Quark mass determinations on the lattice

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- Quark masses – fundamental parameters of the Standard Model.
- Many applications to phenomenology and BSM physics. Example: Higgs partial widths.
  - Couplings proportional to quark masses.
  - Main source of uncertainty in partial widths from m_b, m_c, α_s. [1404.0319]
- Focus on precision charm results by HPQCD collaboration via two different methods.

Outline

- Background
  - Theory background.
  - Lattice determinations.
- Current-current correlator method
  - Time moments of $\langle JJ \rangle$ correlators.
  - Comparison with perturbation theory.
- Regularisation Invariant (RI) methods
  - Summary of method.
  - Results and comparison to $\langle JJ \rangle$ method.
- Summary & Conclusions

Quark mass – definitions

- Quarks are not asymptotic (physical) states.
- Quark masses are scheme and scale dependent, $m_q^{\text{scheme}}(\mu)$.
- Generally will quote results $m_q^{\overline{\text{MS}}}(\mu_{\text{ref}})$.

Lattice determination of quark mass

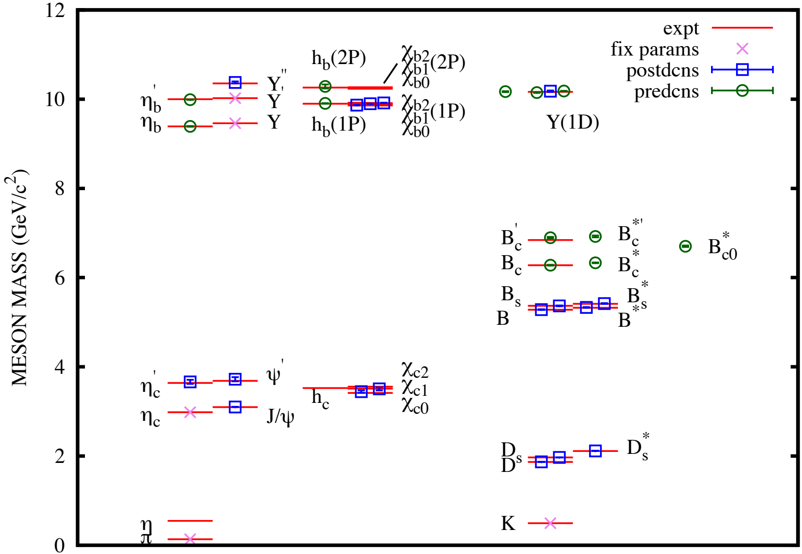
Bare quark masses are input parameters to lattice simulations. These parameters are tuned to reproduce physical quantities, e.g.

- $m_{ud0} \to m_\pi^2$
- $m_{s0} \to m_K^2$
- $m_{c0} \to m_{\eta_c}$

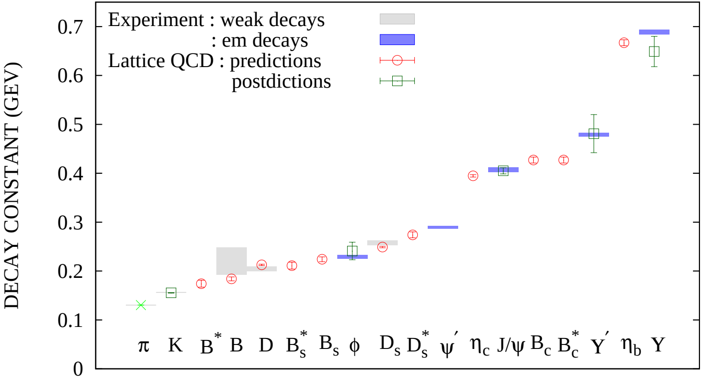
Tuning performed at multiple lattice spacings, defining a continuum trajectory for which $a^2 \to 0$ limit can be taken.

- Rest of physics is then prediction of QCD.
- Parameters can be varied away from physical values.. understand effect of quark mass, quantify systematics, etc.

Meson masses – summary plot



Decay constants – summary plot



$\langle JJ \rangle$-correlator method

Simulating charm

Heavy quarks are challenging to simulate.

- Requires $am_0 < 1$ to keep discretization effects under control.
- Need large enough box to minimize finite-volume effects $\rightarrow N_{\text{site}}$ large.

