Axial currents in nuclei from lattice QCD

William Detmold, MIT

IPPP/nuSTEC workshop, Durham University, Apr 19th 2017

Neutrino-nucleus interactions

- Intensity frontier: precise experiments
  - Sensitivity to probe the rarest interactions of the SM
  - Look for effects where there is no SM contribution
- Important focus of HEP(NP) experimental program
  - Dark matter direct detection
  - **Accelerator neutrino experiments**
  - Charged lepton flavour violation, EDMs, ββ-decay…
- ***Major component is nuclear targets***

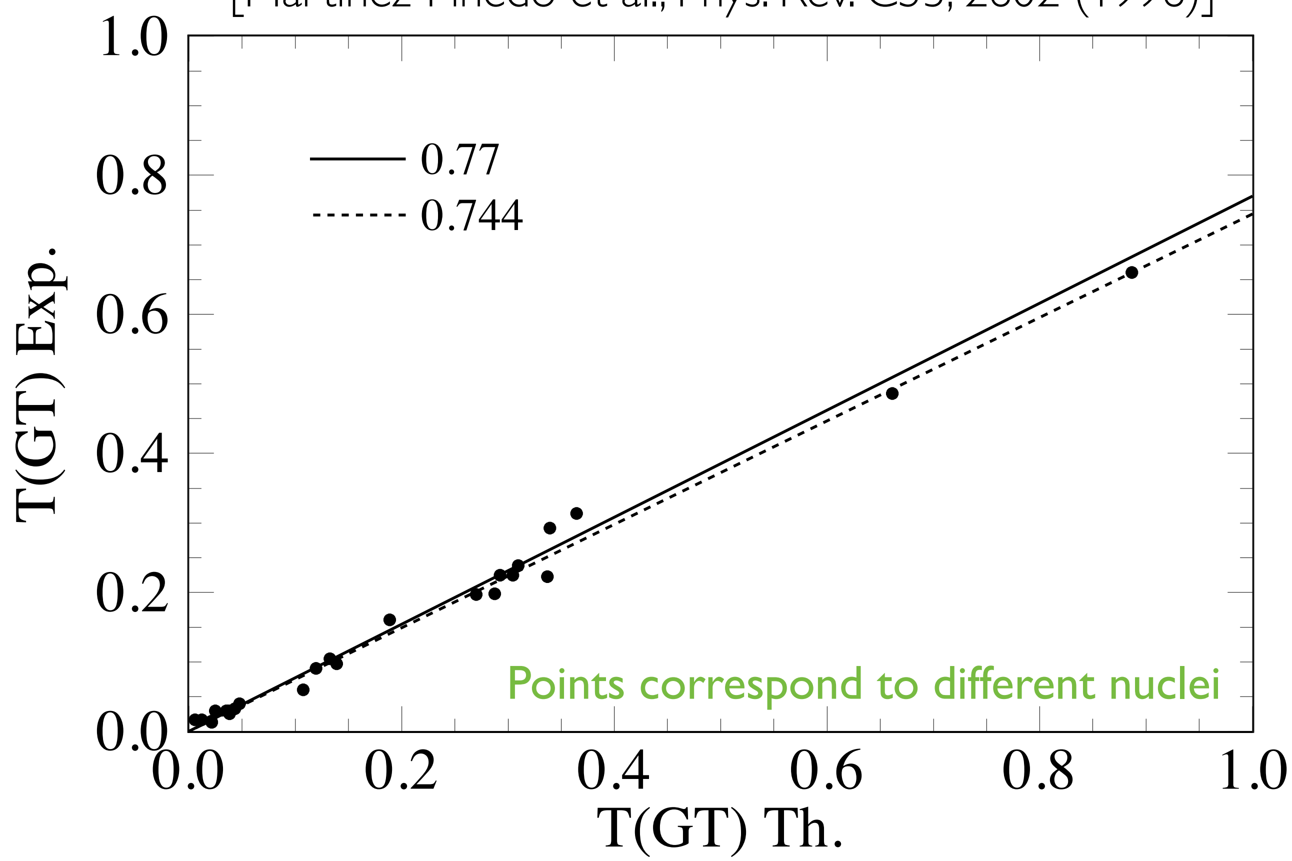
Nuclear uncertainties

- How well do we know nuclear matrix elements?

😢 Stark example of problems: Gamow-Teller transitions in nuclei

- Well measured for large range of nuclei (30<A<60)
- Many nuclear structure calcs (QRPA, shell-model,…) – spectrum well described
- Matrix elements systematically off by 20–30%
- "Correct" by "quenching" axial charge in nuclei ...

[Martinez-Pinedo et al., Phys. Rev. C53, 2602 (1996)]



$$T(GT) \sim \sqrt{\sum_f \langle \boldsymbol{\sigma} \cdot \boldsymbol{\tau} \rangle_{i \to f}}$$

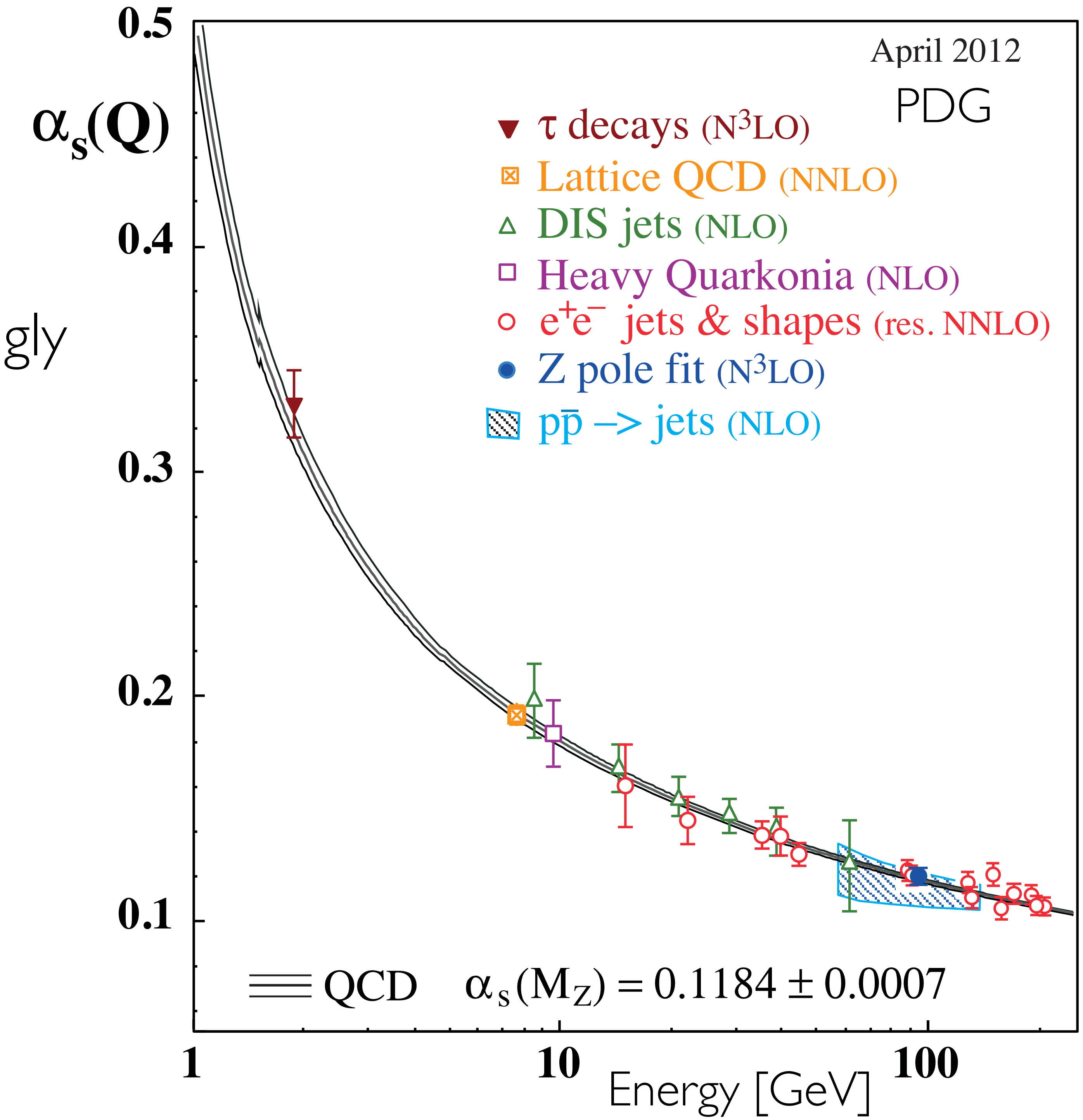
$$\langle \boldsymbol{\sigma\tau} \rangle = \frac{\langle f || \sum_k \boldsymbol{\sigma}^k \boldsymbol{t}_{\pm}^k || i \rangle}{\sqrt{2J_i + 1}}$$

Nuclear theory at the intensity frontier

- Coming need for precision determinations of nuclear matrix elements
  - Must be based on the Standard Model (no hand-waving)
  - Must have fully quantified uncertainties
  - Timeframe and precision goals set by experiment
- Current state is far from this
- *Nuclear physics will become the new flavour physics!*
  - Develop appropriate tools: potential path forward is to use lattice QCD + effective field theory (see Evgeny Epelbaum's talk)

Lattice QCD

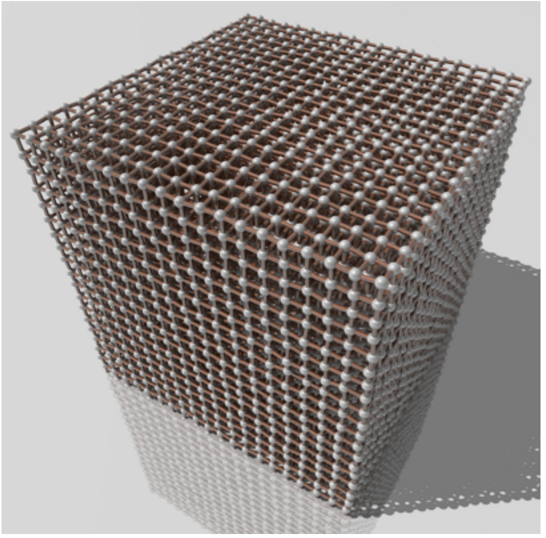
- QCD is the "strong force": quarks and gluons interact strongly
- Interaction strength depends on energy
- At high energy, can use perturbative theory
- At low energies: need non-perturbative approach

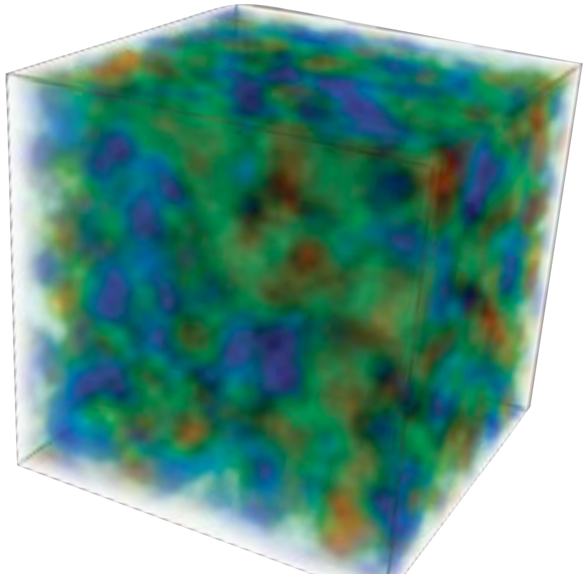


- Lattice QCD: tool to deal with quarks and gluons
  - Correlation functions as functional integral over quark and gluon d.o.f. on R_4

    $$\langle \mathcal{O} \rangle = \int dA_\mu dq d\bar{q}\, \mathcal{O}[q, \bar{q}, A] e^{-S_{QCD}[q, \bar{q}, A]}$$

