Higgs Maxwell Workshop
Royal Society of Edinburgh, 08.02.2017

Lattice Results for Flavour Physics

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University of Southampton

Outline

1. lattice QCD

2. precision flavour physics

3. pushing the frontiers (QED+QCD, rare decays)

UK Lattice community

- Cambridge
- Edinburgh
- Glasgow
- Liverpool
- Oxford
- Plymouth
- Southampton
- Swansea

Various collaborations
(UKQCD, HotQCD, HPQCD, …)

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- QCD flavour phenomenology
- QCD spectra
- BSM models (non-perturbatively)
- finite-T, finite-μ
- developments in quantum field theory, algorithms computing and hardware

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- but there is substantial phenomenological evidence that it can't be the whole story: dark matter, CP-violation, … indicate that there must be sth. else
- despite decades of experimental and theoretical efforts we have not found a smoking gun

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  - ‘bump in the spectrum’
  - SM provides correlation between processes
    experiment + theory to over-constrain SM

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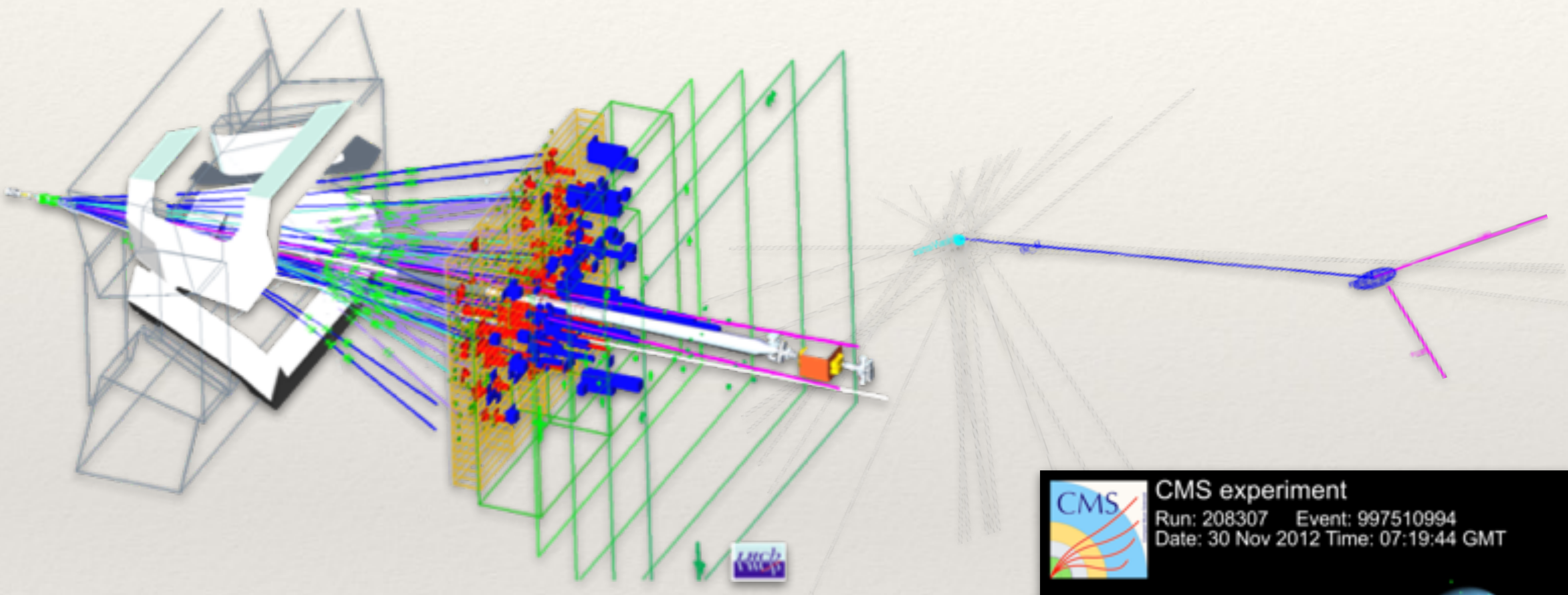
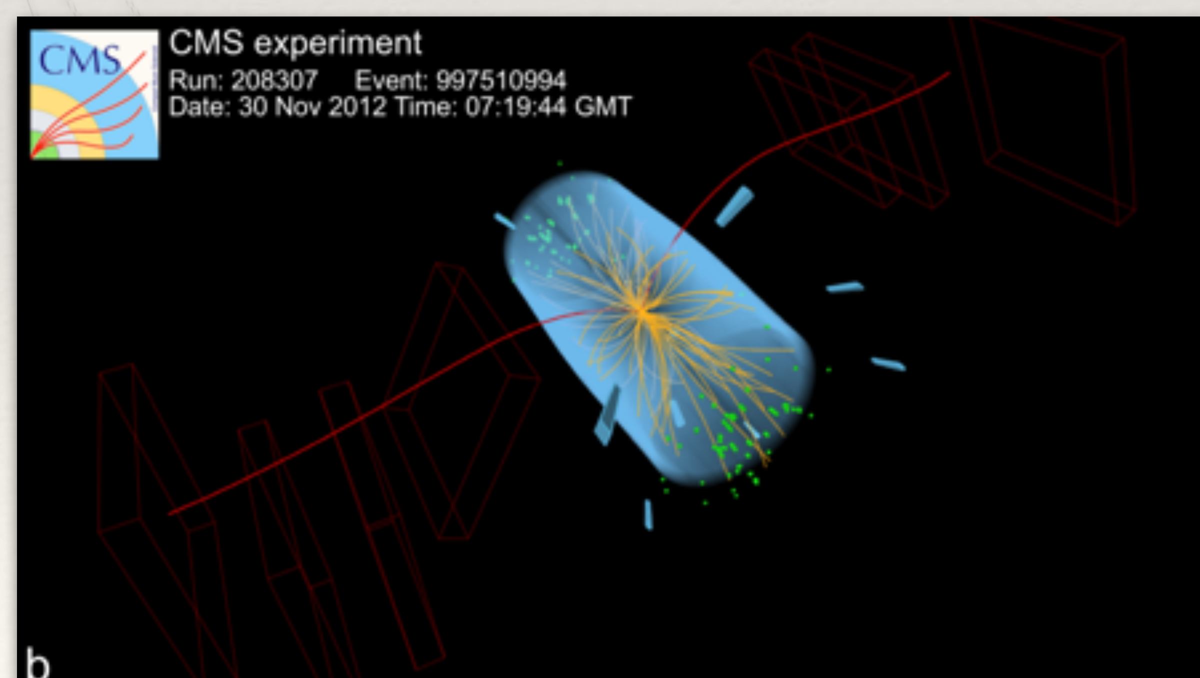
- lattice QCD can in principle provide the relevant input and is becoming increasingly precise in its predictions

$B_s \rightarrow \mu^+\mu^-$

First observed by LHCb, CMS





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Standard Model prediction:

- Loop suppressed in the SM (FCNC) → sensitive to non-SM interaction?

$$\mathrm{Br}(B_s \to \ell^+\ell^-)^{\mathrm{SM}} = \tau_{B_s} \frac{G_F^2 \alpha^2}{16\pi^3} |V_{tb} V_{ts}^*|^2 m_{B_s} m_\ell^2 \beta_\ell(m_{B_s}^2) |C_{10}|^2 f_{B_s}^2$$