These conditions can be satisfied by using a highly improved action (e.g. HISQ).

HISQ action

HISQ action

- No $\mathcal{O}(a^2)$ discretization errors (begin at $\mathcal{O}(\alpha_s a^2)$).
- Significant $\mathcal{O}(\alpha_s a^2)$ effects are in turn suppressed.

$n_f = 4$ simulations

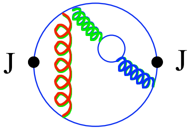
- Charm quarks in the sea.
- Avoid applying perturbation theory at m_c (matching $n_f = 4 \to 3$).

It is increasingly feasible to simulate the b quark relativistically.

Current-current correlators

Calculate time-moments of $J_5 \equiv \bar{\psi}_h \gamma_5 \psi_h$ correlators:

$$G(t) = a^6 \sum_{\mathbf{x}} (am_{0h})^2 \langle J_5(t, \mathbf{x}) J_5(0, 0) \rangle$$



- Currents are absolutely normalized (no Zs required).
- $G(t)$ is UV finite $\rightarrow G(t)_{\mathrm{cont}} = G(t)_{\mathrm{latt}} + \mathcal{O}(a^2)$.

Moments

The time-moments $G_n = \sum_t (t/a)^n G(t)$ can be computed in perturbation theory. For $n \geq 4$,

$$G_n = \frac{g_n(\alpha_{\overline{\mathrm{MS}}}, \mu)}{am_h(\mu)^{n-4}} \, .$$

Basic strategy:

1. Calculate $G_{n,\mathrm{latt}}$ for a variety of lattice spacings and m_{h0}.
2. Compare continuum limit $G_{n,\mathrm{cont}}$ with $G_{n,\mathrm{pert}}$ (at reference scale $\mu = m_h$, say).
3. Determine best-fit values for $\alpha_{\overline{\mathrm{MS}}}(m_h), m_h(m_h)$.

Reduced moments

In practice comparison carried out using reduced moments.

$$R_4 = G_4/G_4^{(0)}$$
$$R_n = \frac{1}{m_{0c}}(G_n/G_n^{(0)})^{1/(n-4)} \quad (n \geq 6)\,.$$

On the perturbative side,

$$R_4 = r_4(\alpha_{\overline{\mathrm{MS}}}, \mu)$$
$$R_n = \frac{1}{m_c(\mu)}\, r_n(\alpha_{\overline{\mathrm{MS}}}, \mu) \quad (n \geq 6)\,.$$

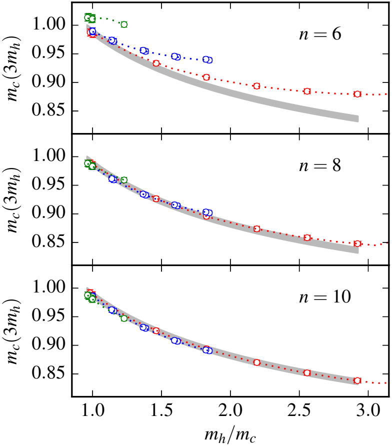
Reference scale is taken as $\mu = 3m_h (= m_c \frac{m_{h0}}{m_{c0}})$.

Some details

- Calculate moments for $n = 4, 6, 8, 10$.
- Three lattice spacings: $a \approx 0.12, 0.09, 0.06$ fm. (MILC)
- Seven input masses from $m_h = m_c - 0.7 m_b$.

All data points fit simultaneously with perturbative R_n expressions $\rightarrow m_c^{\overline{\mathrm{MS}}}(\mu)$, $\alpha_{\overline{\mathrm{MS}}}(\mu)$ for $\mu \approx 3 - 9$ GeV.

