$

Current theory gives us only: $A_I = \langle (\pi\pi)_I | \mathcal{H}_{\text{eff}} | K \rangle$

Normalise to K^+ decay (ω_+, a) and ε_K ,
expand in A_2/A_0 and CP violation:

Formula for ε‘ / ε

a_0, a_2 & a_2^+ from experiment

[Cirigliano, et.al. `11]

a_0 & a_2: isospin amplitudes for isospin conservation

$$\langle \pi^0\pi^0|K^0\rangle = a_0 e^{i\chi_0} + a_2 e^{i\chi_2}/\sqrt{2}$$

$$\langle \pi^+\pi^-|K^0\rangle = a_0 e^{i\chi_0} - a_2 e^{i\chi_2}\sqrt{2}$$

$$\langle \pi^+\pi^0|K^+\rangle = 3a_2^+ e^{i\chi_2^+}/2$$

Current theory gives us only: $A_I = \langle(\pi\pi)_I|\mathcal{H}_{\text{eff}}|K\rangle$

Normalise to K⁺ decay (ω_+, a) and ε_K ,
expand in A_2/A_0 and CP violation:

$$\mathrm{Re}\left(\frac{\epsilon'}{\epsilon}\right) \simeq \frac{\epsilon'}{\epsilon} = -\frac{\omega_+}{\sqrt{2}\,|\epsilon_K|}\left[\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\,(1-\hat{\Omega}_{\text{eff}}) - \frac{1}{a}\,\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2}\right]$$

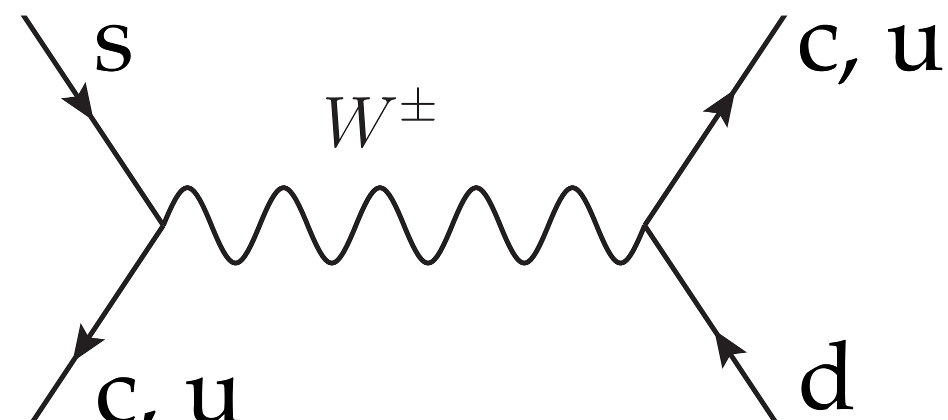
[Buras, MG, Jäger, Jamin `15]

Adjusted to keep electroweak penguins in Im A_0 [Cirigliano, et.al. `11]

Current-Current & CKM

Study Unitarity & CKM Elements to get Im A_I & Re A_I

We use unitarity to eliminate $V_{cs}^* V_{cd} = -V_{us}^* V_{ud} - V_{ts}^* V_{td} Q_2^c$



Current-current interactions:
Two contributions if $\mu > m_c$.

($\propto V_{ts}^* V_{td}$ and $\propto V_{us}^* V_{ud}$)

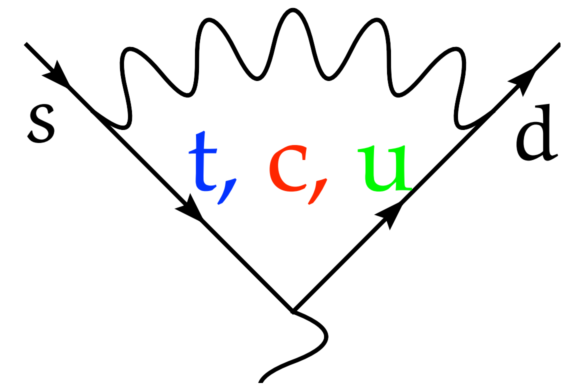
$$V_{us}^* V_{ud} Q_{1/2}^u + V_{cs}^* V_{cd} Q_{1/2}^c \rightarrow$$
$$V_{us}^* V_{ud} (Q_{1/2}^u - Q_{1/2}^c) - V_{ts}^* V_{td} Q_{1/2}^c$$

For $\mu < m_c$: $V_{ts}^* V_{td}$ is absent: $V_{us}^* V_{ud} Q_{1/2}^u$

Penguin & CKM

Penguins: $f(m_u) - f(m_c) = 0$:

Only $V_{ts}^* V_{td}$ contribution

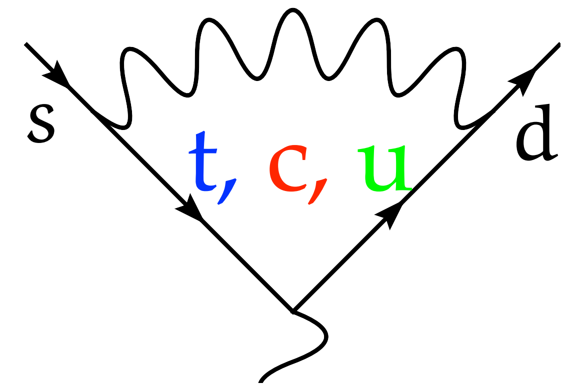


$$\left\{V_{us}^* V_{ud} f(m_u) + V_{cs}^* V_{cd} f(m_c) + V_{ts}^* V_{td} f(m_t)\right\} Q_{\text{Penguin}} \to$$

$$\left\{V_{us}^* V_{ud} \left[f(m_u) - f(m_c)\right] + V_{ts}^* V_{td} \left[f(m_t) - f(m_c)\right]\right\} Q_{\text{Penguin}}$$

