B mixing with Lattice QCD

Christine Davies
University of Glasgow
HPQCD collaboration

Durham UK flavour
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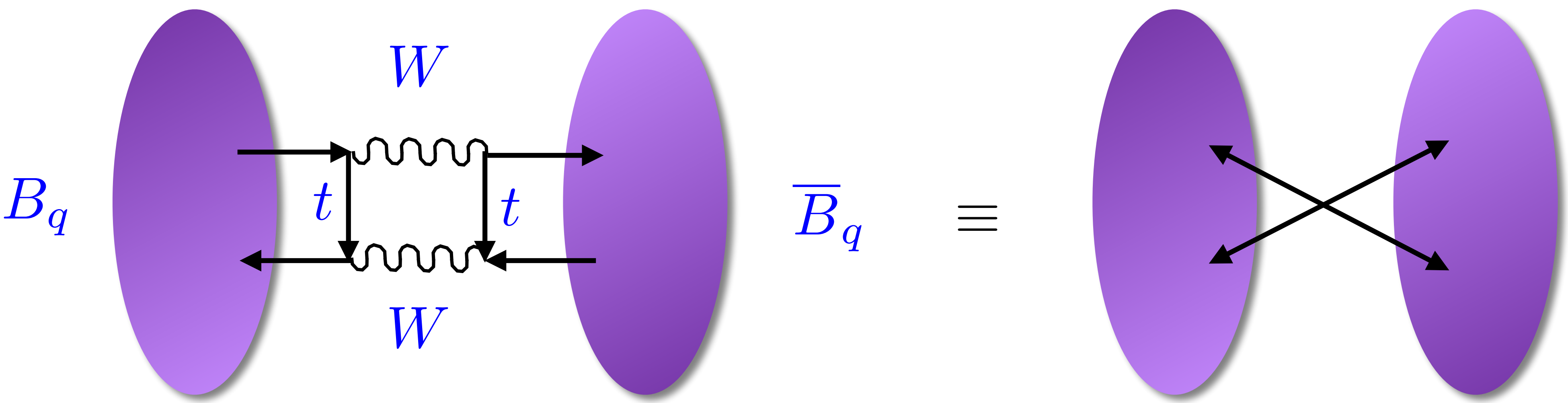
Neutral B (Bs,Bd) mixing - sensitive to new physics

Focus on mass difference of eigenstates, ΔM_q

Electroweak H$_{\text{eff}}$ requires nonperturbative determination of 4-quark operator MEs - use lattice QCD



SM ΔM_q

5 operators cover SM and BSM

$$O_1 = \left[\overline{\Psi}_b^i (V-A) \Psi_q^i\right] \left[\overline{\Psi}_b^j (V-A) \Psi_q^j\right]$$

$$O_2 = \left[\overline{\Psi}_b^i (S-P) \Psi_q^i\right] \left[\overline{\Psi}_b^j (S-P) \Psi_q^j\right] \quad O_4 = \left[\overline{\Psi}_b^i (S-P) \Psi_q^i\right] \left[\overline{\Psi}_b^j (S+P) \Psi_q^j\right]$$

$$O_3 = \left[\overline{\Psi}_b^i (S-P) \Psi_q^j\right] \left[\overline{\Psi}_b^j (S-P) \Psi_q^i\right] \quad O_5 = \left[\overline{\Psi}_b^i (S-P) \Psi_q^j\right] \left[\overline{\Psi}_b^j (S+P) \Psi_q^i\right].$$

Useful insight - vacuum saturation and bag parameters

$$\langle O_i^q \rangle(\mu) = \eta_i^q f_{B_q}^2 M_{B_q}^2 B_{B_q}^{(i)}(\mu)$$

less dependence on m_q, lattice spacing?

decay constant = meson annihiln amp.

$$\eta_1^q = \frac{8}{3}$$

$$\eta_2^q = -\frac{5}{3}\left(\frac{M_{B_q}}{m_b(\mu)+m_q(\mu)}\right)^2$$

$$\eta_3^q = \frac{1}{3}\left(\frac{M_{B_q}}{m_b(\mu)+m_q(\mu)}\right)^2$$

$$\eta_4^q = 2\left[\left(\frac{M_{B_q}}{m_b(\mu)+m_q(\mu)}\right)^2+\frac{1}{6}\right]$$

$$\eta_5^q = \frac{2}{3}\left[\left(\frac{M_{B_q}}{m_b(\mu)+m_q(\mu)}\right)^2+\frac{3}{2}\right]$$

To evaluate use $\mu = m_b$

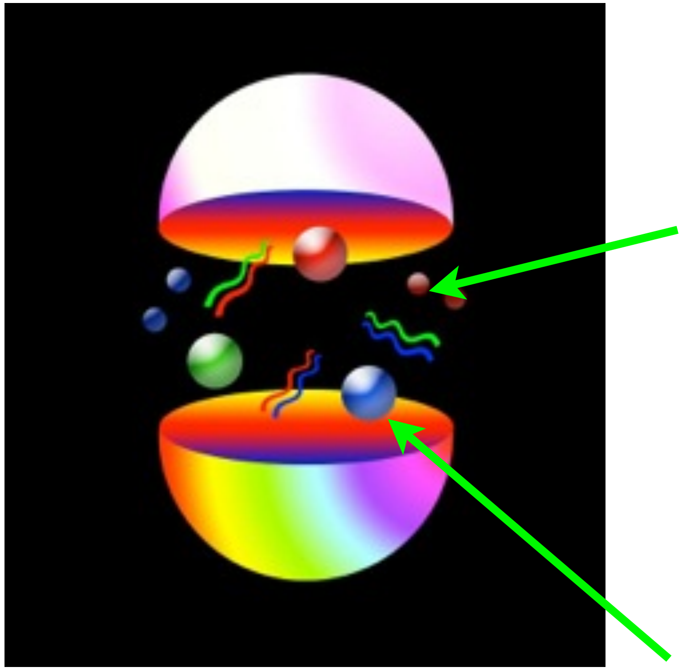
$\overline{m_b}(\overline{m_b})$	m_b/m_s
4.162(48) GeV;	52.55(55)

HPQCD, 1408.4169, 1408.5768

If B ~ 1, expect MEs to vary in size, e.g O3 small

HPQCD Lattice QCD calculation of 4q-ops and annihiln ops done together so B_{Bq} easily determined.
f_B determination done earlier. Well-established lattice results for this.
Here use improved NRQCD for b-quark

Lattice QCD = two-step procedure

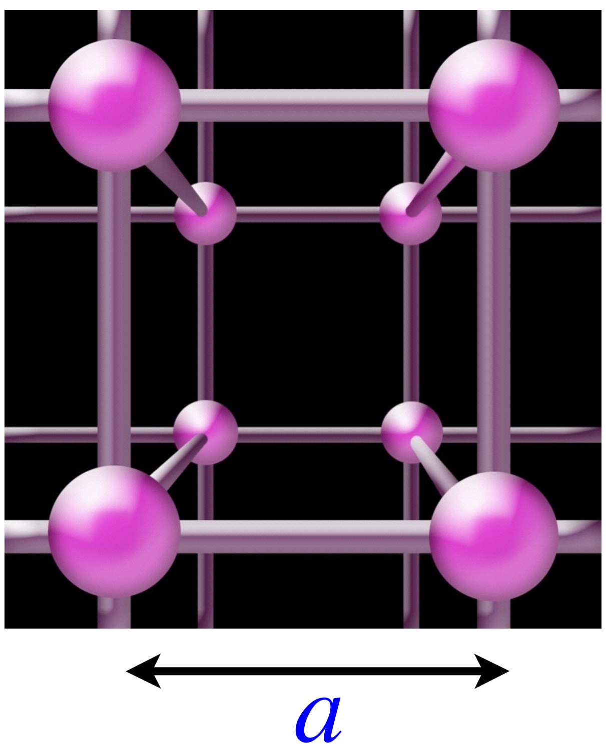


1) Generate sets of gluon fields for Monte Carlo integrn of Path Integral (inc effect of u, d, s, (c) sea quarks)

numerically extremely challenging

2) Calculate valence quark propagators and combine for “hadron correlators”

numerically costly, data intensive



• Fit for masses and matrix elements

• Determine a and fix m_q to get results in physical units.

