Measurement of the CKM angle γ

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UK Flavour Meeting

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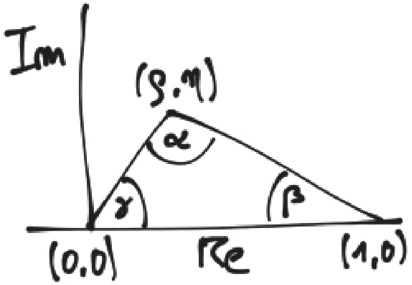
Overview

1. Setup and CKM overview
2. Measuring γ
3. Combination of measurements
4. Status and Future Prospects (also scattered throughout)

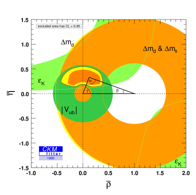


1. Setup and CKM overview

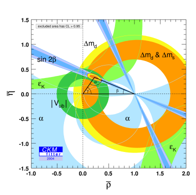
CKM picture is now well verified

- Any discrepancies would be of great importance
- CKM angle γ is the *least well known* constraint

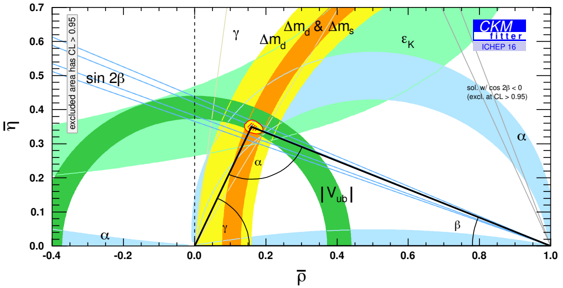
1995



2004



2016 - zoom



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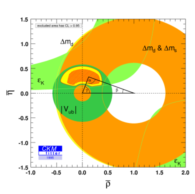

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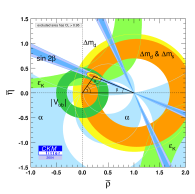


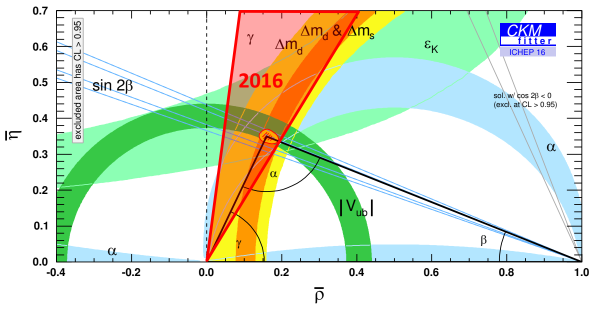
Direct γ measurements

$$\gamma = (72.1^{+5.4}_{-5.8})^\circ$$

Indirect γ extrapolation

$$\gamma = (65.3^{+1.0}_{-2.5})^\circ$$

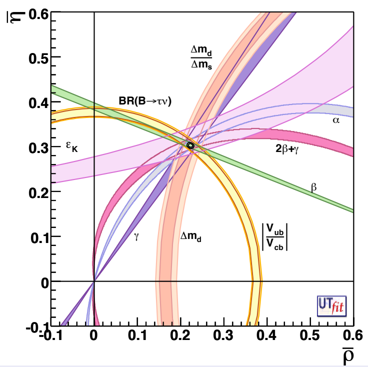
2016 - zoom



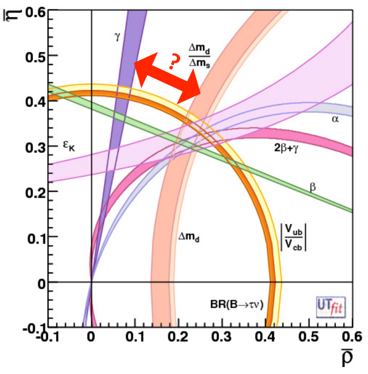
The Ultimate Test

- **γ is a SM benchmark**
  - Only CKM angle accessible at tree level
- **γ is an excellent probe of new physics**
  - Direct vs indirect disagreement
  - Constraints in neutral mixing require γ as input
  - New physics in $C_{1,2}$ can cause sizeable shifts in γ

"The nightmare" - [arXiv:0710.3799]

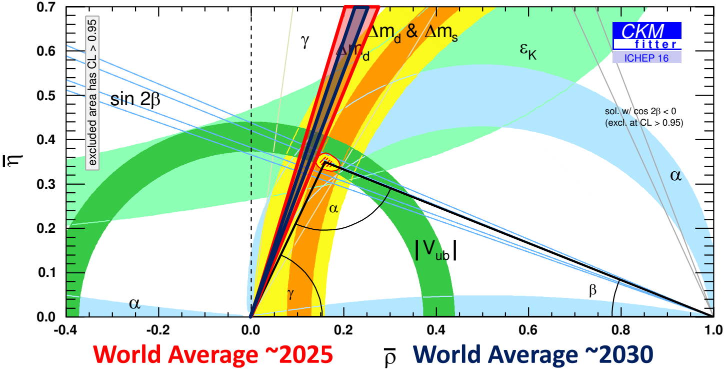


"The dream" - [arXiv:1110.3920]



The Ultimate Test

- LHCb expected precision in 2018 (end of Run 2) $\sim$ 3-4°
- LHCb expected precision in 2023 (end of Run 3) $\sim$ 1.5°
- BelleII expected precision in 2023 (end of Run) $\sim$ 1.5°
- LHCb expected precision in 2029 (end of Run 4) $<$ 1°



2. Measuring γ

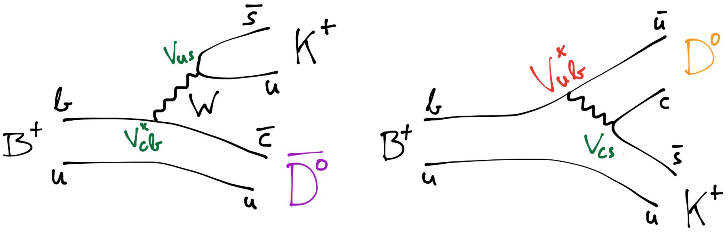
Measuring γ

- γ is the phase between $V_{ub}^* V_{ud}$ and $V_{cb}^* V_{cd}$
  - Require interference between $b \to cW$ and $b \to uW$ to access it
  - No dependence on CKM elements involving the top
  - **Can be measured using tree level B decays**
  - Makes it a benchmark of the SM (no loops)

$$\gamma = \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right)$$

- The "textbook" case is $B^\pm \to \overset{(-)}{D}{}^0 K^\pm$:
  - Transitions themselves have different final states (D^0 and $\bar{D}^0$)
  - Interference occurs when D^0 and $\bar{D}^0$ decay to the same final state f

Reconstruct the $D^0/\bar{D}^0$ in a final state accesible to both to acheieve interference



- **The crucial feature of these (and similar) decays is that the D^0 can be reconstructed in several different final states**

γ from theory

$$\gamma = \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right)$$

- γ is known very well
- Can be determined entirely from tree decays
  - Unique property among all *CP* violation parameters
  - Hadronic parameters can be determined from data
- Neglible theoretical uncertainty (Zupan and Brod 2013)

Theory uncertainty on γ

$\delta\gamma/\gamma \approx \mathcal{O}(10^{-7})$ - [arXiv:1308.5663]

- γ can probe for new physics at extrememly high energy scales (Zupan)
  - (N)MFV new physics scenarios: $\sim \mathcal{O}(10^2)$ TeV
  - gen. FV new physics scenarios: $\sim \mathcal{O}(10^3)$ TeV
- NP contributions to $C_{1,2}$ can cause sizeable shifts ($\mathcal{O}(4^\circ)$) in γ (Brod, Lenz et. al 2014) - [arXiv:1412.1446]

γ from experiment

- γ is NOT known very well
- It is quite challenging to measure
- The decay rates are small

Branching ratio for suppressed γ mode

$$BR(B^- \to DK^-, D \to \pi K) \approx 2 \times 10^{-7}$$

- Small interference effect typically $\sim 10\%$
- Fully hadronic decays - hard to trigger on
- Many channels have a K^0_S in the final state - low efficiency
- Many channels have a π^0 in the final state - very hard at LHCb
- Many different decay channels, many observables and many hadronic unknowns make it statistically challenging

2.1. GLW Method

GLW Method

- CP eigenstates e.g. $D \to KK$, $D \to K^0_S\pi^0$
- Gronau, London, Wyler (1991)

- [Phys. Lett. B253 (1991) 483]
- [Phys. Lett. B265 (1991) 172]

GLW observables

$$A_{CP} = \frac{\Gamma(B^- \to D^0_{CP}K^-) - \Gamma(B^+ \to D^0_{CP}K^+)}{\Gamma(B^- \to D^0_{CP}K^-) + \Gamma(B^+ \to D^0_{CP}K^+)} = \frac{\pm 2r_B \sin(\delta_B)\sin(\gamma)}{1 + r_B^2 \pm 2r_B\cos(\delta_B)\cos(\gamma)} \quad (1)$$

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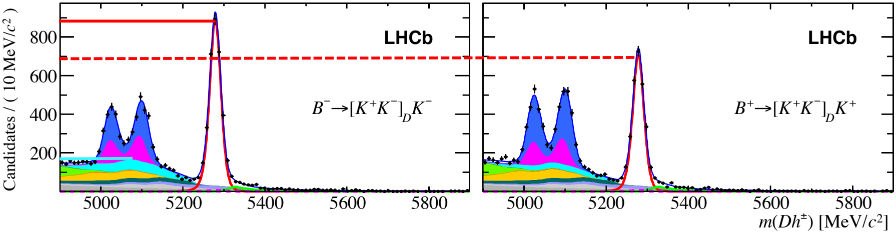

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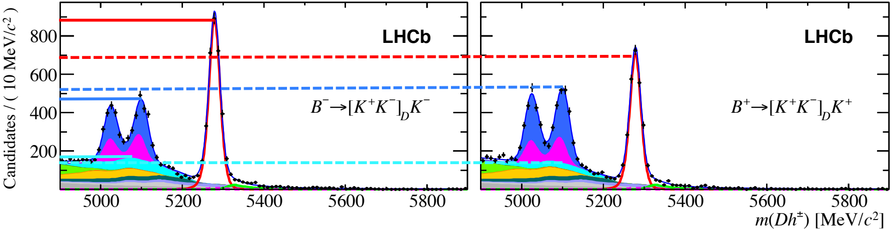

- **LHCb has recently extracted GLW observables from partially reconstructed $B^- \to D^{*0} K^-$ in the same fit** - [arXiv:1708.06370]

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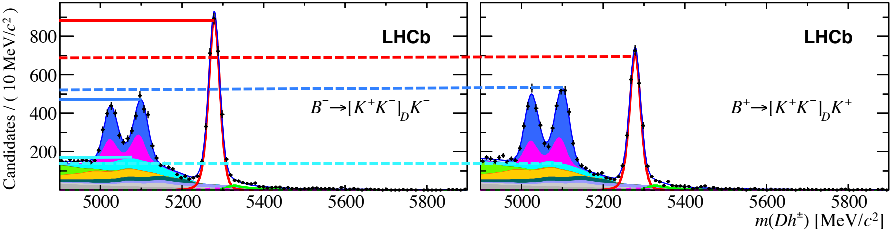

- **LHCb has recently extracted GLW observables from partially reconstructed $B^- \to D^{*0}K^-$ in the same fit** - [arXiv:1708.06370]
- Can extend to quasi-CP-eigenstates ($D^0 \to KK\pi^0$) if fraction of CP content, F^+, is known

GLW Method

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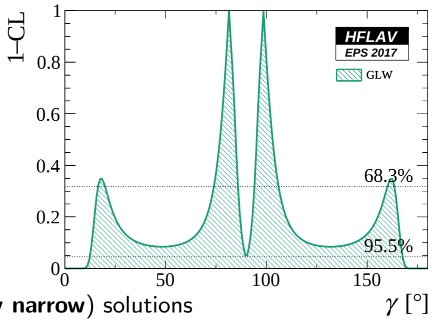

- Multiple (**but very narrow**) solutions
- Require knowledge of F^+ from charm friends