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NLO EW

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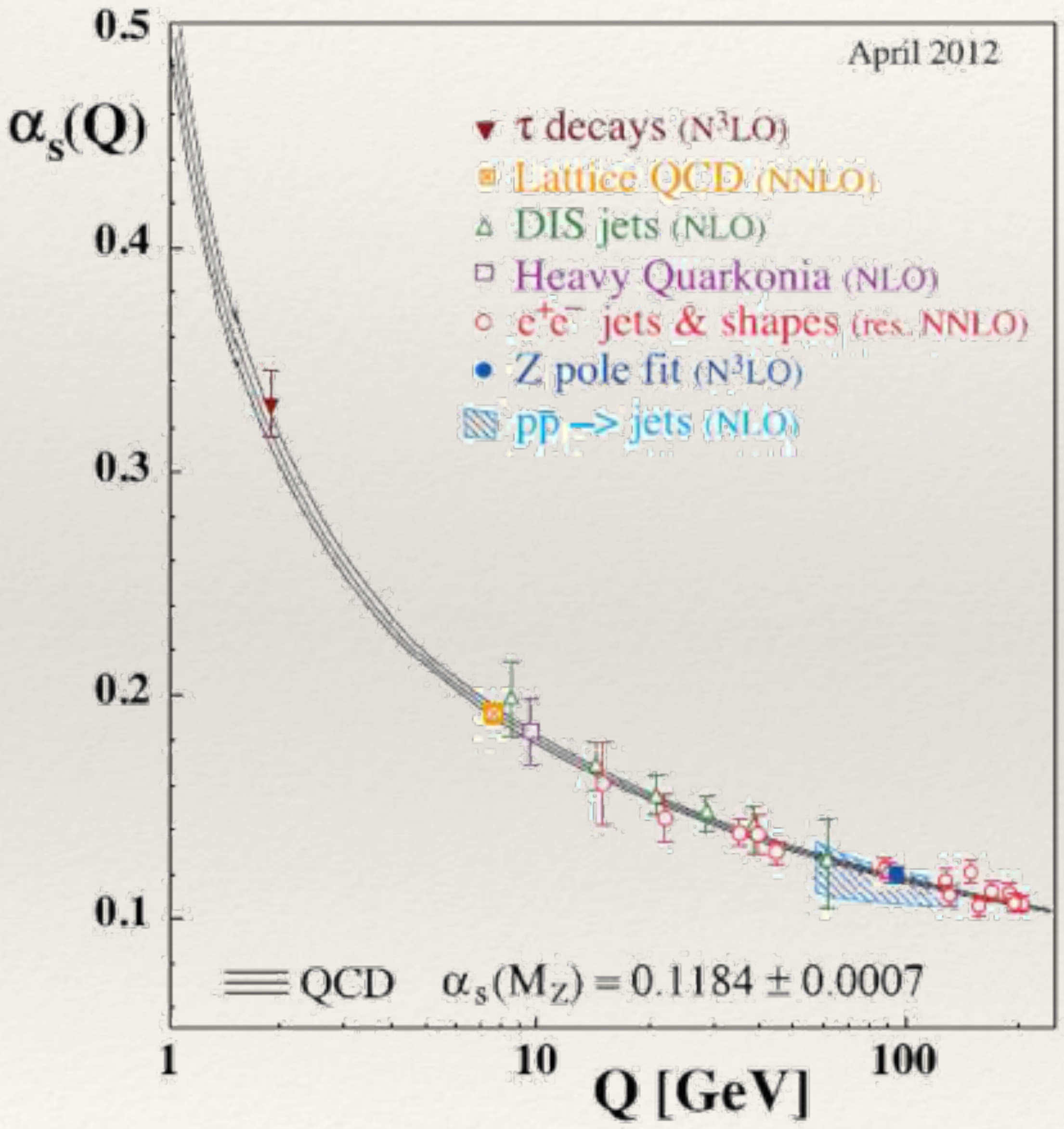
very precise and reliable prediction for the decay constant is needed

QCD

asymptotic freedom



PDG

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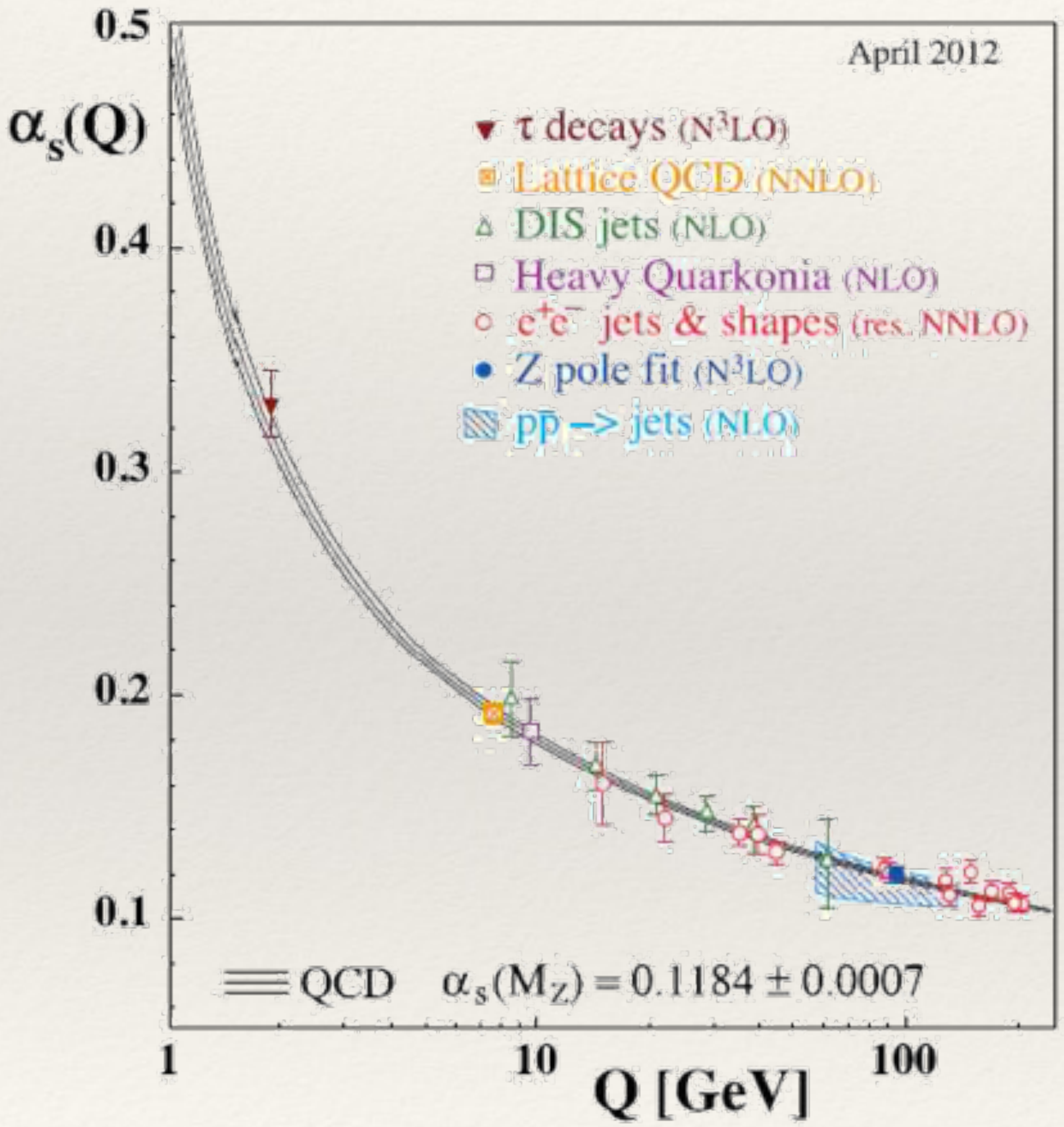

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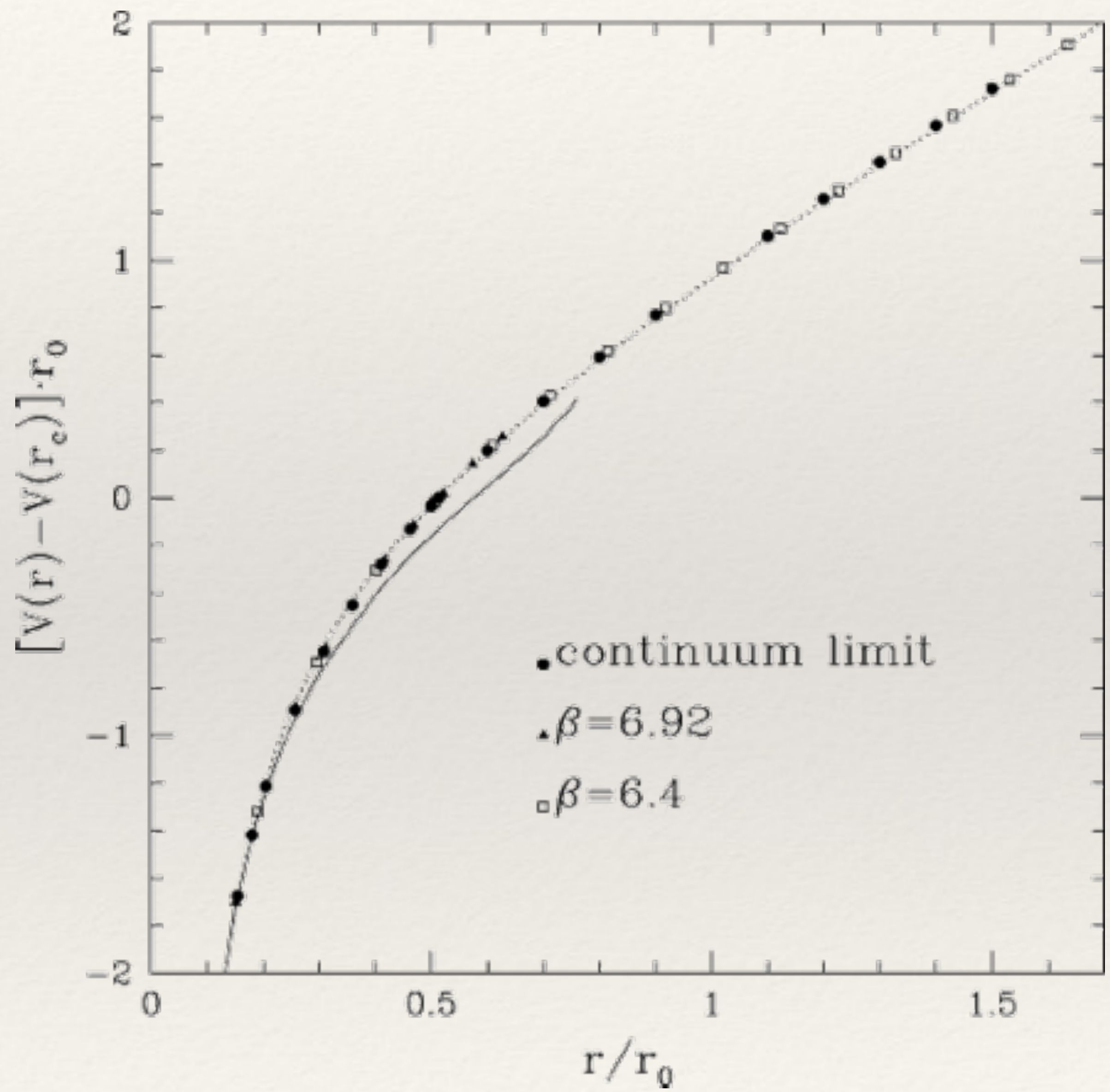
confinement