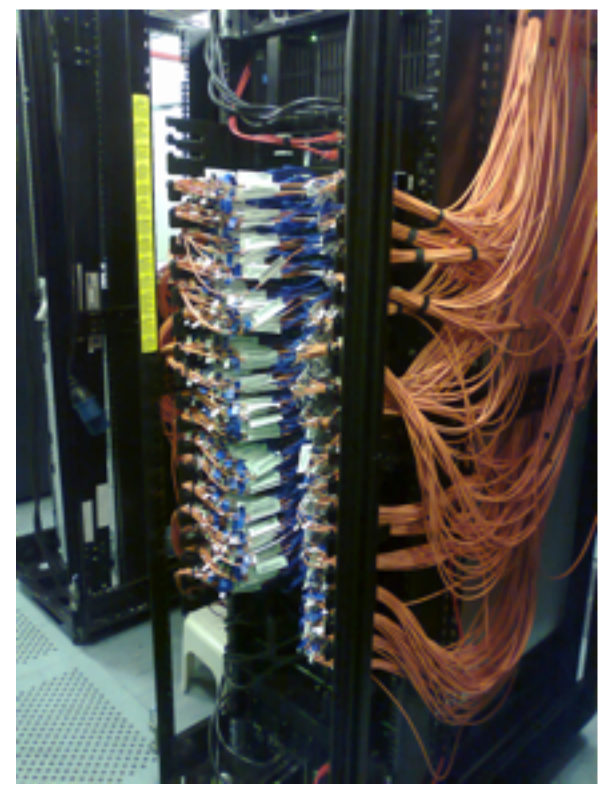
• cost increases as $a \rightarrow 0, m_l \rightarrow phys$ and with statistics, volume.

Using Darwin@Cambridge,

DiRAC

www.dirac.ac.uk

Allows us to calculate quark propagators rapidly and store them for flexible re-use.



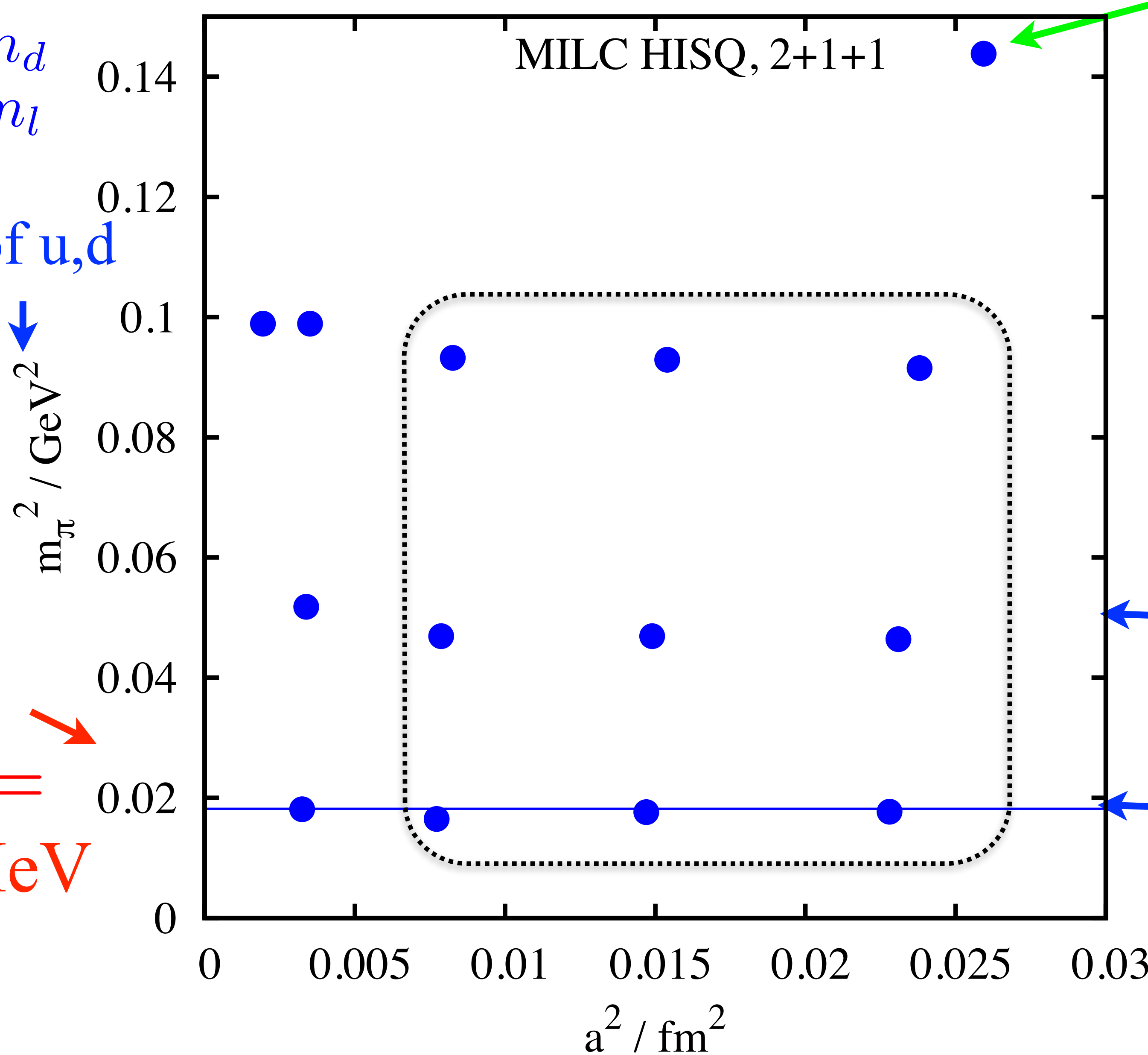
Inversion of 10^7 x 10^7 sparse matrix solves the Dirac equation for the quark propagator on a given gluon field configuration. Must repeat thousands of times for statistical precision.

Example parameters for '2nd generation' calculations now being done with staggered quarks.

$m_u = m_d = m_l$

mass of u,d quarks ↓

real world ↘ $m_{\pi^0} = 135$ MeV



"2nd generation" lattices inc. c quarks in sea and NLO-imp. gluons

HISQ = Highly improved staggered quarks - very accurate discretisation

`E.Follana, et al, HPQCD, hep-lat/0610092.`

$m_{u,d} \approx m_s/10$

$m_{u,d} \approx m_s/27$

Volume:

$m_\pi L > 3$

B meson decay constant calculation from '2-point' function

$$\langle 0|H^\dagger(T)H(0)|0\rangle = \sum_n a_n^2 e^{-m_n T} \xrightarrow[\text{large}]{T} a_0^2 e^{-m_0 T}$$

gives decay constant

masses of all hadrons in H

$$a_0 = \frac{|\langle 0|H|B_q\rangle|}{\sqrt{2M_{B_q}}} \qquad = f_{B_q}\sqrt{\frac{M_{B_q}}{2}} \qquad H = \overline{\psi}_b\gamma_0\gamma_5\psi_q$$

$$= f_{B_q}\sqrt{\frac{M_{B_q}}{2}}\frac{M_{B_q}}{m_b+m_q} \qquad H = \overline{\psi}_b\gamma_5\psi_q$$

