Non-leptonic Multibody B decays

Theory and Prospects

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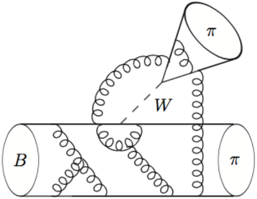
Motivations

- Huge multiplicity of final states (2-body + multi-body), large data sets
- Important input in CKM studies (mostly angles)
- CP violation (SM and new physics)
- Non-trivial hadronic dynamics

  $\Rightarrow$ Perturbative and non-perturbative QCD methods



Non-leptonic B-decay Amplitudes

- Effective Hamiltonian at the hadronic scale $\mu \sim m_B$

$$\mathcal{H}_{\text{eff}} = -\mathcal{L}_{QED+QCD} + \sum_i C_i(\mu)\, \mathcal{O}_i(\mu)$$

- C_i -- Wilson coefficients (UV physics) $\to$ perturbation theory

  Known to NNLL: Bobeth, Misiak, Urban '99; Misiak, Steinhauser '04, Gorbahn, Haisch '04; Gorbahn, Haisch, Misiak '05; Czakon, Haisch, Misiak '06.

- $\mathcal{O}_i$ -- Effective operators (IR physics) [e.g. $\mathcal{O} = (\bar{b}\gamma^\mu u)(\bar{u}\gamma_\mu d)$]

- Amplitudes:

$$\mathcal{A}(B \to M_1 M_2 \cdots) = \sum_i C_i \langle M_1 M_2 \cdots | \mathcal{O}_i | B \rangle$$

The problem is to compute the **operator matrix elements (MEs)**

$\longrightarrow$ non-perturbative, process dependent (non-universal)

Direct CP Violation

$$\mathcal{A}(\bar{B} \to f) \equiv \mathcal{A}_f = \underbrace{\lambda_u}_{\sim e^{i\gamma}} \underbrace{(T_f^u - P_f)}_{\mathcal{A}^u} + \underbrace{\lambda_c}_{\simeq \text{real}} \underbrace{(T_f^c - P_f)}_{\mathcal{A}^c} \qquad \lambda_p = V_{pb} V^\star_{p\{d,s\}}$$

$$T_f^p = \sum_{1,2} C_i^p \langle f | Q_i^p | \bar{B} \rangle \quad \text{(current-current operators)}$$

$$P_f = \sum_{3,\ldots,6} C_i \langle f | Q_i^p | \bar{B} \rangle \quad \text{(penguin operators)}$$

- ▷ In the SM, C_i contain no phases.
- ▷ We write $\mathcal{A}^p = |\mathcal{A}^p| e^{i\delta_p}$. Then:

$$\mathcal{A}_{\text{CP}} \equiv \frac{|\mathcal{A}_f| - |\bar{\mathcal{A}}_{\bar{f}}|}{|\mathcal{A}_f| + |\bar{\mathcal{A}}_{\bar{f}}|} \propto \left| \frac{\lambda_u \mathcal{A}^u}{\lambda_c \mathcal{A}^c} \right| \cdot \sin\gamma \cdot \sin(\delta_c - \delta_u)$$

▶ Look for relative strong phases in interfering amplitudes

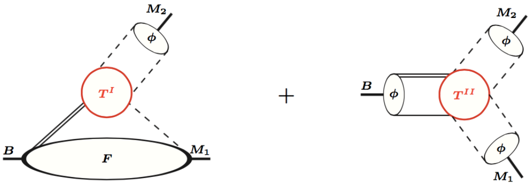
How to deal with MEs

1. Isolate contributions sensitive to IR physics
   - Scale separation -- Factorization -- Effective Field Theory
2. Parametrize them by a few "universal" quantities
   - **Form Factors :** $F^{BM} \sim \langle M|\bar{q}\,\Gamma\, b|B\rangle$
   - **LCDAs :** $\phi_M(u) \sim \int dt\, e^{-it(p\cdot n)u}\, \langle M(p)|\bar{q}(tn)[tn,0]\,\not{n}\gamma_5\, q(0)|0\rangle$
   - **Decay constants :** $f_M \sim \langle M|\bar{q}\,\Gamma\, q|0\rangle$
   - $\cdots$
3. Then, either:
   - Calculate them
   - Extract them from experiment
   - Build observables where they cancel out