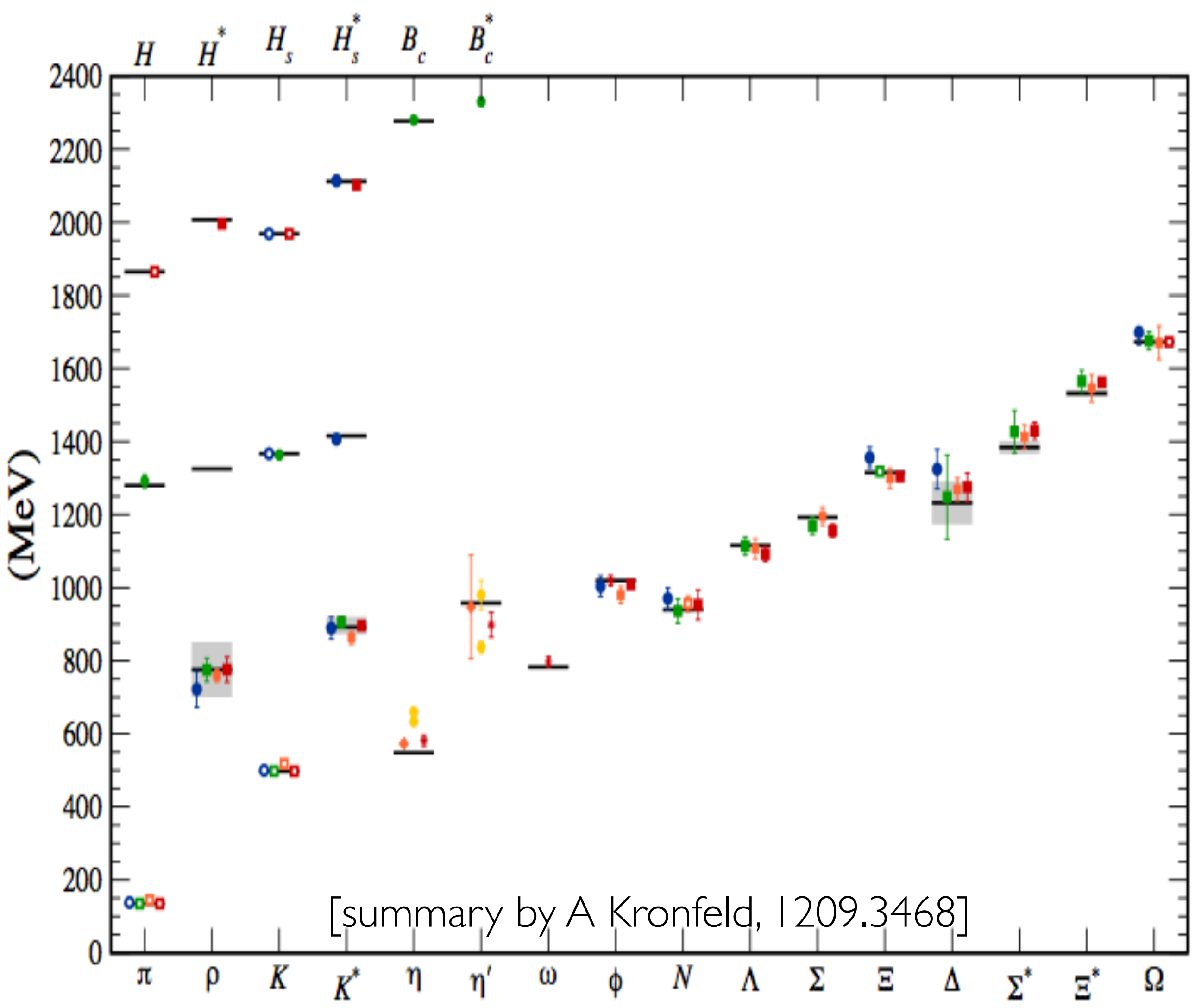
    perform quark integrals exactly
  - Discretise and compactify system
    - Finite but large number of d.o.f (10^{10})
  - Numerically integrate via importance sampling (average over important configurations)
  - Undo the harm done in previous steps
    - Lattice QCD ⇒ QCD



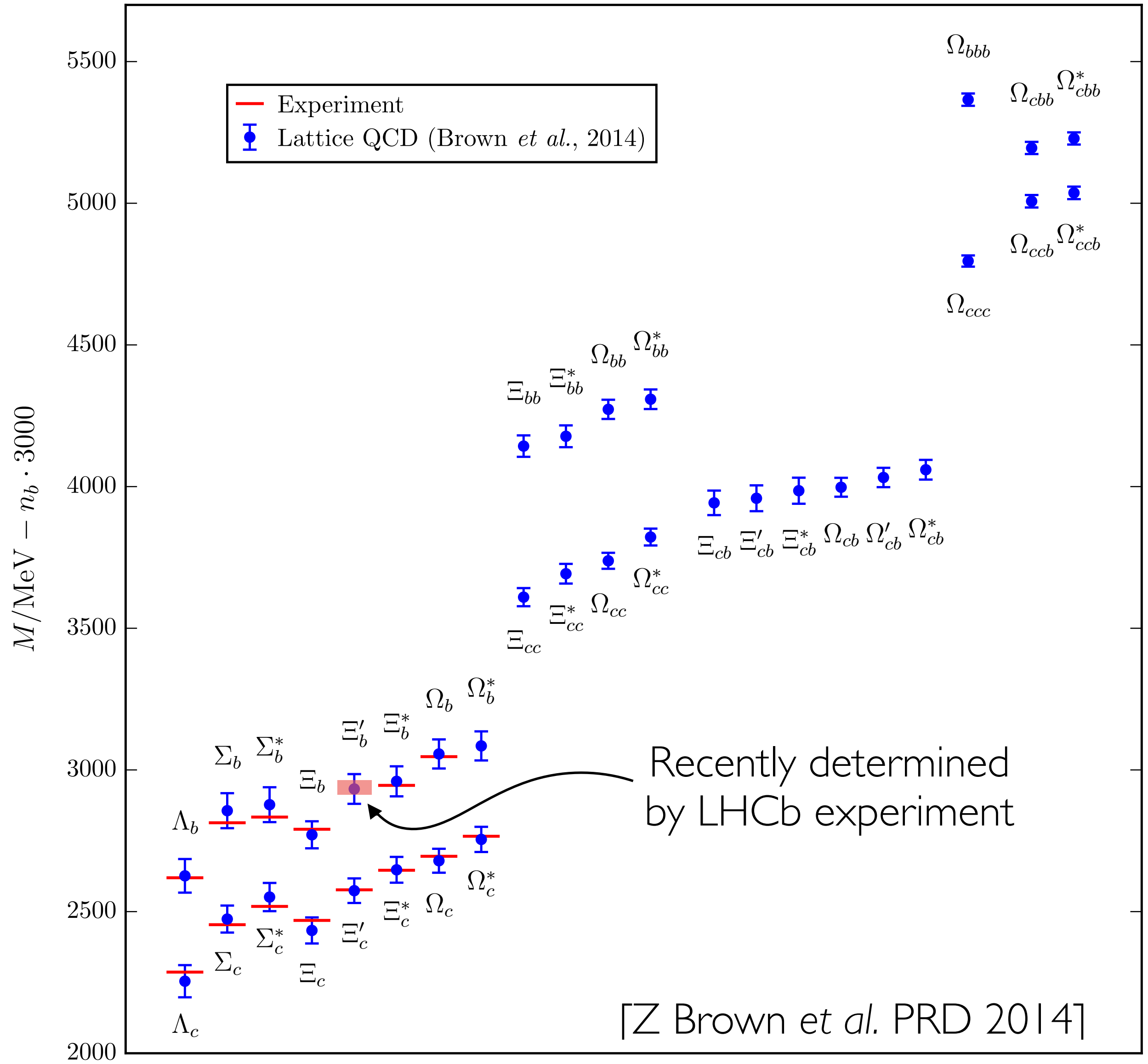


QCD Spectrum

- After 30 years of development
- Ground state hadron spectrum reproduced

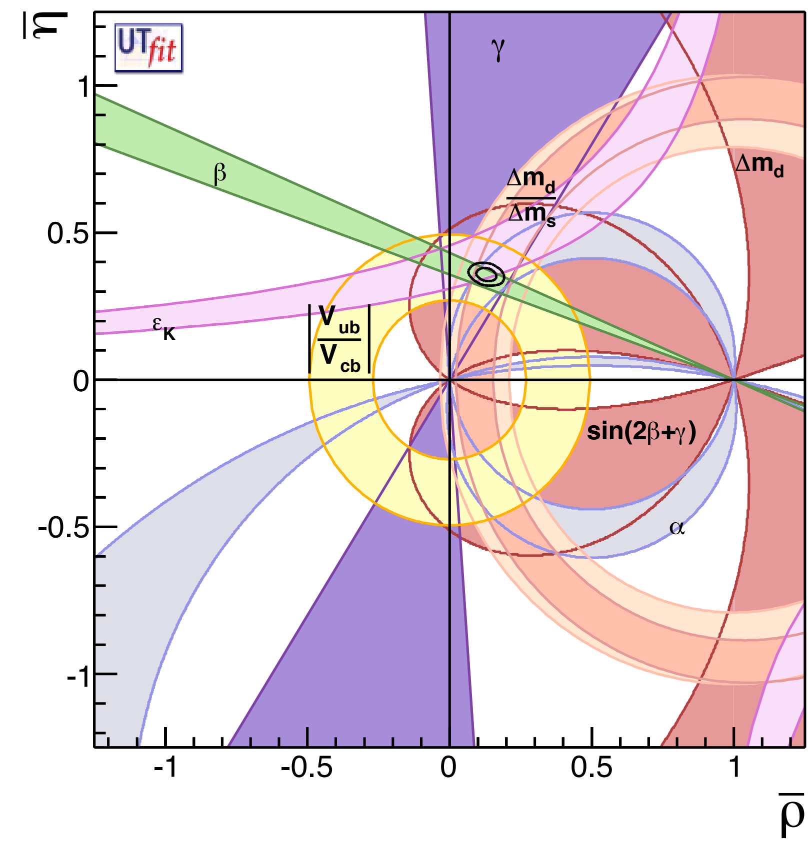


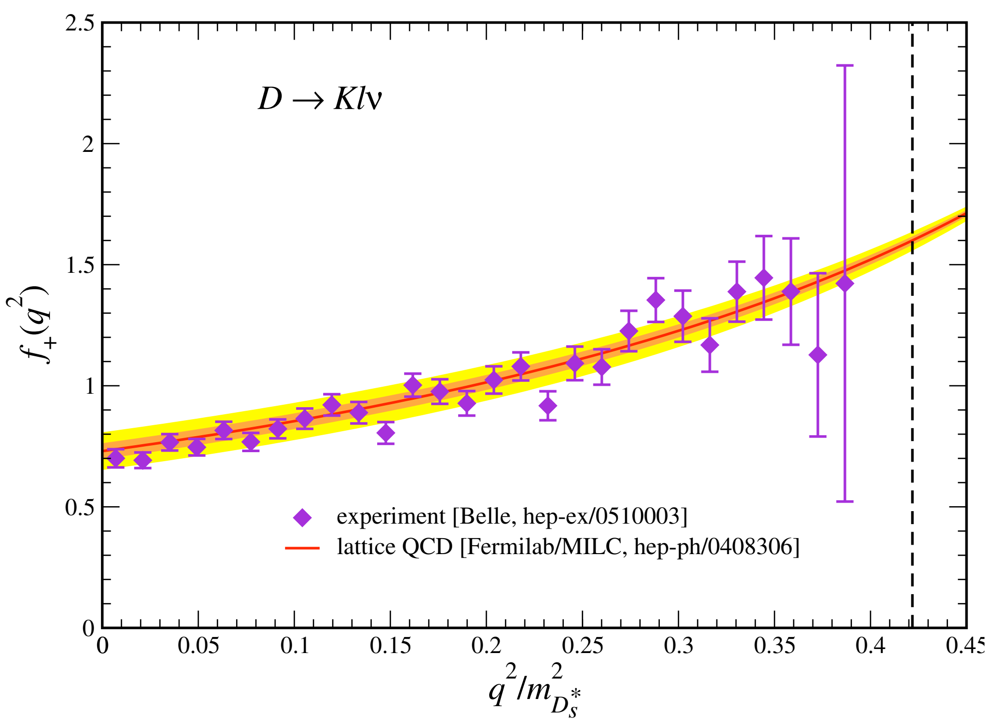
- Predictions for new states with controlled uncertainties