2.2. ADS Method

ADS Method

- CF or DCS decays e.g. $D \to K\pi$
- Atwood, Dunietz, Soni (1997,2001)
- [Phys. Rev. D63 (2001) 036005]
- [Phys. Rev. Lett. 78 (1997) 3257]

ADS observables

$$A_{ADS} = \frac{\Gamma(B^- \to [K^+\pi^-]_D K^-) - \Gamma(B^+ \to [K^-\pi^+]_D K^+)}{\Gamma(B^- \to [K^+\pi^-]_D K^-) + \Gamma(B^+ \to [K^-\pi^+]_D K^+)} = \frac{2r_B r_D \sin(\delta_B + \delta_D)\sin(\gamma)}{r_B^2 + r_D^2 + 2r_B r_D \cos(\delta_B + \delta_D)\cos(\gamma)} \quad (3)$$

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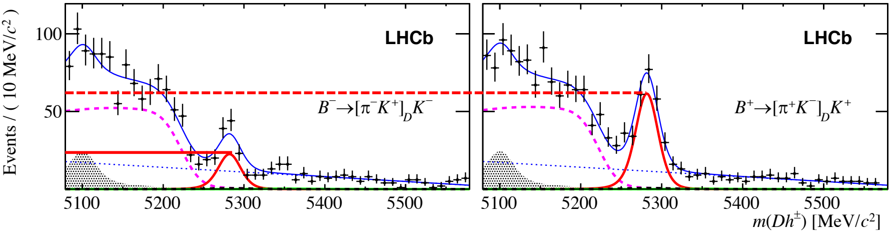

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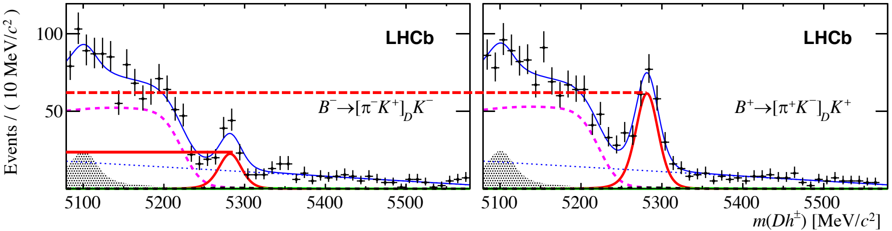

- **Much harder to extract partially reconstructed observables because of $B_s^0 \to D^{(*)0} K^+\pi^-$ backgrounds.**

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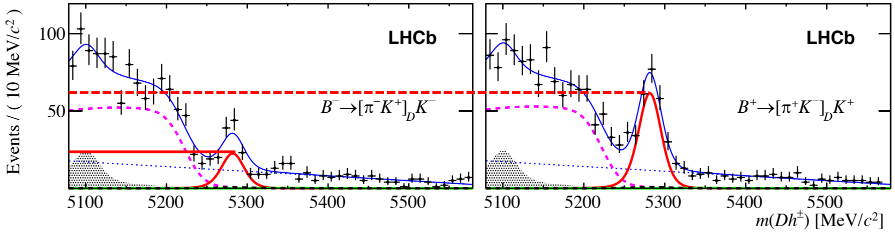

- **Much harder to extract partially reconstructed observables because of $B_s^0 \to D^{(*)0} K^+ \pi^-$ backgrounds.**
- Can extend to multibody-DCS-decays ($D^0 \to K\pi\pi^0$) if dilution from interference, κ_D, is known

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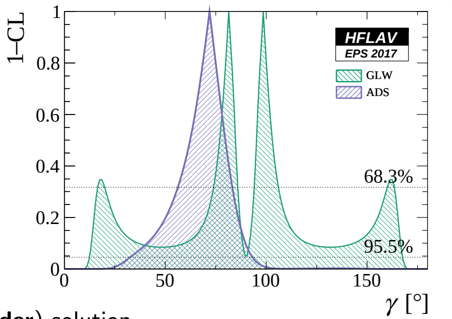


- A single (**yet broader**) solution
- Require knowledge of r_D, δ_D, κ_D from charm friends

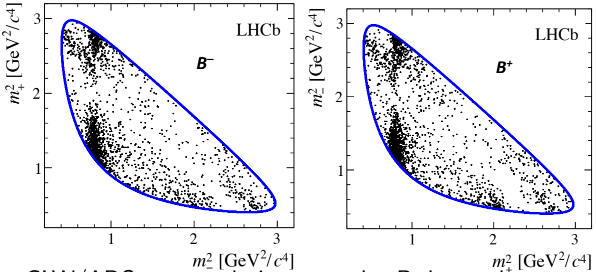
2.3. GGSZ Method

GGSZ Method

- 3-body final states e.g. $D \to K^0_S \pi\pi$
- Giri, Grossman, Soffer, Zupan (2003)
- [Phys. Rev. D68 (2003) 054018]

GGSZ observables (partial rate as function of Dalitz position)

$$d\Gamma_{B^\pm}(\mathbf{x}) = A^2_{(\pm,\mp)} + r_B^2 A^2_{(\mp,\pm)} + 2A_{(\pm,\mp)}A_{(\mp,\pm)}\Big[\underbrace{r_B\cos(\delta_B \pm \gamma)}_{x_\pm}\cos(\delta_{D(\pm,\mp)}) + \underbrace{r_B\sin(\delta_B \pm \gamma)}_{y_\pm}\sin(\delta_{D(\pm,\mp)})\Big] \quad (5)$$



- Essentially a GLW/ADS type analysis across the D decay phase space
- Excellent sensitivity from interference between various contributions

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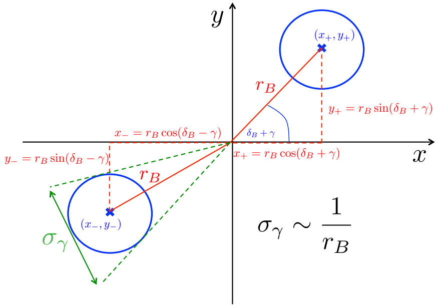

- $x_\pm + iy_\pm = r_B e^{i(\delta_B \pm \gamma)}$
- **Uncertainty on γ is inversely proportional to central value of hadronic unknown!!**
- Fluctuation in nuisance parameter = fluctuation in error on parameter of interest!

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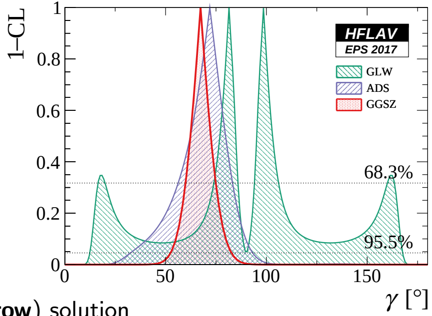
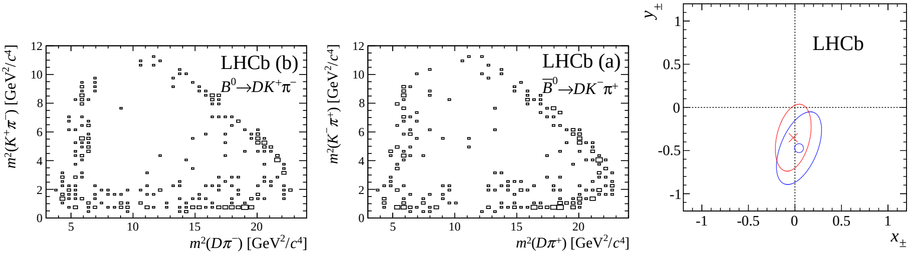


- A single (**and narrow**) solution

2.4. Dalitz (GW) Method

Dalitz (GW) Method

- Use Dalitz structure of **B** decays
- $B^- \to D^0 K^+ \pi^-$
- Gerhson, Williams (2009)
- [Phys. Rev. D80 (2009) 092002]
- Get multiple interfering resonances which increase sensitivity to γ
  - $D_0^*(2400)^-$, $D_2^*(2460)^-$, $K^*(892)^0$, $K^*(1410)^0$, $K_2^*(1430)^0$
- Fit B decay Dalitz Plot for cartesian parameters (similar to GGSZ except for the B not the D)
  - $D \to K^+K^-$, $D \to \pi^+\pi^-$ - GLW-Dalitz (done by LHCb - [arXiv:1602.03455])
  - $D \to K^\pm\pi^\mp$ - ADS-Dalitz (problematic backgrounds from $B_s^0 \to D^{(*)0}K^+\pi^-$)
  - $D \to K_S^0\pi^+\pi^-$ - GGSZ-Dalitz (double Dalitz!)



Current status of measurements

Method		B Decay / D Decay	$B^- \to D^0 K^-$			$B^- \to D^0 K^{*-}$ $[K^{*-} \to K^0_S \pi^-]$			$B^- \to D^{*0} K^-$ $[D^{*0} \to D^0\pi^0],[D^{*0} \to D^0\gamma]$ part-rec			full-rec			$B^0 \to D^0 K^+ \pi^-$ K^{*0} res			Dalitz		
GLW	CP-even	$D^0 \to K^+K^-$	5	✓	✓	5	-	✓	5	-	-	•	✓	✓	3	-	-	3	-	-
		$D^0 \to \pi^+\pi^-$	5	✓	✓	5	-	✓	5	-	-	•	✓	✓	3	-	-	3	-	-
		$D^0 \to K^+K^-\pi^0$	3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
		$D^0 \to \pi^+\pi^-\pi^0$	3	-	✓	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
		$D^0 \to \pi^+\pi^-\pi^+\pi^-$	3	-	-	5	-	-	5	-	-	•	-	-	-	-	-	-	-	-
	CP-odd	$D^0 \to K^0_S\pi^0$	-	✓	✓	-	-	✓	-	-	-	-	✓	✓	-	-	-	-	-	-
		$D^0 \to K^0_S\phi$	-	✓	✓	-	-	✓	-	-	-	-	✓	✓	-	-	-	-	-	-
		$D^0 \to K^0_S\omega$	-	✓	✓	-	-	✓	-	-	-	-	✓	✓	-	-	-	-	-	-
ADS		$D^0 \to K^+\pi^-$	3	✓	✓	5	-	✓	-	-	-	•	✓	✓	3	✓	✓	-	-	-
		$D^0 \to K^+\pi^-\pi^0$	3	✓	✓	-	-	-	-	-	-	-	-	-	-	-	✓	-	-	-
		$D^0 \to K^+\pi^-\pi^+\pi^-$	3	-	-	5	-	-	-	-	-	•	-	-	-	-	✓	-	-	-
GGSZ		$D^0 \to K^0_S\pi^+\pi^-$	3 •	✓	✓	•	✓	✓	-	-	-	•	✓	✓	3 •	✓	✓	-	-	-
		$D^0 \to K^0_S K^+K^-$	3 •	-	✓	•	-	-	-	-	-	•	-	✓	3 •	-	-	-	-	-
		$D^0 \to K^0_S\pi^+\pi^-\pi^0$	•	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
		$D^0 \to K^0_S K^+K^-\pi^0$	•	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-

KEY: 3, 5: LHCb published (fb^{-1}), •: LHCb in progress, ✓: Belle, ✓: BaBar

NOTE: LHCb has a 1• TD result with $B^0_s \to D_s^- K^+$

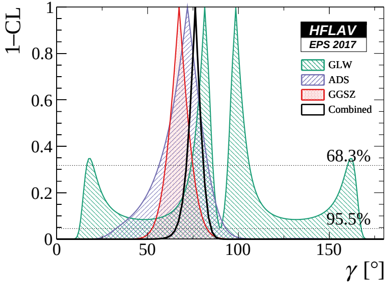
LHCb has a 3 GLW/ADS result with $B^- \to D^0 K^- \pi^+ \pi^-$

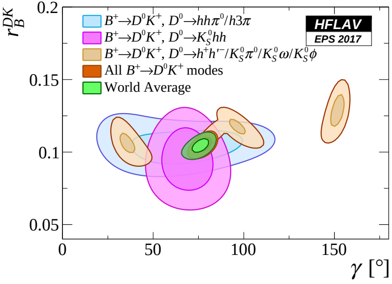
LHCb has a 3 GLS result from $B^- \to D^0 K^-$ with $D^0 \to K^0_S K^\pm \pi^\mp$