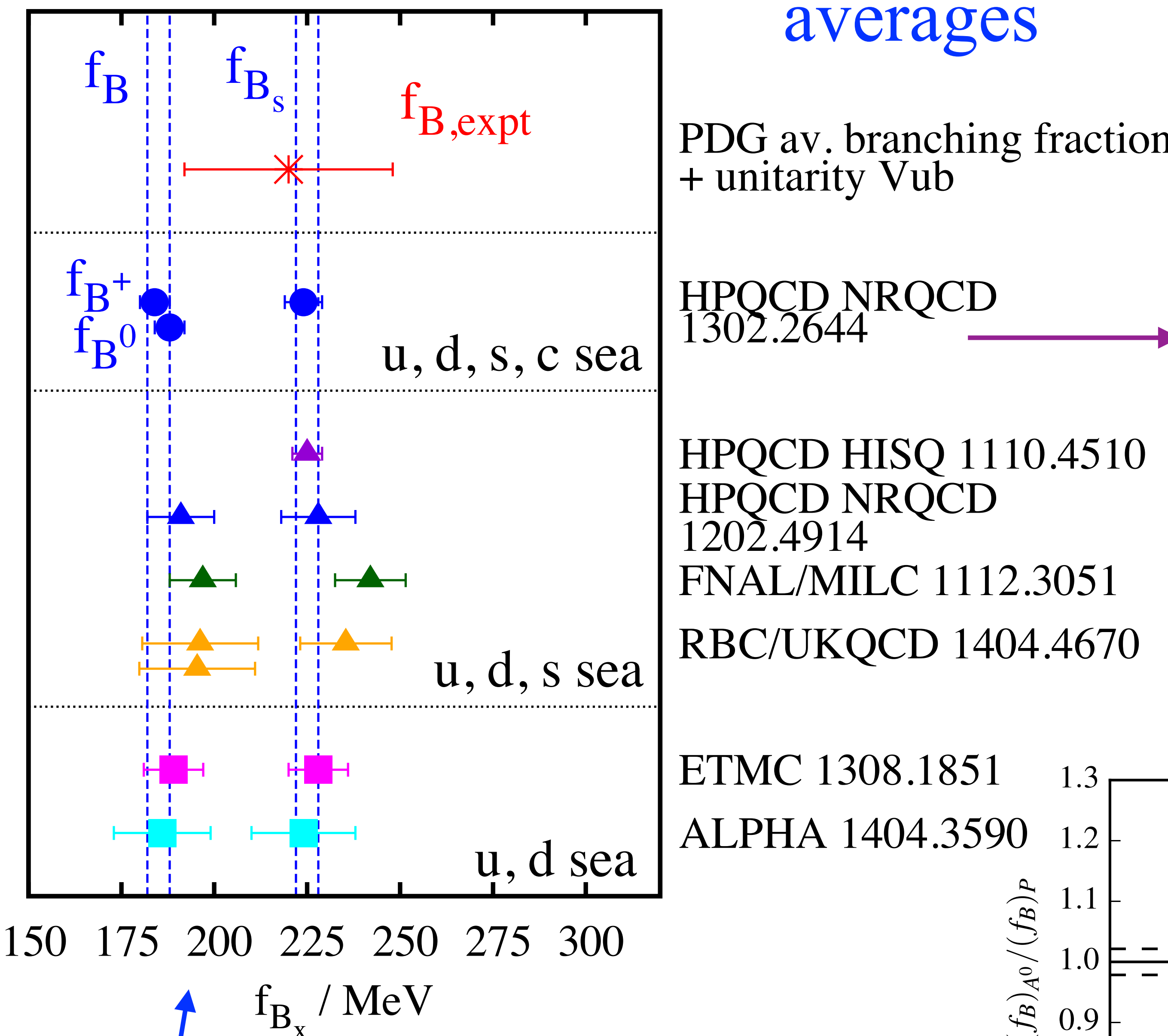
Must match lattice current to continuum for most b quark formalisms. For improved NRQCD done through $\alpha_s\Lambda/m_b$

$$A_0 = (1+\alpha_s z_0)\times(J^{(0)}_{A_0,lat} + (1+\alpha_s z_1)J^{(1)}_{A_0,lat} + \alpha_s z_2 J^{(2)}_{A_0,lat})$$

rel.corrns

R. Dowdall, et al, HPQCD, 1302.2644

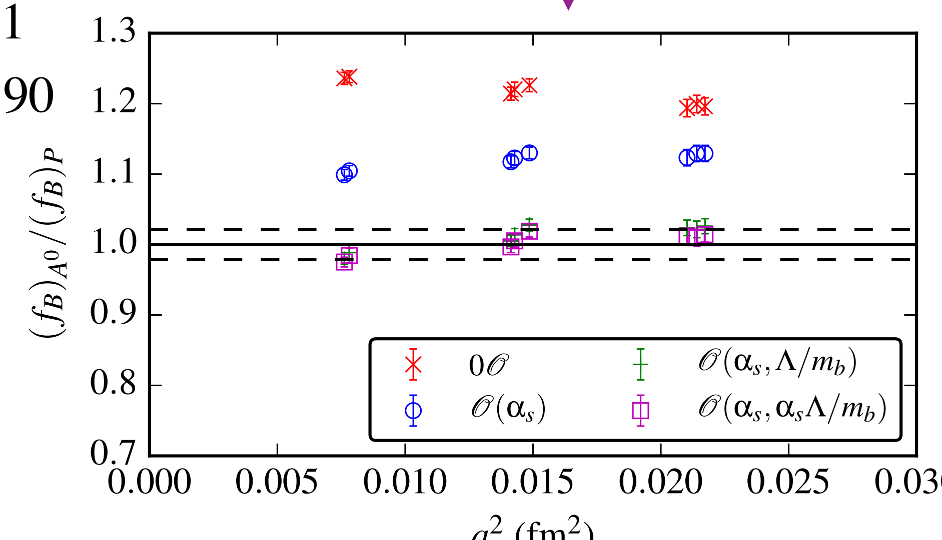
185(3) 225(3) averages B_{av}, B_s decay constant world averages



NEW

C. Hughes, et al, HPQCD, in prep

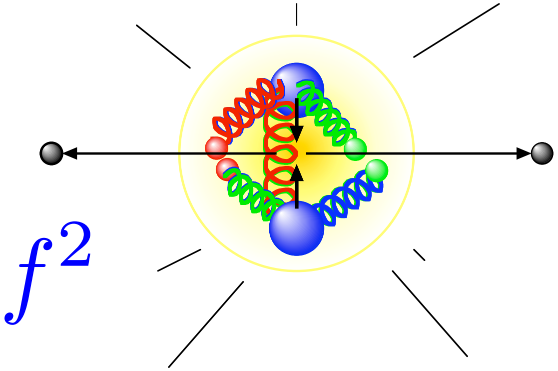
Test by comparing results for A_0 and P to same accuracy - agree within 2% errors



Different b quark and weak current normalisation methods agree

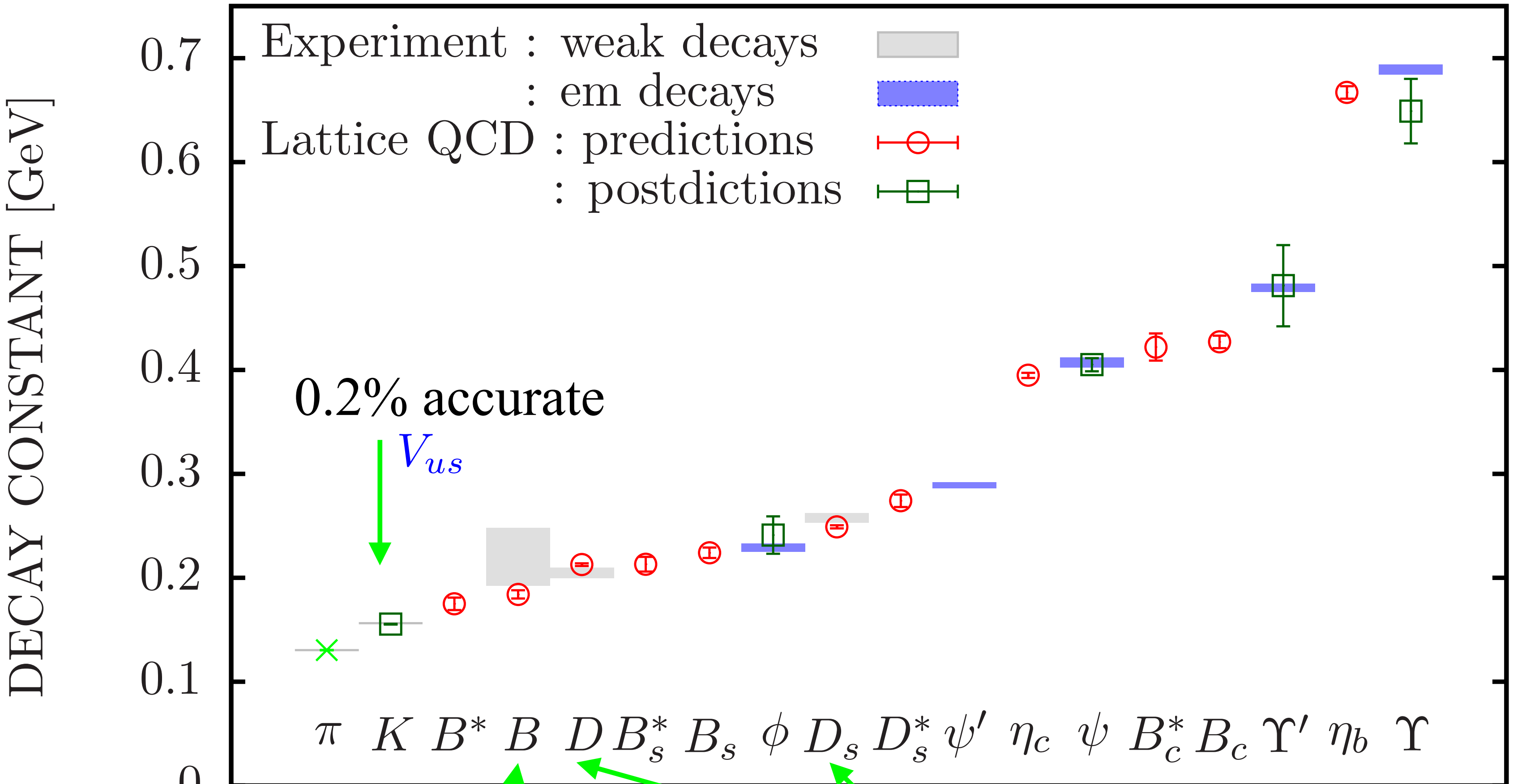
Meson decay constants - big picture

Parameterises hadronic information needed for annihilation rate to W or photon:



$$\Gamma \propto f^2$$

HPQCD 1503.05762



decay constants of vector mesons now being pinned down

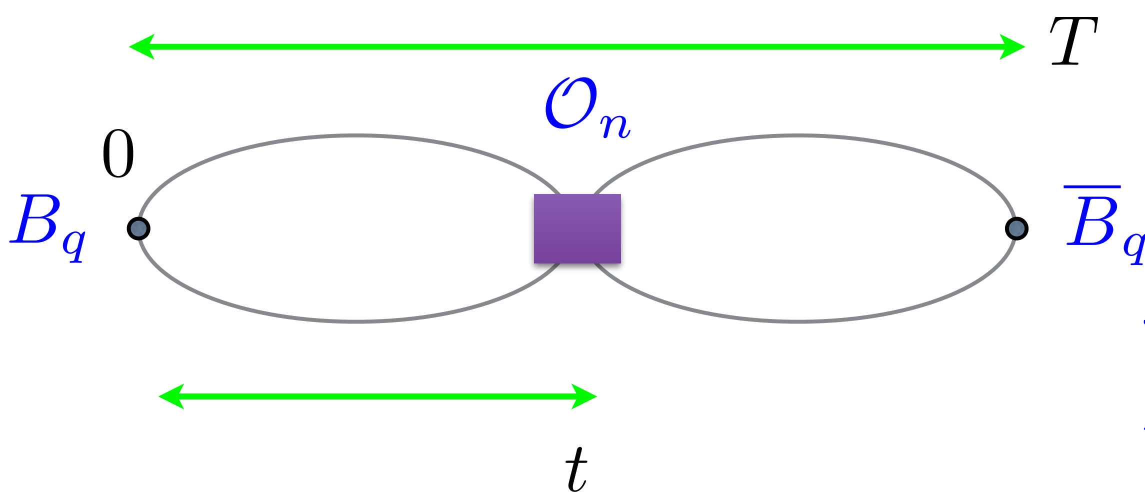
2012

BELLE $B \to \tau\nu$ V_{ub}

0.5% accuracy from lattice QCD now : FNAL/MILC 1407.3772

BES will improve expt. V_{cd} V_{cs}

4-quark operator matrix elements from '3point functions'



Fit 2pt and 3pt together

$$C_{3pt} = \sum_{i,j} a_i J_{ij} a_j e^{-E_i t} e^{-E_j (T-t)}$$

gives

$$\langle B_q | O_n | \overline{B}_q \rangle$$

O_n matched to continuum to order $\alpha_s, \Lambda/m_b$

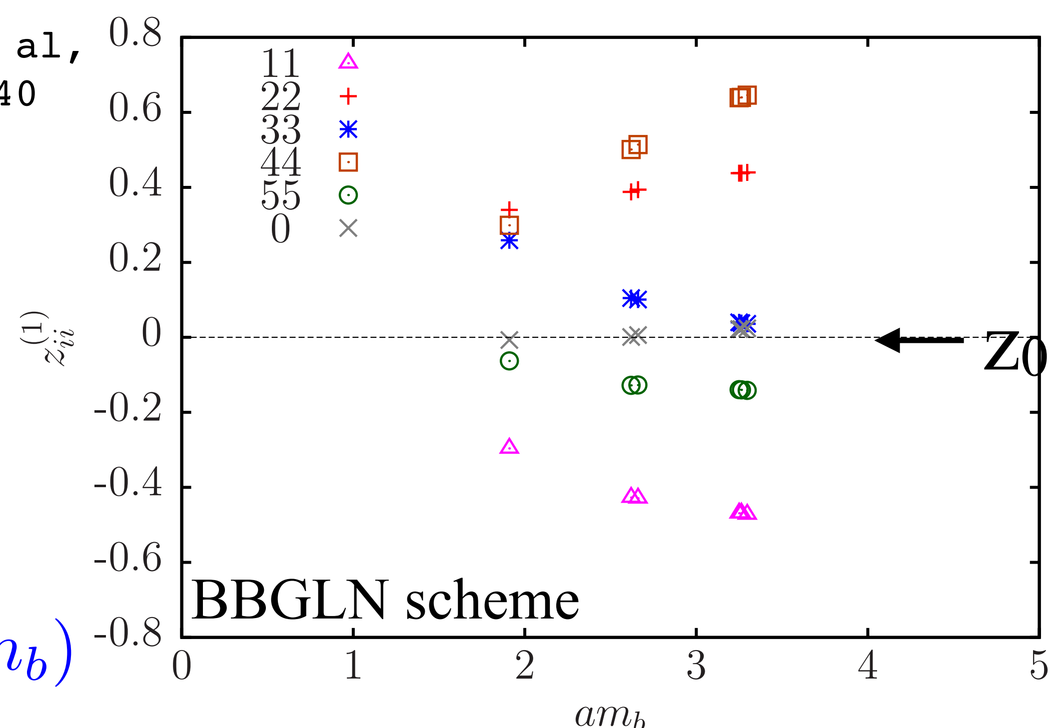
C. Monahan et al, HPQCD,1407.4040

$$\langle O_i \rangle_{\overline{MS}}(m_b) = [1 + \alpha_s z_{ii}^{(1)}] \langle O_{i,lat}^{tree} \rangle$$

