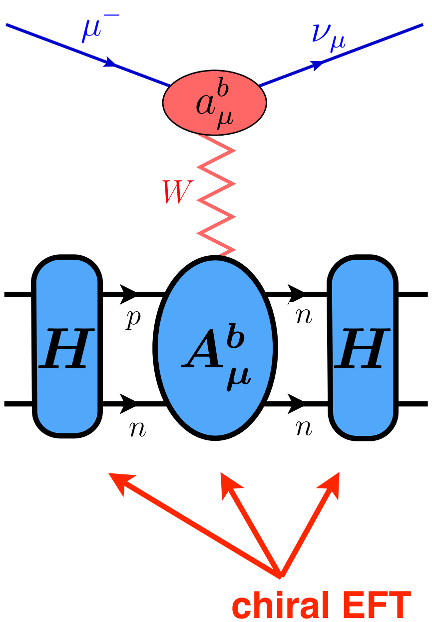
Evgeny Epelbaum, RUB

IPPP/NuSTEC topical meeting on neutrino-nucleus scattering
Durham, UK, April 18-20, 2017

Towards ab-initio chiral EFT calculations of neutrino-nucleus reactions

— based on: Krebs, EE, Meißner, Annals Phys. 378 (2017) 317 —



- Introduction
- Nuclear Hamiltonian
- An overview of electroweak current operators
- Ongoing work on few-N systems
- Summary and outlook

Bundesministerium für Bildung und Forschung
DFG Deutsche Forschungsgemeinschaft
erc European Research Council

The framework

Chiral Perturbation Theory: expansion of the scattering amplitude in powers of Q,

Weinberg, Gasser, Leutwyler, Meißner, ...

$$Q = \frac{\text{momenta of external particles or } M_\pi \sim 140 \text{ MeV}}{\text{breakdown scale } \Lambda_b \; [\sim 600\ldots700 \text{ MeV}]}$$

Write down L_{eff} [π, N, …],
identify relevant diagrams at a given order,
do Feynman calculus,
fit LECs to exp data,
make predictions…

The framework

Chiral Perturbation Theory: expansion of the scattering amplitude in powers of Q,

Weinberg, Gasser, Leutwyler, Meißner, ...

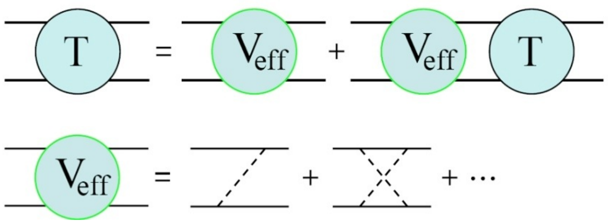
$$Q = \frac{\text{momenta of external particles or } M_\pi \sim 140 \text{ MeV}}{\text{breakdown scale } \Lambda_b \ [\sim 600\ldots700 \text{ MeV}]}$$

Write down $L_{eff}\,[\pi, N, \ldots]$,
identify relevant diagrams at a given order,
do Feynman calculus,
fit LECs to exp data,
make predictions…

Chiral EFT for nuclei: expansion for nuclear forces + resummation (Schrödinger equation)

Weinberg, van Kolck, Kaiser, EE, Glöckle, Meißner, Entem, Machleidt, Krebs, ...

$$\left[\left(\sum_{i=1}^{A} \frac{-\vec{\nabla}_i^2}{2m_N} + \mathcal{O}(m_N^{-3})\right) + \underbrace{V_{2N} + V_{3N} + V_{4N} + \ldots}_{\text{derived in ChPT}}\right]|\Psi\rangle = E|\Psi\rangle$$



- error estimations
- systematically improvable
- unified approach for ππ, πN, NN
- consistent many-body forces and currents

Notice: LS equation with a truncated potential requires infinitely many CTs are needed to absorb all UV divergences from iterations

⟶ one is forced to introduce a finite UV cutoff **[see: Lepage, nucl-th/9607029]**

The framework

- Heavy-baryon chiral Lagrangian with pions and nucleons as the only explicit DOF:

number of derivatives or M_π

$$\mathcal{L} = \mathcal{L}_\pi^{(2)} + \underbrace{\mathcal{L}_\pi^{(4)}}_{l_i} + \mathcal{L}_{\pi N}^{(1)} + \underbrace{\mathcal{L}_{\pi N}^{(2)}}_{c_i} + \underbrace{\mathcal{L}_{\pi N}^{(3)}}_{d_i} + \underbrace{\mathcal{L}_{\pi N}^{(4)}}_{e_i} + \underbrace{\mathcal{L}_{NN}^{(0)}}_{\tilde{C}_{1,2}} + \underbrace{\mathcal{L}_{(\pi)NN}^{(1)}}_{c_D} + \underbrace{\mathcal{L}_{(\pi)NN}^{(2)}}_{C_i} + \underbrace{\mathcal{L}_{(\pi)NN}^{(4)}}_{D_i} + \underbrace{\mathcal{L}_{NNN}^{(0)}}_{c_E} + \ldots$$

Low-energy constants:

— the pionic ones, l_i, are (sufficiently) well known

— the πN LECs relevant for nuclear forces to N⁴LO, $c_{1,2,3,4}$, $d_{1+2,3,5,14-15}$ and $e_{14,17}$, are fairly well known from πN scattering

→ long-range parts of the forces are predicted in a parameter-free way

— exchange currents at N³LO involve further LECs $d_{2,6,8,9,15,21,22,23}$; the e.m. ones are known from radiative π-prod., the axial ones $d_{2,6}$ $d_{15} - 2d_{23}$ are unknown (?)

— (bare) LECs $\tilde{C}_{1,2}$, C_i and D_i are extracted from NN data

— (bare) LECs c_D and c_E can be fixed from A > 2-data

The framework

- Heavy-baryon chiral Lagrangian with pions and nucleons as the only explicit DOF:

number of derivatives or M_π

$$\mathcal{L} = \mathcal{L}_\pi^{(2)} + \underbrace{\mathcal{L}_\pi^{(4)}}_{l_i} + \mathcal{L}_{\pi N}^{(1)} + \underbrace{\mathcal{L}_{\pi N}^{(2)}}_{c_i} + \underbrace{\mathcal{L}_{\pi N}^{(3)}}_{d_i} + \underbrace{\mathcal{L}_{\pi N}^{(4)}}_{e_i} + \underbrace{\mathcal{L}_{NN}^{(0)}}_{\tilde{C}_{1,2}} + \underbrace{\mathcal{L}_{(\pi)NN}^{(1)}}_{c_D} + \underbrace{\mathcal{L}_{(\pi)NN}^{(2)}}_{C_i} + \underbrace{\mathcal{L}_{(\pi)NN}^{(4)}}_{D_i} + \underbrace{\mathcal{L}_{NNN}^{(0)}}_{c_E} + \ldots$$

Low-energy constants:

— the pionic ones, l_i, are (sufficiently) well known

— the πN LECs relevant for nuclear forces to N⁴LO, $c_{1,2,3,4}$, $d_{1+2,3,5,14-15}$ and $e_{14,17}$, are fairly well known from πN scattering

→ long-range parts of the forces are predicted in a parameter-free way

— exchange currents at N³LO involve further LECs $d_{2,6,8,9,15,21,22,23}$; the e.m. ones are known from radiative π-prod., the axial ones $d_{2,6}$ $d_{15} - 2d_{23}$ are unknown (?)

— (bare) LECs $\tilde{C}_{1,2}$, C_i and D_i are extracted from NN data

— (bare) LECs c_D and c_E can be fixed from A > 2-data

- Nuclear potentials & currents are obtained by integrating out pion fields using the chiral expansion **[method of UT, TOPT, matching to S-matrix]**

The framework

- Heavy-baryon chiral Lagrangian with pions and nucleons as the only explicit DOF:

number of derivatives or M_π

$$\mathcal{L} = \mathcal{L}_\pi^{(2)} + \underbrace{\mathcal{L}_\pi^{(4)}}_{l_i} + \mathcal{L}_{\pi N}^{(1)} + \underbrace{\mathcal{L}_{\pi N}^{(2)}}_{c_i} + \underbrace{\mathcal{L}_{\pi N}^{(3)}}_{d_i} + \underbrace{\mathcal{L}_{\pi N}^{(4)}}_{e_i} + \underbrace{\mathcal{L}_{NN}^{(0)}}_{\tilde{C}_{1,2}} + \underbrace{\mathcal{L}_{(\pi)NN}^{(1)}}_{c_D} + \underbrace{\mathcal{L}_{(\pi)NN}^{(2)}}_{C_i} + \underbrace{\mathcal{L}_{(\pi)NN}^{(4)}}_{D_i} + \underbrace{\mathcal{L}_{NNN}^{(0)}}_{c_E} + \ldots$$

Low-energy constants:

— the pionic ones, l_i, are (sufficiently) well known

— the πN LECs relevant for nuclear forces to N⁴LO, $c_{1,2,3,4}$, $d_{1+2,3,5,14-15}$ and $e_{14,17}$, are fairly well known from πN scattering

→ long-range parts of the forces are predicted in a parameter-free way

— exchange currents at N³LO involve further LECs $d_{2,6,8,9,15,21,22,23}$; the e.m. ones are known from radiative π-prod., the axial ones $d_{2,6}$ $d_{15} - 2d_{23}$ are unknown (?)

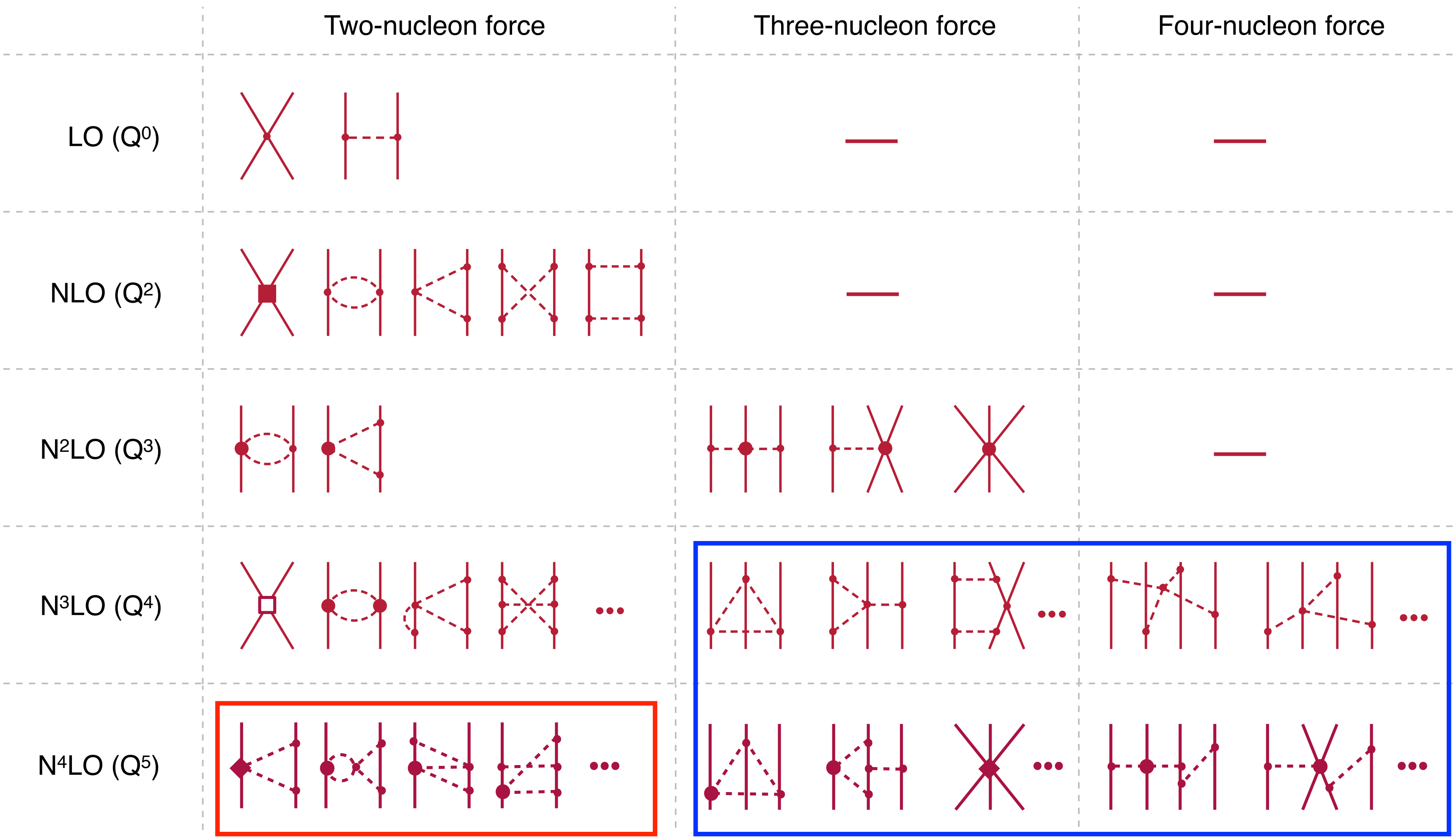
— (bare) LECs $\tilde{C}_{1,2}$, C_i and D_i are extracted from NN data

— (bare) LECs c_D and c_E can be fixed from A > 2-data

- Nuclear potentials & currents are obtained by integrating out pion fields using the chiral expansion **[method of UT, TOPT, matching to S-matrix]**

- Treating Δ(1232) as an explicit DOF allows one to re-sum certain classes of diagrams and to improve convergence of the EFT expansion

Chiral expansion of the nuclear forces [W-counting]



