

Rare B decays on the lattice

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introduction

B

- ▶ Initial state is a pseudoscalar B or B_s meson

$B^+ = (u\bar{b})$, $B^- = (\bar{u}b)$, $B^0 = (d\bar{b})$ and $\bar{B}^0 = (\bar{d}b)$ with mass ~ 5280 GeV

$B_s^0 = (s\bar{b})$ and $\bar{B}_s^0 = (\bar{s}b)$ with mass ~ 5367 GeV

B decays

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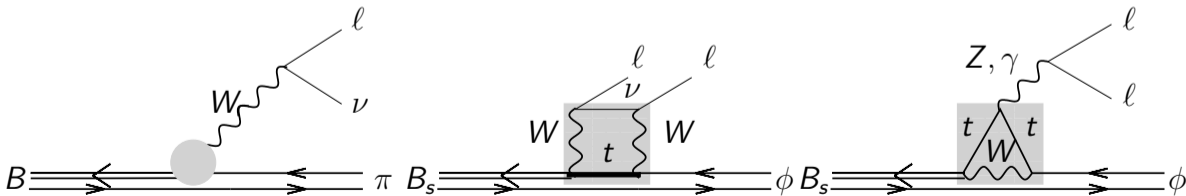
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- ▶ Weak decays of the b -quark

→ Charged flavor changing currents mediated by W^\pm (tree-level)

→ Flavor changing neutral currents (loop-level)



Rare B decays

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 - Flavor changing neutral currents (loop-level)
- ▶ Suppressed in the Standard Model
 - CKM suppressed
 - GIM suppressed (no FCNC)

$$\begin{bmatrix} d' \\ s' \\ b' \end{bmatrix} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \begin{bmatrix} d \\ s \\ b \end{bmatrix} \quad \text{with} \quad \begin{bmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{bmatrix} = \begin{bmatrix} 0.97434 & 0.22506 & 0.00357 \\ 0.22492 & 0.97351 & 0.0411 \\ 0.00875 & 0.0403 & 0.99915 \end{bmatrix}$$

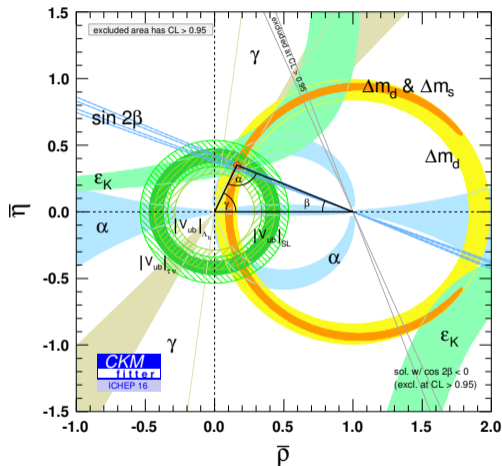
[PDG 2016]

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 - $B^+ = (u\bar{b})$, $B^- = (\bar{u}b)$, $B^0 = (d\bar{b})$ and $\bar{B}^0 = (\bar{d}b)$ with mass ~ 5280 GeV
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 - Flavor changing neutral currents (loop-level)
- ▶ Suppressed in the Standard Model
 - CKM suppressed
 - GIM suppressed (no FCNC)
- ▶ Nonperturbative calculation of form factors
 - Exclusive semi-leptonic decays with one hadronic final state
 - Pseudoscalar or vector (narrow width approximation) final states
 - Only short distance contributions

Why are we interested in rare B decays?

- ▶ Charged current decays allow to determine CKM matrix elements $|V_{cb}|$ and $|V_{ub}|$
- Test unitarity of the CKM matrix
- Precision tests of the Standard Model
- ▶ Searches for / constraints on new physics

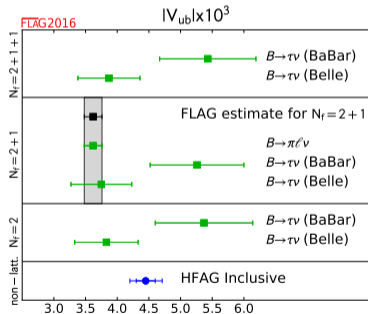
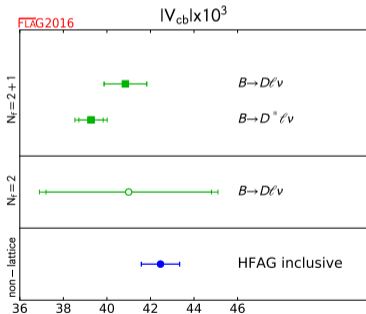


[<http://ckmfitter.in2p3.fr>]