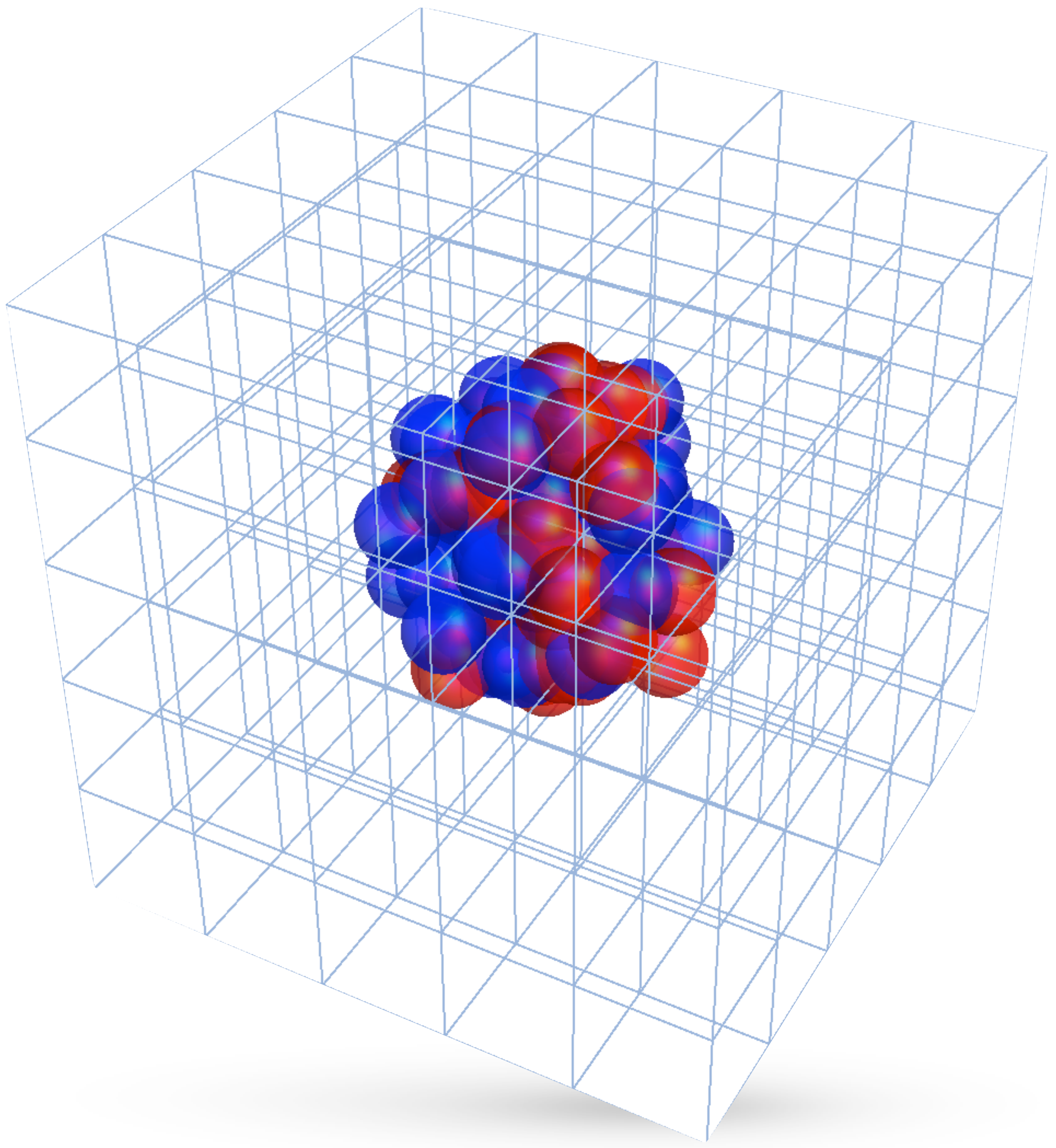
21st century LQCD

- For simple observables LQCD is precision science
  - Combine with experiment to determine SM parameters
    - Verify and test CKM paradigm
  - SM predictions with reliable uncertainty quantification
    - Kaon decays
    - Muon (g-2)

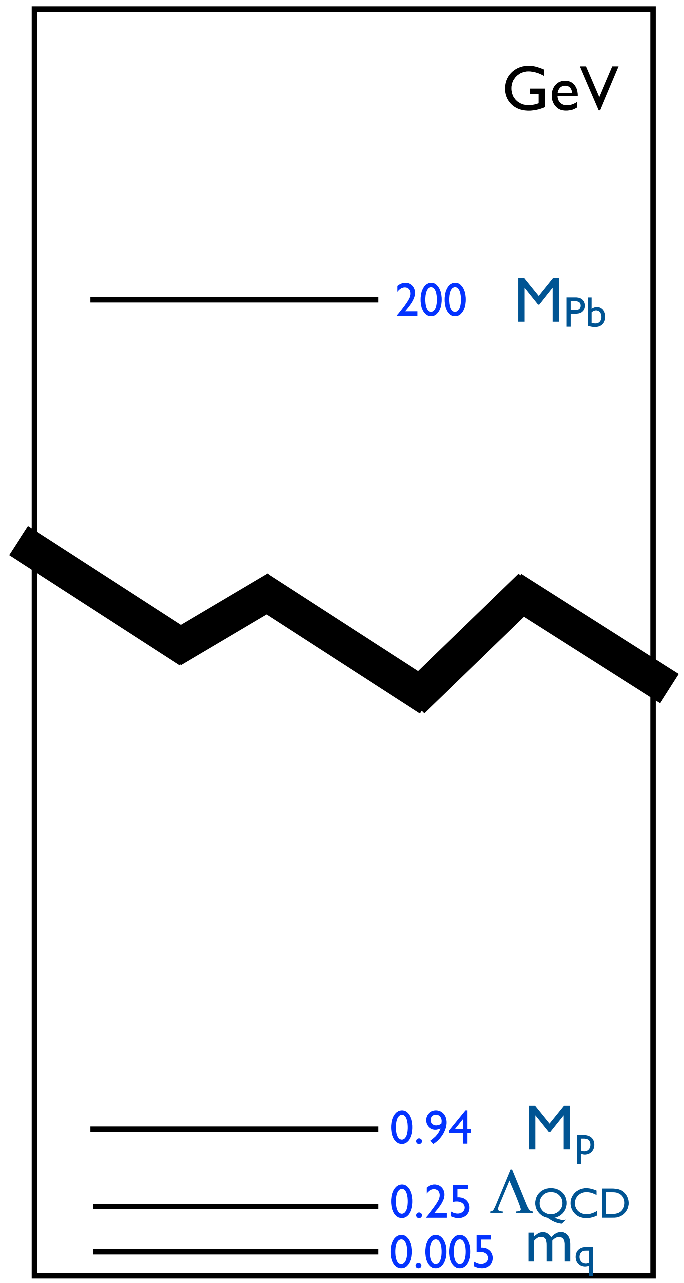




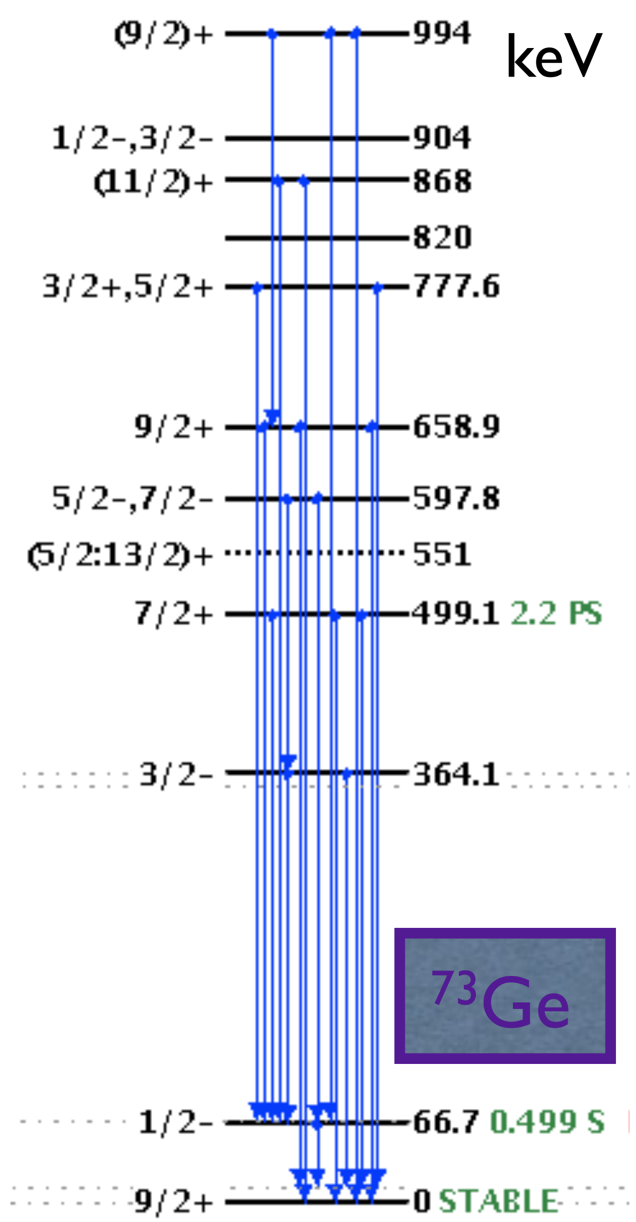
- Nuclei in LQCD are a hard



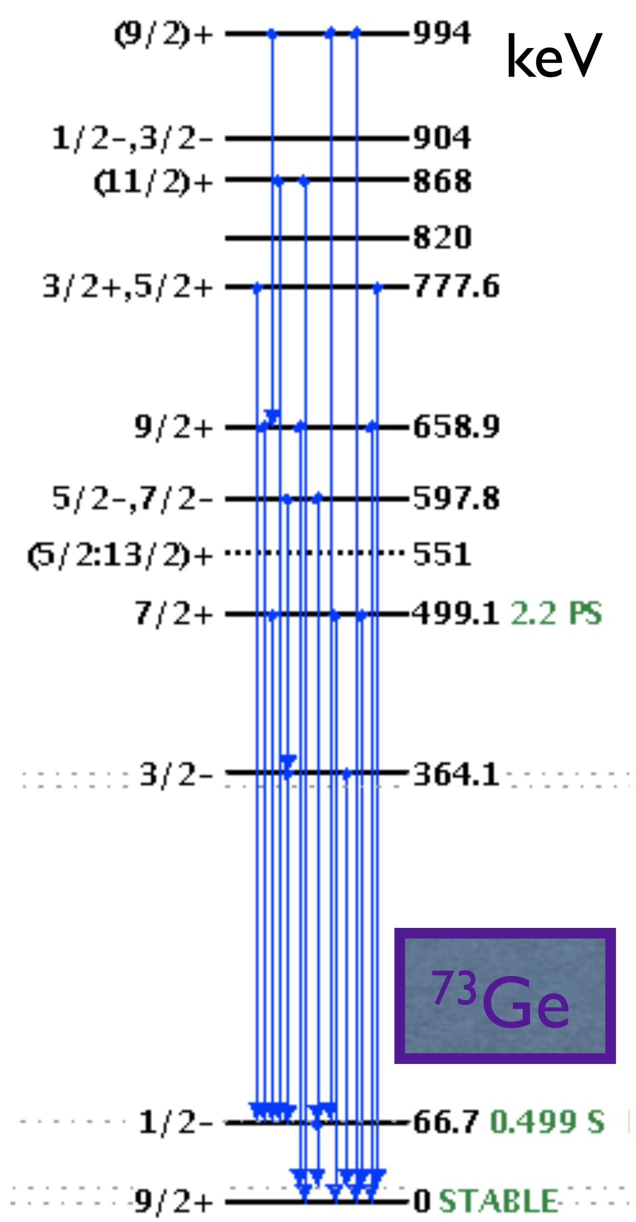
- Nuclei in LQCD are a hard
- Physics at multiple scales