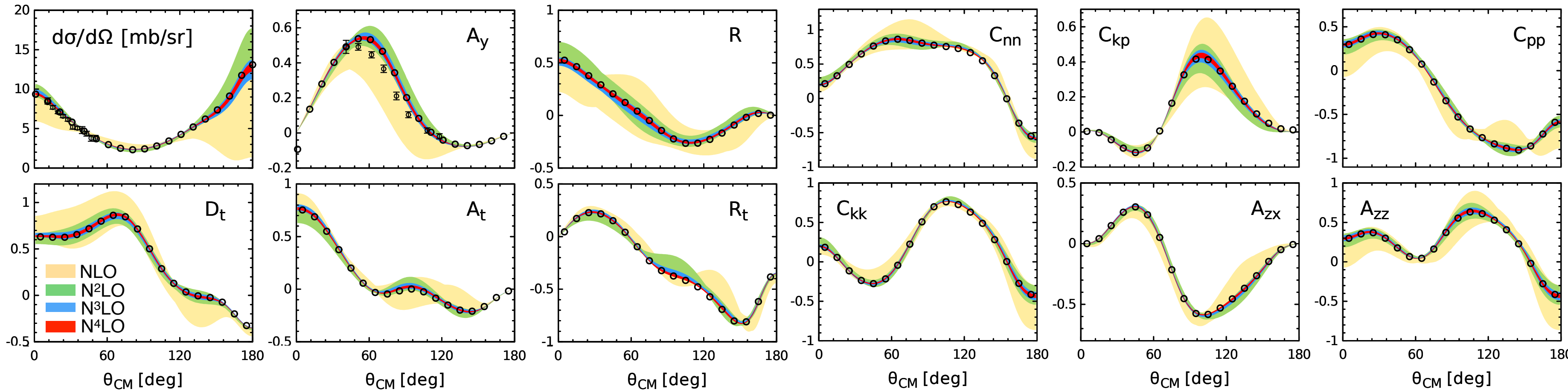
EE, Krebs, Meißner, PRL 115 (2015) 122301
Entem et al., PRC 91 (2015) 014002; arXiv:1703.05454

under investigation…

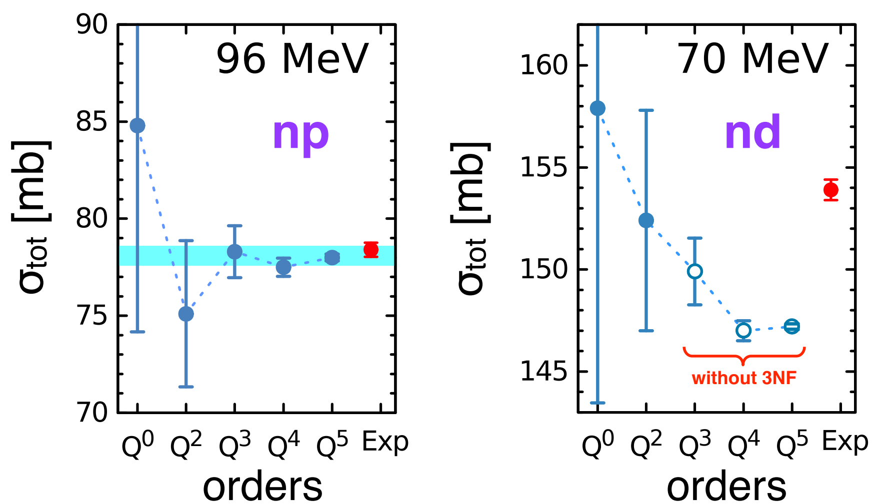
Chiral EFT in the strong sector [W-counting]

- Status in the strong sector: accurate and precise NN forces at fifth order in the chiral expansion [EE, Krebs, Meißner, PRL 115 (2015) 122301; Entem et al., PRC 91 (2015) 014002, arXiv:1703.05454]

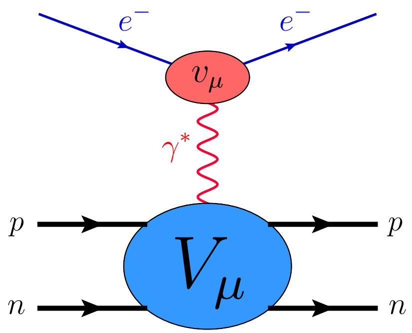
Selected neutron-proton observables at E_{lab} = 143 MeV



- Clear evidence of the chiral 2π exchange
- Simple approach to quantify truncation errors (without relying on Λ-variation)
- Inclusion of the three-nucleon force and extension of calculations to heavier nuclei in progress [LENPIC]

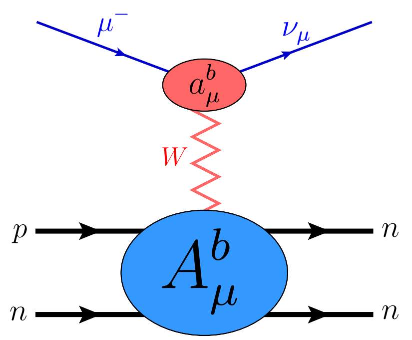


Nuclear current operators in chiral EFT



EM currents:

— Park, Min, Rho '95 (threshold kinematics, incomplete…)
— Kölling, EE, Krebs, Meißner, PRC 80 (09) 045502; 86 (12) 047001
— Jlab-Pisa group (TOPT), Pastore et al. '08 - '11



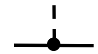
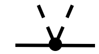
Axial currents:

— Park, Min, Rho '93 (threshold kinematics, incomplete…)
— Krebs, EE, Meißner, Annals Phys. 378 (2017) 317
— Jlab-Pisa group (TOPT), Baroni et al. '16

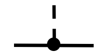
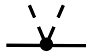
(Our) requirements on the current operators

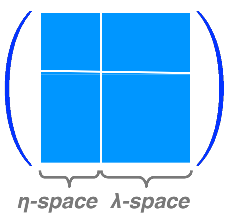
- must be **off-shell consistent with the forces**
- should be **renormalized** (exploit unitary ambiguity)
- (cutoff) regularization of the forces and currents should **maintain the symmetry** (cont. equation)

Method of UT for nuclear forces

- Begin with the $\mathcal{L}_{\text{eff}}[\pi,N]$ without external fields
- Canonical formalism: $\mathcal{L}_{\text{eff}}[\pi,N] \longrightarrow H[\pi,N] =$  +  + ...

Method of UT for nuclear forces

- Begin with the $\mathcal{L}_{\rm eff}\,[\pi,N]$ without external fields
- Canonical formalism: $\mathcal{L}_{\rm eff}\,[\pi,N] \longrightarrow H[\pi,N] =$  +  + ...
- Apply UT in Fock space to decouple purely nucleonic states [model space] from the rest

$$H \rightarrow \tilde{H} = U^\dagger$$ 

$$U = \begin{pmatrix} \tilde{H}_{\rm nucl} & 0 \\ 0 & \tilde{H}_{\rm rest} \end{pmatrix}$$

Using Okubo's minimal parametrization of U in terms of $A = \lambda A \eta$ leads to the

decoupling equation: $$\lambda(H - [A,\ H] - AHA)\eta = 0$$

which is solved perturbatively by means of the chiral expansion

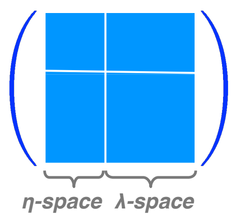
Method of UT for nuclear forces

- Begin with the $\mathcal{L}_{\rm eff}\,[\pi,N]$ without external fields

- Canonical formalism: $\mathcal{L}_{\rm eff}\,[\pi,N]$ ⟶ $H[\pi,N]$ =  + …

- Apply UT in Fock space to decouple purely nucleonic states [model space] from the rest

$$H \;\to\; \tilde{H} \;=\; U^\dagger$$



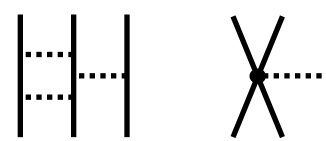
$$U \;=\; \begin{pmatrix} \tilde{H}_{\rm nucl} & 0 \\ 0 & \tilde{H}_{\rm rest} \end{pmatrix}$$

Using Okubo's minimal parametrization of U in terms of $A = \lambda A \eta$ leads to the

decoupling equation: $$\lambda(H - [A,\;H] - AHA)\eta = 0$$

which is solved perturbatively by means of the chiral expansion

- Apply all possible additional UTs on the η-subspace consistent with a given chiral order [6 angles α_i for static N^3LO contributions]



- Renormalizability of the potentials [all 1/(d-4) poles must be canceled by the c.t. from $\mathcal{L}_{\rm eff}$]

⟶ fixes some of the α_i and leads to unique (static) expressions

Exchange currents using the method of UT

- Switch on external sources $s,\ p,\ r_\mu,\ l_\mu$ and consider ***local*** chiral rotations:

$$r_\mu \;\to\; r'_\mu = R\, r_\mu R^\dagger + iR\,\partial_\mu R^\dagger\,, \qquad l_\mu \;\to\; l'_\mu = L\, l_\mu L^\dagger + iL\,\partial_\mu L^\dagger\,,$$
$$s + i\,p \;\to\; s' + i\,p' = R(s + i\,p)L^\dagger\,, \qquad s - i\,p \;\to\; s' - i\,p' = L(s - i\,p)R^\dagger$$

The sources can be conveniently rewritten via $v_\mu = \frac{1}{2}\left(r_\mu + l_\mu\right)\,, \quad a_\mu = \frac{1}{2}\left(r_\mu - l_\mu\right)$ with:

$$v_\mu = v_\mu^{(s)} + \frac{1}{2}\boldsymbol{\tau}\cdot\boldsymbol{v}_\mu, \qquad a_\mu = \frac{1}{2}\boldsymbol{\tau}\cdot\boldsymbol{a}_\mu, \qquad s = s_0 + \boldsymbol{\tau}\cdot\boldsymbol{s}, \qquad p = p_0 + \boldsymbol{\tau}\cdot\boldsymbol{p}$$

Exchange currents using the method of UT

- Switch on external sources $s,\ p,\ r_\mu,\ l_\mu$ and consider ***local*** chiral rotations:

$$r_\mu \;\to\; r'_\mu = R\,r_\mu R^\dagger + iR\,\partial_\mu R^\dagger\,, \qquad l_\mu \;\to\; l'_\mu = L\,l_\mu L^\dagger + iL\,\partial_\mu L^\dagger\,,$$
$$s + i\,p \;\to\; s' + i\,p' = R(s + i\,p)L^\dagger\,, \qquad s - i\,p \;\to\; s' - i\,p' = L(s - i\,p)R^\dagger$$

The sources can be conveniently rewritten via $v_\mu = \frac{1}{2}(r_\mu + l_\mu)\,, \quad a_\mu = \frac{1}{2}(r_\mu - l_\mu)$ with:

$$v_\mu = v_\mu^{(s)} + \frac{1}{2}\boldsymbol{\tau}\cdot\boldsymbol{v}_\mu, \qquad a_\mu = \frac{1}{2}\boldsymbol{\tau}\cdot\boldsymbol{a}_\mu, \qquad s = s_0 + \boldsymbol{\tau}\cdot\boldsymbol{s}, \qquad p = p_0 + \boldsymbol{\tau}\cdot\boldsymbol{p}$$

already known from the strong sector…

- (Naive) attempt: calculate $\tilde{H} \to \tilde{H}[a,v,s,p] = U^\dagger H[a,v,s,p]U$ and extract the nuclear currents via $V_\mu^a(\vec{x}\,) = \frac{\delta\tilde{H}}{\delta v_a^\mu(\vec{x},t)}, \quad A_\mu^a(\vec{x}\,) = \frac{\delta\tilde{H}}{\delta a_a^\mu(\vec{x},t)}$ at $v = a = p = \boldsymbol{s} = 0, \quad s_0 = m_q$.

However, the resulting currents turn out to be non-renormalizable…

⟶ Need to consider a more general class of UTs

Specifically, employ additional η-space UTs $U[a,v,s,p]$ subject to the constraint $U[0,0,m_q,0] = 1$

Notice: the resulting UTs are time-dependent, thus $H' \neq U^\dagger H U$. Indeed:

$$i\frac{\partial}{\partial t}\Psi = H\Psi \quad\longrightarrow\quad i\frac{\partial}{\partial t}\big(U^\dagger(t)\Psi\big) = \left[U^\dagger(t)HU(t) - U^\dagger(t)\left(i\frac{\partial}{\partial t}U(t)\right)\right]\big(U^\dagger(t)\Psi\big)$$

Exchange currents using the method of UT

- Thus, we have:

$$H_{\text{eff}}[a,\dot a,v,\dot v,s,\dot s,p,\dot p] \;=\; U^\dagger[a,v,s,p]U^\dagger_{\text{str}}H[a,v,s,p]U_{\text{str}}U[a,v,s,p] \;+\; \left(i\frac{\partial}{\partial t}U^\dagger[a,v,s,p]\right)U[a,v,s,p]$$

(to the order we are working [leading 1-loop for 2-body operators], can write 33 such UTs…)

Nuclear potentials are given by

$$V \;:=\; H_{\text{eff}}[v=\dot v=a=\dot a=p=\dot p=\dot s=0,\; s=m_q] \;-\; H_0$$

while the current operators in momentum space are defined as (in the Schrödinger picture):

$$V^j_\mu(\vec k,k_0) := \frac{\delta H_{\text{eff}}}{\delta v^\mu_j(\vec k,k_0)}, \qquad A^j_\mu(\vec k,k_0) := \frac{\delta H_{\text{eff}}}{\delta a^\mu_j(\vec k,k_0)}, \qquad P^j(\vec k,k_0) := \frac{\delta H_{\text{eff}}}{\delta p^j(\vec k,k_0)},$$

where the FT of the sources are given by $f(x) =: \int d^4q\, e^{-iq\cdot x}\, f(q)$ with $f=\{v^j_\mu,\; a^j_\mu,\; p^j\}$, H_{eff} is taken at $t=0$ & the functional derivatives are taken at $v=\dot v=a=\dot a=p=\dot p=\dot s=\boldsymbol{s}=0$ & $s_0=m_q$.

- It is straightforward to verify the proper relation to the S-matrix, e.g.:

$$\frac{\delta}{\delta a^{j\mu}(k_0,\vec k)}\langle\alpha|S|\beta\rangle \;=\; -i\,2\pi\delta(E_\alpha-E_\beta-k_0)\langle\alpha|A^j_\mu(k_0,\vec k)|\beta\rangle$$

Exchange currents using the method of UT

- **Manifestations of the symmetry (continuity equation)**

Start with the Schrödinger equation

$$i\frac{\partial}{\partial t}\Psi = H_{\rm eff}[a, \dot a, v, \dot v, s, \dot s, p, \dot p]\Psi$$

and perform a chiral rotation $a, \dot a, v, \dot v, s, \dot s, p, \dot p \quad \longrightarrow \quad a', \dot a', v', \dot v', s', \dot s', p', \dot p'$. For observables to remain unaffected, there must exist a UT on the Fock space such that:

$$i\frac{\partial}{\partial t}U^\dagger\Psi = H_{\rm eff}[a', \dot a', v', \dot v', s', \dot s', p', \dot p'\,]U^\dagger\Psi$$

That is, the Hamiltonians must be unitary equivalent:

$$H_{\rm eff}[a', \dot a', v', \dot v', s', \dot s', p', \dot p'\,] \;=\; U^\dagger H_{\rm eff}[a, \dot a, v, \dot v, s, \dot s, p, \dot p]U \;+\; \left(i\frac{\partial}{\partial t}U^\dagger\right)U$$

Matching both sides of this equation for infinitesimal transformations, one obtains the continuity equation for the axial current:

$$[H_{\rm str}, \boldsymbol{A}_0(\vec k, 0) - \frac{\partial}{\partial k_0}\vec k\cdot\vec{\boldsymbol{A}}(\vec k, k_0) + \frac{\partial}{\partial k_0}\,[H_{\rm str}, \boldsymbol{A}_0(\vec k, k_0)] + m_q\, i\frac{\partial}{\partial k_0}\boldsymbol{P}(\vec k, k_0)] \;=\; \vec k\cdot\vec{\boldsymbol{A}}(\vec k, 0) - m_q\, i\,\boldsymbol{P}(\vec k, 0)$$

The axial currents are linearly dependent on the energy transfer k_0 (at the considered order). This emerges from time derivatives of the UT and is unavoidable if the currents are required to be renormalized.

