The HQE in the charm system

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Based on work in collaboration with
M. Kirk and A. Lenz

Why charm physics?

Charm quark plays a unique role in the Standard Model

- Top quark does not hadronize
- Charm sector offers the only handle on FCNC for up-type quarks
- Strong GIM suppression in charm $\Rightarrow$ highly sensitive to new physics

Huge amount of experimental data in charm sector

But: difficult for theory

Heavy quark expansion in charm?

B-physics: HQE is well established approach, $\Lambda/m_b \sim 0.2 \ll 1$

$$\Delta\Gamma_s^{\rm exp} = (0.086 \pm 0.006)\,{\rm ps}^{-1}, \qquad \Delta\Gamma_s^{\rm SM} = (0.088 \pm 0.020)\,{\rm ps}^{-1}.$$

[HFAG '16] [Artuso, Borissov, Lenz '16]

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BUT: HQE is really an expansion in $\Lambda/\text{momentum release}$

- $\Delta\Gamma_s$ dominated by $\mathrm{D}_s^{(*)+} D_s^{(*)-}$ final state, momentum release $\sim 3.5\ \mathrm{GeV}$
- D decays dominated by $\mathrm{K}\pi^{(1-3)}$ final state, momentum release $\sim 1.7\ \mathrm{GeV}$
- expected expansion parameter is of the order 0.4

Small enough for convergence?

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Shut up and calculate!

Outline

- Introduction to HQE
- D-meson lifetime ratios
- Matrix elements from HQET sum rules
- D mixing within the HQE
  - Naive application
  - Duality violations
  - Higher-dimensional contributions
- Conclusions & outlook

The Heavy Quark Expansion (HQE)

Use optical theorem:

$$\Gamma(D) = \frac{1}{2M_D} \left\langle D \right| \mathrm{Im} \left(i \int d^4x T \left[\mathcal{H}_{\mathrm{eff}}(x) \mathcal{H}_{\mathrm{eff}}(0) \right] \right) \left| D \right\rangle$$

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OPE for small x, i.e. large momentum release

$$\begin{aligned}
\Gamma(D \to f) = & \frac{G_F^2 m_c^5}{192\pi^3} |V_{CKM}|^2 \frac{1}{2M_D} \Bigg[c_3^f \langle D|\bar{c}c|D\rangle \\
& + c_5^f \frac{\langle D|\bar{c} g_s \sigma_{\mu\nu} G^{\mu\nu} c|D\rangle}{m_c^2} \\
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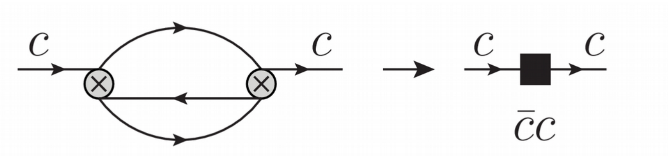

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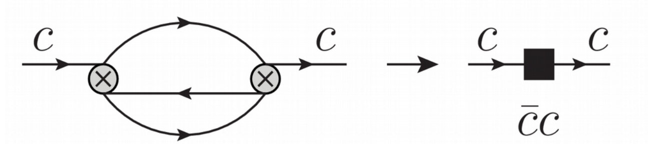

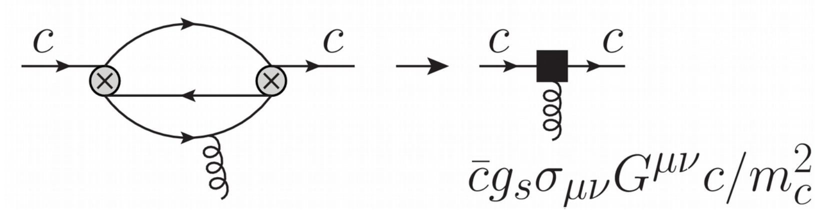

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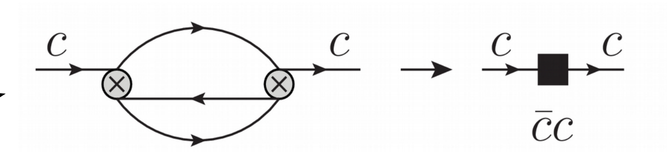
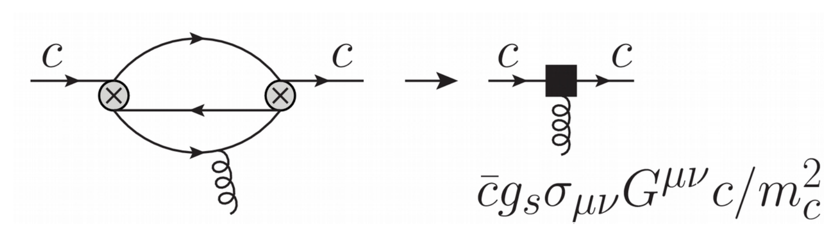
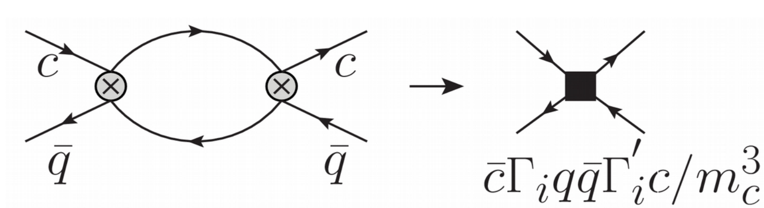






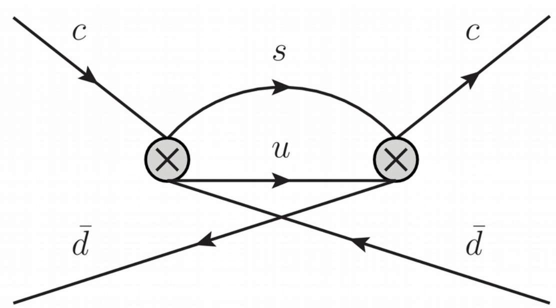
D-meson lifetimes

Large lifetime ratio: $\left(\frac{\tau(D^+)}{\tau(D^0)}\right)_{\text{exp}} = 2.536 \pm 0.019$