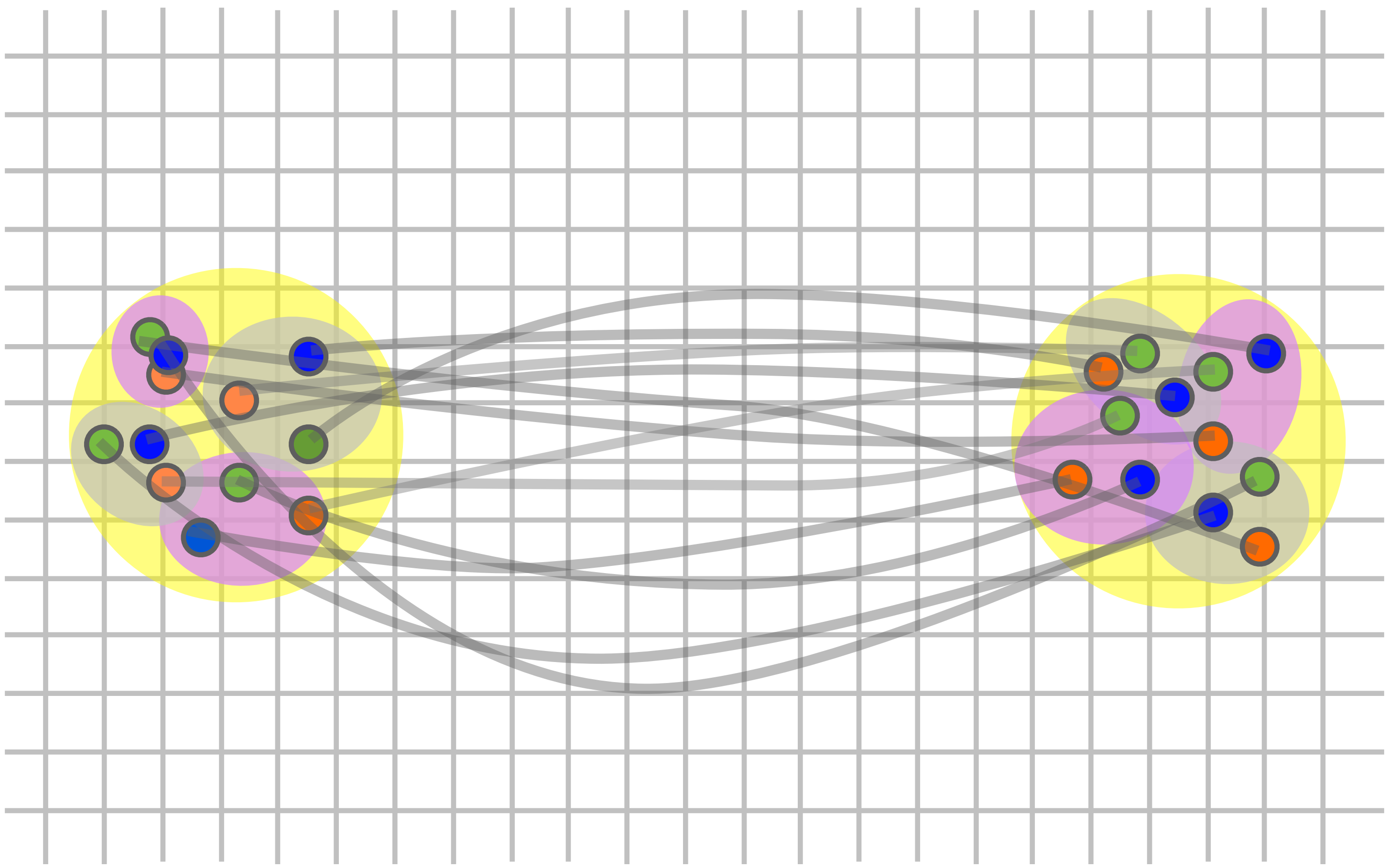
- Nuclei in LQCD are a hard
- Physics at multiple scales



- Nuclei in LQCD are a hard
- Physics at multiple scales
- Two exponentially difficult challenges for LQCD
  - Contraction complexity grows factorially
  - Probabilistic method statistical uncertainty grows exponentially with A (naively)



- Quarks need to be tied together in all possible ways
  - $N_{\text{contractions}} = N_u! N_d! N_s!$ $(\sim 10^{1500}$ for $^{208}Pb)$



- Managed using algorithmic trickery
[WD & Savage, WD & Orginos; Doi & Endres, Günther et al]
  - Study up to N=72 pion systems, A=5 (and 28) nuclei

Case Study

Unphysical nuclei

Unphysical nuclei

- Case study QCD with unphysical quark masses
  - $m_\pi \sim 800$ MeV, $m_N \sim 1{,}600$ MeV
  - $m_\pi \sim 450$ MeV, $m_N \sim 1{,}200$ MeV

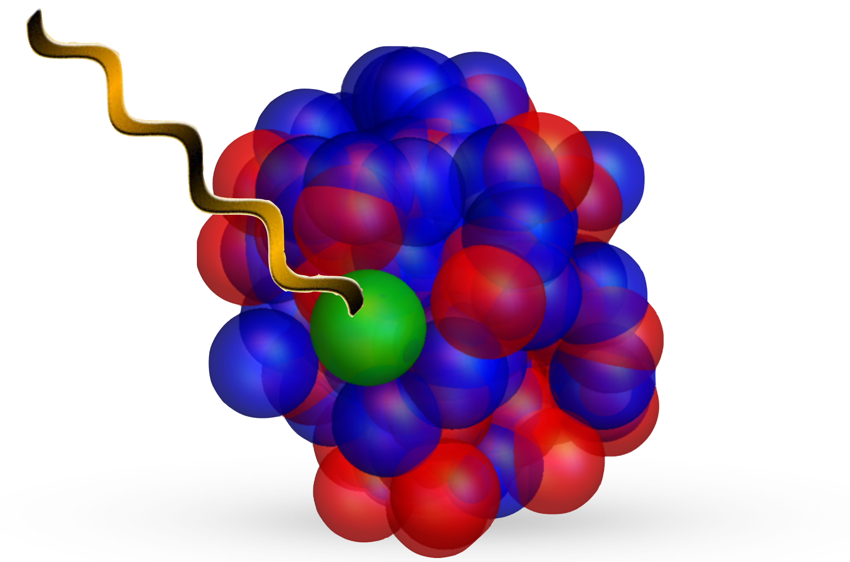
1. Spectrum of light nuclei (A<5)
   [PRD **87** (2013), 034506]
2. Nuclear structure: magnetic moments, polarisabilities (A<5)
   [PRL **113**, 252001 (2014),PRD 92, 114502 (2015)]
3. Nuclear reactions: $np \to d\gamma$
   [PRL **115**, 132001 (2015)]
4. Gamow-Teller transitions: $pp \to de\nu$, $g_A(^3H)$ [arXiv:1610.04545]
5. Double β decay: $pp \to nn$
   [1701.03456,1702.02929]

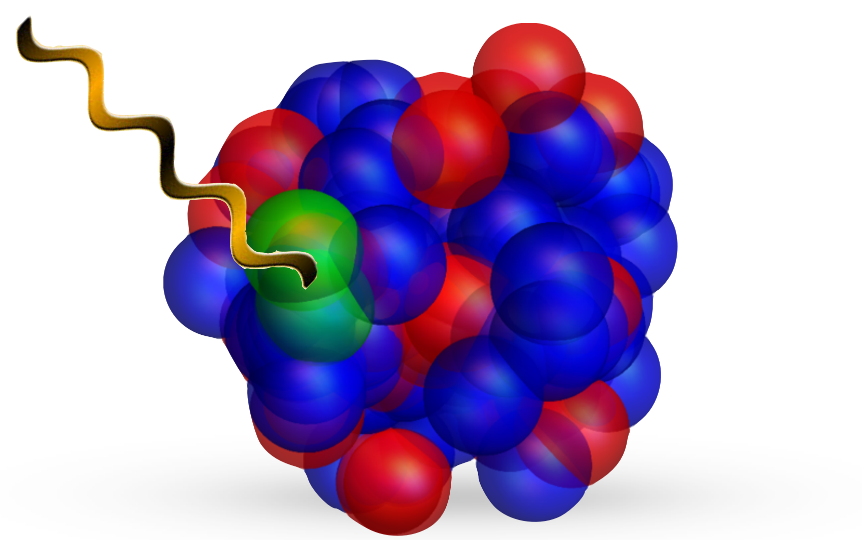
- Case study QCD with unphysical quark masses
  - m_π~800 MeV, m_N~1,600 MeV
  - m_π~450 MeV, m_N~1,200 MeV

1. Spectrum of light nuclei (A<5)
   [PRD **87** (2013), 034506]
2. Nuclear structure: magnetic moments, polarisabilities (A<5)
   [PRL **113**, 252001 (2014),PRD 92, 114502 (2015)]
3. Nuclear reactions: np→dγ
   [PRL **115**, 132001 (2015)]
4. Gamow-Teller transitions: pp→deν, $g_A(^3H)$ [arXiv:1610.04545]
5. Double β decay: pp→nn
   [1701.03456,1702.02929]

Larger nuclei

- What about larger (phenomenologically-relevant) nuclei?
- Nuclear effective field theory:
  - 1-body currents are dominant
  - 2-body currents are sub-leading *but non-negligible*
- Determine one body contributions from single nucleon
- Determine few-body contributions from A=2,3,4...
- Match EFT and many body methods to LQCD to make predictions for larger nuclei



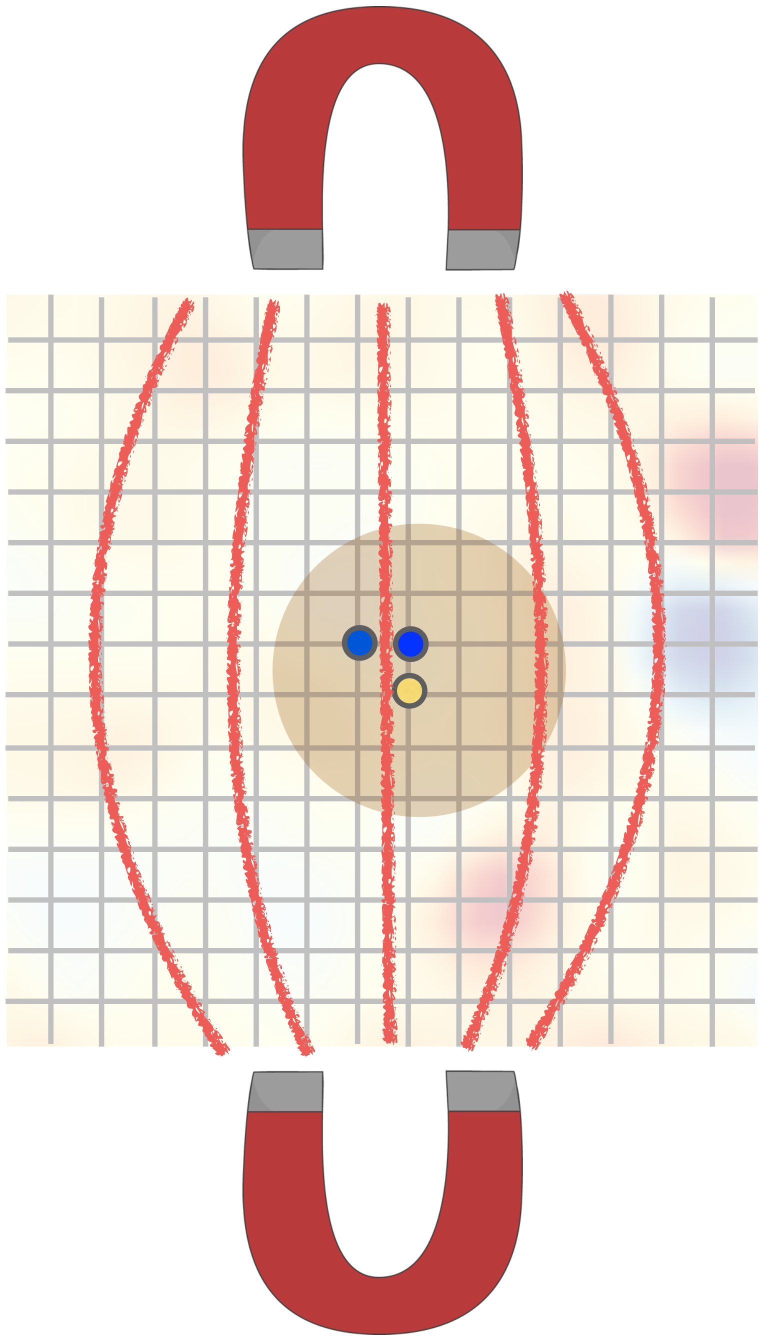


External field method

- Hadron/nuclear energies are modified by presence of fixed external fields
- Eg: fixed B field