Determination of $|V_{cb}|$ and $|V_{ub}|$

- ▶ Commonly $|V_{cb}|$ extracted from $B \rightarrow D^{(*)} \ell \nu$ and $|V_{ub}|$ extracted from $B \rightarrow \pi \ell \nu$
- ▶ Long standing tension between **exclusive** and **inclusive** determinations
- Revisit HQET constraints entering z-parametrizations

[Bigi, Gambino PRD94 (2016) 094008][Bigi, Gambino, Schacht PLB769 (2017) 441-445]



[FLAG2016]

[Fermilab/MILC PRD92 (2015) 034506] [HPQCD PRD92 (2015) 054510]

[Fermilab/MILC PRD89 (2014) 114504]

[Atoui et al. EPJC74 (2014) 2861]

[HPQCD PRD75 (2006) 119906]

[Fermilab/MILC PRD92 (2015) 014024]

[RBC-UKQCD PRD91 (2015) 074510]

Determination of $|V_{cb}|$ and $|V_{ub}|$: explore alternative channels

- ▶ $|V_{ub}|$ from $B \rightarrow \tau\nu$: errors too large
- ▶ $|V_{cb}|/|V_{ub}|$ from exclusive baryonic decays: $\Lambda_b \rightarrow \Lambda_c \ell\nu$ and $\Lambda_b \rightarrow p \ell\nu$
[Detmold, Lehner, Meinel, PRD92 (2015) 034503]
- ▶ $|V_{cb}|$ from $B_s \rightarrow D_s \ell\nu$ and $|V_{ub}|$ from $B_s \rightarrow K \ell\nu$
 - B -factories typically run at the $\Upsilon(4s)$ threshold i.e. B but no B_s mesons are produced
 - Not (yet) experimentally measured with sufficient precision
 - LHC energies are large enough to produce sufficient B_s mesons → **LHCb**
 - Absolute normalization is challenging; ratios are preferred: determine $|V_{cb}|/|V_{ub}|$
- ▶ Use lattice techniques to compute inclusive decays
[Hashimoto PTEP 2017 (2017) 053B03] [Hansen, Meyer, Robaina arXiv:1704.08993]

Searches for new physics (tree-level)

- ▶ Lepton flavor universality violations in \mathcal{R}_D ratios

$$\mathcal{R}_{D^{(*)}^{\tau/\mu}} \equiv \frac{d\Gamma(B \rightarrow D^{(*)}\tau\nu_\tau)/dq^2}{d\Gamma(B \rightarrow D^{(*)}\mu\nu_\mu)/dq^2}$$

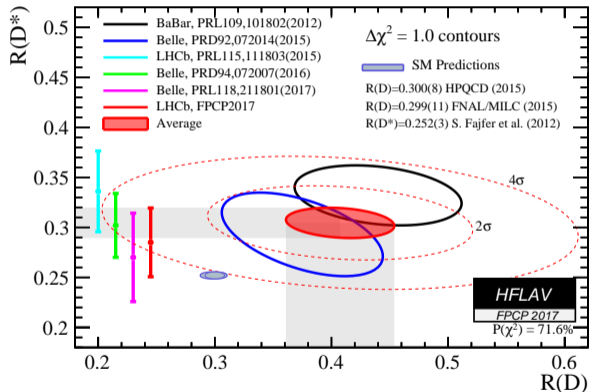
- ▶ Input: form factors over full q^2 range

→ $B \rightarrow D\ell\nu$ [HPQCD PRD92 (2015) 054510]
[Fermilab/MILC PRD92 (2015) 035606]

→ $B \rightarrow D^*\ell\nu$
[Fajfer, Kamenik, Nisandzic PRD85 (2012) 094025]

→ Theoretical uncertainty on $B \rightarrow D^*\ell\nu$
is suspiciously small

[HFLAV website]



Searches for new physics (loop-level)

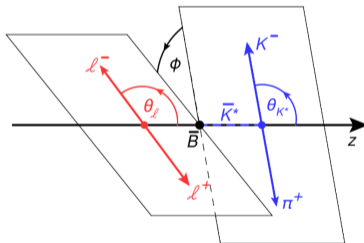
- ▶ $b \rightarrow sl^+l^-$ processes e.g. $B \rightarrow \pi l^+l^-$, $B \rightarrow K^{(*)}l^+l^-$, $B_s \rightarrow \phi l^+l^-$
- ▶ Decomposition into angular variables e.g. $B \rightarrow K^*l^+l^-$

