Challenges and recent progress in leptonic and semi-leptonic charmed meson decays on the lattice

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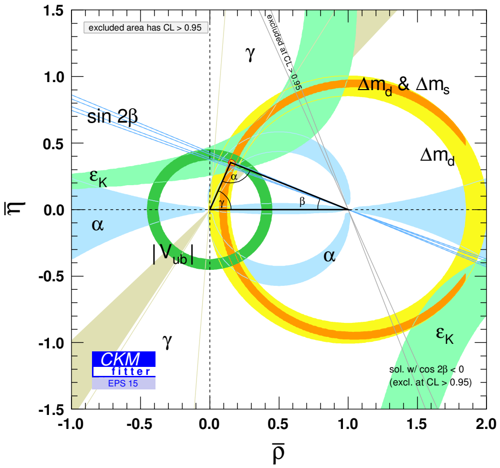
Outline

Motivation

Where to find New Physics?

- Direct searches:
  → Higgs seems to be SM Higgs
  → So far: no 'smoking gun'
- Indirect searches: NP can modify cross sections.
  Constrain SM predictions e.g. in
  → Flavour Sector



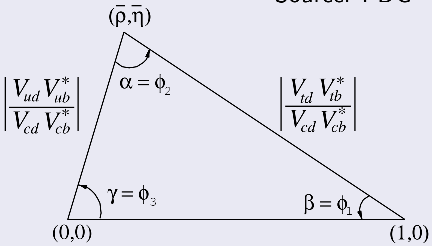
⇒ **Over-constrain the CKM matrix to test SM**

CKM Matrix

- 3 generations
- relates flavour eigenstates (d', s', b') to mass eigenstates (d, s, b)
- complex
  $\Rightarrow$ allows for $\not{CP}$
  via a single phase
- unitary
  e.g. 2nd row:
  $|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 \stackrel{?}{=} 1$

Unitarity Triangle

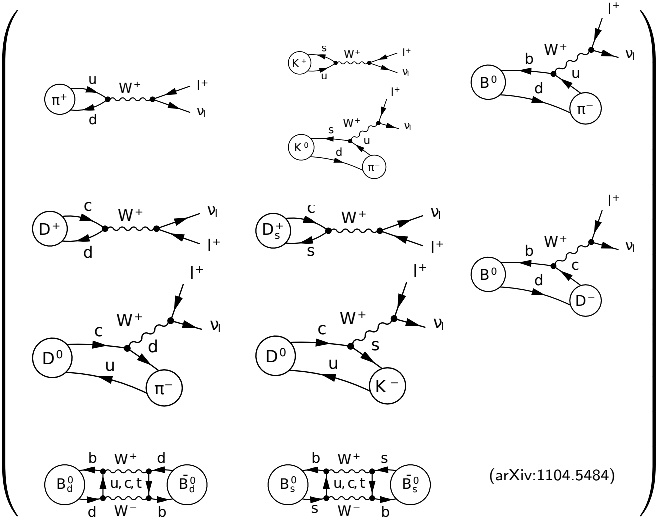
Source: PDG

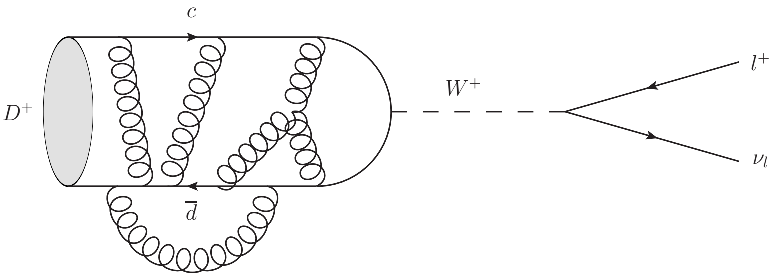


$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$
$$\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} + \frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*} = -1.$$

$\Rightarrow$ Test SM by determining CKM matrix elements

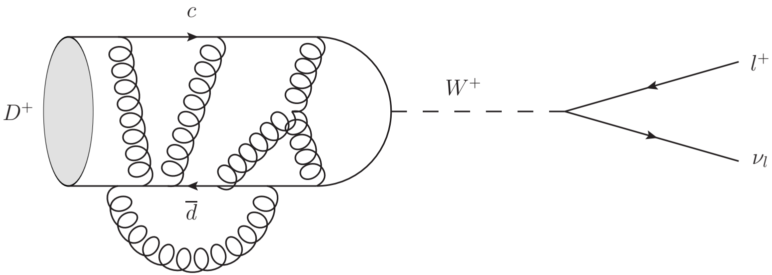
$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \sim$$



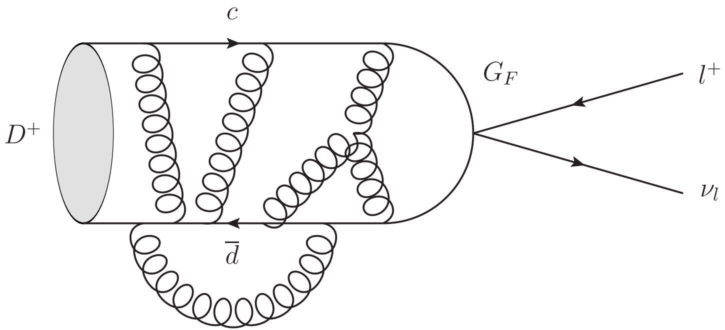


$$\Gamma_{\text{exp}} = V_{\text{CKM}}(\text{STRONG})(\text{WEAK})(\text{EM})$$

Example 1: Leptonic $D_{(s)}$ decays

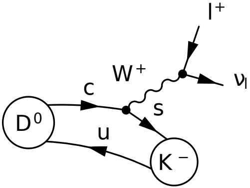


$$\Gamma = V_{\mathrm{CKM}} \underbrace{(\text{STRONG})}_{\text{non-perturbative}} \underbrace{(\text{WEAK})(\text{EM})}_{\text{perturbative}}$$



$$\underbrace{\Gamma(D^+_{(s)} \to l^+\nu_l)}_{\text{experiment}} = |V_{cq}|^2 \, f^2_{D_{(s)}} \underbrace{\frac{G_F^2 \tau_{D_{(s)}}}{8\pi} m_l^2 m_{D_{(s)}} \left(1 - \frac{m_l^2}{m^2_{D_{(s)}}}\right)^2}_{\text{known factors}} + \mathcal{O}(\alpha_{\mathrm{EM}})$$

