CKM ELEMENTS FROM SEMILEPTONIC B DECAYS

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THE CKM MATRIX

Weak and mass eigenstates

$$\begin{pmatrix} d'\\ s'\\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub}\\ V_{cd} & V_{cs} & V_{cb}\\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d\\ s\\ b \end{pmatrix} = \hat{V}_{\rm CKM} \begin{pmatrix} d\\ s\\ b \end{pmatrix}$$

Charged currents (W exchange) mix generations Masses and mixing arise from Yukawa matter-Higgs couplings

 $\hat{V} = \begin{pmatrix} 0.97428 \pm 0.00015 & 0.2253 \pm 0.0007 & 0.00347^{+0.00016}_{-0.00012} \\ 0.2252 \pm 0.0007 & 0.97345^{+0.00015}_{-0.00016} & 0.0410^{+0.0011}_{-0.0007} \\ 0.00862^{+0.00026}_{-0.00020} & 0.0403^{+0.0011}_{-0.0007} & 0.999152^{+0.000030}_{-0.00045} \end{pmatrix}$

CKM matrix has strong hierarchy

V is a unitary matrix with only 4 physical parameters (3 angles and 1 phase)



allow for the determination of V_{cb} and V_{ub} . Exclusive analyses look at specific final states such as X=D,D*, π , ρ

Inclusive vs exclusive B decays



As we aim at high precision, things are not at all simple...

WHY PRECISION CKM STUDIES?

- The SM accomodates flavour & CP violation, but *we have no theory of flavour*: what is the origin of the observed masses and mixing?
- We still expect New Physics not far from the EW scale. Generic NP implies additional flavour and CP violation. Is there anything beyond the CKM? Even without NP at LHC flavour physics may probe scales up to 10³ TeV
- But the CKM mechanism is very successful in general: NP must preserve agreement with data, NP signals will be small
- To uncover small signals of physics beyond CKM, we need precision tests, in many ways a challenge for our QCD understanding

IMPORTANCE OF $|V_{xb}|$



Since several years, exclusive decays prefer smaller $|V_{ub}|$ and $|V_{cb}|$





UNLIKELY PLACE FOR NEW PHYSICS?

The difference in V_{cb} incl vs excl D* with FNAL/MILC form factor is **large**: 3σ or about 8%. The perturbative corrections to inclusive V_{cb} total 5%...

Right Handed currents now excluded since

$$|V_{cb}|_{incl} \simeq |V_{cb}| \left(1 + \frac{1}{2} |\delta|^2\right)$$
$$|V_{cb}|_{B \to D^*} \simeq |V_{cb}| \left(1 - \delta\right)$$
$$|V_{cb}|_{B \to D} \simeq |V_{cb}| \left(1 + \delta\right)$$

Chen,Nam,Crivellin,Buras,Gemmler, Isidori,Mannel,...

$$\delta = \epsilon_R \frac{V_{cb}}{V_{cb}} \approx 0.08$$

Most general SU(2) invariant dim 6 NP (without RH light neutrino) can explain results, but it is incompatible with $Z \rightarrow b\overline{b}$ data

Crivellin, Pokorski 1407.1320

(though this may not apply to the tensor operator Colangelo, De Fazio)

RH CURRENTS DON'T HELP Vub EITHER

- ✦ Can ease |V_{ub}| tension by allowing small righthanded contribution to Standard-Model weak current [Crivellin, PRD81 (2010) 031301]
- RH currents disfavored by Λ_b decays (taking $|V_{cb}|$ from $B \rightarrow D^* | v + HFAG$ to obtain $|V_{ub}|$



Also here SU(2)xU(1) invariant NP cannot explain discrepancies 1407.1320

SEMITAUONIC ANOMALY

$$R(D^{(*)}) = \frac{\mathcal{B}(B \to D^{(*)}\tau\nu)}{\mathcal{B}(B \to D^{(*)}\mu\nu)}$$

Combined discrepancy with SM 4.0σ

about 30% effect on tree-level process!

Lepton flavour universality violation: new scalars, leptoquarks, W'... possible connection with lepton flavour violation in $b \rightarrow sll$

Inconsistent with LEP inclusive measurement

SM predictions?



Celis et al., 1612.07757



- *Simple idea:* inclusive decays do not depend on final state, long distance dynamics of the B meson factorizes. An OPE allows to express it in terms of B meson matrix elements of local operators
- The Wilson coefficients are perturbative, matrix elements of local ops parameterize non-pert physics: *double series in α_s, Λ/m_b*
- Lowest order: decay of a free *b*, linear Λ/m_b absent. Depends on m_{b,c}, 2 parameters at O(1/m_b²), 2 more at O(1/m_b³)...