Results for $n_f = 4$ [1408.4169]

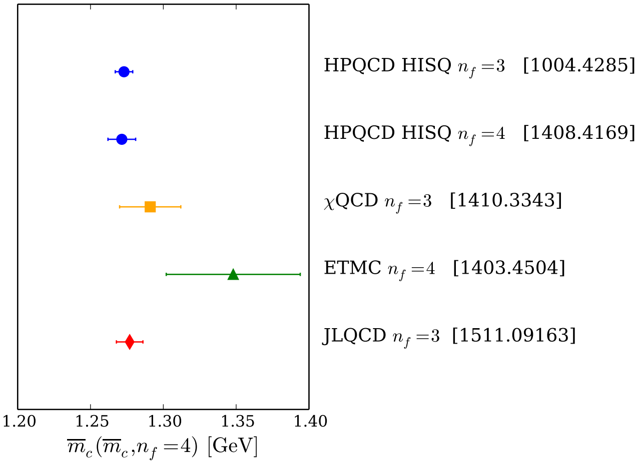


$$m_c(3m_h) = \frac{r_n(\alpha_{\overline{\mathrm{MS}}}, \mu = 3m_h)}{R_n}$$

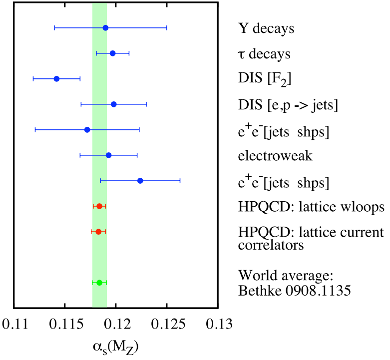
- Discretization effects grow with am_h and decrease with n.
- Grey band shows best-fit $m_c(3m_c)$ evolved perturbatively.

$$\boxed{m_c^{\overline{\mathrm{MS}}}(3\ \mathrm{GeV}) = 0.9851(63)\ \mathrm{GeV}}$$

m_c comparison plot



$\alpha_s^{\overline{\mathsf{MS}}}(m_Z)$



HPQCD $\langle JJ \rangle$ result:

- $\alpha_s^{\overline{\mathrm{MS}}}(m_Z) = 0.1182(7)$
- Agrees with $n_f = 3$ result.
- Agrees well with world average.

Non-perturbative renormalisation (NPR)

NPR method

Trying to determine $Z_m^{\overline{\mathrm{MS}}}(\mu, 1/a)$ st

$$m^{\overline{\mathrm{MS}}}(\mu) = Z_m^{\overline{\mathrm{MS}}}(\mu, 1/a)\, m_0$$

Options:

- Lattice perturbation theory. – difficult!
- Alternatively, use two steps:
  latt $\leftrightarrow$ intermediate(continuum-like) $\leftrightarrow$ $\overline{\mathrm{MS}}$

NPR method

General idea is to renormalize operators using a scheme that is well-defined both in the continuum and on the lattice, e.g. the RI schemes:

Calculate off-shell Green's functions of operator-of-interest with external quark states.

$$G_\Gamma^{ij}(p) = \langle q^i(p) \left(\sum_x \bar{q}(x) \Gamma q(x) \right) \bar{q}^j(-p) \rangle_{\mathrm{amp}}$$

Require that the trace of the renormalized operator takes its tree-level value:

$$\Lambda_\Gamma(p) \equiv \frac{1}{12} \mathrm{Tr} \left[\Gamma \, G_\Gamma(p) \right] \simeq \frac{Z_q(p)}{Z_\Gamma(p)}$$

NPR method (cont.)

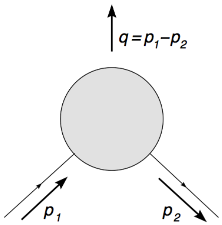
The RI (and $\overline{\mathrm{MS}}$) schemes satisfy $Z_m = Z_S^{-1} = Z_P^{-1}$. Z_m can be extracted from the scalar correlator provided

$$\Lambda_{\mathrm{QCD}} \ll |p| \ll \pi/a$$

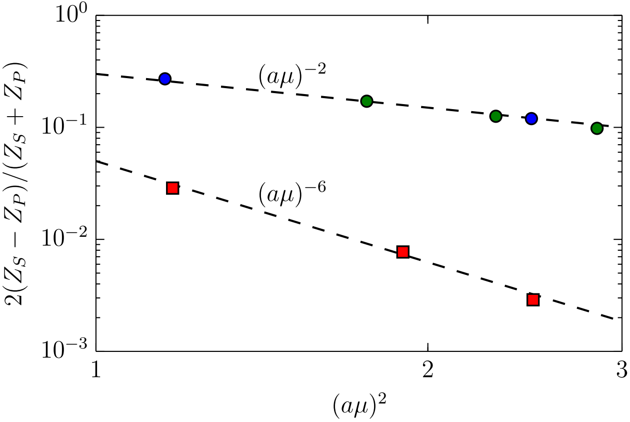
After determining $Z_m^{RI}(p)$, a perturbative calculation can be used to convert $Z^{\overline{\mathrm{MS}}}(p) = C^{\overline{\mathrm{MS}} \leftarrow RI}(p)\, Z_m^{RI}(p)$.