Example 2: Semi-Leptonic decays



$$\frac{d\Gamma(D \to Pl\bar{\nu}_l)}{dq^2} = G_F^2 \left|V_{cx}\right|^2 \left(\mathcal{K}_+(m_i, E_i, q^2)\left|f_+(q^2)\right|^2 + \mathcal{K}_0(m_i, E_i, q^2)\left|f_0(q^2)\right|^2\right)$$

where $x = d, s$, $P = \pi, K$ and $q = p_D - p_P$ is the momentum of the outgoing leptons.

Status summary

Leptonic Decays

HFLAV'16

$$|V_{cd}|\, f_{D^+} = 45.9 \pm 1.1\,\mathrm{MeV} \quad 2.4\% \qquad [\mathrm{FLAG'16}:\ 0.7\%]$$
$$|V_{cs}|\, f_{D_s^+} = 250.3 \pm 4.5\,\mathrm{MeV} \quad 1.8\% \qquad [\mathrm{FLAG'16}:\ 0.5\%]$$

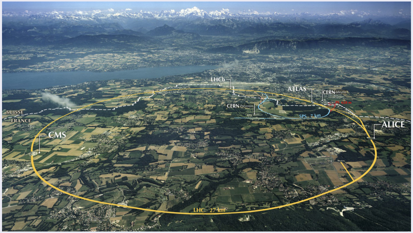
Semi-Leptonic Decays

HFLAV'16

$$|V_{cd}|\, f_+^{\pi}(0) = 0.1426 \pm 0.0019 \quad 1.3\% \qquad [\mathrm{FLAG'16}:\ 4.4\%]$$
$$|V_{cs}|\, f_+^{K}(0) = 0.7226 \pm 0.0034 \quad 0.5\% \qquad [\mathrm{FLAG'16}:\ 2.5\%]$$

Experimental Activity

LHC(b) - Geneva, Switzerland



Belle II - Tsukuba, Japan

BaBar, California, US

- BES III
- CLEO-c

Charm and Lattice

LQCD: From Minkowski to Euclidean

The path integral (Minkowski):

$$\langle \mathcal{O} \rangle_M = \frac{1}{Z} \int \mathcal{D}[\psi, \overline{\psi}]\, \mathcal{D}[U]\, \mathcal{O}[\psi, \overline{\psi}, U]\, e^{iS[\psi,\overline{\psi},U]}$$

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Wick rotate to Euclidean space ($t \to i\tau$):

$$\langle \mathcal{O} \rangle_E = \frac{1}{Z} \int \mathcal{D}[\psi, \overline{\psi}]\, \mathcal{D}[U]\, \mathcal{O}[\psi, \overline{\psi}, U]\, e^{-S_E[\psi,\overline{\psi},U]}$$

LQCD: From Minkowski to Euclidean

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Wick rotate to Euclidean space ($t \to i\tau$):

$$\langle\mathcal{O}\rangle = \frac{1}{Z}\int \mathcal{D}[\psi,\overline{\psi}]\,\mathcal{D}[U]\,\mathcal{O}[\psi,\overline{\psi},U]\,e^{-S_G[U]-S_F[\psi,\overline{\psi},U]}$$

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Integrate out fermionic degrees of freedom:

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}[U]\, \mathcal{O}[\psi,\overline{\psi},U] \left(\prod_f \det D_f \right) e^{-S_G[U]}$$

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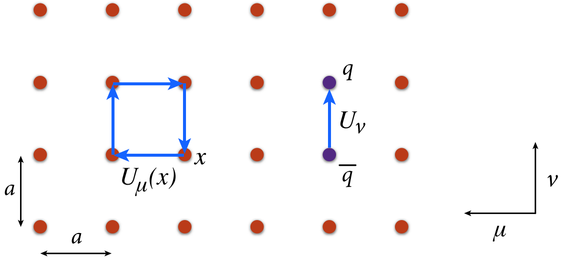
$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}[U]\, \mathcal{O}[\psi, \overline{\psi}, U]\, \underbrace{\left(\prod_f \det D_f \right) e^{-S_G[U]}}_{\text{statistical weight}}$$

Render PI finite dimensional by DISCRETISING in FINITE VOLUME.



source: PDG

- Finite lattice spacing a $\Rightarrow$ UV regulator
- Finite Box of length L $\Rightarrow$ IR regulator

LQCD: Evaluating the Path Integral

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}[U]\, \mathcal{O}[\psi, \overline{\psi}, U] \left(\prod_f \det D_f \right) e^{-S_G[U]}$$

LQCD: Evaluating the Path Integral

$$\langle\mathcal{O}\rangle = \int \mathcal{D}[U]\,\mathcal{O}[\psi,\overline{\psi},U]\,p(U)$$

GOAL: Want to statistically sample the path integral:

$$\langle\mathcal{O}\rangle \approx \frac{1}{N}\sum_{n=0}^{N-1}\mathcal{O}[U_n] \quad \text{where} \quad p(U_n) = \frac{1}{Z}\left(\prod_f \det D_f\right)\, e^{-S[\psi,\overline{\psi},U]}$$