3. Combination of Measurements

Combining measurements to determine γ

- The different methods mentioned previously are complimentary in determining γ
  - GLW - excellent sensitivty but multiple solutions
  - ADS - poorer sensitivty but fewer solutions
  - GGSZ - a single unambigous solution
  - GW - currently fairly weak sensitivity
  - TD - single solution with wide sensitivty (but only measurement with initial state B_s^0)
- Best precision can only be obtained by combining several measurements together
- Requires knowledge of external parameters
  - Particularly in the D system (e.g. r_D, δ_D, κ_D)
  - Extracted from charm data obtained elsewhere (HFLAV, CLEO, LHCb)
- Should also account for D^0-$\bar{D}^0$ mixing (*and K^0 mixing*)
  - Although impact is small for most $B \to DK$-like systems (as $r_D << r_B$)

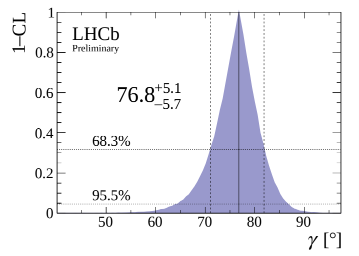




LHCb γ Combination

- Recently updated for EPS [LHCb-CONF-2017-004]
- Combination of all $B \to DK$-**like** modes
  - 85 observables and 37 free parameters
- Frequentist Feldman-Cousins "plugin" procedure
  - $p(\chi^2, N_{\mathrm{dof}}) = 84.8\%$
  - $p(\mathrm{toys}) = (86.8 \pm 0.2)\%$
- Uncertainty $< 10^\circ$ is better than combined B factories
- The most precise single experiment measurement of γ

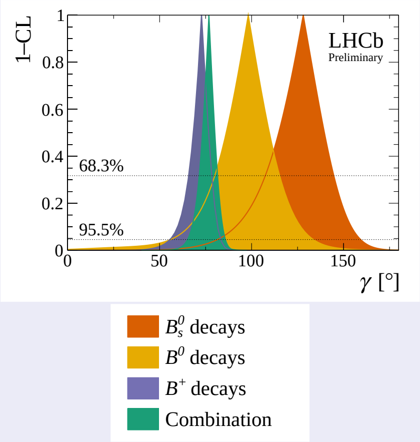
Frequentist - $\gamma = (76.8^{+5.1}_{-5.7})^\circ$



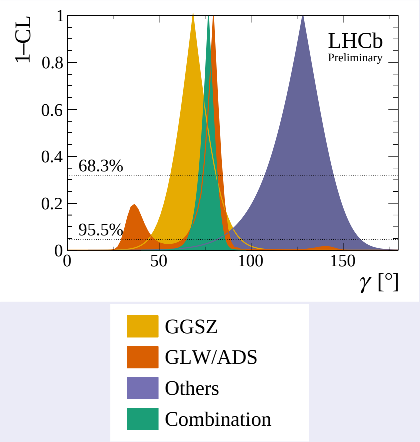
LHCb γ Combination

- Comparison between different decay modes and different analysis methods important
- In certain NP scenarios one can see differences between decays modes
- There shouldn't be any between methods

Comparison split by initial B flavour



Comparison split by analysis method



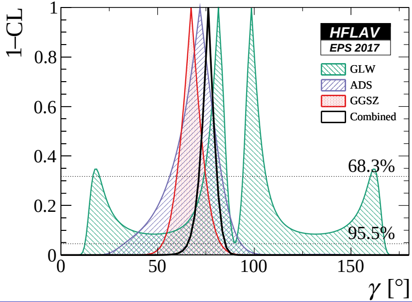
The World Average

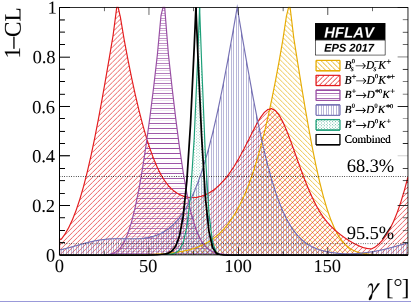
- World average performed for HFLAV ([arXiv:1612.07233]) and PDG
- Add to LHCb results from Belle, BaBar and CDF
  - **Always the most up to date result** (HFLAV)
  - **Only use published results** (PDG)
- A few minor differences (simplifications)
- **132 observables and 33 free parameters**

World Averages

$$\gamma = (76.2^{+4.7}_{-5.2})^\circ \quad \text{(HFLAV)}$$

$$\gamma = (72.8^{+5.3}_{-6.3})^\circ \quad \text{(PDG)}$$





4. Status and Prospects

Current status of measurements

Method		D Decay \ B Decay	$B^- \to D^0 K^-$			$B^- \to D^0 K^{*-}$ $[K^{*-} \to K^0_S \pi^-]$			$B^- \to D^{*0} K^-$ $[D^{*0} \to D^0 \pi^0], [D^{*0} \to D^0 \gamma]$ part-rec			full-rec			$B^0 \to D^0 K^+ \pi^-$ K^{*0} res			Dalitz		
GLW	CP-even	$D^0 \to K^+K^-$	5	✓	✓	5	-	✓	5	-	-	•	✓	✓	3	-	-	3	-	-
		$D^0 \to \pi^+\pi^-$	5	✓	✓	5	-	✓	5	-	-	•	✓	✓	3	-	-	3	-	-
		$D^0 \to K^+K^-\pi^0$	3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
		$D^0 \to \pi^+\pi^-\pi^0$	3	-	✓	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
		$D^0 \to \pi^+\pi^-\pi^+\pi^-$	3	-	-	5	-	-	5	-	-	•	-	-	-	-	-	-	-	-
	CP-odd	$D^0 \to K^0_S\pi^0$	-	✓	✓	-	-	✓	-	-	-	-	✓	✓	-	-	-	-	-	-
		$D^0 \to K^0_S\phi$	-	✓	✓	-	-	✓	-	-	-	-	✓	✓	-	-	-	-	-	-
		$D^0 \to K^0_S\omega$	-	✓	✓	-	-	✓	-	-	-	-	✓	✓	-	-	-	-	-	-
ADS		$D^0 \to K^+\pi^-$	3	✓	✓	5	-	✓	-	-	-	•	✓	✓	3	✓	✓	-	-	-
		$D^0 \to K^+\pi^-\pi^0$	3	✓	✓	-	-	-	-	-	-	-	-	-	-	-	✓	-	-	-
		$D^0 \to K^+\pi^-\pi^+\pi^-$	3	-	-	5	-	-	-	-	-	•	-	-	-	-	✓	-	-	-
GGSZ		$D^0 \to K^0_S\pi^+\pi^-$	3 •	✓	✓	•	✓	✓	-	-	-	•	✓	✓	3 •	✓	✓	-	-	-
		$D^0 \to K^0_S K^+K^-$	3 •	-	✓	•	-	-	-	-	-	•	-	✓	3 •	-	-	-	-	-
		$D^0 \to K^0_S\pi^+\pi^-\pi^0$	•	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
		$D^0 \to K^0_S K^+K^-\pi^0$	•	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-

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- Some gaps to be filled
- Some new columns to be added...

Extensions to other decays

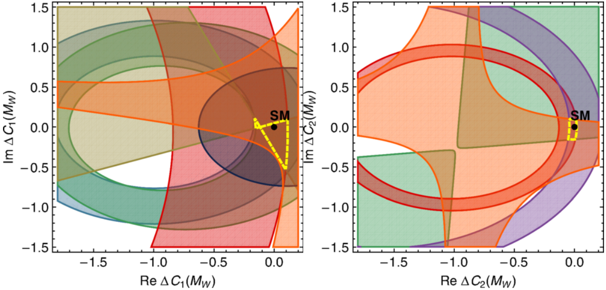
Extension for many other decays(by no means a complete list)

1. Obvious extensions to higher resonant final states
   - $B^{\pm} \to D^0 K^{*\pm}$ ($K^{*\pm} \to K^0_S \pi^{\pm}$)
   - $B^{\pm} \to D^{*0} K^{\pm}$ ($D^{*0} \to D^0 \gamma$ or $D^{*0} \to D^0 \pi^0$)
   - mainly just the B factories so far - LHCb starting for Run 2
   - Similar hadronic parameters as $B^{\pm} \to D^0 K^{\pm}$ but lower yields
2. Extensions to other B decays (swapping spectator quark)
   - $B^0 \to D^0 K^{*0}$ ($K^{*0} \to K^+ \pi^-$ tags initial B flavour)
   - $B^0 \to D^0 K^0_S$ (not self tagging)
   - $B^0_s \to D^0 \phi$ (not self tagging and low rate)
   - $B^{\pm}_c \to D^0 D^{\pm}$ (very low rate - favoured mode not yet seen)
   - under exploration at LHCb with some $B^0 \to D^0 K^{*0}$ published
   - Typically enhanced r_B because favoured diagram is colour supressed
3. Extension into baryon sector (add/swap spectators)
   - $\Lambda^0_b \to D^0 \Lambda$ ($\Lambda \to p \pi^-$)
   - Difficult - either long lived final state or dominated by strong interaction
4. Swap final state $K^{\pm}$ for $\pi^{\pm}$
   - $B^{\pm} \to D^0 \pi^{\pm}$
   - Tried at LHCb - problematic as r_B is very small - statistical difficulties
5. New ideas...

Direct consideration of New Physics

- There is still considerable scope in current constraints on tree-level NP in Wilson coefficients $\mathcal{C}_1$ and $\mathcal{C}_2$ - [Phys. Rev. D92 (2015) 033002]

$$\mathcal{C}_1 = \mathcal{C}_1^{SM} + \Delta\mathcal{C}_1^{NP}$$
$$\mathcal{C}_2 = \mathcal{C}_2^{SM} + \Delta\mathcal{C}_2^{NP}$$



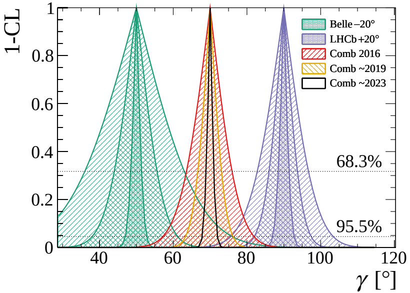
- With sufficient precision on γ we can modify our definition of the supressed / favoured amplitude ratio and fit these directly

Modification of $\mathcal{A}(B^- \to \bar{D}^0 K^-)/\mathcal{A}(B^- \to D^0 K^-)$

$$r_B e^{i\delta_B - \gamma} \to r_B e^{i\delta_B - \gamma} \left[\frac{\mathcal{C}_2^{SM} + r_A \mathcal{C}_1^{SM}}{\mathcal{C}_2^{SM} + r_{A'} \mathcal{C}_1^{SM}} \frac{\mathcal{C}_2^{SM} + \Delta\mathcal{C}_2^{NP} + r_{A'}(\mathcal{C}_1^{SM} + \Delta\mathcal{C}_1^{NP})}{\mathcal{C}_2^{SM} + \Delta\mathcal{C}_2^{NP} + r_A(\mathcal{C}_1^{SM} + \Delta\mathcal{C}_1^{NP})} \right] \quad (8)$$

Prospects

- With Run II of the LHC underway and Belle II starting soon the prospects look good
  - **LHCb and Belle II compliment each other for many measurements**
- We can reasonably expect to half the experimental uncertainty on γ in the next 3 years
- We can reasonably expect to have $\sim 1^\circ$ precision in the next 5-8 years
- In 10-12 years we should be $< 1^\circ$
- Current systematic effects are relatively small:
  - GLW/ADS
    - instrumental charge asymmetries
    - PID calibration
    - Background rates
  - GGSZ
    - efficiency correction over the Dalitz plane
    - amplitude model uncertainties
  - Time-dependent
    - Decay time resolution
    - Decay time acceptance
    - Knowledge of Δm_s, $\Delta\Gamma_s$, Γ_s



- Tree measurements of γ will not be systematically limited for a little while yet