Exchange currents using the method of UT

- **Manifestations of the Poincaré invariance**

boost direction → boost angle

$$\exp\left(-i\,\vec{e}\cdot\vec{K}\theta\right)\boldsymbol{A}_\mu^H(x)\exp\left(i\,\vec{e}\cdot\vec{K}\theta\right)=\Lambda_\mu{}^\nu(\theta)\boldsymbol{A}_\nu^H\left(\Lambda^{-1}(\theta)x\right)$$

boost operator → current operator in the Heisenberg picture

This leads to the (on-shell) relation to be fulfilled by the current in momentum space

$$\delta(E_\alpha-E_\beta-k_0)\,\langle\alpha|\Big([-i\,\vec{e}\cdot\vec{K},\boldsymbol{A}_\mu(\vec{k})]+\boldsymbol{A}_\mu^\perp(\vec{k})-\vec{e}\cdot\vec{\nabla}_k[H_{\rm str},\boldsymbol{A}_\mu(\vec{k})]-\vec{e}\cdot\vec{k}\,\frac{\partial}{\partial k_0}\boldsymbol{A}_\mu(\vec{k})\Big)|\beta\rangle\;=\;0$$

where $\boldsymbol{A}^\perp:=(\vec{e}\cdot\vec{\boldsymbol{A}},\;\vec{e}\boldsymbol{A}_0)$.

Exchange currents using the method of UT

• Manifestations of the Poincaré invariance

boost direction → $\vec{e}$; boost angle → θ; boost operator → $\vec{K}$; current operator in the Heisenberg picture → $\boldsymbol{A}_\mu^H(x)$

$$\exp\left(-i\,\vec{e}\cdot\vec{K}\theta\right)\boldsymbol{A}_\mu^H(x)\exp\left(i\,\vec{e}\cdot\vec{K}\theta\right) = \Lambda_\mu{}^\nu(\theta)\boldsymbol{A}_\nu^H\left(\Lambda^{-1}(\theta)x\right)$$

This leads to the (on-shell) relation to be fulfilled by the current in momentum space

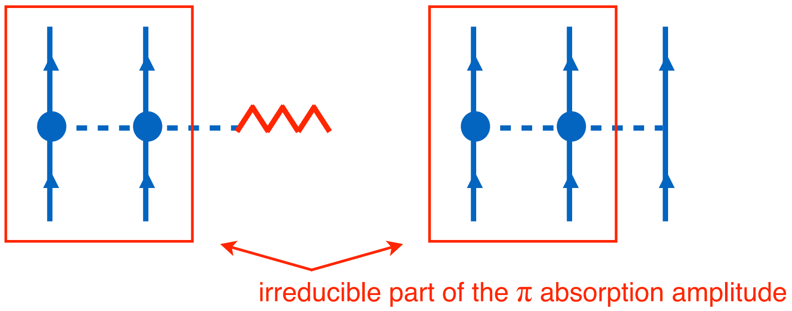
$$\delta(E_\alpha - E_\beta - k_0)\,\langle\alpha|\Big([-i\,\vec{e}\cdot\vec{K},\boldsymbol{A}_\mu(\vec{k})] + \boldsymbol{A}_\mu^\perp(\vec{k}) - \vec{e}\cdot\vec{\nabla}_k[H_{\rm str},\boldsymbol{A}_\mu(\vec{k})] - \vec{e}\cdot\vec{k}\,\frac{\partial}{\partial k_0}\boldsymbol{A}_\mu(\vec{k})\Big)|\beta\rangle = 0$$

where $\boldsymbol{A}^\perp := (\vec{e}\cdot\vec{\boldsymbol{A}},\ \vec{e}\boldsymbol{A}_0)$.

- Unitary ambiguity [33 UT's] is strongly reduced but not completely eliminated by the renormalizability requirement for the currents.

We further require that $\forall$ pion-pole contributions to the axial currents match the corresponding 1π-exchange contributions to the nuclear forces at the pion pole:

$$\lim_{q_i^2\to -M_\pi^2}(q_i^2+M_\pi^2)\left[H_{\rm str} - \vec{\boldsymbol{A}}(-\vec{q}_i)\cdot\left(-\frac{g_A}{2F_\pi^2}\vec{\sigma}_i\boldsymbol{\tau}_i\right)\right] = 0$$



With these constraints, the expressions for the currents are determined unambiguously.

Exchange currents using the method of UT

Summary of the main results of our calculations

[Krebs, EE, Meißner, Annals Phys. 378 (2017) 317]

- worked out completely the axial charge and current operators to Q^4 (N^3LO) relative to the leading 1-body term, i.e.:
  - — at the 2-loop level for 1-body operators,
  - — at the 1-loop level for 2-body operators,
  - — at tree level for 3-body operators

  (…about 250 diagrams…)

- worked out completely the corresponding pseudoscalar currents to the same chiral order

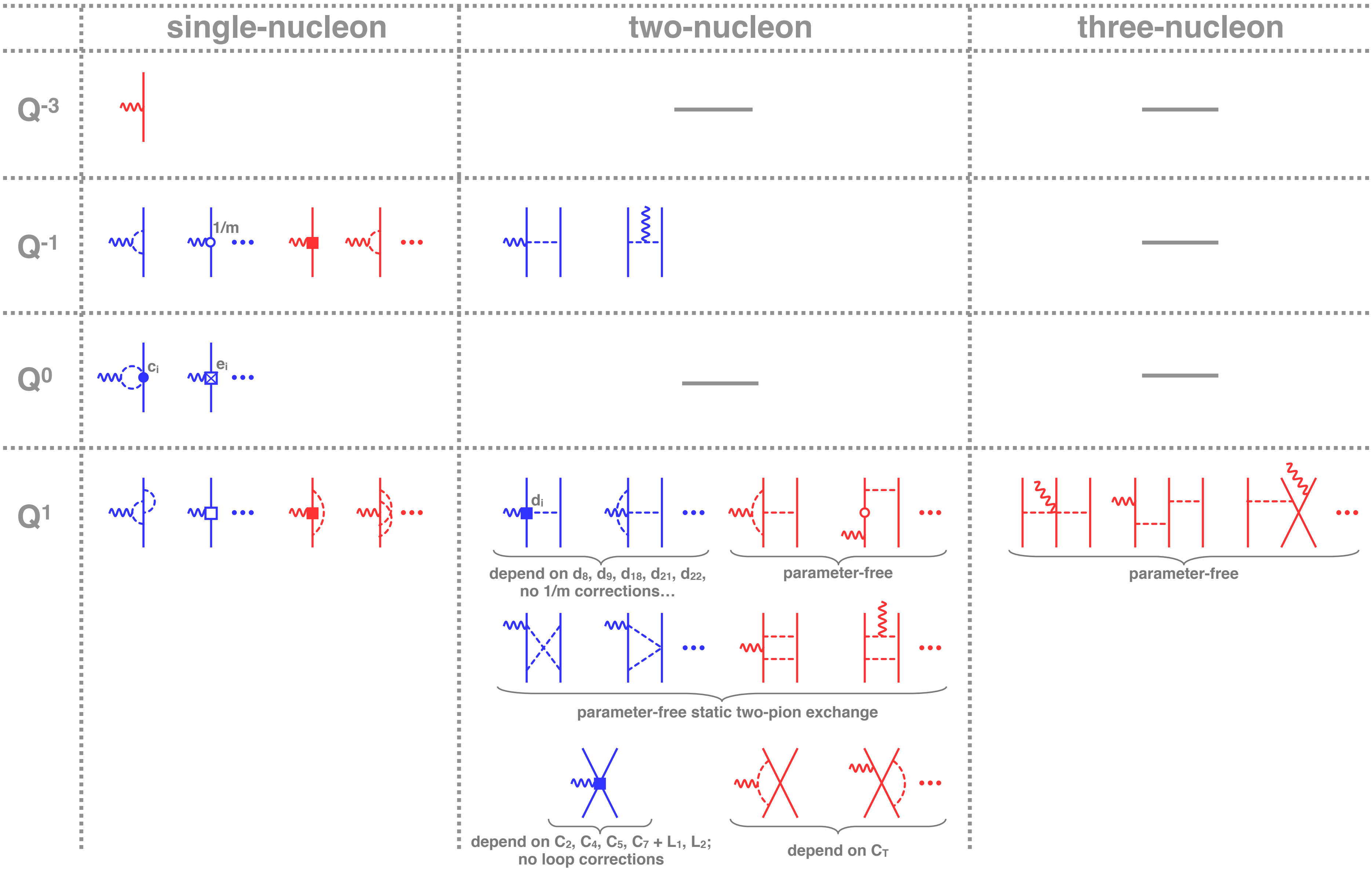
- explicitly verified the validity of the continuity equation for all contributions

- worked out the boost operator (to the required order) and verified the constraints from the Poincaré invariance

Electromagnetic currents

Kölling, EE, Krebs, Meißner, PRC 80 (09) 045502; PRC 86 (12) 047001

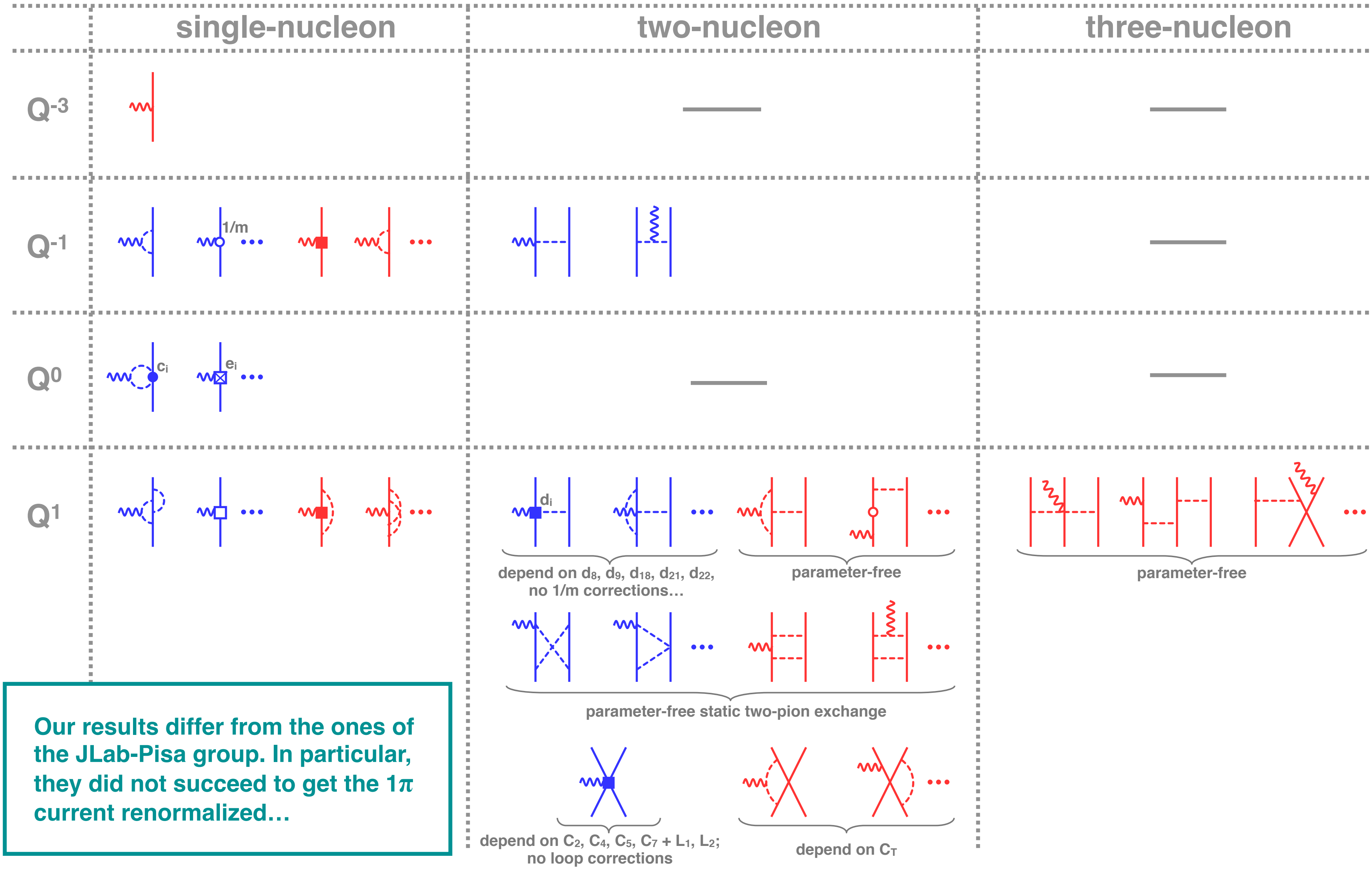
Chiral expansion of the electromagnetic **current** and **charge** operators



Electromagnetic currents

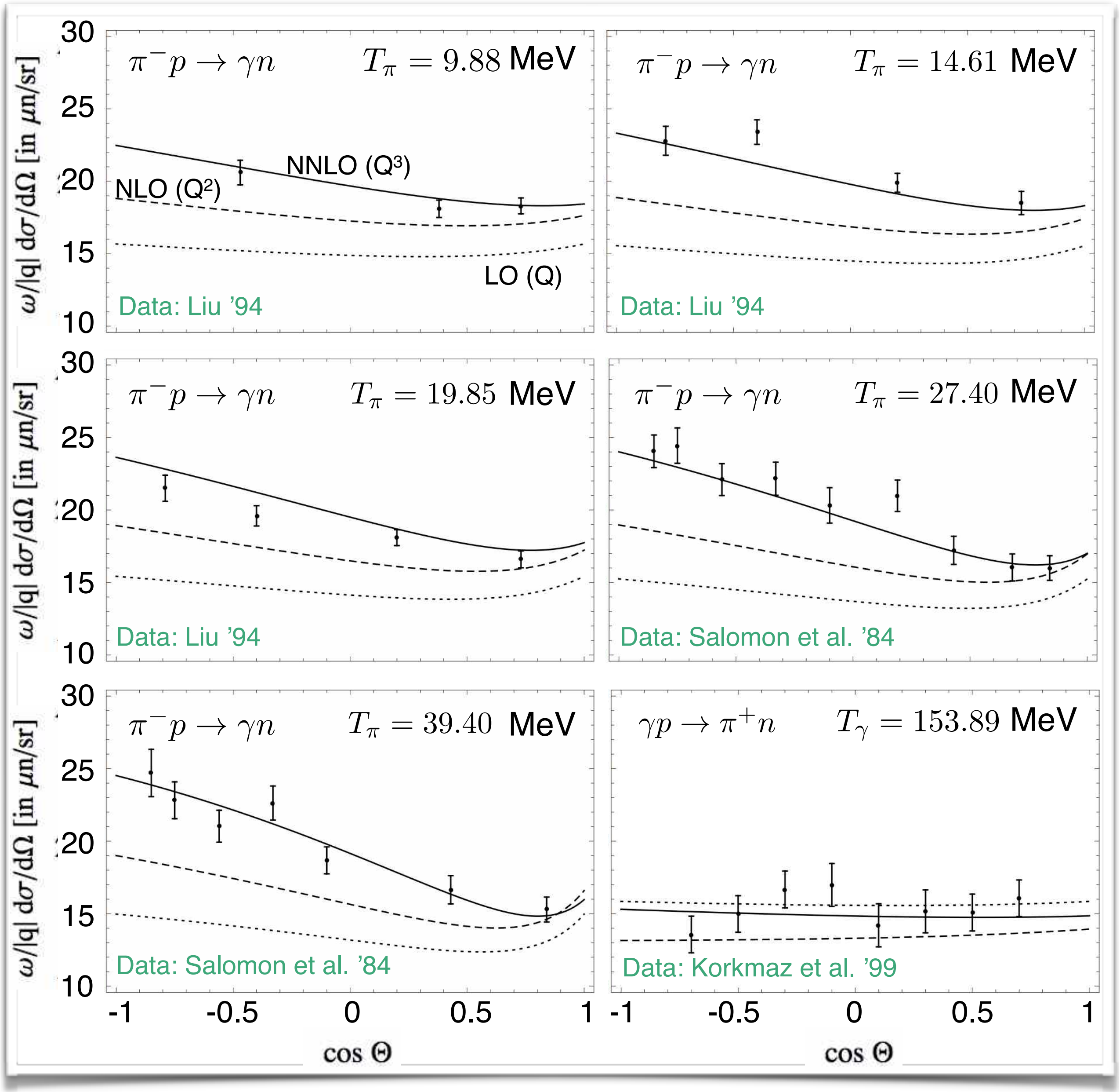
Kölling, EE, Krebs, Meißner, PRC 80 (09) 045502; PRC 86 (12) 047001