$$\frac{1}{d(\Gamma + \bar{\Gamma})/dq^2} \frac{d^4q(\Gamma + \bar{\Gamma})}{dq^2 d\bar{\Omega}} = \frac{9}{32\pi} \left[\frac{3}{4} (1 - F_L) \sin^2 \theta_K + F_L \cos^2 \theta_K \right.$$

$$+ \frac{1}{4} (1 - F_L) \sin^2 \theta_K \cos 2\theta_l - F_L \cos^2 \theta_K \cos 2\theta_l + S_3 \sin^2 \theta_l \cos 2\phi$$

$$+ S_4 \sin 2\theta_K \sin 2\theta_l \cos \phi + S_5 \sin 2\theta_K \sin \theta_l \cos \phi + \frac{4}{3} A_{FB} \sin^2 \theta_K \cos \theta_l$$

$$\left. + S_7 \sin 2\theta_K \sin \theta_l \sin \phi + S_8 \sin 2\theta_K \sin 2\theta_l \sin \phi + S_9 \sin^2 \theta_K \sin^2 \theta_l \sin 2\phi \right]$$

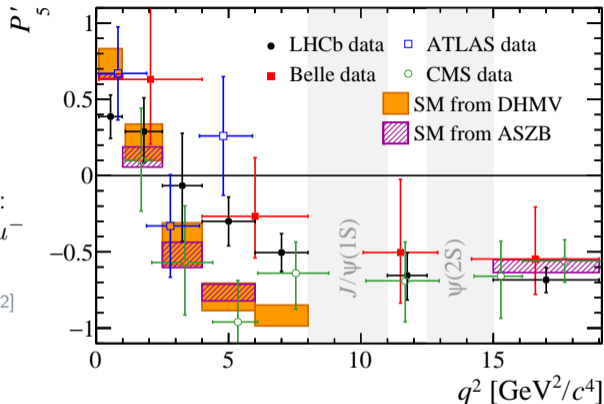


- ▶ F_L , A_{FB} , S_i are functions of Wilson coefficients \Rightarrow sensitive to new physics

- ▶ To reduce hadronic uncertainties, introduce $P'_{i=4,5,6,8} = -\frac{S_{j=4,5,7,8}}{\sqrt{F_L(1-F_L)}}$

Searches for new physics (loop-level)

- ▶ Few sigma deviations from SM expectations seen for branching fractions and angular observables $3 \text{ GeV}^2 \lesssim q^2 \lesssim 8 \text{ GeV}^2$
- ▶ LHCb reported deviations for different processes:
 $B^0 \rightarrow K^0 \mu^+ \mu^-$, $B^+ \rightarrow K^+ \mu^+ \mu^-$, $B^0 \rightarrow K^* \mu^+ \mu^-$
 $B_s \rightarrow \phi \mu^+ \mu^-$, $\Lambda_b \rightarrow \Lambda \mu^+ \mu^-$
 [JHEP 06 (2014) 133][JHEP 11 (2016) 047][JHEP 04 (2017) 142]
 [JHEP 06 (2015) 115][JHEP 09 (2015) 179]
- ▶ Deviations seen by ATLAS, CMS, LHCb, Belle
- ▶ Hinting at new physics in Wilson coefficient C_9 ?
- ▶ Near J/ψ resonance:
 → Are hadronic uncertainties under control?



[LHCb JHEP 02 (2016) 104]

[ATLAS-CONF-2017-023]

[Belle PRL118 (2017) 111801]

[CMS-PAS-BPH-15-008]

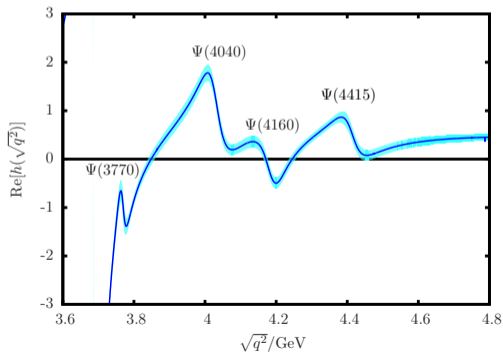
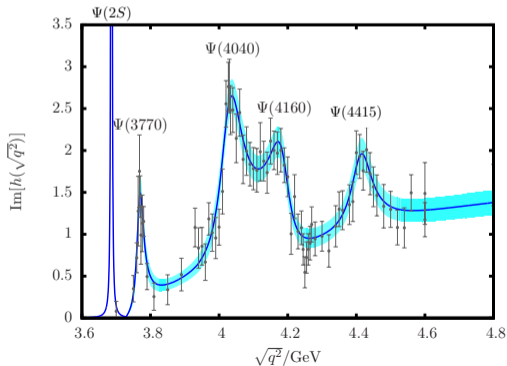
[DHMV JHEP 12 (2014) 125, JHEP 10 (2016) 075]