Penguin & CKM

Penguins: $f(m_u)$ - $f(m_c) = 0$:
Only $V_{ts}^* V_{td}$ contribution



$$\{V_{us}^* V_{ud} f(m_u) + V_{cs}^* V_{cd} f(m_c) + V_{ts}^* V_{td} f(m_t)\} Q_{\mathrm{Penguin}} \rightarrow$$
$$\{V_{us}^* V_{ud} [f(m_u) - f(m_c)] + V_{ts}^* V_{td} [f(m_t) - f(m_c)]\} Q_{\mathrm{Penguin}}$$

$\mu > m_c$: $V_{ts}^* V_{td}\, Q^c_{1/2}$ mixes into $V_{ts}^* V_{td}\, Q_{\mathrm{Penguin}}$ (like usual).

$\mu > m_c$: $V_{us}^* V_{ud}\, (Q^u_{1/2} - Q^c_{1/2})$ does not mix into Q_{Penguin} .

$\mu < m_c$: Match $V_{ts}^* V_{td}\, Q^c_{1/2}$ onto $V_{ts}^* V_{td}\, Q_{\mathrm{Penguin}}$
→ CP violation from Q_{Penguin}
→ CP conserving from $Q^u_{1/2}$ (plus small Q_{Penguin})

Effective Hamiltonian

Currently we use the effective Hamiltonian below the charm:

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}}\, V_{ud} V^*_{us} \sum_{i=1}^{10} \big(z_i(\mu) + \tau\; y_i(\mu) \big)\, Q_i(\mu)\,, \qquad \tau \equiv -\,\frac{V_{td} V^*_{ts}}{V_{ud} V^*_{us}}$$

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current-current

QCD &

electroweak

penguins

$$Q_{1,2/\pm} = (\bar{s}_i u_j)_{V-A} \; (\bar{u}_k d_l)_{V-A}$$

$$Q_{3,\ldots,6} = (\bar{s}_i d_j)_{V-A} \sum_{q=u,d,s} (\bar{q}_k q_l)_{V\pm A}$$

$$Q_{7,\ldots,10} = (\bar{s}_i d_j)_{V-A} \sum_{q=u,d,s} e_q (\bar{q}_k q_l)_{V\pm A}$$

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We have z_i & y_i at NLO [Buras et.al., Ciuchini et. al. `92 `93]

And now also a Lattice QCD calculation of: $\langle(\pi\pi)_I\,|\,Q_i\,|\,K\rangle = \langle Q_i\rangle_I$

by RBC-UKQCD [Blum et. al., Bai et. al. `15]

Im A_2 / Re A_2 – (V-A)x(V-A)

A_2 only contributes in the ratio Im A_2/Re A_2

Let us first consider only (V-A)x(V-A) operators:

$$Q_1 = (\bar{s}_\alpha u_\beta)_{V-A}\,(\bar{u}_\beta d_\alpha)_{V-A} \qquad Q_2 = (\bar{s}u)_{V-A}\,(\bar{u}d)_{V-A}$$

$$Q_9 = \frac{3}{2}\,(\bar{s}d)_{V-A}\sum_{q=u,d,s,c,b} e_q\,(\bar{q}q)_{V-A} \qquad Q_{10} = \frac{3}{2}\,(\bar{s}_\alpha d_\beta)_{V-A}\sum_{q=u,d,s,c,b} e_q\,(\bar{q}_\beta q_\alpha)_{V-A}$$

Isospin limit: $2\,\langle Q_9\rangle_2 = 2\,\langle Q_{10}\rangle_2 = 3\,\langle Q_1\rangle_2 = 3\,\langle Q_2\rangle_2$

Re A_2: $(z_1+z_2)\langle Q_1+Q_2\rangle_2 = z_+\langle Q_+\rangle_2$ Im A_2: $y_9\langle Q_9\rangle_2 + y_{10}\langle Q_{10}\rangle_2$

$$\left(\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2}\right)_{V-A} = \mathrm{Im}\tau\,\frac{3(y_9+y_{10})}{2z_+}\,, \qquad \tau = \frac{V_{ts}^* V_{td}}{V_{us}^* V_{ud}}$$

Im A_0 / Re A_0 – (V-A)x(V-A)

More operators contribute to Im A_0 / Re A_0

$$\mathrm{Re}A_0 = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \left(z_+ \langle Q_+ \rangle_0 + z_- \langle Q_- \rangle_0 \right)$$

Fierz relations for (V-A)x(V-A) give, e.g.: $\langle Q_4 \rangle_0 = \langle Q_3 \rangle_0 + 2 \langle Q_- \rangle_0$

$$\left(\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0} \right)_{V-A} = \mathrm{Im}\tau \, \frac{2 y_4}{(1+q) z_-} + \mathcal{O}(p_3)$$

Is only a function of Wilson coefficients and of the ratio

$$q = (z_+(\mu) \langle Q_+(\mu) \rangle_0) / (z_-(\mu) \langle Q_-(\mu) \rangle_0)$$

Expression with $p_3 = \langle Q_3 \rangle_0 / \langle Q_4 \rangle_0$ and EW penguins given in [Buras, MG, Jäger & Jamin `15]