$$E_{h;j_z}(\mathbf{B}) = \sqrt{M_h^2 + (2n+1)|Q_h eB|} - \boldsymbol{\mu}_h \cdot \mathbf{B} \\ - 2\pi\beta_h^{(M0)}|\mathbf{B}|^2 - 2\pi\beta_h^{(M2)}\langle \hat{T}_{ij} B_i B_j \rangle + \ldots$$

- QCD calculations with multiple fields enable extraction of coefficients of response
  - Magnetic moments, polarisabilities, …
- Not restricted to simple EM fields



[NPLQCD PRL **113**, 252001 (2014)]

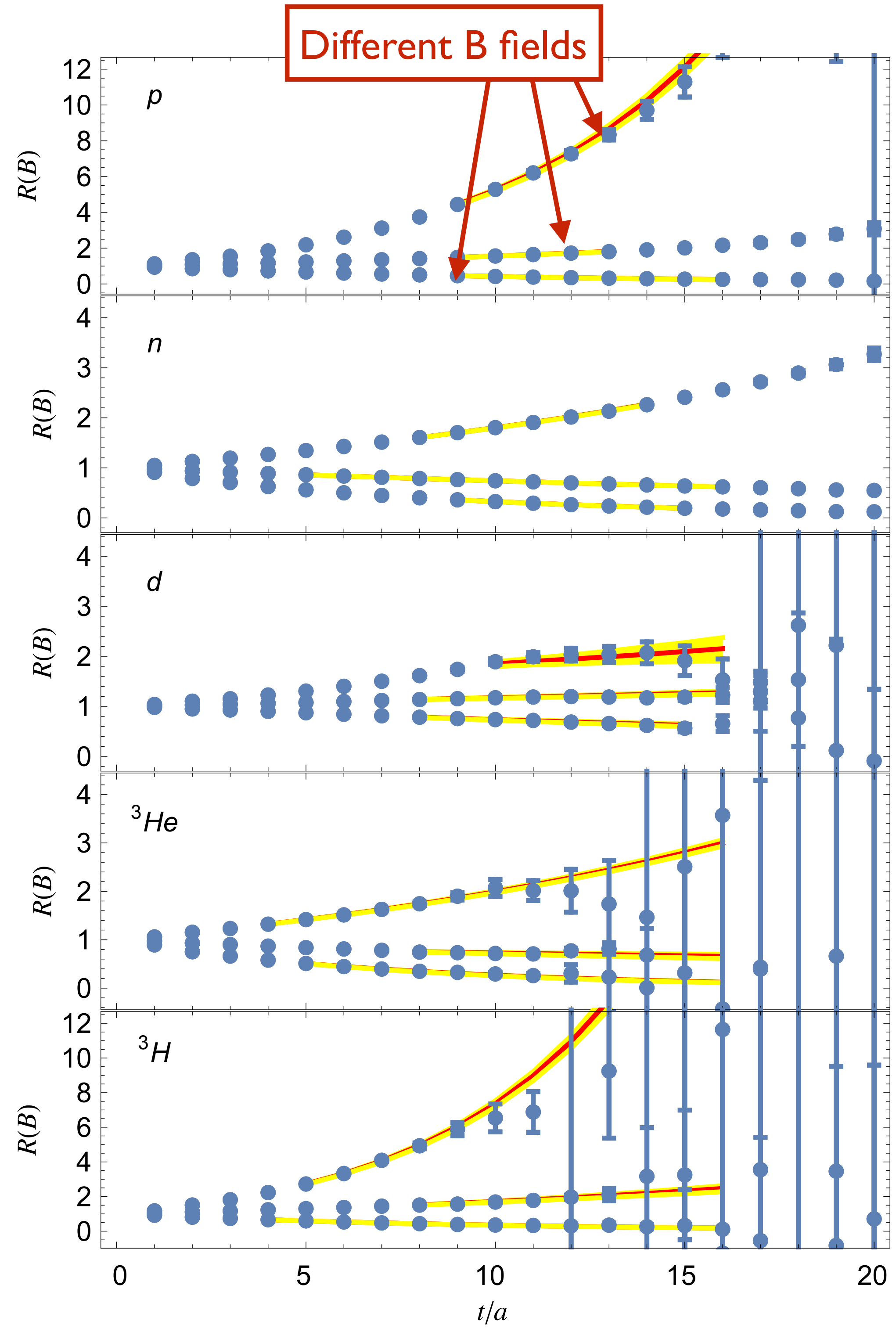
- Magnetic field in *z*-direction (strength quantised by lattice periodicity)
- Magnetic moments from spin splittings

$$\delta E^{(B)} \equiv E_{+j}^{(B)} - E_{-j}^{(B)} = -2\mu|\mathbf{B}| + \gamma|\mathbf{B}|^3 + \ldots$$

- Extract splittings from ratios of correlation functions

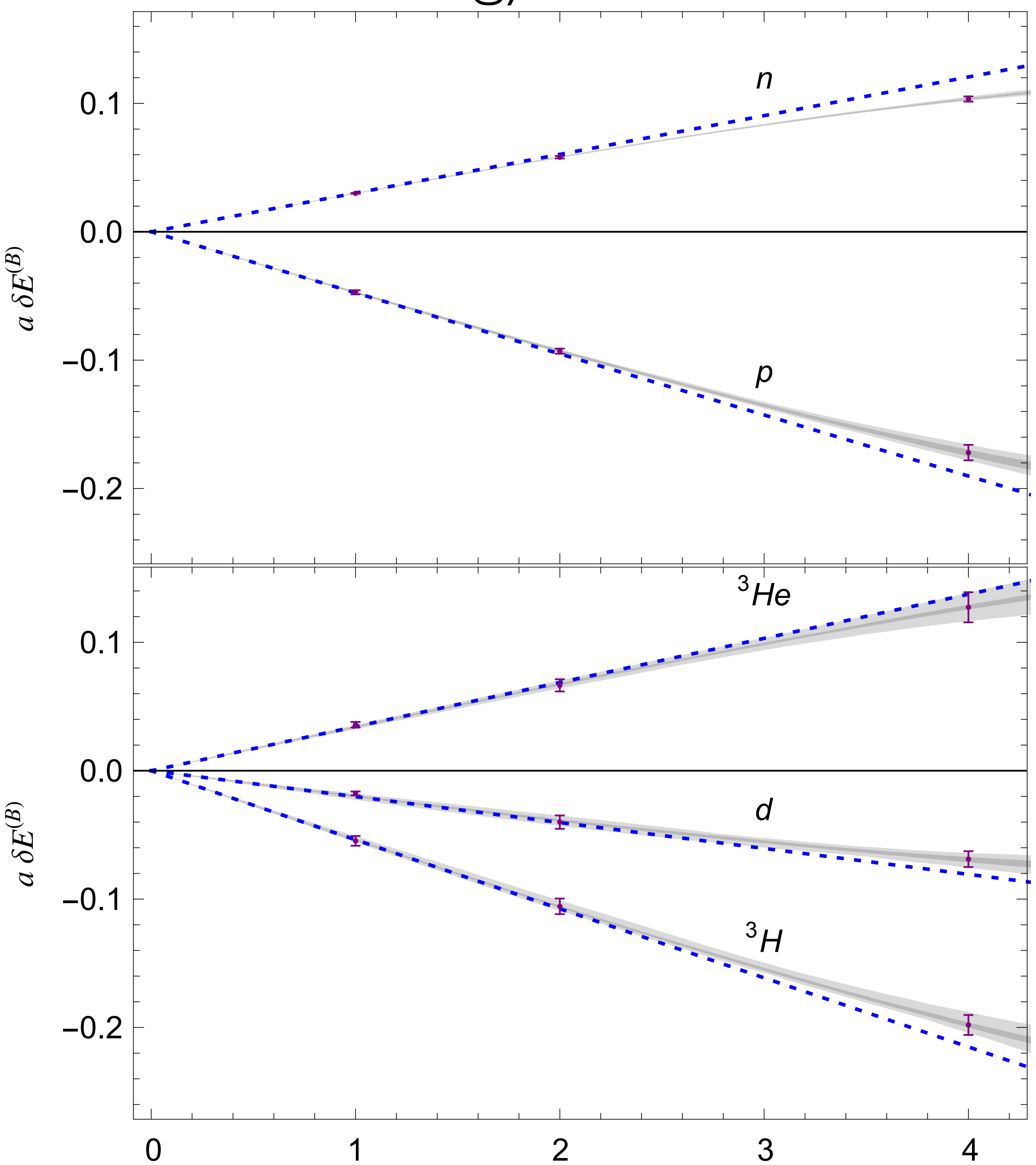
$$R(B) = \frac{C_j^{(B)}(t)\ C_{-j}^{(0)}(t)}{C_{-j}^{(B)}(t)\ C_j^{(0)}(t)} \overset{t\to\infty}{\longrightarrow} Ze^{-\delta E^{(B)}t}$$

- Careful to be in single exponential region of each correlator



[NPLQCD PRL **113**, 252001 (2014)]

