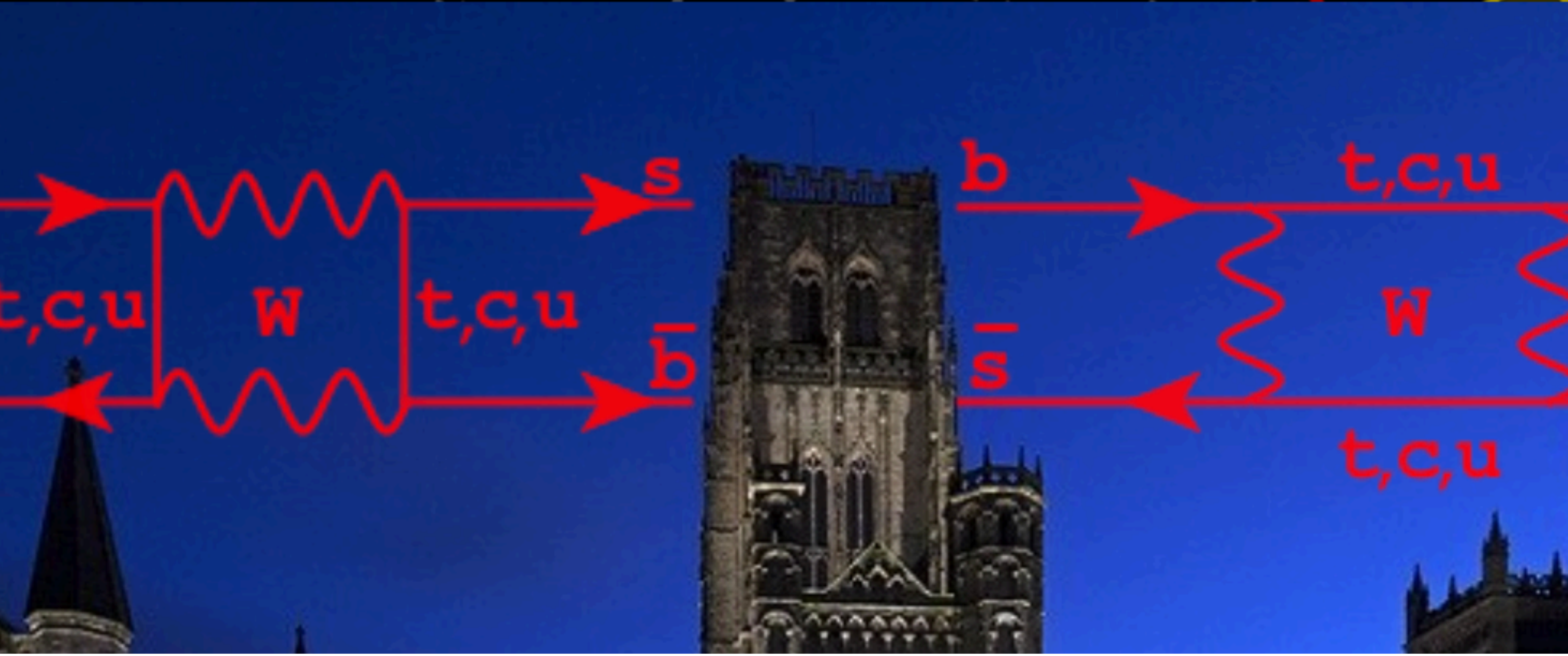
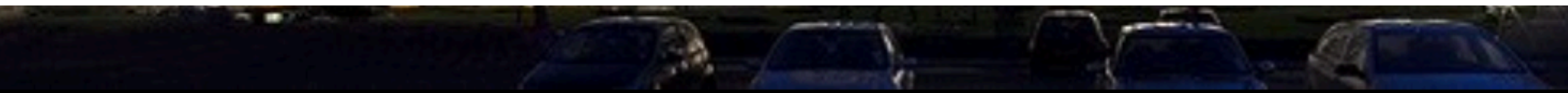


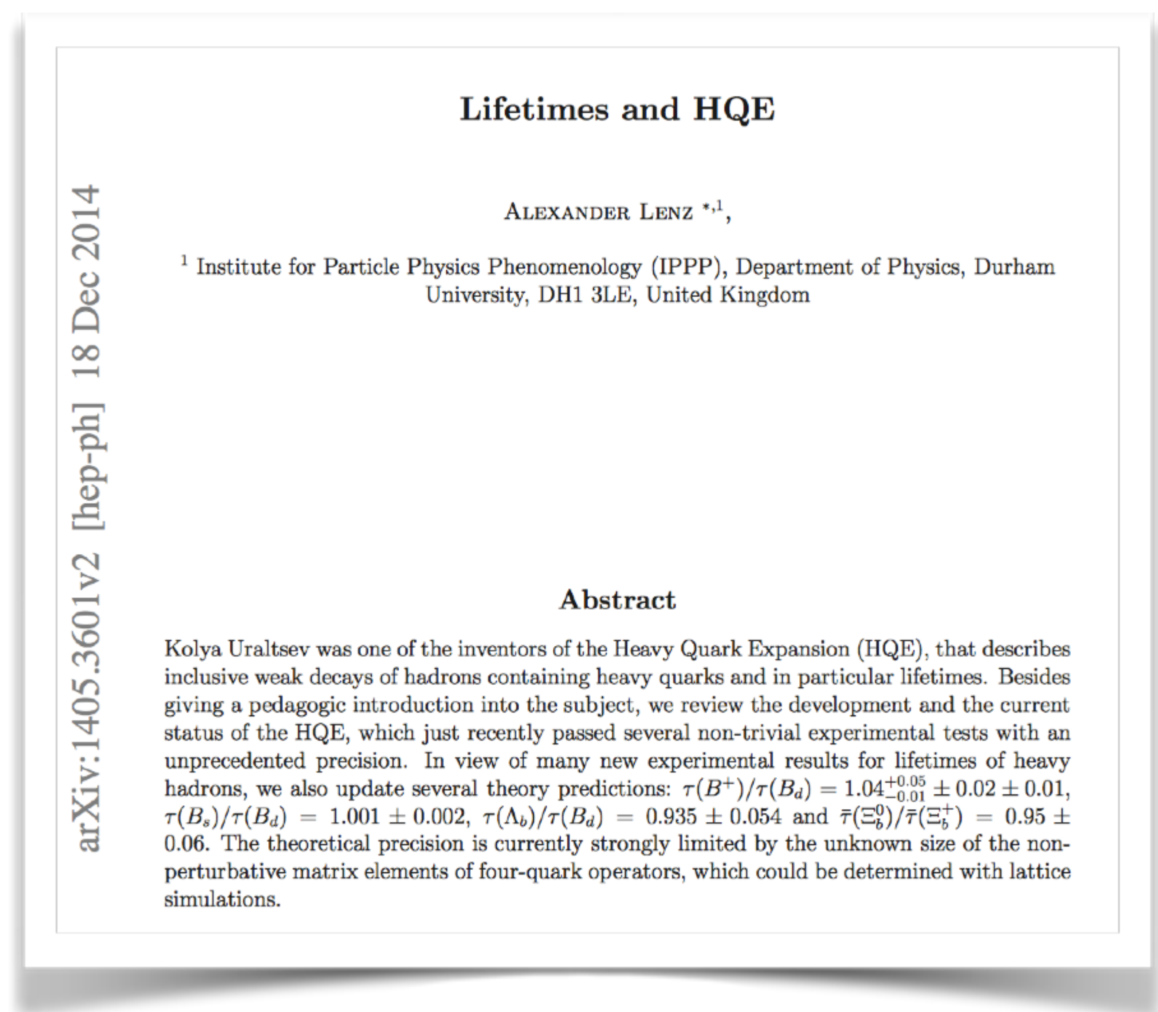
STATUS QUO OF MIXING AND LIFETIMES

Alexander Lenz
Durham - UK Flavour 2017
5.9.2017



CONTENT

- Experimental Status
- The Heavy Quark Expansion (HQE)
- Status before 2017
- News
- Outlook