(V-A)x(V+A) Contributions

Q_6 & Q_8 give the leading contribution to
$\mathrm{Im}A_0$ & $\mathrm{Im}A_2$ respectively

$$\left(\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right)_6 = -\frac{G_F}{\sqrt{2}}\,\mathrm{Im}\lambda_t\, y_6\, \frac{\langle Q_6\rangle_0}{\mathrm{Re}A_0}$$

$$\left(\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2}\right)_8 = -\frac{G_F}{\sqrt{2}}\,\mathrm{Im}\lambda_t\, y_8^{\mathrm{eff}}\, \frac{\langle Q_8\rangle_2}{\mathrm{Re}A_2}$$

Here: Take Re A_0 from data

One can re-express $\langle Q_6\rangle_0$ & $\langle Q_8\rangle_2$ in terms of B_6 & B_8

Prediction for ε‘ / ε

I=2 Similarly for (V-A)x(V-A):

I=0 (V-A)x(V-A) I=2 (V-A)x(V-A)

$$\frac{\varepsilon'}{\varepsilon} = 10^{-4}\left[\frac{\mathrm{Im}\lambda_{\rm t}}{1.4\cdot 10^{-4}}\right]\left[a\left(1-\hat{\Omega}_{\rm eff}\right)\left(-4.1(8)+24.7\,B_6^{(1/2)}\right)+1.2(1)-10.4\,B_8^{(3/2)}\right]$$

(V-A)x(V+A) Matrix elements B_6=0.57(19) and B_8=0.76(5) from Lattice QCD [Blum et. al., Bai et. al. `15]

$$\left(\frac{\epsilon'}{\epsilon}\right)_{\rm SM} = 1.9(4.5)\times 10^{-4}$$

2.9 σ difference

$$\left(\frac{\epsilon'}{\epsilon}\right)_{\rm exp} = 16.6(2.3)\times 10^{-4}$$

Similar findings by Kitahara et.al. 16

quantity	error on ε'/ε
$B_6^{(1/2)}$	4.1
NNLO	1.6
$\hat{\Omega}_{\rm eff}$	0.7
p_3	0.6
$B_8^{(3/2)}$	0.5
p_5	0.4
$m_s(m_c)$	0.3
$m_t(m_t)$	0.3

NLO vs NNLO

Theory prediction only at NLO at the moment

Convergence at m_c is not clear – should calculate next order

Long term use Lattice QCD

Also the error estimate does not include $O(p^2/m_c^2)$ corrections which for $K \to \pi\,\pi$ are expected to be small

Status of ε' / ε NNLO

Energy	Fields	Order
μ_W	g,γ,W,Z,h, u,d,s,c,b,t	NNLO Q_1-Q_6 & Q_{8g} i) NNLO EW Penguins (traditional Basis) ii)
RGE	γ,g,u,d,s,c,b	NNLO Q_1-Q_6 & Q_{8g} iii)
μ_b	γ,g,u,d,s,c,b	NNLO Q_1-Q_6 iv)
RGE	γ,g,u,d,s,c	NNLO Q_1-Q_6 & Q_{8g} iii)
μ_c	γ,g,u,d,s,c	NLO Q_1-Q_{10} v)
RGE	γ,g,u,d,s	NNLO Q_1-Q_6 & Q8g iii)
$M_{Lattice}$	g,u,d,s	NLO Q_1-Q_{10} (traditional Basis) vi)

i) [Misiak, Bobeth, Urban]
ii) [Gambino,Buras, Haisch]
iii)[Gorbahn, Haisch]
iv)[Gorbahn, Brod]
v) [Buras, Jamin, Lautenbacher]
vi)[Blum et. al., Bai et. al. '15]

RG-invariant factorisation

Traditional the contribution of running ($U(\mu,\mu_0)$) and matching ($M(\mu)$) are combined as:

$$\begin{aligned}\langle\vec{Q}\rangle^{(3)}(\mu_L)\vec{C}^{(3)}(\mu_L) &= \langle\vec{Q}\rangle(\mu_L)U^{(3)}(\mu_L,\mu_c)M^{(34)}(\mu_c)U^{(4)}(\mu_c,\mu_b)\\ &\quad M^{(45)}(\mu_b)U^{(5)}(\mu_b,\mu_W)\vec{C}^{(5)}(\mu_W)\end{aligned}$$

Alternatively we can also factorise as

$$\begin{aligned}\langle\vec{Q}\rangle^{(3)}(\mu_L)\vec{C}^{(3)}(\mu) &= \langle\vec{Q}\rangle(\mu_L)^{(3)}u^{(3)}(\mu_L)\\ &\quad u^{(3)^{-1}}(\mu_c)M^{(34)}(\mu_c)u^{(4)}(\mu_c)\\ &\quad u^{(4)^{-1}}(\mu_b)M^{(45)}(\mu_b)u^{(5)}(\mu_b)\\ &\quad u^{(5)^{-1}}(\mu_W)\vec{C}^{(5)}(\mu_W)\end{aligned}$$

or write in terms of scheme and scale independent quantities:

$$\langle\vec{Q}\rangle^{(3)}(\mu_L)\vec{C}^{(3)}(\mu) = \langle\hat{\vec{Q}}\rangle^{(3)}\hat{M}^{(34)}\hat{M}^{(45)}\hat{\vec{C}}^{(5)}$$

RG-invariant factorisation

All hatted quantities $\langle\hat{\vec{Q}}\rangle^{(3)}, \hat{M}^{(34)}, \hat{M}^{(45)}$ and $\hat{\vec{C}}^{(5)}$ and also their products

$$\hat{\vec{C}}^{(3)} = \hat{M}^{(34)}\hat{M}^{(45)}\hat{\vec{C}}^{(5)}$$

are formally scheme and scale independent.

The matrix elements $\langle\hat{\vec{Q}}\rangle$ satisfy $d = 4$ Fierz identities.