INCLUSIVE SEMILEPTONIC B DECAYS

Inclusive observables are double series in Λ/m_b and α_s

$$\begin{split} M_{i} &= M_{i}^{(0)} + \frac{\alpha_{s}}{\pi} M_{i}^{(1)} + \left(\frac{\alpha_{s}}{\pi}\right)^{2} M_{i}^{(2)} + \left(M_{i}^{(\pi,0)} + \frac{\alpha_{s}}{\pi} M_{i}^{(\pi,1)}\right) \frac{\mu_{\pi}^{2}}{m_{b}^{2}} \\ &+ \left(M_{i}^{(G,0)} + \frac{\alpha_{s}}{\pi} M_{i}^{(G,1)}\right) \frac{\mu_{G}^{2}}{m_{b}^{2}} + M_{i}^{(D,0)} \frac{\rho_{D}^{3}}{m_{b}^{3}} + M_{i}^{(LS,0)} \frac{\rho_{LS}^{3}}{m_{b}^{3}} + \dots \\ \mu_{\pi}^{2}(\mu) &= \frac{1}{2M_{B}} \left\langle B \left| \bar{b} \left(i \vec{D} \right)^{2} b \right| B \right\rangle_{\mu} \qquad \mu_{G}^{2}(\mu) = \frac{1}{2M_{B}} \left\langle B \left| \bar{b} \frac{i}{\nu_{2}} \sigma_{\mu\nu} G^{\mu\nu} b \right| B \right\rangle_{\mu} \end{split}$$

OPE valid for inclusive enough measurements, away from perturbative singularities \implies semileptonic width, moments Current fits includes 6 non-pert parameters $m_{b,c} \quad \mu_{\pi,G}^2 \quad \rho_{D,LS}^3$ and all known corrections up to $O(\Lambda^3/m_b^3)$

EXTRACTION OF THE OPE PARAMETERS



Global shape parameters (first moments of the distributions) tell us about $m_{b,} m_c$ and the B structure, total rate about $|V_{cb}|$

OPE parameters describe universal properties of the B meson and of the quarks \rightarrow useful in many applications (rare decays, V_{ub} ,...)

THE CURRENT SEMILEPTONIC FIT

- kinetic scheme calculation based on 1107.3100; hep-ph/0401063
- includes all $O(\alpha_s^2)$ Melnikov, Biswas, Czarnecki, Pak, PG and $O(\alpha_s/m_b^2)$ corrections Becher, Boos, Lunghi, Alberti, Ewerth, Nandi, PG, Mannel, Pivovarov, Rosenthal
- Reliability of the method depends on our ability to control higher order effects. Quark-hadron duality violation would manifest as inconsistency in the fit.
- reassessment of theoretical errors, realistic theoretical correlations following Schwanda, PG, 1307.4551
- external constraints: precise heavy quark mass determinations, mild constraints on μ^2_G from hyperfine splitting and Q^3_{LS} from sum rules

Older fits: Buchmuller & Flaecher (2005), Bauer et al (2004) (1S scheme)

CHARM MASS DETERMINATIONS



Remarkable improvement in last decade. m_c can be used as precise input to fix m_b instead of radiative moments

FIT RESULTS



WITHOUT MASS CONSTRAINTS

 $m_b^{kin}(1 \,\text{GeV}) - 0.85 \,\overline{m}_c(3 \,\text{GeV}) = 3.714 \pm 0.018 \,\text{GeV}$

- results depend little on assumption for correlations and choice of inputs, 1.8% determination of V_{cb}
- 20-30% determination of the OPE parameters
- b mass determination in agreement with recent lattice and sum rules results



HIGHER POWER CORRECTIONS

Proliferation of non-pert parameters starting 1/m⁴: 9 at dim 7, 18 at dim 8

 $2M_B m_1 = \langle ((\vec{p})^2)^2 \rangle$ $2M_B m_2 = g^2 \langle \vec{E}^2 \rangle$ $2M_B m_3 = g^2 \langle \vec{B}^2 \rangle$ $2M_B m_4 = g \langle \vec{p} \cdot \operatorname{rot} \vec{B} \rangle$ $2M_Bm_5 = g^2 \langle \vec{S} \cdot (\vec{E} \times \vec{E})
angle$ $2M_Bm_6 = g^2 \langle \vec{S} \cdot (\vec{B} \times \vec{B})
angle$ $2M_Bm_7 = g \langle (\vec{S} \cdot \vec{p})(\vec{p} \cdot \vec{B})
angle$ $2M_Bm_8 = g \langle (\vec{S} \cdot \vec{B})(\vec{p})^2
angle$ $2M_Bm_9 = g \langle \Delta(\vec{\sigma} \cdot \vec{B})
angle$