This does not include smart new ideas which people often have

Prospects

What happens when we start to become systematically dominated

- For the most sensitive analyses (those using $B^{\pm} \to D^0 K^{\pm}$) the systematics are roughly a factor of 3-5 smaller than the statistical uncertainties
- Many of these systematics can be improved and also **have a statistical dependence**
- However, at $50\,\mathrm{fb}^{-1}$ (or even $300\,\mathrm{fb}^{-1}$ if it ever happens) this will be a serious consideration
- The main systematics for the **different methods** (GLW/ADS/GGSZ/TD etc.) come from **different sources**
- If we start to see differences between the different methods then we probably made a mistake
- If we start to see differences between different decay modes then a mistake is less likely and the difference has physical motivation
- **We will have to start working out the correlation of systematics among different decay modes**
    - Simultaneous methods are under consideration to mitigate this
- We will also need **better external** information - charm inputs from BES III

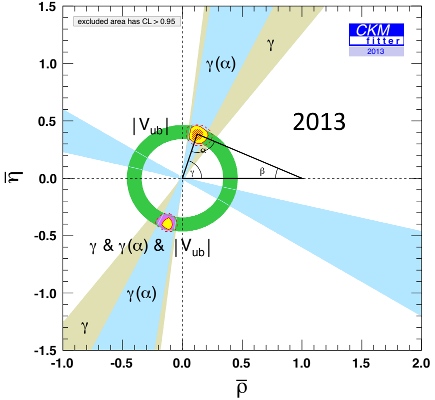
TD measurements could eventually be used as a penguin-free determination of ϕ_s

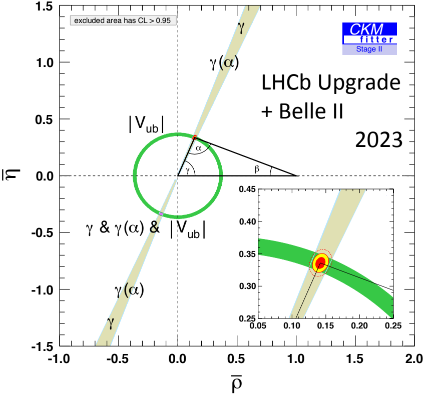
- $B_s^0 \to D_s^- K^+$ measures $(\gamma - 2\beta_s)$

Prospects

- We are approaching the first tree-level precision measurement of the CKM triangle
- Direct measurements of V_{ub} play a crucial role in this as well

[arXiv:1309.2293]





Backup

BACKUP

CKM matrix

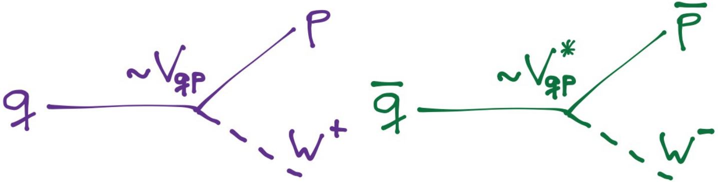
- In the SM quarks can change flavour by emission of a $W^{\pm}$ boson
- Quark mixing in the SM is described by the 3×3 unitary CKM matrix

CKM matrix

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \cdot \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

flavour eigenstates — mass eigenstates

- The matrix elements determine the transition probability



- Parameterised by three mixing angles (θ_{12}, θ_{13}, θ_{23}) and a CP violating phase (δ)

CKM matrix

- The CKM matrix exhibits a clear hierachy, $\sin(\theta_{13}) << \sin(\theta_{23}) << \sin(\theta_{12}) << 1$, so often expressed in Wolfenstein parameterisation (A, λ, ρ, η)

Wolfenstein parametrisation

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

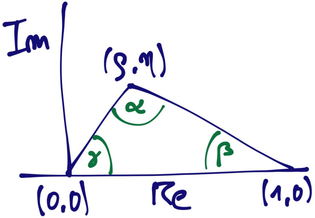
- **Hierachy gives very distinctive behaviour to the flavour sector** of the SM which gives strong constraints on NP
- CKM matrix gives the only source of CP violation in the SM ($m_\nu = \theta_{QCD} = 0$)

Unitarity gives a triangle in the complex plane

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

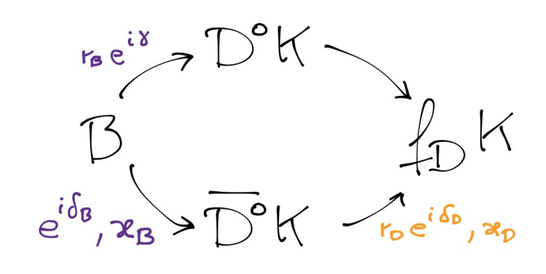
$$\Rightarrow \frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} + 1 + \frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*} = 0$$

- **Area** corresponds to **total CPV** in SM
- SM implies that $\alpha + \beta + \gamma = 180^\circ$



Methods to measure γ

Reconstruct the $D^0/\bar{D}^0$ in a final state accesible to both to acheieve interference



- GLW method
  - CP eigenstates e.g. $D \to KK$
  - Gronau, London, Wyler (1991)
- ADS method
  - CF or DCS decays e.g. $D \to K\pi$
  - Atwood, Dunietz, Soni (1997,2001)
- GGSZ method
  - 3-body final states e.g. $D \to K^0_S\pi\pi$
  - Giri, Grossman, Soffer, Zupan (2003)

- [Phys. Lett. B253 (1991) 483]
- [Phys. Lett. B265 (1991) 172]
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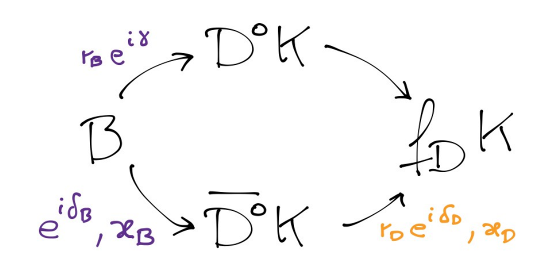

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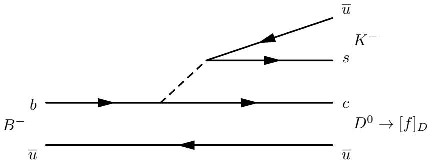
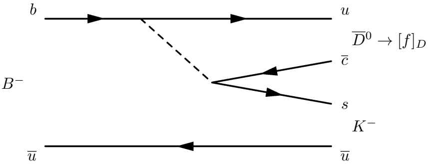
γ with CP eigenstates (GLW)

- Use the $B^{\pm} \to \overset{(-)}{D}{}^{0} K^{\pm}$ case as an example:
  - **Consider only D decays to CP eigenstates, f_{CP}**
  - Favoured: $b \to c$ with strong phase δ_F and weak phase ϕ_F
  - Supressed: $b \to u$ with strong phase δ_S and weak phase ϕ_S

Favoured:



Supressed:



Subsequent amplitude to final state f_{CP} is:

$$A_f = |F|e^{i(\delta_F - \phi_F)} + |S|e^{i(\delta_S - \phi_S)} \tag{9}$$

$$\bar{A}_f = |F|e^{i(\delta_F + \phi_F)} + |S|e^{i(\delta_S + \phi_S)} \tag{10}$$

because strong phases (δ) don't change sign under CP while weak phases (ϕ) do

γ with *CP* eigenstates (GLW)

- Define the *CP* asymmetry as the rate difference between meson with b ($\overline{B}$) and $\overline{b}$ (B)

$$\mathcal{A}_{CP} = \frac{|\bar{A}_{\bar{f}}|^2 - |A_f|^2}{|\bar{A}_{\bar{f}}|^2 + |A_f|^2} \quad \textbf{(experimental observable)}$$

$$= \frac{2|F||S|\sin(\delta_F - \delta_S)\sin(\phi_F - \phi_S)}{|F|^2 + |S|^2 + 2|F||S|\cos(\delta_F - \delta_S)\cos(\phi_F - \phi_S)}$$

- Choose $r_B = \frac{|S|}{|F|}$ (so that $r < 1$) and use strong phase difference $\delta_B = \delta_F - \delta_S$
- γ is the weak phase difference $\phi_F - \phi_S$
- Subsequently the *CP* asymmetry becomes

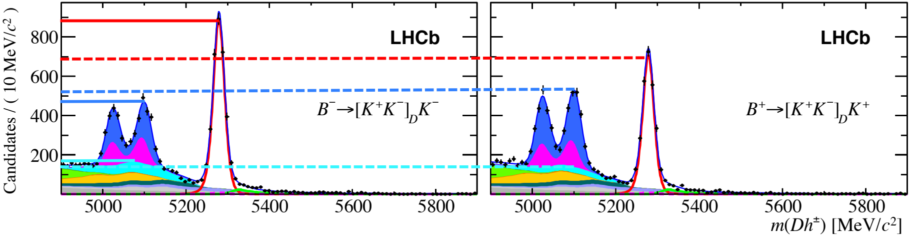
GLW *CP* asymmetry

$$\mathcal{A}_{CP} = \frac{\pm 2 r_B \sin(\delta_B)\sin(\gamma)}{1 + r_B^2 \pm 2 r_B \cos(\delta_B)\cos(\gamma)}$$

- The $+(-)$ sign corresponds to *CP*-even (-odd) final states
- Note that r_B and δ_B (ratio and strong phase difference of favoured and supressed modes) are different for each B decay
- **The value of γ is shared by all such decays**

An example GLW analysis

- GLW analysis of $B^\pm \to DK^\pm$ with $D \to K^+K^-$ - [arXiv:1603.08993]
- CP asymmetry can be seen by eye



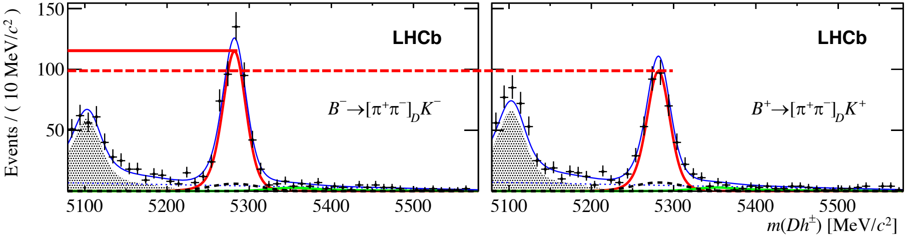
GLW asymmetry for $B^\pm \to DK^\pm$ and $D \to K^+K^-$

$$\mathcal{A}_{CP}^{DK,KK} = \frac{\Gamma(B^- \to [K^+K^-]_D K^-) - \Gamma(B^+ \to [K^+K^-]_D K^+)}{\Gamma(B^- \to [K^+K^-]_D K^-) + \Gamma(B^+ \to [K^+K^-]_D K^+)}$$
$$= 0.087 \pm 0.020 \pm 0.008$$

$\sim 4\sigma$ from zero

An example GLW analysis

- Same story with the $B^{\pm} \to DK^{\pm}$ and $D \to \pi^+\pi^-$
- CP asymmetry can be seen by eye



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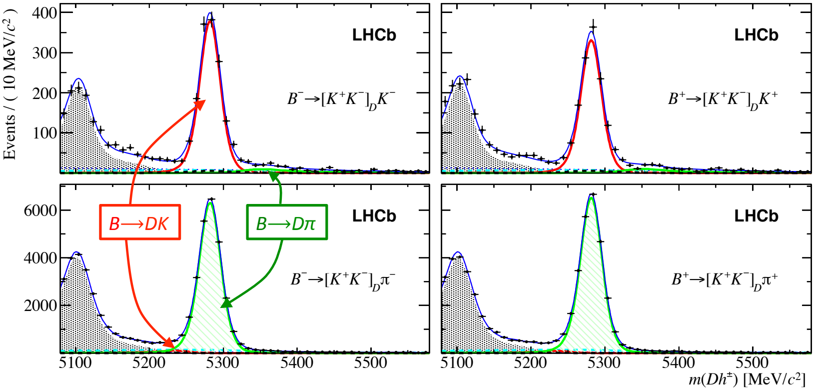
$$= 0.128 \pm 0.037 \pm 0.012$$