arXiv:1405.3601v2 [hep-ph] 18 Dec 2014

Lifetimes and HQE

Alexander Lenz *,[1],

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Abstract

Kolya Uraltsev was one of the inventors of the Heavy Quark Expansion (HQE), that describes inclusive weak decays of hadrons containing heavy quarks and in particular lifetimes. Besides giving a pedagogic introduction into the subject, we review the development and the current status of the HQE, which just recently passed several non-trivial experimental tests with an unprecedented precision. In view of many new experimental results for lifetimes of heavy hadrons, we also update several theory predictions: $\tau(B^+)/\tau(B_d) = 1.04^{+0.05}_{-0.01} \pm 0.02 \pm 0.01$, $\tau(B_s)/\tau(B_d) = 1.001 \pm 0.002$, $\tau(\Lambda_b)/\tau(B_d) = 0.935 \pm 0.054$ and $\bar{\tau}(\Xi_b^0)/\bar{\tau}(\Xi_b^+) = 0.95 \pm 0.06$. The theoretical precision is currently strongly limited by the unknown size of the non-perturbative matrix elements of four-quark operators, which could be determined with lattice simulations.

EXPERIMENTAL STATUS

b-hadron species	average lifetime	lifetime ratio
B^0	**1.518 ± 0.004 ps**	
B^+	**1.638 ± 0.004 ps**	$B^+/B^0 = 1.076 \pm 0.004$
B_s^0	**1.509 ± 0.004 ps**	$B_s^0/B^0 = 0.994 \pm 0.004$
B_{sL}	**1.414 ± 0.006 ps**	
B_{sH}	**1.619 ± 0.009 ps**	
B_c^+	**0.510 ± 0.009 ps**	
Λ_b	**1.470 ± 0.009 ps**	$\Lambda_b/B^0 = 0.968 \pm 0.006$
Ξ_b^-	**1.571 ± 0.040 ps**	
Ξ_b^0	**1.479 ± 0.030 ps**	$\Xi_b^0/\Xi_b^- = 0.929 \pm 0.028$
Ω_b^-	**1.64 +0.18 −0.17 ps**	

HFLAV 2017

Matt Needham

CP violation parameter in B^0 mixing	
$\lvert q/p\rvert = 1.0009 \pm 0.0013$ $A_{SL} = -0.0019 \pm 0.0027$ $\mathrm{Re}(\varepsilon_B)/(1+\lvert\varepsilon_B\rvert^2) = -0.0005 \pm 0.0007$	from measurements at the $Y(4S)$
$\mathbf{\lvert q/p\rvert = 1.0010 \pm 0.0008}$ $\mathbf{A_{SL} = -0.0021 \pm 0.0017}$ $\mathbf{Re(\varepsilon_B)/(1+\lvert\varepsilon_B\rvert^2) = -0.0005 \pm 0.0004}$	world average

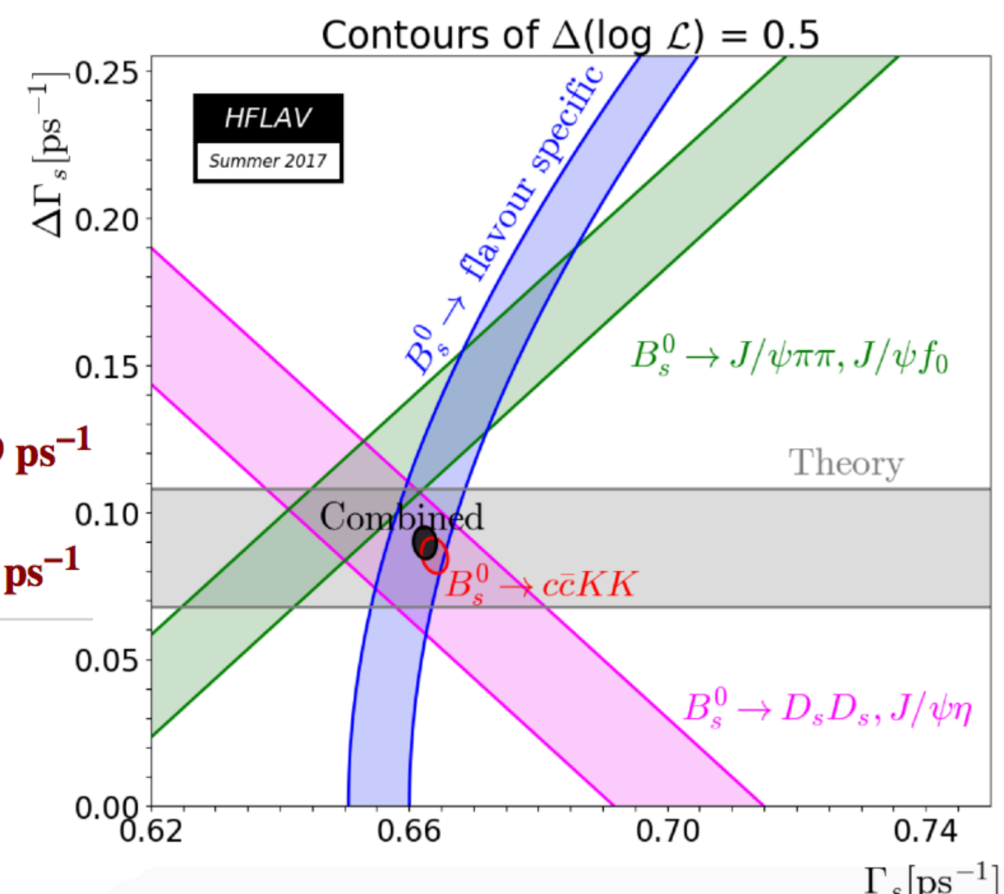
CP violation parameter in B_s mixing	
$\mathbf{\lvert q/p\rvert = 1.0003 \pm 0.0014}$ $\mathbf{A_{SL} = -0.0006 \pm 0.0028}$	world average

Fit results from ATLAS, CDF, CMS, D0 and LHCb data	without constraint from effective lifetime measurements	with constraints I and II	**with constraints I, II and III**
Γ_s	0.6640 ± 0.0020 ps^{-1}	0.6627 ± 0.0020 ps^{-1}	**0.6625 ± 0.0018 ps^{-1}**
$1/\Gamma_s$	1.506 ± 0.005 ps	1.509 ± 0.004 ps	**1.509 ± 0.004 ps**
$\tau_{\mathrm{Short}} = 1/\Gamma_L$	1.415 ± 0.007 ps	1.414 ± 0.006 ps	**1.414 ± 0.006 ps**
$\tau_{\mathrm{Long}} = 1/\Gamma_H$	1.609 ± 0.010 ps	1.618 ± 0.010 ps	**1.619 ± 0.009 ps**
$\Delta\Gamma_s$	$+0.085 \pm 0.006$ ps^{-1}	$+0.089 \pm 0.006$ ps^{-1}	**$+0.090 \pm 0.005$ ps^{-1}**
$\Delta\Gamma_s/\Gamma_s$	$+0.128 \pm 0.009$	$+0.135 \pm 0.008$	**$+0.135 \pm 0.008$**
correlation $\varrho(\Gamma_s, \Delta\Gamma_s)$	−0.193	−0.153	**−0.082**

$$\Delta m_d = 0.5065 \pm 0.0019 \text{ ps}^{-1}$$

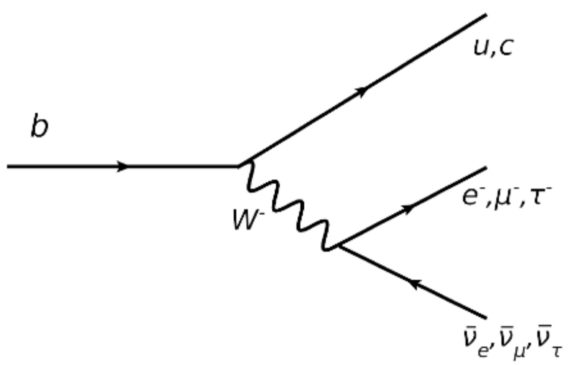
$$\Delta m_s = 17.757 \pm 0.021 \text{ ps}^{-1}$$