Mannel, Turczyk, Uraltsev 1009.4622

can be estimated by Lowest Lying State Saturation approx by truncating

$$B|O_1O_2|B\rangle = \sum_{n} \langle B|O_1|n\rangle \langle n|O_2|B\rangle$$
see also Heinonen Mannel 1407 4384

and relating higher dimensional to lower dimensional matrix elements, e.g.

$$\rho_D^3 = \epsilon \,\mu_\pi^2 \qquad \rho_{LS}^3 = -\epsilon \,\mu_G^2 \qquad \epsilon \sim 0.4 \text{GeV}$$

excitation energy to P-wave states. LLSA might set the scale of effect, but large corrections to LLSA have been found in some cases (Mannel, Uraltsev, PG, 2012) In LLSA *good convergence* of the HQE.

We used LLSA as loose constraint (60% gaussian uncertainty, dimensional estimate for vanishing matrix elements) in the fit including higher powers

SENSITIVITY TO HIGHER POWER CORRECTIONS

PG,Healey,Turczyk 1606.06174



DEPENDENCE ON LLSA UNCERTAINTY



if we rescale all LLSA uncertainties by a factor ξ the results change very little

PROSPECTS OF INCLUSIVE V_{cb}

- Theoretical uncertainties already dominant
- $O(\alpha_s/m_b^3)$ calculation under way
- $O(1/mQ^{4,5})$ effects need further investigation but small effect on V_{cb}
- NNNLO corrections to total width feasible, needed for 1% uncertainty?
- Electroweak (QED) corrections
- New observables in view of Belle-II: FB asymmetry proposed by S.Turczyk could be measured already by Babar and Belle now
- Lattice QCD information on local matrix elements is the next frontier, e.g.

$$M_{H_Q} = m_Q + \bar{\Lambda} + \frac{\mu_\pi^2 - a_H \mu_G^2}{2m_Q} + \dots$$

CUTS IN $B \rightarrow X_u l v$

Experiments often use kinematic cuts to avoid the $b \rightarrow clv$ background:

 $m_X < M_D$ $E_\ell > (M_B^2 - M_D^2)/2M_B$

$$q^2 > (M_B - M_D)^2 \dots$$

The cuts destroy convergence of the OPE that works so well in $b \rightarrow c$. OPE expected to work only away from pert singularities

Rate becomes sensitive to *local* b-quark wave function properties like Fermi motion. Dominant nonpert contributions can be resummed into a <u>SHAPE FUNCTION</u> $f(k_+)$. Equivalently the SF is seen to emerge from soft gluon resummation



HOW TO ACCESS THE SF?

$$\frac{d^{3}\Gamma}{dp_{+}dp_{-}dE_{\ell}} = \frac{G_{F}^{2}|V_{ub}|^{2}}{192\pi^{3}} \int dk C(E_{\ell}, p_{+}, p_{-}, k)F(k) + O\left(\frac{\Lambda}{m_{b}}\right)$$

Subleading SFs

OPE constraints e.g. at q²=0 $\int_{-\infty}^{\overline{\Lambda}} k^2 F(k) dk = \frac{\mu_{\pi}^2}{3} + O(\frac{\Lambda^3}{m_b})$ etc.

Predictions based on
resummed pQCDOPE constraints +
parameterizationDress Gluonwithout/with resummationExponentiation, ADFRGGOU, BLNP

Fit semileptonic (and radiative) data SIMBA, NNVub

|Vub| DETERMINATIONS

Inclusive: 5% total error

HFAG 2014	Average IV
DGE	4.52(16)(16)
BLNP	4.45(16)(22)
GGOU	4.51(16)(15)

UT fit (without direct V_{ub}): $V_{ub}{=}3.66(12)\;10^{\text{-}3}$

Recent experimental results are theoretically cleanest (2%) but based on background modelling. Signal simulation also relies on theoretical models...