RI/SMOM scheme

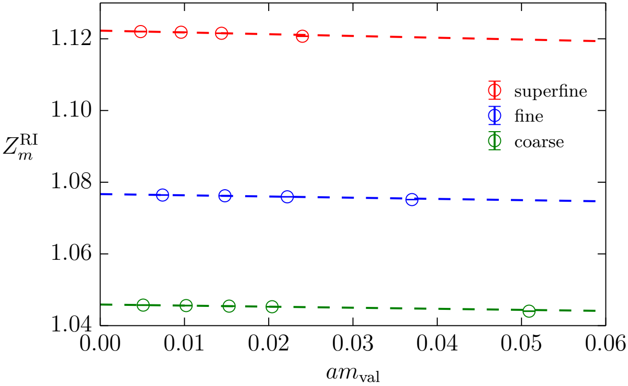
- Momentum flow suppresses infrared effects. $p_1^2 = p_2^2 = (p_1 - p_2)^2$
- $p_1 \sim (x, x, 0, 0)$, $p_2 \sim (0, x, x, 0)$ for $x = 2, 3, 4$
- Other advantages:
  - Reduced mass dependence.
  - SMOM $\rightarrow \overline{\text{MS}}$ matching factors closer to 1.



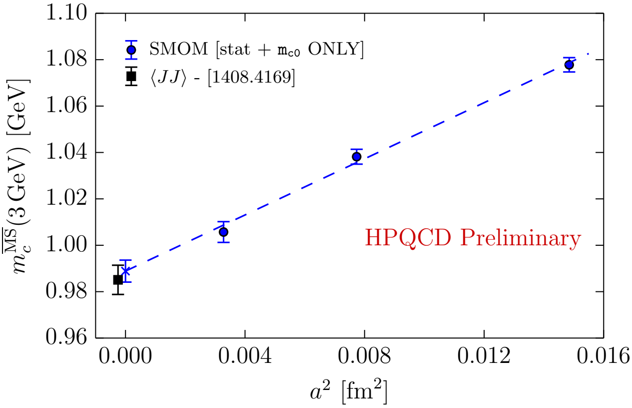
$Z_S - Z_P$



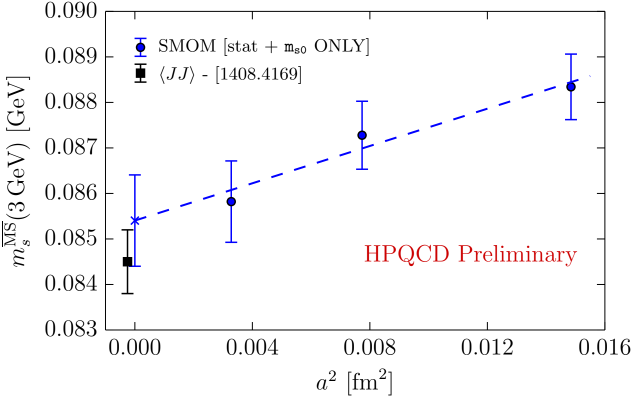
Z_m chiral extrapolation



Continuum extrapolations



Continuum extrapolations



Summary

- Accurate determinations of quark masses are of fundamental importance for (B)SM physics.

- LQCD simulations provide an effective and controlled way to determine quark masses.
  - Systematically improveable.
  - Multiple complementary approaches $\rightarrow$ assess systematics, check consistency.

- Reviewed HPQCD results via two alternative methods.
  - Compare time moments of $\langle JJ \rangle$-correlators to perturbation theory.
  - Renormalise scalar operator directly on lattice in RI/SMOM scheme + perturbative matching step to $\overline{\text{MS}}$. (preliminary)

- Results from these methods at present achieve 1–2% precision and are in good agreement.