Nucleon axial form factors and structure

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University of Cyprus and The Cyprus Institute

IPPP/NuSTEC topical meeting on neutrino-nucleus scattering

Outline

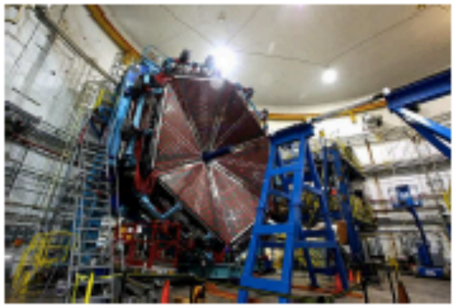
1. **Introduction and Motivation**
   - Current status of simulations
   - Low-lying baryon masses
   - Evaluation of matrix elements in lattice QCD

2. **Nucleon charges: g_A, g_s, g_T**

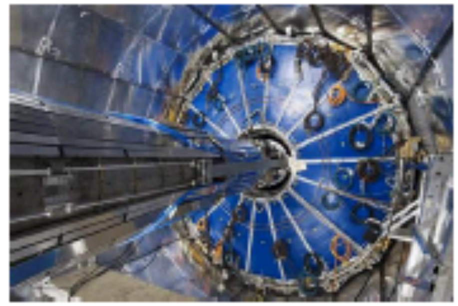
3. **Nucleon axial form factors: $G_A(Q^2)$ and $G_p(Q^2)$**

4. **Conclusions**

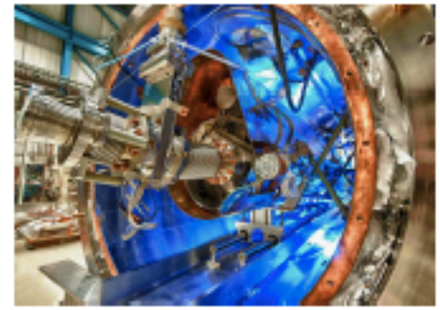
JLAB (12GeV Upgrade)



RHIC (BNL)



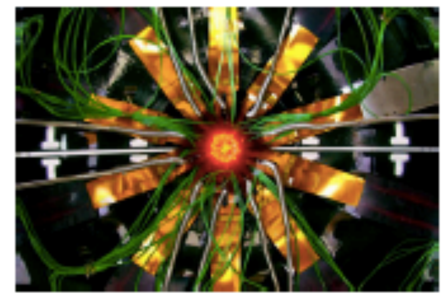
FERMILAB



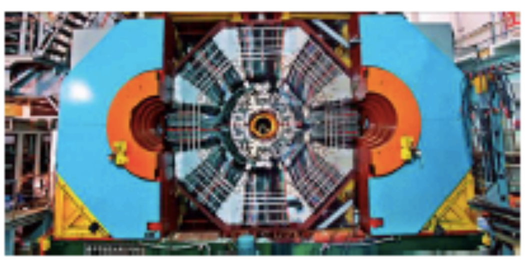
JPARC

Rich experimental activities in major facilities

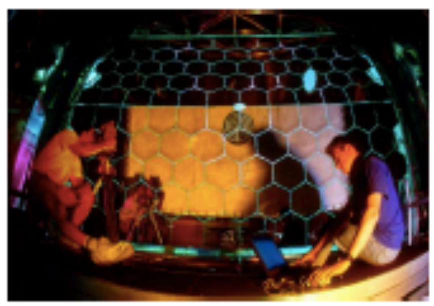
ALICE



BES III



COMPASS



PSI

MAMI

With simulations at the physical point lattice QCD can provide essential input for the experimental programs.

Quantum ChromoDynamics (QCD)

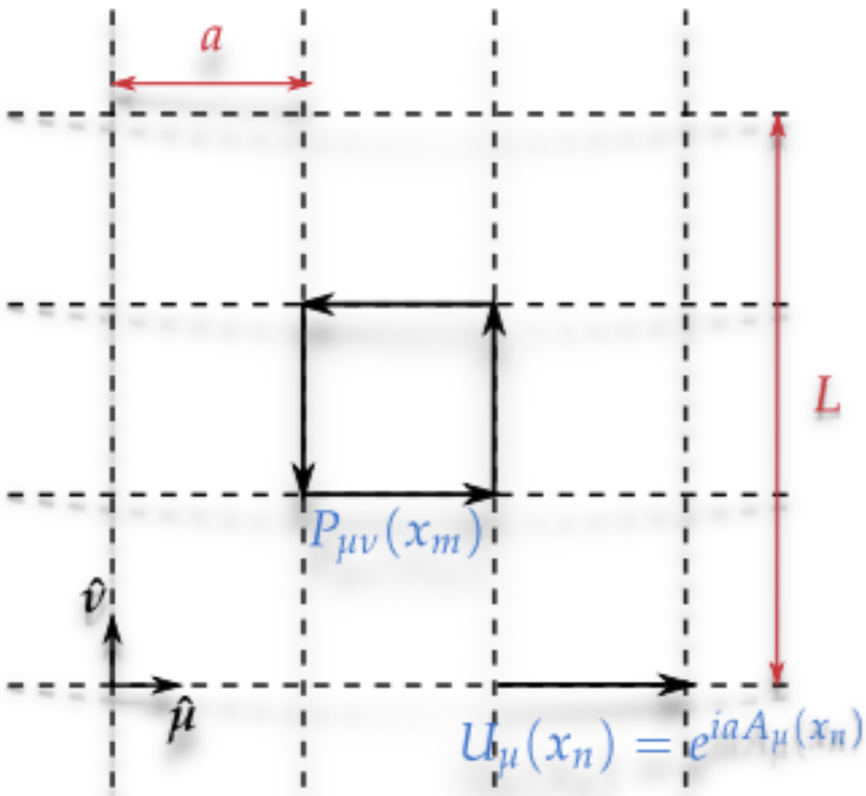
QCD-Gauge theory of the strong interaction

Lagrangian: formulated in terms of quarks and gluons

$$\mathcal{L}_{QCD} = -\frac{1}{4} F^{a}_{\mu\nu} F^{a\,\mu\nu} + \sum_{f=u,d,s,c,b,t} \bar{\psi}_f \left(i\gamma^\mu D_\mu - m_f \right) \psi_f$$

$$D_\mu = \partial_\mu - ig \frac{\lambda^a}{2} A^a_\mu$$

Choice of fermion discretisation scheme e.g. Clover, Twisted Mass, Staggered, Domain Wall, Overlap
Each has its advantages and disadvantages



Eventually,

- all discretization schemes must agree in the continuum limit $a \to 0$
- observables extrapolated to the infinite volume limit $L \to \infty$

Questions we would like to address

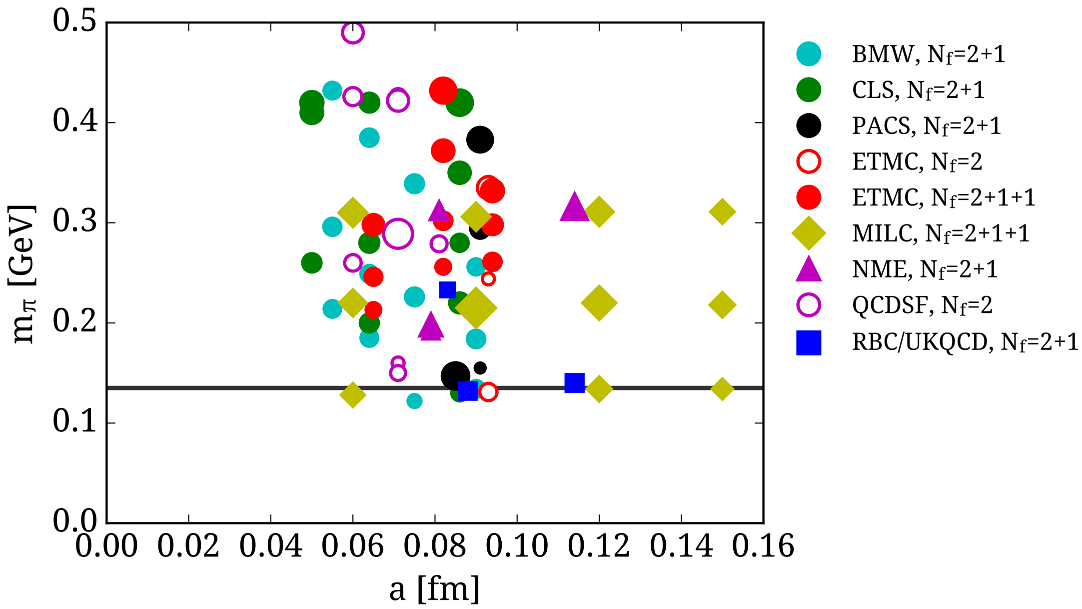
With simulations at the physical value of the pion mass there is a number of interesting questions we want to address:

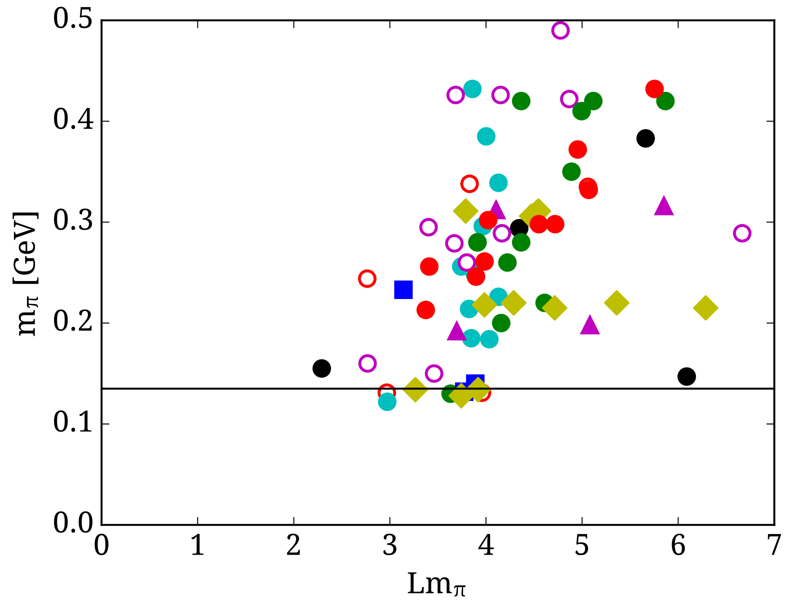
- Can we reproduce known quantities including the excited spectrum of the nucleon and its associated resonances?
- Can we resolve the long-standing issue of the spin content of the nucleon?
- Can we determine accurately fundamental properties of the proton such as its charge and axial radii?
- Can we provide input for experimental searches for new physics?

In this talk I will address two topics:

- The nucleon charges g_A, g_S and g_T
- The nucleon axial form factors $G_A(Q^2)$ and $G_p(Q^2)$

Status of simulations





Size of the symbols according to the value of $m_\pi L$: smallest value $m_\pi L \sim 3$ and largest $m_\pi L \sim 6.7$.

Computational resources

Juelich SuperComputing Centre, Germany

Peak performance: 5.9 Petaflop/s

458 752 cores

Our time allocation: 65 Million core-h

Swiss National Supercomputing Centre, Switzerland

Peak performance: 7.8 PFlops/s

42 176 cores

Tesla Graphic cards

Our time allocation: 2 Million node-h

(equiv. to 200 Million core-h)