Necco & Sommer NPB 622 (2002)

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Euclidean space-time Boltzmann factor

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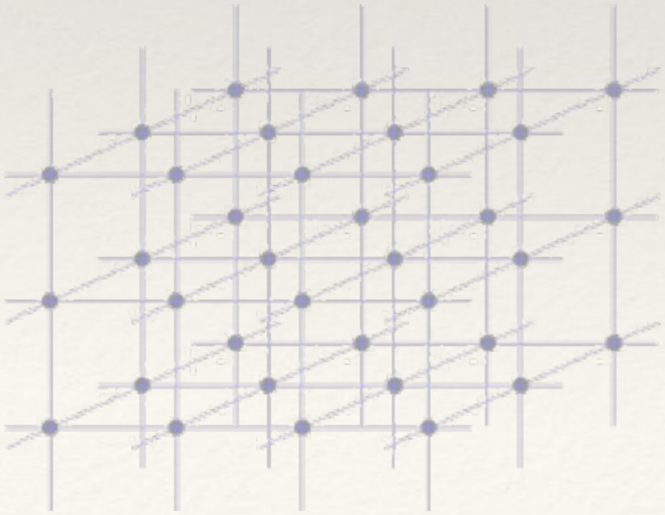

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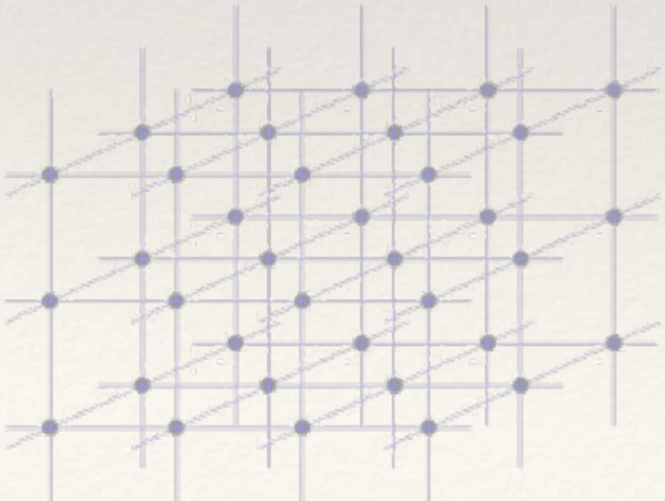

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$\propto L^{-1}$ $\propto a^{-1}$

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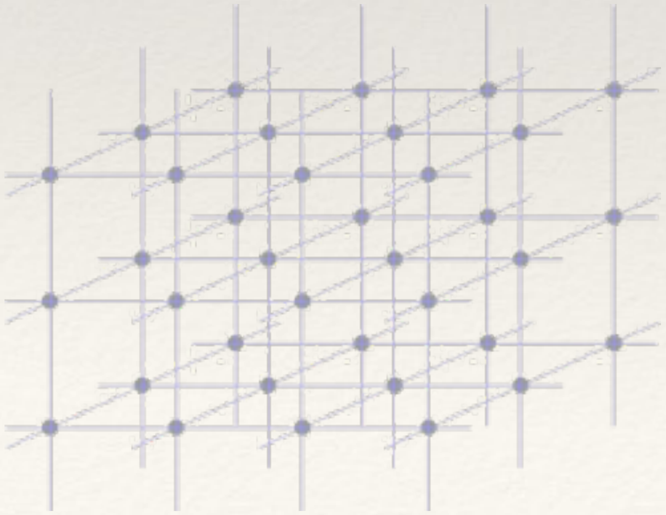

finite volume, space-time grid (IR and UV regulators)
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→ well defined, finite dimensional Euclidean path integral

→ from first principles

Lattice QCD

- gauge-invariant regularisation (Wilson 1974)
- naively: replace derivatives by finite differences, integrals by sums
- finite volume lattice path integral still over large number of degrees of freedom $> O(10^{10})$
- Evaluate discretised path integral by means of Markov Chain Monte Carlo on state-of-the-art HPC installations
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Euclidean correlation function

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two-point function

$$= \sum_{n}|\langle 0|\mathcal{O}_{B_s}(0)|n\rangle|^2\, e^{-E_n t_x}$$

$$\overset{t\to\infty}{=} |\langle 0|\mathcal{O}_{B_s}(0)|B_s\rangle|^2\, e^{-m_{B_s} t_x}$$

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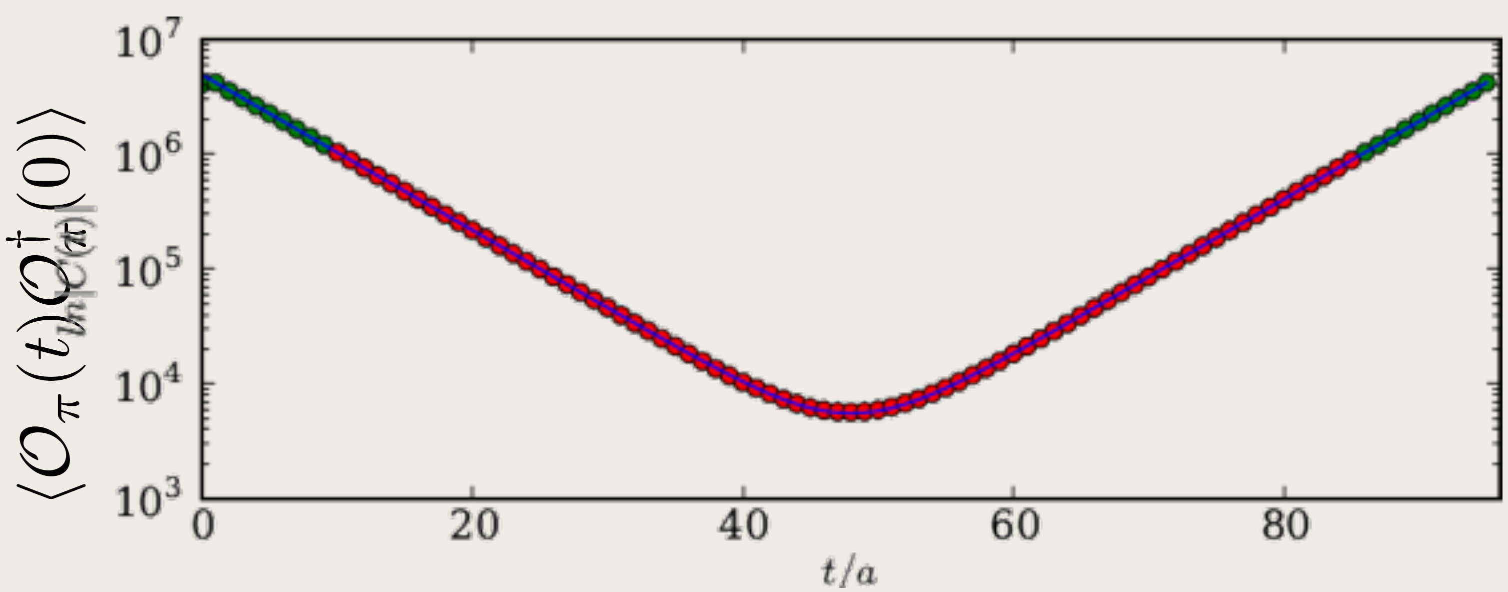
two-point function

extract physical properties from fits to simulation data:

- normalisation → matrix element (e.g. decay constant)
- time-dependence → particle spectrum (e.g. meson mass)
- stat. errors from MC sampling over N field configurations

$$\langle \mathcal{O}\mathcal{O}^\dagger\rangle = \frac{1}{N}\sum_{n=1}^{N}[\mathcal{O}\mathcal{O}^\dagger]_n$$