Chiral expansion of the electromagnetic **current** and **charge** operators



Our results differ from the ones of the JLab-Pisa group. In particular, they did not succeed to get the 1π current renormalized…

The low-energy constants



LECs entering the 1**π** current:

$\bar{l}_6,\ \bar{d}_8,\ \bar{d}_9,\ \bar{d}_{18},\ \bar{d}_{21},\ \bar{d}_{22}$

$\bar{l}_6$ - known from the **π** sector

$\bar{d}_{18}$ - known from GTD

$\bar{d}_{22}$ - from the axial radius:

$$\bar{d}_{22} = 2.2 \pm 0.2\ \text{GeV}^{-2}$$

$\bar{d}_9,\ \bar{d}_{21},\ \bar{d}_{22}$ - contribute to charged pion photoproduction (radiative capture)

Fearing et al.'00
Till Wolf, master thesis, Bochum, 2013

LEC [GeV⁻²]	Fearing *et al.*	Wolf
$\bar{d}_9$	2.5 ± 0.8	2.2 ± 0.9
$\bar{d}_{20}$	-1.5 ± 0.5	-3.2 ± 0.5
$2\bar{d}_{21} - \bar{d}_{22}$	5.7 ± 0.8	6.8 ± 1.0

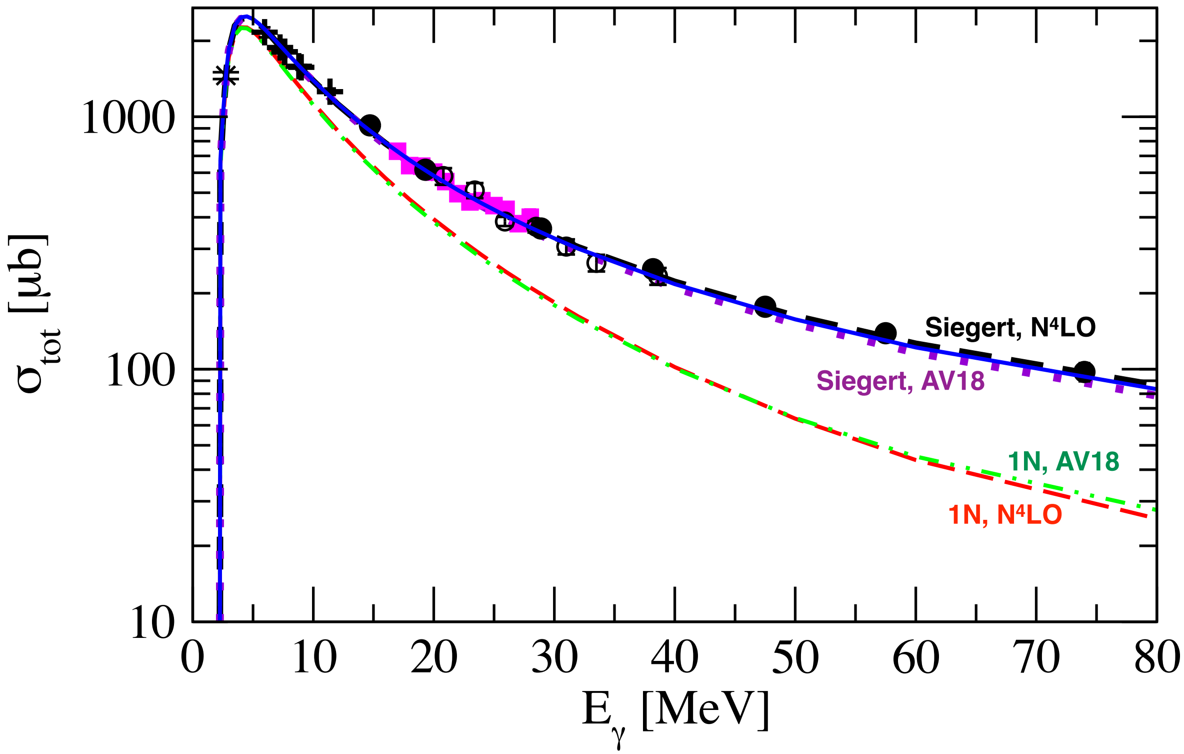
Some d_i's have been determined by Gasparyan, Lutz '10 (ChPT + disp. relations)

Electromagnetic exchange currents

Skibinski, Golak, Topolnicki, Witala, EE, Krebs, Kamada, Meißner, Nogga, PRC 93 (2016) 064002

- To maintain consistency between currents and forces (symmetry), we generate regularized longitudinal terms in the current via the continuity equation (i.e. Siegert approach).
- Transverse terms in the currents are to be regularized and included explicitly (in progress…)

Total cross section for the deuteron photo-disintegration reaction $\gamma + d \rightarrow p + n$

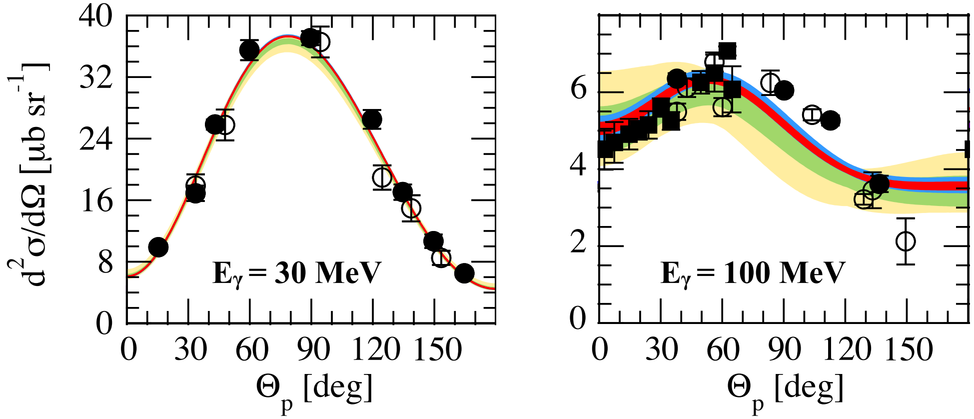


Electromagnetic exchange currents

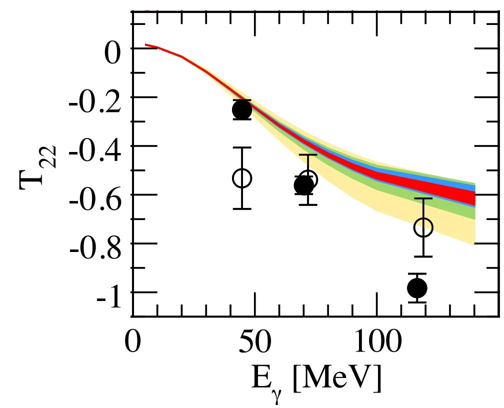
Skibinski, Golak, Topolnicki, Witala, EE, Krebs, Kamada, Meißner, Nogga, PRC 93 (2016) 064002

Deuteron photo-disintegration: $\gamma + d \rightarrow p + n$

Differential cross section

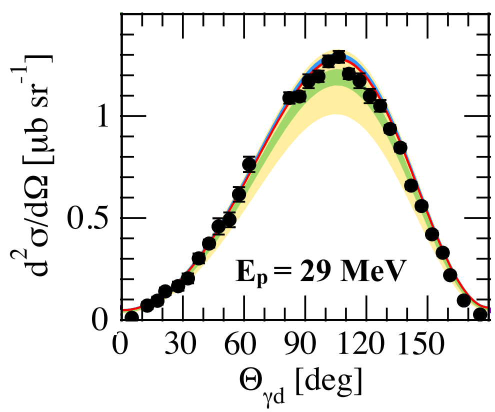


Deuteron analyzing power T_{22}

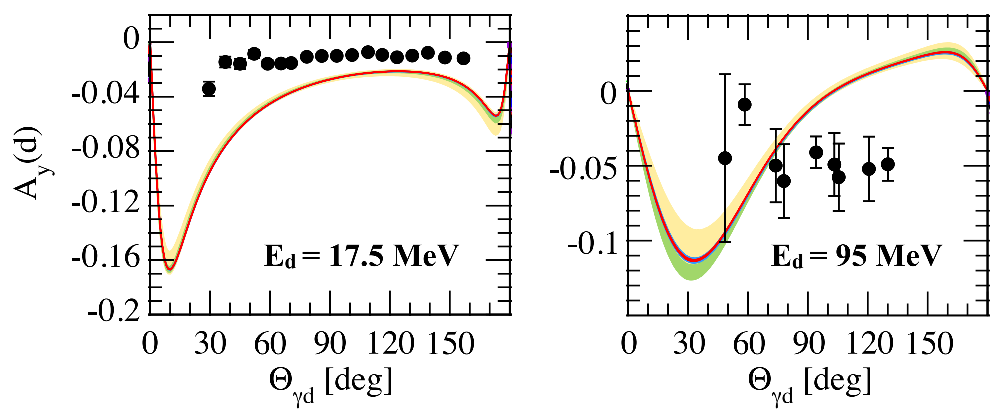


Nucleon-deuteron radiative capture: $p(n) + d \rightarrow {}^3H({}^3He) + \gamma$