Energy shift vs B

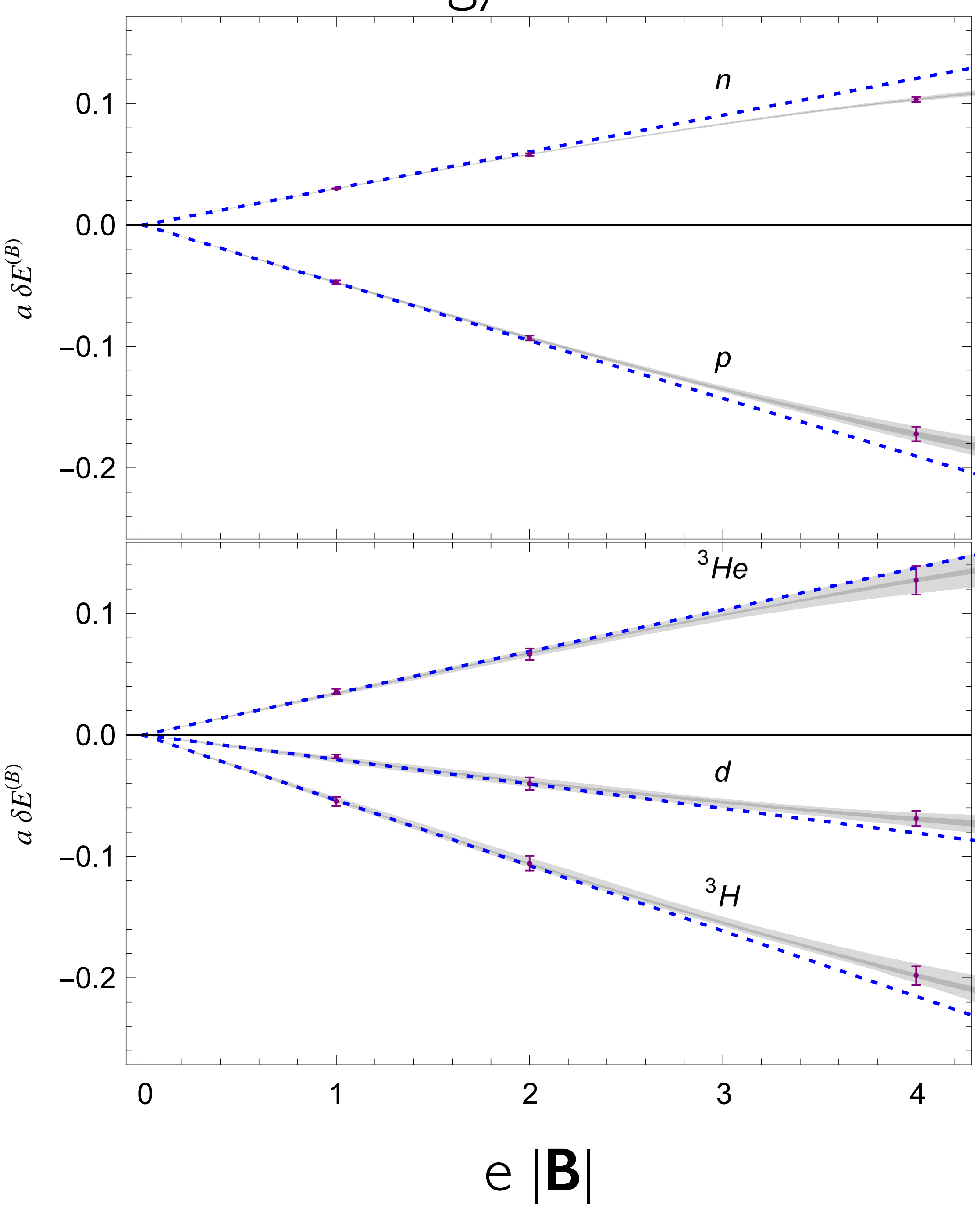


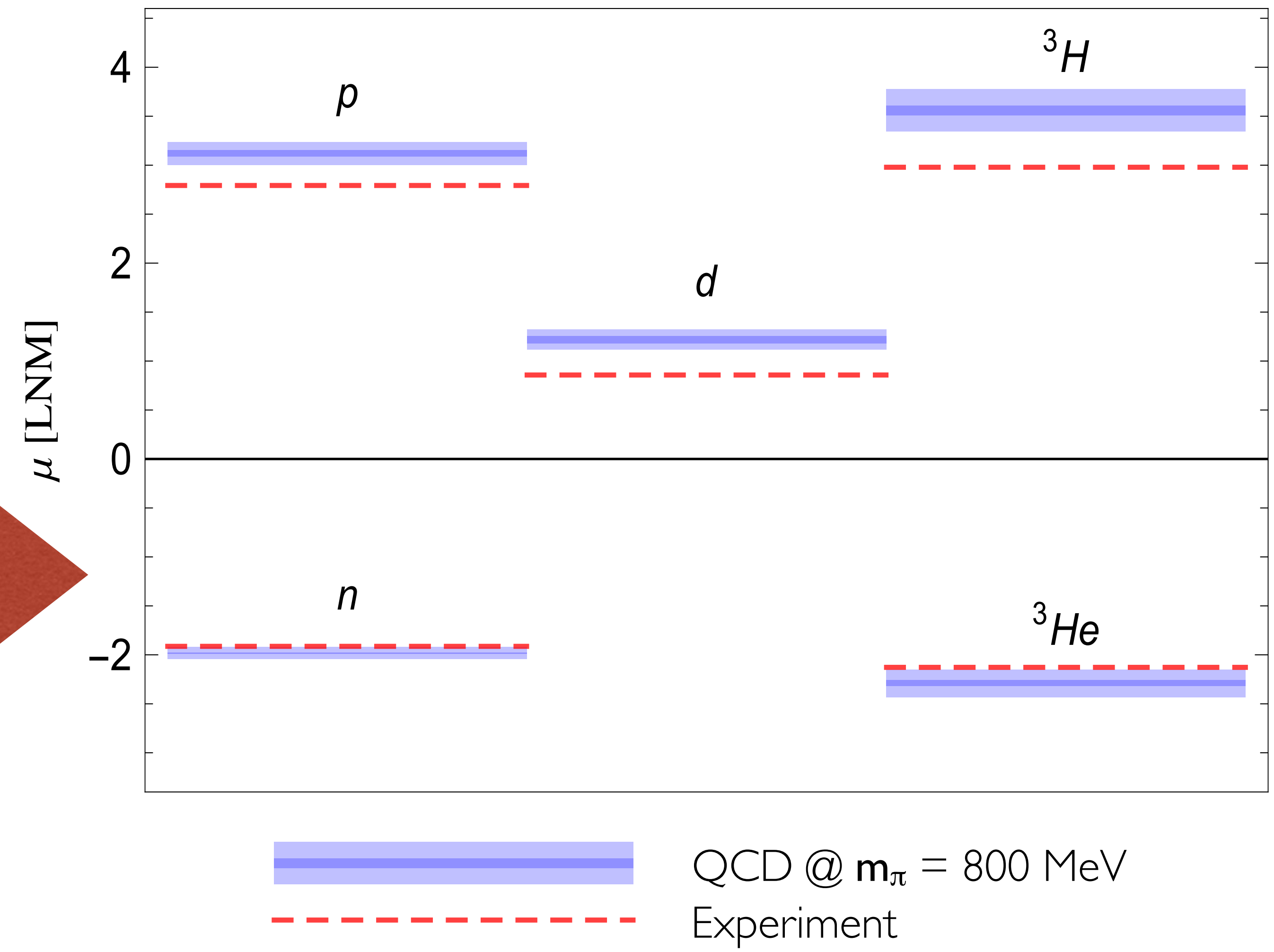
e |**B**|

[NPLQCD PRL **113**, 252001 (2014)]

Magnetic moments of nuclei

Energy shift vs B





	n	p	d	3	3
μ	-1.98(1)(2)	3.21(3)(6)	1.22(4)(9)	-2.29(3)(12)	3.56(5)(18)

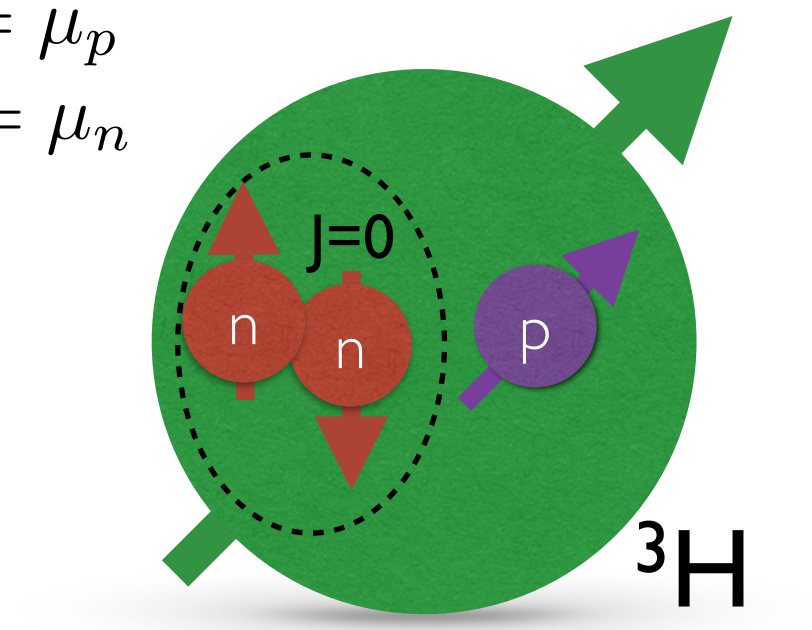
In units of appropriate nuclear magnetons (heavy M_N)

[NPLQCD PRL **113**, 252001 (2014)]

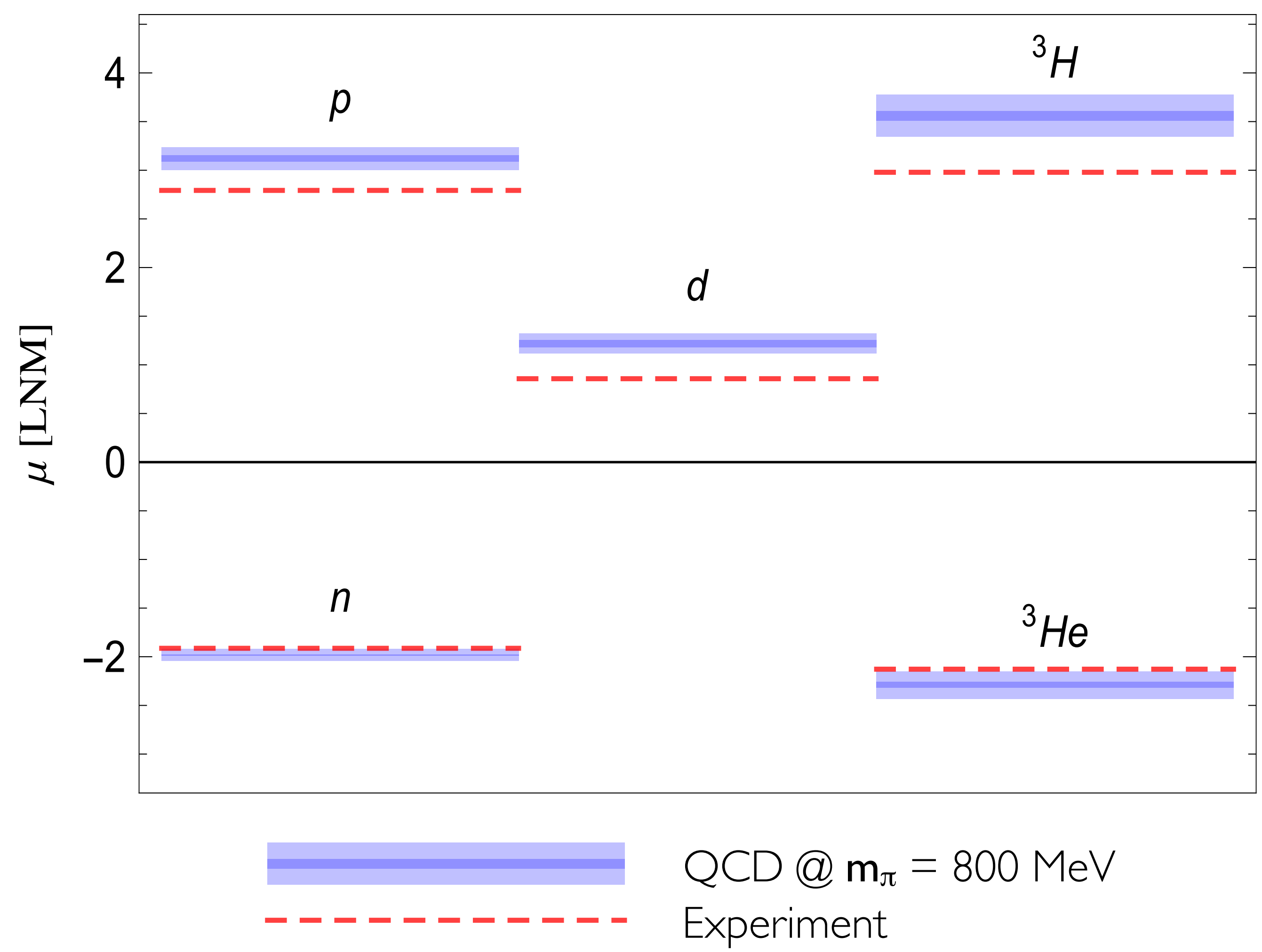
- Numerical values are surprisingly interesting

- Shell model expectations

$$\mu_d = \mu_p + \mu_n$$
$$\mu_{^3\mathrm{H}} = \mu_p$$
$$\mu_{^3\mathrm{He}} = \mu_n$$



- Lattice results appear to suggest heavy quark nuclei are shell-model like!