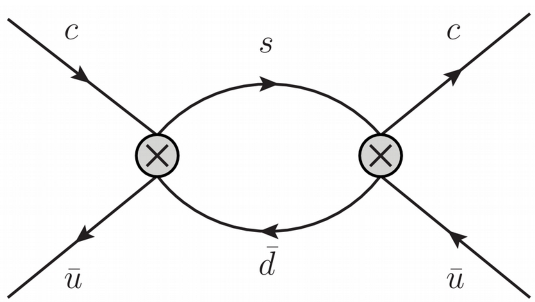
Dominant contribution from spectator effects:

- Phase-space enhancement of $16\pi^2$,
  2 → 2 process instead of 1 → 3

Known at NLO in QCD:
[Beneke, Buchalla, Greub, Lenz, Nierste '02]
[Ciuchini, Franco, Lubicz, Mescia '01]
[Franco, Lubicz, Mescia, Tarantino '02]



Pauli interference



Weak annihilation

Studied in [Lenz, TR '13] including NLO QCD and $1/m_c$ corrections.
Large hadronic uncertainties from missing lattice input!

Non-perturbative input

Need hadronic matrix elements of the dimension-six operators:

$$Q^q = \bar{c}\gamma_\mu(1-\gamma_5)q\ \bar{q}\gamma^\mu(1-\gamma_5)c, \qquad Q_S^q = \bar{c}(1-\gamma_5)q\ \bar{q}(1+\gamma_5)c,$$
$$T^q = \bar{c}\gamma_\mu(1-\gamma_5)T^a q\ \bar{q}\gamma^\mu(1-\gamma_5)T^a c, \qquad T_S^q = \bar{c}(1-\gamma_5)T^a q\ \bar{q}(1+\gamma_5)T^a c.$$

Commonly parametrized through Bag parameters:

$$\left\langle D^+|Q^d - Q^u|D^+\right\rangle = f_D^2 M_D^2\, B_1, \qquad \left\langle D^+|Q_S^d - Q_S^u|D^+\right\rangle = f_D^2 M_D^2\, B_2,$$
$$\left\langle D^+|T^d - T^u|D^+\right\rangle = f_D^2 M_D^2\, \epsilon_1, \qquad \left\langle D^+|T_S^d - T_S^u|D^+\right\rangle = f_D^2 M_D^2\, \epsilon_2.$$

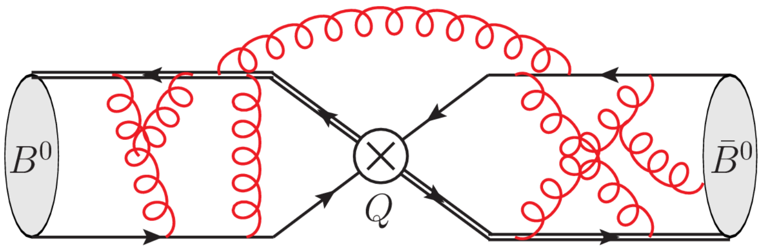
Inspired by vacuum saturation approximation (VSA):

$$\langle D|\bar{c}\Gamma q\bar{q}\Gamma' c|D\rangle = \sum_X \langle D|\bar{c}\Gamma q|X\rangle\,\langle X|\bar{q}\Gamma' c|D\rangle \approx \langle D|\bar{c}\Gamma q|0\rangle\,\langle 0|\bar{q}\Gamma' c|D\rangle$$

This yields:

$$\mathrm{B}_i^{\mathrm{VSA}} = 1 \pm \tfrac{1}{N_c},\ \ \epsilon_i^{\mathrm{VSA}} = 0 \pm \tfrac{1}{N_c}.$$

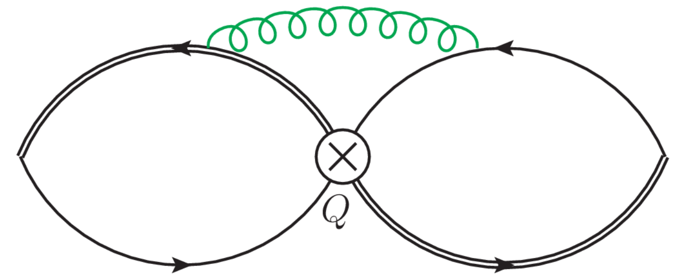
Sum rule determination



Sum rule

Quark-hadron duality

Analyticity



Hadronic matrix element	Correlation function
Characteristic scale: Λ_{QCD}	Characteristic scale: 'virtuality' ω
$\alpha_s\left(\Lambda_{\mathrm{QCD}}\right) \sim \mathcal{O}(1)$	Choose ω s.t. $\alpha_s(\omega) \ll 1$
$\Rightarrow$ non-perturbative	$\Rightarrow$ perturbatively calculable

$$\mathrm{F}^2(\mu)\langle\tilde{\mathcal{O}}(\mu)\rangle\, e^{-\frac{\overline{\Lambda}}{t_1}-\frac{\overline{\Lambda}}{t_2}} = \int_0^{\omega_c} d\omega_1 d\omega_2\, e^{-\frac{\omega_1}{t_1}-\frac{\omega_2}{t_2}}\, \rho_{\tilde{\mathcal{O}}}^{\mathrm{OPE}}(\omega_1,\omega_2).$$