$$+\alpha_s z_{ij}^{(1)} \langle O_{j,lat}^{tree} \rangle$$

$$O_{i,lat}^{tree} = O_{i,lat}^{(0)} + O_{i,lat}^{(1)}$$

$\mathcal{O}(\Lambda/m_b)$

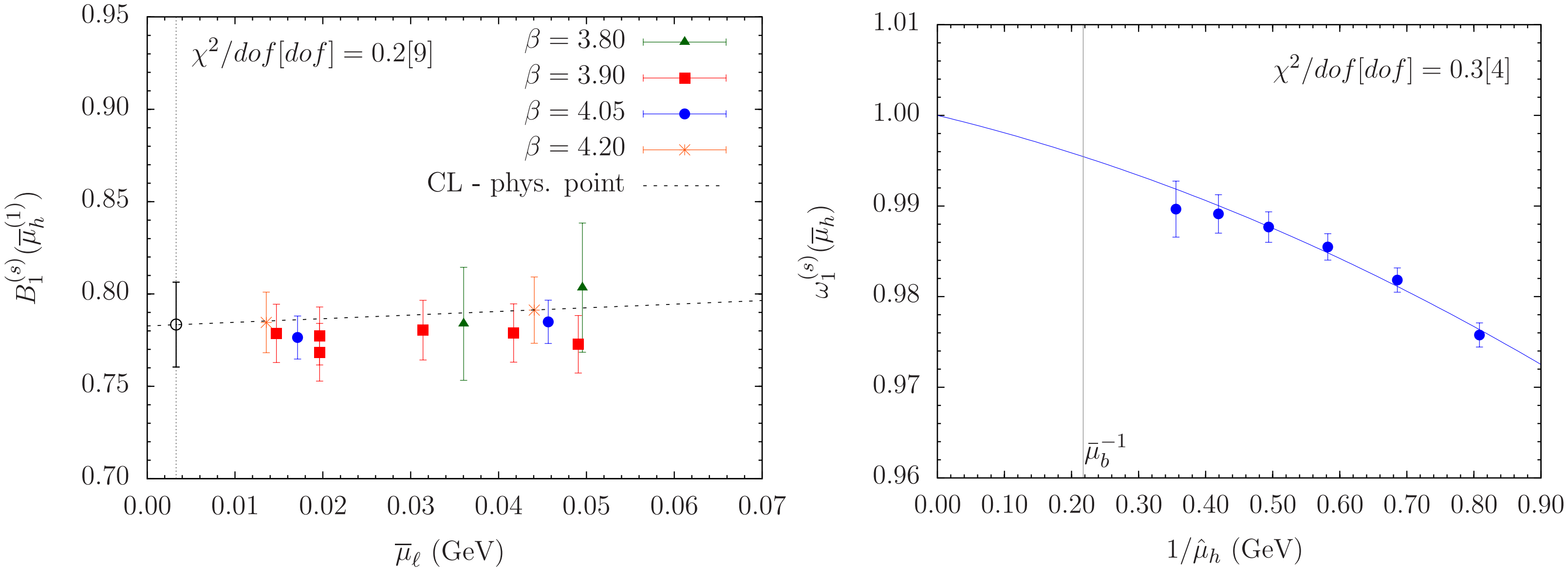


Divide by 2-point op at same order to get B_{Bq}

z_{ij} are smaller than z_{ii}

Previous results

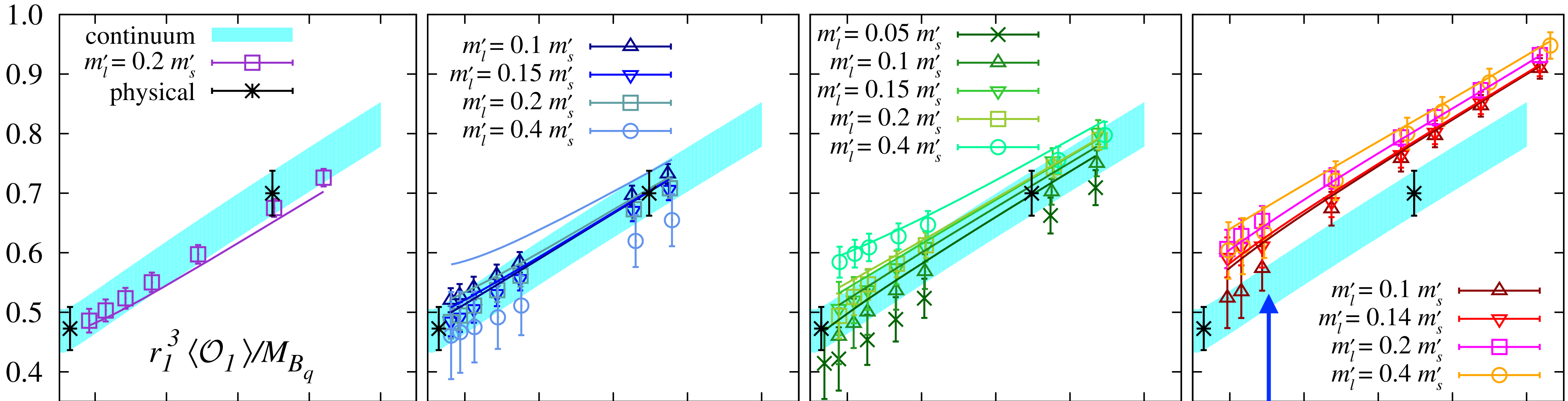
ETM,1308.1851



n_f=2 twisted mass (*NO* s quark in sea); a=0.1fm to 0.05fm; B params; RI-MOM matching and then extrapolate to b.

FNAL/MILC,1602.03560.

n_f=2+1 asqtad + clover b; a=0.12fm to 0.045 fm; 4q-op ME, pert match

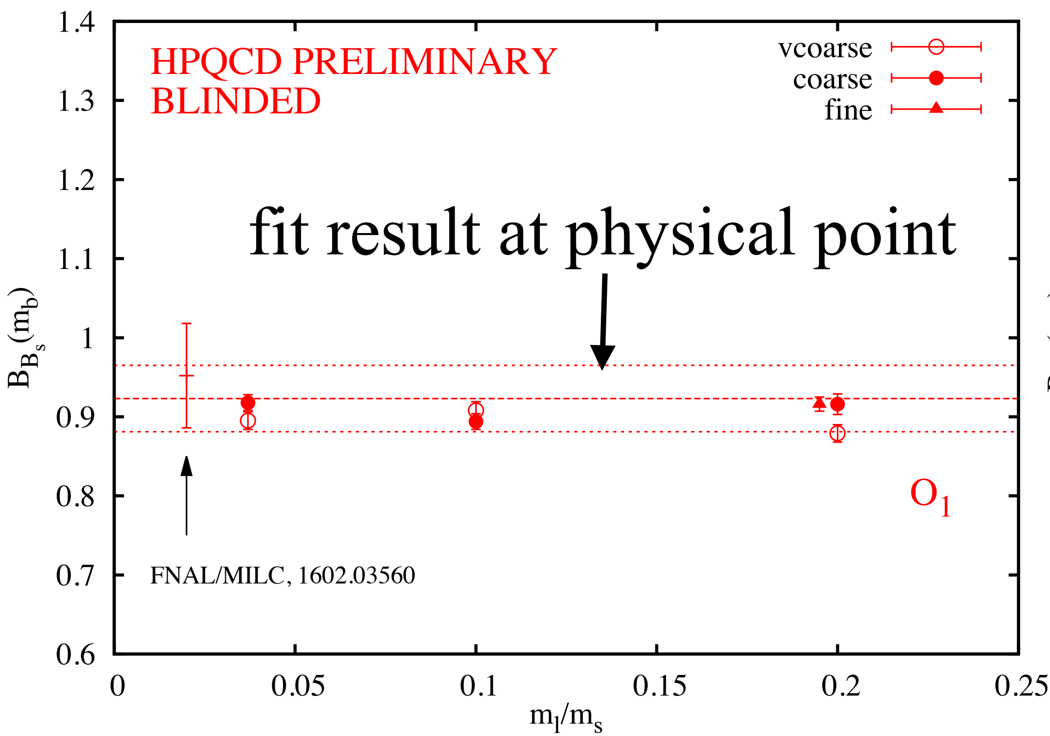


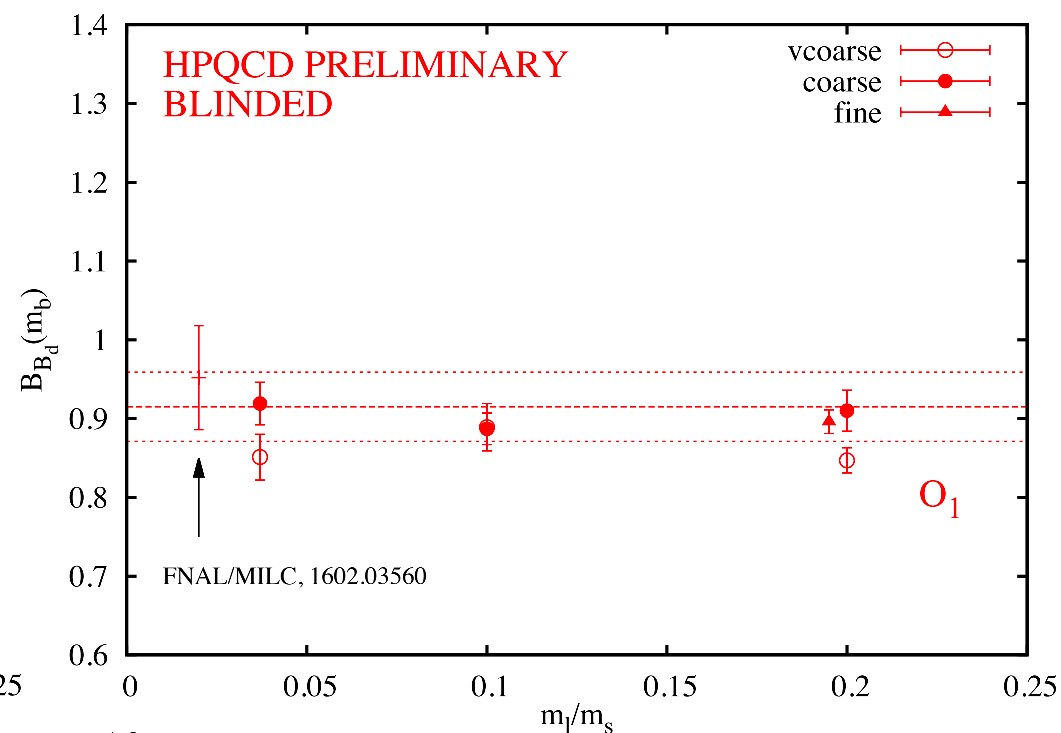
m_q

a. inc.