NEW Babar endpoint analysis 1611.05624

High sensitivity of the BR on the shape of the signal in the endpoint region. <u>Single most precise measurement to date, not yet in HFAG</u>

GGOU:
$$|V_{ub}| = (3.96 \pm 0.10_{exp} \pm 0.17_{th}) \times 10^{-3}$$



What happens if same is done in other BaBar analyses? What's going on with BLNP? NB Belle multivariate analysis uses GGOU+DN for the inclusive part

FUNCTIONAL FORMS

 $\mathbf{2}$

 $\widehat{F}(k) \begin{bmatrix} \operatorname{GeV}^{-1} \end{bmatrix}$

0.5

0

0

0.2

0.4

0.6



About 100 forms considered in GGOU, large variety, double max discarded. Small uncertainty (1-2%) on V_{ub}

Only 2 parameters FF, is that good enough? A more systematic method by Ligeti et al. arXiv:0807.1926 Plot shows 9 SFs that satisfy all the first three moments

0.8

 $k \, [\text{GeV}]$

1

 $c_3 = \pm 0.15, c_4 = 0$ $c_3 = 0, c_4 = \pm 0.15$

1.2

1.4

1.6

 $c_3 = \pm 0.1, c_4 = \pm 0.1$

The NNVub Project

K.Healey, C. Mondino, PG, 1604.07598



- Use Artificial Neural Networks to parameterize shape functions without bias and extract V_{ub} from theoretical constraints and data, together with HQE parameters in a model independent way (without assumptions on functional form). Similar to NNPDF. Applies to b→ulv, b→sy, b→sl+l-
- Belle-II will be able to measure some kinematic distributions, thus constraining directly the shape functions. NNVub will provide a flexible tool to analyse data.

Selection of NN replicas trained on the first three moments only. They are not sufficient. We know photon spectrum in bsgamma: single peak dominance, not too steep

Beware: sampling can be biased by implementation!



0.5

0.0 -2.0

-1.5

-1.0



-0.5

0.0

0.5

10³ |V_{ub}| Comparison with 2007 Paper Comparison with В 2007 paper, same Α inputs 3.75 4.00 4.25 4.50 4.57 4.75 4.12 3.93 GGOU(2007)NNVub $|V_{ub}| \times 10^3$ $|V_{ub}| \times 10^3 [15]$ Experimental cuts (in GeV or GeV^2) $4.30(20)\binom{26}{27}$ 4.29(20)(21)(22) $M_X < 1.55, E_\ell > 1.0$ Babar [44] $(4.05(23))^{(19)}_{(20)}$ $4.09(23)\binom{18}{19}$ $M_X < 1.7, E_{\ell} > 1.0$ Babar [44] $M_X \leq 1.7, q^2 > 8, E_\ell > 1.0 \text{ Babar}[44] | 4.23(23)(\frac{22}{28}) | 4.32(23)(\frac{27}{30})$ $4.47(26)\binom{22}{27}$ $4.50(26)\binom{18}{25}$ $E_{\ell} > 2.0$ Babar [41] $4.58(27)\binom{10}{11}$ $4.60(27)(^{10}_{11})$ $E_{\ell} > 1.0$ Belle [45]

Inputs for constraints from sl fit by Alberti et al, 2014 with full uncertainties and correlations

Prospects @ Belle-II

- Learning from kinematic distributions, e.g. M_X spectrum
- OPE parameters checked/ improved in b→ulv (moments): global NN+OPE fit
- alternative approach SIMBA Tackmann, Ligeti, Stewart
- check signal dependence at endpoint
- full phase space implementation of α_s^2 and α_s/m_b^2 corrections
- model/exclude high q² tail



At Belle-II we can expect to bring inclusive Vub at almost the same level as Vcb

EXCLUSIVE $B \rightarrow D^* \ell v$

At zero recoil, where rate vanishes, the ff is

$$\mathcal{F}(1) = \eta_A \left[1 + O\left(\frac{1}{m_c^2}\right) + \dots \right]$$

Thanks to measurement of slopes and shape parameters, exp error only ~1.3% extrapolation to zero recoil with CLN parameterization

The ff F(I) cannot be experimentally determined. Lattice QCD is the best hope to compute it. <u>Only one</u> unquenched Lattice calculation published:

 $F(I) = 0.906(I3) \implies |V_{cb}| = 39.25(49)_{exp}(53)_{lat}(19)_{QED} 10^{-3}$

Bailey et al 1403.0635 (FNAL/MILC)