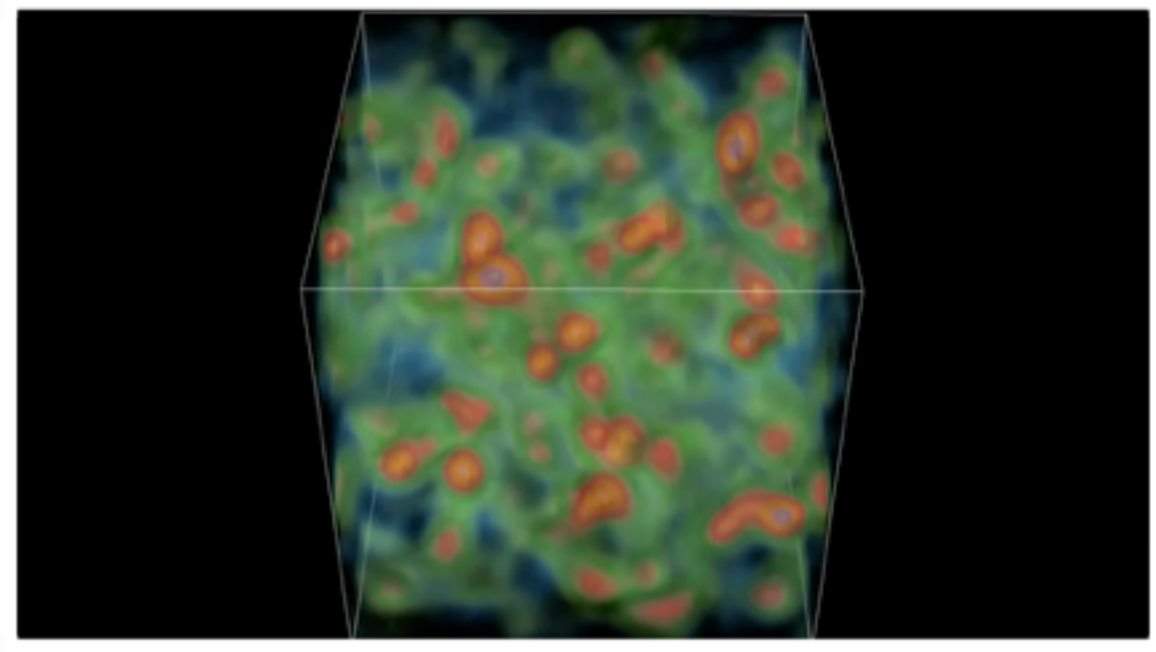
(bootstrap, jackknife error analysis, autocorrelation analysis, …)



State of the art of lattice QCD simulations

What we can do

- simulations of QCD with dynamical (sea) *u,d,s,c* quarks with masses as found in nature → $N_f = 2,\ 2+1,\ 2+1+1$
- bottom only as valence quark
- cut-off $a^{-1} \leq 4\text{GeV}$
- volume $L \leq 6fm$



action density of RBC/UKQCD physical point DWF ensemble

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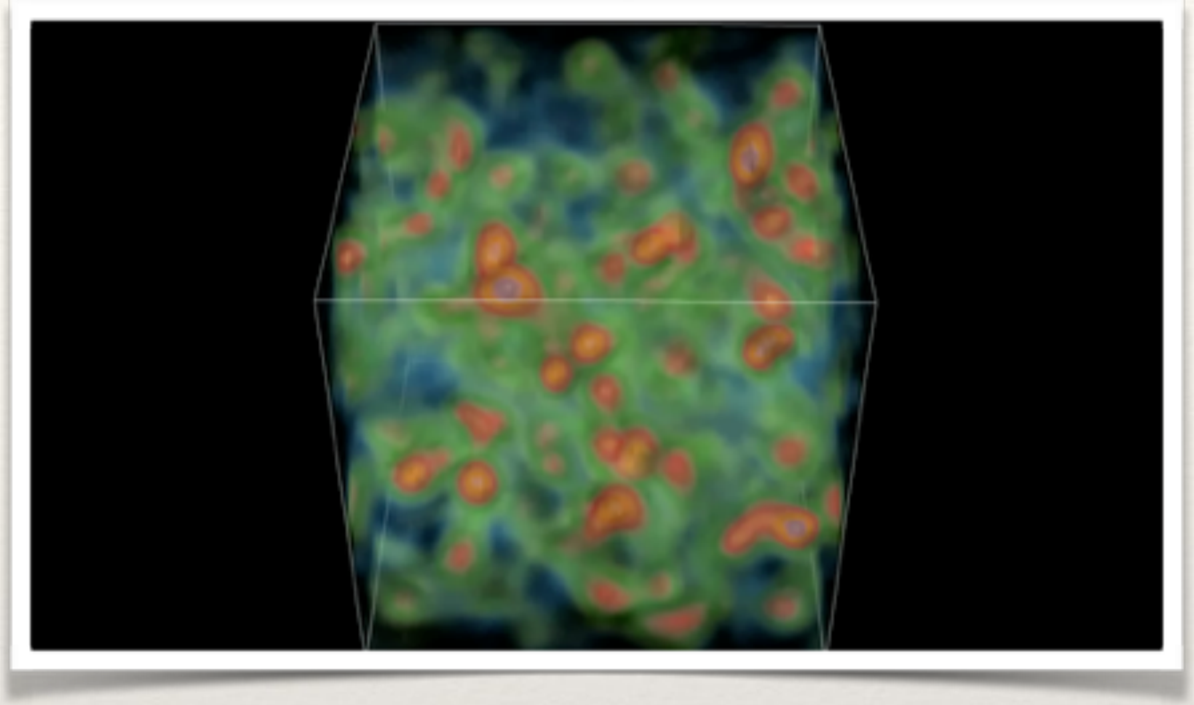

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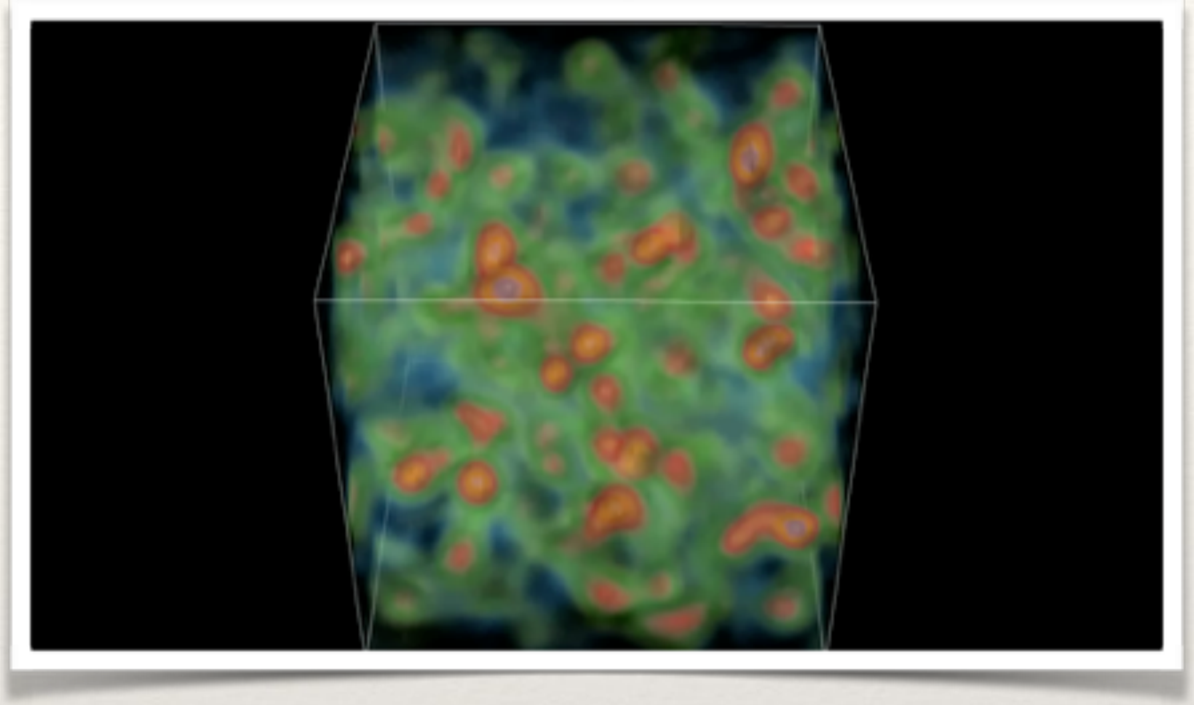

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Parameter tuning

start from *educated guesses* and:

- tune light quark mass am_l such that $\dfrac{am_\pi}{am_P} = \dfrac{m_\pi^{PDG}}{m_P^{PDG}}$
- tune strange quark mass such that $\dfrac{am_\pi}{am_K} = \dfrac{m_\pi^{PDG}}{m_K^{PDG}}$
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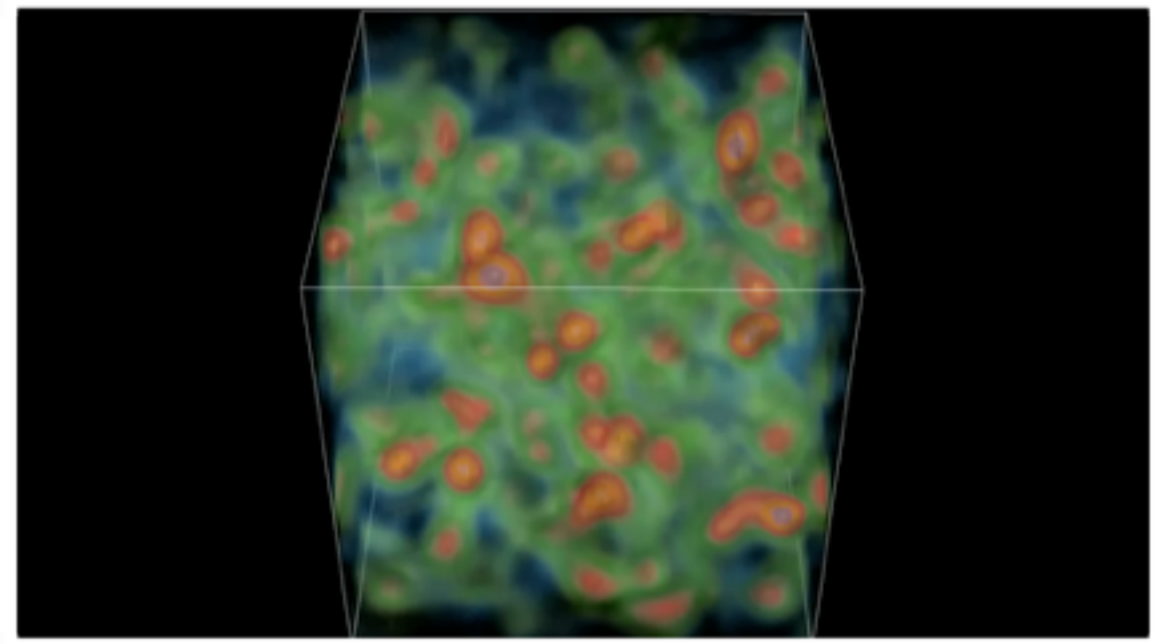

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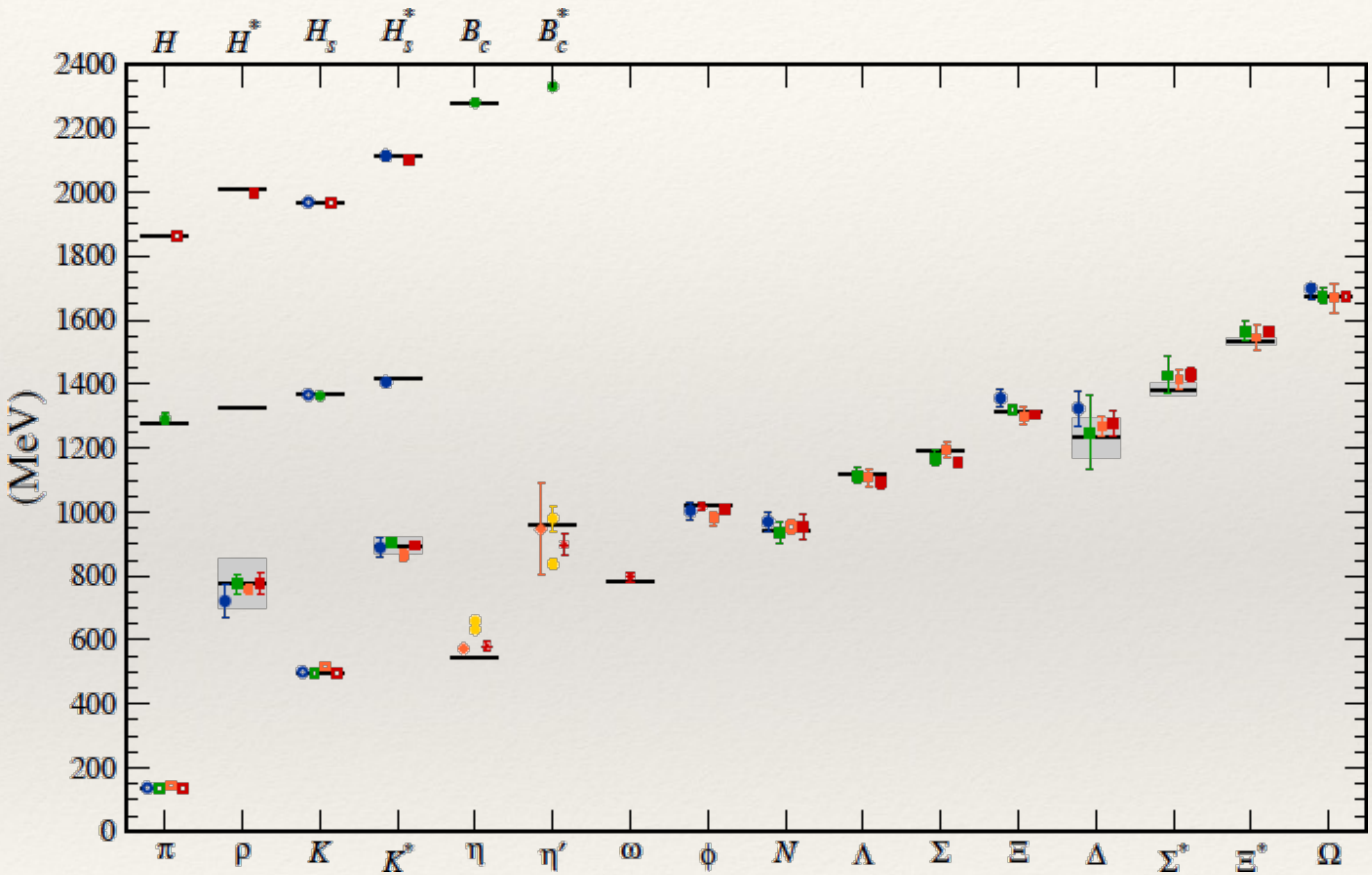
IMPORTANT:
once the QCD-parameters are *tuned* no further parameters need to be fixed and we can make fully predictive simulations of QCD

benchmark - the hadron spectrum



Kronfeld, Ann. Rev. of Nucl. Part. Sci 2012 62

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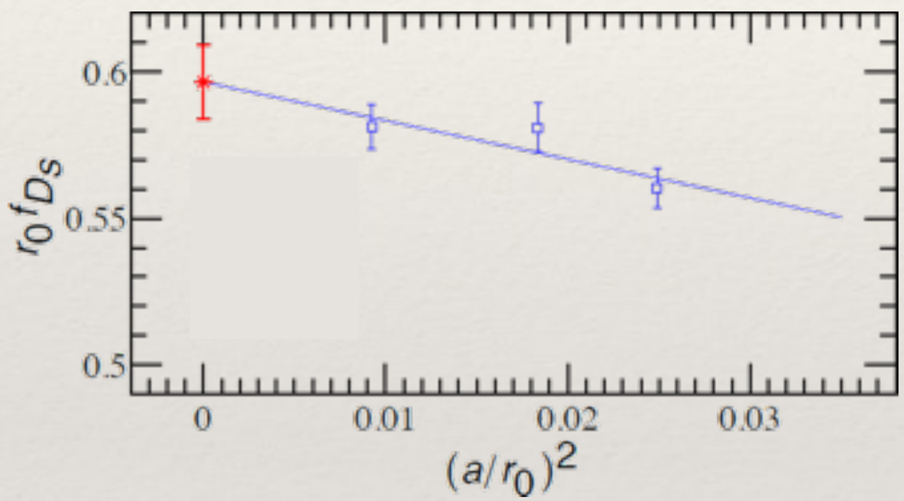