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Example: D_s meson at rest

$$\left\langle 0 \middle| \mathcal{O}_{D_s}(t) \mathcal{O}^\dagger_{D_s}(0) \middle| 0 \right\rangle = \int \mathcal{D}[U]\; \mathcal{O}_{D_s}(t) \mathcal{O}^\dagger_{D_s}(0)\; p(U)$$

Extracting physics from correlation functions

Relate numerical evaluation of PI

$$\left\langle 0 \middle| \mathcal{O}_{D_s}(t)\mathcal{O}^\dagger_{D_s}(0) \middle| 0 \right\rangle = \int \mathcal{D}[U]\ \mathcal{O}_{D_s}(t)\mathcal{O}^\dagger_{D_s}(0)\ p(U)$$

Extracting physics from correlation functions

Relate numerical evaluation of PI

$$\left\langle 0 \right| \mathcal{O}_{D_s}(t)\mathcal{O}^\dagger_{D_s}(0) \left| 0 \right\rangle = \int \mathcal{D}[U]\ \mathcal{O}_{D_s}(t)\mathcal{O}^\dagger_{D_s}(0)\ p(U)$$

to physical quantities

$$\begin{aligned}
\left\langle 0 \right| \mathcal{O}_{D_s}(t)\mathcal{O}^\dagger_{D_s}(0) \left| 0 \right\rangle &= \sum_{\mathbf{x},n} \left\langle 0 \right| \mathcal{O}_{D_s}(x) \left| n \right\rangle \left\langle n \right| \mathcal{O}^\dagger_{D_s}(0) \left| 0 \right\rangle \\
&= \sum_{n} \left| \left\langle 0 \right| \mathcal{O}_{D_s}(0) \left| n \right\rangle \right|^2 e^{-E_n t} \\
&\overset{t\to\infty}{=} \left| \left\langle 0 \right| \mathcal{O}_{D_s}(0) \left| D_s \right\rangle \right|^2 e^{-m_{D_s} t}
\end{aligned}$$

- time behaviour: masses and energies
- normalisation: MEs (decay constants, form factors)

Lattice vs Continuum

Simulation:

- 2+1+1, 2+1, 2
- finite lattice spacing a
- finite volume L^3
- lattice regularised
- Some bare input quark masses am_l, am_s, am_h Often: $m_\pi^{\mathrm{sim}} > m_\pi^{\mathrm{phys}}$
- Isospin symmetric

Real World (SM):

- 6 distinct quark flavours
- $a = 0$
- $L = \infty$
- some continuum scheme
- $m_l = m_l^{\mathrm{phys}}$ $m_s = m_s^{\mathrm{phys}}$ $m_h = m_c^{\mathrm{phys}}$
- $m_u \neq m_d$, EW, EM

$\Rightarrow$ **We need to retrieve the continuum theory from the lattice simulations.**

LQCD: A Calculation

1. Create sets of U_n called *ensembles*
$p(U) = \frac{1}{Z} \left(\prod_f \det D_f\right) e^{-S[\psi,\overline{\psi},U]}$
2. Carry out measurements on these ensembles
$\left\langle 0 \middle| \mathcal{O}_{D_s}(t)\mathcal{O}^\dagger_{D_s}(0) \middle| 0 \right\rangle = \int \mathcal{D}[U]\ \mathcal{O}_{D_s}(t)\mathcal{O}^\dagger_{D_s}(0)\ p(U)$
3. Fit correlators to extract physics
$\left\langle 0 \middle| \mathcal{O}_{D_s}(t)\mathcal{O}^\dagger_{D_s}(0) \middle| 0 \right\rangle \overset{t\to\infty}{=} \left| \langle 0| \mathcal{O}_{D_s}(x) |D_s\rangle \right|^2 e^{-m_{D_s} t}$
4. Analysis:
$\Rightarrow$ From Lattice to Continuum
$\Rightarrow$ Statistical Error Analysis
$\Rightarrow$ Estimate of Systematic Errors

Differences in lattice calculations

Ensemble Properties

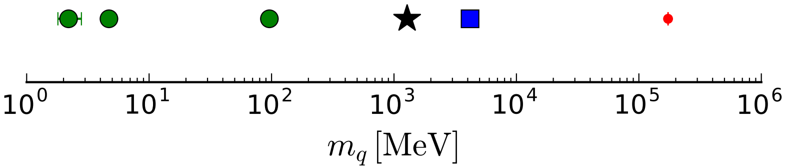
- Lattice dimensions: a, L, m_π
- Number of flavours in the sea
- Fermion formulation(s) [sea]
- Gauge formulation

Analysis

- Fermion formulation(s) [valence]
- Scale setting + Experimental Inputs
- Heavy quark treatment
- Renormalisation
- Many technical details

Final result must be independent of these choices!