I.9% error (adding in quadrature)
 ~2.9σ or ~8% from inclusive determination
 NEW HPQCD F(1)=0.862(35) preliminary, CKM 2016
 NB Heavy Quark Sum Rules estimate F(1)=0.86(2) PG, Mannel, Uraltsev 2012

$B \rightarrow D$ form factors $f_+ f_0$ from lattice



NEW BELLE SPECTRUM 1510.03657

provided in a parametrization independent way





USING QUARK-HADRON DUALITY. DISPERSION RELATIONS→ GLOBAL QHD The BGL, BCL, CLN parametrizations include in various ways these constraints **Stronger** constraints use HQET relations to estimate some of the other channels Global fit to $B \rightarrow Dlv$

D.Bigi, PG arXiv:1606.08030



Global fit to $B \rightarrow Dlv$

• $|V_{cb}| = 40.49(0.97) 10^{-3}$ compatible with both inclusive and $B \rightarrow D^*$, same for BGL, BCL parametrizations

- Constrained fit with **strong unitarity bounds** (weak bounds lead to similar results with slightly larger errors)
- CLN relies too heavily on HQET: it has intrinsic uncertainties that can no longer be neglected
- fit assumes no correlation between FNAL and HPQCD, 3% syst error on Babar data, correct treatment of last bin, no finite size bin effect.
- Non-zero recoil lattice results are <u>crucial</u>: only zero recoil leads to |V_{cb}|=39.6(2.0) 10⁻³ (BGL)
- Possible improvements from more precise data (Belle-II, reanalysis of Babar data) and lattice calculations
- *R(D)=0.299(3)* 2σ from HFAG average

PROSPECTS FOR EXCLUSIVE V_{cb}

- Need for more lattice calculations and extension of $\mathbf{B} \rightarrow \mathbf{D}^*$ ff to non-zero recoil. Matching at $1/m_Q^3$ for lattice discretization effects under study by FNAL/MILC. Simulations at physical pion mass and $m_b a \leq 1$?
- All B→D^{*} analyses based on CLN: errors somewhat underestimated. However the spectrum is measured precisely and extrapolation to zero-recoil are a small effect. New Belle 1702.01521 permits alternative approach!
- Heavy quark sum rules favor smaller F(1)=0.86(2) leading to agreement with inclusive. Difficult to improve, how good are the BPS arguments used?
- QED/EW corrections: SD log OK, SD remainder tiny if G_µ employed, soft/ collinear radiation subtracted out by Photos, intermediate photons (IR finite) are structure dependent: lattice calculations? exp cuts? relevance of Coulomb enhancement for B^o decay rate?
- New channels (Bc, Bs, Λ_b) at Belle-II and LHCb, can also be combined for unitarity bounds, better understanding of D^{**}

RECENT LATTICE $B \rightarrow \pi$ **RESULTS**

30

FNAL/MILC 1503.07839



FNAL/MILC 3.72(16) 10⁻³ only 4.3% error 2.2σ from inclusive **RBC/UKQCD 3.61(32) 10⁻³** 1.9σ from inclusive LCSR 3.32(26) 10⁻³ 2.9σ from inclusive

 $B \rightarrow \rho(\omega) lv$ with LCSR ~3.2(4) 10⁻³ Bharucha, Straub, Zwicky 2015 $|V_{ub}| \times 10^3$ **LHCb** studied $\Lambda_b \rightarrow plv / \Lambda_b \rightarrow \Lambda_c plv$ depends on V_{cb} employed but low

RBC/UKQCD 1501.05373



RECENT LATTICE RESULTS 1503.07839



Prospects: further improvements in LQCD, much more data @ BelleII, $B_s \rightarrow Klv$ and other channels @Belle-II and LHCb, $B \rightarrow \pi \pi lv vs B \rightarrow \rho lv$??



- Improvements of OPE approach to s.l. decays continue. No sign of inconsistency in this approach so far, competitive mb-mc determination.
- Exclusive/incl. tension in V_{cb} remains (2.9σ, 8%) only in the D* channel. The D channel is becoming competitive and is compatible with both. The remaining tension calls for new lattice analyses and new data
- Exclusive/incl tension in V_{ub} might be receding because of new FNAL/ MILC and HPQCD results and of preliminary Babar results. Significant progress will come with Belle-II (using NNVub/SIMBA frameworks) and further LHCb data.
- New physics explanations look quite constrained for both V_{ub} and V_{cb} .
- After 40 years of *b* physics, things may not be simple (we are becoming sensitive to nasty details) but are still exciting: this seems to bode well for the future!