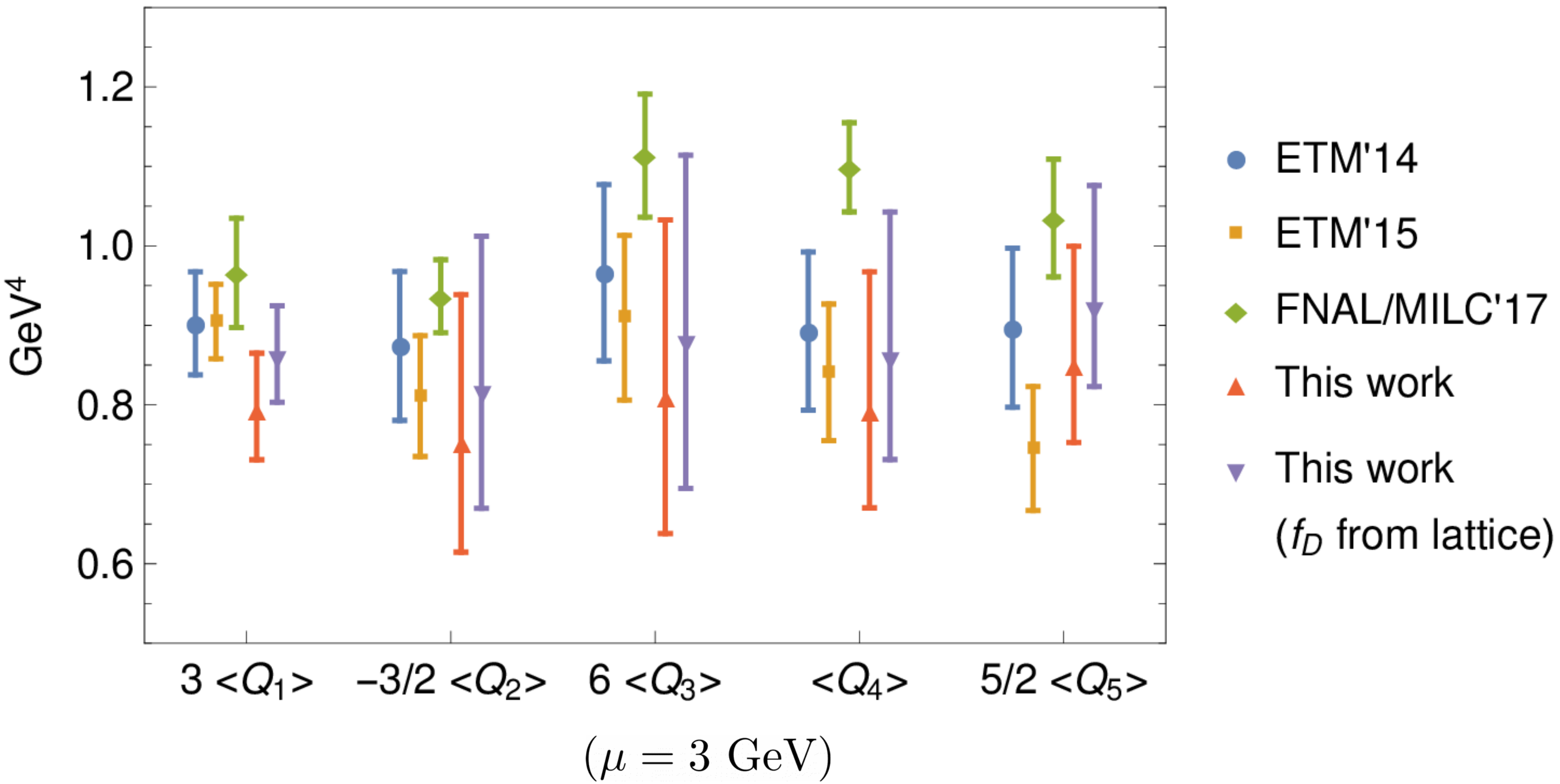
$$\begin{aligned}\rho_{\tilde{\mathcal{O}}}^{\mathrm{OPE}}(\omega_1,\omega_2) = {} & \rho_{\tilde{\mathcal{O}}}^{\mathrm{pert}}\left(\frac{\omega_1}{\omega_2}\right)\omega_1^2\omega_2^2 + \rho_{\tilde{\mathcal{O}}}^{\langle\bar{q}q\rangle}\left(\frac{\omega_1}{\omega_2}\right)\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle\left[\omega_2^2\delta(\omega_1)+\omega_1^2\delta(\omega_2)\right] + \\ & \rho_{\tilde{\mathcal{O}}}^{\langle G^2\rangle}\left(\frac{\omega_1}{\omega_2}\right)\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle + \rho_{\tilde{\mathcal{O}}}^{\langle\bar{q}G^2q\rangle}\left(\frac{\omega_1}{\omega_2}\right)\langle g_s\bar{q}\sigma_{\mu\nu}G^{\mu\nu}q\rangle\left[\delta(\omega_1)+\delta(\omega_2)\right] \\ & + \ldots\end{aligned}$$

B mixing Bag parameter B_1 determined recently
[Grozin, Klein, Mannel, Pivovarov '16]

Preliminary results (Mixing)

Comparison of D-mixing matrix elements with recent lattice results

[Lenz, Kirk, TR, 17xx.yyyyy]