Europe's Fastest GPU SuperComputer

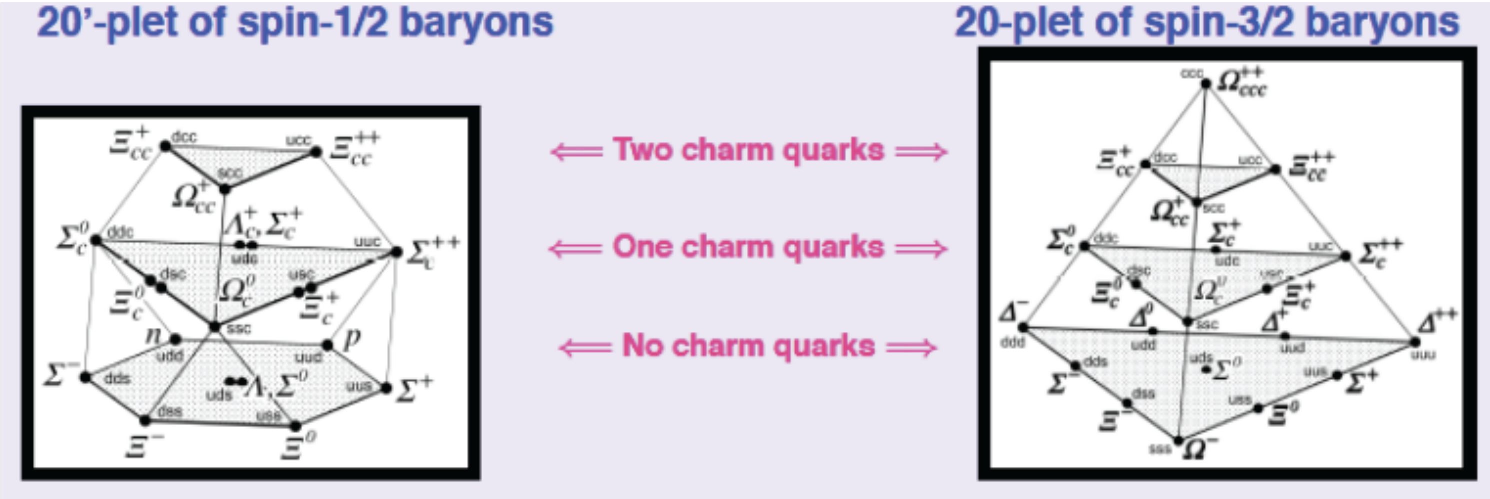
Gauss Centre, Stuttgart, Germany

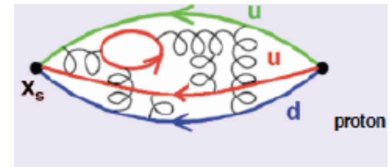
Peak performance: 7.42 Petaflop/s

185 088 cores

Our time allocation: 48 Million core-h

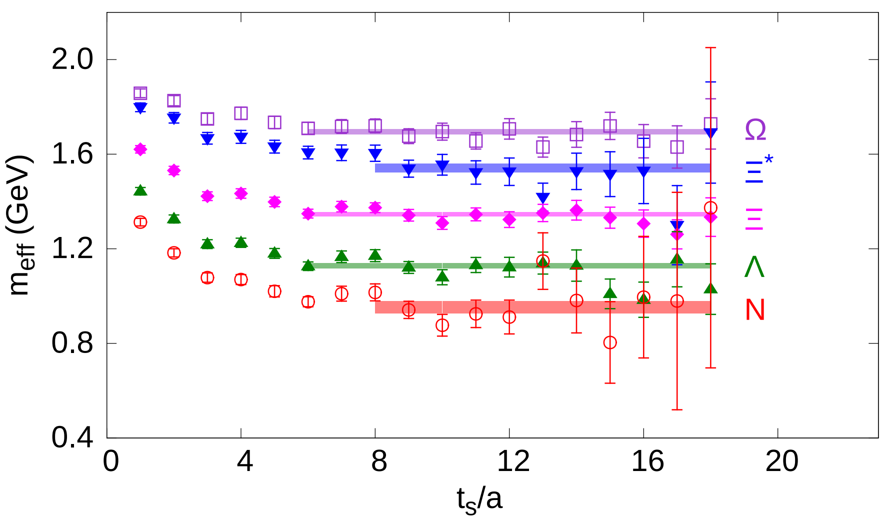
Hadron masses



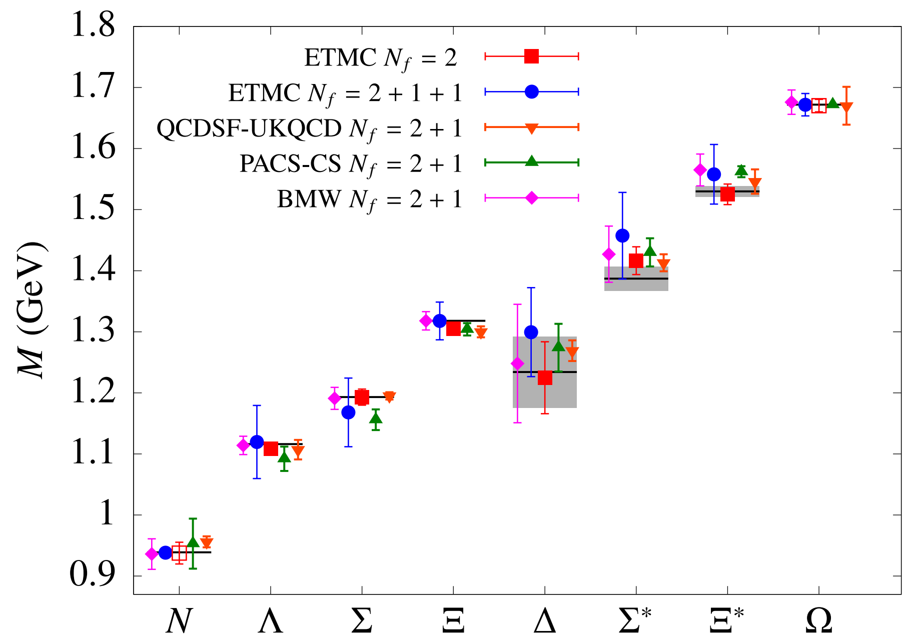


$$C(\vec{p}, t_s) = \sum_{\vec{x}_s} e^{-i\vec{x}_s\cdot\vec{p}} \langle J_p(\vec{x}_s, t_s)\bar{J}_p(0)\rangle = \sum_{n=0}^{\infty} A_n e^{-E_n(\vec{p})t_s} \overset{t_s\to\infty}{\to} A_0 e^{-E_0(\vec{p})t_s} \overset{\vec{p}=\vec{0}}{=} \mathcal{A} e^{-m_p t_s}$$

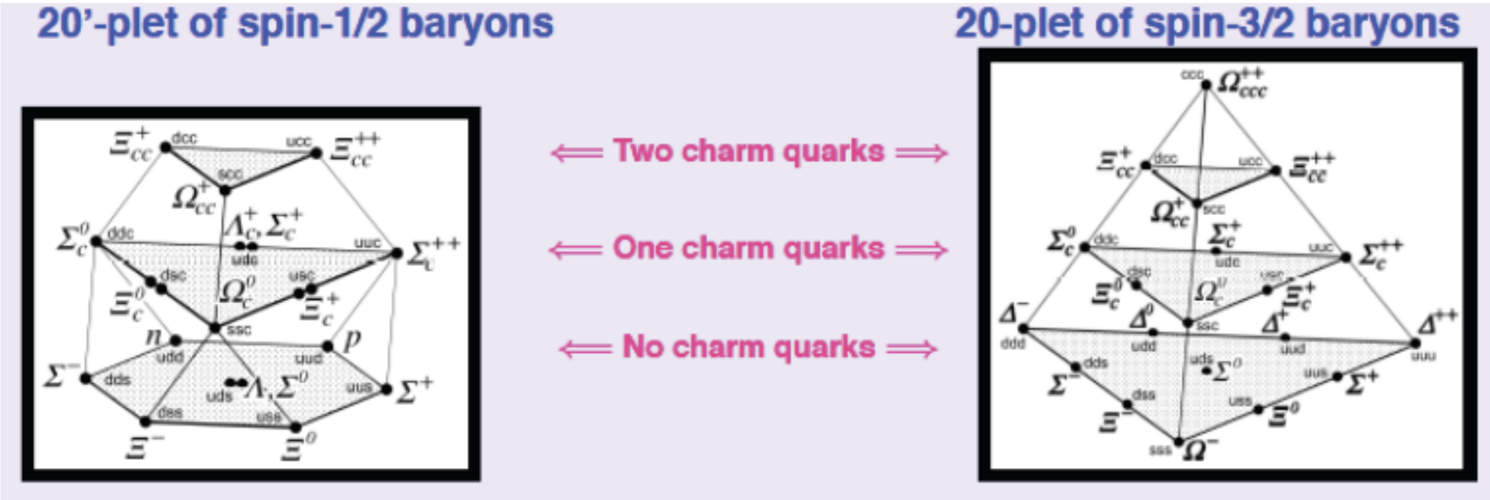
$$\to am_p^{\mathrm{eff}}(t_s) \equiv \log\left(\frac{C(\vec{0}, t_s)}{C(\vec{0}, t_s + a)}\right) \overset{t_s\to\infty}{\to} am_p$$

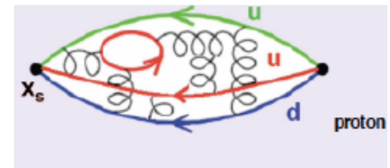


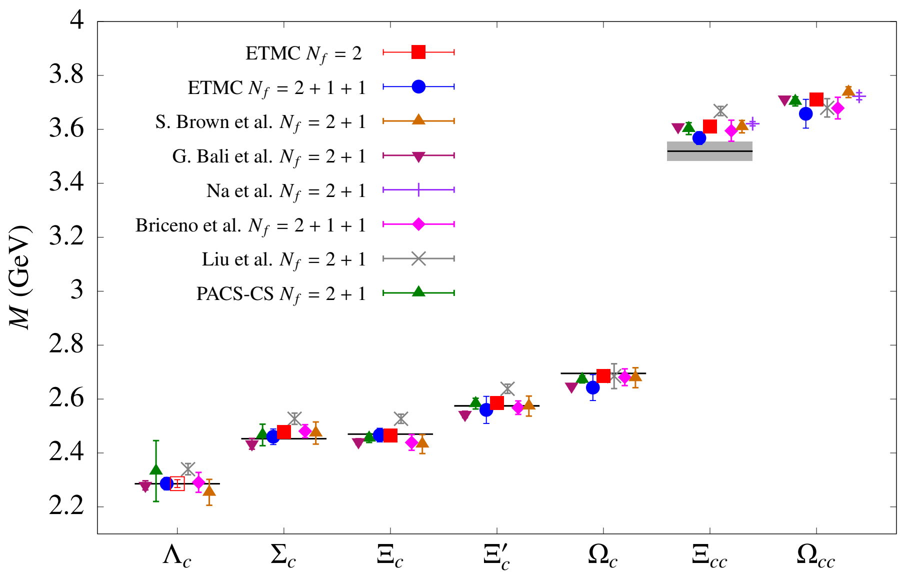
Using $N_f = 2$ simulations at a physical value of the pion mass



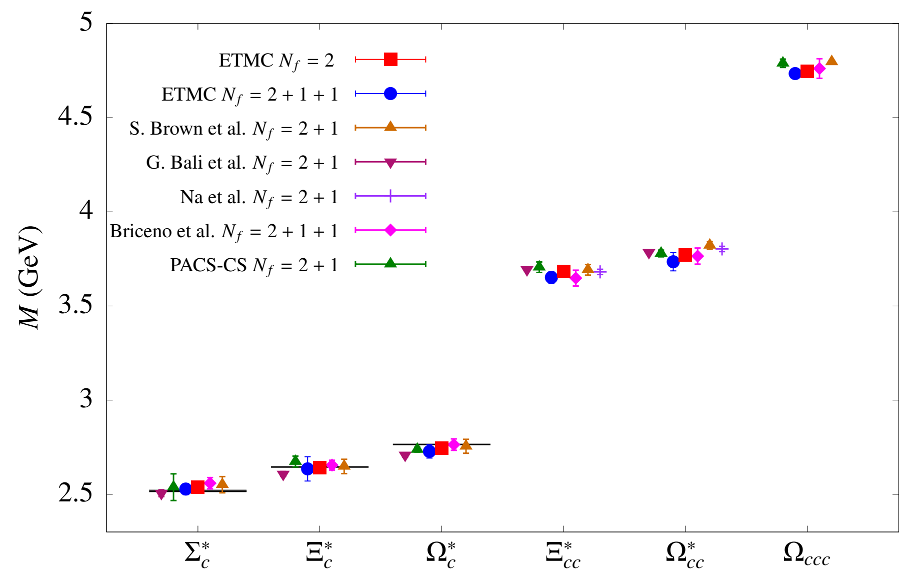
Hadron masses







spin-1/2

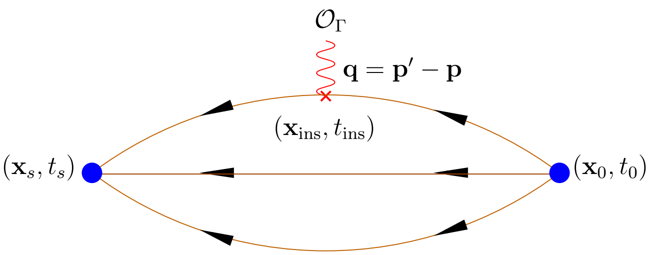


spin-3/2

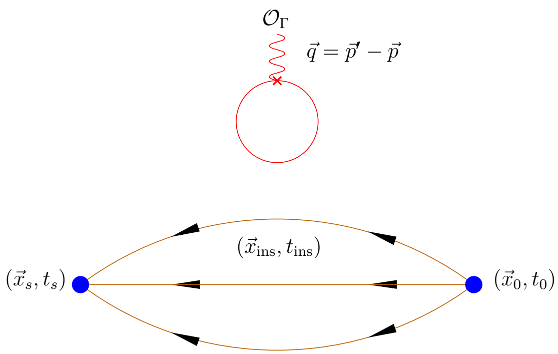
Evaluation of matrix elements

Three-point functions:

$$G^{\mu\nu}(\Gamma, \vec{q}, t_s, t_{\rm ins}) = \sum_{\vec{x}_s, \vec{x}_{\rm ins}} e^{i\vec{x}_{\rm ins}\cdot\vec{q}}\, \Gamma_{\beta\alpha}\, \langle J_\alpha(\vec{x}_s, t_s)\mathcal{O}^{\mu\nu}_\Gamma(\vec{x}_{\rm ins}, t_{\rm ins})\bar{J}_\beta(\vec{x}_0, t_0)\rangle$$



connected contribution



disconnected contribution

Evaluation of matrix elements

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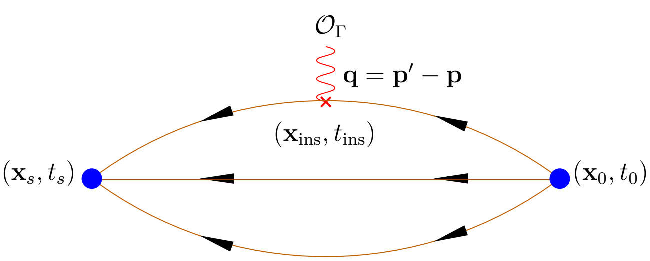
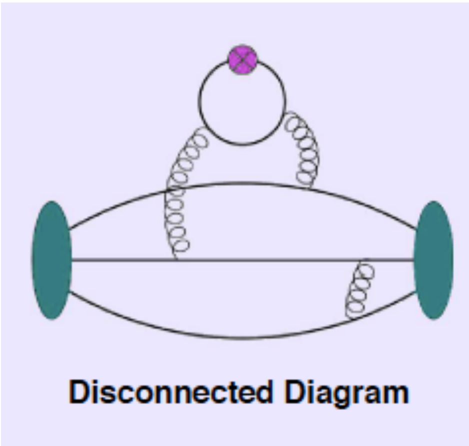


connected contribution



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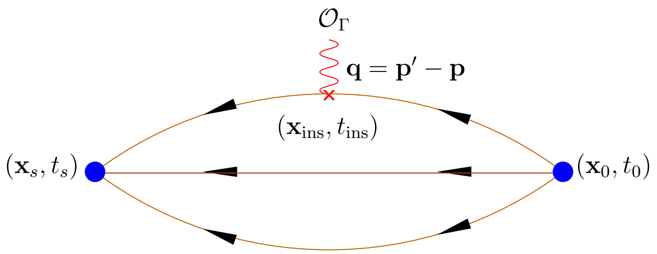
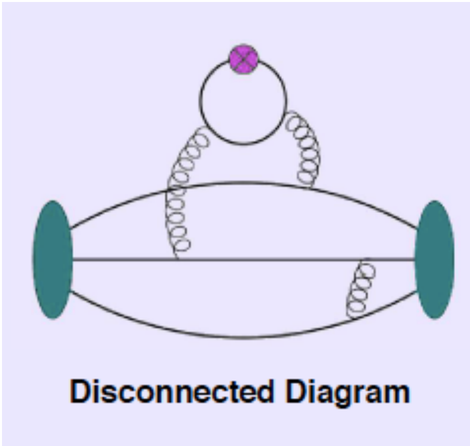


connected contribution



- Plateau method:

$$R(t_s, t_{\rm ins}, t_0) \xrightarrow[(t_s - t_{\rm ins})\Delta \gg 1]{(t_{\rm ins} - t_0)\Delta \gg 1} \mathcal{M}[1 + \ldots e^{-\Delta(\mathbf{p})(t_{\rm ins} - t_0)} + \ldots e^{-\Delta(\mathbf{p}')(t_s - t_{\rm ins})}]$$

- Summation method: Summing over $t_{\rm ins}$:

$$\sum_{t_{\rm ins}=t_0}^{t_s} R(t_s, t_{\rm ins}, t_0) = \text{Const.} + \mathcal{M}[(t_s - t_0) + \mathcal{O}(e^{-\Delta(\mathbf{p})(t_s - t_0)}) + \mathcal{O}(e^{-\Delta(\mathbf{p}')(t_s - t_0)})].$$

  Excited state contributions are suppressed by exponentials decaying with $t_s - t_0$, rather than $t_s - t_{\rm ins}$ and/or $t_{\rm ins} - t_0$
  However, one needs to fit the slope rather than to a constant or take differences and then fit to a constant
  L. Maiani, G. Martinelli, M. L. Paciello, and B. Taglienti, Nucl. Phys. B293, 420 (1987); S. Capitani *et al.*, arXiv:1205.0180

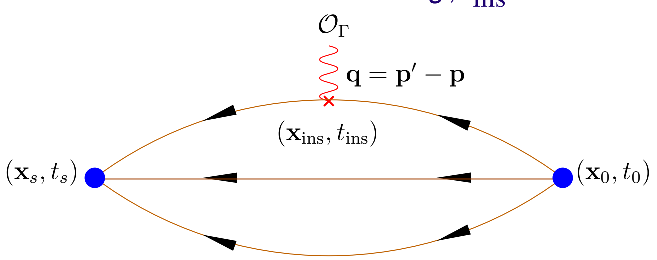
- Fit keeping the first excited state, T. Bhattacharya *et al.*, arXiv:1306.5435

All should yield the same answer in the end of the day!