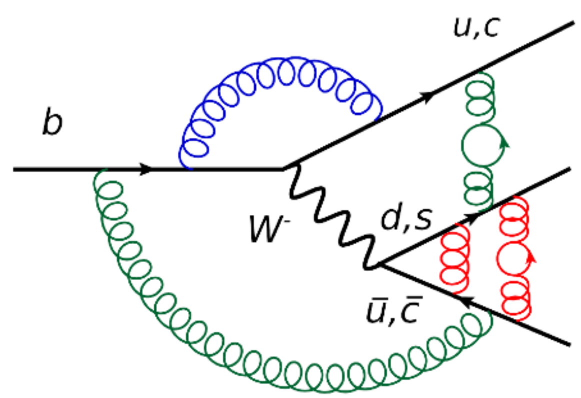
$s \times \Delta\Gamma_d/\Gamma_d = -0.002 \pm 0.010$	from DELPHI, BABAR, Belle, ATLAS and LHCb



HEAVY QUARK EXPANSION I - LIFETIMES

- Free quark decay





$$\Gamma_b = \frac{G_F^2 m_b^5}{192\pi^3} |V_{cb}|^2 c_{3,b}$$

$$c_{3,b} = \begin{cases} 9 \\ 2.97 \\ 3.25 \\ 4.66 \end{cases} \quad \text{for} \quad \begin{cases} m_c = 0, \\ m_c^{\mathrm{Pole}}, m_b^{\mathrm{Pole}} \\ \bar{m}_c(\bar{m}_c), \bar{m}_b(\bar{m}_b) \\ \bar{m}_c(\bar{m}_b), \bar{m}_b(\bar{m}_b) \end{cases}$$

$$\tau_b = 2.60 \text{ ps} \quad \text{for } \bar{m}_c(\bar{m}_b),\ \bar{m}_b(\bar{m}_b)$$

- Effective Hamiltonian *(e.g. Buras, Les Houches)*

Free quark decay is an expansion in $\alpha_s(m_b) \ln \frac{m_b^2}{M_W^2} > 1$ instead of $\alpha_s(m_b) \approx 0.2$

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} \left[\sum_{q=u,c} V_c^q (C_1 Q_1^q + C_2 Q_2^q) - V_p \sum_{j=3} C_j Q_j \right] \qquad Q_2 = c_\alpha \gamma_\mu (1-\gamma_5) \bar{b}_\alpha \times d_\beta \gamma^\mu (1-\gamma_5) \bar{u}_\beta$$

sums up large logarithms to all orders!

Wilson coefficients are known up to NNLO-QCD!

e.g. Gorbahn, Haisch 2004

Use $\mathcal{H}_{eff}$ to calculate total decay rates

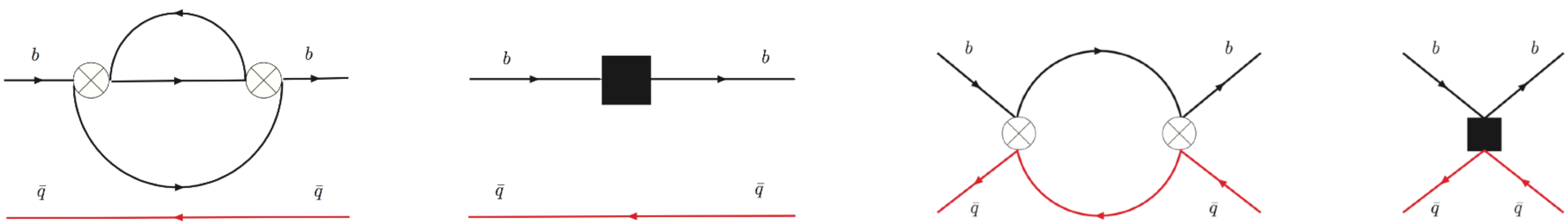
HEAVY QUARK EXPANSION II - LIFETIMES

$$\Gamma(B \to X) = \frac{1}{2m_B} \sum_X \int_{\mathrm{PS}} (2\pi)^4 \delta^{(4)}(p_B - p_X) |\langle X|\mathcal{H}_{eff}|B\rangle|^2$$

Assume:

- *mb is large compared to hadronic scale*
- *decay rate is a Taylor series in 1/mb*

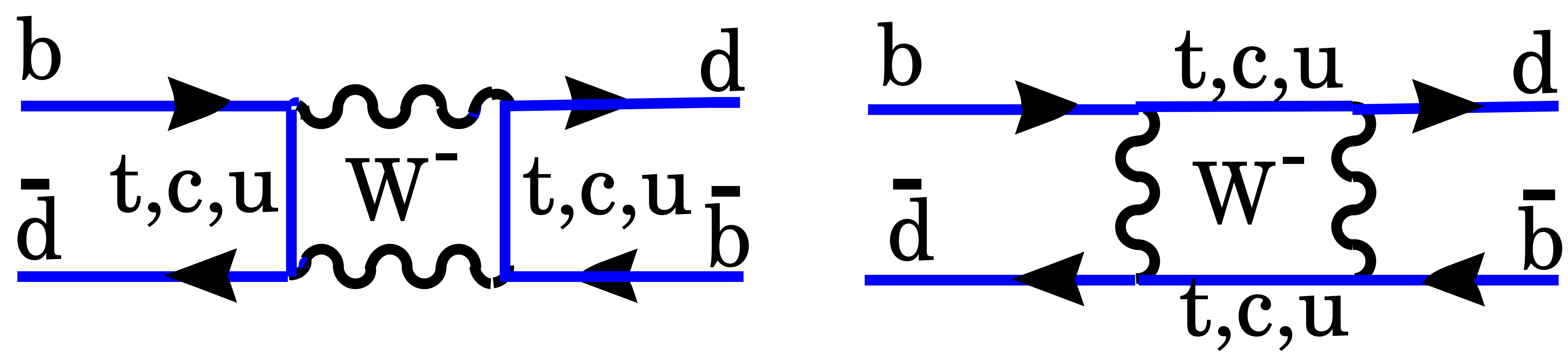
$$\boxed{\Gamma = \frac{G_F^2 m_b^5}{192\pi^3}|V_{cb}|^2 \left[c_{3,b} \frac{\langle B|\bar{b}b|B\rangle}{2M_B} + \frac{c_{5,b}}{m_b^2} \frac{\langle B|\bar{b}g_s\sigma_{\mu\nu}G^{\mu\nu}b|B\rangle}{2M_B} + \frac{c_{6,b}}{m_b^3} \frac{\langle B|(\bar{b}q)_\Gamma(\bar{q}b)_\Gamma|B\rangle}{M_B} + \ldots \right]}$$



Remarks:

- *leading term (=free quark decay) is universal*
- *different B mesons differ from the 3rd term on*
- *lifetime predictions need: non-perturbative matrix elements and perturbative Wilson coefficients*

MIXING OBSERVABLES



$|M_{12}|$, $|\Gamma_{12}|$ and $\phi = \arg(-M_{12}/\Gamma_{12})$ can be related to three observables:

- **Mass difference:** $\Delta M := M_H - M_L \approx 2|M_{12}|$ **(off-shell)**
  $|M_{12}|$: heavy internal particles: t, SUSY, ...