$\sim 3\sigma$ from zero

An example GLW analysis - $B^{\pm} \to D^0 K^{\pm}$, $D^0 \to K^+ K^-$

[arXiv:1603.08993]



Charge asymmetries

$$A_h^f = \frac{\Gamma(B^- \to [f]_D h^-) - \Gamma(B^+ \to [f]_D h^+)}{\Gamma(B^- \to [f]_D h^-) + \Gamma(B^+ \to [f]_D h^+)}$$

Kaon/pion ratio

$$R_{K/\pi}^f = \frac{\Gamma(B^{\pm} \to [f]_D K^{\pm})}{\Gamma(B^{\pm} \to [f]_D \pi^{\pm})}$$

γ with CF and DCS decays (ADS)

- A 2-body D decay to final state f accesible to both D^0 and $\bar{D}^0$ can be
  - Cabibbo-favoured (CF) - $D^0 \to \pi^- K^+$
  - Doubly-Cabibbo-supressed (DCS) - $\bar{D}^0 \to \pi^- K^+$
- Introduces 2 new hadronic parameters:
  - r_D - ratio of magnitudes for D^0 and $\bar{D}^0$ decay to f
  - δ_D - relative phase for D^0 and $\bar{D}^0$ decay to f
- Leads to an additional (modified) asymmetry defintion and an additional ratio observable

ADS asymmetry

$$\mathcal{A}_{ADS} = \frac{2 r_D r_B \sin(\delta_B + \delta_D) \sin(\gamma)}{r_D^2 + r_B^2 + 2 r_B r_D \cos(\delta_B + \delta_D) \cos(\gamma)}$$

ADS ratio

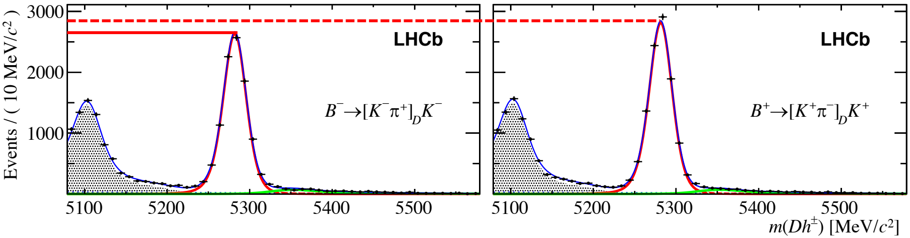
$$\mathcal{R}_{ADS} = \frac{|\bar{A}_{\bar{f}}|^2 + |A_f|^2}{|\bar{A}_f|^2 + |A_{\bar{f}}|^2} = r_B^2 + r_D^2 + 2 r_B r_D \cos(\delta_B + \delta_D) \cos(\gamma)$$

- Hadronic parameters r_D and δ_D can be de independently determined (using CLEO data and HFAG averages)
- Combining all information for various decays allows determination of γ, r_B and δ_B

An example ADS analysis

- ADS analysis of $B^{\pm} \to DK^{\pm}$ with $D \to K^{\pm}\pi^{\mp}$ which has **two asymmetries**
  1. "Favoured:"
     - $B^- \to [D^0 \to K^-\pi^+]K^-$ **AND** $B^- \to [\bar{D}^0 \to K^-\pi^+]K^-$
     - (fav. B with fav. D) **AND** (sup. B with sup. D)

Dominated by doubly favoured



Favoured ADS asymmetry for $B^{\pm} \to DK^{\pm}$ and $D \to K^{\pm}\pi^{\mp}$

$$\mathcal{A}_{CP}^{DK,K\pi} = \frac{\Gamma(B^- \to [K^-\pi^+]_D K^-) - \Gamma(B^+ \to [K^+\pi^-]_D K^+)}{\Gamma(B^- \to [K^-\pi^+]_D K^-) + \Gamma(B^+ \to [K^+\pi^-]_D K^+)}$$

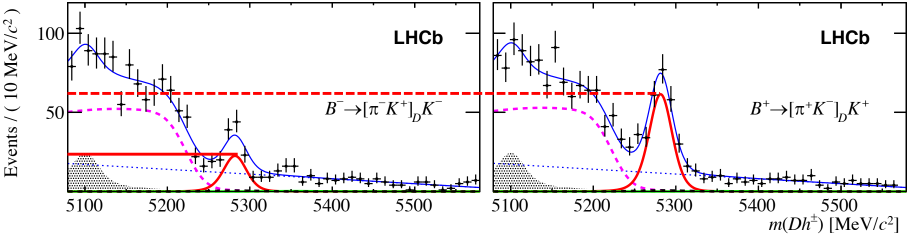
$$= -0.0194 \pm 0.0072 \pm 0.0060$$

$\sim 2\sigma$ from zero

An example ADS analysis

- ADS analysis of $B^{\pm} \to DK^{\pm}$ with $D \to K^{\pm}\pi^{\mp}$ which has **two asymmetries**
  2. Supressed:
     - $B^{-} \to [D^{0} \to K^{+}\pi^{-}]K^{-}$ **AND** $B^{-} \to [\bar{D}^{0} \to K^{+}\pi^{-}]K^{-}$
     - (fav. B with sup. D) **AND** (sup. B with fav. D)

Both of similar size so large CP effect



Favoured ADS asymmetry for $B^{\pm} \to DK^{\pm}$ and $D \to K^{\mp}\pi^{\pm}$

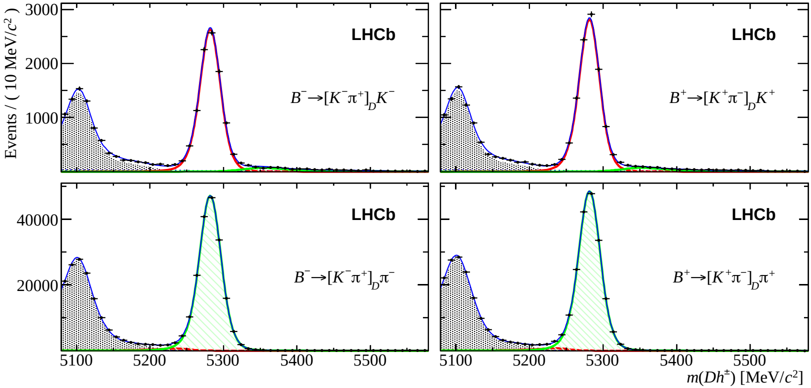
$$\mathcal{A}_{\rm ADS}^{DK,\pi K} = \frac{\Gamma(B^{-} \to [K^{+}\pi^{-}]_{D}K^{-}) - \Gamma(B^{+} \to [K^{-}\pi^{+}]_{D}K^{+})}{\Gamma(B^{-} \to [K^{+}\pi^{-}]_{D}K^{-}) + \Gamma(B^{+} \to [K^{-}\pi^{+}]_{D}K^{+})}$$
$$= -0.403 \pm 0.056 \pm 0.011$$

$\sim 7\sigma$ from zero

An example ADS analysis - $B^{\pm} \to D^0 K^{\pm}$, $D^0 \to K^{\pm}\pi^{\pm}$

- Favoured mode

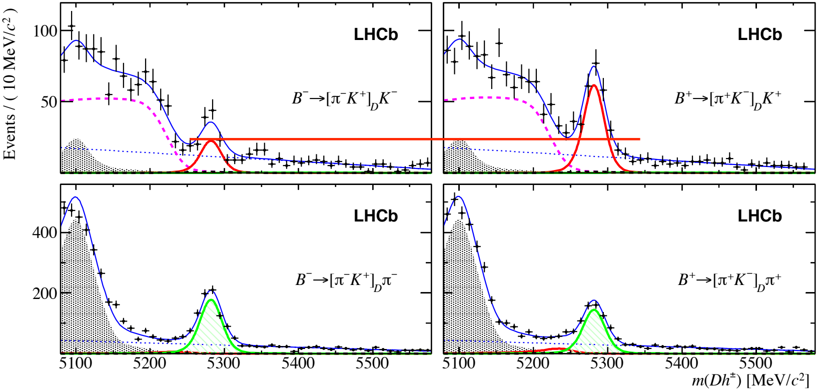
[arXiv:1603.08993]



An example ADS analysis - $B^{\pm} \to D^0 K^{\pm}$, $D^0 \to K^{\pm}\pi^{\pm}$

- Suppressed mode

[arXiv:1603.08993]



An example ADS analysis - $B^{\pm} \to D^0 K^{\pm}$, $D^0 \to K^{\pm}\pi^{\pm}$

- Define observables as yield ratios (many systematics cancel)
- Along with the GLW observables build a system of equations to overconstrain the parameters

ADS ratios of favoured to suppressed

$$R^{\bar{f}}_{\mathrm{ADS}} = \frac{\Gamma(B^- \to [\bar{f}]_D h^-) + \Gamma(B^+ \to [f]_D h^+)}{\Gamma(B^- \to [f]_D h^-) + \Gamma(B^+ \to [\bar{f}]_D h^+)}$$

Corresponding charge asymmetries

$$A^{\bar{f}}_{\mathrm{ADS}} = \frac{\Gamma(B^- \to [\bar{f}]_D h^-) - \Gamma(B^+ \to [f]_D h^+)}{\Gamma(B^- \to [\bar{f}]_D h^-) + \Gamma(B^+ \to [f]_D h^+)}$$

- Relatively trivial extension to multibody D decays ($D \to 4\pi$, $D \to K3\pi$, $D \to KK\pi^0$, $D \to \pi\pi\pi^0$, $D \to K\pi\pi^0$), multibody B decays ($B^{\pm} \to DK^{\pm}\pi^+\pi^-$) and other initial B states ($B^0 \to DK^{*0}$)

Aside: Multibody final states

- **The GLW/ADS formalisms are fairly trivally extended to multibody final states**

GLW

- Multibody quasi-CP states
- $D \to K^+ K^- \pi^0$
- $D \to \pi^+ \pi^- \pi^0$
- $D \to \pi^+ \pi^- \pi^+ \pi^-$
- Account for the fraction, F^+ of CP-even content

ADS

- Multibody DCS decys
- $D \to K^+ \pi^- \pi^0$
- $D \to K^+ \pi^- \pi^+ \pi^-$
- Account for the dilution, κ, from interference between resonances

$$\kappa e^{i\delta_D} = \frac{\int A_f(x) A_{\bar{f}(x) dx}}{\sqrt{\int A_f^2(x) dx \int A_{\bar{f}}^2(x) dx}}$$

quasi-GLW asymmetry

$$\mathcal{A}_{CP} = \frac{\pm 2(2F^+ + 1) r_B \sin(\delta_B) \sin(\gamma)}{1 + r_B^2 \pm 2(2F^+ + 1) r_B \cos(\delta_B) \cos(\gamma)}$$

quasi-ADS asymmetry

$$\mathcal{A}_{CP} = \frac{2\kappa r_D r_B \sin(\delta_B + \delta_D) \sin(\gamma)}{r_D^2 + r_B^2 + 2\kappa r_B r_D \cos(\delta_B + \delta_D) \cos(\gamma)}$$

- We can also construct partial rate ratios as additional observables

γ with 3-body self-conjugate states (GGSZ)

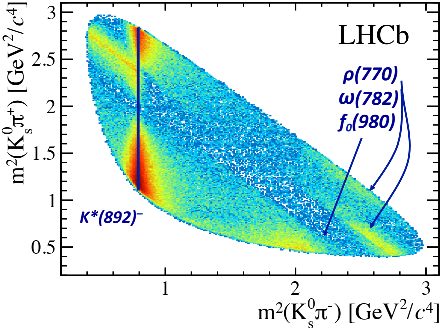
- Now get additional sensitivity over the 3-body phase space
- Idea is to perform a GLW/ADS type analysis across the D decay phase space
- For example $D^0 \to K^0_S \pi^+\pi^-$ has contributions from
  - Singly-Cabibbo-suppressed decay $D^0 \to K^0_S \rho^0$
  - Doubly-Cabibbo-suppressed decay $D^0 \to K^{*+}\pi^-$
  - Interference between them enhances sensitivity and resolves amigiuties in γ determination