Factorization formula for $B \to M_1 M_2$

To leading power in the heavy-quark limit Beneke, Buchalla, Neubert, Sachrajda '99

$$\langle M_1 M_2 | \mathcal{O} | B \rangle = F^{BM_1} \int du\, T^{I}(u) \phi_{M_2}(u) + \int d\omega\, du\, dv\, T^{II}(\omega, u, v) \phi_B(\omega) \phi_{M_1}(u) \phi_{M_2}(v)$$



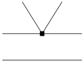
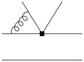
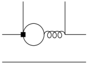
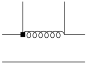
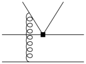
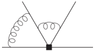
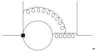
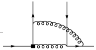
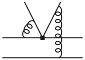
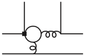
- Vertex corrections: $T^{I}(u) = 1 + \mathcal{O}(\alpha_s)$
- Spectator scattering: $T^{II}(\omega, u, v) = \underbrace{\mathcal{O}(\alpha_s)}_{real} + \mathcal{O}(\alpha_s^2/\pi)$ -- (pow. supp. if M_1 heavy)
- Strong phases are perturbative [$\mathcal{O}(\alpha_s)$] or power suppressed [$\mathcal{O}(\Lambda/m_b)$].
- $A_{\rm CP} = \mathcal{O}(\alpha_s(m_b)/\pi) + \mathcal{O}(\Lambda/m_b)$ -- But ... $\alpha_s(m_b)/\pi \sim \Lambda/m_b$!!

Perturbative calculation

Two hard-scattering kernels for each operator insertion: T^{I} (vertex), T^{II} (spectator)

$$\langle M_1 M_2 | \mathcal{O}_i | B \rangle \simeq F^{BM_1} T_i^{I} \otimes \phi_{M_2} + T_i^{II} \otimes \phi_B \otimes \phi_{M_1} \otimes \phi_{M_2}$$

and two classes of topological amplitudes: "Tree", "Penguin".

	T^{I}, tree	T^{I}, penguin	T^{II}, tree	T^{II}, penguin
LO: $\mathcal{O}(1)$				
NLO: $\mathcal{O}(\alpha_s)$ BBNS '99-'04		 		
NNLO: $\mathcal{O}(\alpha_s^2)$	 Bell '07,'09 Beneke, Huber, Li '09	  Kim, Yoon '11, Bell Beneke, Huber, Li '15	 Beneke, Jager '05 Kivel '06, Pilipp '07	 Beneke, Jager '06 Jain, Rothstein, Stewart '07

Two-Body decays

- Not covered here
- All I would say can be found in :

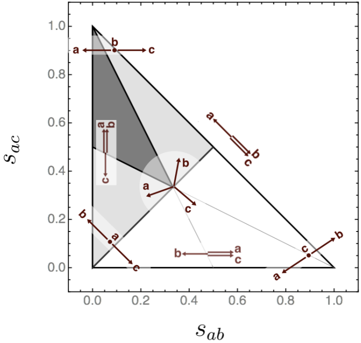
J. Virto, "Charmless Non-Leptonic Multi-Body B decays,"
PoS FPCP 2016, 007 (2017), arXiv:1609.07430 [hep-ph].

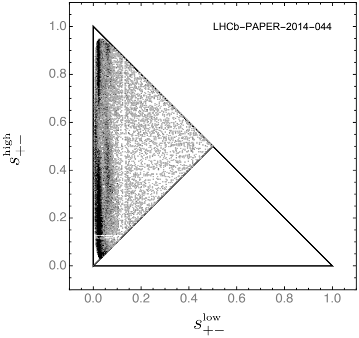
Three-body B decays

- Model-independent treatment of vector resonances:
  - $B \to \rho\ell\nu \quad \longrightarrow \quad B \to [\pi\pi]\ell\nu$
  - $B \to K^*\ell\ell \quad \longrightarrow \quad B \to [K\pi]\ell\ell$
  - Finite-width effects, interference (S-wave pollution, etc.)
- More complicated kinematics $\longrightarrow$ more observables
- Larger phase space: different kinematic regimes, different theory descriptions
- Kinematic distributions $\longrightarrow$ tests of EFT expansions & Factorization
- E-dependent rescattering effects $\longrightarrow$ large strong phases

  $\longrightarrow$ Large localized CP asymmetries
- Huge data sets
- Many applications: CKM parameters, tests of factorization, New Physics, spectroscopy, meson-meson scattering,...