Differential cross section



Deuteron vector analyzing power

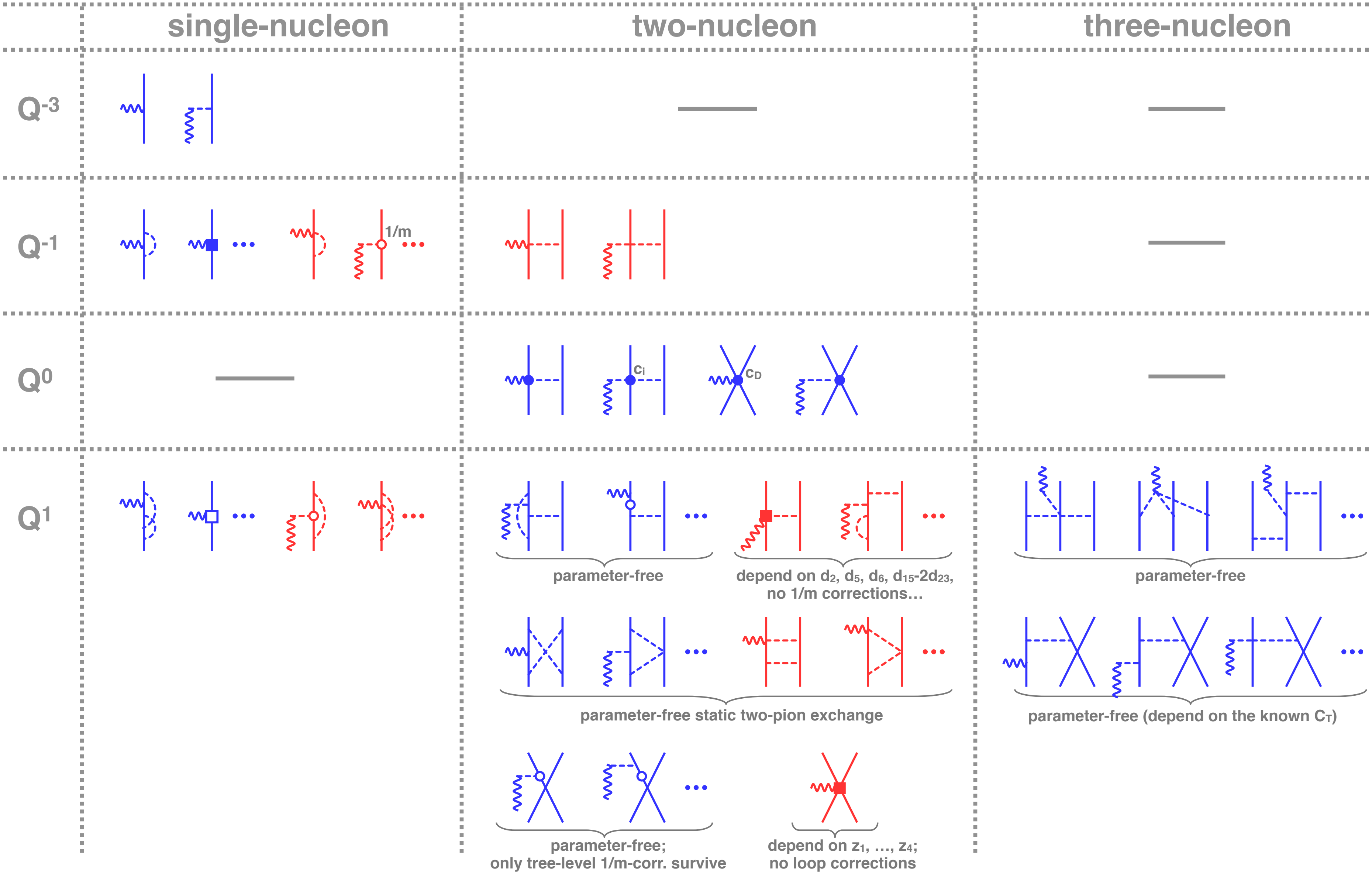


… explicit inclusion of exchange currents in progress…

Exchange axial currents

Krebs, EE, Meißner, Annals Phys. 378 (2017) 317

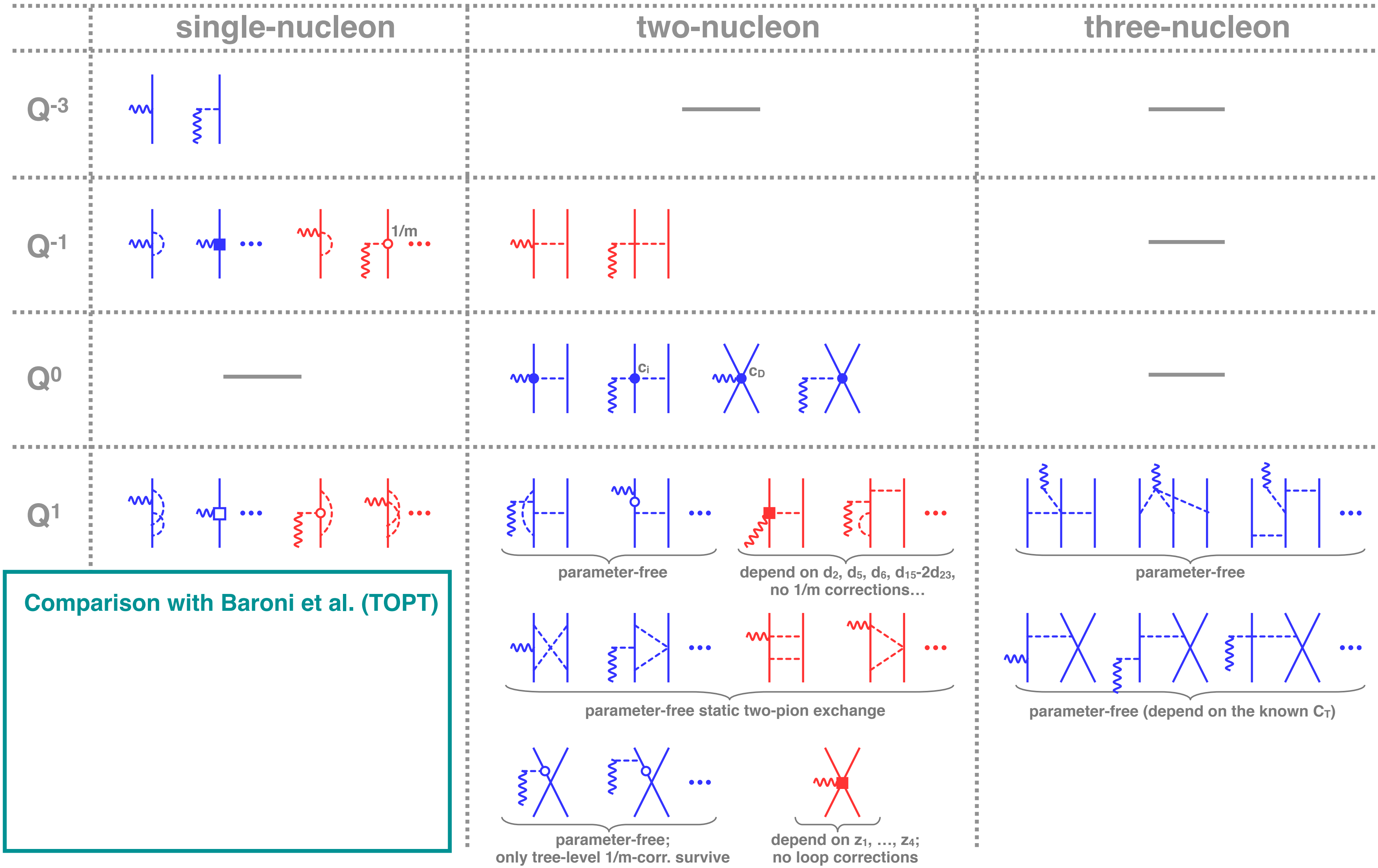
Chiral expansion of the axial **current** and **charge** operators



Exchange axial currents

Krebs, EE, Meißner, Annals Phys. 378 (2017) 317

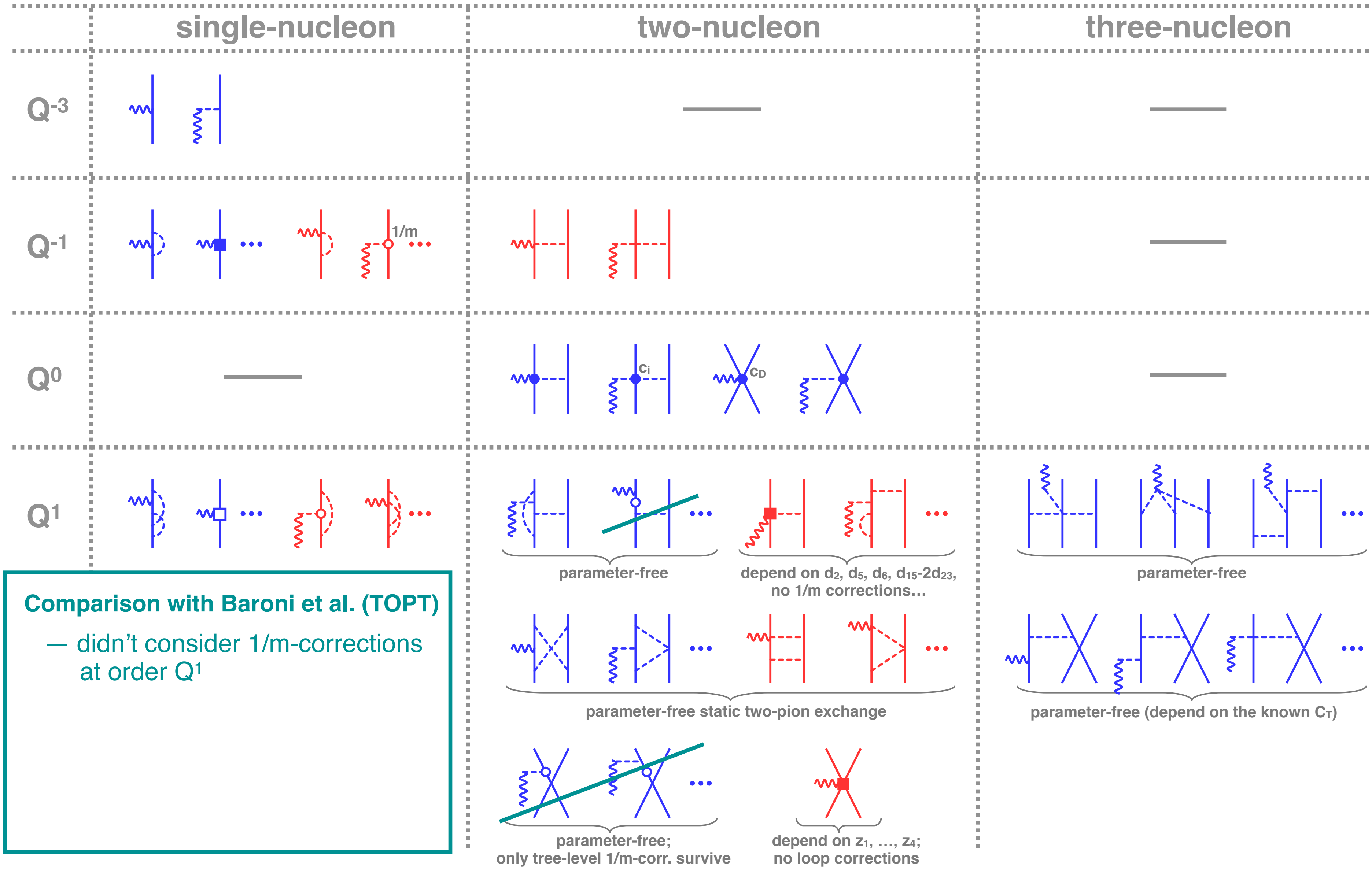
Chiral expansion of the axial current and charge operators



Exchange axial currents

Krebs, EE, Meißner, Annals Phys. 378 (2017) 317

Chiral expansion of the axial **current** and **charge** operators



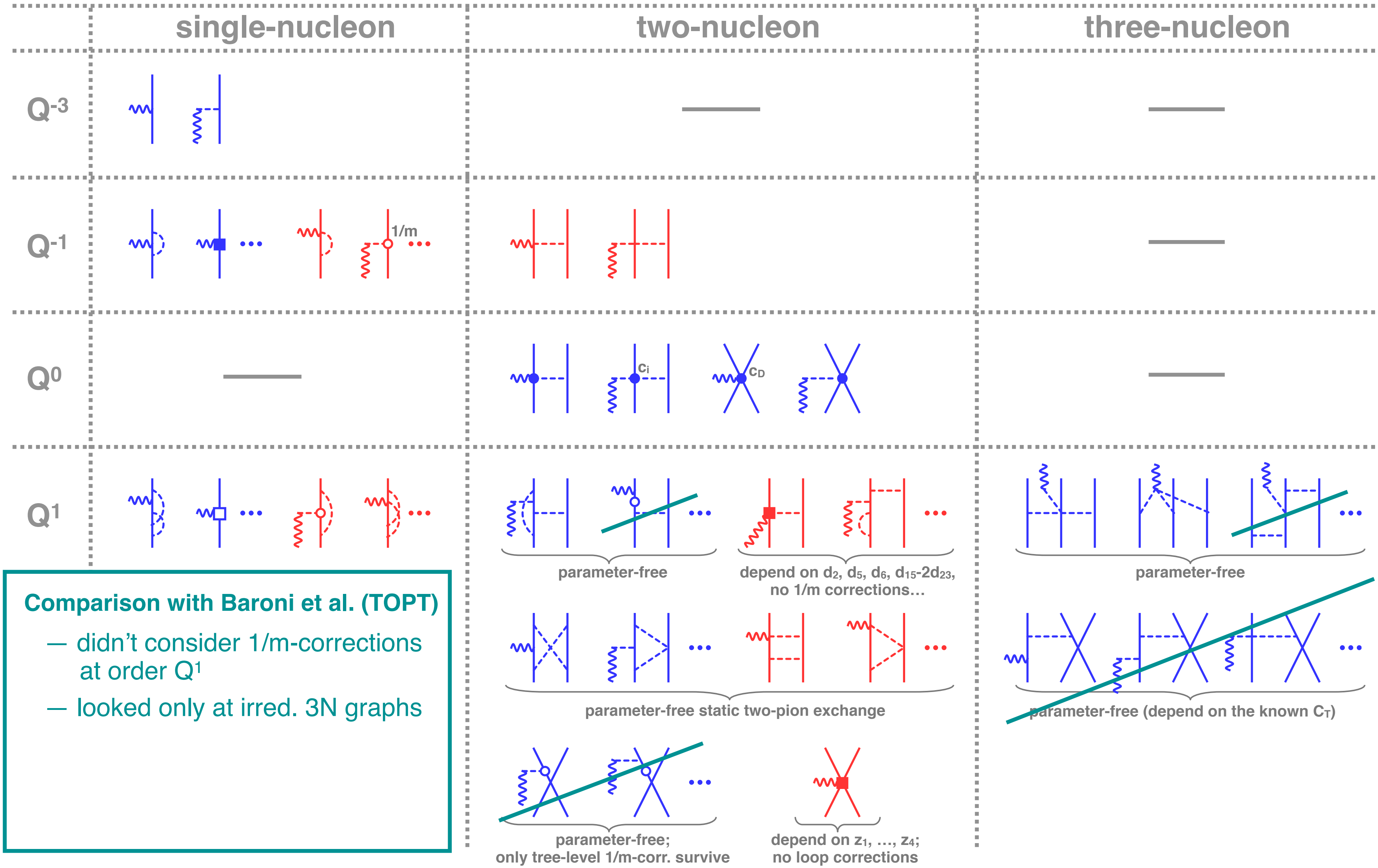
Comparison with Baroni et al. (TOPT)

— didn't consider 1/m-corrections at order Q^1

Exchange axial currents

Krebs, EE, Meißner, Annals Phys. 378 (2017) 317

Chiral expansion of the axial **current** and **charge** operators



Comparison with Baroni et al. (TOPT)

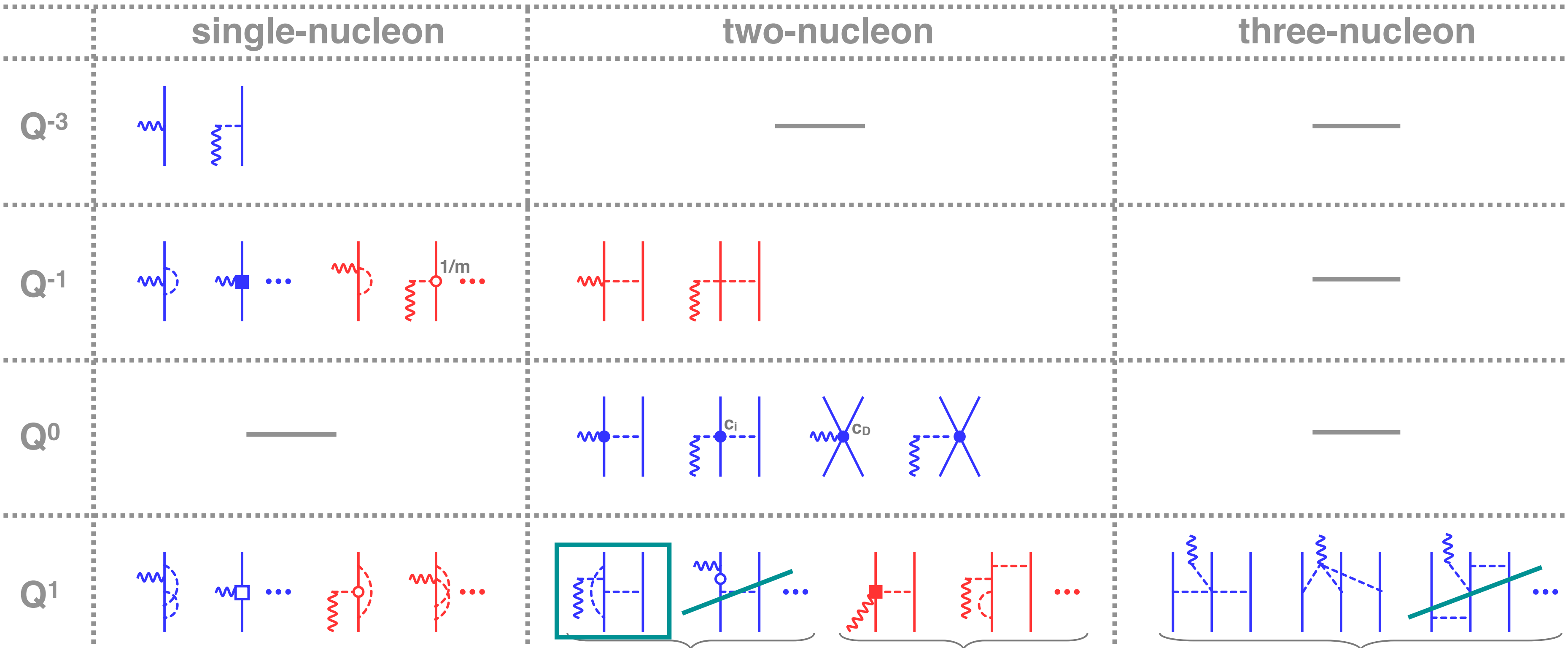
— didn't consider 1/m-corrections at order Q^1