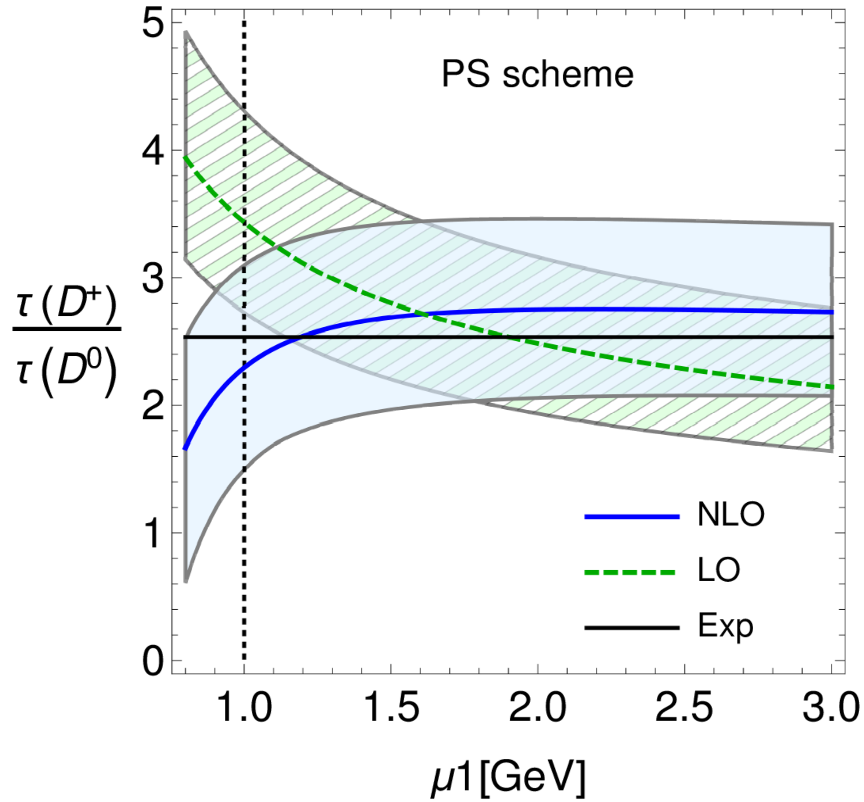
Lifetime difference

$$\begin{aligned}
\overline{B}_1(3\,\text{GeV}) &= 0.902\,^{+0.077}_{-0.051} = 0.902\,^{+0.018}_{-0.018}\,(\text{sum rule})\,^{+0.075}_{-0.048}\,(\text{matching}),\\
\overline{B}_2(3\,\text{GeV}) &= 0.739\,^{+0.124}_{-0.073} = 0.739\,^{+0.015}_{-0.015}\,(\text{sum rule})\,^{+0.123}_{-0.072}\,(\text{matching}),\\
\overline{\epsilon}_1(3\,\text{GeV}) &= -0.132\,^{+0.041}_{-0.046} = -0.132\,^{+0.025}_{-0.026}\,(\text{sum rule})\,^{+0.033}_{-0.038}\,(\text{matching}),\\
\overline{\epsilon}_2(3\,\text{GeV}) &= -0.005\,^{+0.032}_{-0.032} = -0.005\,^{+0.011}_{-0.012}\,(\text{sum rule})\,^{+0.030}_{-0.030}\,(\text{matching}).
\end{aligned}$$

$$\begin{aligned}
\left.\frac{\tau(D^+)}{\tau(D^0)}\right|_{\text{exp}} &= 2.536 \pm 0.019,\\
\left.\frac{\tau(D^+)}{\tau(D^0)}\right|_{\overline{\text{MS}}} &= 2.59^{+0.72}_{-0.77} = 2.59\,^{+0.70}_{-0.66}\,(\text{had.})\,^{+0.12}_{-0.38}\,(\text{scale}) \pm 0.09\,(\text{param.}),\\
\left.\frac{\tau(D^+)}{\tau(D^0)}\right|_{\text{PS}} &= 2.68^{+0.74}_{-0.82} = 2.68\,^{+0.72}_{-0.68}\,(\text{had.})\,^{+0.11}_{-0.45}\,(\text{scale}) \pm 0.10\,(\text{param.}),\\
\left.\frac{\tau(D^+)}{\tau(D^0)}\right|_{\text{1S}} &= 2.54^{+0.81}_{-0.99} = 2.54\,^{+0.78}_{-0.74}\,(\text{had.})\,^{+0.22}_{-0.65}\,(\text{scale}) \pm 0.10\,(\text{param.}),\\
\left.\frac{\tau(D^+)}{\tau(D^0)}\right|_{\text{kin}} &= 2.52^{+0.72}_{-0.76} = 2.52\,^{+0.70}_{-0.66}\,(\text{had.})\,^{+0.13}_{-0.37}\,(\text{scale}) \pm 0.10\,(\text{param.}).
\end{aligned}$$

[Lenz, Kirk, TR, 17xx.yyyyy]

Lifetime difference



Good convergence:

NLO QCD +28%, 1/mc -34%.

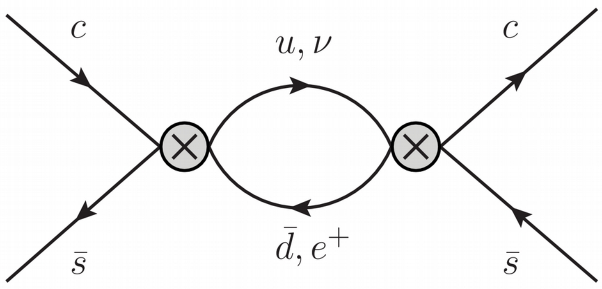
Good behaviour under scale variation above about 1 GeV.

Possible future improvements:

- Dimension seven matrix elements and NLO matching coefficients
- Lattice input for dimension six matrix elements
- Could reduce uncertainty by a factor two!

More lifetimes and semileptonic rates

D_s^+: Cabibbo favoured spectator effects in semileptonic rates, used in [Lenz, TR, '13] to constrain combinations of matrix elements



$$\left[\frac{\Gamma(D_s^+ \to X e^+ \nu)}{\Gamma(D^0 \to X e^+ \nu)}\right]_{\text{exp}} = 0.821 \pm 0.054$$

$$\left[\frac{\Gamma(D_s^+ \to X e^+ \nu)}{\Gamma(D^0 \to X e^+ \nu)}\right]_{\text{th}} = 1 + A(B_1^s - B_2^s) + B(\epsilon_1^s - \epsilon_2^s) + \ldots$$

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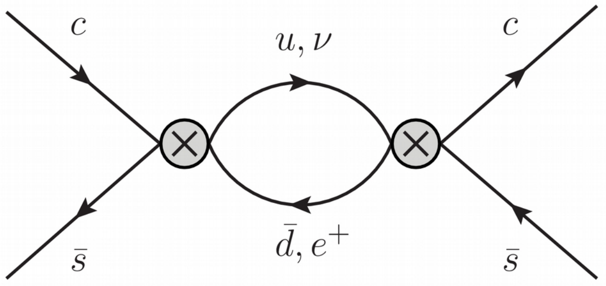

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➡

$$\left(\frac{\overline{\tau}(D_s^+)}{\tau(D^0)}\right)_{\rm exp} = 1.292 \pm 0.019,$$
$$\left(\frac{\overline{\tau}(D_s^+)}{\tau(D^0)}\right)_{\overline{\rm MS}} = 1.19 \pm 0.12^{(\rm hadronic)} \pm 0.04^{(\rm scale)} \pm 0.01^{(\rm exp)}.$$

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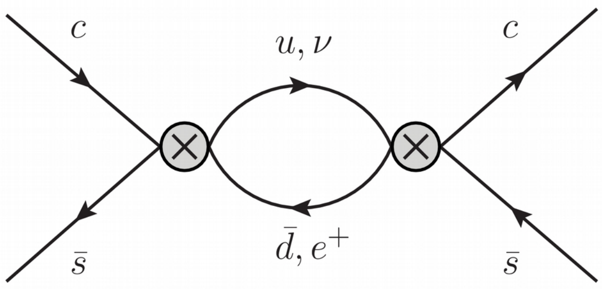