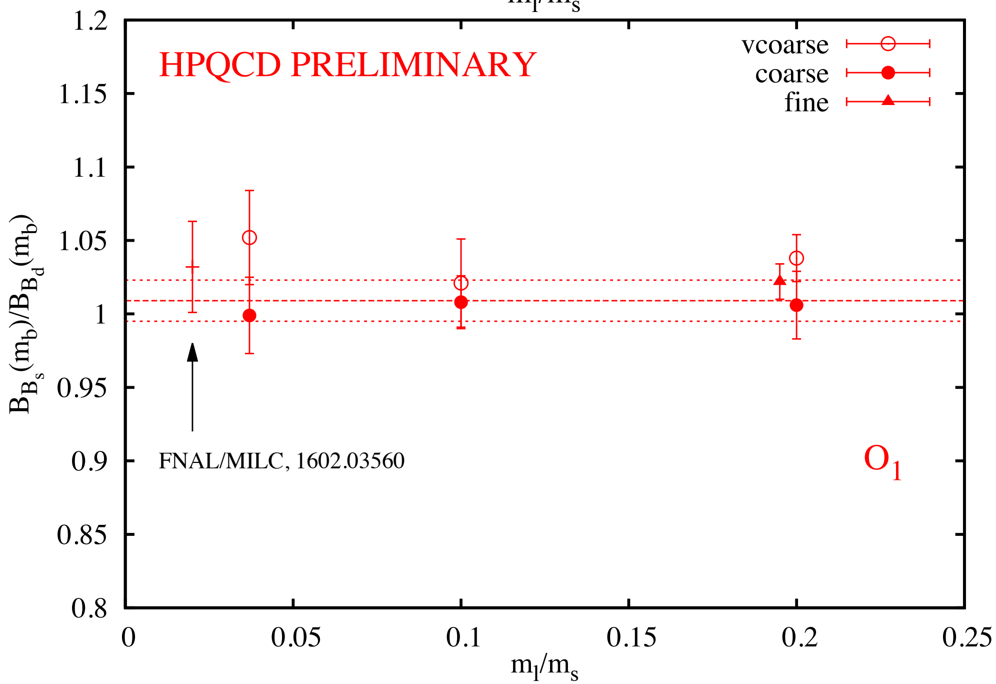
physical result from fit as function of a and m

HPQCD preliminary - 3pt amplitude currently blinded



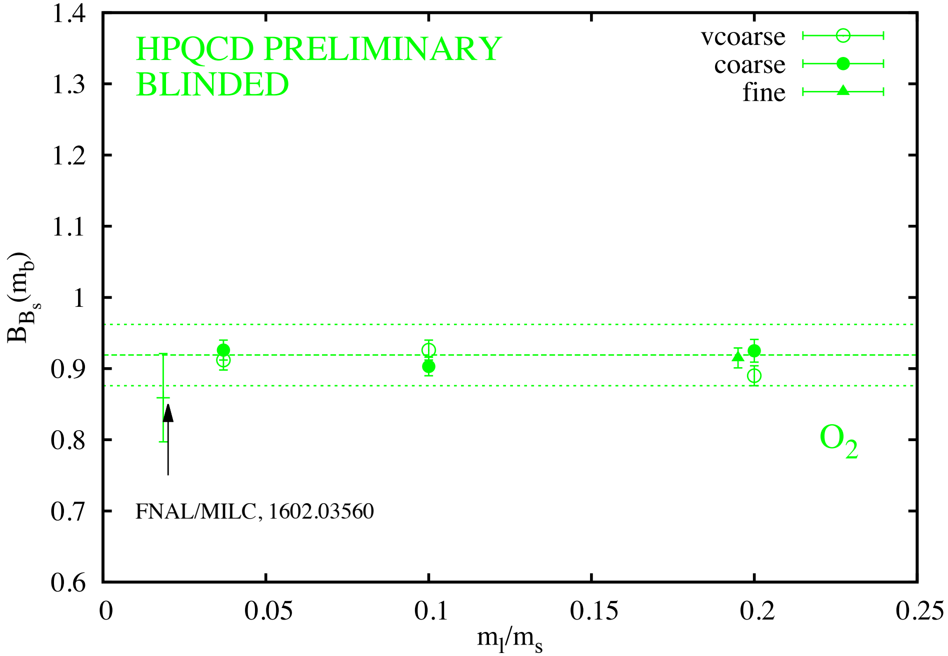


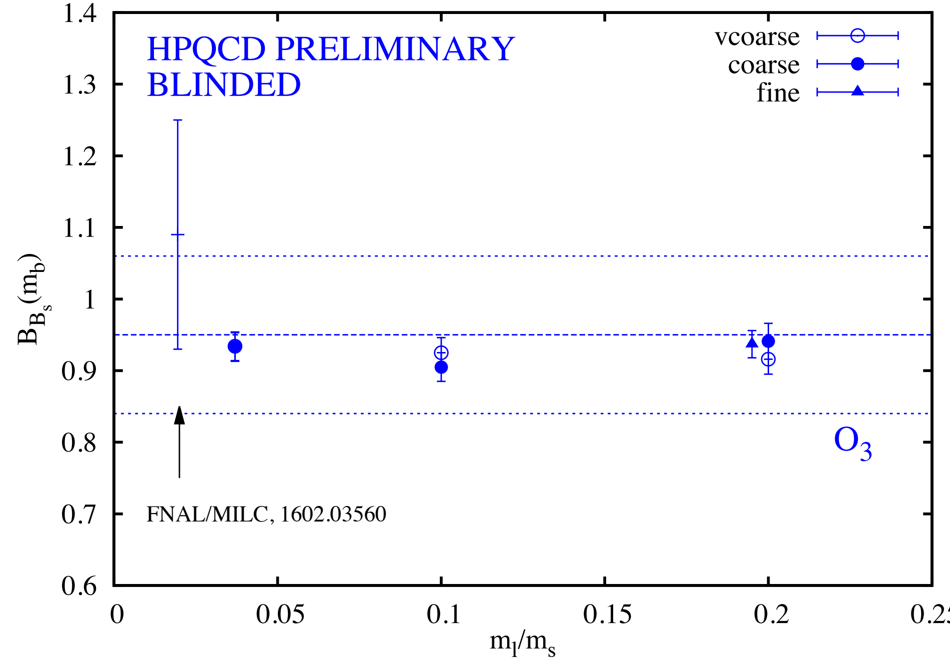
n_f=2+1+1 HISQ + improved NRQCD b; a=0.15fm to 0.09 fm; B params, pert match