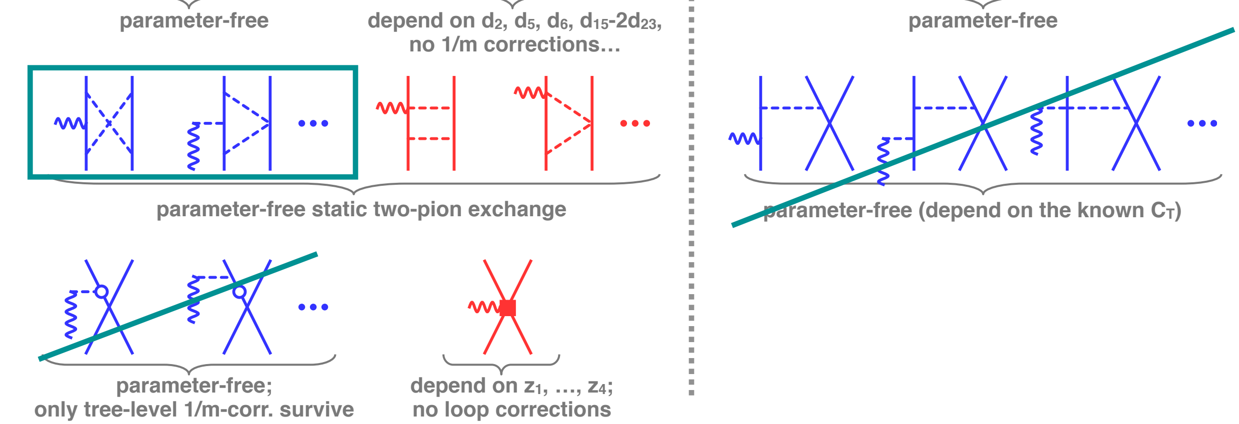
— looked only at irred. 3N graphs

Exchange axial currents

Krebs, EE, Meißner, Annals Phys. 378 (2017) 317

Chiral expansion of the axial **current** and **charge** operators





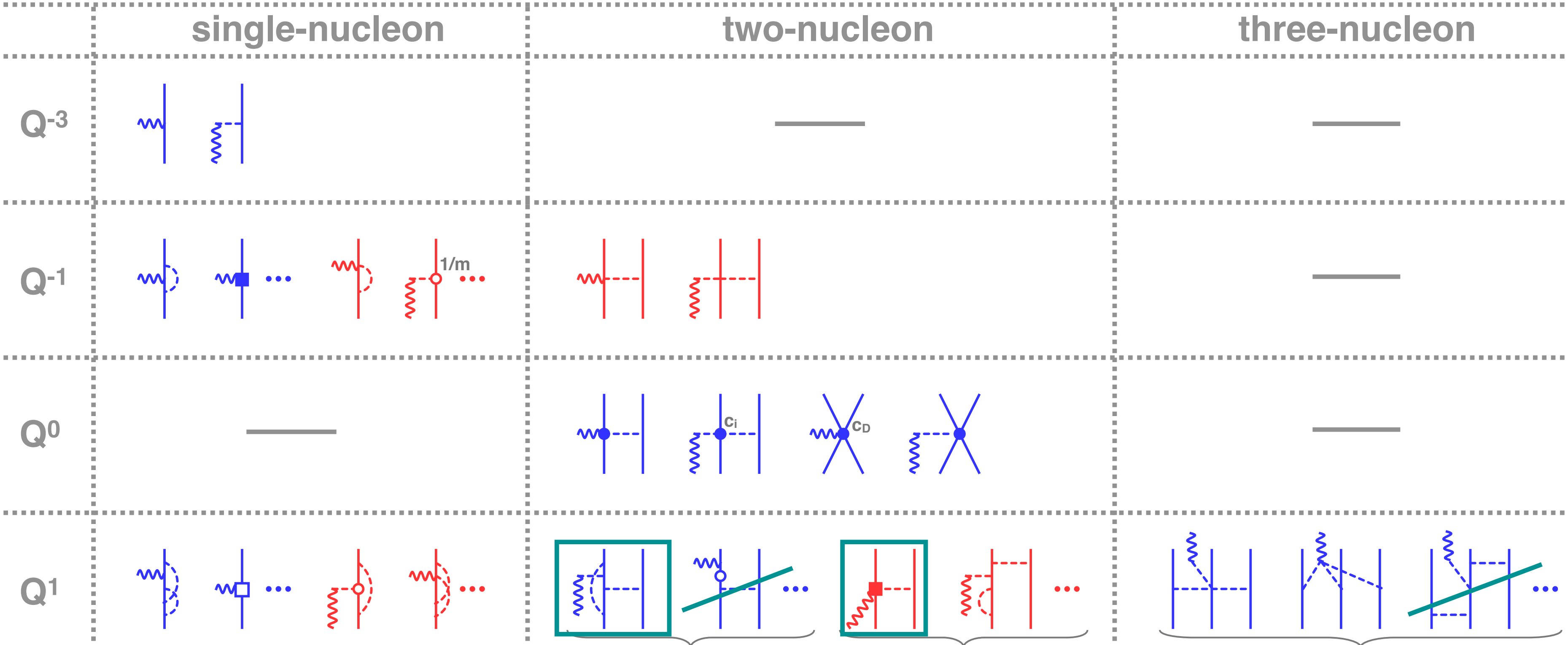
Comparison with Baroni et al. (TOPT)

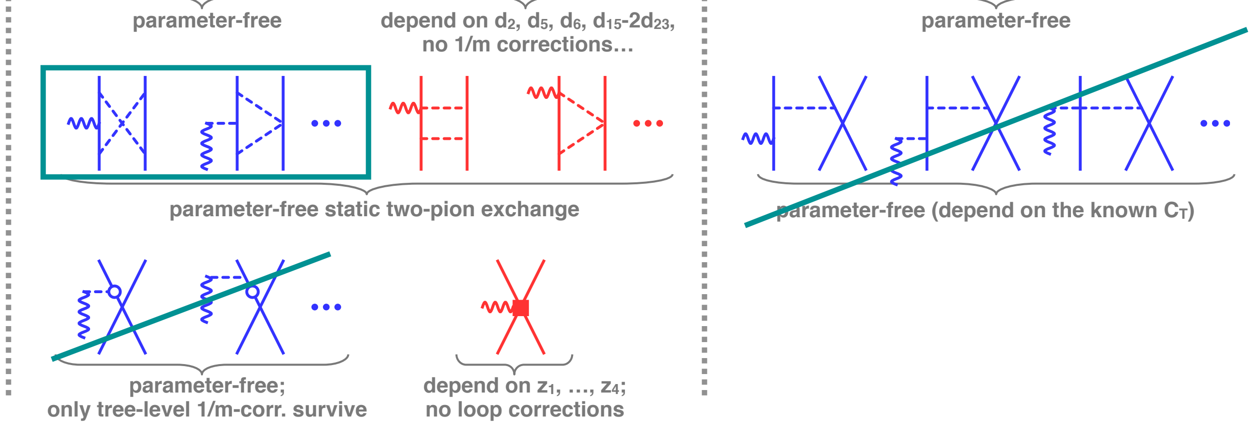
- didn't consider 1/m-corrections at order Q^1
- looked only at irred. 3N graphs
- different results for π-exchange current contributions

Exchange axial currents

Krebs, EE, Meißner, Annals Phys. 378 (2017) 317

Chiral expansion of the axial current and charge operators





Comparison with Baroni et al. (TOPT)

- didn't consider 1/m-corrections at order Q^1
- looked only at irred. 3N graphs
- different results for π-exchange current contributions
- differences in tree-level 1π-terms

Tritium β-decay [Skibinski et al., in progress]

- Half-life of ^3H (up to known radiative corrections): $$ft = \frac{K}{G_V^2 \langle F \rangle^2 + g_A^2 \langle GT \rangle^2} = 1129.6 \pm 3.0 \text{ s}$$

  ⟶ constraints on the Gamow-Teller ME

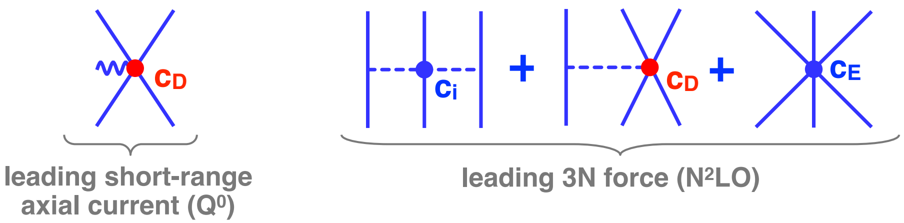
Tritium β-decay [Skibinski et al., in progress]

- Half-life of ^{3}H (up to known radiative corrections): $$ft = \frac{K}{G_V^2 \langle F \rangle^2 + g_A^2 \langle GT \rangle^2} = 1129.6 \pm 3.0 \text{ s}$$

  ⟶ constraints on the Gamow-Teller ME

- Using 1N current, the ***ft*** value is off by ~ 5% ⟵ exchange current contribution!
  Up to Q^1 (i.e. N^3LO), no LECs except for known c_i and $\mathbf{c_D}$ involved. Fixing $\mathbf{c_D}$ in the strong sector allows one to predict ***ft***!
  (it is crucial to maintain the symmetry)

  ⟶ test axial exchange currents



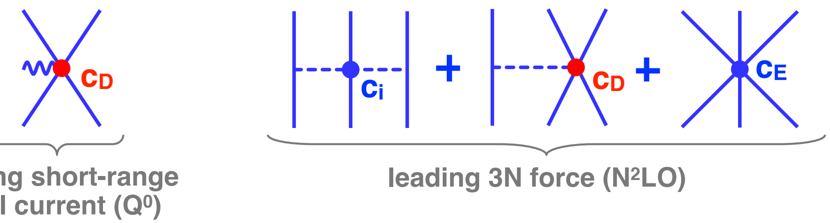
Tritium β-decay [Skibinski et al., in progress]

- Half-life of ^{3}H (up to known radiative corrections): $$ft = \frac{K}{G_V^2 \langle F \rangle^2 + g_A^2 \langle GT \rangle^2} = 1129.6 \pm 3.0 \text{ s}$$

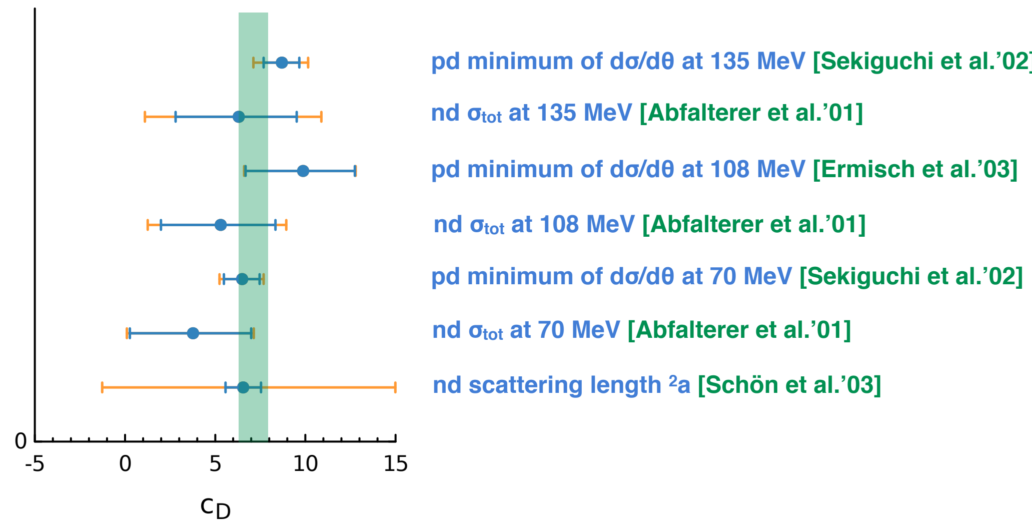
  ⟶ constraints on the Gamow-Teller ME

- Using 1N current, the ***ft*** value is off by ~ 5% ⟵ exchange current contribution!
  Up to Q^1 (i.e. N^{3}LO), no LECs except for known c_i and c_D involved. Fixing c_D in the strong sector allows one to predict ***ft***!
  (it is crucial to maintain the symmetry)

  ⟶ test axial exchange currents



- Within the LENPIC, work is in progress on the determination of c_D. ⟶

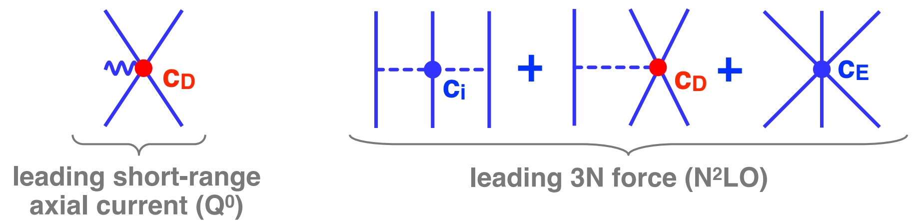


LENPIC: Low Energy Nuclear Physics International Collaboration

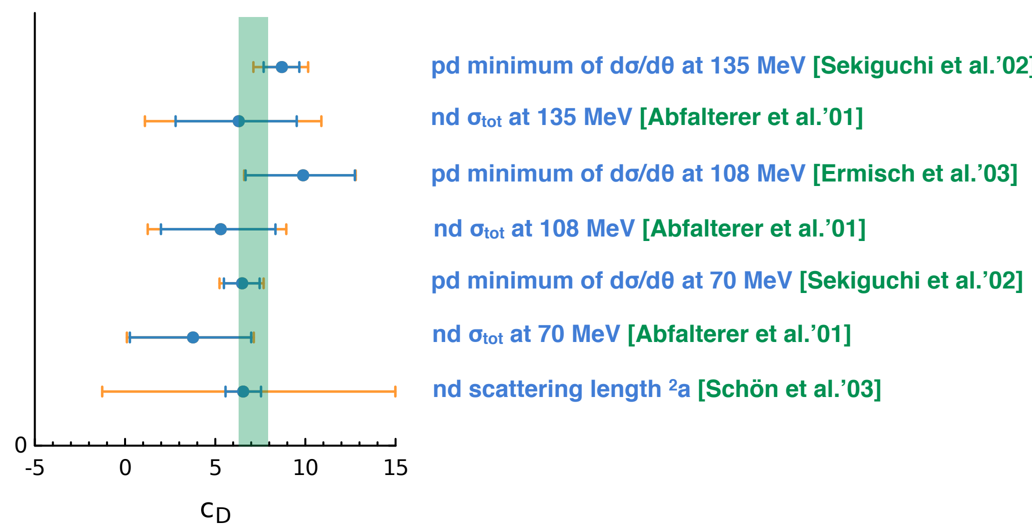
Tritium β-decay [Skibinski et al., in progress]

- Half-life of ^{3}H (up to known radiative corrections): $$ft = \frac{K}{G_V^2 \langle F \rangle^2 + g_A^2 \langle GT \rangle^2} = 1129.6 \pm 3.0 \text{ s}$$
  ⟶ constraints on the Gamow-Teller ME

- Using 1N current, the ***ft*** value is off by ~ 5% ⟵ exchange current contribution!
  Up to Q^1 (i.e. N^{3}LO), no LECs except for known c_i and c_D involved. Fixing c_D in the strong sector allows one to predict ***ft***!
  (it is crucial to maintain the symmetry)
  ⟶ test axial exchange currents



- Within the LENPIC, work is in progress on the determination of c_D.