- **Decay rate difference:** $\Delta\Gamma := \Gamma_L - \Gamma_H \approx 2|\Gamma_{12}| \cos\phi$ **(on-shell)**
  $|\Gamma_{12}|$: light internal particles: u, c, ... (almost) no NP!!!

- **Flavor specific/semi-leptonic CP asymmetries:** e.g. $B_q \to Xl\nu$ (*semi-leptonic*)

$$a_{sl} \quad \equiv \quad a_{fs} = \frac{\Gamma(\overline{B}_q(t) \to f) - \Gamma(B_q(t) \to \overline{f})}{\Gamma(\overline{B}_q(t) \to f) + \Gamma(B_q(t) \to \overline{f})} = \left|\frac{\Gamma_{12}}{M_{12}}\right| \sin\phi$$

HEAVY QUARK EXPANSION III

Total decay rate can be expanded in inverse powers of mb

$$\Gamma = \Gamma_0 + \frac{\Lambda^2}{m_b^2}\Gamma_2 + \frac{\Lambda^3}{m_b^3}\Gamma_3 + \frac{\Lambda^4}{m_b^4}\Gamma_4 + \ldots$$

Each term in the series can be further expanded in the strong coupling

$$\Gamma_j = \Gamma_j^{(0)} + \frac{\alpha_s(\mu)}{4\pi}\Gamma_j^{(1)} + \frac{\alpha_s^2(\mu)}{(4\pi)^2}\Gamma_j^{(2)} + \ldots$$

Each term is a product of a perturbative function and the matrix element of ***Delta B = 0 operators*** *(lattice - Davies, sum rules - Rauh, Lenz)*

Mixing obeys a similar HQE

$$\Gamma_{12}^q = \left(\frac{\Lambda}{m_b}\right)^3 \Gamma_3 + \left(\frac{\Lambda}{m_b}\right)^4 \Gamma_4 + \ldots$$

Now ***Delta B = 2 operators appear***

STATUS BEFORE 2017

	$\Gamma_3^{(0)}$	$\Gamma_3^{(1)}$	$\Gamma_3^{(2)}$	$<\lvert\dim 6\rvert>$	$\Gamma_4^{(0)}$	$\Gamma_4^{(1)}$	$<\lvert\dim 7\rvert>$
B+	1985 -1996 ✓	2002 ✓	✗	2001 ✗	2003 ✓	✗	✗
Bs	1985 -1996 ✓	2002 ✓	✗	2001 ✗	2003 ✓	✗	✗
G12s	1985 -1996 ✓	1998 -2006 ✓	✗	-2016 ✗	1996 ✓	✗	✗
G12d	1985 -1996 ✓	2003 -2006 ✓	✗	-2016 ✗	2003 ✓	✗	✗

STATUS BEFORE 2017

$$\frac{\tau(B^+)}{\tau(B_d)}^{\mathrm{HQE\,2014}} = 1.04^{+0.07}_{-0.03}\,,$$

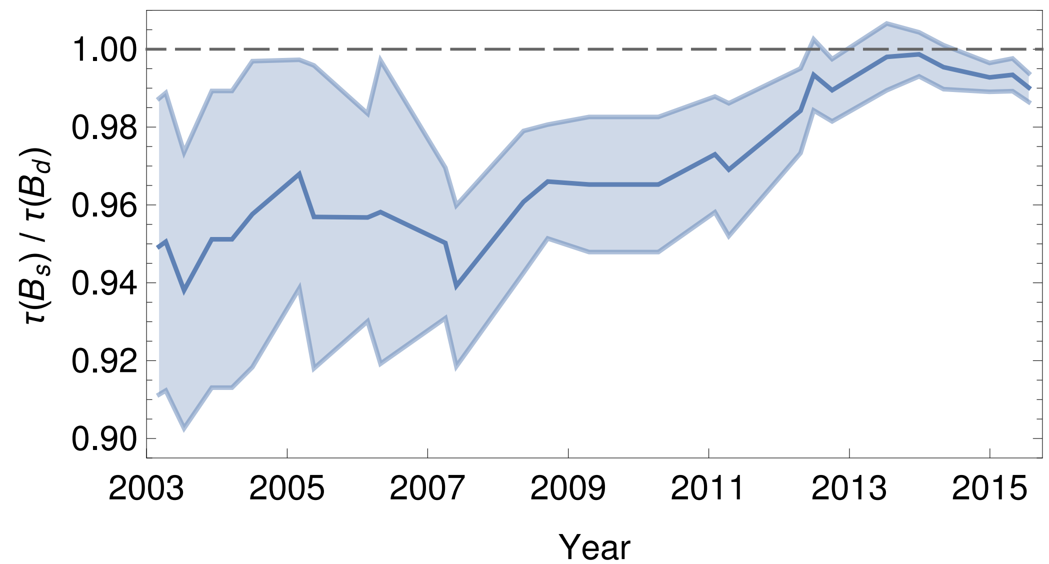
$$\frac{\tau(B_s)}{\tau(B_d)}^{\mathrm{HQE\,2014}} = 1.001 \pm 0.002\,,$$

$$\frac{\tau(\Lambda_b)}{\tau(B_d)}^{\mathrm{HQE\,2014}} = 0.935 \pm 0.054\,,$$

$$\frac{\bar\tau(\Xi_b^0)}{\bar\tau(\Xi_b^+)}^{\mathrm{HQE\,2014}} = 0.95 \pm 0.06\,.$$

**Large uncertainties due to old non-perturbative input*

**Perfect cancellation in Bs lifetime - test of NP models*



see talk of Leslie

Observable	SM – conservative	SM – aggressive	Experiment
ΔM_s	(18.3 ± 2.7) ps^{-1}	(20.11 ± 1.37) ps^{-1}	(17.757 ± 0.021) ps^{-1}
$\Delta\Gamma_s$	(0.088 ± 0.020) ps^{-1}	(0.098 ± 0.014) ps^{-1}	(0.082 ± 0.006) ps^{-1}
a_{sl}^s	$(2.22 \pm 0.27) \cdot 10^{-5}$	$(2.27 \pm 0.25) \cdot 10^{-5}$	$(-7.5 \pm 4.1) \cdot 10^{-3}$

Ideal for NP searches - experimental precision > theory precision!

On the ultimate precision of meson mixing observables
Thomas Jubb, Matthew Kirk, Alexander Lenz, Gilberto Tetlalmatzi-Xolocotzi
Published in Nucl.Phys. B915 (2017) 431-453

THEORY UNCERTAINTIES IN MIXING

$\Delta\Gamma_s^{SM}$	This work
Central value	0.088 ps^{-1}
$\delta(B_{\tilde{R}_2})$	14.8%
$\delta(f_{B_s}\sqrt{B})$	13.9%
$\delta(\mu)$	8.4%
$\delta(V_{cb})$	4.9%
$\delta(\tilde{B}_S)$	2.1%
$\delta(B_{R_0})$	2.1%
$\delta(\bar{z})$	1.1%
$\delta(m_b)$	0.8%
$\delta(B_{\tilde{R}_1})$	0.7%
$\delta(B_{\tilde{R}_3})$	0.6%
$\delta(B_{R_1})$	0.5%
$\delta(B_{R_3})$	0.2%
$\delta(m_s)$	0.1%
$\delta(\gamma)$	0.1%
$\delta(\alpha_s)$	0.1%
$\delta(\lvert V_{ub}/V_{cb}\rvert)$	0.1%
$\delta(\bar{m}_t(\bar{m}_t))$	0.0%
$\sum\delta$	22.8%