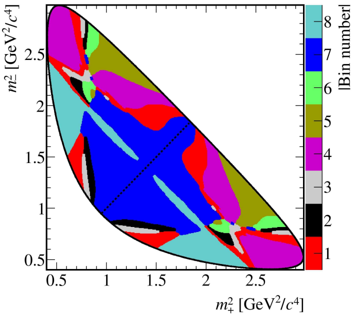
Partial B rate as function of Dalitz position $(+,-) = (m_{K^0_S\pi^+}, m_{K^0_S\pi^-})$

$$
\begin{aligned}
d\Gamma_{B^\pm}(\mathbf{x}) = \; & A^2_{(\pm,\mp)} + r_B^2 A^2_{(\mp,\pm)} \\
& + 2A_{(\pm,\mp)}A_{(\mp,\pm)}\Big[\underbrace{r_B\cos(\delta_B \pm \gamma)}_{x_\pm}\cos(\delta_{D(\pm,\mp)}) + \underbrace{r_B\sin(\delta_B \pm \gamma)}_{y_\pm}\sin(\delta_{D(\pm,\mp)})\Big]
\end{aligned}
$$

- Model-dependent - Fit Dalitz plot with full amplitude model for $(x_\pm, y_\pm)$
- Model-independent - Choose binning scheme in Dalitz plane to minimize δ_D variation across bin and fit simultaneously in each bin for $(x_\pm, y_\pm)$

Examples of the $D^0 \to K^0_S\pi^+\pi^-$ Dalitz distribution

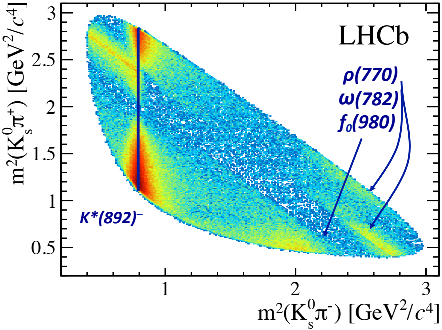


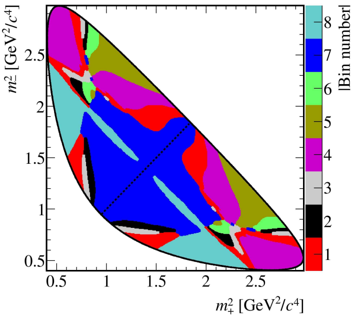


- Model-dependent - Fit Dalitz plot with full amplitude model for $(x_\pm, y_\pm)$
- Model-independent - Choose binning scheme in Dalitz plane to minimize δ_D variation across bin and fit simultaneously in each bin for $(x_\pm, y_\pm)$
- **Sensitivity to γ by comparing D Dalitz distributions for B^+ and B^-**
- In other words *CP* asymmetry in bins of Dalitz space

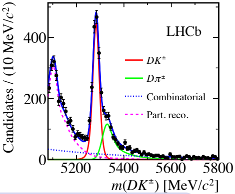
An example GGSZ analysis

- Requires a self-conjugate 3-body final state ($D^0 \to K^0_S \pi^- \pi^+$, $D^0 \to K^0_S K^- K^+$)
- The basic idea is to perform a GLW/ADS type analysis in each bin of the D decay phase space
- Compare Dalitz distribution for B^+ and B^-
  - Model dependent: use a Dalitz model describing all the intermediate resonances and fit for $x_\pm$, $y_\pm$
  - Model independent: define bins which maximise sensitivity to $x_\pm$, $y_\pm$



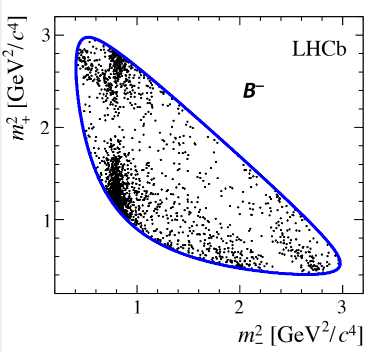


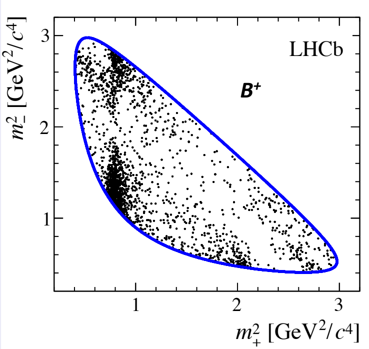
An example GGSZ analysis - $B^{\pm} \to DK^{\pm}$, $D \to K^0_S \pi^+ \pi^-$



- First fit invariant B mass distrbution
- Project (cut) signal candidates into Dalitz plane
- Requires experimental efficiency and background distributions in DP
- Control channels used are $B^{\pm} \to D\pi^{\pm}$ and $B^{-} \to D\mu^{-}\nu$

Compare bin by bin differences for signal candidates in the Dalitz plane

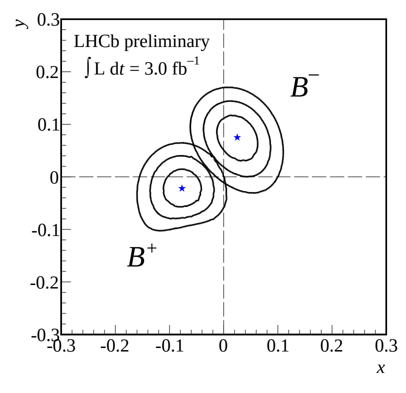




An example GGSZ analysis

- GGSZ analyses have excellent standalone sensitivity with a single solution

$B^{\pm} \to D^0(\to K_S^0 hh)K^{\pm}$



"Third" uncertainty arises from:

- MD: - amplitude model uncertainty
- MI: - knowledge of strong phase in DP bins

$$x_+ = -0.077 \pm 0.024 \pm 0.010 \pm 0.004$$
$$y_+ = -0.022 \pm 0.025 \pm 0.004 \pm 0.010$$
$$x_- = 0.025 \pm 0.025 \pm 0.010 \pm 0.005$$
$$y_- = 0.075 \pm 0.029 \pm 0.005 \pm 0.014$$

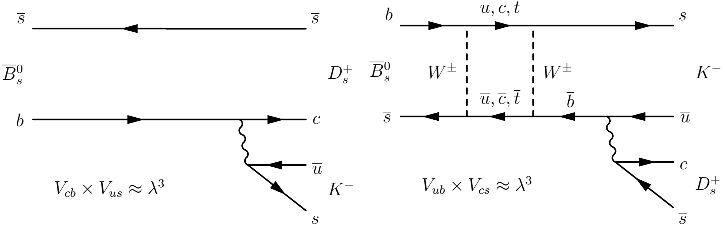
Extensions to other decays

Extension for many other decays with inclusion of relevant coherence factors, κ

1. Obvious extensions to higher resonant final states
   - $B^\pm \to D^0 K^{*\pm}$ ($K^{*\pm} \to K^0_S \pi^\pm$)
   - $B^\pm \to D^{*0} K^\pm$ ($D^{*0} \to D^0 \gamma$ or $D^{*0} \to D^0 \pi^0$)
   - mainly just the B factories so far - LHCb starting for Run 2
   - Similar hadronic parameters as $B^\pm \to D^0 K^\pm$ but lower yields
2. Extensions to other B decays (swapping spectator quark)
   - $B^0 \to D^0 K^{*0}$ ($K^{*0} \to K^+ \pi^-$ tags initial B flavour)
   - $B^0 \to D^0 K^0_S$ (not self tagging)
   - $B^0_s \to D^0 \phi$ (not self tagging and low rate)
   - $B^\pm_c \to D^0 D^\pm$ (very low rate - favoured mode not yet seen)
   - under exploration at LHCb with some $B^0 \to D^0 K^{*0}$ published
   - Typically enhanced r_B because favoured diagram is colour supressed
3. Extension into baryon sector (add/swap spectators)
   - $\Lambda^0_b \to D^0 \Lambda$ ($\Lambda \to p \pi^-$)
   - Difficult - either long lived final state or dominated by strong interaction
4. Swap final state $K^\pm$ for $\pi^\pm$
   - $B^\pm \to D^0 \pi^\pm$
   - Tried at LHCb - problematic as r_B is very small - statistical difficulties
5. Other methods
   - Time-dependent method ($B^0_s \to D^\mp_s K^\pm$ etc.)
   - Dalitz method (multibody B decays e.g. $B^0 \to D^0 K^\pm \pi^\mp$)
   - Both tried at LHCb

The time-dependent method with $B_s^0 \to D_s^{\mp} K^{\pm}$

- B_s^0 and $\overline{B}_s^0$ can both decay to same final state $D_s^{\mp} K^{\pm}$ (one via $b \to cW$, the other via $b \to uW$)
- Intereference acheived by neutral B_s^0 mixing (requires knowledge of $-2\beta_s \equiv \phi_s$)
  - Weak phase difference is $(\gamma - 2\beta_s)$



- Requires tagging the initial B_s^0 flavour
- Requires a time-dependent analysis to observe the meson oscillations
- Fit the decay-time-dependent decay rates
- Also requires knowledge of Γ_s, $\Delta\Gamma_s$, Δm_s

The time-dependent method with $B_s^0 \to D_s^{\mp} K^{\pm}$

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Time-dependent decay rate for initial B_s^0 or $\bar{B}_s^0$ at $t = 0$

$$\frac{\mathrm{d}\Gamma_{B_s^0\to f}(t)}{\mathrm{d}t} \propto e^{-\Gamma_s t}\left[\cosh\left(\frac{\Delta\Gamma_s t}{2}\right) + A_f^{\Delta\Gamma_s}\sinh\left(\frac{\Delta\Gamma_s t}{2}\right) + C_f\cos\left(\Delta m_s t\right) - S_f\sin\left(\Delta m_s t\right)\right]$$

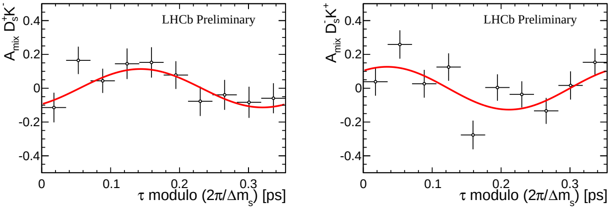
$$\frac{\mathrm{d}\Gamma_{\bar{B}_s^0\to f}(t)}{\mathrm{d}t} \propto e^{-\Gamma_s t}\left[\cosh\left(\frac{\Delta\Gamma_s t}{2}\right) + A_f^{\Delta\Gamma_s}\sinh\left(\frac{\Delta\Gamma_s t}{2}\right) - C_f\cos\left(\Delta m_s t\right) + S_f\sin\left(\Delta m_s t\right)\right]$$

Time-dependent rate asymmetry

$$\mathcal{A}_{CP}(t) = \frac{\Gamma_{\bar{B}_s^0\to f}(t) - \Gamma_{B_s^0\to f}(t)}{\Gamma_{\bar{B}_s^0\to f}(t) + \Gamma_{B_s^0\to f}(t)} = \frac{S_f\sin(\Delta m_s t) - C_f\cos(\Delta m_s t)}{\cosh(\frac{\Delta\Gamma_s t}{2}) + A_f^{\Delta\Gamma_s}\sinh(\frac{\Delta\Gamma_s t}{2})}$$

The time-dependent method with $B_s^0 \to D_s^{\mp} K^{\pm}$

- Fit for decay-time-dependent asymmetry

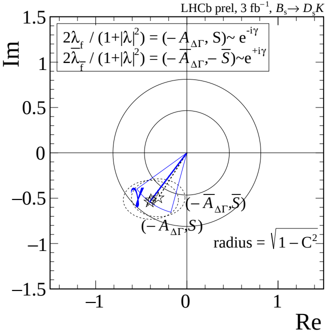


Variable definitions

$$C_f = -C_{\bar{f}} = \frac{1 - r_B^2}{1 + r_B^2}$$

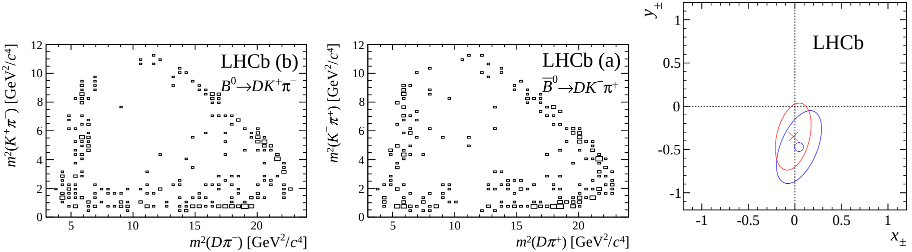
$$A^{\Delta\Gamma_s}_{f(\bar{f})} = \frac{-2r_B \cos(\gamma - 2\beta_s \mp \delta_B)}{1 + r_B^2}$$

$$S_{f(\bar{f})} = \frac{\pm 2r_B \sin(\gamma - 2\beta_s \mp \delta_B)}{1 + r_B^2}$$