- Two independent invariants, *e.g.* $s_{ab} = \frac{(p_a+p_b)^2}{m_B^2}$ and $s_{ac} = \frac{(p_a+p_c)^2}{m_B^2}$



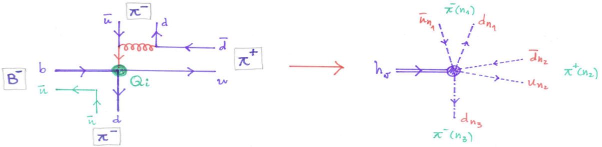


- Different kinematic regions with different factorization properties.
- Can also trade s_{ac} by angle θ_c : $2s_{ac} = (1 - s_{ab})(1 - \cos\theta_c)$, and do PWE :

$$\mathcal{A}(s_{ab}, s_{ac}) = \sum_{\ell=0}^{\infty}(2\ell + 1)\mathcal{A}^{(\ell)}(s_{ab})P_\ell(\cos\theta_c)$$

Central region

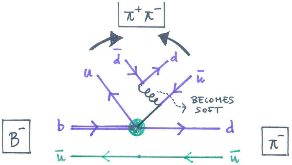
⋆ Three collinear directions n_1, n_2, n_3, disconnected at the leading power.

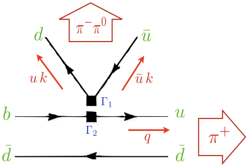


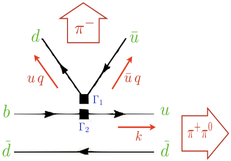
$$\langle \pi^- \pi^+ \pi^- | \mathcal{O}_i | \bar{B} \rangle = F^{B\to\pi} \int du\, dv\, T_i^I(u,v)\, \phi_\pi(u)\, \phi_\pi(v)$$
$$+ \int d\omega\, du\, dv\, dy\, T_i^{II}(\omega,u,v,y)\, \phi_B(\omega)\, \phi_\pi(u)\, \phi_\pi(v)\, \phi_\pi(y)$$

- ▷ Power $(1/m_b^2)$ & α_s suppressed with respect to two-body.
- ▷ At leading order/power/twist all convolutions are finite $\to$ factorization ✓
- ▷ Some pieces proven at NLO: Factorization of $B \to \pi\pi$ form factors Böer, Feldmann, van Dyk '16 and 2π LCDAs Diehl, Feldmann, Kroll, Vogt '99
- ▷ $A_{\rm CP} = \mathcal{O}(\alpha_s(m_b)/\pi) + \mathcal{O}(\Lambda/m_b)$ -- Like two-body !
- ▷ But this region might not exist for $m_B = 5$ GeV Krankl, Mannel, JV '15

- Breakdown of factorization at resonant edges requires new NP functions.
- 3-body decay resembles 2-body, but with new $(\pi\pi)$ "compound object":







- Operators are the same as in 2-body, but final states different:

$$\langle \pi^-_{\bar{n}}\, \pi^+_{\bar{n}}\, \pi^-_{n} | \mathcal{O} | B \rangle = F^{B\to\pi} \int du\, T_1(u,\zeta,s)\, \phi_{\pi\pi}(u,\zeta,s) + F^{B\to\pi\pi}(\zeta,s) \int du\, T_2(u,\zeta,s)\, \phi_\pi(u)$$

- New non-perturbative input: (Contains NP strong phases!!)
  - Generalized Distribution Amplitudes (GDAs) Diehl, Polyakov, Gousset, Pire, Grozin...
  - Generalized Form Factors (GFFs) Faller, Feldmann, Khodjamirian, Mannel, van Dyk...