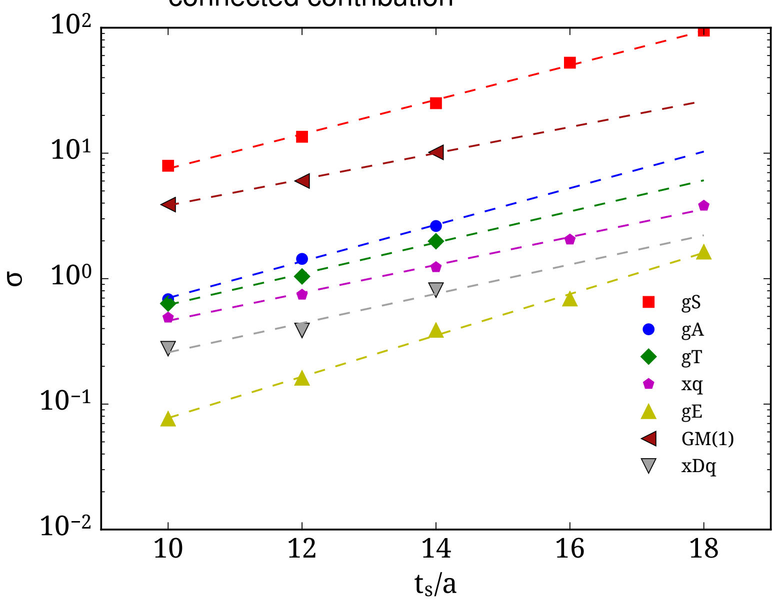
Evaluation of matrix elements

Three-point functions:

$$G^{\mu\nu}(\Gamma, \vec{q}, t_s, t_{\rm ins}) = \sum_{\vec{x}_s, \vec{x}_{\rm ins}} e^{i\vec{x}_{\rm ins}\cdot\vec{q}}\,\Gamma_{\beta\alpha}\,\langle J_\alpha(\vec{x}_s, t_s)\mathcal{O}^{\mu\nu}_\Gamma(\vec{x}_{\rm ins}, t_{\rm ins})\bar{J}_\beta(\vec{x}_0, t_0)\rangle$$



connected contribution



- $\mathcal{M}$ the desired matrix element
- t_s, $t_{\rm ins}$, t_0 the sink, insertion and source time-slices
- $\Delta(\mathbf{p})$ the energy gap with the first excited state

To ensure ground state dominance need multiple sink-source time separations ranging from 0.9 fm to 1.5 fm

Nucleon isovector charges: g_A, g_s, g_T

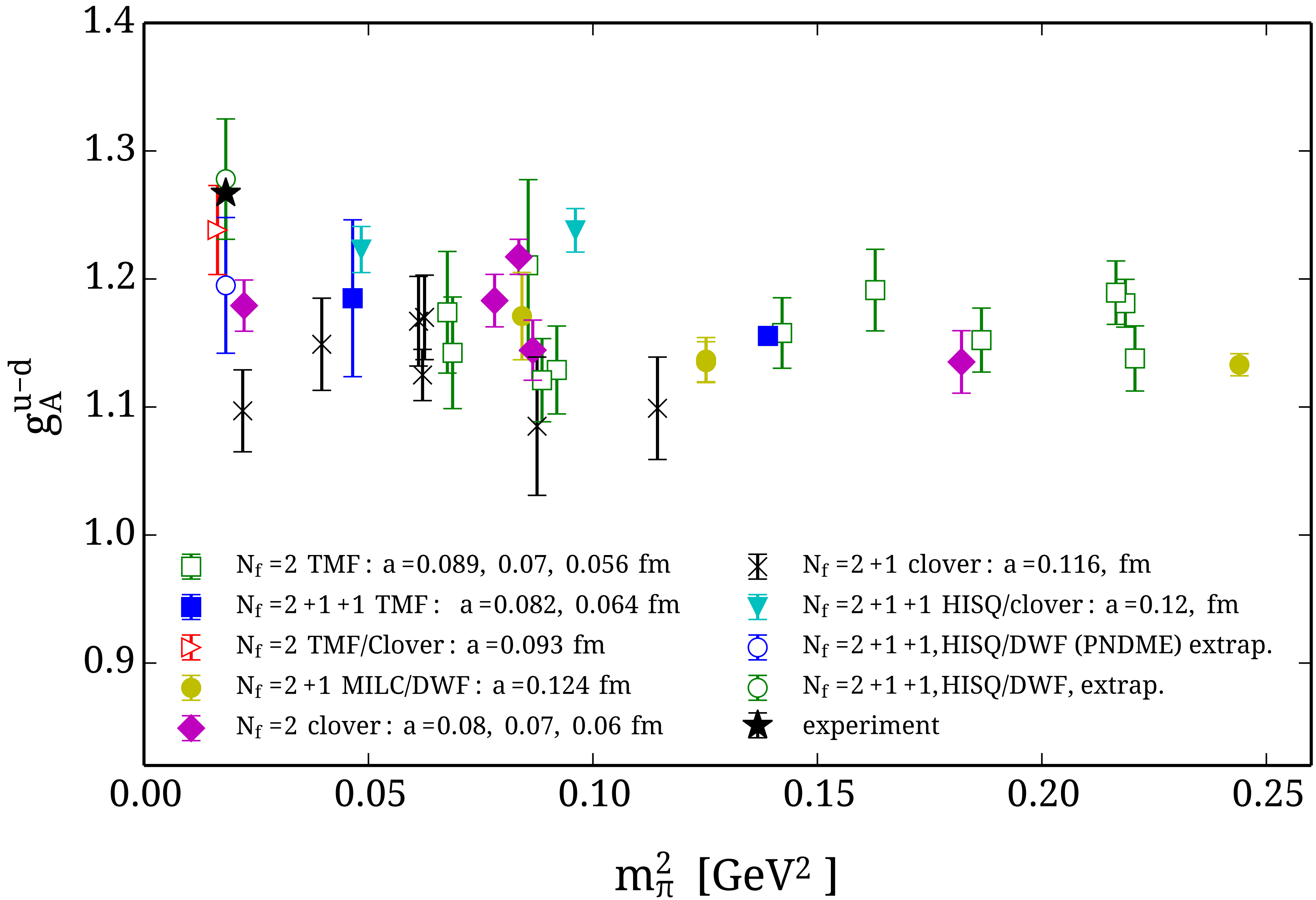
- axial-vector operator: $\mathcal{O}^a_A = \bar{\psi}(x)\gamma^\mu\gamma_5\frac{\tau^a}{2}\psi(x)$
- scalar operator: $\mathcal{O}^a_S = \bar{\psi}(x)\frac{\tau^a}{2}\psi(x)$
- pseudoscalar: $\mathcal{O}^a_p = \bar{\psi}(x)\gamma_5\frac{\tau^a}{2}\psi(x)$
- tensor operator: $\mathcal{O}^a_T = \bar{\psi}(x)\sigma^{\mu\nu}\frac{\tau^a}{2}\psi(x)$

$\Longrightarrow$ extract from matrix element: $\langle N(\vec{p}')\mathcal{O}_X N(\vec{p})\rangle|_{q^2=0}$

• Axial charge g_A • Scalar charge g_S • Pseudoscalar charge g_p, • Tensor charge g_T

(i) isovector combination has no disconnect contributions; (ii) g_A well known experimentally, Goldberger-Treiman relation yields g_p, g_T to be measured at JLab, Predict g_S

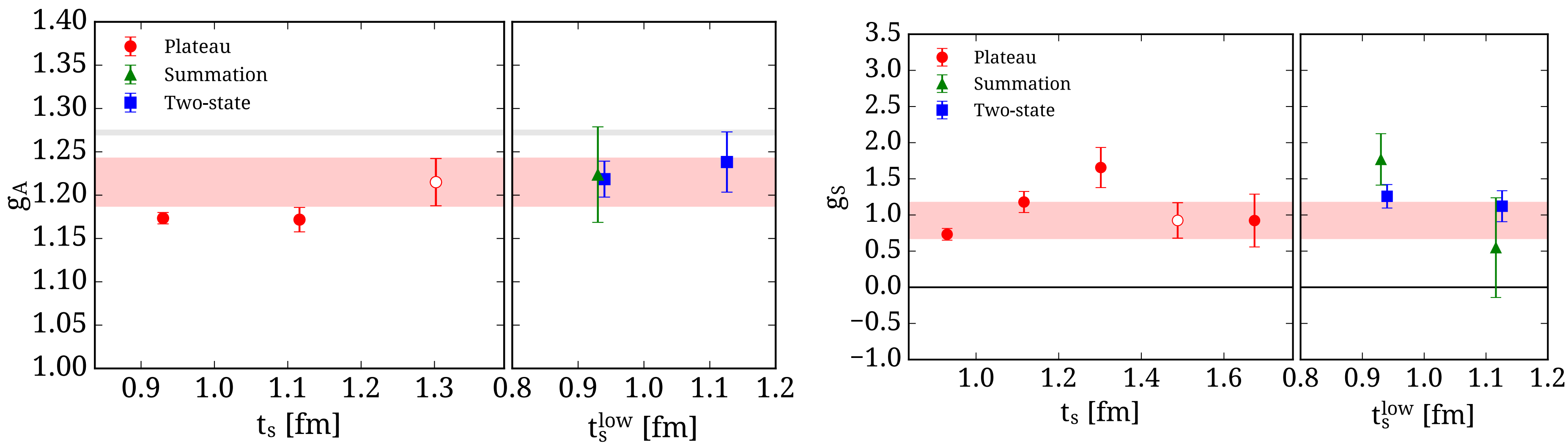
Nucleon axial charge g_A



g_A well known experimentally. It is an iso-vector quantity extracted at zero momentum transfer $\longrightarrow$ straight forward to compute in lattice QCD.

Nucleon charges: g_A, g_S

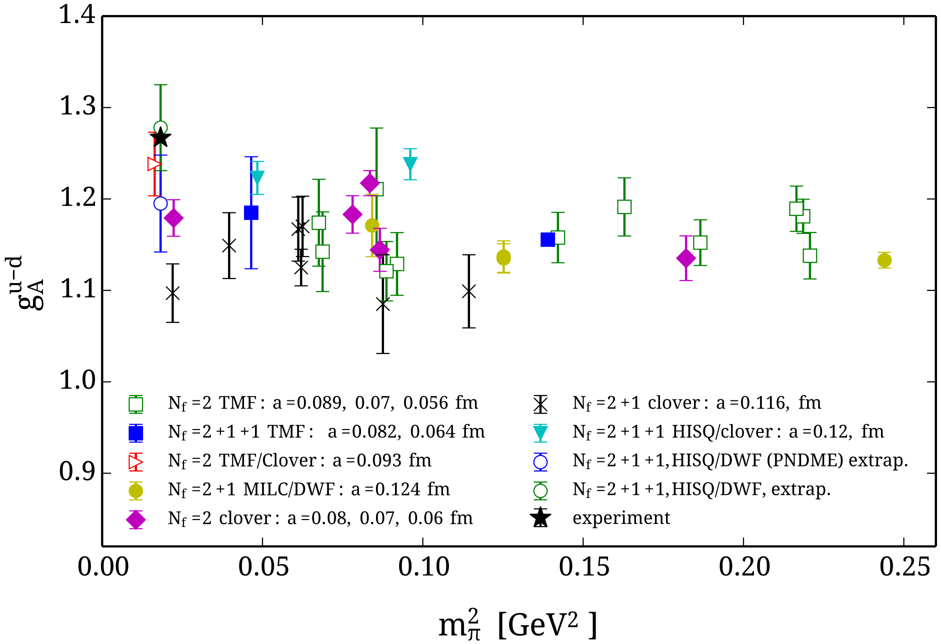
- $N_f = 2$ twisted mass plus clover, $48^3 \times 96$, a = 0.093(1) fm, m_π = 131 MeV
- $\sim$ 9260 statistics for $t_s/a = 10, 12, 14$, $\sim$ 48000 for $t_s/a = 16$ and $\sim$ 70000 for $t_s/a = 18$
- 5 sink-source time separations ranging from 0.9 fm to 1.7 fm

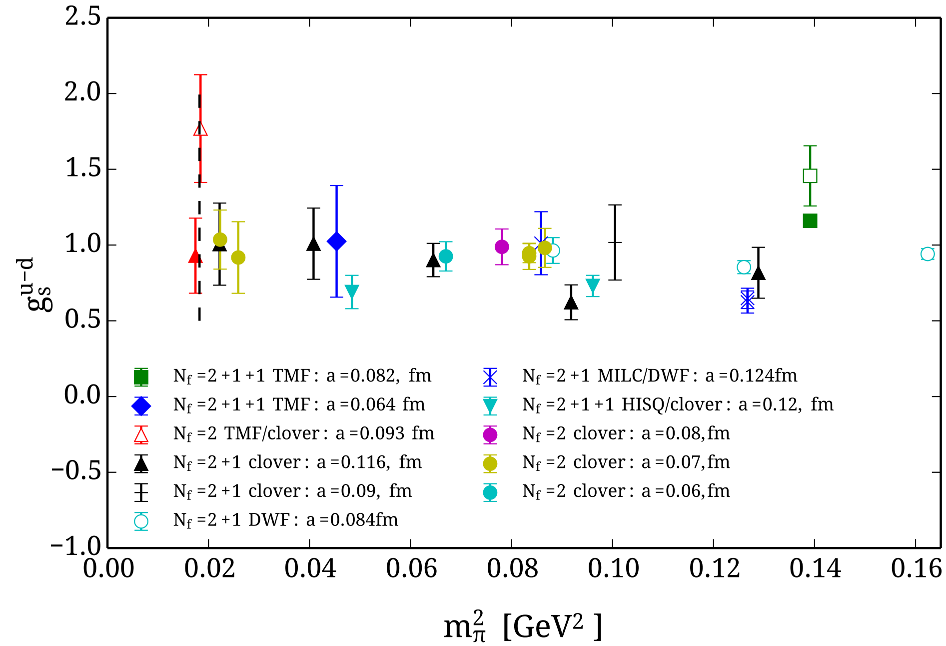


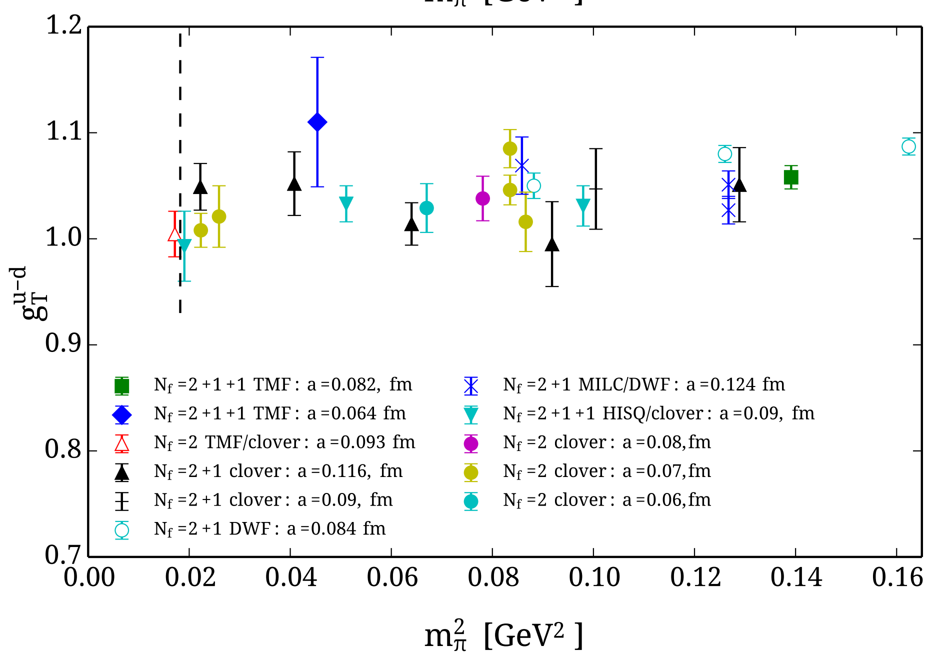
A. Abdel-Rehim *et al.* (ETMC):1507.04936, 1507.05068, 1411.6842, 1311.4522