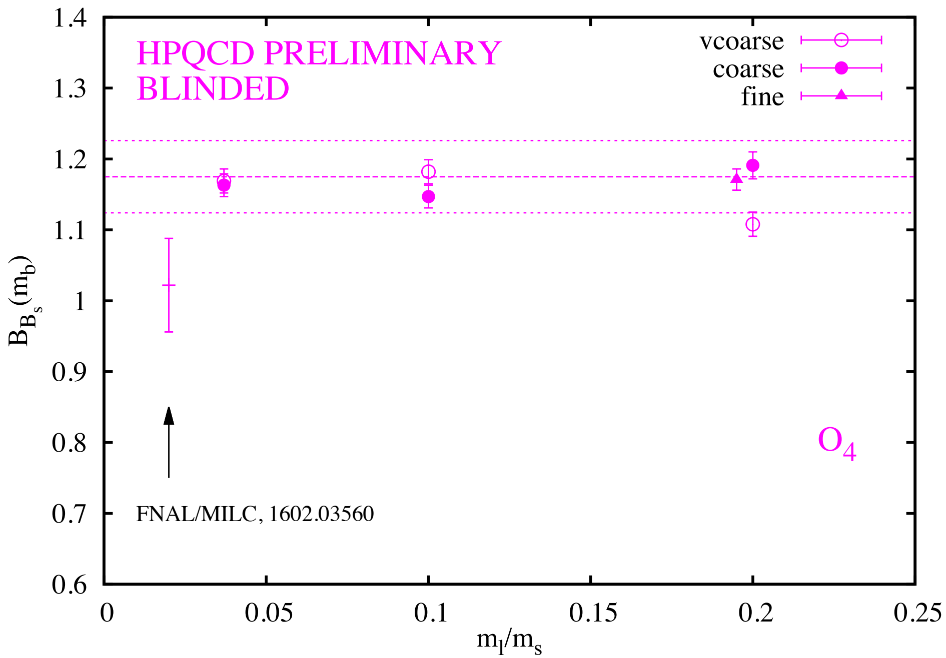


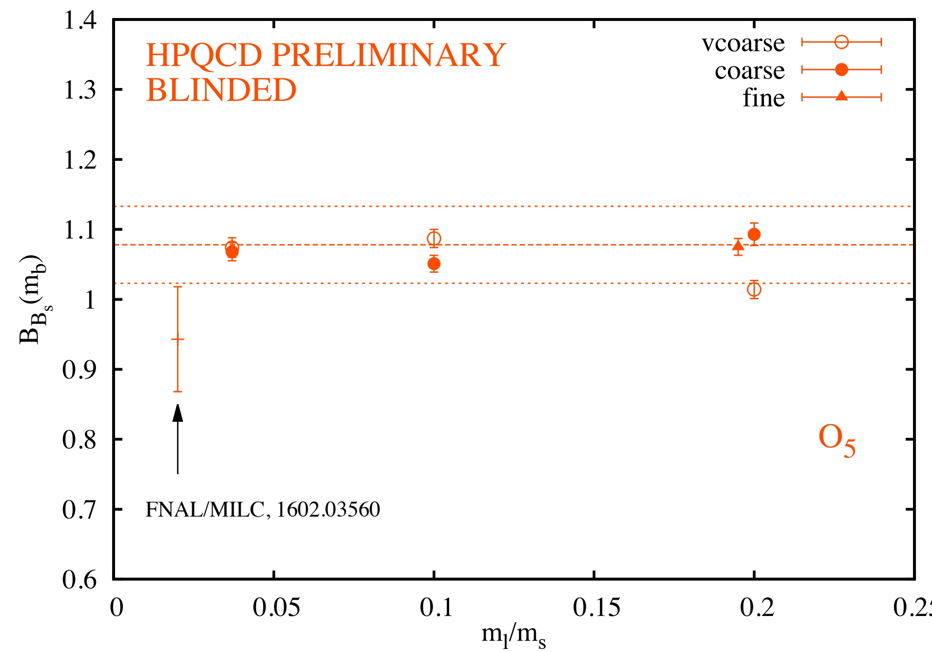
Bag parameters are indeed very insensitive to m_q consistent with benign chiral pert. th. behaviour

Further results - all blinded (so normalisation is not correct)









Chiral pert. theory for dependence on m_π - log terms

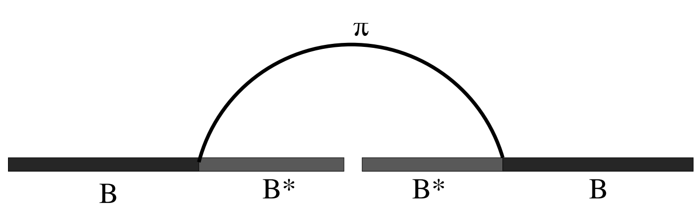
$$f_B = f_{B,0}\left(1 - \frac{3+9g^2}{4}\frac{m_\pi^2}{\Lambda_\chi^2}\ln(\frac{m_\pi^2}{\mu^2})\right)$$

$$B_B^{(n)} = B_{B.0}^{(n)}\left(1 - \frac{1-3g^2X_n}{2}\frac{m_\pi^2}{\Lambda_\chi^2}\ln(\frac{m_\pi^2}{\mu^2})\right)$$

$g^2 = 0.20(7)$ is B*Bπ coupling from lattice QCD

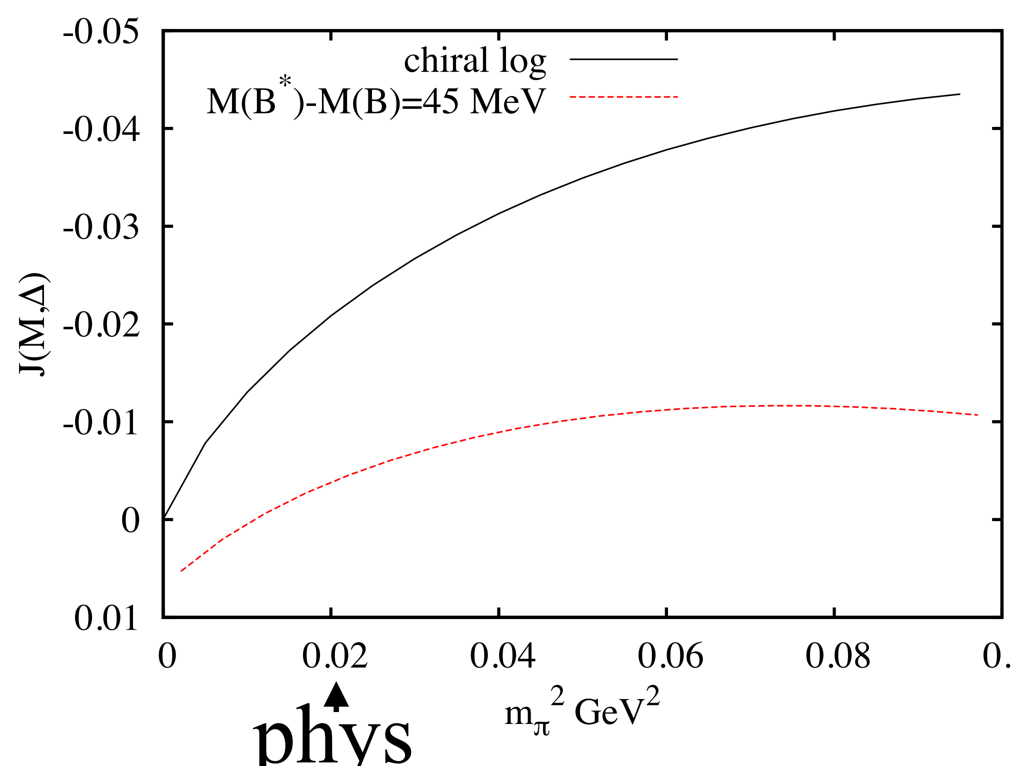
(there are also m_π^2 polynomial terms)

g^2 term from "sunset" diag.



Inc. B*-B splitting modifies log considerably

For n > 1 g^2 term multiplies B* bag. For n=1 X=1.

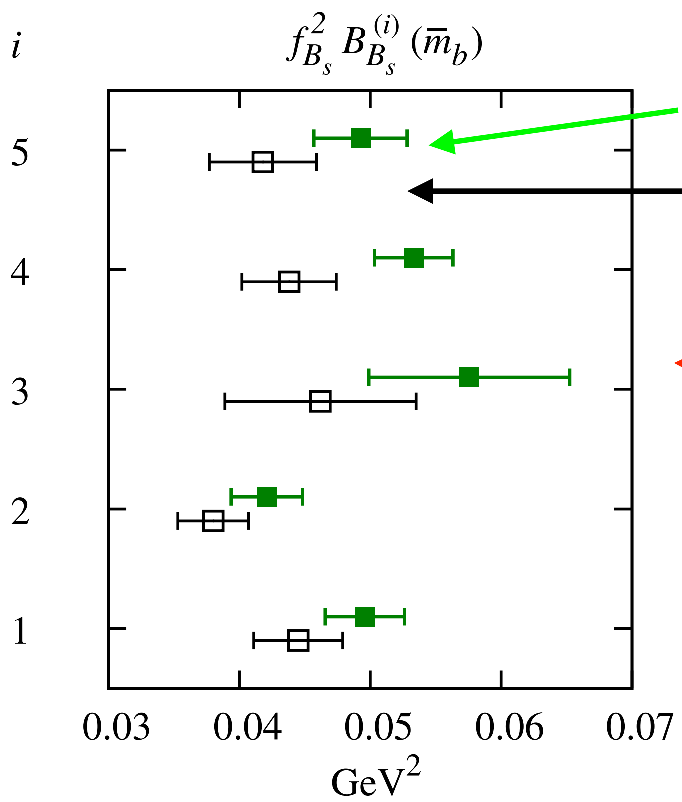


Preliminary Error budget for O_1

	B_{B_s}	B_{B_s}/B_{B_d}
data/stats	1.2%	1.3%
light-quark mass	0.6%	0.6%
lattice-spacing	1.1%	0.01%
α_s^2	4.2%	0.1%
total	4.6%	1.4%

matching error dominates - must allow for coeffs varying with a*m_b. Cancels in ratio

Current status (plots from FNAL/MILC 1602.03560)