Dalitz methods

- Study Dalitz structure of 3-body B **decays** with $B^0 \to D^0 K^+ \pi^-$
  - In principle has excellent sensitivity to γ
  - "GW method"? (Gershon-Williams - [arXiv:0909.1495])
- Get multiple interfering resonances which increase sensitivity to γ
  - $D_0^*(2400)^-$, $D_2^*(2460)^-$, $K^*(892)^0$, $K^*(1410)^0$, $K_2^*(1430)^0$
- Fit B decay Dalitz Plot for cartesian parameters (similar to GGSZ except for the B not the D)
  - $D \to K^+K^-$, $D \to \pi^+\pi^-$ - GLW-Dalitz (done by LHCb - [arXiv:1602.03455])
  - $D \to K^\pm\pi^\mp$ - ADS-Dalitz (problematic backgrounds from $B_s^0 \to DK^\pm\pi^\mp$)
  - $D \to K_S^0\pi^+\pi^-$ - GGSZ-Dalitz (double Dalitz!)



Methods to measure γ

GLW observables

$$A_{CP} = \frac{\Gamma(B^- \to D^0_{CP}K^-) - \Gamma(B^+ \to D^0_{CP}K^+)}{\Gamma(B^- \to D^0_{CP}K^-) + \Gamma(B^+ \to D^0_{CP}K^+)} = \frac{\pm 2 r_B \sin(\delta_B)\sin(\gamma)}{1 + r_B^2 \pm 2 r_B \cos(\delta_B)\cos(\gamma)} \quad (11)$$

$$R_{CP} = \frac{\Gamma(B^- \to D^0_{CP}K^-) + \Gamma(B^+ \to D^0_{CP}K^+)}{\Gamma(B^- \to D^0 K^-) + \Gamma(B^+ \to D^0 K^+)} = 1 + r_B^2 \pm 2 r_B \cos(\delta_B)\cos(\gamma) \quad (12)$$

ADS observables

$$A_{ADS} = \frac{\Gamma(B^- \to [K^+\pi^-]_D K^-) - \Gamma(B^+ \to [K^-\pi^+]_D K^+)}{\Gamma(B^- \to [K^+\pi^-]_D K^-) + \Gamma(B^+ \to [K^-\pi^+]_D K^+)} = \frac{2 r_B r_D \sin(\delta_B + \delta_D)\sin(\gamma)}{r_B^2 + r_D^2 + 2 r_B r_D \cos(\delta_B + \delta_D)\cos(\gamma)} \quad (13)$$

$$R_{ADS} = \frac{\Gamma(B^- \to [K^+\pi^-]_D K^-) + \Gamma(B^+ \to [K^-\pi^+]_D K^+)}{\Gamma(B^- \to [K^-\pi^+]_D K^-) + \Gamma(B^+ \to [K^+\pi^-]_D K^+)} = r_B^2 + r_D^2 + 2 r_B r_D \cos(\delta_B + \delta_D)\cos(\gamma) \quad (14)$$

GGSZ observables (partial rate as function of Dalitz position)

$$x_\pm = r_B \cos(\delta_B \pm \gamma) \quad (15)$$

$$y_\pm = r_B \sin(\delta_B \pm \gamma) \quad (16)$$

$$d\Gamma_{B^\pm}(\mathbf{x}) = A^2_{(\pm,\mp)} + r_B^2 A^2_{(\mp,\pm)} + 2A_{(\pm,\mp)}A_{(\mp,\pm)}\left[\underbrace{r_B\cos(\delta_B \pm \gamma)}_{x_\pm}\cos(\delta_{D(\pm,\mp)}) + \underbrace{r_B\sin(\delta_B \pm \gamma)}_{y_\pm}\sin(\delta_{D(\pm,\mp)})\right] \quad (17)$$

Methods to measure γ

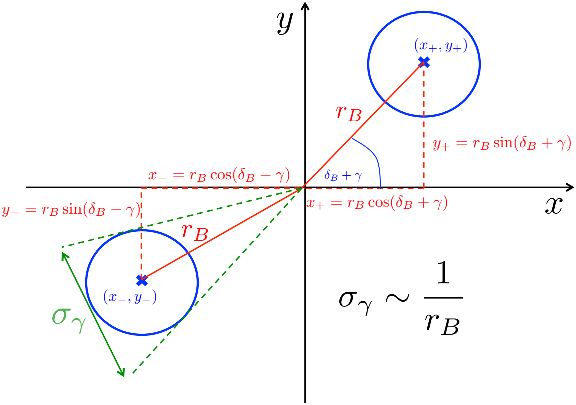
- GLW method
  - CP eigenstates e.g. $D \to KK$
  - Gronau, London, Wyler (1991)
  - [Phys. Lett. B253 (1991) 483]
  - [Phys. Lett. B265 (1991) 172]
- ADS method
  - CF or DCS decays e.g. $D \to K\pi$
  - Atwood, Dunietz, Soni (1997,2001)
  - [Phys. Rev. D63 (2001) 036005]
  - [Phys. Rev. Lett. 78 (1997) 3257]
- GGSZ method
  - 3-body final states e.g. $D \to K^0_{\mathrm{S}}\pi\pi$
  - Giri, Grossman, Soffer, Zupan (2003)
  - [Phys. Rev. D68 (2003) 054018]
- TD method
  - Interference through B^0_s mixing phase $= (\gamma - 2\beta_s)$
  - Gronau, London, Wyler (1991)
  - [Phys. Lett. B253 (1991) 483]
  - [Phys. Lett. B265 (1991) 172]
- Dalitz method
  - CP eigenstates e.g. $D \to KK$
  - Gronau, London, Wyler (1991)
  - [Phys. Lett. B253 (1991) 483]
  - [Phys. Lett. B265 (1991) 172]

The cartesian coordinates

Cartesian definition

$$x_\pm + iy_\pm = r_B e^{i(\delta_B \pm \gamma)}$$
$$x_\pm = r_B \cos(\delta_B \pm \gamma)$$
$$y_\pm = r_B \sin(\delta_B \pm \gamma)$$

- Use these for fit stability
- Ease of combianation
- Good statistical behaviour



Uncertainty on γ is inversely proportional to central value of hadronic unknown!!

- Fluctuation in nuisance parameter = fluctuation in error on parameter of interest!

GLS method

- Use and ADS-like method with singly-Cabibbo suppressed decays - $D \to K_S^0 K^\pm \pi^\mp$
- Use Dalitz Plot for 3-body D decay
- Currently poor sensitivity to γ as the rate is incredibly low

LHCb γ combination inputs

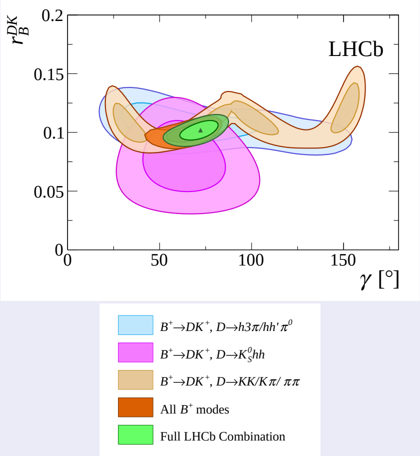
	B decay	D decay	Type	$\int \mathcal{L}$	Ref.
LHCb Inputs	$B^+ \to DK^+$	$D \to hh$	GLW/ADS	$3\,\mathrm{fb}^{-1}$	[arXiv:1603.08993]
	$B^+ \to DK^+$	$D \to h\pi\pi\pi$	GLW/ADS	$3\,\mathrm{fb}^{-1}$	[arXiv:1603.08993]
	$B^+ \to DK^+$	$D \to hh\pi^0$	GLW/ADS	$3\,\mathrm{fb}^{-1}$	[arXiv:1504.05442]
	$B^+ \to DK^+$	$D \to K^0_S hh$	GGSZ	$3\,\mathrm{fb}^{-1}$	[arXiv:1405.2797]
	$B^+ \to DK^+$	$D \to K^0_S K\pi$	GLS	$3\,\mathrm{fb}^{-1}$	[arXiv:1402.2982]
	$B^0 \to D^0 K^{*0}$	$D \to K\pi$	ADS	$3\,\mathrm{fb}^{-1}$	[arXiv:1407.3186]
	$B^+ \to DK^+\pi\pi$	$D \to hh$	GLW/ADS	$3\,\mathrm{fb}^{-1}$	[arXiv:1505.07044]
	$B^0_s \to D_s^\mp K^\pm$	$D_s^+ \to hhh$	TD	$1\,\mathrm{fb}^{-1}$	[arXiv:1407.6127] *
	$B^0 \to D^0 K^+\pi^-$	$D \to hh$	GLW-Dalitz	$3\,\mathrm{fb}^{-1}$	[arXiv:1602.03455]
	$B^0 \to D^0 K^{*0}$	$D \to K^0_S\pi\pi$	GGSZ	$3\,\mathrm{fb}^{-1}$	[arXiv:1604.01525]
	Decay	**Parameters**	**Source**		**Ref.**
Auxilliary Inputs	D^0–$\bar{D}^0$ mixing		HFlAv	-	[arXiv:1412.7515]
	$D \to K\pi\pi\pi$	$(\delta_D,\ \kappa_D,\ r_D)$	CLEO+LHCb	-	[arXiv:1602.07430] *
	$D \to \pi\pi\pi\pi$	(F^+)	CLEO	-	[arXiv:1504.05878]
	$D \to K\pi\pi^0$	$(\delta_D,\ \kappa_D,\ r_D)$	CLEO+LHCb	-	[arXiv:1602.07430]
	$D \to hh\pi^0$	(F^+)	CLEO	-	[arXiv:1504.05878]
	$D \to K^0_S K\pi$	$(\delta_D,\ \kappa_D)$	CLEO	-	[arXiv:1203.3804] *
	$D \to K^0_S K\pi$	(r_D)	CLEO	-	[arXiv:1203.3804]
	$D \to K^0_S K\pi$	(r_D)	LHCb	-	[arXiv:1509.06628]
	$B^0 \to D^0 K^{*0}$	$(\kappa_B,\ \bar{R}_B,\ \bar{\Delta}_B)$	LHCb	-	[arXiv:1602.03455]
	$B^0_s \to D_s^+ K^-$	(ϕ_s)	LHCb	-	[arXiv:1411.3104]
Combination:					[arXiv:1611.03076]

New or updated since last combination

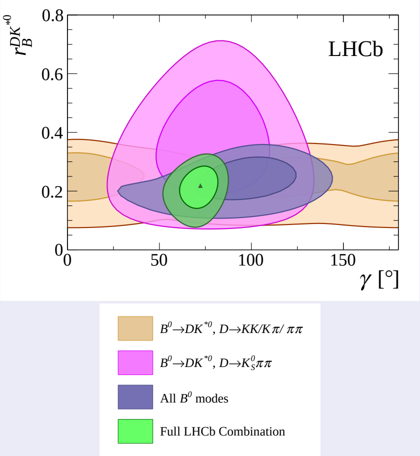
LHCb γ Combination

Naive statistical treatement (profile likelihood method) - plots for demonstrative purposes only