A problem of scales: a, L and m_c



Finite Volume effects

Require: $m_\pi L \gtrsim 4$
For $m_\pi^{\mathrm{phys}} \approx 140\,\mathrm{MeV}$:

$$aN = L \gtrsim 5.9\,\mathrm{fm}$$

Discretisation effects

Require: $am_q \ll 1$
$\overline{m}_c(3\,\mathrm{GeV}) \approx 1\,\mathrm{GeV}$:

$$a^{-1} \gtrsim 2.0\,\mathrm{GeV} \quad \text{or} \quad a \lesssim 0.1\,\mathrm{fm}$$

Combining this we require $N \gtrsim 60$. (i.e. lattice size: $N^3 \times (2N)$)

Lattice charm challenges: Lattice spacings

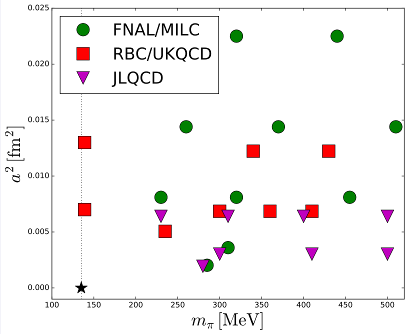
- In principle:

$$\mathcal{O}(a) = \mathcal{O}(a=0) + C_{CL}^1 a + C_{CL}^2 a^2 + \cdots$$

- Most actions are $O(a)$-improved, i.e. $C_{CL}^1 \equiv 0$
- Some actions only have even powers of a ($C_{CL}^{2n+1} \equiv 0$)
- We need multiple lattice spacings.
  BUT:
  $\Rightarrow$ At fixed V (same m_π) number of lattice points must increase.
  $\Rightarrow$ Autocorrelation times $\uparrow \infty$ as $a \downarrow 0$

Ensembles used for charm physics

$N_f = 2+1$



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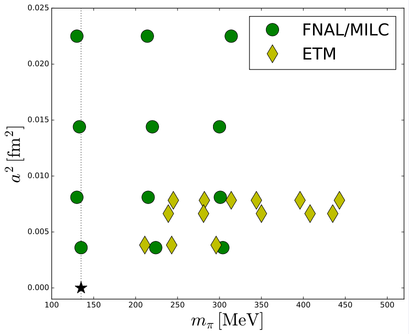
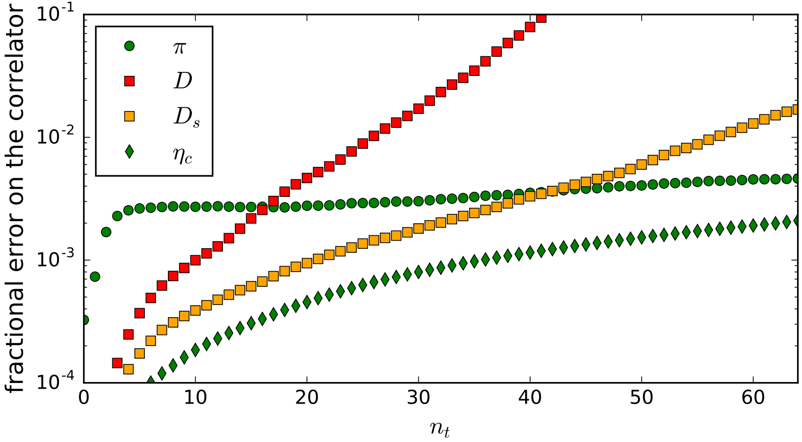


- Different fermion actions
- Only two sets of ensembles with **physical pion masses**

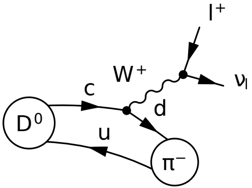
Lattice charm challenges: Signal-to-noise



$\Rightarrow$ Signal-to-noise decreases as $m_h - m_l$ increases

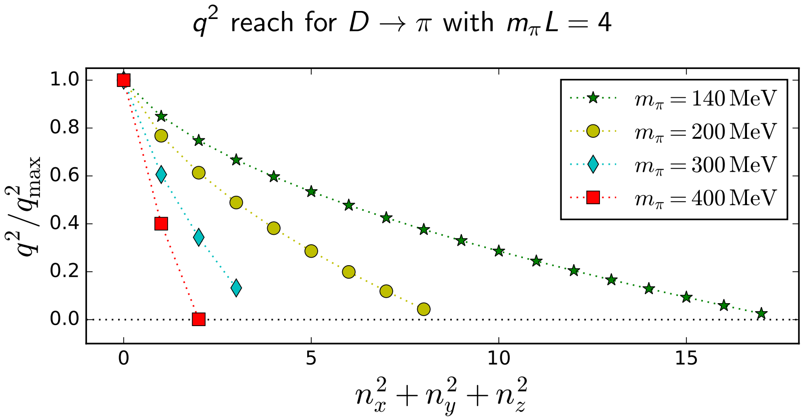
$\Rightarrow$ Signal-to-noise decreases as $m_l \downarrow m_l^{\mathrm{phys}}$

- Momentum transfer $q_\mu = (p_D - p_\pi)_\mu$
- $m_{D^0} \approx 1.865\,\mathrm{GeV}$, $m_{\pi^\pm} \approx 0.140\,\mathrm{GeV}$
- Assume $L \sim 6\,\mathrm{fm}$, $a^{-1} \sim 2.0\,\mathrm{GeV}$
- Quantised momenta (periodic BCs) $\mathbf{p} = \frac{2\pi}{L}\mathbf{n}$
- Signal $\downarrow$ as $|n| \uparrow$



$$0 \leq q^2 \leq q^2_{\mathrm{max}} = (m_D - m_\pi)^2 \sim (1.7\,\mathrm{GeV})^2$$

Easy to simulate at q^2_{max}
BUT experiment is best at $q^2 = 0$.