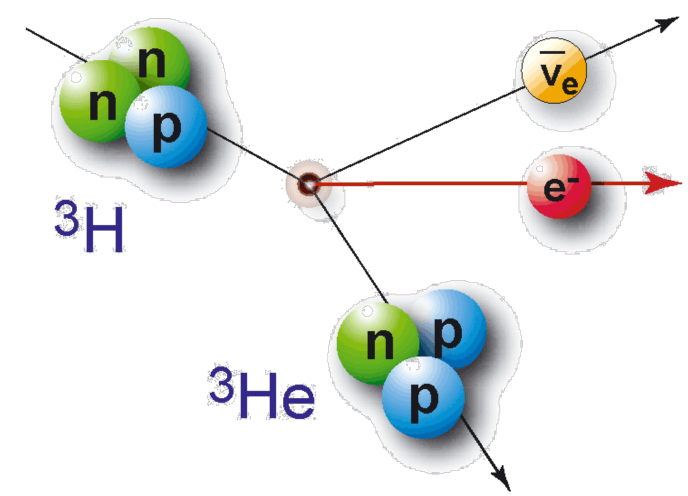
	n	p	d	3	3
μ	-1.98(1)(2)	3.21(3)(6)	1.22(4)(9)	-2.29(3)(12)	3.56(5)(18)

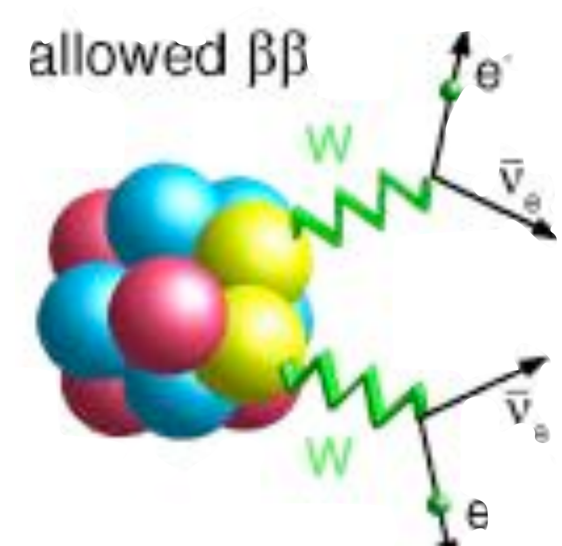
In units of appropriate nuclear magnetons (heavy M_N)

[NPLQCD PRL **113**, 252001 (2014)]

Gamow-Teller matrix elements

- Background axial field
- Axial coupling to NN system
  - $pp \rightarrow de^{+}\boldsymbol{\nu}$ fusion
  - Muon capture: MuSun @ PSI
  - $d\,\boldsymbol{\nu} \rightarrow n\,n\,e+$: SNO
- Tritium half-life
  - Understand multi-body contributions to $\langle \mathbf{GT} \rangle$: better predictions for decay rates of larger nuclei
- Second order: $\beta\beta$ decay





- Tritium decay half life

$$\boxed{\frac{(1+\delta_R) f_V}{K/G_V^2}}\boxed{t_{1/2}} = \frac{1}{\boxed{\langle \mathbf{F} \rangle^2} + f_A/f_V\, g_A^2 \boxed{\langle \mathbf{GT} \rangle^2}}$$

half-life

vector ME

axial ME

known from theory or expt.

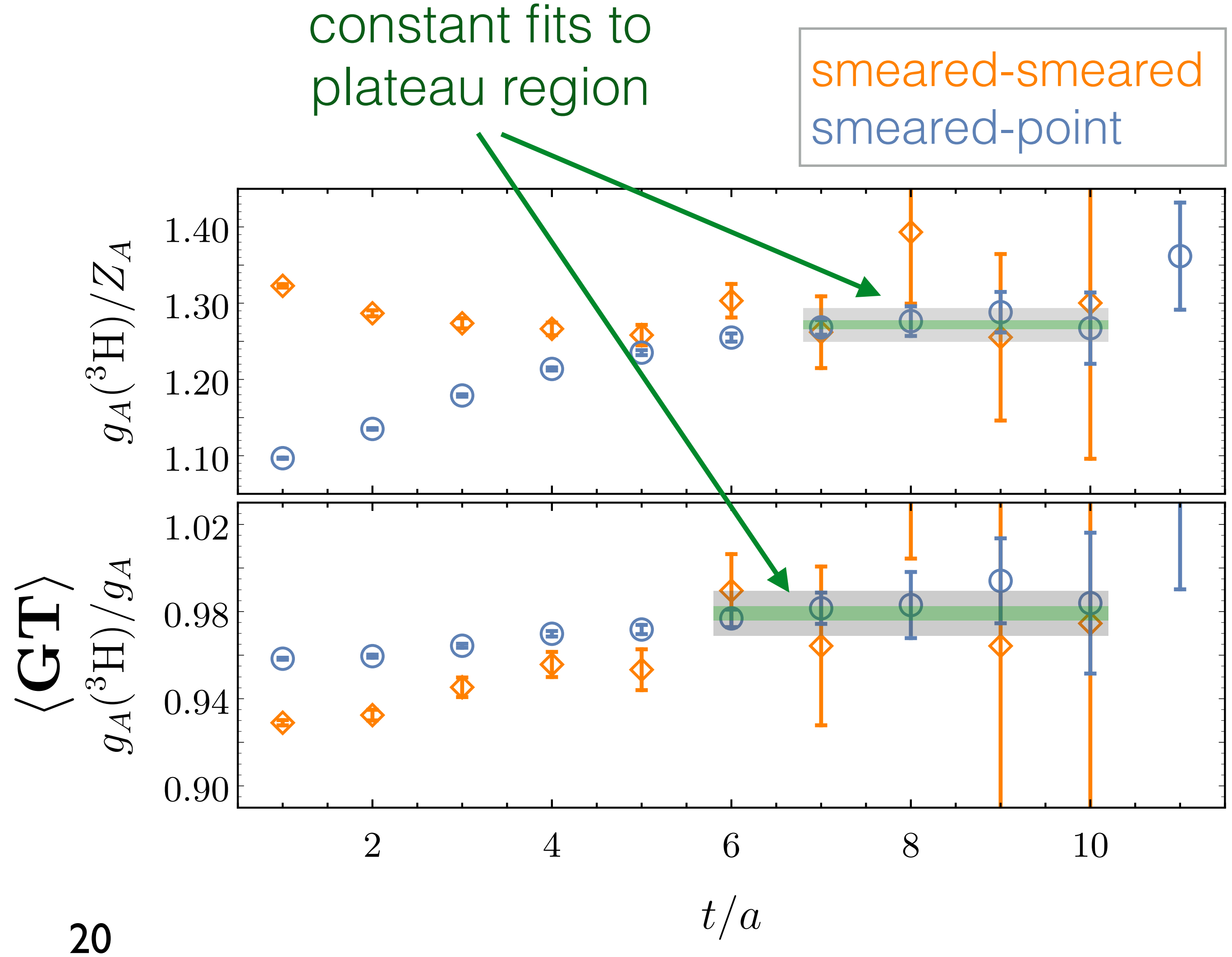
- Biggest uncertainty in

$$g_A \langle \mathbf{GT} \rangle = \langle {}^{\mathbf{3}}\mathrm{He} | \overline{\mathbf{q}} \gamma_{\mathbf{k}} \gamma_{\mathbf{5}} \tau^{-} \mathbf{q} | {}^{\mathbf{3}}\mathrm{H} \rangle$$

- Form ratios of correlators to cancel leading time-dependence:

$$\frac{\overline{R}_{^3\mathrm{H}}(t)}{\overline{R}_p(t)} \xrightarrow{t\to\infty} \frac{g_A(^3\mathrm{H})}{g_A} = \langle \mathbf{GT} \rangle$$

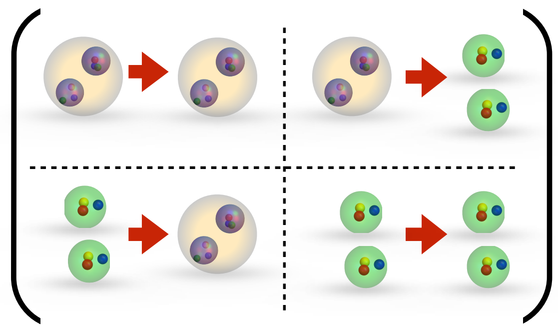
constant fits to plateau region



pp fusion

- Axial background field mixes $^3S_1$, $^1S_0$ states

H =



- Extract matrix element through linear response of $^3S_1 \rightarrow {}^1S_0$ correlators to the background field

matrix elt. is linear in λ_u

$$\boxed{C^{(^3S_1,\,^1S_0)}_{\lambda_u;\lambda_d=0}(t)} = \boxed{\lambda_u \sum_{\tau=0}^{t} \sum_{\mathbf{x}} \langle 0|\chi^3_{^3S_1}(\mathbf{x},t)\, A^u_3(\tau)\, \chi^\dagger_{^1S_0}(0)|0\rangle} \; \boxed{+\, c_2\lambda_u^2 + c_3\lambda_u^3},$$

correlator formed with background field coupling to u quark

irrelevant consts.

- Calculate correlators at multiple values of λ_u, λ_d
  ➡ extract matrix element pieces

- Form ratios of compound correlators to cancel leading time-dependence

transition pieces linear in λ_u-λ_d

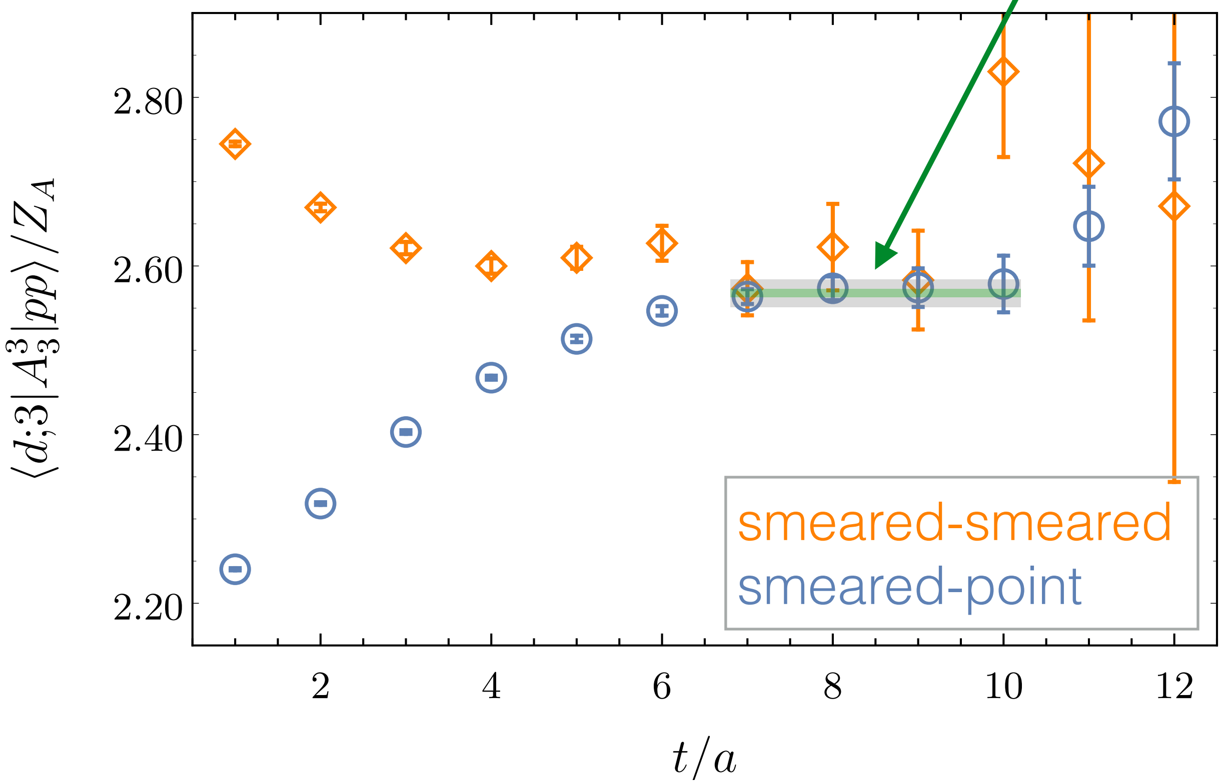
$$R_{^3S_1,^1S_0}(t) = \frac{\boxed{C^{(^3S_1,^1S_0)}_{\lambda_u,\lambda_d=0}(t)\Big|_{\mathcal{O}(\lambda_u)} - C^{(^3S_1,^1S_0)}_{\lambda_u=0,\lambda_d}(t)\Big|_{\mathcal{O}(\lambda_d)}}}{\boxed{\sqrt{C^{(^3S_1,^3S_1)}_{\lambda_u=0,\lambda_d=0}(t)C^{(^1S_0,^1S_0)}_{\lambda_u=0,\lambda_d=0}(t)}}}$$