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Observable	HQE estimation	Experiment
$\tau(\Xi_c^+)/\tau(\Lambda_c^+)$	~ 2.1	2.21 ± 0.15
$\tau(\Xi_c^+)/\tau(\Xi_c^0)$	~ 3.2	3.95 ± 0.48
$\Gamma(\Xi_c^+ \to e^+ \text{ anything})/\Gamma(\Lambda_c^+ \to e^+ \text{ anything})$	~ 1.8	$---$
$\Gamma(\Xi_c^0 \to e^+ \text{ anything})/\Gamma(\Lambda_c^+ \to e^+ \text{ anything})$	~ 1.8	$---$

D mixing

$$2\mathrm{M}_D \left(M - \frac{i\Gamma}{2}\right)_{12} = \left\langle D^0 | H^{|\Delta C|=2} | \overline{D}^0 \right\rangle + \sum_n \frac{\left\langle D^0 | H^{|\Delta C|=1} | n \right\rangle \left\langle n | H^{|\Delta C|=1} | \overline{D}^0 \right\rangle}{M_D - E_n + i0}$$

Short-distance part only contributes to M_{12}

Long-distance part contributes to both

HQE for long-distance part:

Dimension six at NLO in QCD:

[Beneke, Buchalla, Greub, Lenz, Nierste '98]

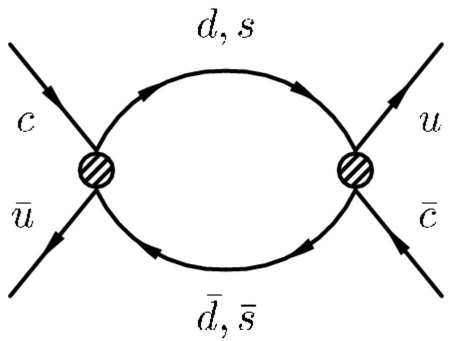
[Beneke, Buchalla, Lenz, Nierste '03]

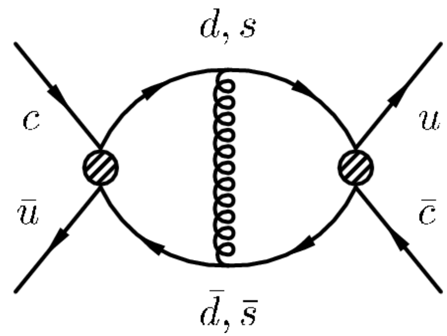
[Ciuchini, Franco, Lubicz, Mescia, Tarantino '03]

Dimension seven at LO:

[Beneke, Buchalla, Dunietz '96]

$$\Gamma_{12} = \frac{G_F^2 m_c^5}{192\pi^3} |V_{CKM}|^2 \frac{1}{2M_D} \left[\sum_i c_{6,i} \frac{\langle D | Q_i | \overline{D} \rangle}{m_c^3} + \sum_i c_{7,i} \frac{\langle D | R_i | \overline{D} \rangle}{m_c^4} + \mathcal{O}\left(\frac{1}{m_c^5}\right) \right].$$





D-mixing results

$$x_D \equiv \frac{\Delta M_D}{\Gamma_D}, \qquad y_D \equiv \frac{2\Delta\Gamma_D}{\Gamma_D},$$

$$x_D^{\exp} = (0.32 \pm 0.14) \qquad y_D^{\exp} = (0.69^{+0.06}_{-0.07})\%,$$ [HFAG '16]

$$x_D^{\mathrm{SD}} \sim 10^{-7}, \qquad y_D^{\mathrm{D=6,7\ NLO}} \leq 4.7 \cdot 10^{-7} \ldots 1.6 \cdot 10^{-6}.$$ [Bobrowski, Lenz, Riedl, Rohrwild '10]

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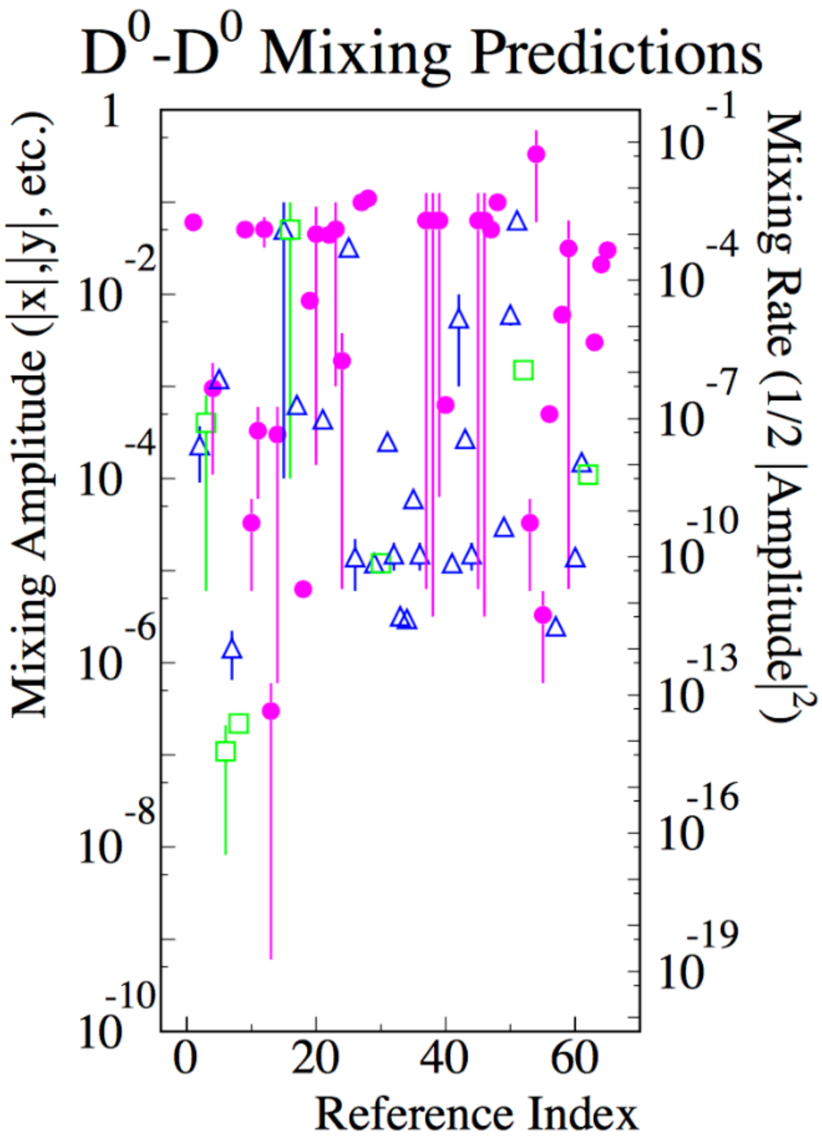
No! Must investigate origin of this suppression.