BACK-UP SLIDES

CHARM MASS DEPENDENCE



FIT PERFORMED WITH ETM CHARM MASS: $m_c(3GeV)=1.056(16)GeV$ V_{cb} only slightly smaller

CHECK: BOTTOM MASS



The fit gives $m_b^{kin}(1\text{GeV})=4.553(20)\text{GeV}$ scheme translation error $m_b^{kin}(1\text{GeV})=m_b(m_b)+0.37(3)\text{GeV}$ $\overline{m}_b(\overline{m}_b)=4.183(37)\text{GeV}$

EXCITATION ENERGY DEPENDENCE



UUT analysis in CMFV models



LQCD calculations for $|V_{ub}|$: recent progress

> Disclaimer: the list is not meant to be inclusive. I am focusing on the publicized results.

Lattice Group	Fermilab/MILC	HPQCD	RBC/UKQCD	Alpha	Detmold et al.
Process	$B o \pi \ell \nu$	$B_s \to K \ell \nu$	$B \to \pi \ell \nu$	$(B_s \to K \ell \nu)$	$\Lambda_b \to p \ell \nu$
	$(B_s \rightarrow K \ell \nu)$	$(B \to \pi \ell \nu)$	$B_s \to K \ell \nu$		
Gauge ensembles	MILC asqtad	MILC asqtad	Domain-Wall	CLS	Domain-Wall
Sea flavors	2+1	2+1	2+1	2	2+1
a (fm)	0.045-0.12	0.09-12	0.086-0.11	0.049-0.076	0.086-0.11
M_{π}	$\geq 177~{ m MeV}$	$\geq 354~{\rm MeV}$	$\geq 289~{ m MeV}$	$\geq 310~{\rm MeV}$	$\geq 295~{\rm MeV}$
<i>l</i> -quark action	asqtad	HISQ	Domain-Wall	Imprv. Wilson	Domain-Wall
b-quark action	Fermilab Clover	NRQCD	RHQ	Lat. HQET	RHQ
χΡΤ	NNLO,SU(2), hard- π	$HP\chi PT+$	NLO,SU(2), hard- π		
q^2 -extrapolation	functional BCL	modified z	synthetic BCL		modified-z
Ref.	arXiv:1503.07839	arXiv:1406.2279	arXiv:1501.05373v2	arXiv:1411.3916	arXiv:1306.0446
	arXiv:1312.3197	f _o		1601 04277	arXiv:1503.01421v2
		1601.04277			arXiv:1504.01568

Du, MITP workshop 2015

(): work in progress

THEORETICAL ERRORS



Theoretical errors are generally the **dominant** ones in the fits. We estimate them in a **conservative** way, mimicking higher orders by varying the parameters by fixed amounts: $m_{c,b}$ 8MeV, $\alpha_{s}(m_{b})$ 0.018, 7% in $1/m^{2}$ parameters, 30% in $1/m^{3}$ parameters New corrections have been within theor. uncertainties so far.

FORM FACTORS

$$\langle D(p')|V^{\mu}|\overline{B}(p)\rangle = f_{+}(q^{2})(p+p')^{\mu} + f_{-}(q^{2})(p-p')^{\mu}$$
 $q^{2} = (p-p')^{2}$

$$\frac{d\Gamma}{dq^2}(B \to Dl\nu_l) = \frac{\eta_{ew}^2 G_F^2 |V_{cb}|^2 m_B \lambda^{1/2}}{192\pi^3} \left(1 - \frac{m_l^2}{q^2}\right)^2 \left[c_+^l f_+(q^2)^2 + c_0^l f_0(q^2)^2 + r_0^l f_0(q^2)^2\right]$$

$$r = m_D/m_B, \ \lambda = (q^2 - m_B^2 - m_D^2)^2 - 4m_B^2 m_D^2$$

$$\eta_{ew} = 1 + \alpha/\pi \ln M_Z/m_b \approx 1.0066$$

$$c_+^l = \frac{\lambda}{m_B^4} \left(1 + \frac{m_l^2}{2q^2}\right), \qquad c_0^l = (1 - r^2)^2 \frac{3m_l^2}{2q^2}$$

$$f_0(q^2) = f_+(q^2) + \frac{q^2}{m_B^2 - m_D^2} f_-(q^2)$$

$$f_+(0) = f_0(0)$$

UNITARITY CONSTRAINTS

$$\begin{pmatrix} -g^{\mu\nu} + \frac{q^{\mu}q^{\nu}}{q^2} \end{pmatrix} \Pi^T(q^2) + \frac{q^{\mu}q^{\nu}}{q^2} \Pi^L(q^2) \equiv i \int d^4x \, e^{iqx} \langle 0|TJ^{\mu}(x)J^{\dagger\nu}(0)|0\rangle$$
$$\chi^L(q^2) = \frac{\partial \Pi^L}{\partial q^2}, \qquad \chi^T(q^2) = \frac{1}{2} \frac{\partial^2 \Pi^T}{\partial (q^2)^2}$$