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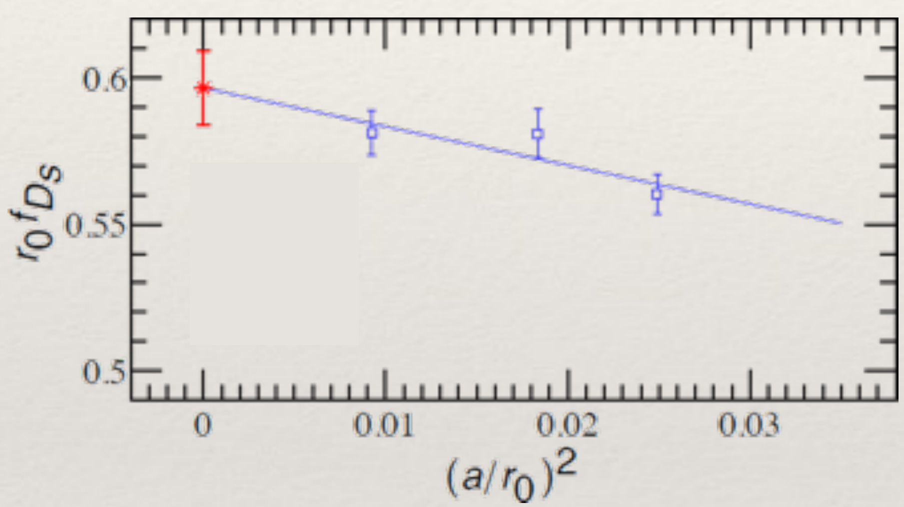

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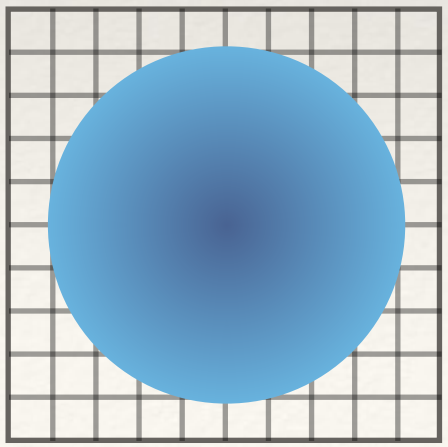

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more complicated for processes with several hadrons in initial or final state

Lüscher Commun.Math.Phys. 105 (1986) 153-188, Nucl. Phys. B354, 531 (1991)

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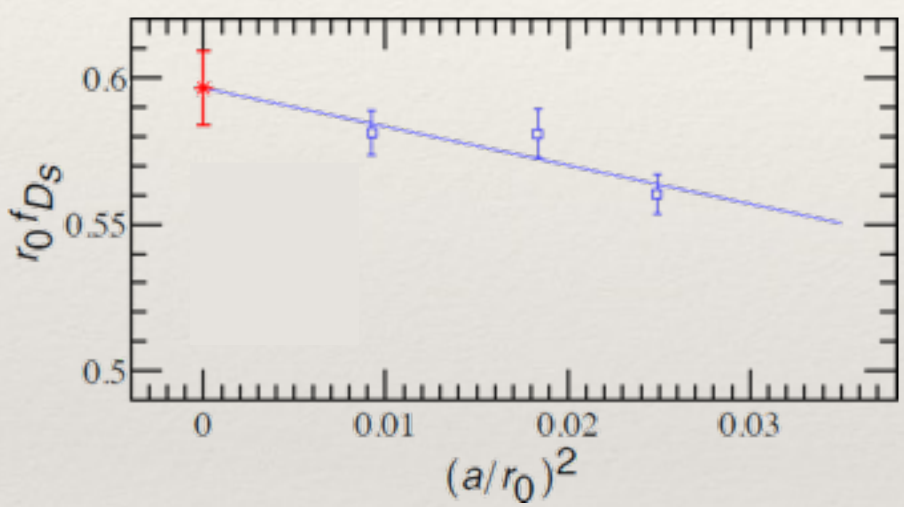

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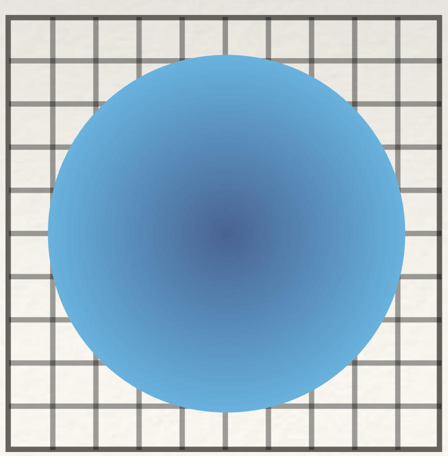

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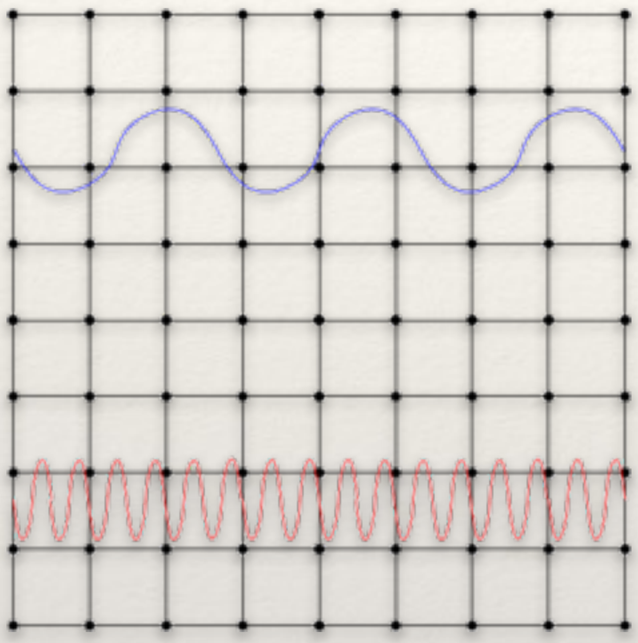
quite crude, in practice more complicated

a state-of-the-art lattice



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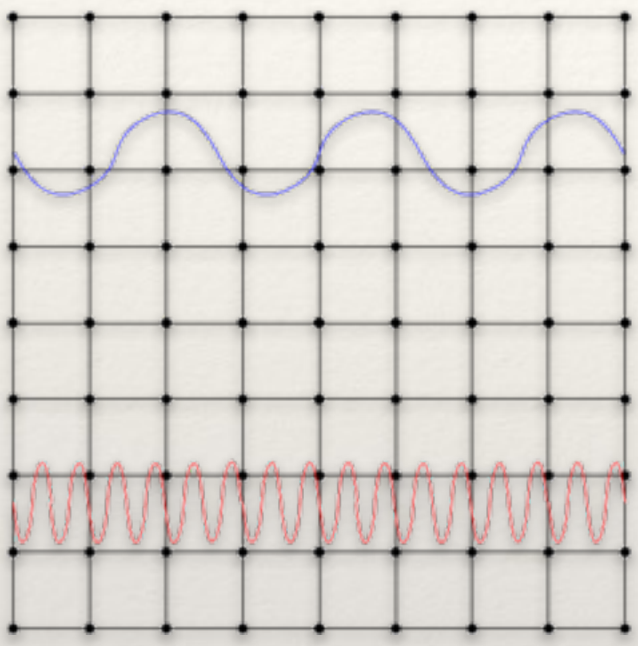

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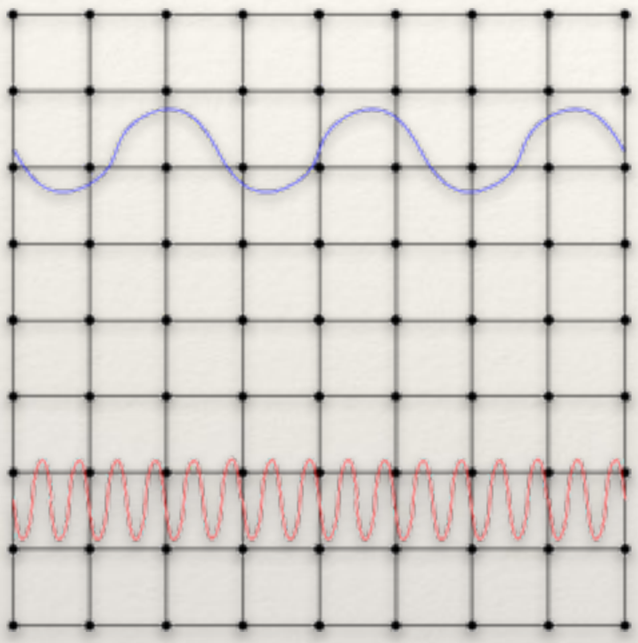

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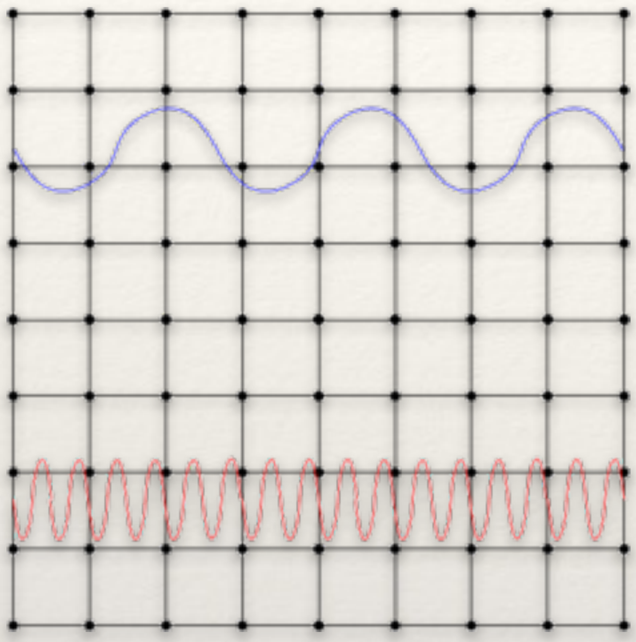
Fulfilling all the constraints for light hadrons is just starting to happen (e.g. first $96^3 \times 192$ have been generated (MILC)) in the meantime most collaborations