Overview of predictions for D mixing



GIM cancellations

$$\lambda_q = V_{cq} V_{uq}^*$$

[Bobrowski, Lenz, Riedl, Rohrwild '10]

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$$\begin{aligned}&0.048 - 3.5\cdot 10^{-6} i \\ &\approx \lambda^{2.0} - \lambda^{8.4} i\end{aligned}$$

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Large phases possible!!

[Bobrowski, Lenz, Riedl, Rohrwild '10]

Duality violations?

Huge sensitivity to any mechanism that breaks GIM cancellation!

Effects of O(few %) duality violation:

- lifetimes: few %
- mixing: oder(s) of magnitude, DVs are naturally SU(3) breaking

ΔM_D expected to be less sensitive to DV than $\Delta\Gamma_D$.

$$\Delta M_D = \frac{1}{2\pi} PV \int d\omega \frac{\Delta\widetilde{\Gamma}_D(\omega)}{\omega}$$

DV should cancel to some extend in the dispersion integral.

$\mathrm{y}_D^{\mathrm{exp}} > x_D^{\mathrm{exp}} \gg x_D^{\mathrm{HQE}}, y_D^{\mathrm{HQE}}$ seems like the natural scenario for a breakdown of the HQE in mixing due to duality violations.

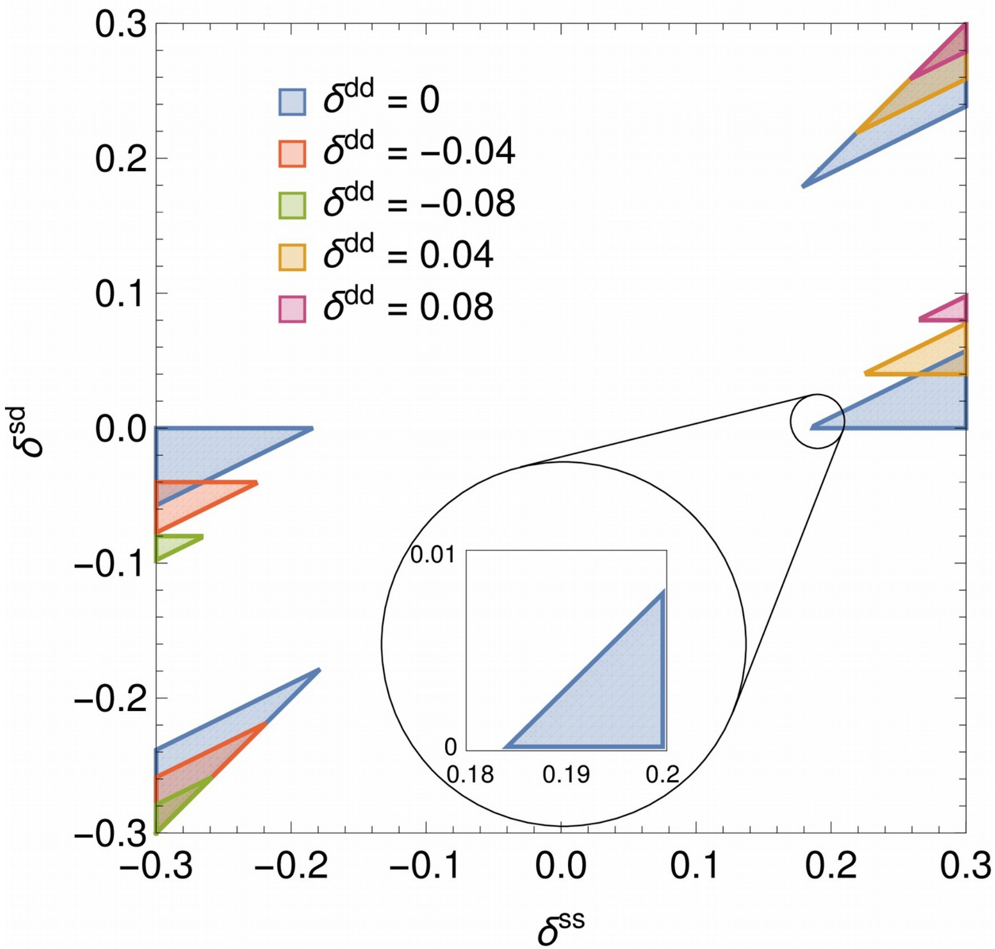
Duality violations?

$$\Gamma_{12}^{ss} \to \Gamma_{12}^{ss}(1+\delta^{ss})$$

$$\Gamma_{12}^{sd} \to \Gamma_{12}^{sd}(1+\delta^{sd})$$

$$\Gamma_{12}^{dd} \to \Gamma_{12}^{dd}(1+\delta^{dd})$$

Duality violation of the order 20% can explain $\Delta\Gamma_D^{\rm exp}$ and are not yet probed with D meson lifetimes.



[Jubb, Kirk, Lenz, Tetlalmatzi-Xolocotzi'16]