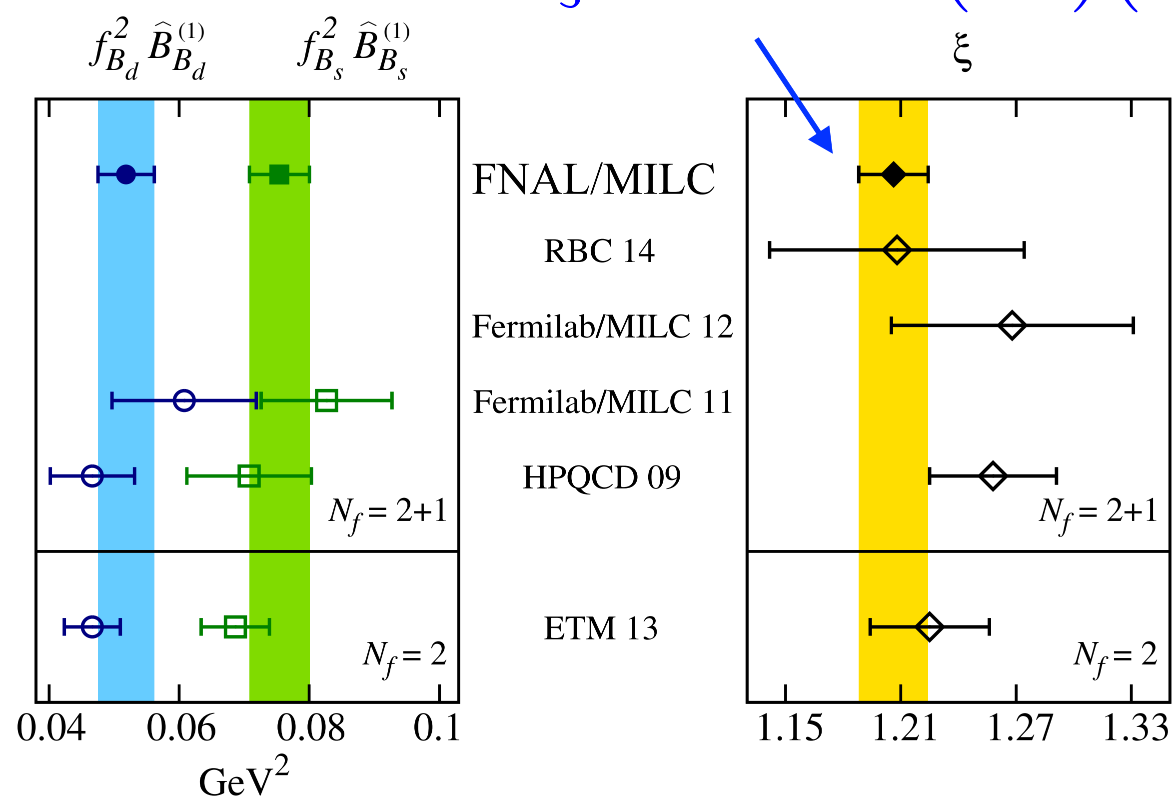


FNAL/MILC,1602.03560.

ETM,1308.1851

Uncty largest for O_3 since it mixes with much larger O_1

$$\xi = 1.206(18)(6)$$

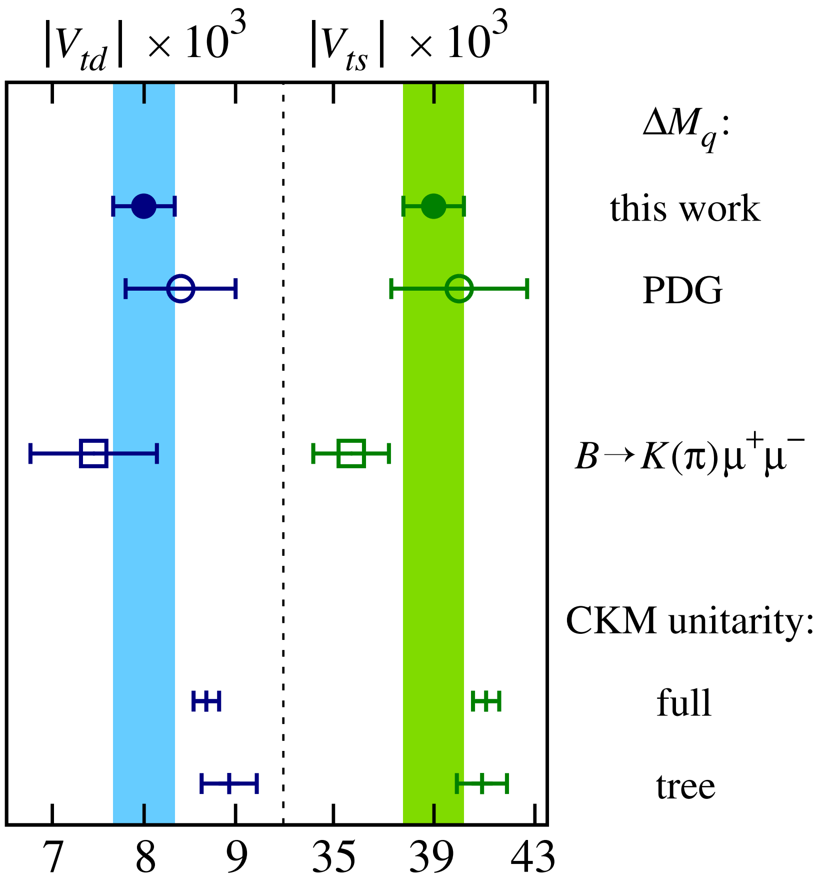


$$\xi = \frac{f_{B_s}}{f_{B_d}}\sqrt{\frac{B_{B_s}}{B_{B_d}}}$$

$$\frac{f_{B_s}}{f_B} = 1.205(7)$$

R. Dowdall, et al, HPQCD, 1302.2644

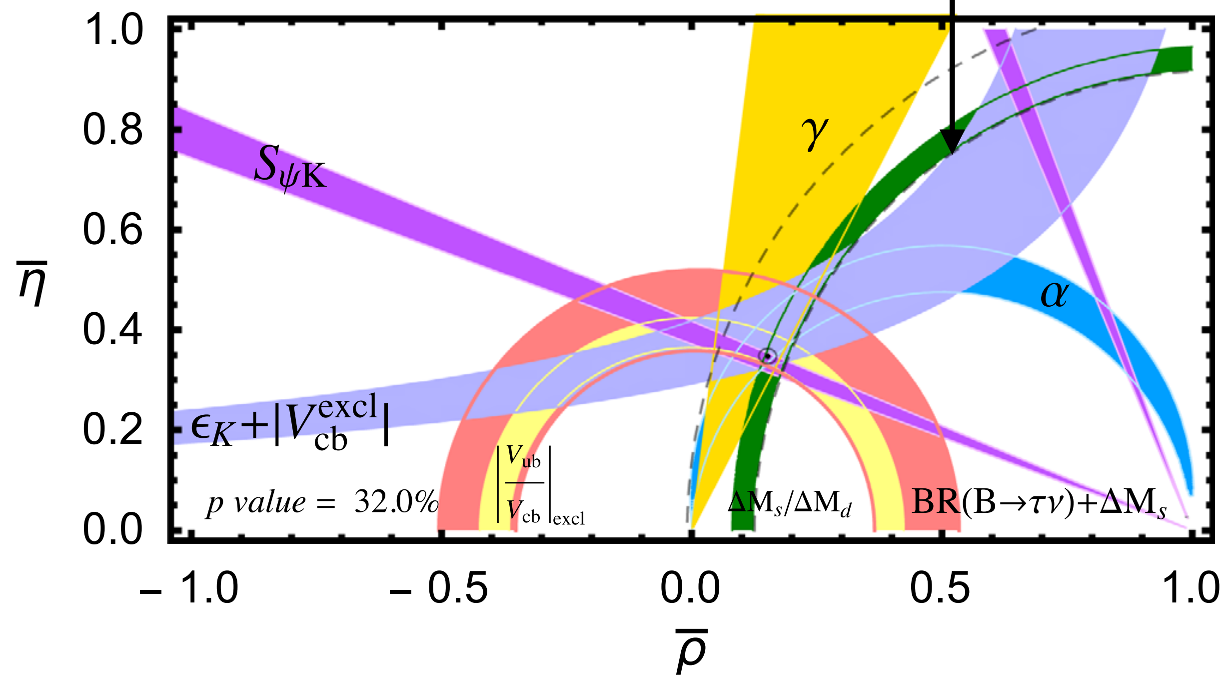
Current status (plots from FNAL/MILC 1602.03560)



Combining O_1 results with expt for ΔM_q gives CKM elements

$$\Delta M_q = \frac{G_F^2 m_W^2 M_{B_q}^2}{6\pi^2} S_0(x_t)\eta_{2B}|V_{tq}^* V_{tb}|^2 f_{B_q}^2 \hat{B}_{B_q}^{(1)}$$

2σ tension with CKM unitarity



Conclusion

- HPQCD results for MEs of $\Delta B = 2$ operators almost complete. Calculations have sea u,d,s,c and include physical u/d quark masses. Calculation allows direct determination of bag parameters, useful because they have little dependence on m_q, a etc.

- Errors will be similar or better to existing results from FNAL/MILC. Errors dominated by perturbative matching uncertainty.

Future

- Dimension 7 ops. for $\Delta\Gamma_s$ (Wingate talk, LAT17) - large matching errors, but existing errors may be halved.