[ASZB JHEP 08 (2016) 098, EPJC 75 (2015) 382]

plot: [Gershon arXiv:1707.05290]

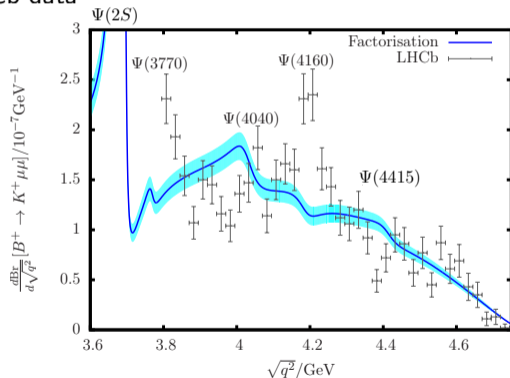
Searches for new physics (loop-level)

- ▶ Hadronic uncertainties: charm resonances [Lyon and Zwicky, arXiv:1406.0566]
 - SM predictions rely on factorization approximation (FA)
 - In the FA, charm-resonance contributions equal charm vacuum polarization
 - Extract charm vacuum polarization via dispersion relation from BESII-data ($e^+e^- \rightarrow \text{hadrons}$)



Searches for new physics (loop-level)

- ▶ Hadronic uncertainties: charm resonances [Lyon and Zwicky, arXiv:1406.0566]
 - Derived SM prediction for $B \rightarrow K\ell\ell$ based on FA and lattice QCD [HPQCD PRD88 (2013) 054509] disagrees with LHCb data

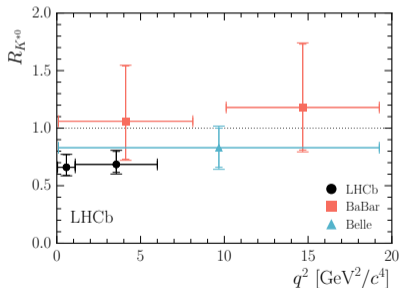
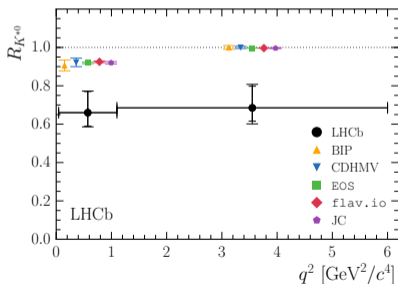


- ▶ Explore experimentally e.g. phase difference between short- and long-distance amplitude [LHCb EPJC77 (2017) 161]

Searches for new physics (loop-level)

- ▶ Lepton flavor universality violations in \mathcal{R}_K ratios

$$\mathcal{R}_{K^*}^{\mu/e} \equiv \frac{d\Gamma(B^0 \rightarrow K^{*0} \mu^+ \mu^-)/dq^2}{d\Gamma(B^0 \rightarrow K^{*0} e^+ e^-)/dq^2}$$



[BIP EPJC76 (2016) 440]

[CDHMV JHEP 10 (2016) 0751]

[EOS PRD95 (2017) 035029]

[flav.io JHEP08 (2016) 098]

[JC PRD93 (2016) 014028]

[BaBar PRD86 (2012) 032012]

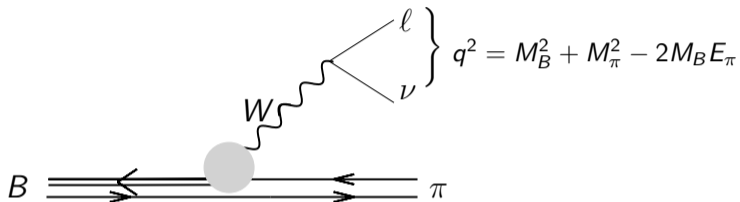
[Belle PRL103 (2009) 171801]

[LHCb JHEP 08 (2017) 055]

plots: [LHCb JHEP 08 (2017) 055]

calculation

$|V_{ub}|$ from exclusive semileptonic $B \rightarrow \pi \ell \nu$ decay



- Conventionally parametrized by (neglecting term $\propto m_\ell^2 f_0^2$)

$$\frac{d\Gamma(B \rightarrow \pi \ell \nu)}{dq^2} = \frac{G_F^2}{192\pi^3 M_B^3} \left[(M_B^2 + M_\pi^2 - q^2)^2 - 4M_B^2 M_\pi^2 \right]^{3/2} \times |f_+(q^2)|^2 \times |V_{ub}|^2$$