Dominant uncertainties from hadronic MEs:

$$\langle R_2\rangle = -\frac{2}{3}\left[\frac{M_{B_s}^2}{m_b^{\mathrm{pow}2}} - 1\right] M_{B_s}^2 f_{B_s}^2 B_{R_2}, \qquad R_2 = \frac{1}{m_b^2}\,\bar{s}_\alpha \overleftarrow{D}_\rho \gamma^\mu (1-\gamma_5) D^\rho b_\alpha\, \bar{s}_\beta \gamma_\mu (1-\gamma_5) b_\beta$$

Dim 7 has never been done

-**HPQCD** works on lattice - see talk Davies

-**Rauh, Kirk, Lenz** with QCD sum rules

$$\langle Q\rangle \equiv \langle \bar{B}_s^0|Q|B_s^0\rangle = \tfrac{8}{3} M_{B_s^0}^2 f_{B_s}^2 B(\mu) \qquad Q = \bar{s}^\alpha \gamma_\mu (1-\gamma_5) b^\alpha \times \bar{s}^\beta \gamma^\mu (1-\gamma_5) b^\beta$$

Dim 6 is done on the lattice

newest results (**Fermilab MILC 1602:03560**)

indicate a small tension with experiment

CP violation in the Bs system

Marina Artuso, Guennadi Borissov, Alexander Lenz

Rev.Mod.Phys. 88 (2016) no.4,045002

NEWS

	$\Gamma_3^{(0)}$	$\Gamma_3^{(1)}$	$\Gamma_3^{(2)}$	$<\lvert\dim 6\rvert>$	$\Gamma_4^{(0)}$	$\Gamma_4^{(1)}$	$<\lvert\dim 7\rvert>$
B+	1985 -1996 ✓	2002 ✓	✗	2001 ✗	2003 ✓	✗	✗
Bs	1985 -1996 ✓	2002 ✓	✗	2001 ✗	2003 ✓	✗	✗
G12s	1985 -1996 ✓	1998 -2006 ✓	✗	-2016 ✗	1996 ✓	✗	✗
G12d	1985 -1996 ✓	2003 -2006 ✓	✗	-2016 ✗	2003 ✓	✗	✗

HPQCD: see talk by Christine Davies

Sum rules: this talk and Thomas Rauhs talk yesterday

WORK IN PROGRESS

all dim-6 Delta B = 0,2 operators

1 dim-6 Delta B =2 operator

IPPP/17/65
August 25, 2017

Dimension-six matrix elements for meson mixing and lifetimes from sum rules

M. Kirk, A. Lenz and T. Rauh

IPPP, Department of Physics, University of Durham, DH1 3LE, United Kingdom

Abstract

The hadronic matrix elements of dimension-six $\Delta F = 0,2$ operators are crucial inputs for the theory predictions of mixing observables and lifetime ratios in the B and D system. We determine them using HQET sum rules for three-point correlators. The results of the required three-loop computation of the correlators and the one-loop computation of the QCD-HQET matching are given in analytic form. For mixing matrix elements we find very good agreement with recent lattice results and comparable theoretical uncertainties. For lifetime matrix elements we present the first ever determination in the D meson sector and the first determination of $\Delta B = 0$ matrix elements with uncertainties under control - superseeding preliminary lattice studies stemming from 2001 and earlier. With our state-of-the-art determination of the bag parameters we predict: $\tau(B^+)/\tau(B_d^0) = 1.079^{+0.021}_{-0.027}$, $\tau(B_s^0)/\tau(B_d^0) = 0.999 \pm 0.002$, $\tau(D^+)/\tau(D^0) = 2.7^{+0.7}_{-0.8}$ and $\Delta\Gamma_s = 0.xxx \pm 0.xxx \mathrm{ps}^{-1}$, in excellent agreement with the most recent experimental averages.

PHYSICAL REVIEW D **94,** 034024 (2016)

B^0-$\bar{B}^0$ mixing at next-to-leading order

Andrey G. Grozin
Budker Institute of Nuclear Physics SB RAS, Novosibirsk 630090, Russia and Novosibirsk State University, Novosibirsk 630090, Russia

Rebecca Klein, Thomas Mannel, and Alexei A. Pivovarov
Theoretische Elementarteilchenphysik, Naturwiss.- techn. Fakultät, Universität Siegen, 57068 Siegen, Germany
(Received 4 July 2016; published 11 August 2016)

We compute the perturbative corrections to the heavy quark effective theory sum rules for the matrix element of the $\Delta B = 2$ operator that determines the mass difference of B^0, $\bar{B}^0$ states. Technically, we obtain analytically the nonfactorizable contributions at order α_s to the bag parameter that first appear at the three-loop level. Together with the known nonperturbative corrections due to vacuum condensates and $1/m_b$ corrections, the full next-to-leading order result is now available. We present a numerical value for the renormalization group invariant bag parameter that is phenomenologically relevant and compare it with recent lattice determinations.

Three-loop HQET vertex diagrams for B^0–$\bar{B}^0$ mixing

Andrey G. Grozin and Roman N. Lee

*Budker Institute of Nuclear Physics,
Novosibirsk 630090, Russia*
E-mail: A.G.Grozin@inp.nsk.su, R.N.Lee@inp.nsk.su

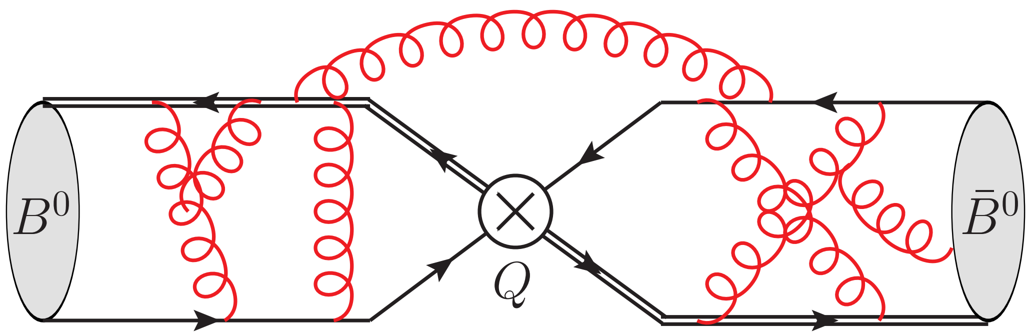
Master integrals

Abstract: Three-loop vertex diagrams in HQET needed for sum rules for B^0–$\bar{B}^0$ mixing are considered. They depend on two residual energies. An algorithm of reduction of these diagrams to master integrals has been constructed. All master integrals are calculated exactly in d dimensions; their ε expansions are also obtained.

Keywords: NLO Computations, B-Physics.

JHEP02(2009)047

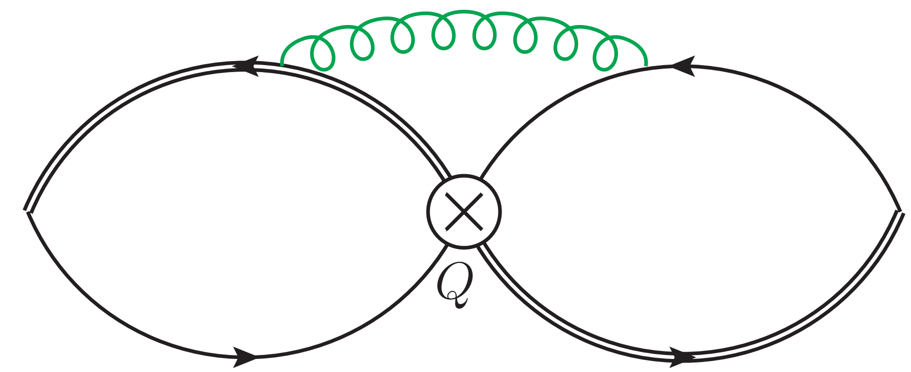
WORK IN PROGRESS



Sum rule

Quark-hadron duality

Analyticity



Hadronic matrix element

Characteristic scale: Λ_{QCD}

$\alpha_s\left(\Lambda_{\text{QCD}}\right) \sim \mathcal{O}(1)$

$\Rightarrow$ non-perturbative

Correlation function

Characteristic scale: 'virtuality' ω

Choose ω s.t. $\alpha_s\left(\omega\right) \ll 1$

$\Rightarrow$ perturbatively calculable

- Do all dim 6 and dim 7 operators

operator Q done by
Grozin, Klein, Mannel, Pivovarov
hep-ph/1606.06054

- 3 loop diagrams with FIRE reduced (2 external momenta)
- Master integrals known: Grozin, Lee; hep-ph/0812.4522
- Expect to reduce uncertainty by a factor of up to two!