Summary of results on nucleon charges: g_A, g_S, g_T

Isovector





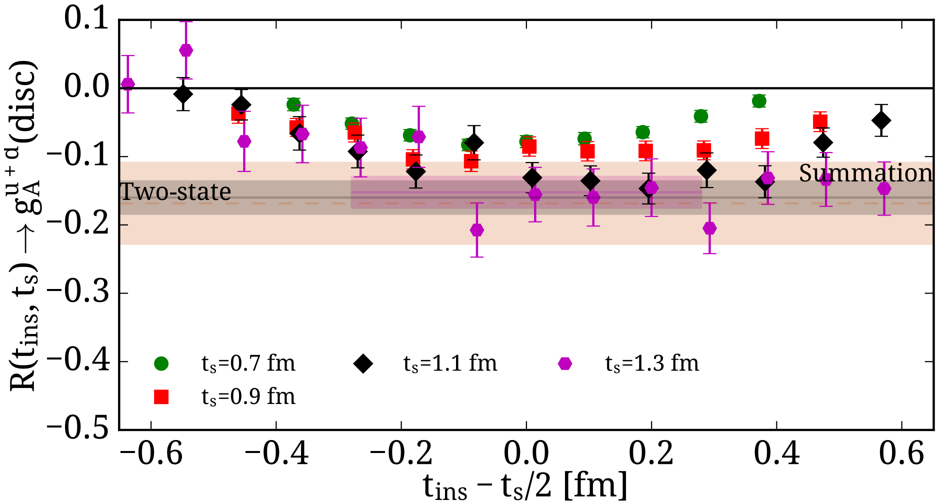


At the physical point we find from the plateau method:

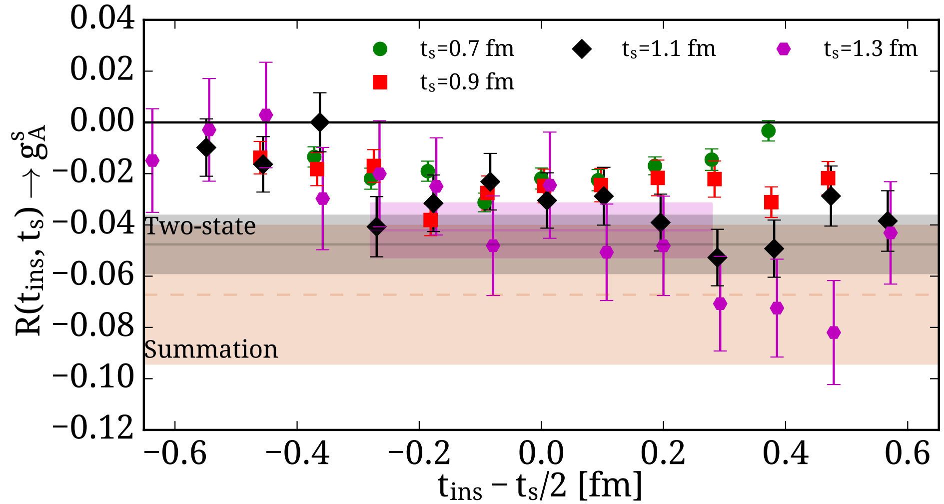
- $g_A^{\rm isov} = 1.21(3)(3)$, $g_S^{\rm isov} = 0.93(25)(33)$, $g_T^{\rm isov} = 1.00(2)(1)$
- g_A further study for larger t_s. Important to keep constant error
- New analysis of COMPASS and Belle data: $g_T^{u-d} = 0.81(44)$, M. R. A. Courtoy, A. Bacchettad, M. Guagnellia, arXiv: 1503.03495
- For g_S increasing the sink-source time separation to ~ 1.5 fm is crucial

Disconnected contributions to g_A^q

- $N_f = 2$ twisted mass fermions with a clover term at a physical value of the pion mass, $48^3 \times 96$ and $a = 0.093(1)$ fm
- Intrinsic quark spin: $\Delta\Sigma^q = g_A^q$



Disconnected isoscalar axial charge



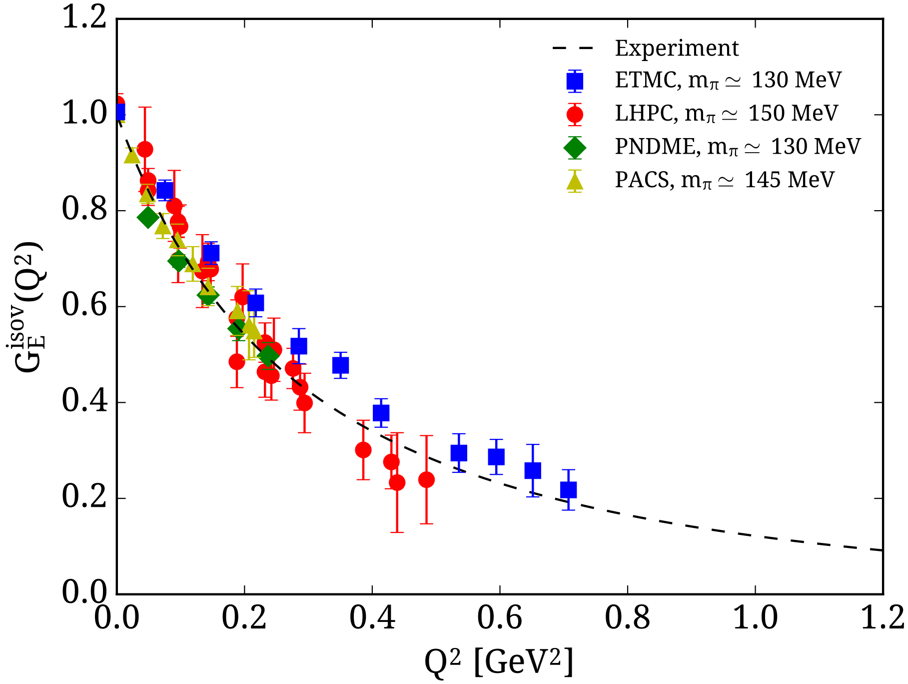
Strange axial charge

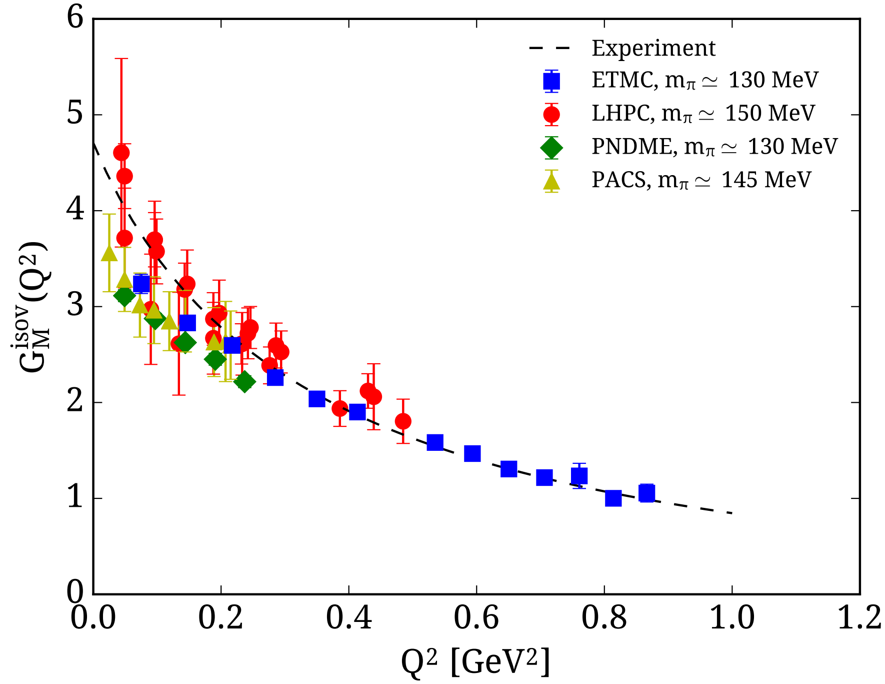
We find from the plateau method:

- $g_A^{u+d} = 0.595(28)$ (conn.) $-0.15(2)$ (disconn.) with 854,400 statistics
- Combining with the isovector we find: $g_A^u = 0.828(21)$, $g_A^d = -0.387(21)$
- $g_A^s = -0.042(10)$ with 861,200 statistics
- $g_A^c = -0.007(6)$ with 861,200 statistics

Recent results on the electric and magnetic form factors

Isovector form factors

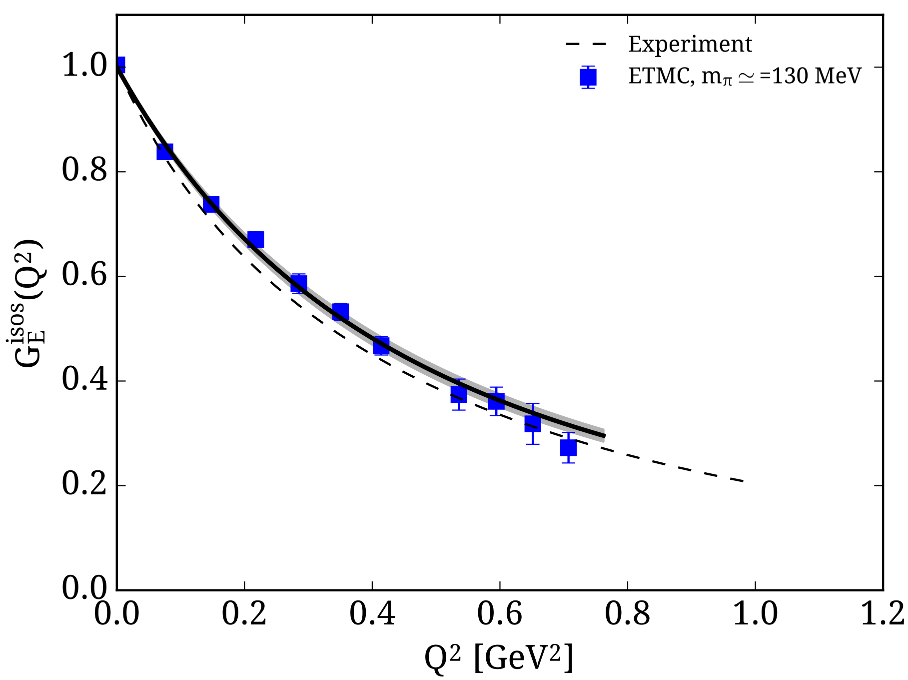


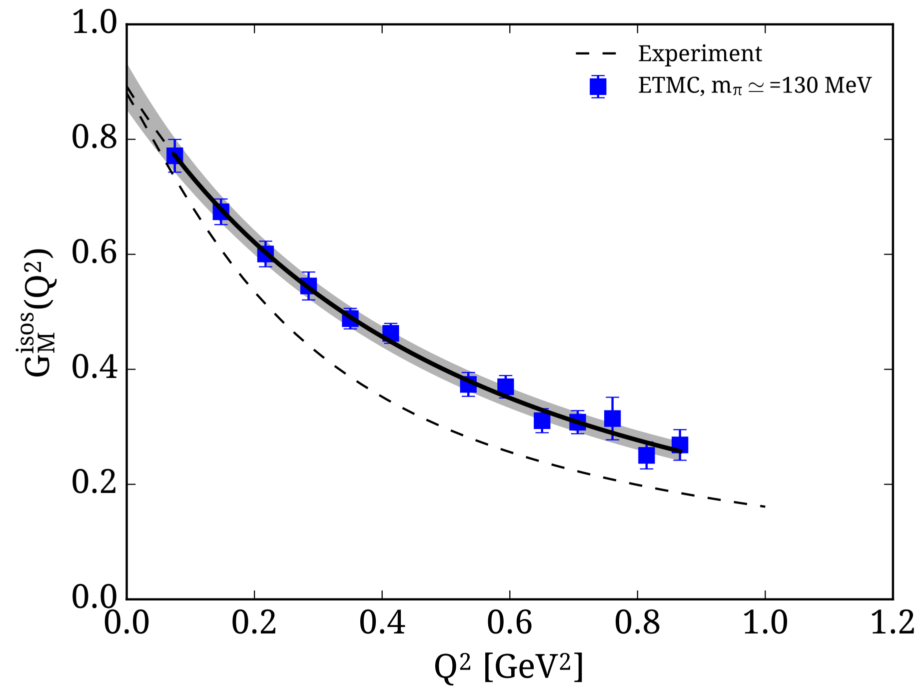


- ETMC using $N_f = 2$ twisted mass fermions (TMF), $a = 0.093$ fm, $48^3 \times 96$ G_E with $t_s = 1.7$ fm and 66,000 statistics, G_M with $t_s = 1.3$ fm and 9,300 statistics
- LHPC using $N_f = 2+1$ clover fermions, $a = 0.116$ fm, 48^4, summation method with 3 values of t_s from 0.9 fm to 1.4 fm and $\sim 7,800$ statistics, 1404.4029
- PNDME mixed action HISQ $N_f = 2+1+1$ and clover valence, $a = 0.087$ fm, $64^3 \times 96$, summation method with 3 values of t_s from 0.9 fm to 1.4 fm and $\sim 7,00$ HP and $\sim 85,000$ NP, Yong-Chull Jang, Lattice 2016
- PACS using $N_f = 2+1$ clover fermions, $a = 0.085$ fm, $96^3 \times 192$, $t_s = 1.3$ fm, 9,300 statistics, Y. Kuramashi, Lattice 2016