$\hat{\vec{C}}^{(3)}$ is μ *independent*, but shows *residual* μ *dependence*.

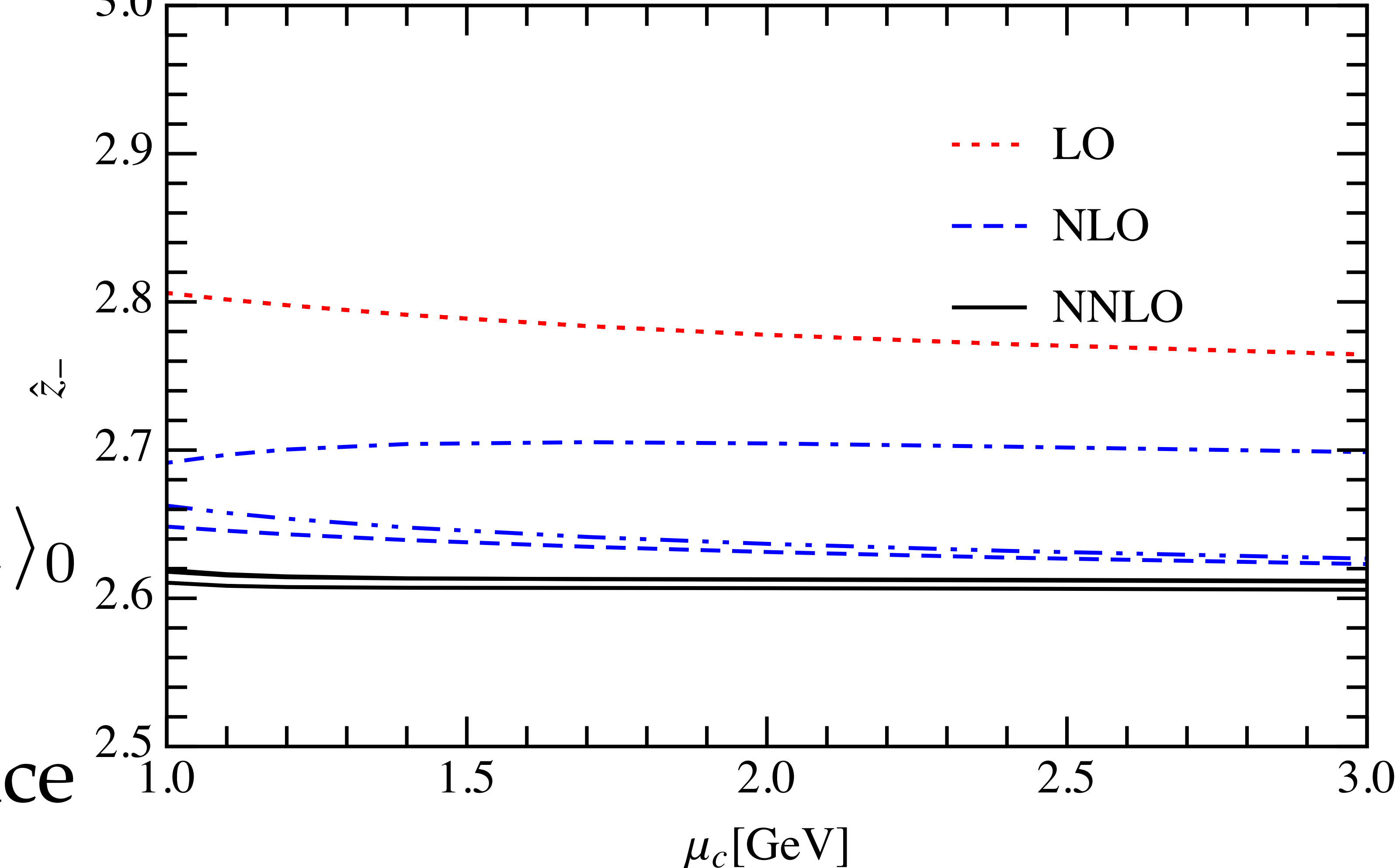
Plot this for the $\hat{y}(\mu_c)$ (the ones $\propto \mathrm{Im}(V_{ts}^* V_{td})$):
and for $\hat{z}(\mu_c)$ (relevant for Re A_0 and Re A_2)

Use different RGE running (numerical or via Λ_{MS})
from $\alpha_s(M_Z)$ at LO, NLO & NNLO