WORK IN PROGRESS

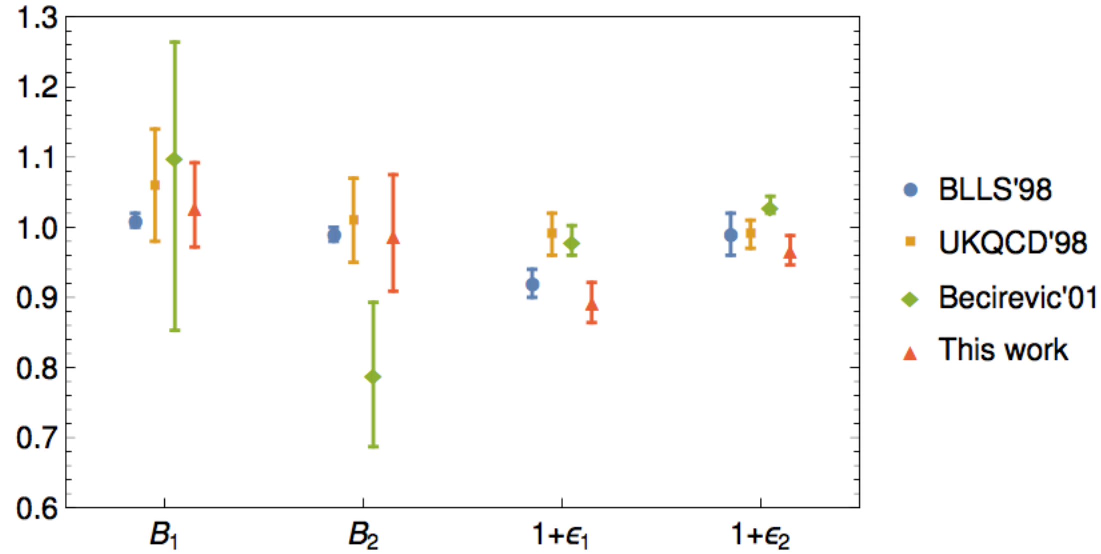


Figure 6: Comparison of our results for the $\Delta B = 0$ Bag parameters at the scale $\overline{m}_b(\overline{m}_b)$ to the HQET sum rule results [13] and the lattice values of UKQCD'98 [20] and Becirevic'01 [21].

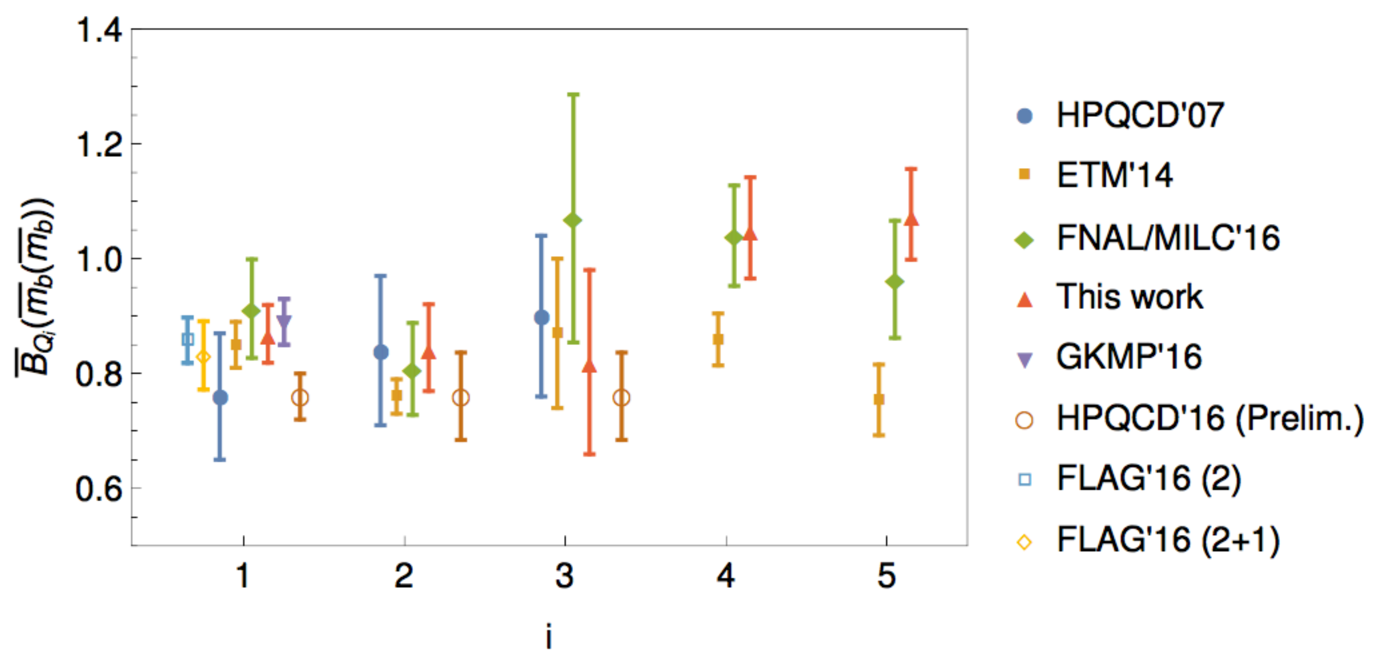


Figure 4: Comparison of our results for the $\Delta B = 2$ Bag parameters at the scale $\overline{m}_b(\overline{m}_b)$ to the lattice values of HPQCD'07 [2], ETM'14 [3] and FNAL/MILC'16 [4], the FLAG averages [59] and the sum rule result GKMP'16 [10].

WORK IN PROGRESS

Numerical results:

$$\frac{\tau(B^+)}{\tau(B_d)} = 1.079^{+0.021}_{-0.027}$$

$$\frac{\tau(B_s)}{\tau(B_d)} = 0.999 \pm 0.002$$

$$\frac{\tau(D^+)}{\tau(D^0)} = 2.7^{+0.7}_{-0.8}$$

$$\Delta\Gamma_s = (0.079 \pm 0.020)\,\mathrm{ps}^{-1}$$

Remarks:

- *Mixing: confirmation of lattice results with slightly worse precision*
- *Lifetime: by far most precise available results*

  B+ and Bs agree perfectly with experiment

  Indication for convergence of HQE even in the charm sector

  now it is up to lattice to do the lifetime matrix elements

TAKE HOME MESSAGES

Status Quo

- HQE seems to be in a very good shape:
  lifetimes and mixing confirm HQE - no sign of duality violation
- Even a convergence in the D system seems to be plausible - understand D-mixing

Improvements

- Lifetime of Bs should be known more precisely
- Need lattice results for dim 6 and 7 operators for Delta B,C = 0,2
- NNLO calculations will soon be necessary
- Do baryon lifetimes

END

TEST OF UNDERLYING THEORY ASSUMPTIONS: DUALITY

1970 Blom, Gilman for e-p scattering

1979 Poggio, Quinn, Weinberg for e+e- to hadrons

Basic idea: Sum overall hadrons = quark level

Our definition: duality violation is deviation from HQE

$$\Gamma = \Gamma_0 + \frac{\Lambda^2}{m_b^2}\Gamma_2 + \frac{\Lambda^3}{m_b^3}\Gamma_3 + \frac{\Lambda^4}{m_b^4}\Gamma_4 + \ldots$$