Recent results on the electric and magnetic form factors

Isoscalar form factors - connected contributions





- ETMC using $N_f = 2$ twisted mass fermions (TMF), $a = 0.093$ fm, $48^3 \times 96$ G_E with $t_s = 1.7$ fm and 66,000 statistics, G_M with $t_s = 1.3$ fm and 9,300 statistics

Recent results on the electric and magnetic form factors

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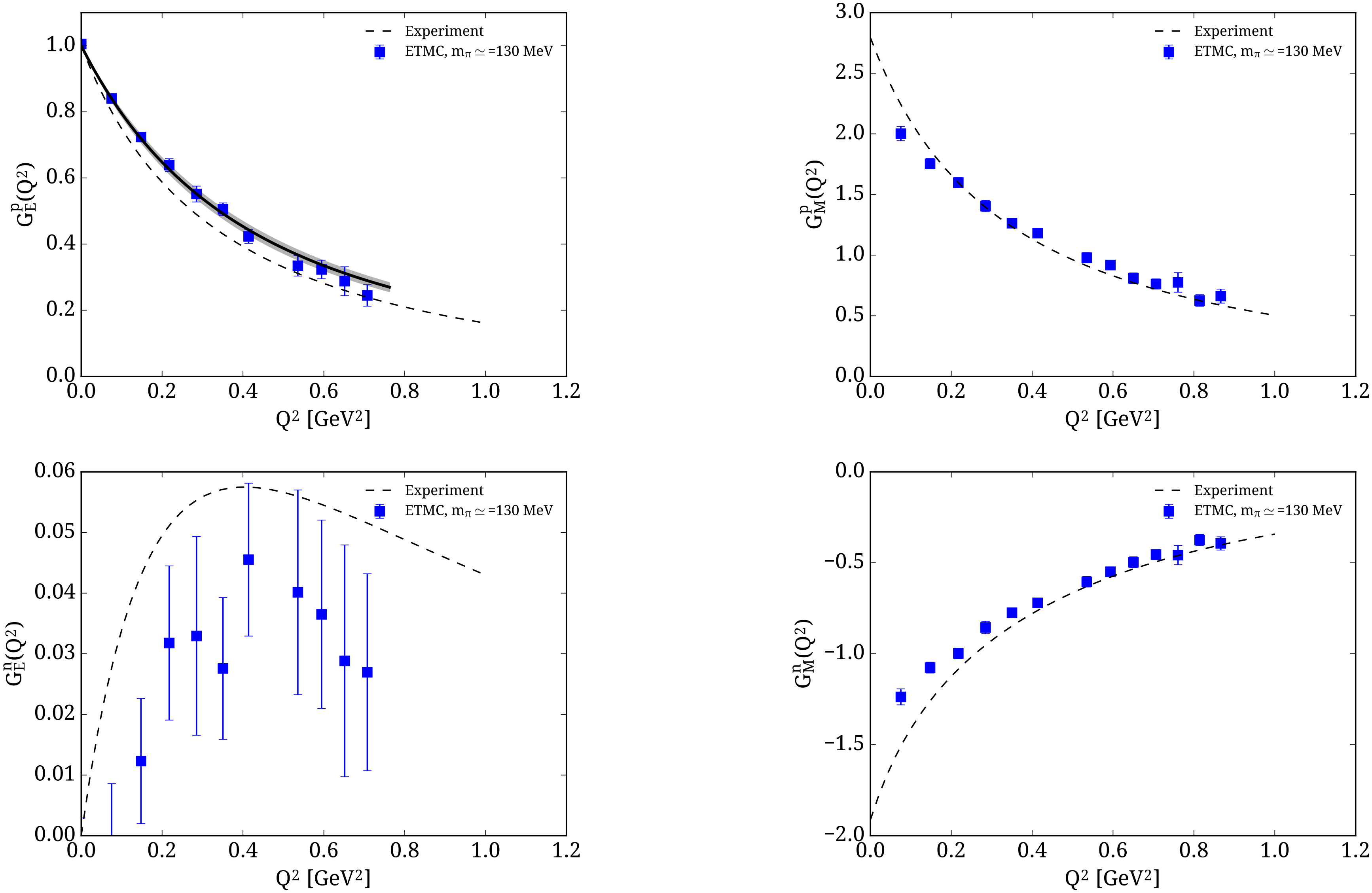

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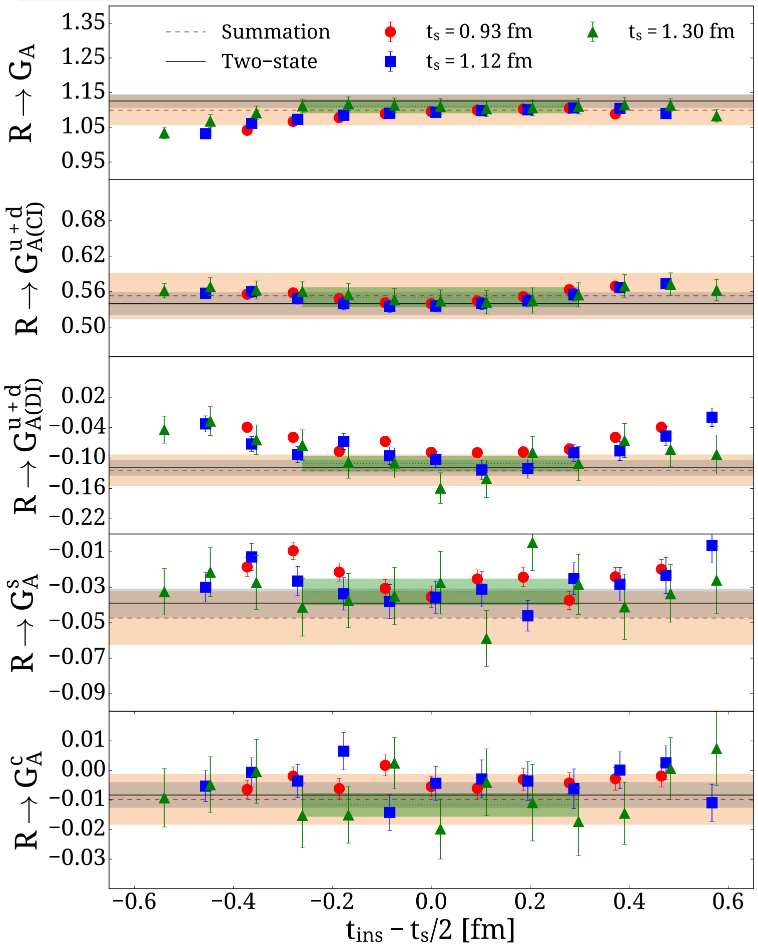
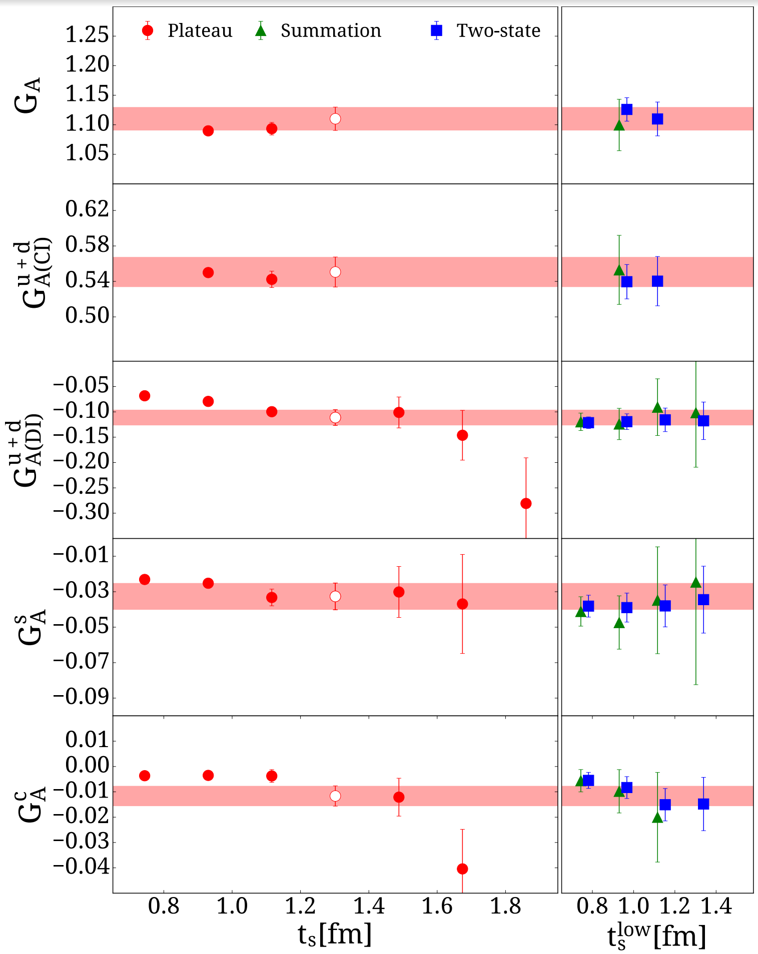
Nucleon axial form factors

$$\langle N(p',s')|A_\mu|N(p,s)\rangle = i\sqrt{\frac{m_N^2}{E_N(\vec{p}')E_N(\vec{p})}}\bar{u}_N(p',s')\left(\gamma_\mu G_A(Q^2) - i\frac{Q_\mu}{2m_N}G_p(Q^2)\right)\gamma_5 u_N(p,s)$$

ETMC using $N_f = 2$ twisted mass fermions (TMF), $a = 0.093$ fm, $48^3 \times 96$

- For the connected: largest $t_s = 1.3$ fm and 9,300 statistics
- For the disconnected: all t_s, $\mathcal{O}(850,000)$ statistics





Nucleon axial form factors

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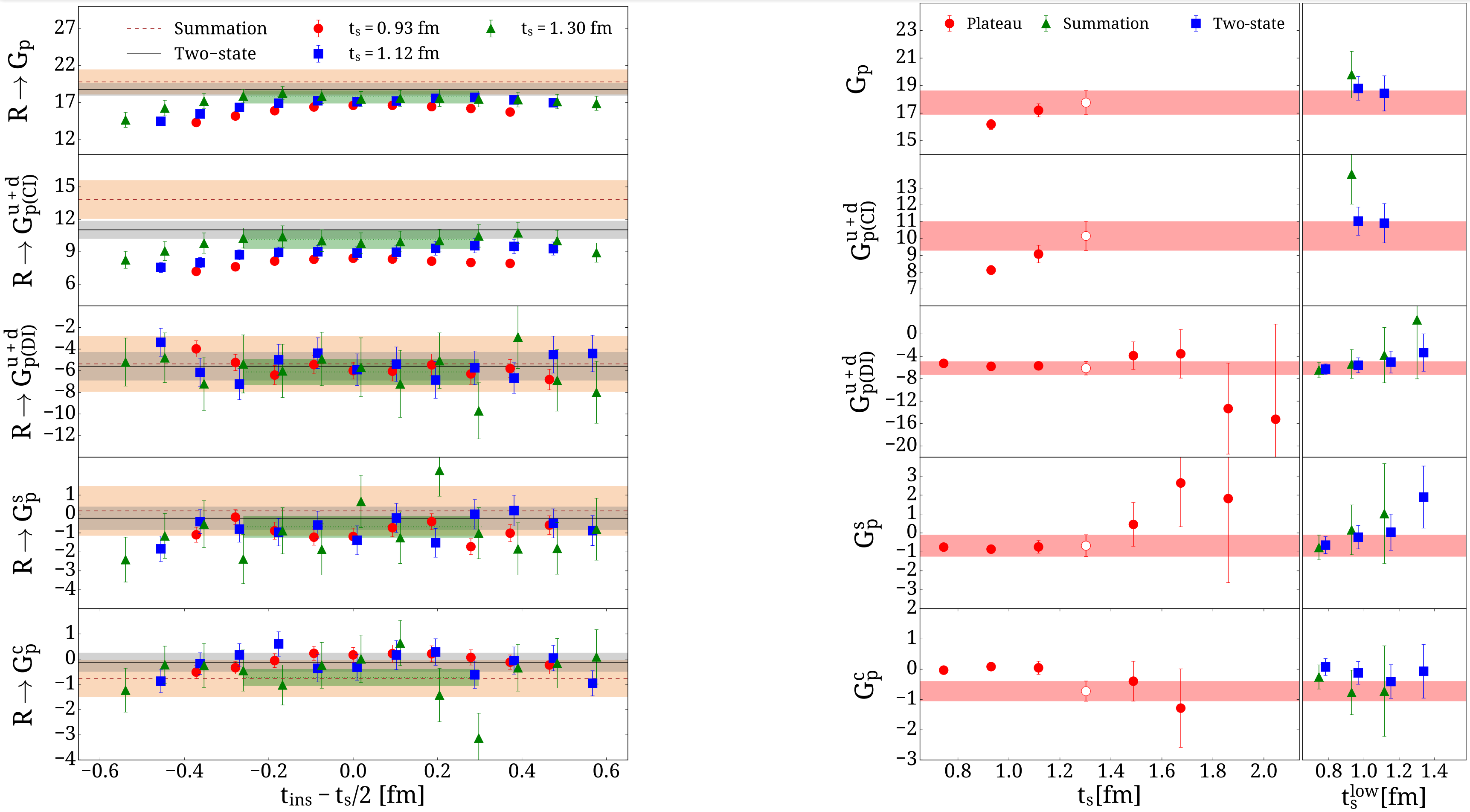