experiment

known

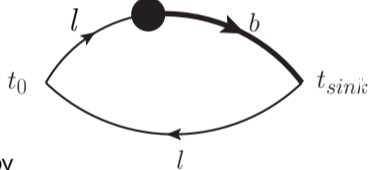
nonperturbative input

CKM

$B \rightarrow \pi l \nu$ form factors

- ▶ Parametrize the hadronic matrix element for the flavor changing vector current in terms of the form factors $f_+(q^2)$ and $f_0(q^2)$

$$\langle \pi(k) | \bar{u} \gamma^\mu b | B(p) \rangle = f_+(q^2) \left(p^\mu + k^\mu - \frac{M_B^2 - M_\pi^2}{q^2} q^\mu \right) + f_0(q^2) \frac{M_B^2 - M_\pi^2}{q^2} q^\mu$$

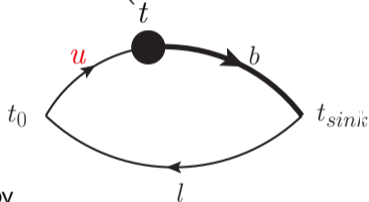


- ▶ Calculate 3-point function by
 - Inserting a quark source for a “light” propagator at t_0
 - Allow it to propagate to t_{sink} , turn it into a sequential source for a b quark
 - Use another “light” quark propagating from t_0 and contract both at t

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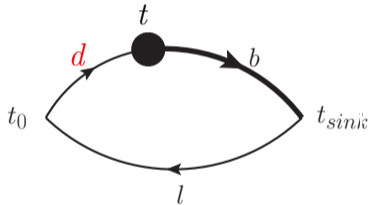


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 - Allow it to propagate to t_{sink} , turn it into a sequential source for a b quark
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- ▶ On the lattice u and d quarks are degenerate (l); physically the daughter quark is a u -quark

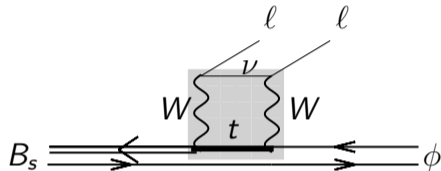
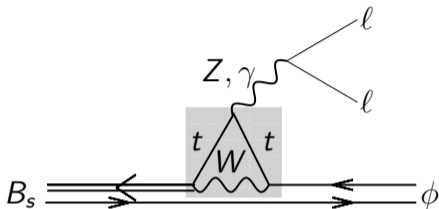
$B \rightarrow \pi l^+ l^-$ form factor

- ▶ If the daughter quark is a d -quark, we have a FCNC decay at loop-level
- ▶ Dominant contributions at short distance: f_0 , f_+ , and f_T

$$\langle \pi(k) | i \bar{d} \sigma^{\mu\nu} b(p) | B \rangle = 2 \frac{p^\mu k^\nu - p^\nu k^\mu}{M_B + M_\pi} f_T(q^2)$$



$B_s \rightarrow \phi l^+ l^-$ form factors



- ▶ Vector final state treated in narrow width approximation
- ▶ Effective Hamiltonian

$$\mathcal{H}_{\text{eff}}^{b \rightarrow s} = -\frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_i^{10} C_i O_i^{(r)}$$

- ▶ Leading contributions at short distance

$$O_7^{(r)} = \frac{m_b e}{16\pi^2} \bar{s} \sigma^{\mu\nu} P_{R(L)} b F_{\mu\nu}$$

$$O_9^{(r)} = \frac{e^2}{16\pi^2} \bar{s} \gamma^\mu P_{L(R)} b \bar{l} \gamma_\mu l$$

$$O_{10}^{(r)} = \frac{e^2}{16\pi^2} \bar{s} \gamma^\mu P_{L(R)} b \bar{l} \gamma_\mu \gamma^5 l$$

Seven form factors

$$\langle \phi(k, \lambda) | \bar{s} \gamma^\mu b | B_s(p) \rangle = f_V(q^2) \frac{2i \epsilon^{\mu\nu\rho\sigma} \epsilon_\nu^* k_\rho p_\sigma}{M_{B_s} + M_\phi}$$

$$\langle \phi(k, \lambda) | \bar{s} \gamma^\mu \gamma_5 b | B_s(p) \rangle = f_{A_0}(q^2) \frac{2M_\phi \epsilon^* \cdot q}{q^2} q^\mu$$

$$+ f_{A_1}(q^2) (M_{B_s} + M_\phi) \left[\epsilon^{*\mu} - \frac{\epsilon^* \cdot q}{q^2} q^\mu \right]$$