- More units of $\mathbf{n}$ needed as $m_\pi \downarrow m_\pi^{\mathrm{phys}}$ and $L \uparrow$
- Need $\mathbf{n} \sim (3, 2, 2)$ to reach $q^2 = 0$ for m_π^{phys}

Applications of charm physics on the lattice

- Extracting V_{cs} and V_{cd} from leptonic decay constants and semi-leptonic form factors
  $\Rightarrow$ Test CKM unitarity of second row

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- b-physics:
  $b \to c$ $(R_D,\ R_{D^*})$
  Combine relativistic sim's $(m_h < m_b)$ with RHQ sim's at m_b.
  inclusive vs exclusive B?
  $B - \bar{B}$ mixing

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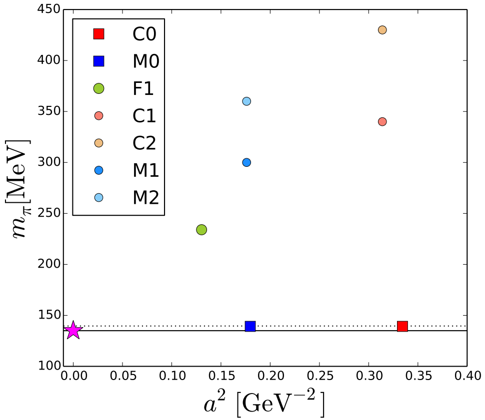
- Exploiting the GIM mechanism on the lattice
- Connected charm contribution to the Hadronic Vacuum Polarisation

Results

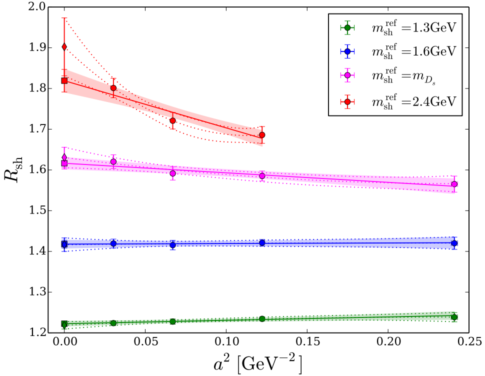
f_D and f_{D_s}

- 3 lattice spacings ($a_{\min} = 0.07\mathrm{fm}$)
- 2 physical pion mass ensembles
- $N_f = 2 + 1$
- DWF for sea and valence



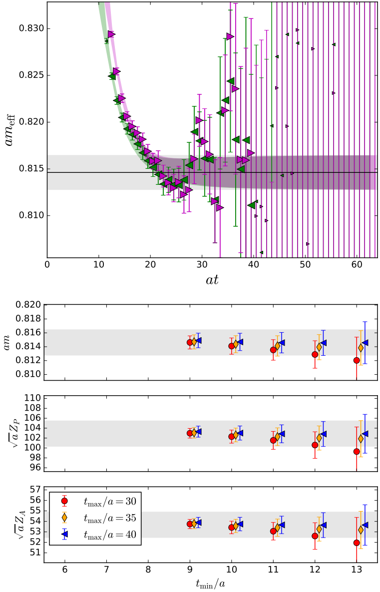
f_D and f_{D_s}

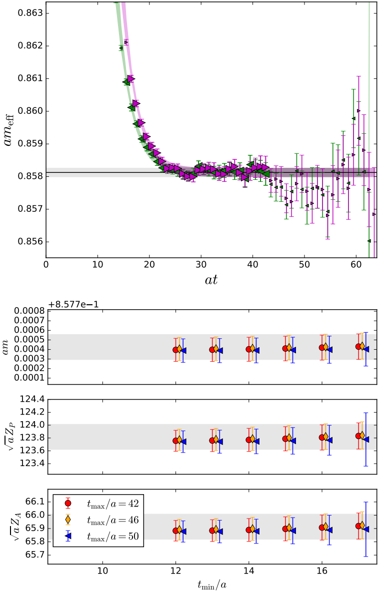
- 3 lattice spacings ($a_{\min} = 0.07\mathrm{fm}$)
- 2 physical pion mass ensembles
- $N_f = 2+1$
- DWF for sea and valence
- Slight modification of DW parameters for charm. $am_h \lesssim 0.4$ $\Rightarrow$ Mixed Action



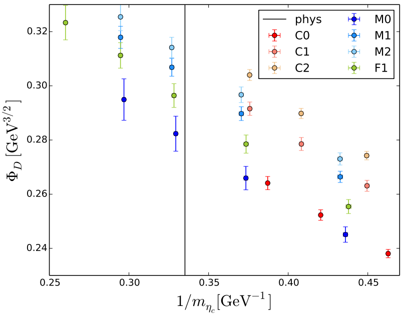
Tested in Pilot Study: arXiv:1602.04118

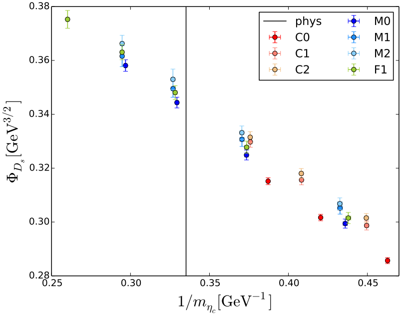
arXiv:1701.02644: Correlation functions





$$\Phi_P = f_P \sqrt{m_P}$$





$\Rightarrow$ mild a^2 behaviour

$\Rightarrow$ mild m_π^2 behaviour

- $\Rightarrow$ Slight extrapolation in am_h on coarse ensembles needed.
- $\Rightarrow$ Linear in m_π^2, a^2 and $1/m_H$