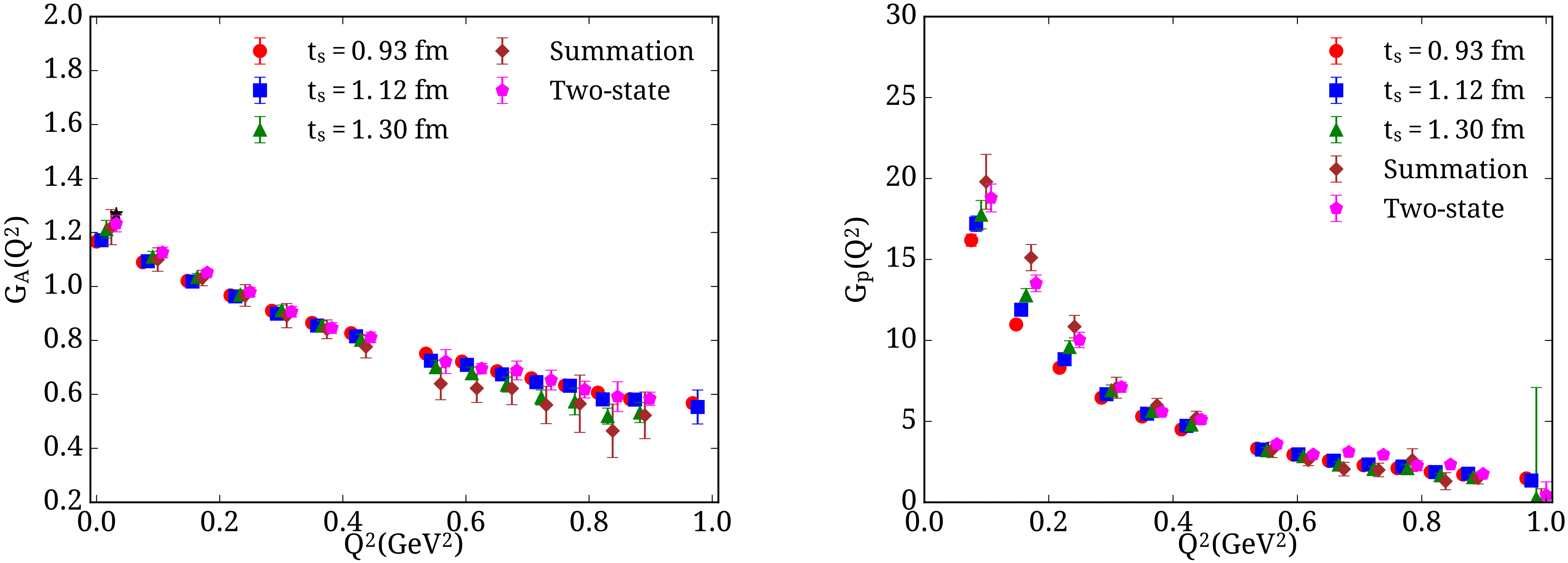
Nucleon axial form factors

$$\langle N(p',s')|A_\mu|N(p,s)\rangle = i\sqrt{\frac{m_N^2}{E_N(\vec{p}')E_N(\vec{p})}}\bar{u}_N(p',s')\left(\gamma_\mu G_A(Q^2) - i\frac{Q_\mu}{2m_N}G_p(Q^2)\right)\gamma_5 u_N(p,s)$$

ETMC using $N_f = 2$ twisted mass fermions (TMF), $a = 0.093$ fm, $48^3 \times 96$

- For the connected: largest $t_s = 1.3$ fm and 9,300 statistics
- For the disconnected: all t_s, $\mathcal{O}(850,000)$ statistics

Isovector



Nucleon axial form factors

$$\langle N(p', s')|A_\mu|N(p, s)\rangle = i\sqrt{\frac{m_N^2}{E_N(\vec{p}')E_N(\vec{p})}}\bar{u}_N(p', s')\left(\gamma_\mu G_A(Q^2) - i\frac{Q_\mu}{2m_N}G_p(Q^2)\right)\gamma_5 u_N(p, s)$$

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- For the connected: largest $t_s = 1.3$ fm and 9,300 statistics
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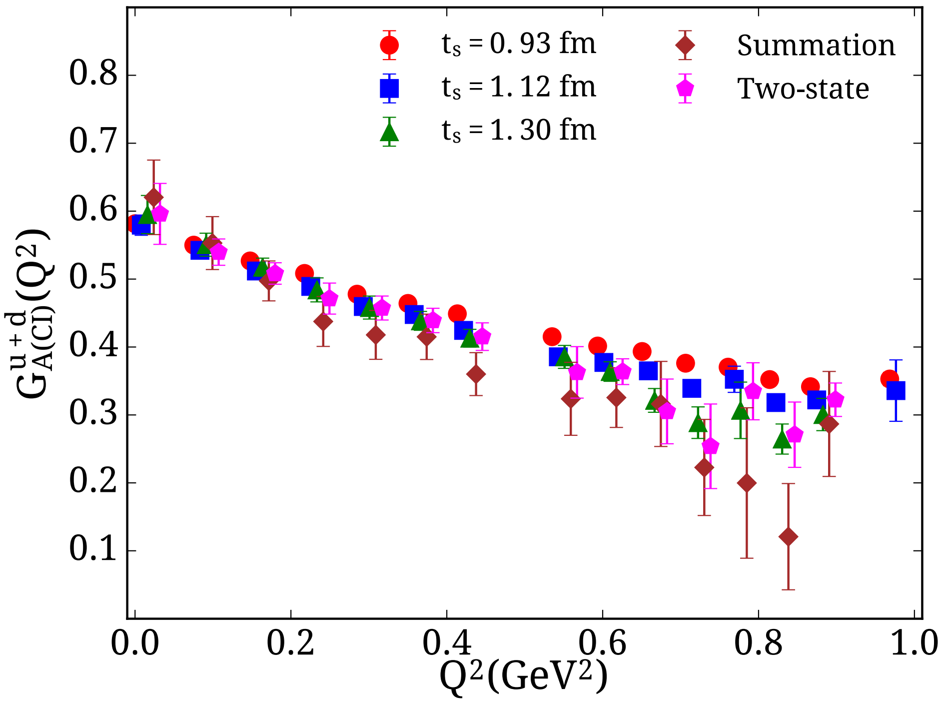
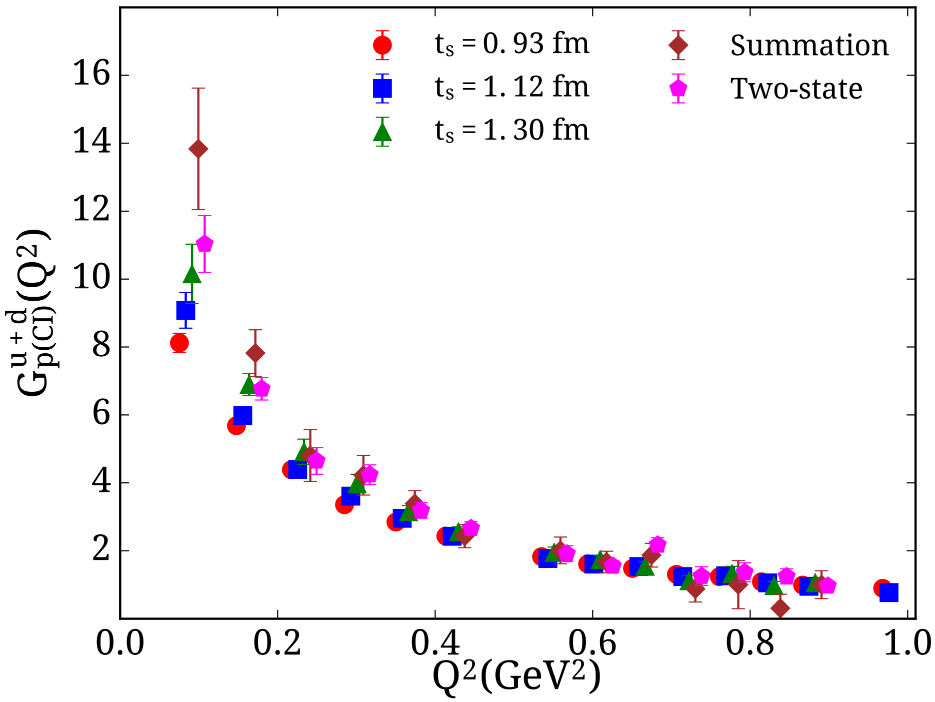
Isoscalar connected

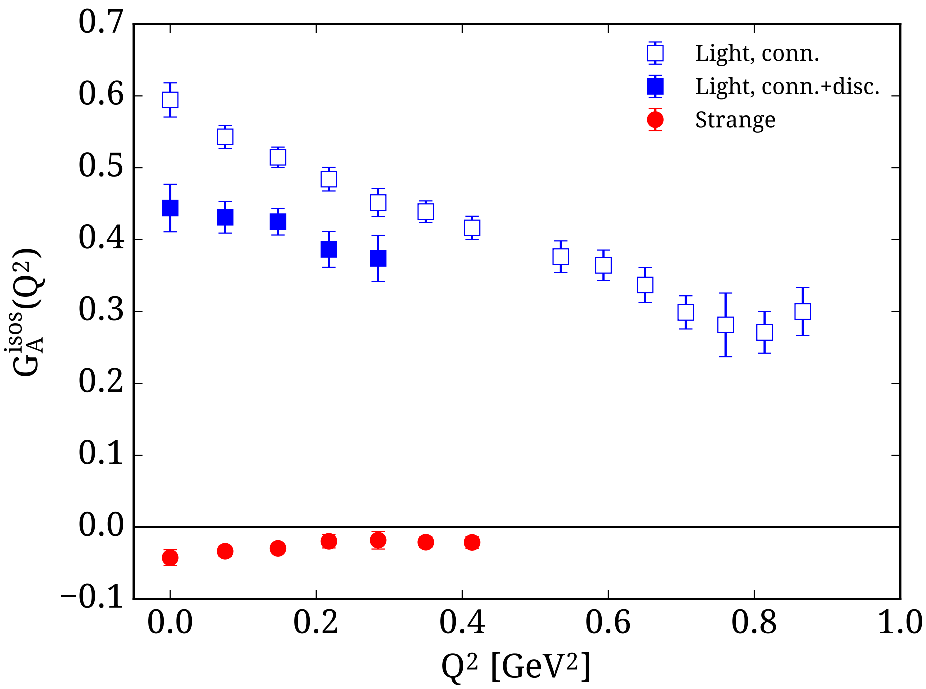


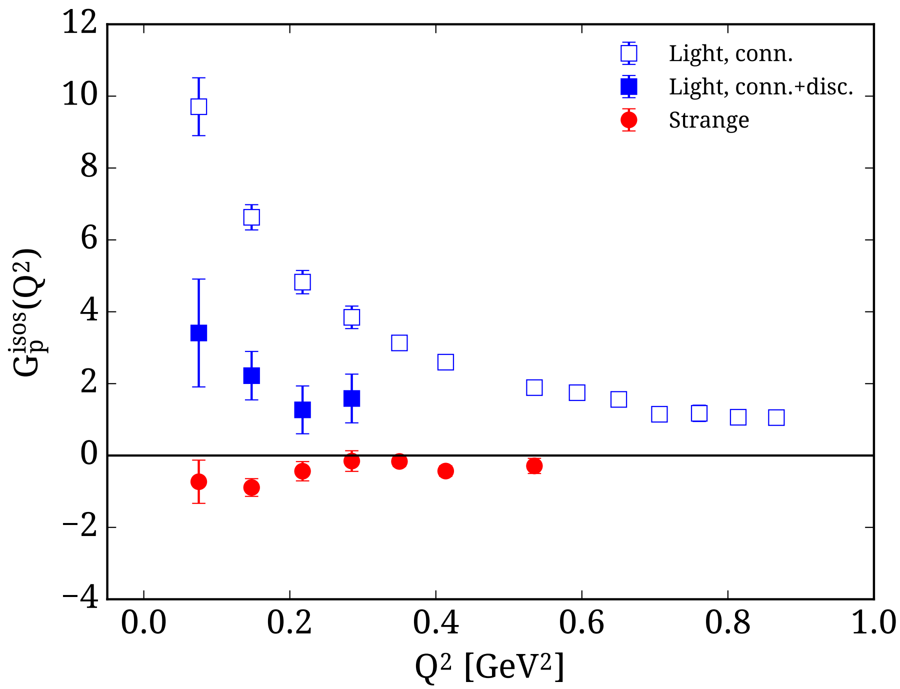


Isoscalar and strange nucleon axial form factors

ETMC using $N_f = 2$ twisted mass fermions (TMF), $a = 0.093$ fm, $48^3 \times 96$
- For the connected: largest $t_s = 1.3$ fm and 9,300 statistics
- For the disconnected: all t_s, $\mathcal{O}(850, 000)$ statistics