The Real Part of A_0 & A_2 is dominated by z_+ & z_-

$$\mathrm{Re}A_2 = \hat{z}_+ \langle \hat{Q}_+ \rangle_2$$

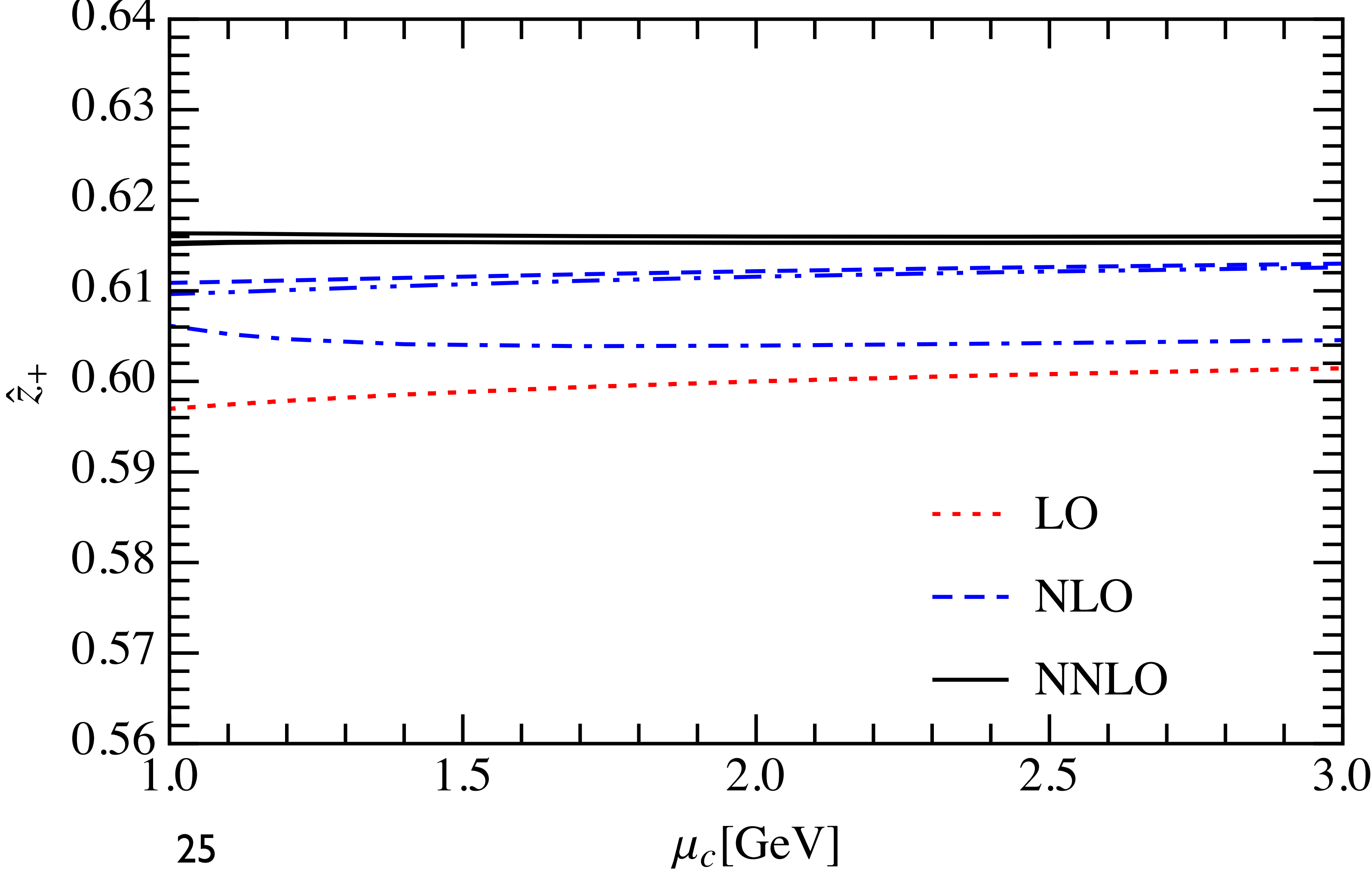
$$\mathrm{Re}A_0 = \hat{z}_+ \langle \hat{Q}_+ \rangle_0 + \hat{z}_- \langle \hat{Q}_- \rangle_0$$



The residual μ_c dependence reduces order by order

At NLO there is still a dependence on the implementation of α_s Running.

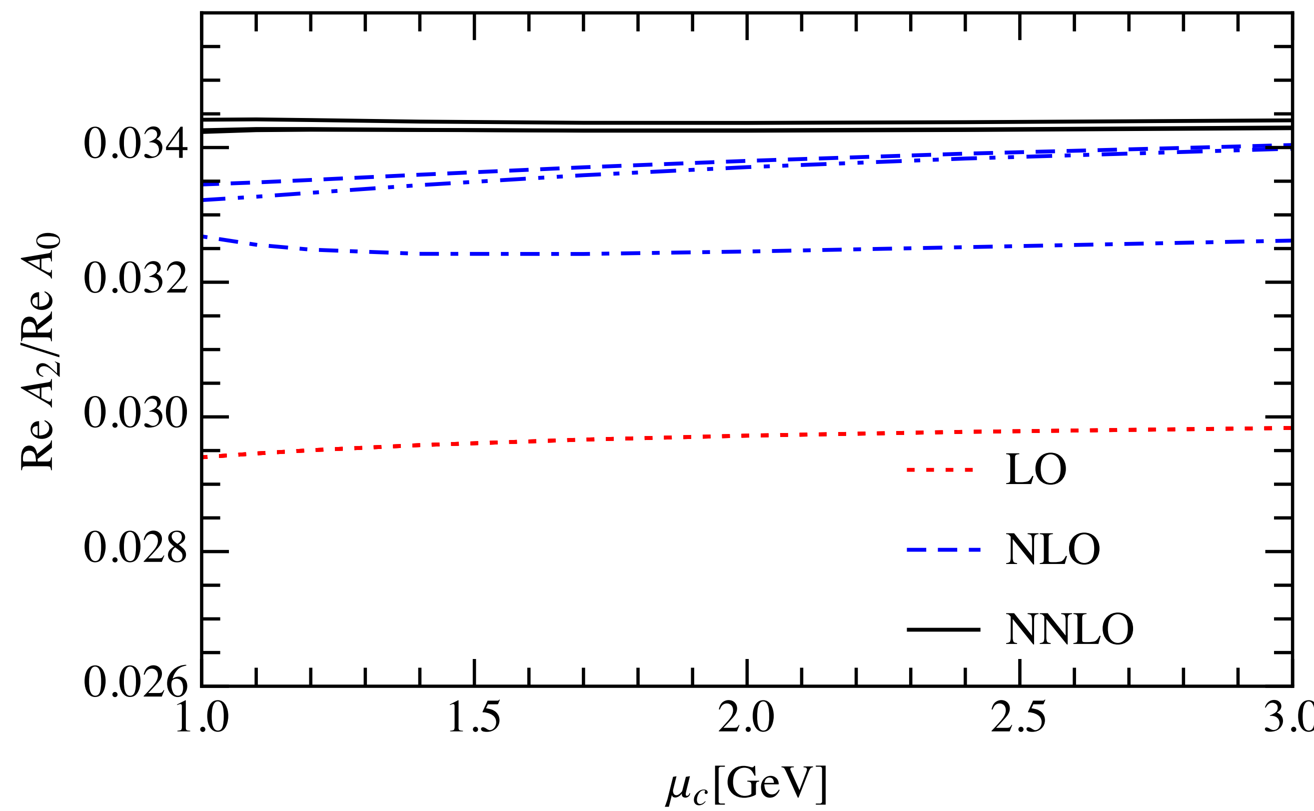
Shift probably due to running down from M_Z