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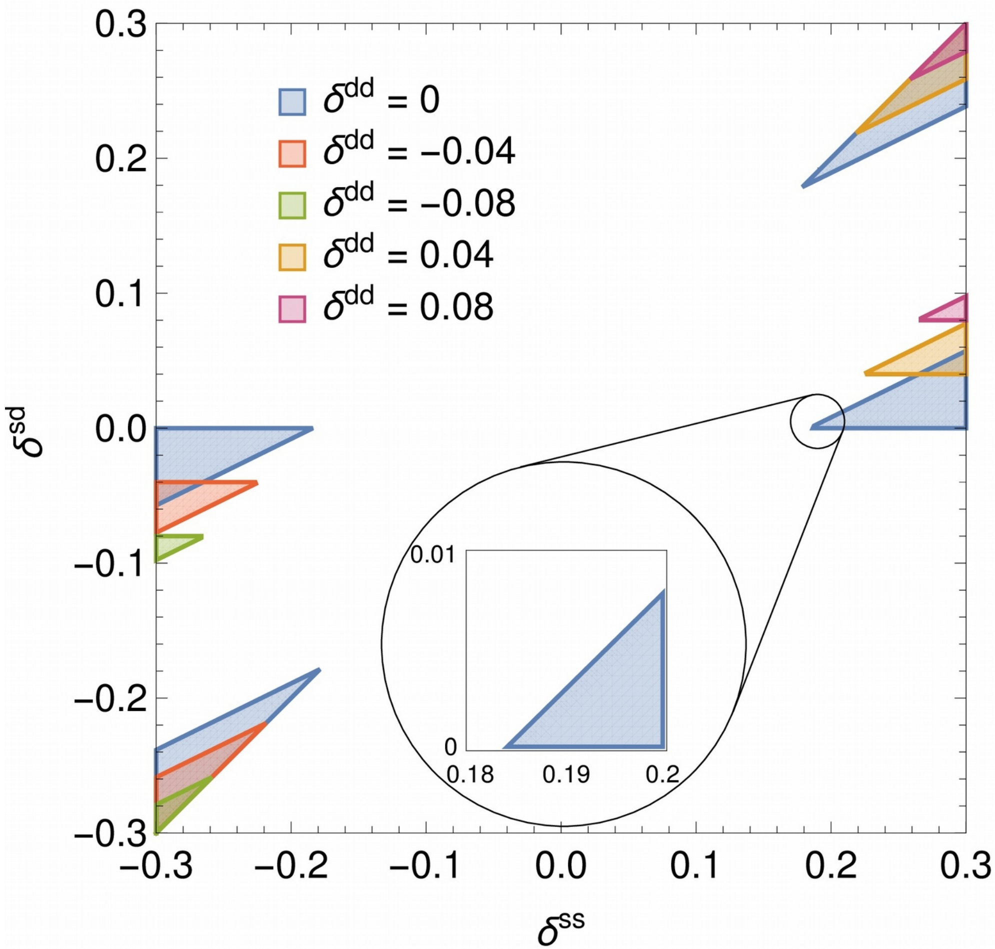
The final verdict?



[Jubb, Kirk, Lenz, Tetlalmatzi-Xolocotzi'16]

The final verdict?

D^0–$\overline{D}^0$ oscillations as a probe of quark–hadron duality

I.I. Bigi [a], N.G. Uraltsev [a,b,*]

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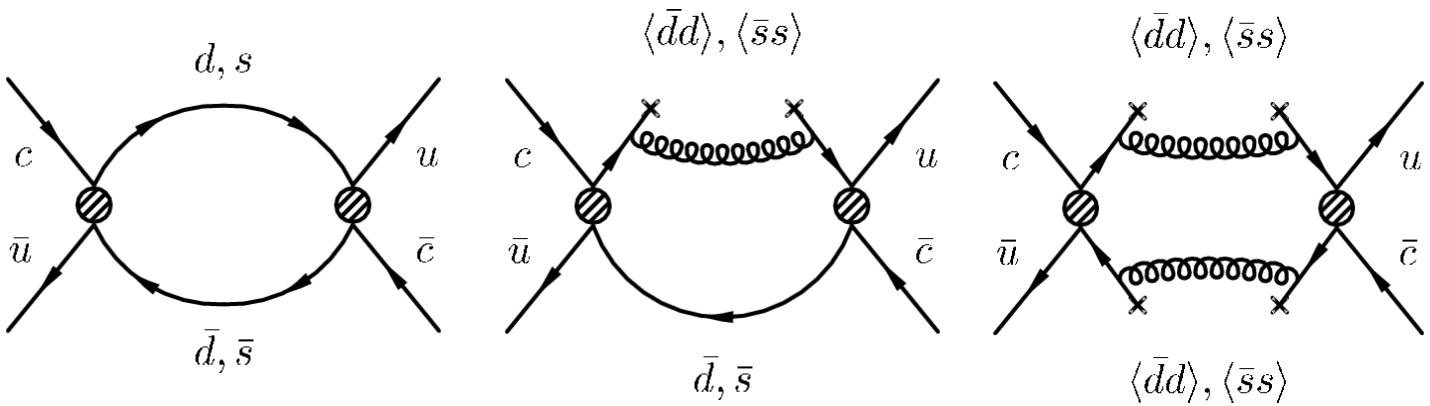
Abstract

It is usually argued that the Standard Model predicts slow D^0–$\overline{D}^0$ oscillations with $\Delta M_D, \Delta\Gamma_D \leqslant 10^{-3}\,\Gamma_D$ and that New Physics can reveal itself through ΔM_D exceeding $10^{-3}\,\Gamma_D$. It is believed that the bulk of the effect is due to long distance dynamics that cannot be described at the quark level. We point out that in general the OPE yields soft GIM suppression scaling only like $(m_s/\mu_{\mathrm{hadr}})^2$ and even like m_s/μ_{hadr} rather than m_s^4/m_c^4 of the simple quark box diagram. Such contributions can actually yield $\Delta M_D, \Delta\Gamma_D \sim \mathcal{O}(10^{-3})\,\Gamma_D$ without invoking additional long distance effects. They are reasonably suppressed as long as the OPE and local duality are qualitatively applicable in the $1/m_c$ expansion. We stress the importance of improving the sensitivity on $\Delta\Gamma_D$ as well as ΔM_D in a dedicated fashion as a laboratory for analyzing the onset of quark–hadron duality and comment on the recent preliminary study on $\Delta\Gamma_D$ by the FOCUS group.