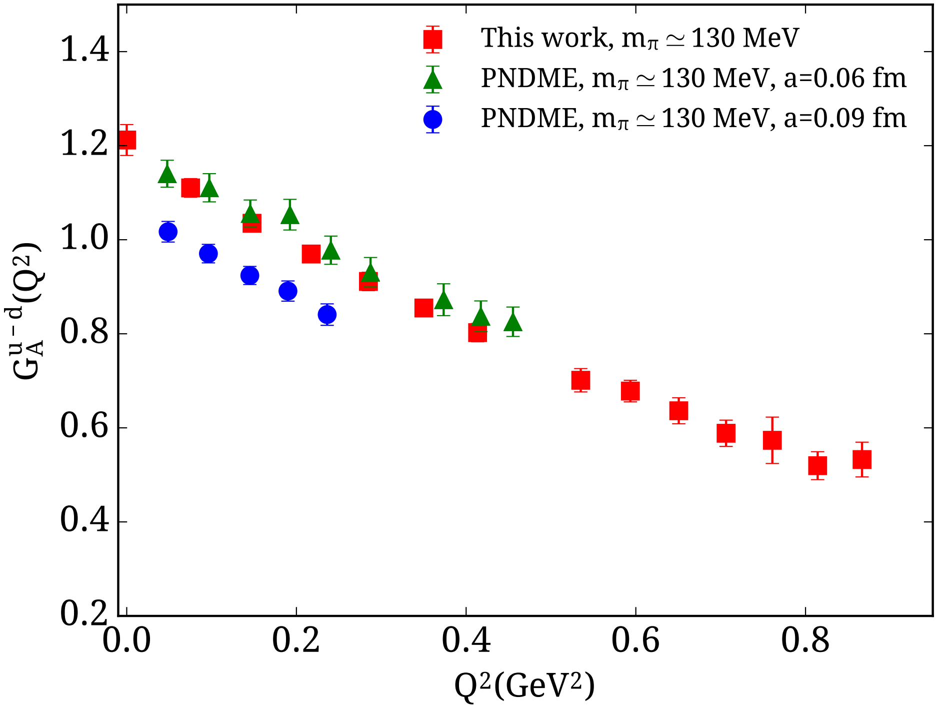
Isoscalar and strange form factors

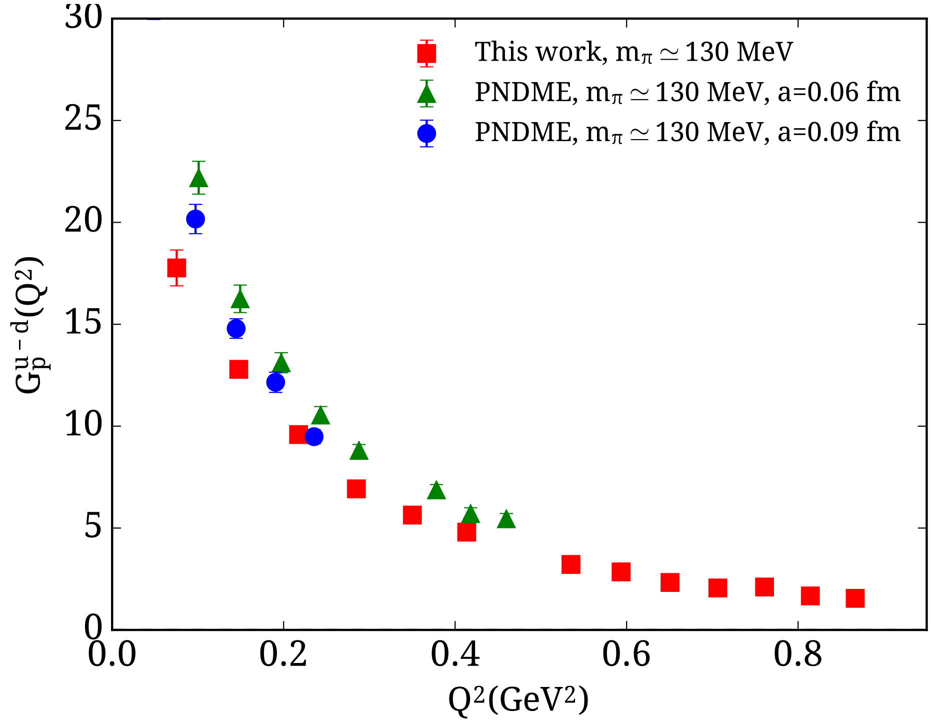




Recent results on nucleon axial form factors

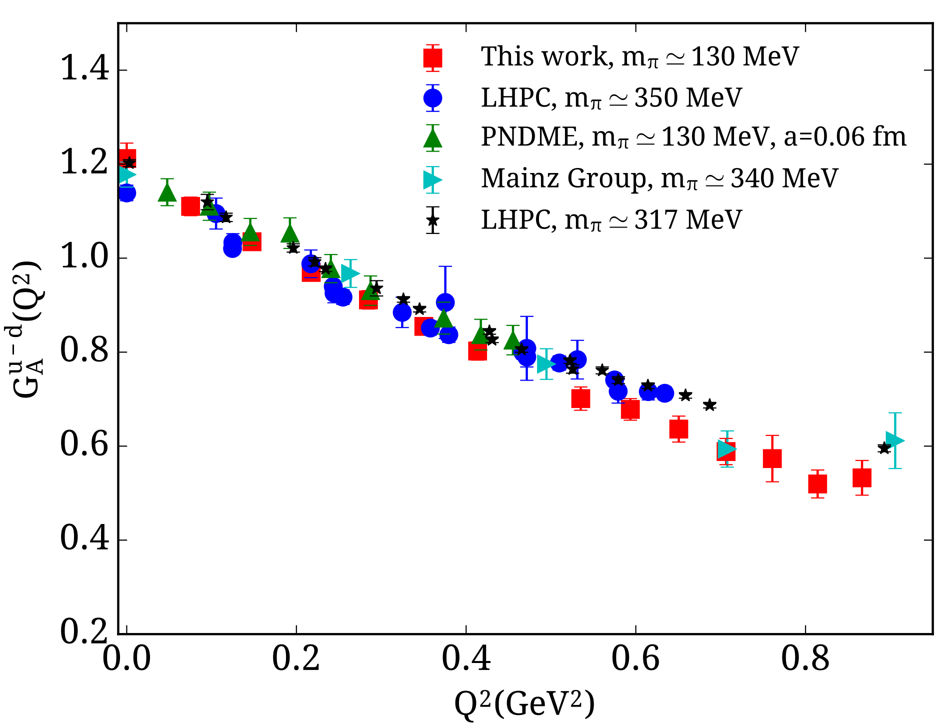
Isovector form factors

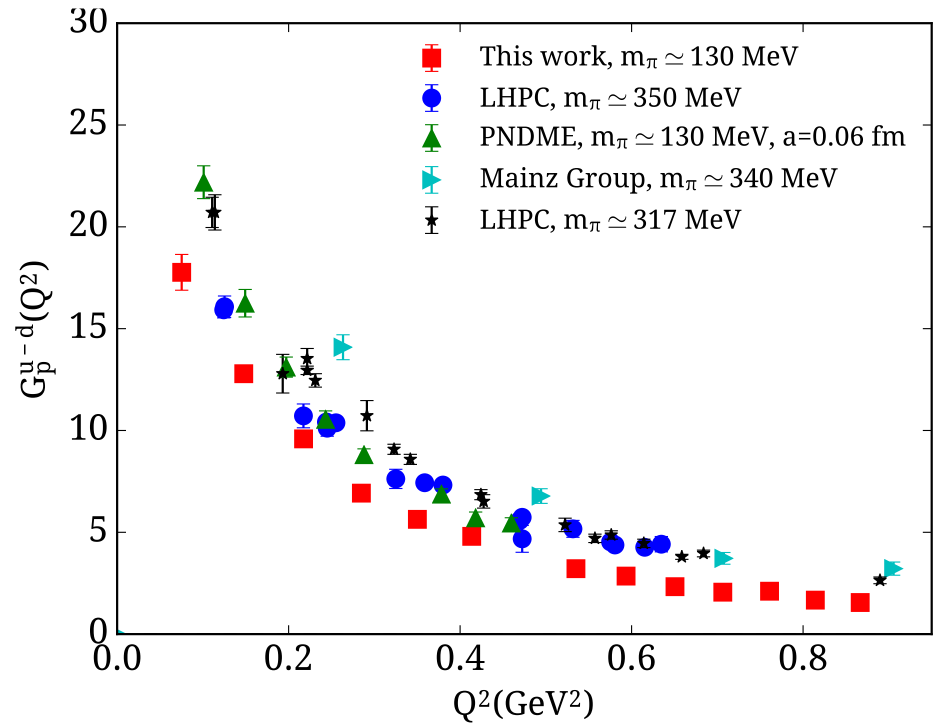




- ETMC using $N_f = 2$ twisted mass fermions (TMF), $a = 0.093$ fm, $48^3 \times 96$, C. A. *et al.* (2016), arXiv:1702.00984
- PNDME using mixed action of $N_f = 2 + 1 + 1$ HISQ sea fermions and clover valence and $a = 0.09$ ($64^3 \times 96$), and 0.06 ($96^3 \times 192$) fm, Yong-Chull Jang, *et al.* (2016)

Comparison with other lattice QCD results

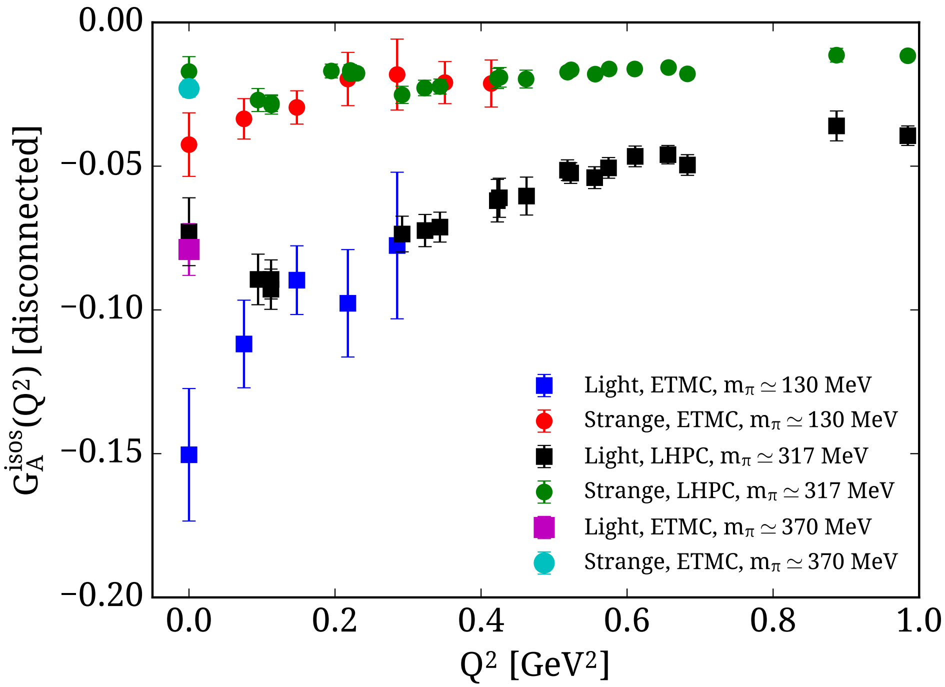


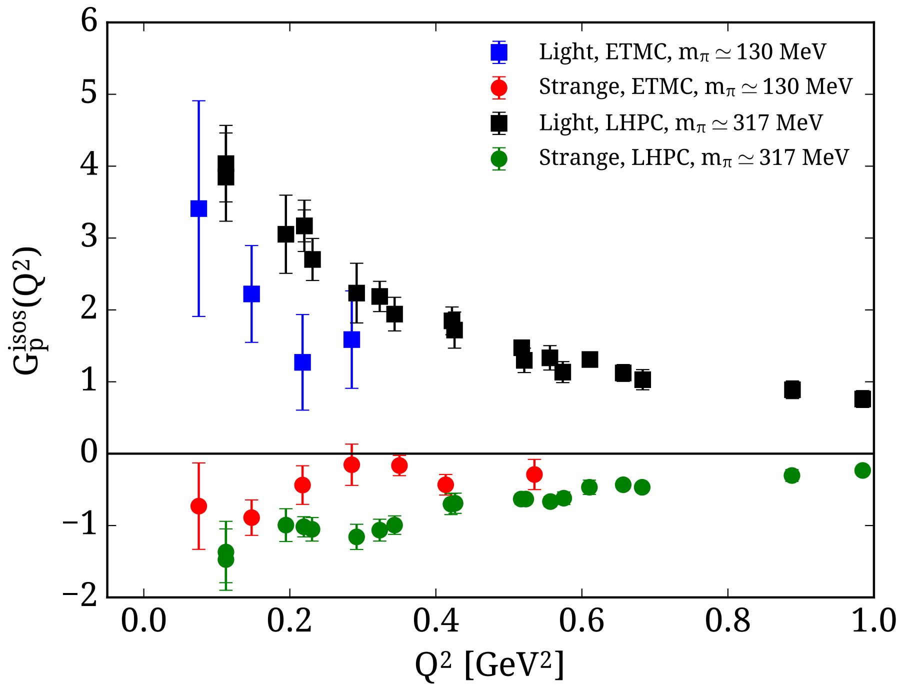


- ETMC using $N_f = 2$ twisted mass fermions (TMF), $a = 0.093$ fm, $48^3 \times 96$, C. A. *et al.* (2016), arXiv:1702.00984
- LHPC mixed action of $N_f = 2+1$ staggered sea and domain wall fermions, $a = 0.12$ fm, $28^3 \times 64$, Bratt *et al.* (2010), arXiv:1001.3620
- LHPC using $N_f = 2+1$ clover fermions, $a = 0.114$ fm, $32^3 \times 96$, J. Green *et al.*, (2017), arXiv:1703.06703
- PNDME using mixed action of $N_f = 2+1+1$ HISQ sea fermions and clover valence and $a = 0.09$ ($64^3 \times 96$), and 0.06 fm, Yong-Chull Jang, *et al.* Lattice 2016
- Mainz using $N_f = 2$ clover with $a = 0.05$ fm $48^3 \times 96$ Lattice 2016

Comparison of nucleon disconnected axial form factors

Disconnected contributions

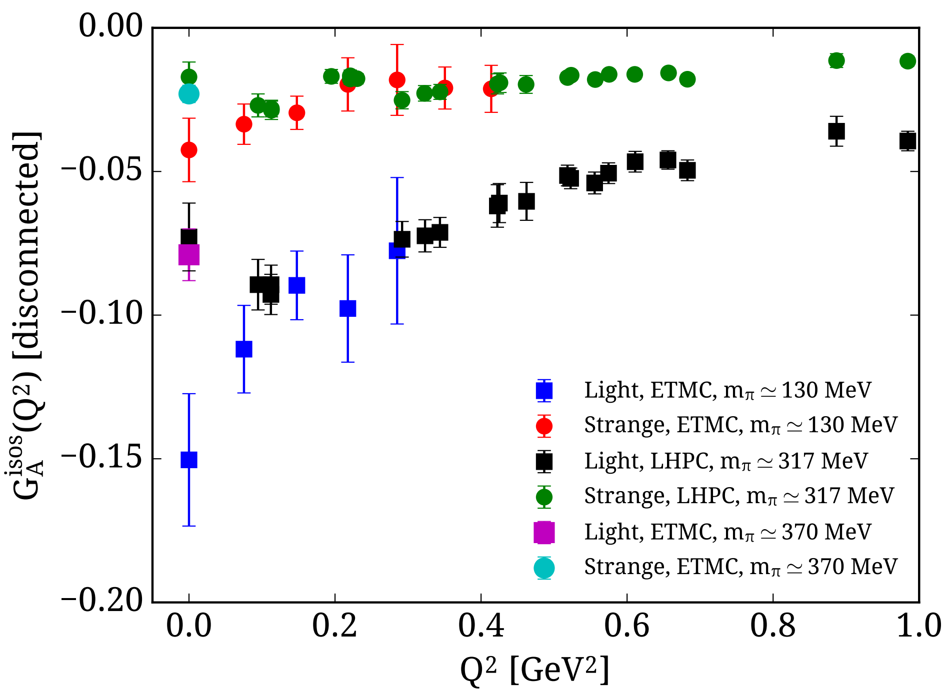


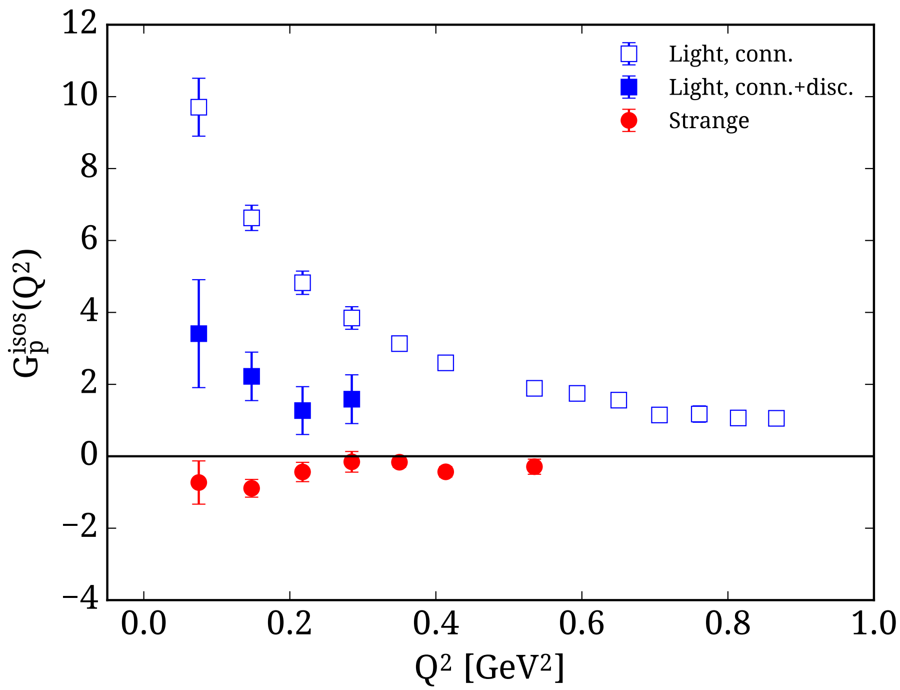


- ETMC using $N_f = 2$ twisted mass fermions (TMF), $a = 0.093$ fm, $48^3 \times 96$, $a = 0.093$ fm, $m_\pi = 131$ MeV, 855,000 statistics
- LHPC using $N_f = 2 + 1$ clover fermions, $32^3 \times 96$, $a = 0.114$ fm, $m_\pi = 317$ MeV, 98,700 statistics