Generalized Distribution Amplitudes

Krankl, Mannel, JV '15

- Definition: $[k_{12} = k_1 + k_2\ ;\ s = k_{12}^2\ ;\ k_1 = \zeta k_{12}\ ;\ k_2 = (1-\zeta) k_{12}]$

$$\phi_{\pi\pi}^q(z,\zeta,s) = \int \frac{dx^-}{2\pi} e^{iz(k_{12}^+ x^-)} \langle \pi^+(k_1)\pi^-(k_2)|\bar{q}(x^- n_-)\not{n}_+ q(0)|0\rangle$$

- Normalization (local correlator):

$$\int dz\, \phi_{\pi\pi}(z,\zeta,s) = (2\zeta - 1)\, F_\pi(s) \quad \text{(pion vector FF)}$$

- Double Gegenbauer + Partial Wave Expansion:

$$\Phi_\parallel^{I=1}(u,\zeta,k^2) = 6u\bar{u} \sum_{n=0,2,\cdots}^{\infty} \sum_{\ell=1,3,\cdots}^{n+1} B_{n\ell}^\parallel(k^2)\, C_n^{3/2}(u-\bar{u})\, \beta_\pi(k^2) P_\ell^{(0)}(\cos\theta_\pi)$$

where $B_{01}^\parallel(k^2) = F_\pi(k^2)$

- $I = 0$ involves n odd and ℓ even (but normalization is zero).

Leading order formula

$$\mathcal{A}(s_\pm^{\rm low}, s_\pm^{\rm high}) = \frac{G_F}{\sqrt{2}} \Big\{ \left[\lambda_u(a_2 - a_4^u) - \lambda_c a_4^c\right] m_B^2 f_+(s_\pm^{\rm low})\,(1 - s_\pm^{\rm low} - 2 s_\pm^{\rm high})\, F_\pi(s_\pm^{\rm low})$$

$$+ \left[\lambda_u(a_1 + a_4^u) + \lambda_c a_4^c\right] f_\pi\, m_\pi \left[F_t^{l=0}(s_\pm^{\rm low}, s_\pm^{\rm high}) + F_t^{l=1}(s_\pm^{\rm low}, s_\pm^{\rm high})\right] \Big\}$$

- $\lambda_{u,c}$ are (complex) CKMs, and $a_{1,2,4}$ are (real) WCs
- F_π is the vector pion form factor
- $F_t^{l=0}$ and $F_t^{l=1}$ are isoscalar and isovector $B \to \pi\pi$ form factors

▷ Note the interplay of weak and strong phases

$B \to \pi\pi$ form factors from 2π-LCDAs

- Correlation function

$$\Pi^5(p^2, k^2, q^2, q\cdot\bar{k}) = i\int d^4x\, e^{iq\cdot x}\langle\pi^+(k_1)\pi^0(k_2)|\mathrm{T}\{\bar{u}(x)im_b\gamma_5 b(x), \bar{b}(0)im_b\gamma_5 d(0)\}|0\rangle$$

- Unitarity relation

$$2\mathrm{Im}\Pi^5 = (2\pi)\delta(p^2 - m_B^2)\underbrace{\langle\pi^+(k_1)\pi^0(k_2)|\bar{u}im_b\gamma_5 b|\bar{B}(p)\rangle}_{\sqrt{q^2}F_t(q^2,k^2,q\cdot k)}\underbrace{\langle\bar{B}(p)|\bar{b}im_b\gamma_5 d|0\rangle}_{m_B^2 f_B} + \cdots$$

$$= (2\pi)\,\delta(p^2 - m_B^2)\, m_B^2 f_B\, \sqrt{q^2}\, F_t(q^2, k^2, q\cdot k) + \cdots$$

- Dispersion relation + LCOPE + Borel + duality

$$m_B^2 f_B\, \sqrt{q^2}\, F_t(q^2, k^2, q\cdot\bar{k})\, e^{-m_B^2/M^2} = \Pi^5_{OPE}(M^2, q^2, k^2, q\cdot\bar{k})$$