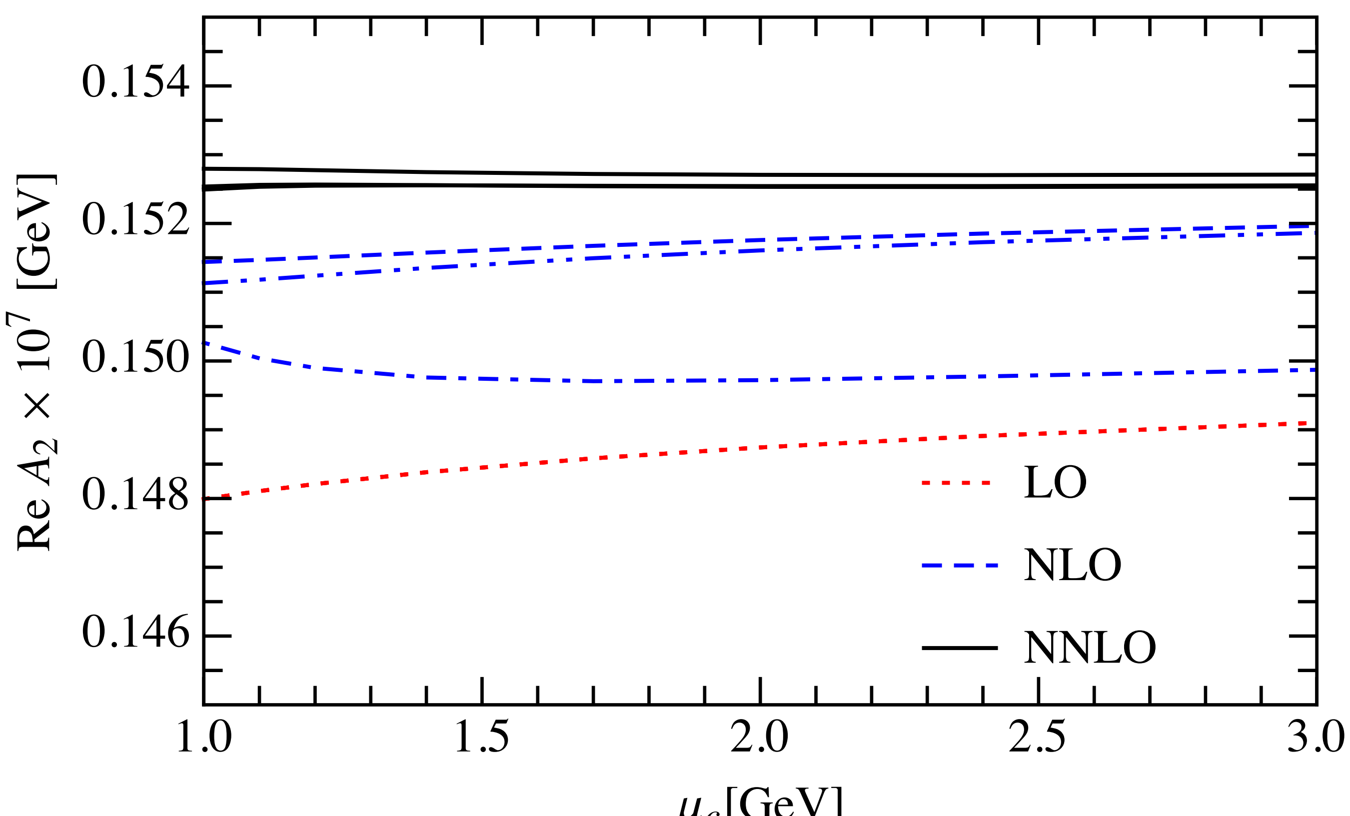


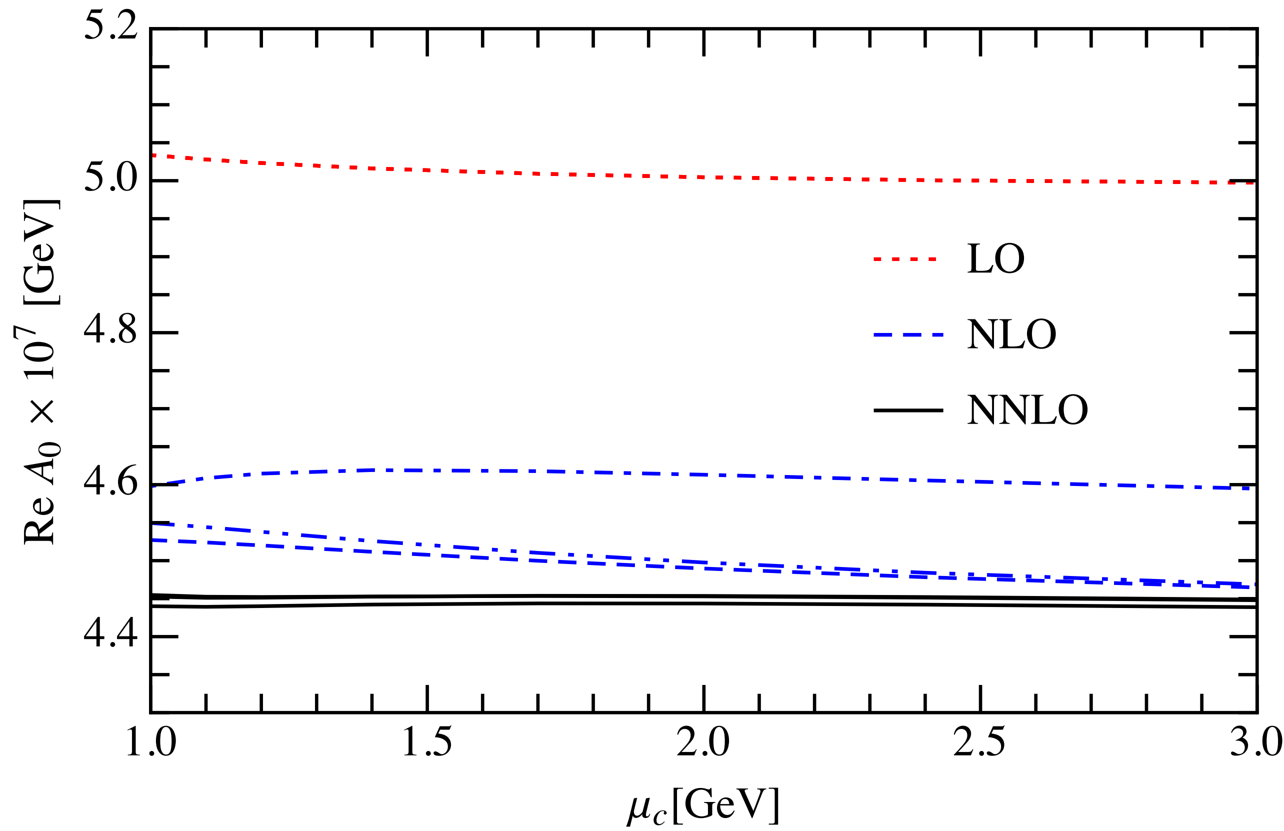
Transform Lattice RISMOM matrix elements to $\hat{q}$ scheme

$\mathrm{Re}\, A_0 = 33.2 \times 10^{-8}$ GeV