Comparison of nucleon disconnected axial form factors

Disconnected contributions





Large disconnected contributions

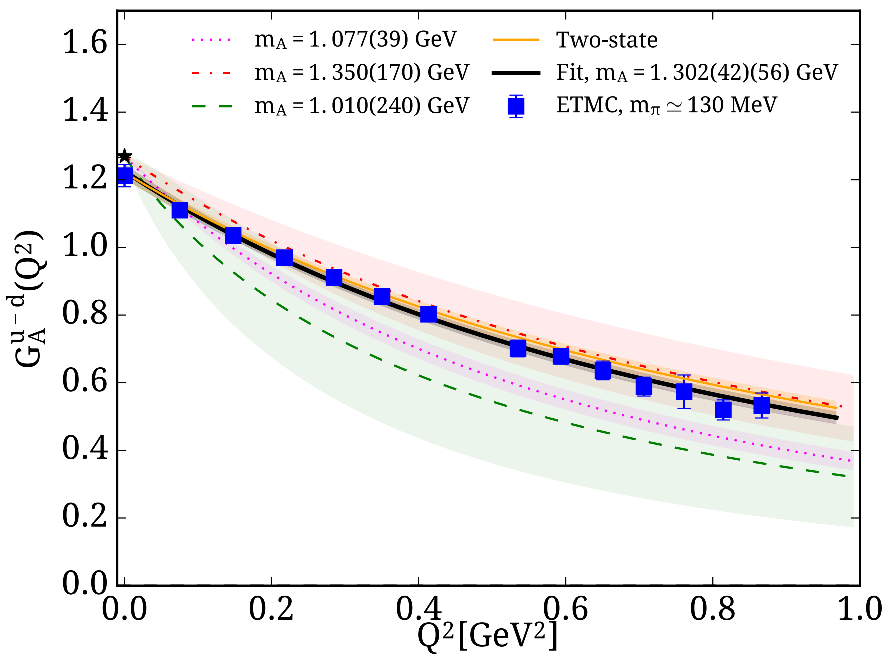
- ETMC using $N_f = 2$ twisted mass fermions (TMF), $a = 0.093$ fm, $48^3 \times 96$, $a = 0.093$ fm, $m_\pi = 131$ MeV, 855,000 statistics
- LHPC using $N_f = 2 + 1$ clover fermions, $32^3 \times 96$, $a = 0.114$ fm, $m_\pi = 317$ MeV, 98,700 statistics

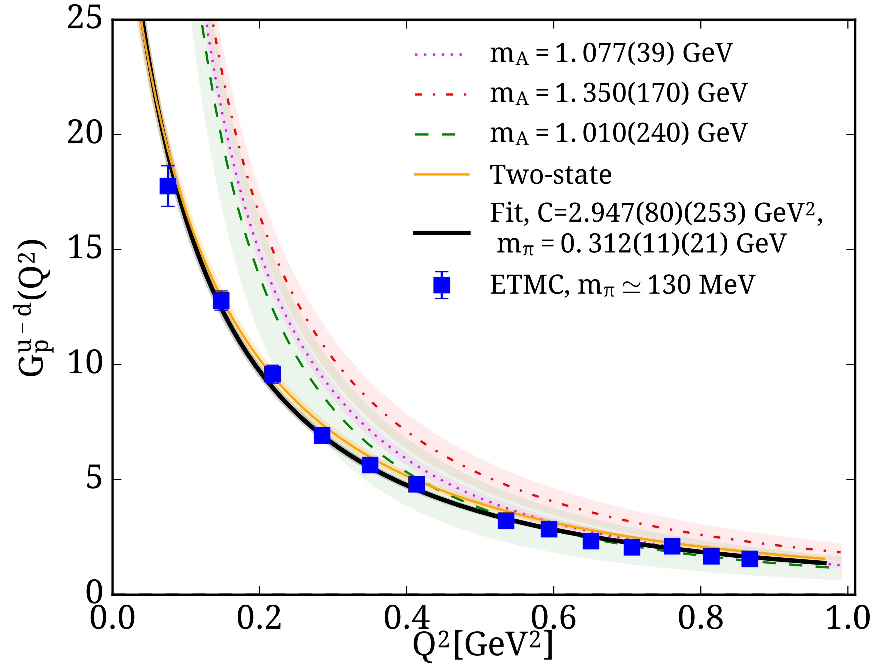
Fits to isovector lattice QCD results

Perform fits to lattice results using:

$$G_A(Q^2) = \frac{g_A}{(1+\frac{Q^2}{m_A^2})^2}, \qquad G_p(Q^2) = G_A(Q^2)\frac{C}{Q^2+m_p^2}$$

If pole dominance is satsfied then $C = 4m_N^2$ and $m_p = m_\pi \rightarrow$ allow to vary

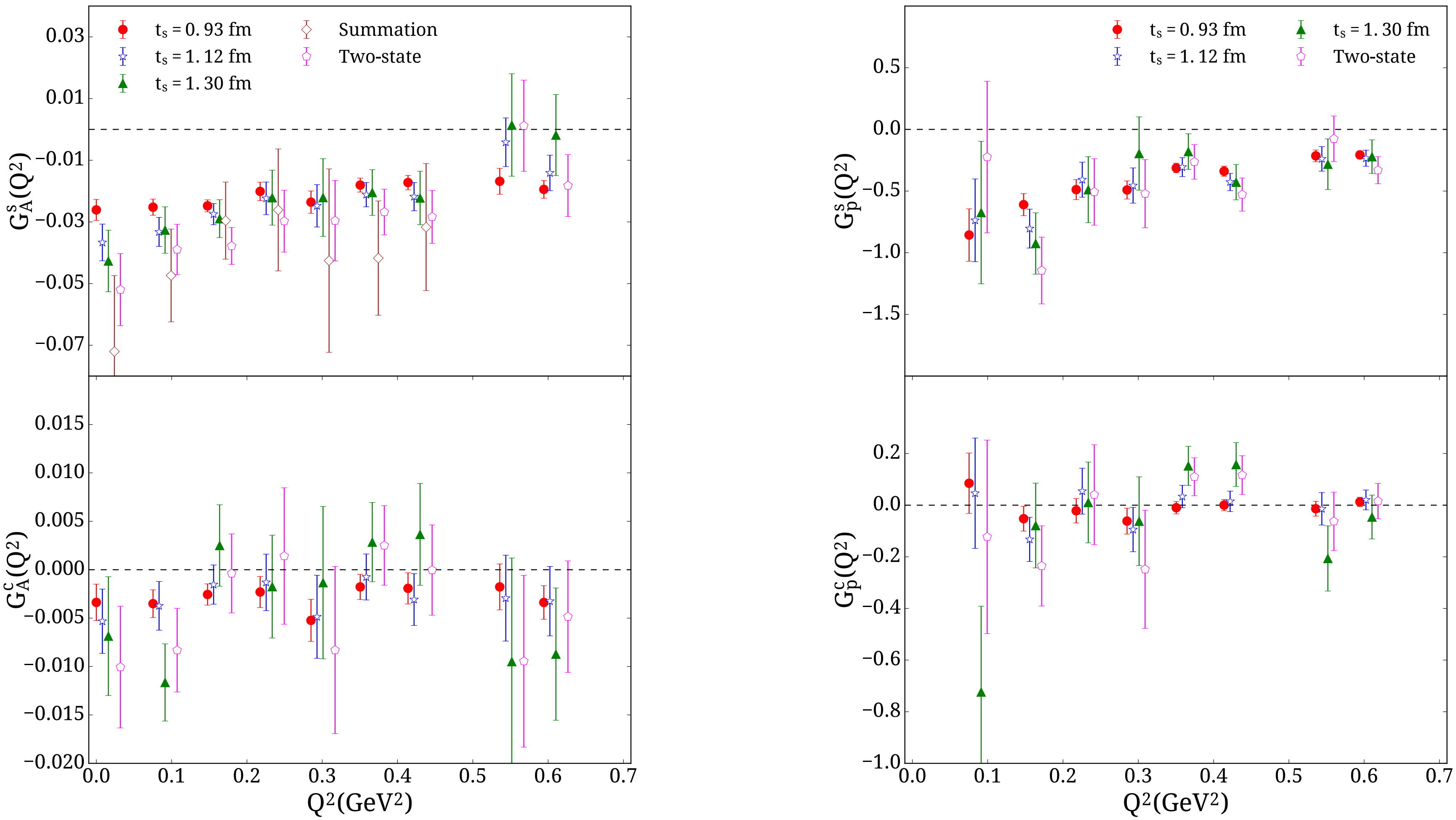




Experimental bands

- $m_A = 1.077(39)$ GeV A. Liesenfeld *et al.*, using pion-photoproduction data, Phys. Lett., B468:20, 1999.
- $m_A = 1.350(170)$ GeV using $\mu - \nu$ charged current quasielastic scattering (MiniBooNE), Aguilar-Arevalo *et al.*, Phys. Rev. D 81 (2010).
- $m_A = 1.010(240)$ GeV using neutrino-nucleus scattering, Aaron S. Meyer, Minerba Betancourt, Richard Gran, and Richard J. Hill, Phys. Rev., D93(11) 113015 (2016)

Strange and charm quark axial form factors



Strange and charm quark axial form factors

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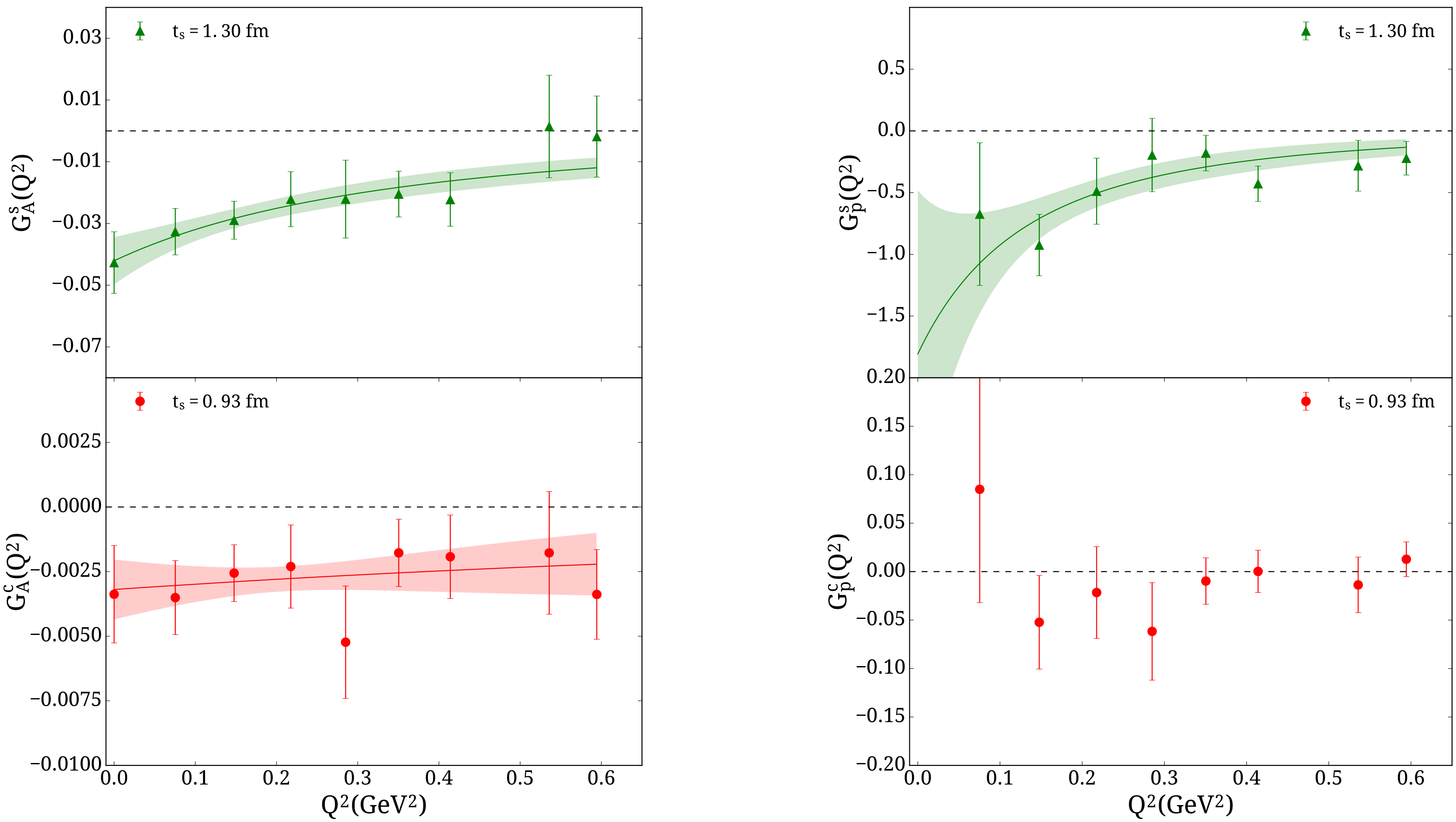
$$G_A(Q^2) = \frac{g_A}{(1+\frac{Q^2}{m_A^2})^2}, \qquad G_p(Q^2) = G_A(Q^2)\frac{C}{Q^2+m_p^2}$$



Conclusions and Future Perspectives

- Computation of g_A at the physical point allows direct comparison with experiment
- Provide predictions for g_s, g_T, etc.
- Computation of $G_A(Q^2)$ and $G_p(Q^2)$
  - Disconnected contributions non-zero and especially large for G_p
  - Excited state contributions and finite volume dependence need to be further investigated especially for G_p
- A number of collaborations are in process of finalizing their results including the physical point.

Extended Twisted Mass Collaboration

European Twisted Mass Collaboration (ETMC)

Cyprus (Univ. of Cyprus, Cyprus Inst.), France (Orsay, Grenoble), Germany (Berlin/Zeuthen, Bonn, Frankfurt, Hamburg, Münster), Italy (Rome I, II, III, Trento), Netherlands (Groningen), Poland (Poznan), Spain (Valencia), Switzerland (Bern), UK (Liverpool)

Collaborators:
S. Bacchio, M. Constantinou, J. Finkenrath, K. Hadjiyiannakou, K.Jansen, Ch. Kallidonis, G. Koutsou, A. Vaquero