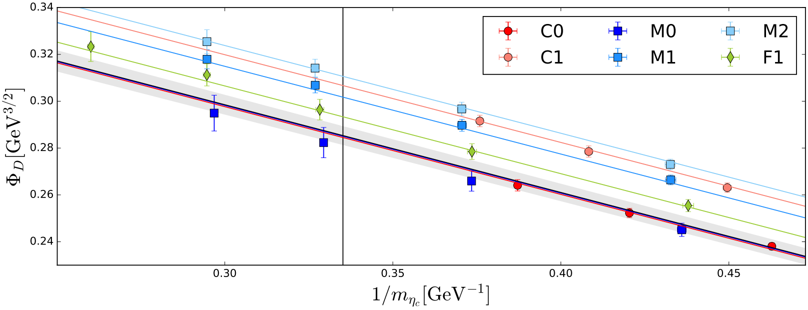
Choose fit form linear in m_π^2, a^2 and $1/m_H$:

$$\begin{aligned}\mathcal{O}(a, m_\pi, m_c) = \; & \mathcal{O}^{\rm phys}(0, m_\pi^{\rm phys}, m_c^{\rm phys}) \\ & + C_{CL}(\Delta m_H^{-1})\, a^2 \\ & + C_\chi(\Delta m_H^{-1}) \left[m_\pi^2 - m_\pi^{2\,\rm phys} \right] \\ & + C_h^0\, \Delta m_H^{-1}\end{aligned}$$

where

$$\Delta m_H^{-1} = 1/m_H - 1/m_H^{\rm phys} \qquad m_H = D, D_s, \eta_c$$

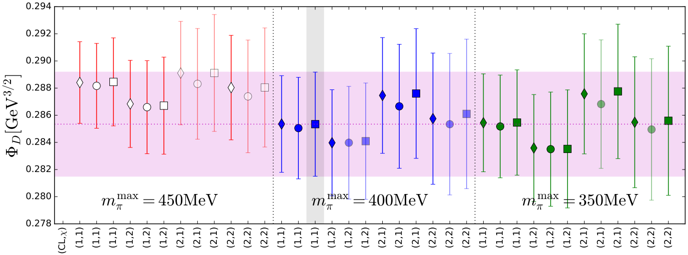
$$C_{CL,\chi}(\Delta m_H^{-1}) = C^0_{CL,\chi} + C^1_{CL,\chi} \Delta m_H^{-1}$$



$$\Phi_D = 0.2853(38)\,\mathrm{GeV}^{3/2}$$

$$C_{CL}^{0} = -0.003(11)\,\mathrm{GeV}^{7/2}$$

$$C_\chi = 0.230(22)\,\mathrm{GeV}^{-1/2}, \quad C_h = -0.3747(97)\,\mathrm{GeV}^{5/2}$$



- fix heavy quark with $H = D(\diamond)$, $D_s(\bigcirc)$, and $\eta_c^{\mathrm{connected}}(\square)$
- Pion mass cuts (350, 400, 450 MeV)
- w and w/o mass dependent CL and m_π coefficients

$$\Phi_D = 0.2853(38)_{\mathrm{stat}} \left(^{+24}_{-18}\right)_{\mathrm{fit}} \ \mathrm{GeV}^{3/2}$$

arXiv:1701.02644: Systematic Errors summary

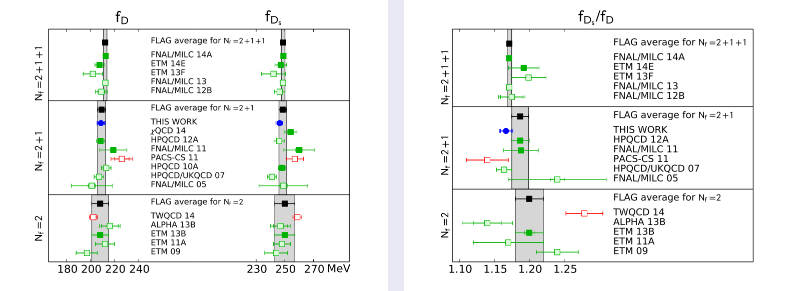
observable	central	stat	total sys	fit sys	renormalisation	finite volume	$m_u \neq m_d$	other
		$\times 10^4$				$\times 10^4$		
Φ_D [GeV$^{3/2}$]	0.2853	38	$^{+29}_{-24}$	$^{+24}_{-18}$	11	10	4.7	-
Φ_{D_s} [GeV$^{3/2}$]	0.3457	26	$^{+18}_{-26}$	$^{+\ 3}_{-19}$	14	6	4.4	7.1
f_{D_s}/f_D	1.1667	77	$^{+57}_{-43}$	$^{+44}_{-23}$	-	35	8	3

$$f_D = 208.7(2.8)_{\rm stat} \left(^{+2.1}_{-1.8}\right)_{\rm sys} \ \mathrm{MeV}$$

$$f_{D_s} = 246.4(1.9)_{\rm stat} \left(^{+1.3}_{-1.9}\right)_{\rm sys} \ \mathrm{MeV}$$

Leptonic Decay Constants: Summary of works

FLAG Review ($arXiv$: 1607.00299) + arXiv:1701.02644



N_f	f_D[MeV]	f_{D_s}[MeV]	f_{D_s}/f_D
2+1+1	212.15(1.45)	248.83(1.27)	1.1716(32)
2+1	209.2(3.3)	249.8(2.3)	1.187(12)
2	208(7)	250(7)	1.20(2)

Leptonic Decay Constants: Summary of works

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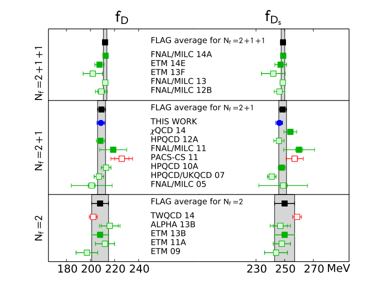
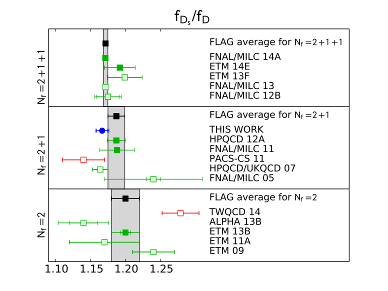




N_f	f_D[MeV]	f_{D_s}[MeV]	f_{D_s}/f_D
2+1+1	0.68%	0.51%	0.27%
2+1	1.58%	0.92%	1.01%
2	3.37%	2.80%	1.67%

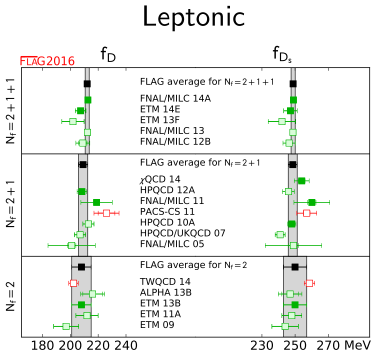
EM corrections become important!