$B \to \pi\pi$ form factors from 2π-LCDAs

Cheng, Khodjamirian, JV, 1709.00173

- In this case:

$$\Pi^5_{OPE}(M^2, q^2, k^2, q\cdot\bar{k}) = \frac{m_b^2}{\sqrt{2}}\int_{u_0}^{1}\frac{du}{u^2}\, e^{-s(u)/M^2}\,(m_b^2 - q^2 + u^2 k^2)\,\Phi_{\parallel}^{I=1}(u, q\cdot\bar{k}, k^2)$$

- **SUM RULE :**

$$\sqrt{q^2}F_t(q^2, k^2, \zeta) = \frac{m_b^2}{\sqrt{2}m_B^2 f_B}\int_{u_0}^{1}\frac{du}{u^2}\, e^{\frac{m_B^2 - s(u)}{M^2}}\,(m_b^2 - q^2 + u^2 k^2)\,\Phi_{\parallel}^{I=1}(u, \zeta, k^2)$$

- **Gegenbauer + Partial Wave Expansions :**

$$\sqrt{q^2}F_t^{(\ell)}(q^2, k^2) = -\frac{6m_b^2}{\sqrt{2}f_B m_B^2}\frac{\beta_\pi(k^2)}{\sqrt{2\ell+1}}\sum_{\substack{n=\ell-1 \\ n\ \text{even}}}^{\infty} B_{n\ell}^{\parallel}(k^2)\int_{u_0}^{1}\frac{du}{u}\,\bar{u}\, e^{\frac{m_B^2 - s(u)}{M^2}}\,(m_b^2 - q^2 + u^2 k^2)\, C_n^{3/2}(u - \bar{u})$$

- $B_{01}^{\parallel}(k^2) = F_\pi(k^2)$ -- but for the sum rule we need higher moments.
- Narrow-ρ dominance on $\Phi_\parallel$ leads to $B \to \rho$ form factor from ρ-LCDA. ✓

  [$\Phi_\parallel \longleftrightarrow \phi_\rho$ Polyakov '98, K. Vos, JV w.i.p]

- Correlation function

$$F_\mu(k,q) = i \int d^4x e^{ik\cdot x} \langle 0 | \mathrm{T}\{\bar{d}(x)\gamma_\mu u(x), m_b \bar{u}(0)\gamma_5 b(0)\} | \bar{B}^0(q+k)\rangle$$

- Unitarity relation

$$2\mathrm{Im}F_\mu(k,q) = m_b \int d\tau_{2\pi} \underbrace{\langle 0|\bar{d}\gamma_\mu u|\pi(k_1)\pi(k_2)\rangle}_{F_\pi^\star(k^2)} \underbrace{\langle \pi(k_1)\pi(k_2)|\bar{u}\gamma_5 b|\bar{B}^0(q+k)\rangle}_{F_t(k^2,q^2,\cos\theta_\pi)} + \cdots$$

$$= q_\mu \frac{s\sqrt{q^2}[\beta_\pi(s)]^2}{4\sqrt{6}\pi\sqrt{\lambda}} F_\pi^\star(k^2)\, F_t^{(\ell=1)}(k^2,q^2) + \cdots$$

<u>Corollary</u> : $F_\pi^\star(s)\, F_t^{(\ell=1)}(s,q^2)$ is real for all $s < 16 m_\pi^2 \Rightarrow$

Phase($F^{B\to\pi\pi}_{P-wave}$) = Phase(vector pion form factor)

Important for CP violation!!!   [See also Kang, Kubis, Hanhart, Meissner '13]

$B \to \pi\pi$ form factors from B-meson LCDAs

Cheng, Khodjamirian, JV, 1701.01633

- Dispersion relation + LCOPE + Borel + duality

$$-\int_{4m_\pi^2}^{s_0^{2\pi}} ds\, e^{-s/M^2}\, \frac{s\,\sqrt{q^2}\,[\beta_\pi(s)]^2}{4\sqrt{6}\pi^2\sqrt{\lambda}}\, F_\pi^\star(s)\, F_t^{(1)}(s,q^2) = f_B m_B^2 m_b \left\{ \int_0^{\sigma_0^{2\pi}} d\sigma\, e^{-s(\sigma,q^2)/M^2} \times \right.$$

$$\left. \times \left[\frac{\sigma}{\bar\sigma}\phi_+^B(\sigma m_B) - \frac{\sigma}{\bar\sigma}\left[\phi_+^B(\sigma m_B) - \phi_-^B(\sigma m_B)\right] - \frac{1}{\bar\sigma m_B}\bar\Phi_\pm^B(\sigma m_B)\right] + \Delta A_0^{BV}(q^2,\sigma_0^{2\pi},M^2) \right\}$$