Higher-dimensional effects

GIM cancellations softer at higher orders in the HQE:

[Georgi '92]
[Ohl, Ricciardi, Simmons '93]
[Bigi, Uraltsev '01]
[Bobrowski, Lenz, Riedl, Rohrwild '10]



$(\Gamma_{ss} - 2\Gamma_{sd} + \Gamma_{dd})$	$\bar{z}^3, \ \alpha_s \bar{z}^2$	$\alpha_s \bar{z}^{3/2} \frac{\overline{\Lambda}^3}{m_c^3}$	$\alpha_s^2 \bar{z} \frac{\overline{\Lambda}^6}{m_c^6}$
$(\Gamma_{sd} - \Gamma_{dd})$	$\bar{z}$	$\alpha_s \bar{z}^{1/2} \frac{\overline{\Lambda}^3}{m_c^3}$	$\alpha_s^2 \frac{\overline{\Lambda}^6}{m_c^6}$
Γ_{dd}	1	0	$\alpha_s^2 \frac{\overline{\Lambda}^6}{m_c^6}$

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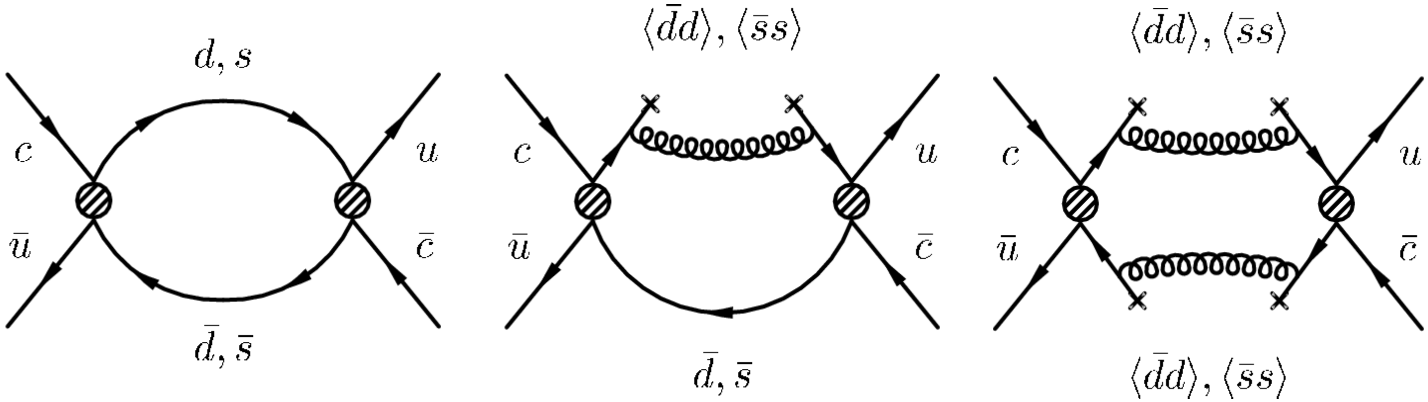

$(\Gamma_{ss} - 2\Gamma_{sd} + \Gamma_{dd})$	$\lambda^{9.7},\ \lambda^{7.1}$	$\lambda^{7.0-7.4}$	$\lambda^{7.4-8.3}$
$(\Gamma_{sd} - \Gamma_{dd})$	$\lambda^{3.2}$	$\lambda^{3.7-4.2}$	$\lambda^{4.2-5.1}$
Γ_{dd}	1	0	$\lambda^{4.2-5.1}$

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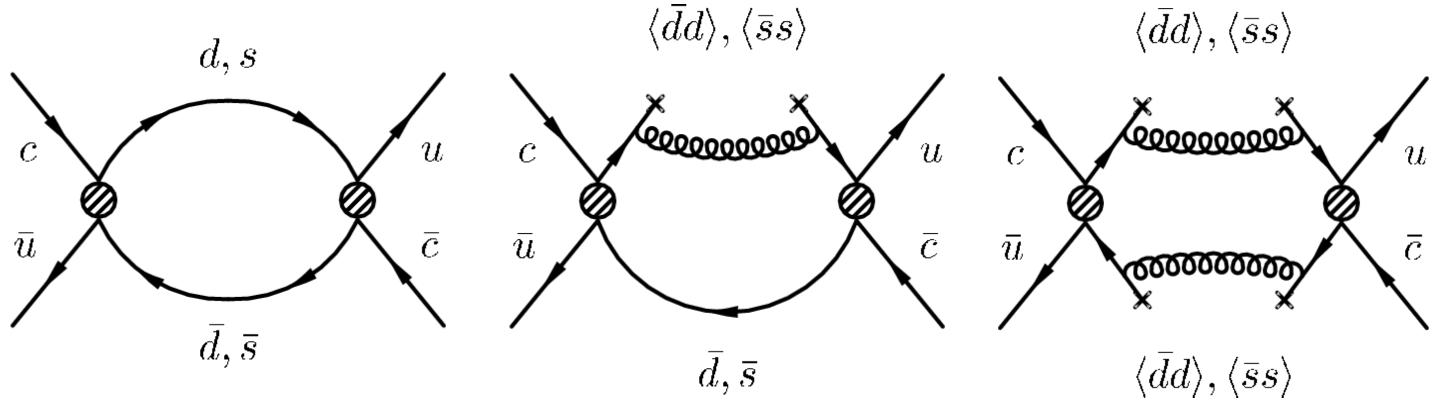

$(\Gamma_{ss} - 2\Gamma_{sd} + \Gamma_{dd})$	$\lambda^{9.7},\ \lambda^{7.1}$	Further enhanced by $\bar{z}^{-1/2} = \lambda^{-1.6}$?	$\lambda^{7.4-8.3}$
$(\Gamma_{sd} - \Gamma_{dd})$	$\lambda^{3.2}$	[Bigi, Uraltsev '01]	$\lambda^{4.2-5.1}$
Γ_{dd}	1	0	$\lambda^{4.2-5.1}$

Conclusions & outlook

- HQE provides good description of lifetimes in charm sector

$$\left.\frac{\tau(D^+)}{\tau(D^0)}\right|_{\mathrm{exp}} = 2.536 \pm 0.019\,, \qquad \left.\frac{\tau(D^+)}{\tau(D^0)}\right|_{\mathrm{HQE}} = 2.7^{+0.7}_{-0.8}\,,$$

$$\left.\frac{\overline{\tau}(D_s^+)}{\tau(D^0)}\right|_{\mathrm{exp}} = 1.292 \pm 0.019\,, \qquad \left.\frac{\overline{\tau}(D_s^+)}{\tau(D^0)}\right|_{\mathrm{HQE}} = 1.19 \pm 0.13\,.$$

- Matrix elements from HQET sum rules. What about lattice?

- Huge GIM cancellations in D-mixing
- 'Naive application' of HQE fails by orders of magnitude
- Very sensitive to SU(3) breaking effects
- Duality violations vs. higher-dimensional contributions vs. NP