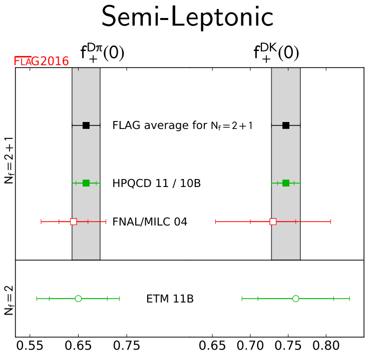
Leptonic decay constants: Ongoing works

Presented at Lattice'17

- JLQCD - Domain Wall fermions
  Preliminary, no full systematic yet. $m_\pi^{\mathrm{min}} = 230\,\mathrm{MeV}$
  $f_D = 212.8(1.7)_{\mathrm{stat}}(3.6)_{\mathrm{scale}}\,\mathrm{MeV}$
  $f_{D_s} = 244.0(0.8)_{\mathrm{stat}}(4.1)_{\mathrm{scale}}\,\mathrm{MeV}$
- FNAL/MILC - Highly Improved Staggered Quarks
  ($N_f = 2+1+1, m_\pi^{\mathrm{phys}}$), - preliminary
  $f_D = 212.7 \pm 0.3_{\mathrm{stat}} \pm 0.3_{\mathrm{sys}} \pm 0.2_{f_\pi,\mathrm{PDG}}\,\mathrm{MeV}$
  $f_{D_s} = 250.0 \pm 0.3_{\mathrm{stat}} \pm 0.2_{\mathrm{sys}} \pm 0.2_{f_\pi,\mathrm{PDG}}\,\mathrm{MeV}$
- α + RQCD - Wilson (CLS effort) - open BCs
- RBC/UKQCD - Domain Wall Fermions (new setup)
- Combined RBC/UKQCD/JLQCD data set

Summary of recent calculations: arXiv:1702.05360





- So far: Few published results for semi-leptonic form factors
- BUT: Ongoing activity

New ETM result: arXiv:1706.03017

ETM

- $N_f = 2 + 1 + 1$
- Twisted Mass
- 3 $a \in (0.062, 0.089)\,\mathrm{fm}$
- $m_\pi = 220 - 500\,\mathrm{MeV}$
- $L = 2.0 - 3.0\,\mathrm{fm}$
- $O(a)$-improved
- Momenta via twisted BCs
  $p_i = \frac{2\pi}{L}(\theta_i + n_i)$
  Democratic, 7 values of θ_i

  $\Rightarrow$ Many kinematical points from different combinations of twist

New ETM result: arXiv:1706.03017

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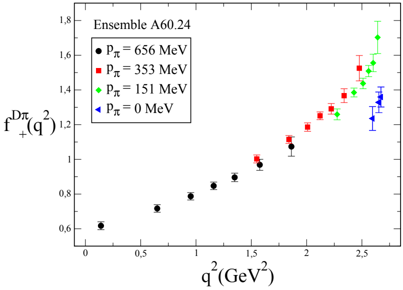

- On same ensemble
  (i.e. at same a, L, m_π, m_D)
- Why the disagreement?
  FSEs?

New ETM result: arXiv:1706.03017

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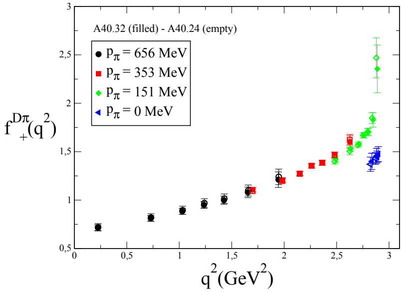
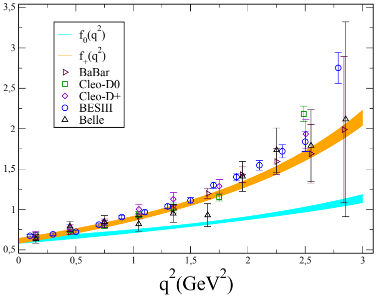


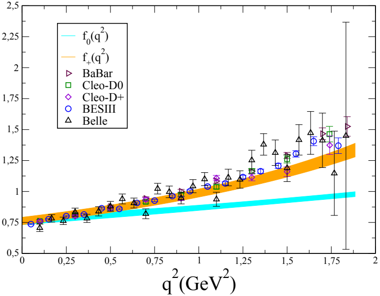
- Filled and open symbols only differ in volume.
- Discretisation effects due to breaking of hypercubic symmetry

New ETM result: arXiv:1706.03017

$D \to \pi$



$D \to K$



$$f_+^{D\pi}(0) = 0.612(35)$$

$$f_+^{DK}(0) = 0.765(31)$$

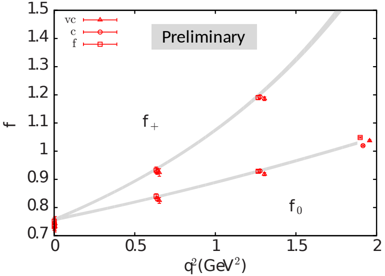
- Full range of q^2
- Chiral continuum limit
- WARNING: Evidence of hypercubic symmetry breaking effects