SATISFY UNSUBTRACTED DISP REL, PERT CALCULATION FOR $q^2=0$ Boyd, Grinstein, Lebed 1995

 $\chi^{T}(0) = [5.883 + 0.552_{\alpha_{s}} + 0.050_{\alpha_{s}^{2}}] \ 10^{-4} \,\text{GeV}^{-2} = 6.486(48) \ 10^{-4} \,\text{GeV}^{-2}$ $\chi^{L}(0) = [5.456 + 0.782_{\alpha_{s}} - 0.034_{\alpha_{s}^{2}}] \ 10^{-3} = 6.204(81) \ 10^{-3}$

USING UP-TO-DATE QUARK MASSES AND SLOOP CALCULATION Grigo et al 2012

$$\tilde{\chi}^T(0) = \chi^T(0) - \sum_{n=1,2} \frac{f_n^2(B_c^*)}{M_n^4(B_c^*)}$$

BOUND STATE

Type	Mass (GeV)	Decay constants (GeV)
1-	6.329(3)	0.422(13)
1-	6.920(20)	0.300(30)
1-	7.020	
1-	7.280	
0^{+}	6.716	
0^{+}	7.121	

UNITARITY CONSTRAINTS



BGL PARAMETERIZATION: TRUNCATE EXPANSION AT n=NPROBLEMS AT THRESHOLD AND WITH LARGE q^2 SCALING

BCL PARAMETERIZATION BOURELLY CAPRINI LELLOUCH 2008

$$f_{+}(z) = \frac{1}{1 - q^2/M_{+}^2} \sum_{n=0}^{N} a_n^{+} \left[z^n - (-1)^{n-N-1} \frac{n}{N+1} z^{N+1} \right]$$
(BCL)

STRONG UNITARITY CONSTRAINTS

If one knows something about the other channels the constraints become tighter In the heavy quark limit all $B^{(*)} \rightarrow D^{(*)}$ form factors either vanish or are prop to the Isgur-Wise function $H \propto \infty$

$$\sum_{i=1}^{n} \sum_{n=0}^{\infty} b_{in}^2 \le 1 \qquad \sum_{n} b_{in} z^n = c_i(z) f_+(z)$$

CAPRINI LELLOUCH NEUBERT CLN 1998

$$\begin{aligned} f_{+}(z) &\simeq f_{+}(0) \left[1 - 8\rho_{1}^{2}z + (51\rho_{1}^{2} - 10)z^{2} - (252\rho_{1}^{2} - 84)z^{3}) \right] \\ \frac{f_{0}(z)}{f_{+}(z)} &\simeq \left(\frac{2\sqrt{r}}{1+r} \right)^{2} \frac{1+w}{2} 1.0036 \left[1 - 0.0068w_{1} + 0.0017w_{1}^{2} - 0.0013w_{1}^{3} \right] \\ w_{1} &= w - 1 \end{aligned}$$

CLN exploit NLO HQET relations between form factors to reduce to only 2 parameters... but 1/m² corrections can be sizable For ex at zero recoil

$$\frac{F_{D^*}(z=0)}{f_+(z=0)} = 0.948 \neq 0.860(14)$$

NLO HQET LATTICE (FNAL)
$$\frac{f_+(0)}{f_0(0)} = 0.775 \neq 0.753(3)$$

NLO HQET LATTICE (FNAL)

3%

CLN parameterization has intrinsic uncertainties that can no longer be neglected. We use HQET expressions only in derivation of unitarity bounds and have checked that results are unaffected