- Being validated in ^{3}H β-decay, the theory can be used to predict the μ capture rate on ^{2}H (being measured in MuSun@PSI) & to study few-N weak reactions (μ capture, pp fusion, ν scattering, …)

LENPIC: Low Energy Nuclear Physics International Collaboration

Summary and outlook

Nuclear Hamiltonian:

— derivation of contributions up to N^3LO completed already in 2011; derivation of N^4LO corrections done for V_{2N} and almost done for V_{3N} (new LECs…) and V_{4N}

— accurate & precise NN potentials at N^4LO are available, implementation of many-body forces beyond N^2LO in progress **[LENPIC]**

Electroweak current operators:

— have been worked out completely to N^3LO

— 1N contributions expressible in terms of form factors

— some πN LECs in 1π axial charge at N^3LO are unknown…
[lattice QCD? ν-induced π-production? resonance saturation? large-N_c?…]

— 2N short-range e.m. current/axial charge involve a few new LECs

Next steps (in progress):

— Precision tests of the theory for ^3H β decay & μ capture (validation)

— Extension to other processes, heavier nuclei, N^4LO, explicit Δ's, …

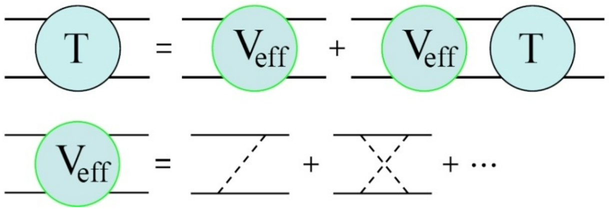
spares…

Nuclear chiral EFT

Nuclear chiral EFT

Weinberg, van Kolck, Kaiser, EE, Glöckle, Meißner, Entem, Machleidt, Krebs, ...

$$\left[\left(\sum_{i=1}^{A} \frac{-\vec{\nabla}_i^2}{2m_N} + \mathcal{O}(m_N^{-3})\right) + \underbrace{V_{2N} + V_{3N} + V_{4N} + \ldots}_{\text{derived in ChPT}}\right]|\Psi\rangle = E|\Psi\rangle$$



LS equation is linearly divergent already at LO

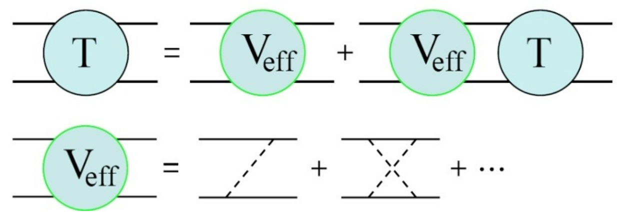
⟶ infinitely many CTs are needed to absorb all UV divergences from iterations!

Nuclear chiral EFT

Nuclear chiral EFT

Weinberg, van Kolck, Kaiser, EE, Glöckle, Meißner, Entem, Machleidt, Krebs, ...

$$\left[\left(\sum_{i=1}^{A} \frac{-\vec{\nabla}_i^2}{2m_N} + \mathcal{O}(m_N^{-3})\right) + \underbrace{V_{2N} + V_{3N} + V_{4N} + \ldots}_{\text{derived in ChPT}}\right] |\Psi\rangle = E|\Psi\rangle$$



LS equation is linearly divergent already at LO

→ infinitely many CTs are needed to absorb all UV divergences from iterations!

Commonly used approach [EGM, EM, EKM, Gezerlis et al.'14, Piarulli et al.'15, Carlsson et al.'16, ...]:

- Introduce a finite UV regulator $\Lambda \sim \Lambda_b$ ($\Lambda_b \sim 600$ MeV)
- Include short-range operators in V_{NN} according to NDA ← minimal possible set; alternatives have been proposed…
- Solve the LS equation & tune the **bare** LECs $C_i(\Lambda)$ to data (implicit renormalization)
- (Numerical) self-consistency checks via error analysis and Λ-variation

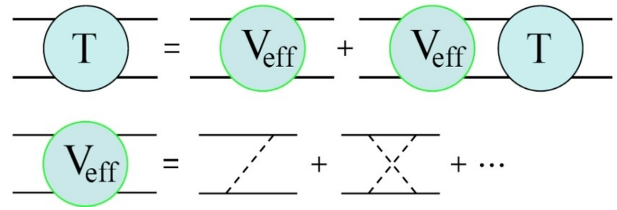
See: Lepage, „How to renormalize the Schrödinger equation“, nucl-th/9607029 and talk@INT in 2000

Nuclear chiral EFT

Nuclear chiral EFT

Weinberg, van Kolck, Kaiser, EE, Glöckle, Meißner, Entem, Machleidt, Krebs, ...

$$\left[\left(\sum_{i=1}^{A} \frac{-\vec{\nabla}_i^2}{2m_N} + \mathcal{O}(m_N^{-3})\right) + \underbrace{V_{2N} + V_{3N} + V_{4N} + \ldots}_{\text{derived in ChPT}}\right]|\Psi\rangle = E|\Psi\rangle$$



LS equation is linearly divergent already at LO

⟶ infinitely many CTs are needed to absorb all UV divergences from iterations!

Commonly used approach [EGM, EM, EKM, Gezerlis et al.'14, Piarulli et al.'15, Carlsson et al.'16, ...]:

- Introduce a finite UV regulator $\Lambda \sim \Lambda_b$ ($\Lambda_b \sim 600$ MeV)
- Include short-range operators in V_{NN} according to NDA ⟵ minimal possible set; alternatives have been proposed…
- Solve the LS equation & tune the **bare** LECs $C_i(\Lambda)$ to data (implicit renormalization)
- (Numerical) self-consistency checks via error analysis and Λ-variation

  See: Lepage, „How to renormalize the Schrödinger equation“, nucl-th/9607029 and talk@INT in 2000

(Some) alternatives:

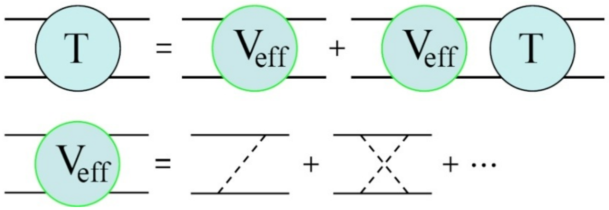
- renormalizable approach based on the Lorentz invariant $\mathcal{L}_{\rm eff}$ [EE, Gegelia '12 - '16]
- RG-based approach to determine the counting for contacts [Birse]
- non-perturbative „renormalization“ within the non relativistic framework using $\Lambda \gg \Lambda_b$ [van Kolck, Pavon Valderrama, Long, …]

Nuclear chiral EFT

Nuclear chiral EFT

Weinberg, van Kolck, Kaiser, EE, Glöckle, Meißner, Entem, Machleidt, Krebs, ...

$$\left[\left(\sum_{i=1}^{A}\frac{-\vec{\nabla}_i^2}{2m_N}+\mathcal{O}(m_N^{-3})\right)+\underbrace{V_{2N}+V_{3N}+V_{4N}+\ldots}_{\text{derived in ChPT}}\right]|\Psi\rangle = E|\Psi\rangle$$



LS equation is linearly divergent already at LO

⟶ infinitely many CTs are needed to absorb all UV divergences from iterations!

Commonly used approach [EGM, EM, EKM, Gezerlis et al.'14, Piarulli et al.'15, Carlsson et al.'16, ...]:

— Introduce a finite UV regulator $\Lambda \sim \Lambda_b$ ($\Lambda_b \sim 600$ MeV)

— Include short-range operators in V_{NN} according to NDA ⟵ minimal possible set; alternatives have been proposed…

— Solve the LS equation & tune the **bare** LECs $C_i(\Lambda)$ to data (implicit renormalization)

— (Numerical) self-consistency checks via error analysis and Λ-variation

See: Lepage, „How to renormalize the Schrödinger equation“, nucl-th/9607029 and talk@INT in 2000

(Some) alternatives:

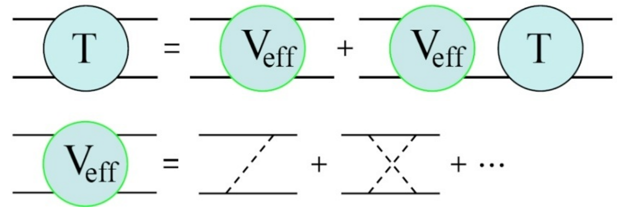
- renormalizable approach based on the Lorentz invariant $\mathcal{L}_{\text{eff}}$ [EE, Gegelia '12 - '16]
- RG-based approach to determine the counting for contacts [Birse]
- ~~non-perturbative „renormalization“ within the non relativistic framework using $\Lambda \gg \Lambda_b$~~ [van Kolck, Pavon Valderrama, Long, ...] is more in spirit of peratization [EE, Gegelia EPJA 41 (09) 341]

Nuclear chiral EFT

Nuclear chiral EFT

Weinberg, van Kolck, Kaiser, EE, Glöckle, Meißner, Entem, Machleidt, Krebs, ...

$$\left[\left(\sum_{i=1}^{A}\frac{-\vec{\nabla}_i^2}{2m_N}+\mathcal{O}(m_N^{-3})\right)+\underbrace{V_{2N}+V_{3N}+V_{4N}+\ldots}_{\text{derived in ChPT}}\right]|\Psi\rangle = E|\Psi\rangle$$



LS equation is linearly divergent already at LO

⟶ infinitely many CTs are needed to absorb all UV divergences from iterations!

Commonly used approach [EGM, EM, EKM, Gezerlis et al.'14, Piarulli et al.'15, Carlsson et al.'16, ...]:

— Introduce a finite UV regulator $\Lambda \sim \Lambda_b$ ($\Lambda_b \sim 600$ MeV)

— Include short-range operators in V_{NN} according to NDA ⟵ minimal possible set; alternatives have been proposed…

— Solve the LS equation & tune the **bare** LECs $C_i(\Lambda)$ to data (implicit renormalization)

— (Numerical) self-consistency checks via error analysis and Λ-variation

See: Lepage, „How to renormalize the Schrödinger equation“, nucl-th/9607029 and talk@INT in 2000

(Some) alternatives:

- renormalizable approach based on the Lorentz invariant $\mathcal{L}_{\text{eff}}$ [EE, Gegelia '12 - '16]
- RG-based approach to determine the counting for contacts [Birse]
- ~~non-perturbative „renormalization“ within the non relativistic framework using $\Lambda >> \Lambda_b$~~ [van Kolck, Pavon Valderrama, Long, ...] is more in spirit of peratization [EE, Gegelia EPJA 41 (09) 341]

See also: *Nuclear Effective Field Theories — the crux of the matter*, open discussion by Mike Birse and EE at the KITP program „Frontiers in Nuclear Physics“, August 22 - November 4, 2016, available at http://online.kitp.ucsb.edu/online/nuclear16/

Comparison with Baroni et al.

For ^{3}H β-decay ($\vec{k} \simeq 0$), the N^{3}LO contribution to the 2N current by Baroni et al. is:

$$\vec{A}^a_{\text{Baroni et al.}} = \frac{g_A^3}{32\pi F_\pi^4}\tau_2^a \left[W_1(q_1)\vec{\sigma}_1 + W_2(q_1)\vec{q}_1\,\vec{\sigma}_1\cdot\vec{q}_1 + Z_1(q_1)\left(2\frac{\vec{q}_1\,\vec{\sigma}_2\cdot\vec{q}_1}{q_1^2+M_\pi^2} - \vec{\sigma}_2\right)\right]$$
$$+ \frac{g_A^5}{32\pi F_\pi^4}\tau_1^a W_3(q_1)(\vec{\sigma}_2\times\vec{q}_1)\times\vec{q}_1 - \frac{g_A^3}{32\pi F_\pi^4}[\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2]^a Z_3(q_1)\frac{\vec{\sigma}_1\times\vec{q}_1\vec{\sigma}_2\cdot\vec{q}_1}{q_1^2+M_\pi^2} + 1\leftrightarrow 2\,,$$

Baroni et al., PRC93 (2016) 015501; PRC94 (2016) 024003

where the various loop functions are defined as:

$$W_1(q_1) = \frac{1}{2}A(q_1)\left[4(1-2g_A^2)M_\pi^2 + (1-5g_A^2)q_1^2\right] + \frac{1}{2}M_\pi\left[g_A^2\left(\frac{4M_\pi^2}{4M_\pi^2+q_1^2} - 9\right) + 1\right],$$

$$W_2(q_1) = \frac{M_\pi\left(4(2g_A^2+1)M_\pi^2 + (3g_A^2+1)q_1^2\right)}{2q_1^2(4M_\pi^2+q_1^2)} - \frac{A(q_1)\left(4(2g_A^2+1)M_\pi^2 + (g_A^2-1)q_1^2\right)}{2q_1^2},$$

$$W_3(q_1) = -\frac{4A(q_1)}{3} - \frac{1}{6M_\pi},$$

← does not exist in the chiral limit! (in contrast to our results)

$$Z_1(q_1) = 2A(q_1)(2M_\pi^2+q_1^2) + 2M_\pi,$$

$$Z_3(q_1) = \frac{1}{2}A(q_1)(4M_\pi^2+q_1^2) + \frac{M_\pi}{2},$$

$$A(q_1) = \frac{1}{2q_1}\arctan\left(\frac{q_1}{2M_\pi}\right).$$

Comparison with Baroni et al.

For ^{3}H β-decay ($\vec{k} \simeq 0$), the N^{3}LO contribution to the 2N current by Baroni et al. is:

$$\vec{A}^a_{\text{Baroni et al.}} = \frac{g_A^3}{32\pi F_\pi^4}\tau_2^a \left[W_1(q_1)\vec{\sigma}_1 + W_2(q_1)\vec{q}_1\,\vec{\sigma}_1\cdot\vec{q}_1 + Z_1(q_1)\left(2\frac{\vec{q}_1\,\vec{\sigma}_2\cdot\vec{q}_1}{q_1^2+M_\pi^2} - \vec{\sigma}_2\right)\right]$$
$$+ \frac{g_A^5}{32\pi F_\pi^4}\tau_1^a W_3(q_1)(\vec{\sigma}_2\times\vec{q}_1)\times\vec{q}_1 - \frac{g_A^3}{32\pi F_\pi^4}[\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2]^a Z_3(q_1)\frac{\vec{\sigma}_1\times\vec{q}_1\vec{\sigma}_2\cdot\vec{q}_1}{q_1^2+M_\pi^2} + 1\leftrightarrow 2\,,$$