- weaken the finite volume effects by simulating unphysically heavy pions
- extrapolate from coarser lattices relying on assumptions for functional form of cutoff effects

heavy quarks on the lattice



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$$\boxed{m_b \gg a^{-1} > m_c \gg \Lambda_{QCD} \gg L^{-1}}$$

- OK for c quark
- b beyond reach in current simulations

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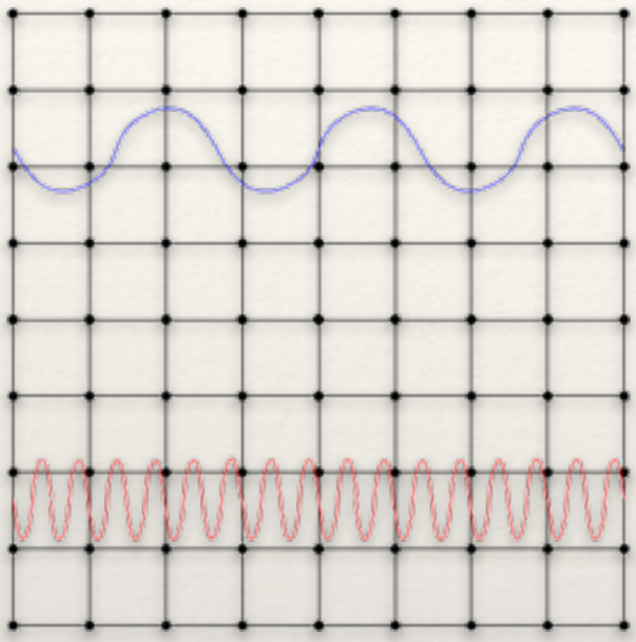

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  - (lattice)-heavy quark effective theory
  - NRQCD
  - relativistic heavy quark (lattice) actions
    (El-Khadra et al. PRD 55 3933 1997, Aoki et al. PTP 109 383 2003, Christ, Lin PRD 76 074505 2007)
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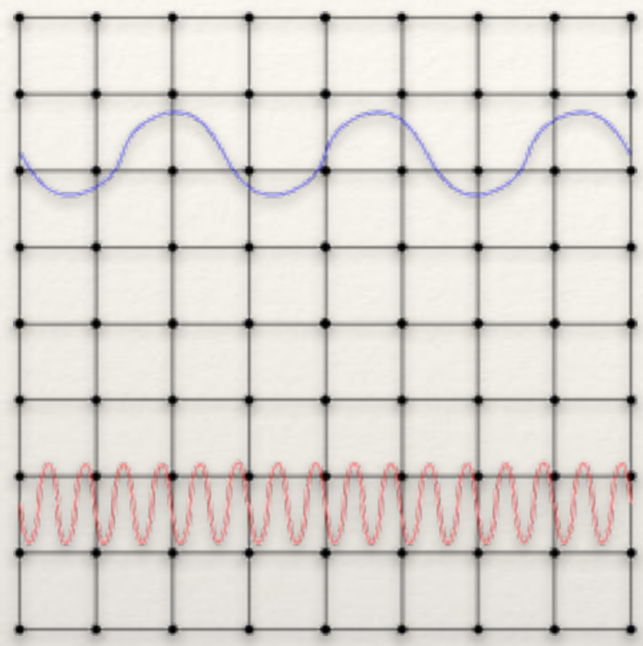

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b on the lattice therefore much less standard than u, d, s, c but if care is taken robust and rather precise predictions for b-quark physics can be made

Lattice pheno - what's possible

- **Standard:**
  - meson ME with single incoming and/or outgoing pseudo-scalar states
    $\pi, K, D_{(s)}, B_{(s)} \to \mathrm{QCD-vacuum}, \; \pi \to \pi, K \to \pi, D \to K, B \to \pi, ..., \; B_K, (B_D), B_B$
  - QCD parameters: quark masses, strong coupling constant
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  - two initial/final hadronic states, one channel $\pi\pi \to \pi\pi, K\pi \to K\pi, K \to \pi\pi, \ldots$
  - elm. effects in spectra
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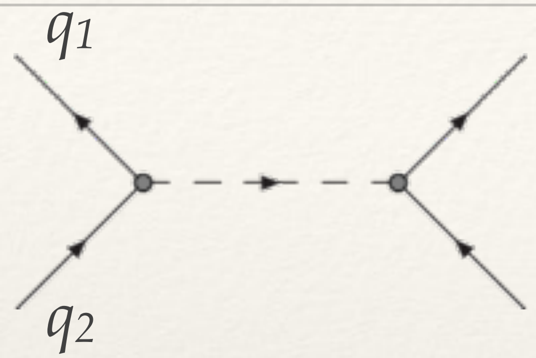
- **Very challenging - new ideas needed/no clue:**
  - multi-channel final states (hadronic D, B) (e.g. Hansen, Sharpe PRD86, 016007 (2012))
  - transition MEs with unstable in/out states (Briceño et al. arXiv:1406.5965)
  - electromagnetic effects in hadronic MEs

Quark Flavour Physics

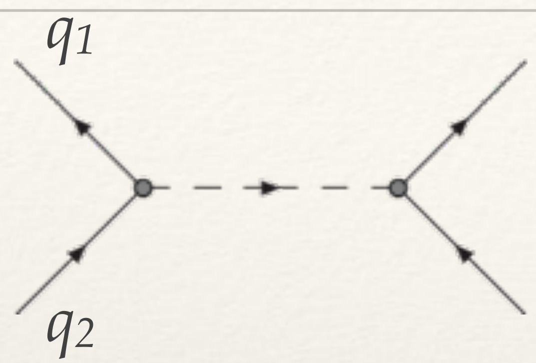


$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

3x3 unitary matrix

4 unknown parameters

Quark Flavour Physics



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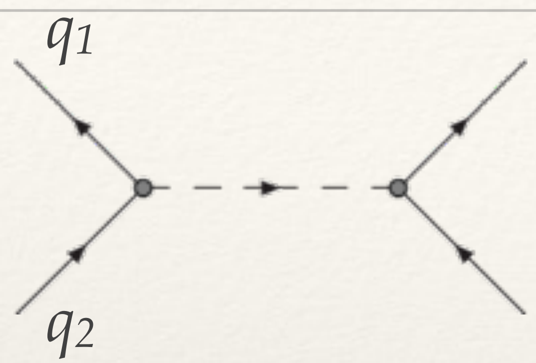
- high energy reach
- inconsistencies → failure of the SM?