RESULTS

exp data	lattice data	N,par	$10^3 \times V_{cb} $	χ^2/dof	R(D)
all	all	2,BGL	40.62(98)	22.1/26	0.302(3)
all	all	3,BGL	40.47(97)	18.2/24	0.299(3)
all	all	4, BGL	40.49(97)	19.0/22	0.299(3)
Belle	all	3,BGL	40.92(1.12)	11.6/14	0.300(3)
BaBar	all	3,BGL	40.11(1.55)	12.6/14	0.301(4)
all	FNAL	3,BGL	40.17(1.05)	10.4/18	0.293(4)
all	HPQCD	3,BGL	$40.51^{+1.82}_{-1.11}$	10.1/18	0.299(7)
all	all	CLN	40.85(95)	77.1/29	0.305(3)
all	f_+ only	CLN	40.33(99)	20.0/23	0.305(3)
all	all	2, BCL	40.49(98)	18.2/26	0.299(3)
all	all	3,BCL	40.48(96)	18.2/24	0.299(3)
all	all	4,BCL	40.48(97)	17.9/22	0.299(3)

WEAK vs STRONG BOUNDS



Figure 2: Form factor $f_{+}(z)$ in the N = 4 BGL fit to lattice data for $f_{+,0}(z)$ with weak (brown band) and strong (gray band) unitarity constraints. The N = 2 band (independent of unitarity constraints) is shown in dashed lines for comparison. FNAL/MILC synthetic data are shown in red, HPQCD in blue. On the right, enlarged detail of the tail.

A GLOBAL COMPARISON 09

0907.5386, Phys Rept

GGOU

8



 $|V_{\rm ub}|10^{3}$

4.0

3.5

1

2

3

4

analysis

5

6

7

- * Overall good agreement SPREAD WITHIN THEORY ERRORS
- * NNLO BLNP still missing: will push it up a bit
- * Systematic offset of central values: normalization? to be investigated

EFT view of b→cτv

Data can be best described by (a combination of) following operators



[Freytsis, Ligeti, Ruderman, 1506.08896]

Models for b→cτv

Only tree-level NP can compete with tree-level SM!

Charged scalars: extra Higgs(es)

- Non-minimal flavour structure (e.g. type III)
- Scalar form factor F₀ enhanced in B→Dτν, absent in B→Dℓν



Coloured bosons - LQs

 Fierzed basis of operators: →scalar/vector/tensor

[Sakaki, Tanaka, Tayduganov, Watanabe, 1309.0301] [Bauer, Neubert, 1511.01900] [Li, Yang, Zhang, 1605.09308]

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LQ models for b→cτv

Vector U₃ (3,3)_{2/3}

A weak triplet state couples at tree-level only to LH fermions

$$\mathcal{L}_{U_{3}} = g_{ij} \bar{Q}_{i} \gamma^{\mu} \tau^{A} U_{3\mu}^{A} L_{j}$$

$$\overset{[\text{Paller, NK, 1511,01900]}}{\text{[Barbieri, 1sidori, Pattori, Senia, 1512.01560]}}$$

$$\mathcal{L}_{U_{3}} = U_{3\mu}^{(2/3)} \left[\underbrace{\mathcal{V}g\mathcal{U}_{ij} \bar{u}_{i} \gamma^{\mu} P_{L} \nu_{j}}_{(\sqrt{2}\mathcal{V}g)_{ij} \bar{u}_{i} \gamma^{\mu} P_{L} \nu_{j}} \underbrace{-g_{ij} \bar{d}_{i} \gamma^{\mu} P_{L} \ell_{j}}_{(\sqrt{2}\mathcal{V}g)_{ij} \bar{u}_{i} \gamma^{\mu} P_{L} \ell_{j}} \underbrace{-\text{LH currents for Puzzle b->sll!}}_{\text{H currents for Puzzle b->sll!} + U_{3\mu}^{(5/3)} (\sqrt{2}\mathcal{V}g)_{ij} \bar{u}_{i} \gamma^{\mu} P_{L} \ell_{j} \underbrace{-\text{charm and top}}_{(\sqrt{2}g\mathcal{U})_{ij} \bar{d}_{i} \gamma^{\mu} P_{L} \nu_{j} + \text{h.c.}} \underbrace{-\text{B} \rightarrow \text{Kvv}}$$

$$\mathcal{L}_{\rm SL} = -\left[\frac{4G_F}{\sqrt{2}}\mathcal{V}_{cb}\mathcal{U}_{\tau i} + \underbrace{g^*_{b\tau}\mathcal{V}g\mathcal{U}_{ci}}_{M_U^2}\right](\bar{c}\gamma^{\mu}P_Lb)(\bar{\tau}\gamma_{\mu}P_L\nu_i)$$

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