$$- f_{A_2}(q^2) \frac{\epsilon^* \cdot q}{M_{B_s} + M_\phi} \left[k^\mu + p^\mu - \frac{M_{B_s}^2 - M_\phi^2}{q^2} q^\mu \right]$$

$$q_\nu \langle \phi(k, \lambda) | \bar{s} \sigma^{\nu\mu} b | B_s(p) \rangle = 2f_{T_1}(q^2) \epsilon^{\mu\rho\tau\sigma} \epsilon_\rho^* k_\tau p_\sigma,$$

$$q_\nu \langle \phi(k, \lambda) | \bar{s} \sigma^{\nu\mu} \gamma^5 b | B_s(p) \rangle = if_{T_2}(q^2) \left[\epsilon^{*\mu} (M_{B_s}^2 - M_\phi^2) - (\epsilon^* \cdot q) (p + k)^\mu \right]$$

$$+ if_{T_3}(q^2) (\epsilon^* \cdot q) \left[q^\mu - \frac{q^2}{M_{B_s}^2 - M_\phi^2} (p + k)^\mu \right]$$

Other (recent) lattice calculations

- ▶ Further details on charged current semileptonic B decays [see talk by Chris Bouchard](#)
- ▶ HPQCD
 - Form factors f_0 , f_+ , and f_T for $B \rightarrow K\ell^+\ell^-$ [PRL111 (2013) 162002, Erratum: PRL112 (2014) 149902]
[PRD88 (2013) 054509, Erratum: PRD88 (2013) 079901]
- ▶ Fermilab/MILC
 - Tensor form factor f_T for $B \rightarrow \pi\ell^+\ell^-$ [PRL115 (2015) 152002]
 - Form factors f_0 , f_+ , and f_T for $B \rightarrow K\ell^+\ell^-$ [PRD93 (2016) 025026]
- ▶ Horgan, Liu, Meinel, and Wingate
 - 7 form factors for $B \rightarrow K^*\ell^+\ell^-$ and $B_s \rightarrow \phi\ell^+\ell^-$ [PRD89 (2014) 090501][PoS Lattice2014 (2015) 372]
 - Angular analysis [PRL112 (2014) 212003]

Challenges

- ▶ b -quark (4.18 GeV) is ~ 1000 times heavy than d -quark (4.7 MeV) and larger than a^{-1}
 - simulate b -quark with effective action
 - requires renormalization of mixed action
 - Fermilab-action/RHQ, NRQCD, HQET
 - extrapolate to physical b -quark
 - allows for full nonperturbative renormalization
 - ETMC ratio method, heavy HISQ, heavy DWF
- ▶ Vector final states are unstable in QCD \rightarrow narrow width approximation
 - \rightarrow Chiral perturbation theory cannot guide extrapolations of data at unphysically heavy pions
 - \rightarrow Resonances in range of kinematic extrapolation (hadronic uncertainties)
 - \rightarrow Pioneering work on $B \rightarrow K^* \rightarrow K\pi$ (cf. Leskovec, Meinel); new ideas [Hansen, Meyer, Robaina arXiv:1704.08993]
- ▶ Simulations with physical light and bottom quarks troubled by poor signal-to-noise ratio
 - \rightarrow So far form factors only at q_{\max}^2 calculated [HPQCD PRD93 (2016) 034502]
- ▶ Long distance contributions

RBC-UKQCD's project

Target quantities

- ▶ Decay constants f_B and f_{B_s}
- ▶ $B^0 - \bar{B}^0$ mixing matrix elements
- ▶ Semileptonic form factors with charged and neutral flavor changing currents