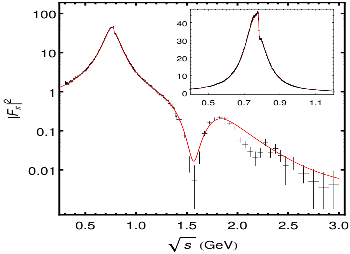
- ρ-dominance + zero-width limit:

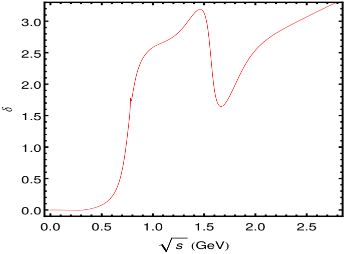
$$F_\pi^\star(s) \simeq \frac{f_\rho g_{\rho\pi\pi} m_\rho/\sqrt{2}}{m_\rho^2 - s + i\sqrt{2}\Gamma_\rho(s)} \quad , \quad F_t^{(1)}(s,q^2) \simeq -\frac{\beta_\pi(s)\sqrt{\lambda}}{\sqrt{3q^2}}\, \frac{m_\rho g_{\rho\pi\pi} A_0^{B\rho}(q^2)}{m_\rho^2 - s - i\sqrt{2}\Gamma_\rho(s)}$$

$$LHS = 2f_\rho m_\rho A_0^{B\rho}(q^2) \int_{4m_\pi^2}^{s_0^{2\pi}} ds\, e^{-s/M^2} \underbrace{\left[\frac{\sqrt{s}\,\Gamma_\rho(s)/\pi}{(m_\rho^2 - s)^2 + s\Gamma_\rho^2(s)}\right]}_{\xrightarrow{\Gamma_\rho\to 0}\ \delta(s-m_\rho^2)} \xrightarrow{\Gamma_\rho\to 0} 2f_\rho m_\rho A_0^{B\rho}(q^2)\, e^{-s/m_\rho^2}$$

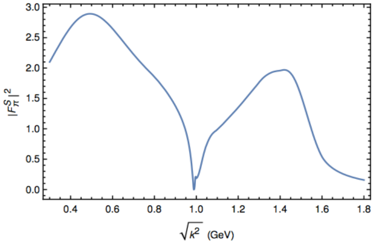
hep-ph/0611193 ✓

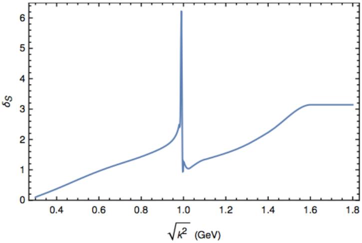
- $F_\pi(s)$: Data ($e^+e^- \to \pi\pi(\gamma)$ [BaBar] or $\tau \to \pi\pi\nu_\tau$ [Belle])





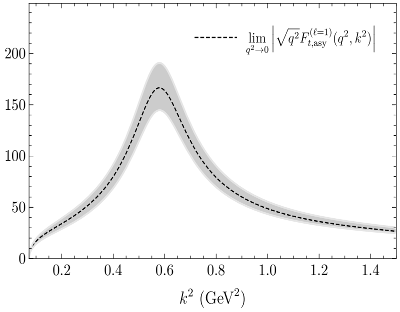
- $F_\pi^S(s)$: Dispersive methods [e.g. 1309.3564]



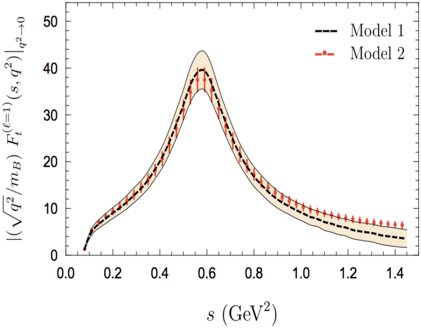


$B \to \pi\pi$ form factor ($F_t^{\ell=1}$)