Baroni et al., PRC93 (2016) 015501; PRC94 (2016) 024003

where the various loop functions are defined as:

$$W_1(q_1) = \frac{1}{2}A(q_1)\left[4(1-2g_A^2)M_\pi^2 + (1-5g_A^2)q_1^2\right] + \frac{1}{2}M_\pi\left[g_A^2\left(\frac{4M_\pi^2}{4M_\pi^2+q_1^2} - 9\right) + 1\right],$$

$$W_2(q_1) = \frac{M_\pi\left(4(2g_A^2+1)M_\pi^2 + (3g_A^2+1)q_1^2\right)}{2q_1^2(4M_\pi^2+q_1^2)} - \frac{A(q_1)\left(4(2g_A^2+1)M_\pi^2 + (g_A^2-1)q_1^2\right)}{2q_1^2},$$

$$W_3(q_1) = -\frac{4A(q_1)}{3} - \frac{1}{6M_\pi},$$

← does not exist in the chiral limit! (in contrast to our results)

$$Z_1(q_1) = 2A(q_1)(2M_\pi^2+q_1^2) + 2M_\pi,$$

$$Z_3(q_1) = \frac{1}{2}A(q_1)(4M_\pi^2+q_1^2) + \frac{M_\pi}{2},$$

$$A(q_1) = \frac{1}{2q_1}\arctan\left(\frac{q_1}{2M_\pi}\right).$$

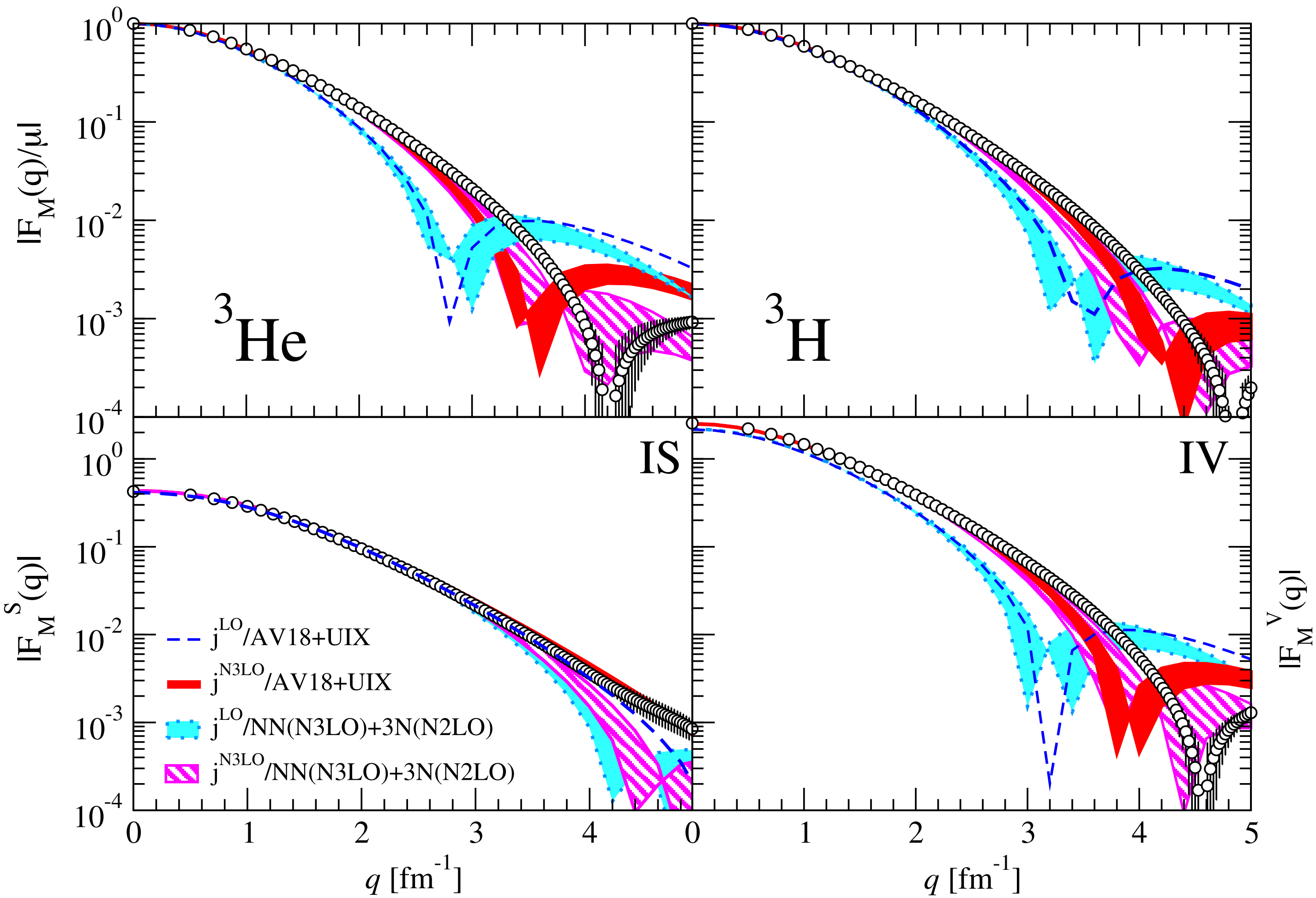
We find:

$$\vec{A}^a_{\text{Baroni et al.}} - \vec{A}^a_{\text{KEM}} = -\frac{g_A^5 A(q_1)\left(\vec{\sigma}_2\tau_1^a q_1^4 + 2\vec{q}_1(6M_\pi^2+q_1^2)\vec{q}_1\cdot\vec{\sigma}_2\tau_1^a\right)}{96\pi F_\pi^4 q_1^2} + \text{rational function of } \vec{q}_1 + 1\leftrightarrow 2$$

→ the currents have different long-range parts!

Magnetic form factors of ^{3}He, ^{3}H

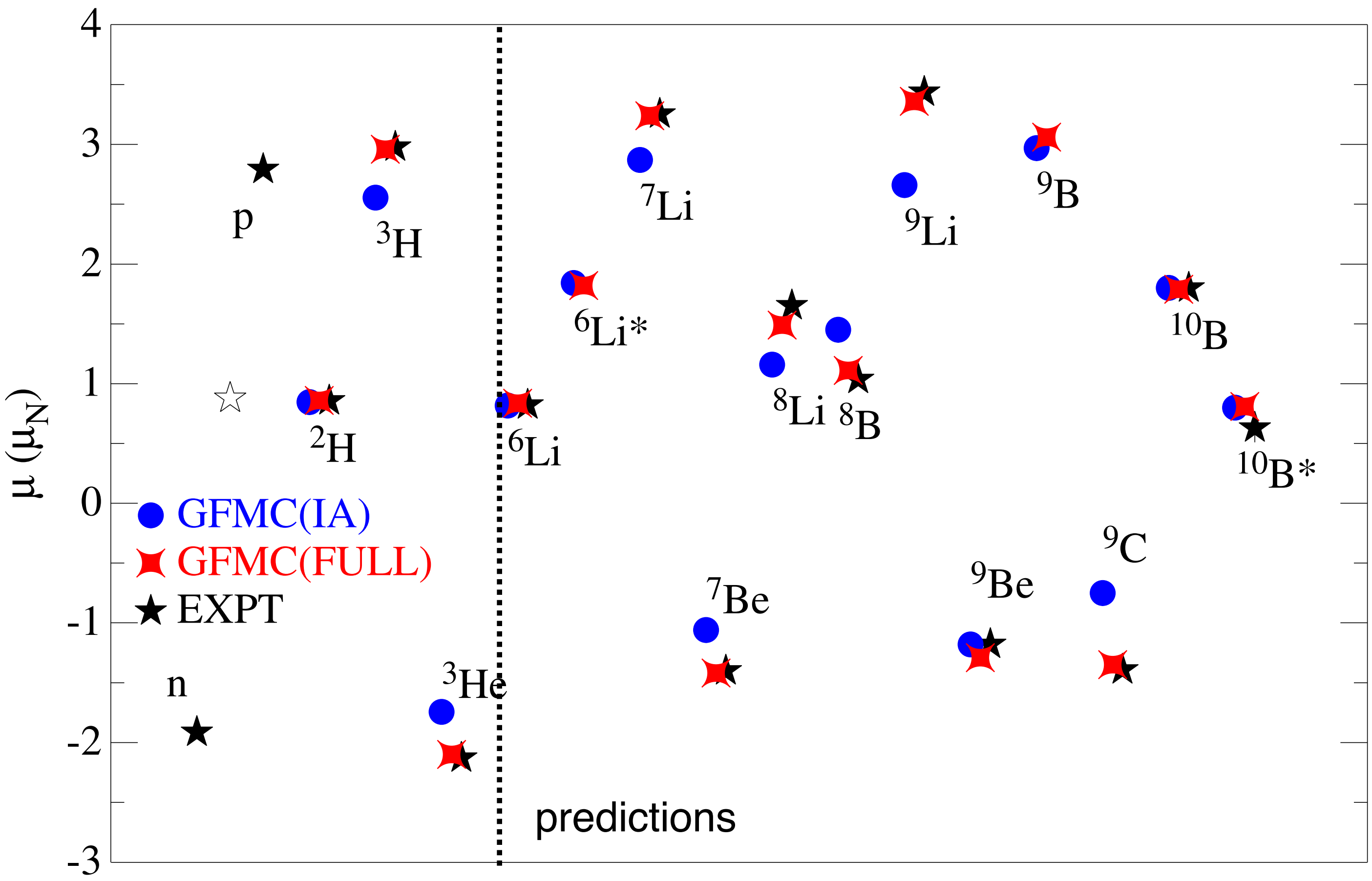
Piarulli, Girlanda, Marcucci, Pastore, Schiavilla, Viviani, Phys. Rev. C87 (2013) 014006



- ^{3}He/^{3}H m.m's used to fix EM LECs; ~15% correction from two-body currents
- Exchange currents crucial to improve agreement with exp data

Magnetic moments of light nuclei

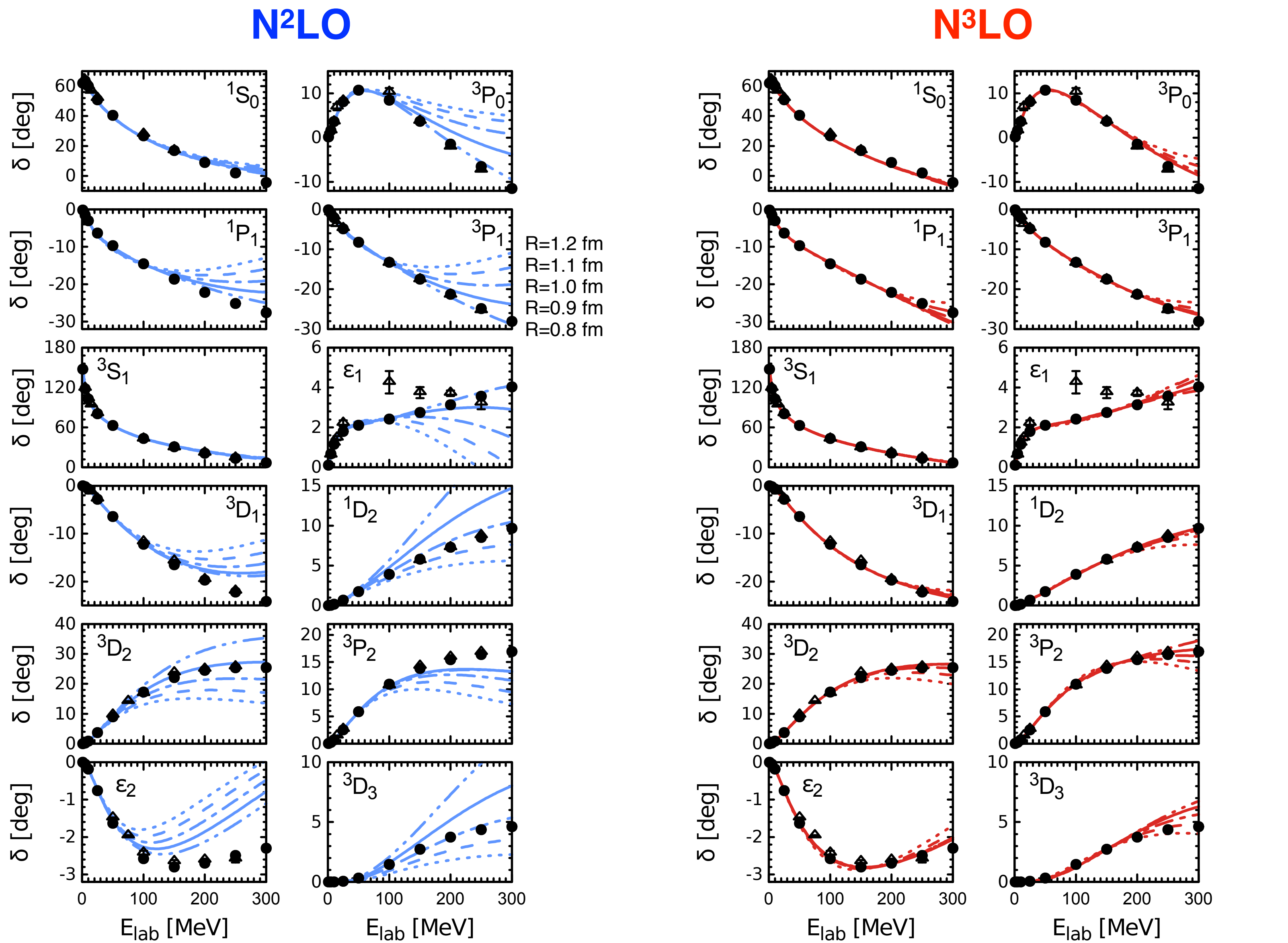
Pastore, Pieper, Schiavilla, Wiringa, Phys. Rev. C87 (2013) 035503



- Hybrid GFMC calculations using AV18 + Urbana 3NF
- Magnetic moments of A = 2, 3 nuclei used to fix EM LECs
- Theoretical uncertainties?

NN phase shifts: cutoff dependence

EE, Krebs, Meißner, EPJ A51 (2015) 5, 53; PRL 115 (2015) 122301



Regulator (in)dependence

How do our results depend on the specific form of the regulator $f\left(\frac{r}{R}\right) = \left[1 - \exp\left(-\frac{r^2}{R^2}\right)\right]^n$

and/or additional spectral function regularization $V_C(q) = \frac{2}{\pi}\int_{2M_\pi}^{\Lambda_{\rm SFR}} d\mu\, \mu\, \frac{\rho_C(\mu)}{\mu^2+q^2}$

Selected phase shifts (in deg.) for different values of Λ_{SFR} and n at $N^3LO_{[R = 0.9\ fm]}$

Lab. energy	NPWA	our result	DR, $n=5$	DR, $n=7$	SFR, 1.0 GeV	SFR, 1.5 GeV	SFR, 2.0 GeV
proton-proton $^1S_0$ phase shift							
10 MeV	55.23	55.22 ± 0.08	55.22	55.22	55.22	55.22	55.22
100 MeV	24.99	24.98 ± 0.60	24.98	24.98	24.98	24.98	24.98
200 MeV	6.55	6.56 ± 2.2	6.55	6.56	6.56	6.56	6.57
neutron-proton $^3S_1$ phase shift							
10 MeV	102.61	102.61 ± 0.07	102.61	102.61	102.61	102.61	102.61
100 MeV	43.23	43.22 ± 0.30	43.28	43.20	43.17	43.21	43.22
200 MeV	21.22	21.2 ± 1.4	21.2	21.2	21.2	21.2	21.2
proton-proton $^3P_0$ phase shift							
10 MeV	3.73	3.75 ± 0.04	3.75	3.75	3.75	3.75	3.75
100 MeV	9.45	9.17 ± 0.30	9.15	9.18	9.18	9.17	9.17
200 MeV	−0.37	-0.1 ± 2.3	−0.1	−0.1	−0.1	−0.1	−0.1
proton-proton $^3P_1$ phase shift							
10 MeV	−2.06	-2.04 ± 0.01	−2.04	−2.04	−2.04	−2.04	−2.04
100 MeV	−13.26	-13.42 ± 0.17	−13.43	−13.41	−13.41	−13.42	−13.42
200 MeV	−21.25	-21.2 ± 1.6	−21.2	−21.2	−21.2	−21.2	−21.2
proton-proton $^3P_2$ phase shift							
10 MeV	0.65	0.65 ± 0.01	0.66	0.65	0.65	0.65	0.65
100 MeV	11.01	11.03 ± 0.50	10.97	11.06	11.07	11.05	11.04
200 MeV	15.63	15.6 ± 1.9	15.6	15.5	15.5	15.5	15.6

→ negligible regulator dependence (compared to the estimated theor. accuracy)