$\mathrm{Re}\, A_2 = 1.48 \times 10^{-8}$ GeV

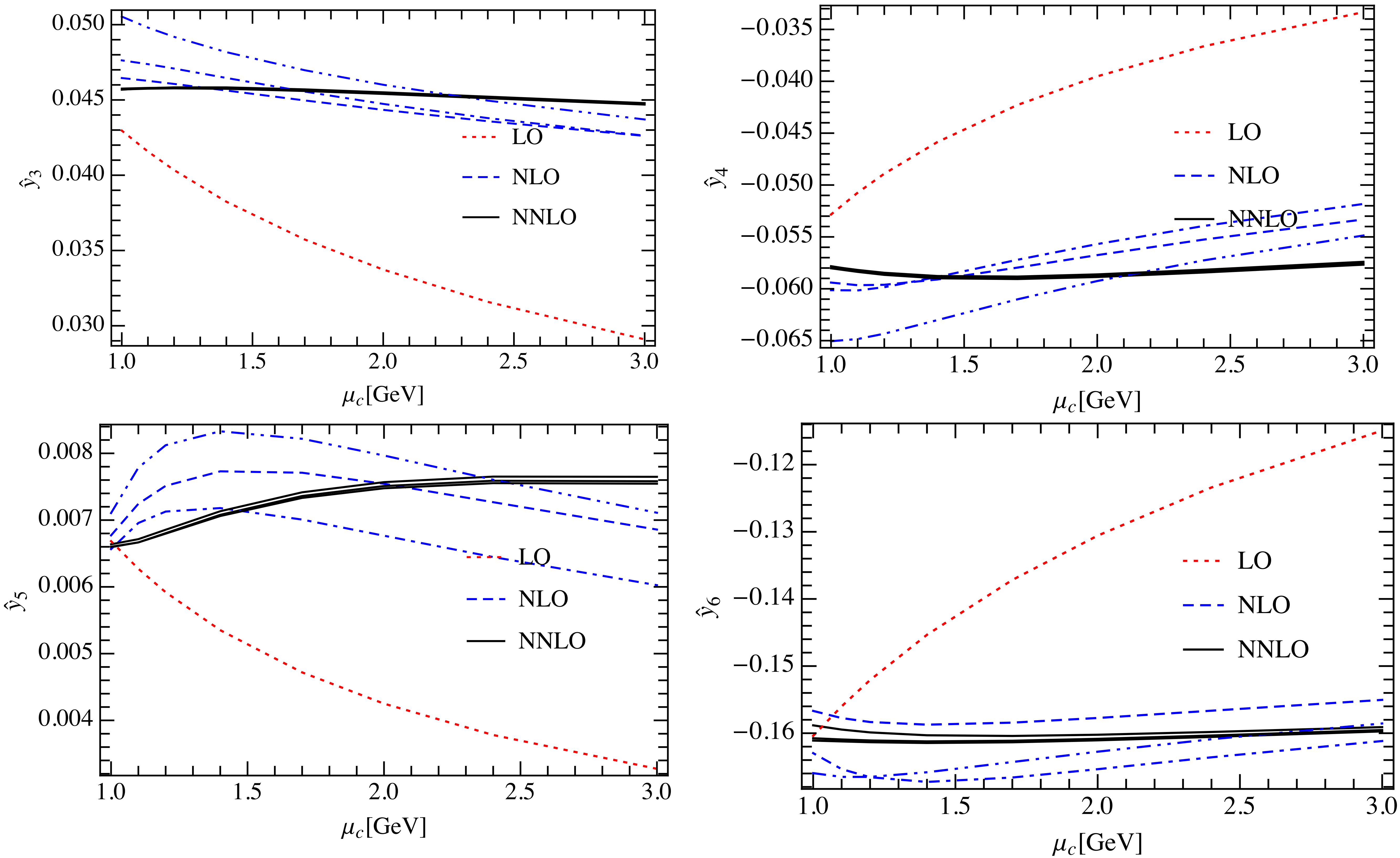
Lattice input to Re A_0 has still 20% / 25% stat / sys. uncertainty