$$B \rightarrow \pi l \nu, B_s \rightarrow K l \nu$$

$$B \rightarrow D^{(*)} l \nu, B_s \rightarrow D_s^{(*)} l \nu, \dots$$

$$B \rightarrow K^{(*)} l^+ l^-, B_s \rightarrow \phi l^+ l^-, \dots$$

→ Ratios $R(D^{(*)}), R(K^{(*)}), \dots$

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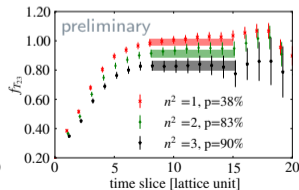
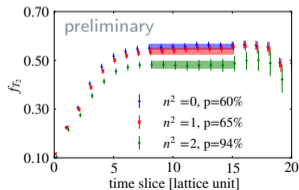
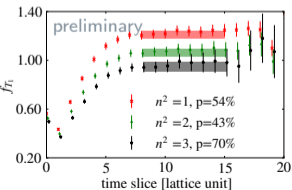
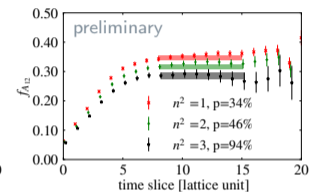
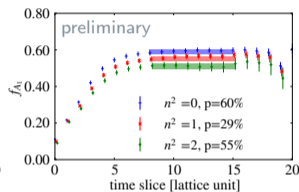
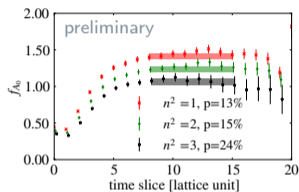
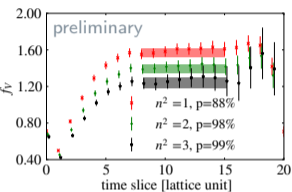
$$B \rightarrow K^{(*)} l^+ l^-, B_s \rightarrow \phi l^+ l^-, \dots$$

→ Ratios $R(D^{(*)}), R(K^{(*)}), \dots$

Set-up

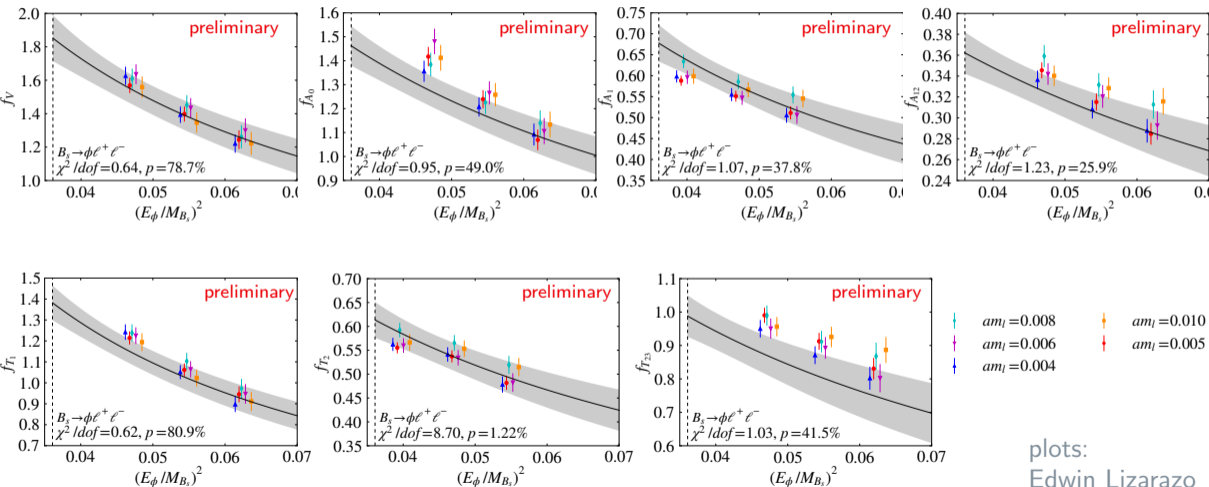
- ▶ RBC-UKQCD's 2+1 flavor domain-wall fermion and Iwasaki gauge field ensembles
 - 3 lattice spacings incl. one with phys. pions
- ▶ Unitary DWF for light and strange quarks
- ▶ Heavy Möbius DWF for charm quarks
- ▶ RHQ for bottom quarks
- ▶ Operators $O(a)$ -improved at 1-loop/treelevel
- ▶ Mostly nonperturbative renormalization

$B_s \rightarrow \phi ll$: Seven form factors ($a^{-1} = 1.784$ GeV, $am_l^{\text{sea}} = 0.005$, $am_s = 0.03224$)



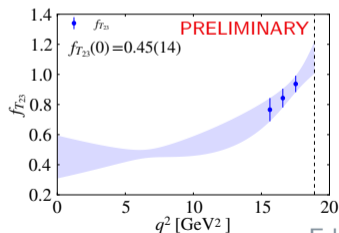
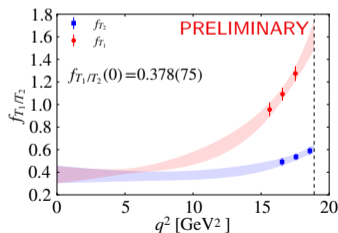
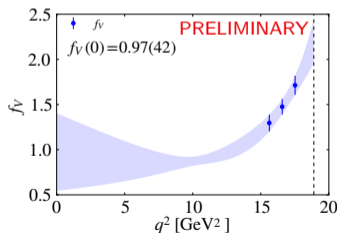
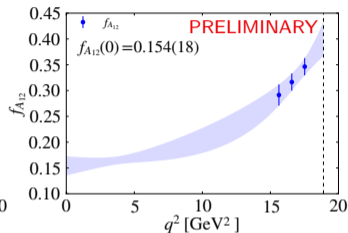
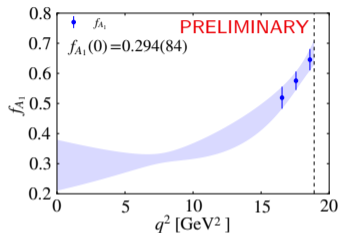
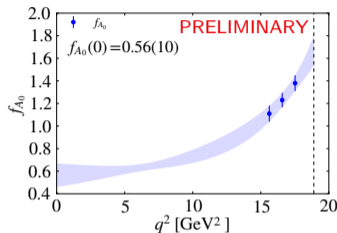
plots:
Edwin Lizarazo