$B^+ \to D^0 K^+$ system



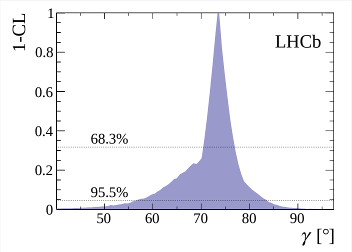
$B^0 \to D^0 K^{*0}$ system

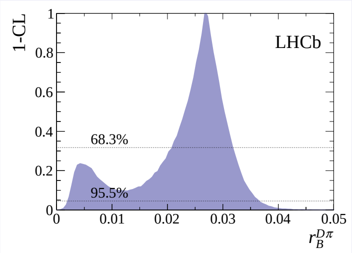


On inclusion of $B \to D\pi$-like modes

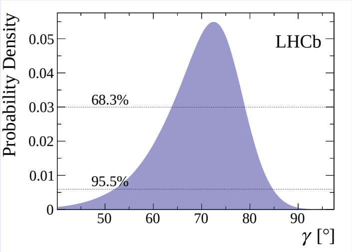
- $r_B^{D\pi}$ expectation ~ 0.005 (favoured enhanced by V_{ud}/V_{us}, suppressed reduced by V_{cd}/V_{cs})

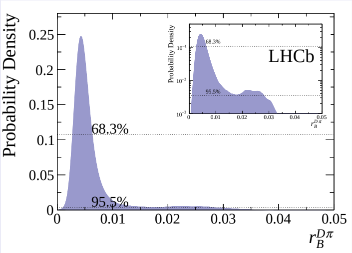
Frequentist





Bayesian

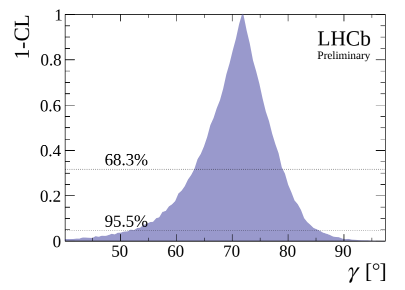




On inclusion of $B \to D\pi$-like modes

- The *a priori* sensitivity gain is rather minimal
- Enforcing a constraint on $r_B^{D\pi}$ using a theory prediction $r_B^{D\pi} = 0.0053 \pm 0.0007$ ([arXiv:1606.09129] - Kenzie, Martinelli, Tuning)
- Recovers similar results to DK-mode combination

Using external constraint on $r_B^{D\pi}$:
$\gamma = (71.8^{+7.2}_{-8.6})^\circ$



HFIAv γ combination inputs (1/4)

B decay	D decay	Method	Experiment
$B^- \to DK^-$	$D \to K^+K^-$, $D \to \pi^+\pi^-$, $D \to K_S^0\pi^0$, $D \to K_S^0\omega$, $D \to K_S^0\phi$	GLW	BABAR
$B^- \to DK^-$	$D \to K^+K^-$, $D \to \pi^+\pi^-$, $D \to K_S^0\pi^0$, $D \to K_S^0\omega$, $D \to K_S^0\phi$	GLW	Belle
$B^- \to DK^-$	$D \to K^+K^-$, $D \to \pi^+\pi^-$	GLW	CDF
$B^- \to DK^-$	$D \to K^+K^-$, $D \to \pi^+\pi^-$	GLW	LHCb
$B^- \to D^*K^-$ $D^* \to D\gamma\ (\pi^0)$	$D \to K^+K^-$, $D \to \pi^+\pi^-$, $D \to K_S^0\pi^0$, $D \to K_S^0\omega$, $D \to K_S^0\phi$	GLW	BABAR
$B^- \to D^*K^-$ $D^* \to D\gamma\ (\pi^0)$	$D \to K^+K^-$, $D \to \pi^+\pi^-$, $D \to K_S^0\pi^0$, $D \to K_S^0\omega$, $D \to K_S^0\phi$	GLW	Belle
$B^- \to DK^{*-}$	$D \to K^+K^-$, $D \to \pi^+\pi^-$, $D \to K_S^0\pi^0$, $D \to K_S^0\omega$, $D \to K_S^0\phi$	GLW	BABAR
$B^- \to DK^{*-}$	$D \to K^+K^-$, $D \to \pi^+\pi^-$,	GLW	LHCb
$B^- \to DK^-\pi^+\pi^-$	$D \to K^+K^-$, $D \to \pi^+\pi^-$	GLW	LHCb

HFIAv γ combination inputs (2/4)

$B^- \to DK^-\pi^+\pi^-$	$D \to K^+K^-$, $D \to \pi^+\pi^-$	GLW	LHCb
$B^- \to DK^-$	$D \to \pi^+\pi^-\pi^0$	GLW-like	*BABAR*
$B^- \to DK^-$	$D \to h^+h^-\pi^0$	GLW-like	LHCb
$B^- \to DK^-$	$D \to \pi^+\pi^-\pi^+\pi^-$	GLW-like	LHCb
$B^- \to DK^-$	$D \to K^\pm\pi^\mp$	ADS	*BABAR*
$B^- \to DK^-$	$D \to K^\pm\pi^\mp$	ADS	Belle
$B^- \to DK^-$	$D \to K^\pm\pi^\mp$	ADS	CDF
$B^- \to DK^-$	$D \to K^\pm\pi^\mp$	ADS	LHCb
$B^- \to DK^-$	$D \to K^\pm\pi^\mp\pi^0$	ADS	*BABAR*
$B^- \to DK^-$	$D \to K^\pm\pi^\mp\pi^0$	ADS	Belle
$B^- \to DK^-$	$D \to K^\pm\pi^\mp\pi^0$	ADS	LHCb
$B^- \to DK^-$	$D \to K^\pm\pi^\mp\pi^+\pi^-$	ADS	LHCb
$B^- \to D^*K^-$, $D^* \to D\gamma$	$D \to K^\pm\pi^\mp$	ADS	*BABAR*
$B^- \to D^*K^-$, $D^* \to D\pi^0$	$D \to K^\pm\pi^\mp$	ADS	*BABAR*

HFIAv γ combination inputs (3/4)

$B^- \to DK^{*-}$	$D \to K^\pm\pi^\mp$	ADS	*BABAR*
$B^- \to DK^{*-}$	$D \to K^\pm\pi^\mp$	ADS	LHCb
$B^- \to DK^-\pi^+\pi^-$	$D \to K^\pm\pi^\mp$	ADS	LHCb
$B^- \to DK^-$	$D \to K^0_s\pi^+\pi^-$	GGSZ MD	*BABAR*
$B^- \to DK^-$	$D \to K^0_s\pi^+\pi^-$	GGSZ MD	Belle
$B^- \to D^*K^-$	$D \to K^0_s\pi^+\pi^-$	GGSZ MD	*BABAR*
$D^* \to D\gamma\ (\pi^0)$			
$B^- \to D^*K^-$	$D \to K^0_s\pi^+\pi^-$	GGSZ MD	Belle
$D^* \to D\gamma\ (\pi^0)$			
$B^- \to DK^{*-}$	$D \to K^0_s\pi^+\pi^-$	GGSZ MD	*BABAR*
$B^- \to DK^{*-}$	$D \to K^0_s\pi^+\pi^-$	GGSZ MD	Belle
$B^- \to DK^-$	$D \to K^0_s\pi^+\pi^-$	GGSZ MI	LHCb
$B^- \to DK^-$	$D \to K^0_sK^+\pi^-$	GLS	LHCb
$B^0 \to DK^{*0}$	$D \to K^\pm\pi^\mp$	ADS	LHCb
$B^0 \to DK^+\pi^-$	$D \to h^+h^-$	GLW-Dalitz	LHCb
$B^0 \to DK^+\pi^-$	$D \to K^0_sh^+h^-$	GGSZ MI	LHCb
$B^0_s \to D^\mp_sK^\pm$	$D^+_s \to h^+h^-\pi^+$	TD	LHCb

HFIAv γ combination inputs (4/4)

Decay	Parameters	Source
$D \to K^\pm\pi^\mp$	$r_D^{K\pi}$, $\delta_D^{K\pi}$	HFAG
$D \to K^\pm\pi^\mp\pi^+\pi^-$	$\delta_D^{K3\pi}$, $\kappa_D^{K3\pi}$, $r_D^{K3\pi}$	CLEO+LHCb
$D \to \pi^+\pi^-\pi^+\pi^-$	$F_{\pi\pi\pi\pi}$	CLEO
$D \to K^\pm\pi^\mp\pi^0$	$\delta_D^{K2\pi}$, $\kappa_D^{K2\pi}$, $r_D^{K2\pi}$	CLEO+LHCb
$D \to h^+h^-\pi^0$	$F_{\pi\pi\pi^0}$, $F_{KK\pi^0}$	CLEO
$D \to K_S^0K^+\pi^-$	$\delta_D^{K_SK\pi}$, $\kappa_D^{K_SK\pi}$, $r_D^{K_SK\pi}$	CLEO
	$r_D^{K_SK\pi}$	LHCb
$B^0 \to DK^{*0}$	$\kappa_B(DK^{*0})$, $\overline{R}_B^{DK^{*0}}$, $\overline{\Delta}_B^{DK^{*0}}$	LHCb
$B_s^0 \to D_s^\mp K^\pm$	ϕ_s	HFAG

The World Average

- A simultaneous determination of γ, r_B and δ_B for each decay allows us to extract the breakdown of favoured and suppressed decay branching fractions

Parameter	Value
γ	$(72.8^{+5.3}_{-6.3})^\circ$
$r_B^{D^0K^\pm}$	(0.103 ± 0.005)
$r_B^{D^0K^{*\pm}}$	(0.13 ± 0.05)
$r_B^{D^{*0}K^\pm}$	$(0.12 \pm 0.02$
$r_B^{D^0K^{*0}}$	(0.22 ± 0.04)
$\delta_B^{D^0K^\pm}$	$(137.4^{+5.3}_{-5.9})^\circ$
$\delta_B^{D^0K^{*\pm}}$	$(129^{+25}_{-33})^\circ$
$\delta_B^{D^{*0}K^\pm}$	$(311^{+13}_{-17})^\circ$
$\delta_B^{D^0K^{*0}}$	$(194^{+27}_{-22})^\circ$

Decay	Favoured BR	Suppressed BR
$B^\pm \to D^0K^\pm$	$(3.69 \pm 0.17) \times 10^{-4}$	$(3.91 \pm 0.42) \times 10^{-6}$
$B^\pm \to D^0K^{*\pm}$	$(5.30 \pm 0.40) \times 10^{-4}$	$(8.96 \pm 6.92) \times 10^{-6}$
$B^\pm \to D^{*0}K^\pm$	$(4.20 \pm 0.34) \times 10^{-4}$	$(6.05 \pm 2.07) \times 10^{-6}$
$B^0 \to D^0K^{*0}$	$(4.50 \pm 0.60) \times 10^{-5}$	$(2.18 \pm 0.84) \times 10^{-6}$

Prospects

- There are several established methods / channels that LHCb haven't exploited yet
  - $B^{\pm} \to D^{*0} K^{\pm}$ - work is underway on both GGSZ and GLW/ADS methods
  - $B^{\pm} \to D^{0} K^{*\pm}$ - first GLW/ADS results shown at CKM (other methods expect inclusion for Moriond or Summer)
  - Any *CP*-odd GLW decays ($D^0 \to K^0_S \pi^0$, $D^0 \to K^0_S \omega$) - neutrals are **very hard** for LHCb
- There are also ideas for new methods / channels
  - ADS-Dalitz and GGSZ-Dalitz (double Dalitz)
  - Simultaneous GGSZ analysis (reduce systematic uncertainties)
  - TD analyses with $B^0_s \to D_s^{+\star +} K^-$
  - Use decays from B^0, B^0_s, B^+_c (low branching fractions and production rates but topology means larger values of r_B)
  - Use decays from Λ^0_b (difficult final state Λ)
- We struggle to keep up with our data as it is
  - The TD $B^0_s \to D^-_s K^+$ analysis was only just updated to full $3\,\mathrm{fb}^{-1}$ (still only a CONF and not yet a PAPER)
  - This wasn't included for the last LHCb combination but now we already have at least an equivalent size dataset for 2015+2016 and will soon have a much bigger dataset with 2017
- Belle II will start taking data soon