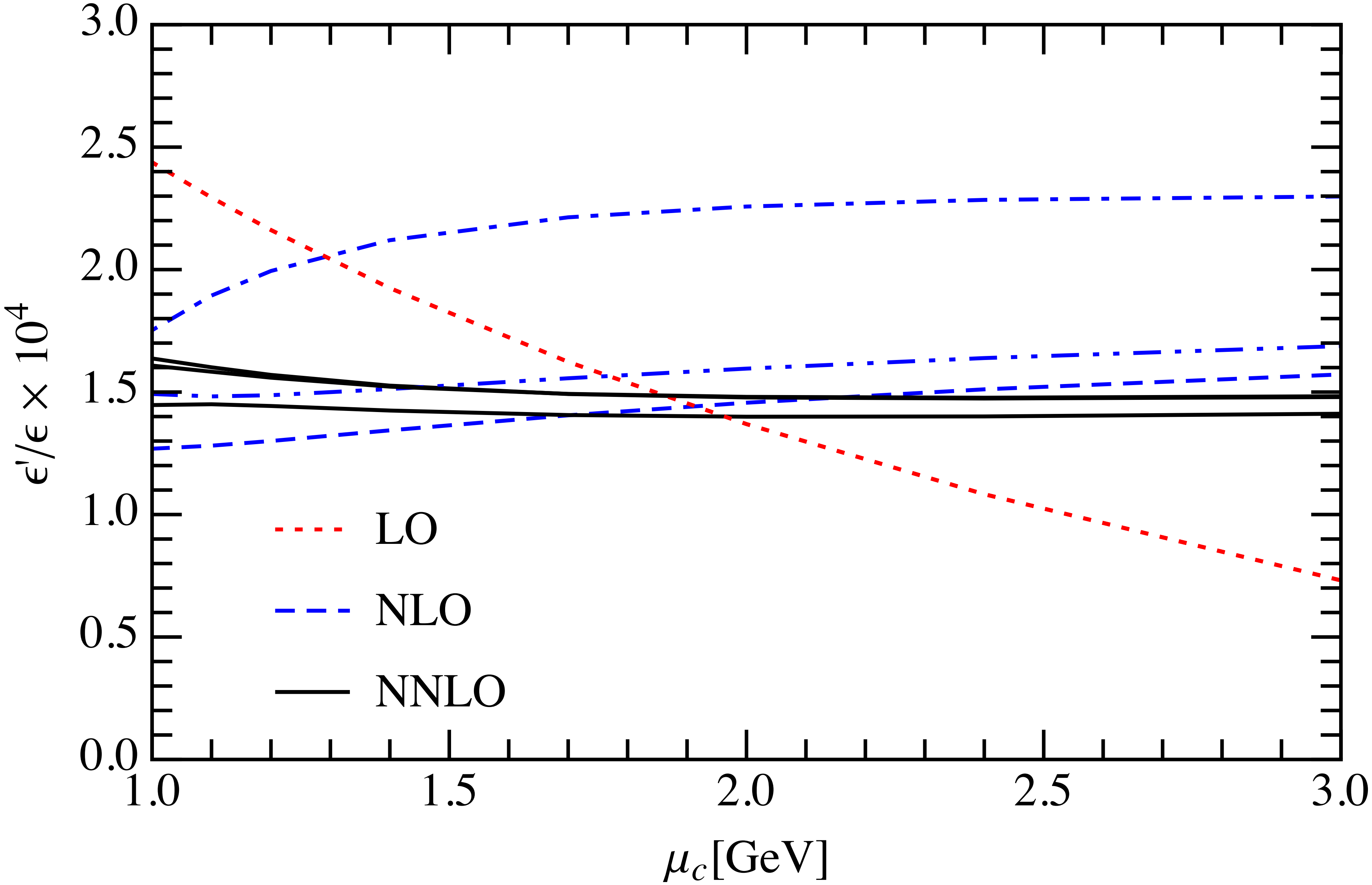






QCD Penguin scale uncertainty is reduced from NLO to NNLO





Plot residual μ_c dependence of the QCD contribution to $\varepsilon' / \varepsilon$

Uncertainty is significantly reduced by going to NNLO

There are still improvements:

e.g. better α_s implementation & better incorporation of subleading corrections – will not change the overall picture

Conclusion

Perturbative calculations for $K \to \pi \bar{\upsilon} \upsilon$ under very good control, with only sub-leading non-perturbative effects.

Ongoing Lattice efforts improve the estimate of non-perturbative effects for $K \to \pi \bar{\upsilon} \upsilon$.

New perturbative NNLO calculation removes large part of the perturbative uncertainty in ε'_K.

Interesting tension with experiment.