$B_s \rightarrow \phi ll$: Seven form factors vs. q^2



plots:
Edwin Lizarazo

$B_s \rightarrow \phi ll$: First attempt to use z-parametrization



plots:
Edwin Lizarazo

conclusion

Conclusion

- ▶ Rare B decays allow many tests of the Standard Model and exhibit tantalizing signals
- ▶ Yet more data and improved theoretical predictions are needed
- ▶ Further lattice calculations are on the way but analysis takes time
- ▶ However, general code allows to share the computational costs for many processes of interest
 - $B \rightarrow \pi l \nu$, $B \rightarrow \pi ll$
 - $B \rightarrow K^* ll$
 - $B \rightarrow D^{(*)} l \nu$
 - $B_s \rightarrow K^{(*)} l \nu$, $B_s \rightarrow K^{(*)} ll$
 - $B_s \rightarrow D_s^{(*)} l \nu$
 - ...

Resources and Acknowledgments

USQCD: Ds, Bc, and π^0 cluster (Fermilab), qcd12s cluster (Jlab)

RBC qcdcl (RIKEN) and cuth (Columbia U)

UK: ARCHER, cirrus (EPCC) and DiRAC (UKQCD)



appendix

Set-up

- ▶ RBC-UKQCD's 2+1 flavor domain-wall fermion and Iwasaki gauge action ensembles
 - Three lattice spacings $a \sim 0.11$ fm, 0.08 fm, 0.07 fm; one ensemble with physical pions
[PRD 78 (2008) 114509][PRD 83 (2011) 074508][PRD 93 (2016) 074505][arXiv:1701.02644]
- ▶ Unitary and partially quenched domain-wall up/down quarks
[Kaplan PLB 288 (1992) 342], [Shamir NPB 406 (1993) 90]
- ▶ Domain-wall strange quarks at/near the physical value
- ▶ Charm: Möbius domain-wall fermions optimized for heavy quarks [Boyle et al. JHEP 1604 (2016) 037]
 - Simulate 3 or 2 charm-like masses then extrapolate/interpolate
- ▶ Effective relativistic heavy quark (RHQ) action for bottom quarks
[Christ et al. PRD 76 (2007) 074505], [Lin and Christ PRD 76 (2007) 074506]
 - Builds upon Fermilab approach [El-Khadra et al. PRD 55 (1997) 3933]
 - Allows to tune the three parameters ($m_0 a$, c_P , ζ) nonperturbatively [PRD 86 (2012) 116003]
 - Smooth continuum limit; heavy quark treated to all orders in $(m_b a)^n$

2+1 Flavor Domain-Wall Iwasaki ensembles

L	$a^{-1}(\text{GeV})$	am_l	am_s	$M_\pi(\text{MeV})$	# configs.	#sources	
24	1.784	0.005	0.040	338	1636	1	[PRD 78 (2008) 114509]
24	1.784	0.010	0.040	434	1419	1	[PRD 78 (2008) 114509]
32	2.383	0.004	0.030	301	628	2	[PRD 83 (2011) 074508]
32	2.383	0.006	0.030	362	889	2	[PRD 83 (2011) 074508]
32	2.383	0.008	0.030	411	544	2	[PRD 83 (2011) 074508]
48	1.730	0.00078	0.0362	139	40	81/1*	[PRD 93 (2016) 074505]
64	2.359	0.000678	0.02661	139	—	—	[PRD 93 (2016) 074505]
48	2.774	0.002144	0.02144	234	70	24	[arXiv:1701.02644]

* All mode averaging: 81 “sloppy” and 1 “exact” solve [Blum et al. PRD 88 (2012) 094503]

► Lattice spacing determined from combined analysis [Blum et al. PRD 93 (2016) 074505]

► a : ~ 0.11 fm, ~ 0.08 fm, ~ 0.07 fm