Cheng, Khodjamirian, JV, 1709.00173



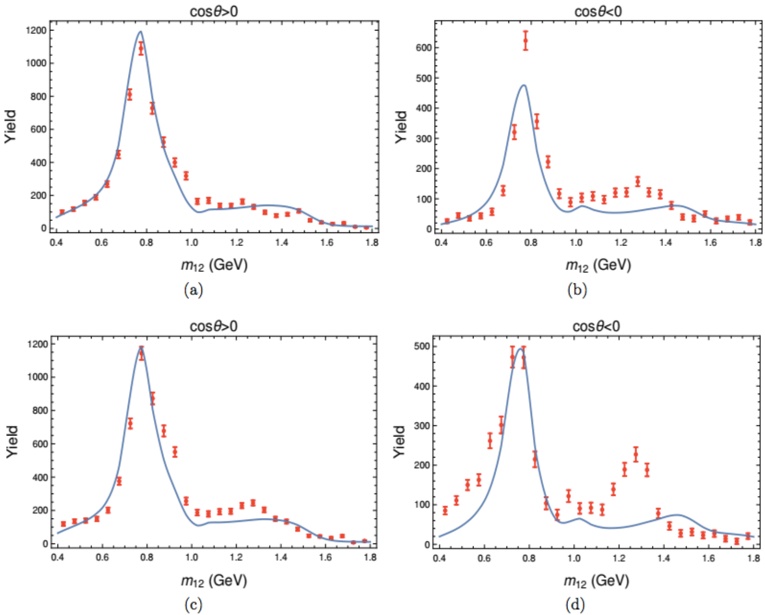
Cheng, Khodjamirian, JV, 1701.01633



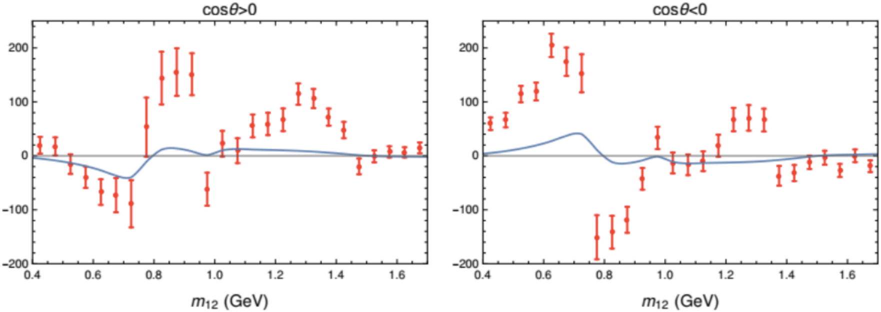
- Both approaches give consistent results
- Corrections to narrow-ρ approximation at the level of 10 - 20 %

$B^{\pm} \to \pi^{\mp}\pi^{\pm}\pi^{\pm}$ -- Dalitz Plot projections

Klein, Mannel, JV, Vos '17



$B^{\pm} \to \pi^{\mp}\pi^{\pm}\pi^{\pm}$ -- CP violation



- Probably need to understand much better $F_t^{I=0}$ and phase of F_π.
- Only a first exploratory analysis.

Outlook

So far :

- There is a plan for 3-body decays Krankl, Mannel, JV '15
- Non-perturbative input available and improving Shan, Khodjamirian, JV '17
  (important: F^S_π, $F_t^{I=0}$ and phase of F_π)
- First serious estimations of finite-width effects in quasi-2-body K.Vos, JV, w.i.p
- Need more investigation of leading order results vs data Klein, Mannel, JV, Vos '17

Prospects :

- Collateral applications
  $B \to K\pi$ form factors $\Rightarrow B \to K\pi\ell\ell$!! Descotes-Genon, Khodjamirian, JV, w.i.p
- Form factor extraction from data : $B \to \pi\pi\ell\nu$ etc. (Belle-2) Faller et.al. '13
- Study of 2π-DAs : $B \to D\pi\pi$ Huber, JV, Vos, w.i.p (also $K\pi$ etc.)
- Soft corners (soft-pion theorems) Mannel, JV, w.i.p
- $\cdots$