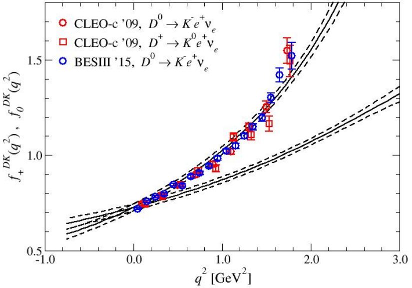
HPQCD

- $N_f = 2+1+1$
- HISQ
- 3 $a \in (0.09, 0.15)\,\mathrm{fm}$
- $m_\pi^{\mathrm{min}} = m_\pi^{\mathrm{phys}}$
- Momenta from twisting

JLQCD

- $N_f = 2+1$
- Domain Wall Fermions
- 3 $a \in (0.044, 0.079)\,\mathrm{fm}$
- $m_\pi^{\mathrm{min}} = 230\,\mathrm{MeV}$
- Fourier Momenta





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Summary

Conclusions and Outlook

Good control

- Relativistic charm formulations
- f_D, f_{D_s} at m_π^{phys} and 3a
- Charm quark mass
- Charm HVP

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Near-future goals

- Independent calculations of semi-leptonic form factors at full q^2 range and different kinematics
- Neutral Meson Mixing

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Challenges

- Extrapolations to b from charm data
- Finer lattices
- Tame Signal-to-noise problem
  $\Rightarrow$ due to mass difference
  $\Rightarrow$ due to large momenta
- Long distance contributions
- Include EM corrections

ADDITIONAL SLIDES

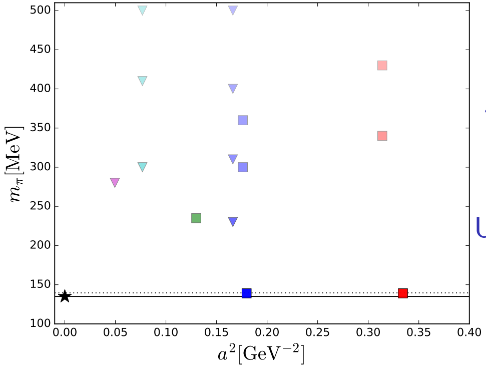
- $\mathbb{Z}_2$-Wall sources
- placed on many source planes
- binned into one effective measurements per config

Name	hits/conf	confs	total
C0	48	88	4224
C1	32	100	3200
C2	32	101	3232
M0	32	80	2560
M1	32	83	2656
M2	16	77	1232
F1	48	82	3936

- $\mathbb{Z}_2$-Wall sources
- placed on many source planes
- binned into one effective measurements per config
- strange quark mass slightly mistuned on some ensembles

Name	am_s^{phys}	am_s^{sim}
C0	0.03580(16)	0.0362
C1	0.03224(18)	
C2	0.03224(18)	
M0	0.02539(17)	0.02661
M1	0.02477(18)	
M2	0.02477(18)	
F1	0.02132(17)	0.02144

in collaboration with Guido Cossu, Brendan Fahy, Shoji Hashimoto



Both: Domain Wall Fermions

JLQCD: (triangles)

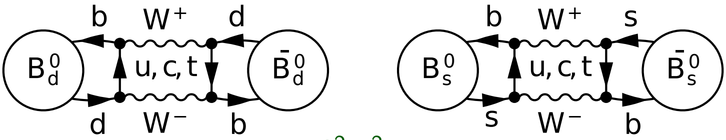
Fine lattices: $a^{-1} = 2.4 - 4.5\,\mathrm{GeV}$

UKQCD: (squares)

Physical Pion masses

Combine data sets for well controlled chiral and continuum limit.

$D - \overline{D}$-mixing long distance dominated for charm BUT short distance dominated for $B - \overline{B}$.

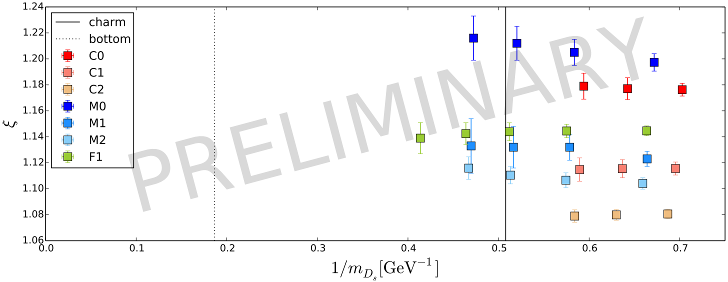


$$\Delta m_q = |V^*_{tq} V^*_{tb}| \times \frac{G_F^2 m_W^2}{16\pi^2 m_{B_q}} S_0(x_t)\eta_B \times \mathcal{M}_q$$

$$\begin{aligned}\mathcal{M}_q &= \langle \bar{B}^0_q | \left[\bar{b}\gamma^\mu(1-\gamma_5)q\right] \left[\bar{b}\gamma^\mu(1-\gamma_5)q\right] | B^0_q \rangle \\ &= \langle \bar{B}^0_q | O_{VV+AA} | B^0_q \rangle \\ &= \frac{8}{3} m^2_{B_q} f^2_{B_q} B_{B_q}\end{aligned}$$

FNAL/MILC arXiv:1706.04622: result for short distance contribution to $D - \bar{D}$ mixing

$$B_P = \frac{\langle P^0 | O_{VV+AA} | \bar{P}^0 \rangle}{\frac{8}{3} f_P^2 m_P^2}, \qquad \xi = \frac{f_{hs}\sqrt{B_{hs}}}{f_{hl}\sqrt{B_{hl}}}$$



Fits by A. Khamseh

- Data at charm: percent level precision
- Mild heavy mass dependence
- More data on disk
- Extrapolate to b