Actual expansion parameter is momentum release $\frac{\Lambda}{M_i^2 - M_f^2}$

Taylor expansion of exp[-1/x] in x does give zero

Channel	Expansion parameter x	Numerical value	$\exp[-1/x]$
$b \to c\bar{c}s$	$\frac{\Lambda}{\sqrt{m_b^2-4m_c^2}} \approx \frac{\Lambda}{m_b}\left(1+2\frac{m_c^2}{m_b^2}\right)$	$0.054 - 0.58$	$9.4 \cdot 10^{-9} - 0.18$
$b \to c\bar{u}s$	$\frac{\Lambda}{\sqrt{m_b^2-m_c^2}} \approx \frac{\Lambda}{m_b}\left(1+\frac{1}{2}\frac{m_c^2}{m_b^2}\right)$	$0.045 - 0.49$	$1.9 \cdot 10^{-10} - 0.13$
$b \to u\bar{u}s$	$\frac{\Lambda}{\sqrt{m_b^2-4m_u^2}} = \frac{\Lambda}{m_b}$	$0.042 - 0.48$	$4.2 \cdot 10^{-11} - 0.12$

Best candidate:

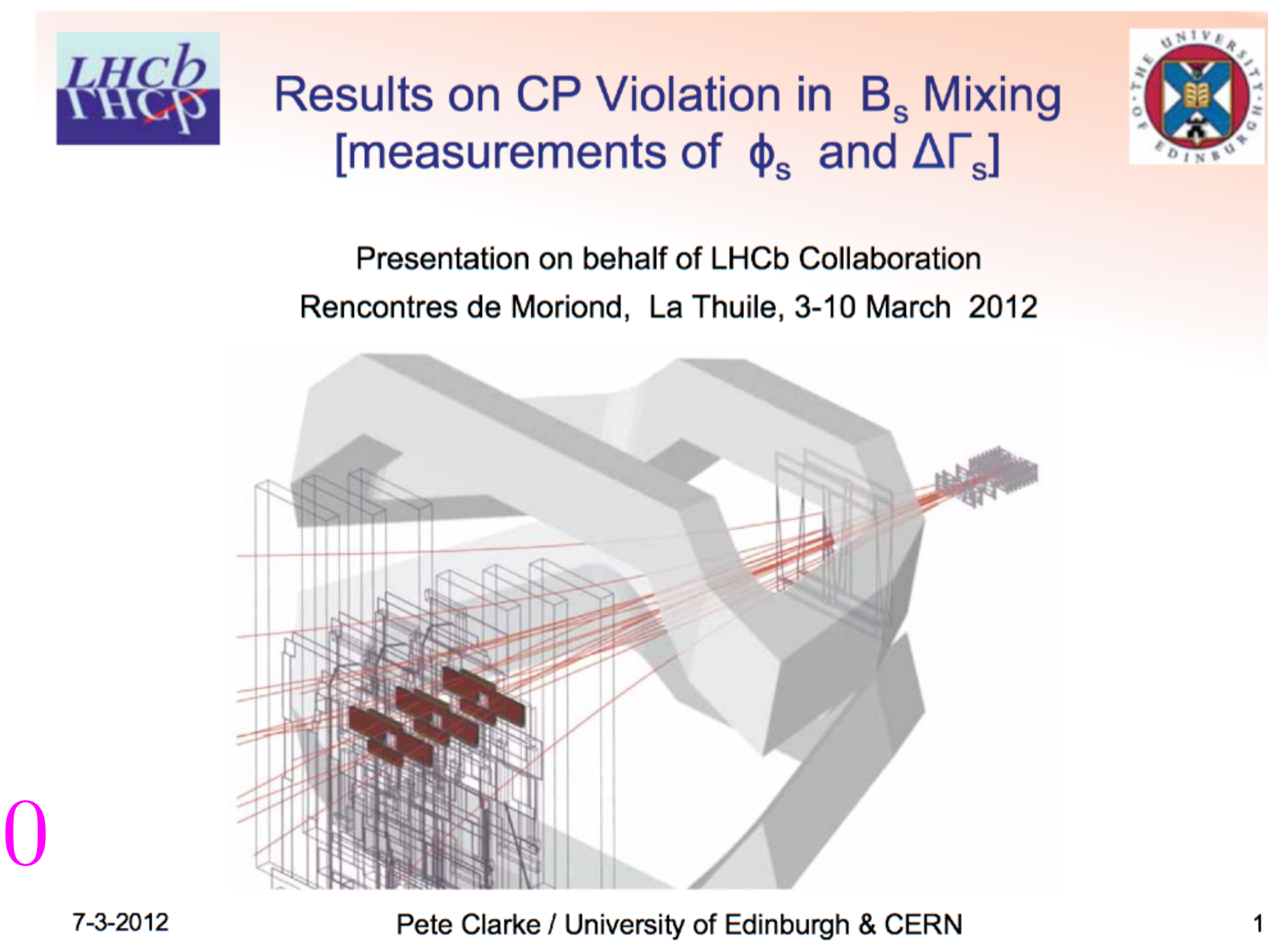
$$b \to c\bar{c}s$$

DUALITY VIOLATION

- Many historic hints for possible duality violation: missing charm puzzle, Λ_b-lifetime, di-muon asymmetry,…
- Duality cannot be proofed - solution of QCD necessary: test whether duality based predictions agree with experiment
- Since Moriond 2012:

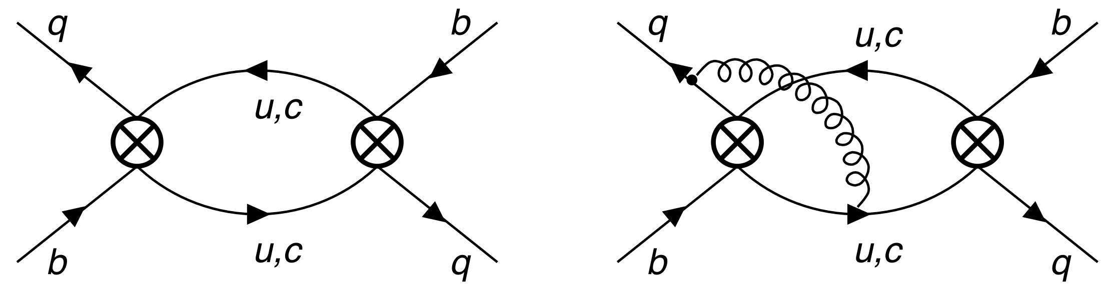
  size of duality violations is severely constrained by perfect agreement of experiment and theory for

$$\frac{\left(\frac{\Delta\Gamma_s}{\Delta M_s}\right)^{\mathrm{SM}}}{\left(\frac{\Delta\Gamma_s}{\Delta M_s}\right)^{\mathrm{Exp}}} = 0.99 \pm 0.20$$