diagonal pieces with no field

- Fit a constant to the 'effective matrix element plot' at late times

$$R_{^3S_1,^1S_0}(t+1) - R_{^3S_1,^1S_0}(t) \xrightarrow{t\to\infty} \frac{\langle ^3S_1; J_z=0|A_3^3|^1S_0; I_z=0\rangle}{Z_A}$$

constant fit to plateau region



pp fusion

- Low-energy cross section for $pp \to de^+\nu$ dictated by the matrix element

$$\left|\langle d; j \,|A_k^-|\, pp\rangle\right| \equiv g_A C_\eta \sqrt{\frac{32\pi}{\gamma^3}}\,\Lambda(p)\,\delta_{jk}$$

C_η	Sommerfield factor
γ	Deuteron binding mtm
$r_1,\ \rho$	Effective ranges
a_{pp}	pp scattering length
$\Gamma(0,\chi)$	Incomplete gamma func.
	$\chi = \alpha M_p/\gamma$

- Relate $\Lambda(0)$ to extrapolated LEC using EFT

$$\Lambda(0) = \frac{1}{\sqrt{1-\gamma\rho}}\{e^\chi - \gamma a_{pp}[1-\chi e^\chi \Gamma(0,\chi)] + \frac{1}{2}\gamma^2 a_{pp}\sqrt{r_1\rho}\} - \frac{1}{2g_A}\gamma a_{pp}\sqrt{1-\gamma\rho}\,\boxed{L_{1,A}^{sd-2b}}$$

extrapolated lattice value

- Determine $L_{1,A}$ (two body contribution - N^{2}LO $\not\pi$EFT in dibaryon approach) and compute other observables using it
  - npdγ suggests weak mass dependence of two-body counterterms so extrapolate to physical point

Butler and Chen, Phys. Lett. B520, 87 (2001)
Detmold and Savage, Nucl. Phys. A743, 170 (2004).

- Fusion cross section dictated by

$$\Lambda(0) = 2.6585(6)(72)(25)$$

$$\Lambda(0) = 2.652(2)$$ (models/EFT)

E. G. Adelberger et al., Rev. Mod. Phys. 83, 195 (2011)

- Relevant counter-term in EFT

$$L_{1,A} = 3.9(0.1)(1.0)(0.3)(0.9)\ \mathrm{fm}^3$$

$$L_{1,A} = 3.6(5.5)\ \mathrm{fm}^3$$ (reactor expts.)

M. Butler, J.-W. Chen, and P. Vogel, Phys. Lett. B549

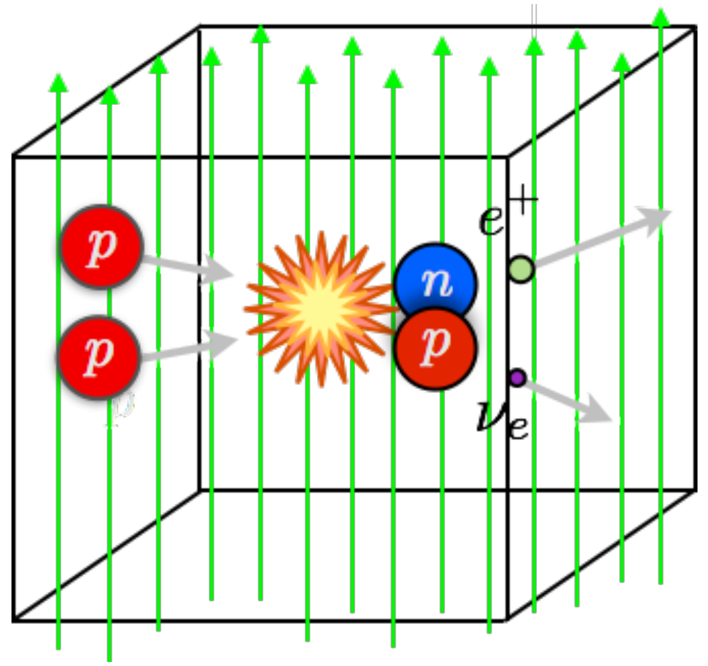


Fig: Z Davoudi

NPLQCD arXiv:1701.03456, 1702.XXXXX

- Background axial field to second order
  - nn→pp transition matrix element

$$M_{GT}^{2\nu} = 6 \int d^4x d^4y \langle pp | T \left[J_3^+(x) J_3^+(y) \right] | nn \rangle$$

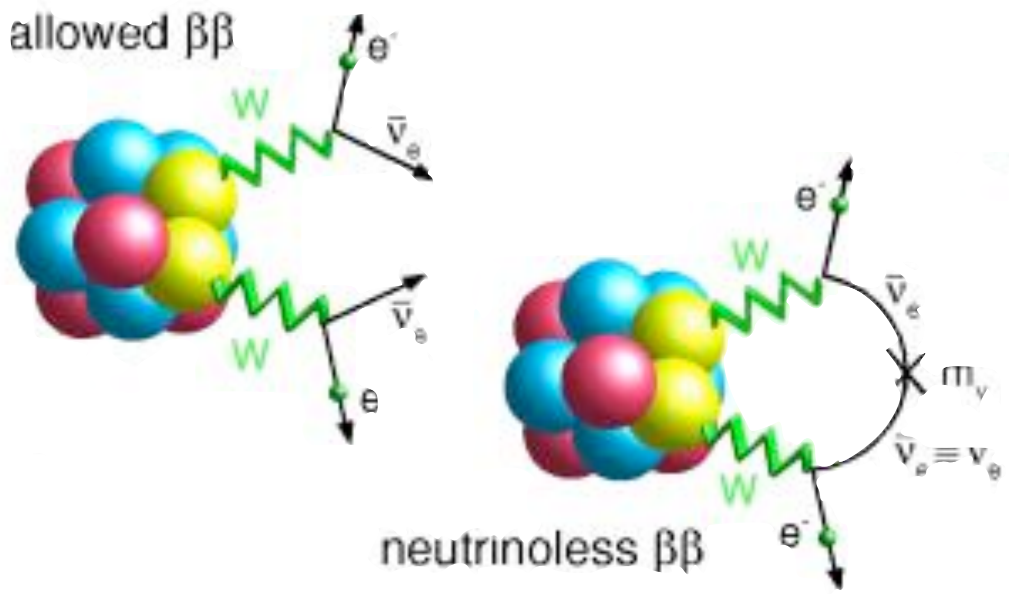
introduces a host of technical LQCD complications

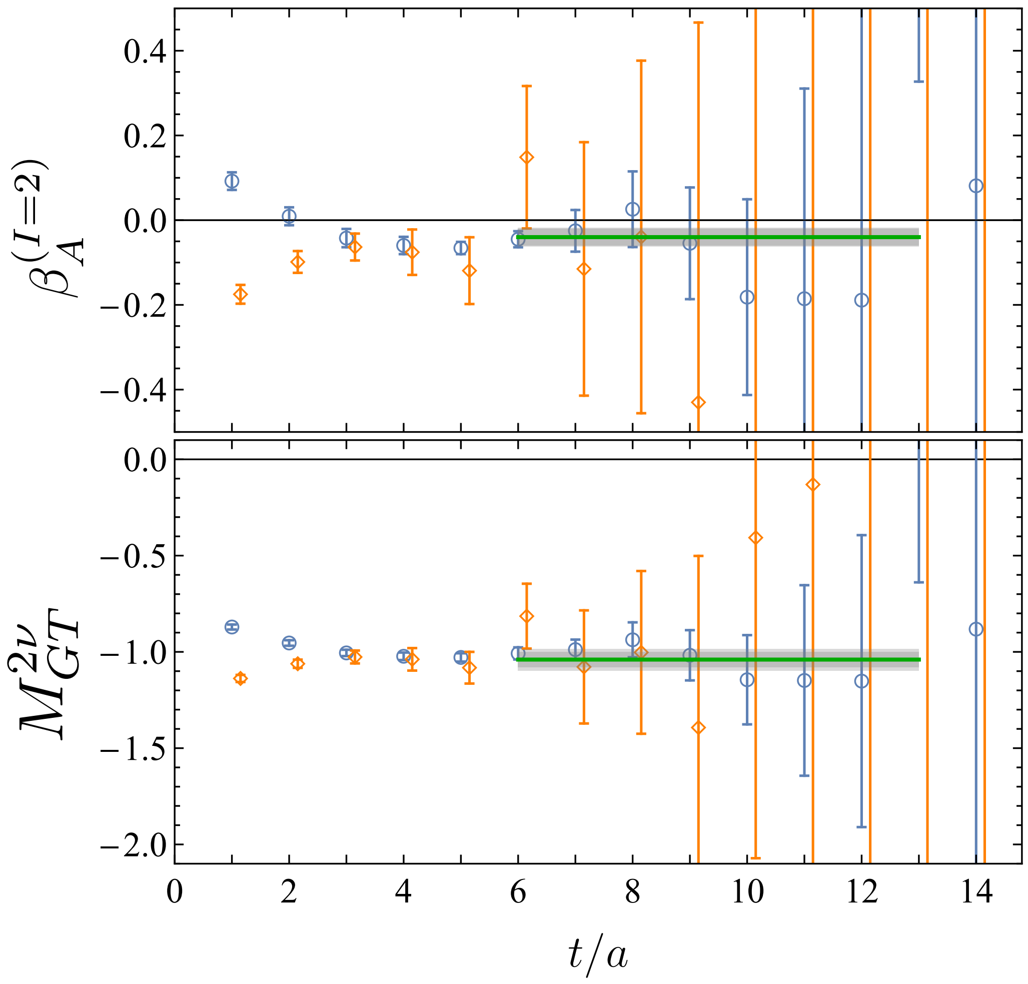
  - Non-negligible deviation from long distance deuteron intermediate state contribution

Isotensor axial polarisability

$$M_{GT}^{2\nu} = -\frac{|M_{pp\to d}|^2}{E_{pp} - E_d} + \beta_A^{(I=2)}$$

  - Quenching of g_A in nuclei is insufficient!
  - TBD: connect to EFT for larger systems





- Nuclei are under serious study directly from QCD
  - Spectroscopy and structure
  - Electroweak interactions: axial charges, *pp* fusion, $\beta\beta$ decay
- Prospect of a quantitative connection to QCD
- ***Potential obstacles***
  - physical mass: will get there (faster computers, new algorithms)
  - larger (A,Z) - relies on convergence of EFT
  - elastic FFs at larger mtm transfer - EFT no help at 1.5 GeV!
  - quasi-elastic region: unstable resonances are hard
    [see talk of Max Hansen]
  - DIS region :access through moments of PDFs

fin

Acknowledgements