QUANTIFY THE POSSIBLE SIZE OF DUALITY VIOLATIONS

$$\Gamma_{12}^{q} =$$



We expect duality violations to be more pronounced if the final state phase space is becoming smaller

our ansatz:

$$\Gamma_{12}^{s,cc} \to \Gamma_{12}^{s,cc}(1+4\delta) \; ,$$

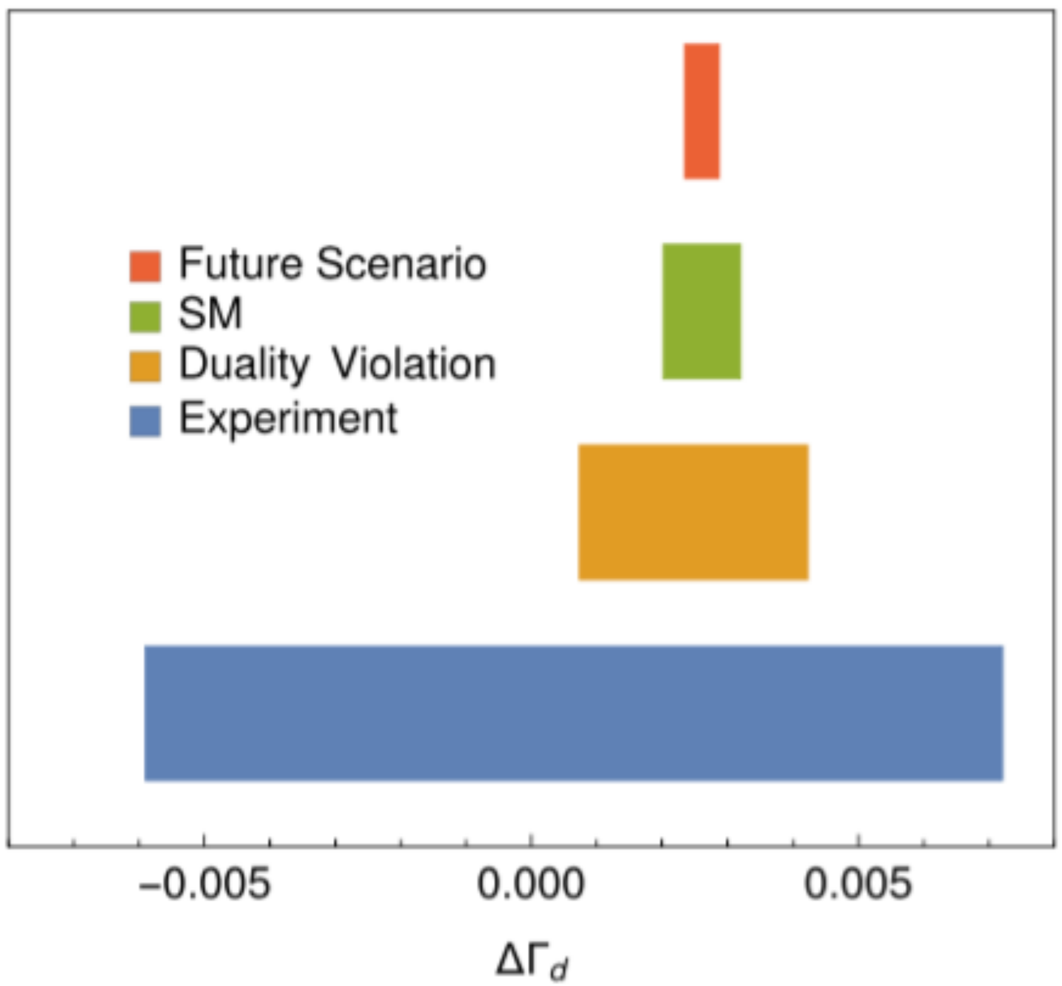
$$\Gamma_{12}^{s,uc} \to \Gamma_{12}^{s,uc}(1+\delta) \; ,$$

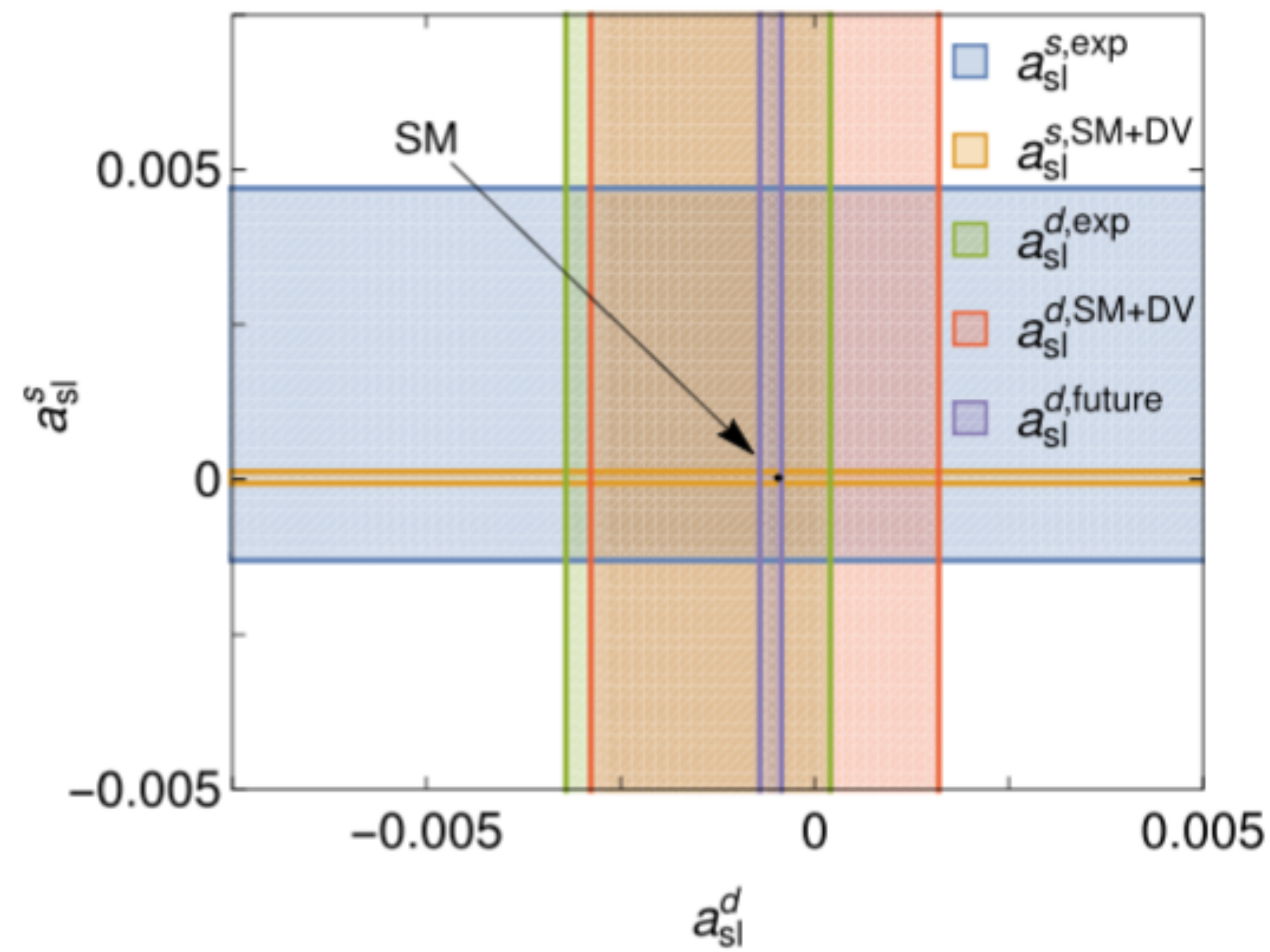
$$\Gamma_{12}^{s,uu} \to \Gamma_{12}^{s,uu}(1+0\delta) \; .$$

We get the following dependence of mixing observables

Observable	B_s^0	B_d^0
$\frac{\Delta\Gamma_q}{\Delta M_q}$	$48.1(1+3.95\delta)\cdot 10^{-4}$	$49.5(1+3.76\delta)\cdot 10^{-4}$
$\Delta\Gamma_q$	$0.0880(1+3.95\delta)\ \mathrm{ps}^{-1}$	$2.61(1+3.759\delta)\cdot 10^{-3}\ \mathrm{ps}^{-1}$
a_{sl}^{q}	$2.225(1-22.3\delta)\cdot 10^{-5}$	$-4.74(1-24.5\delta)\cdot 10^{-4}$

QUANTIFY THE POSSIBLE SIZE OF DUALITY VIOLATIONS





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