Quark Flavour Physics

100GeV



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2GeV

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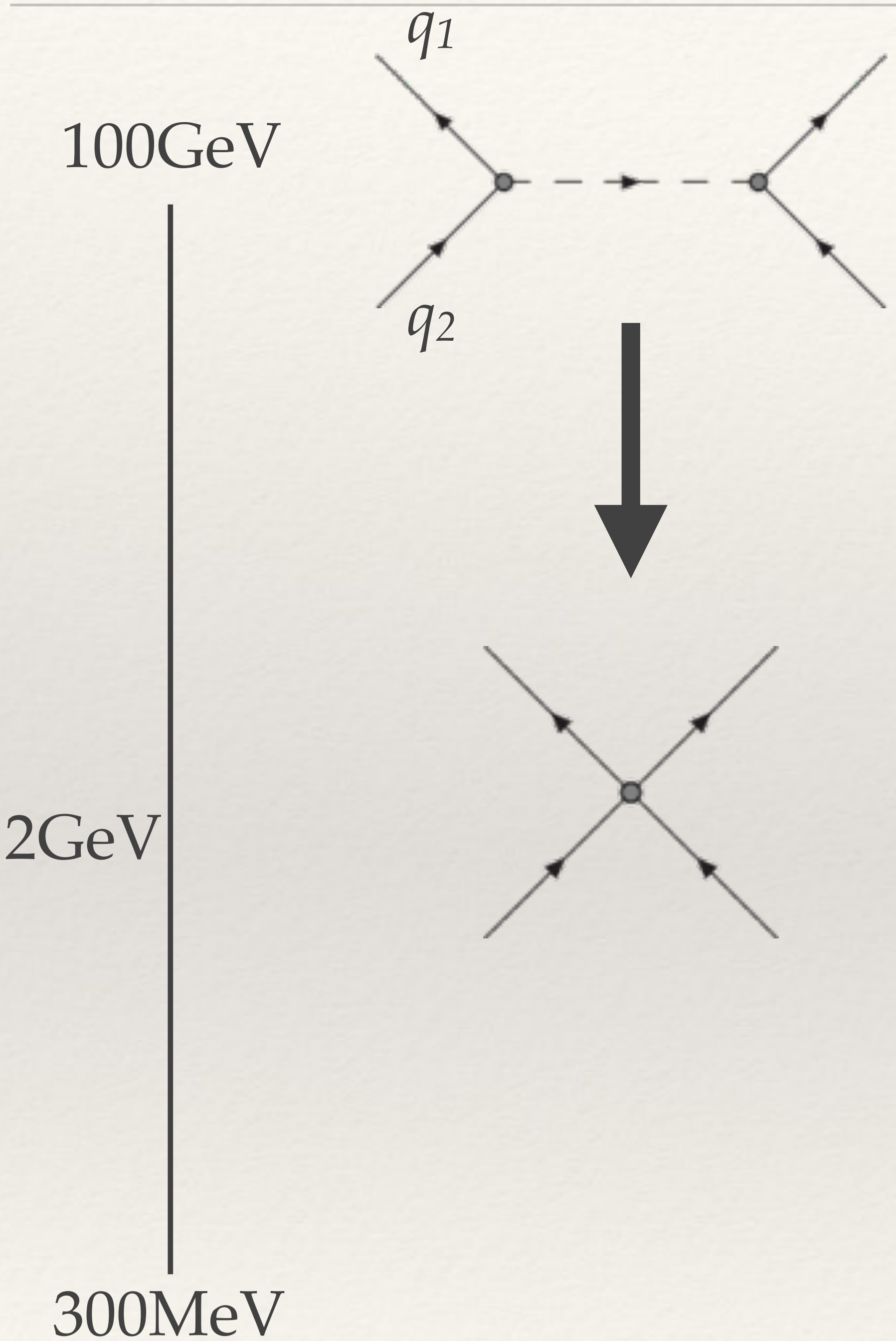
300MeV

Quark Flavour Physics



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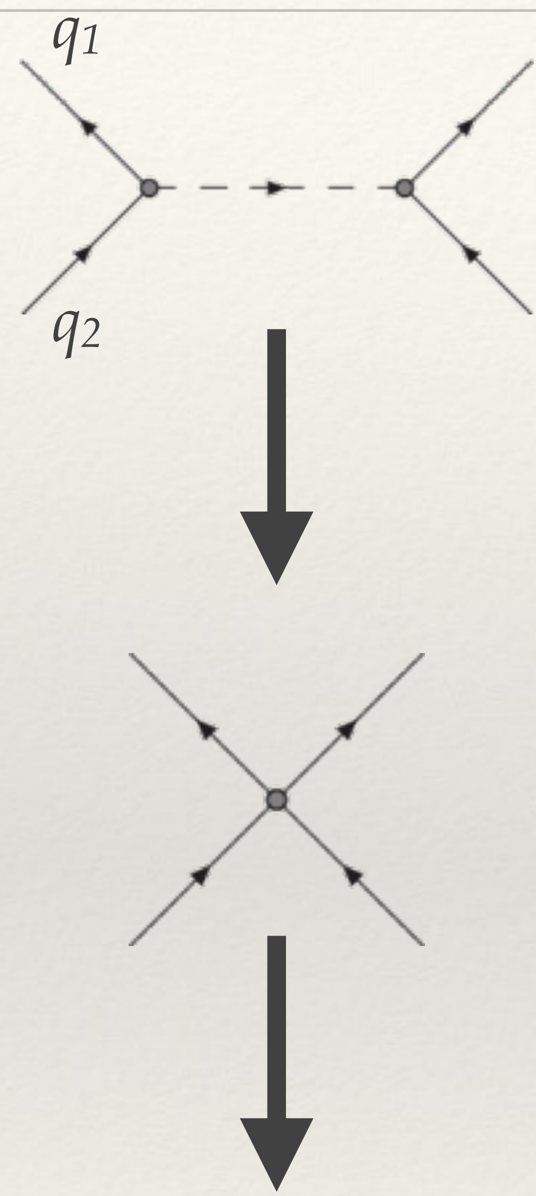

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300MeV $\langle \mathrm{had}_f | H_W | \mathrm{had}_i \rangle$

$\langle 0 | H_W | \mathrm{had}_i \rangle$

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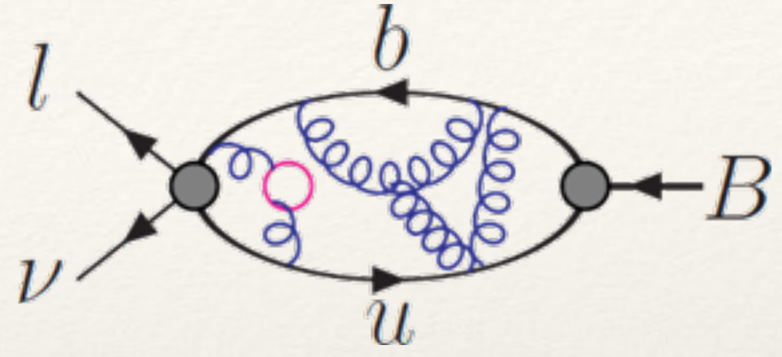
e.g tree level leptonic B decay:



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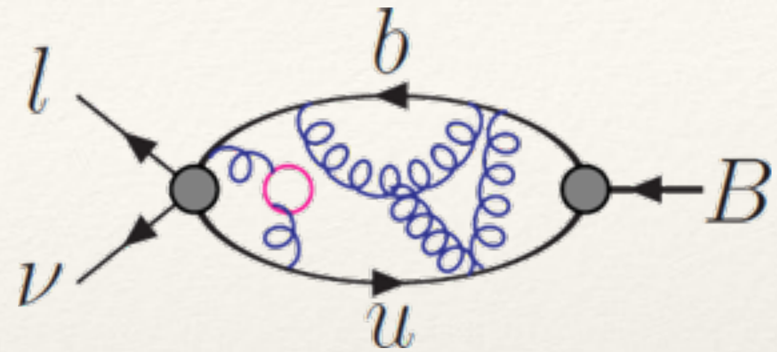

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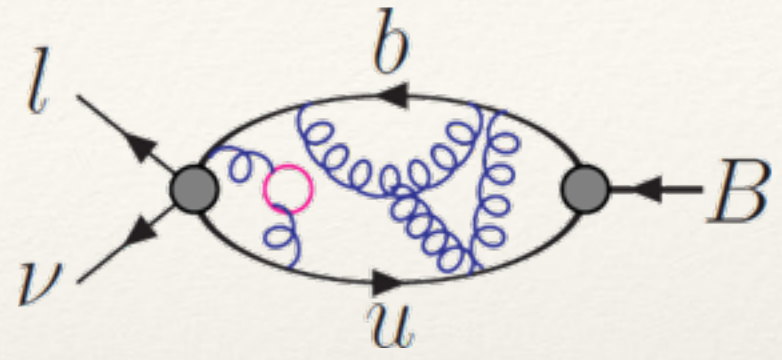

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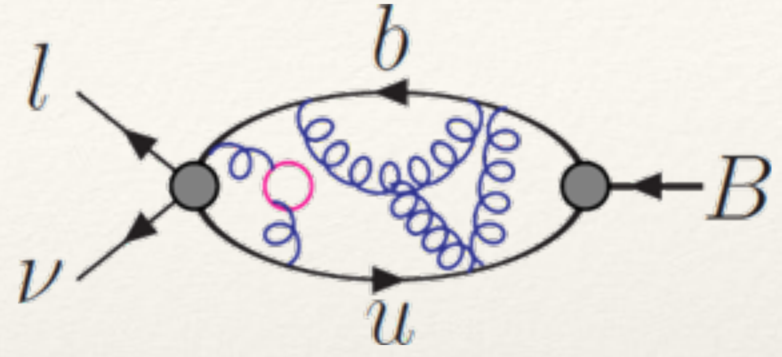

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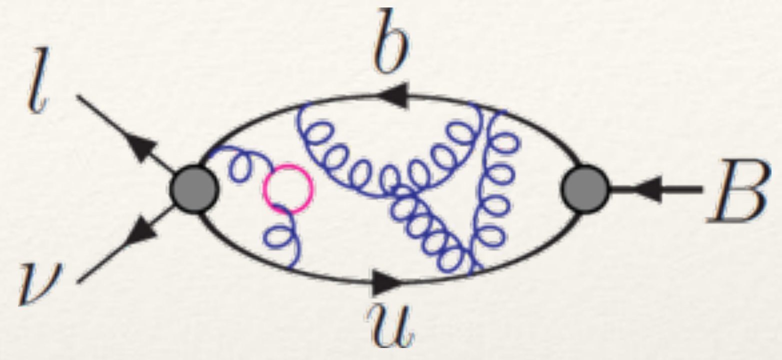

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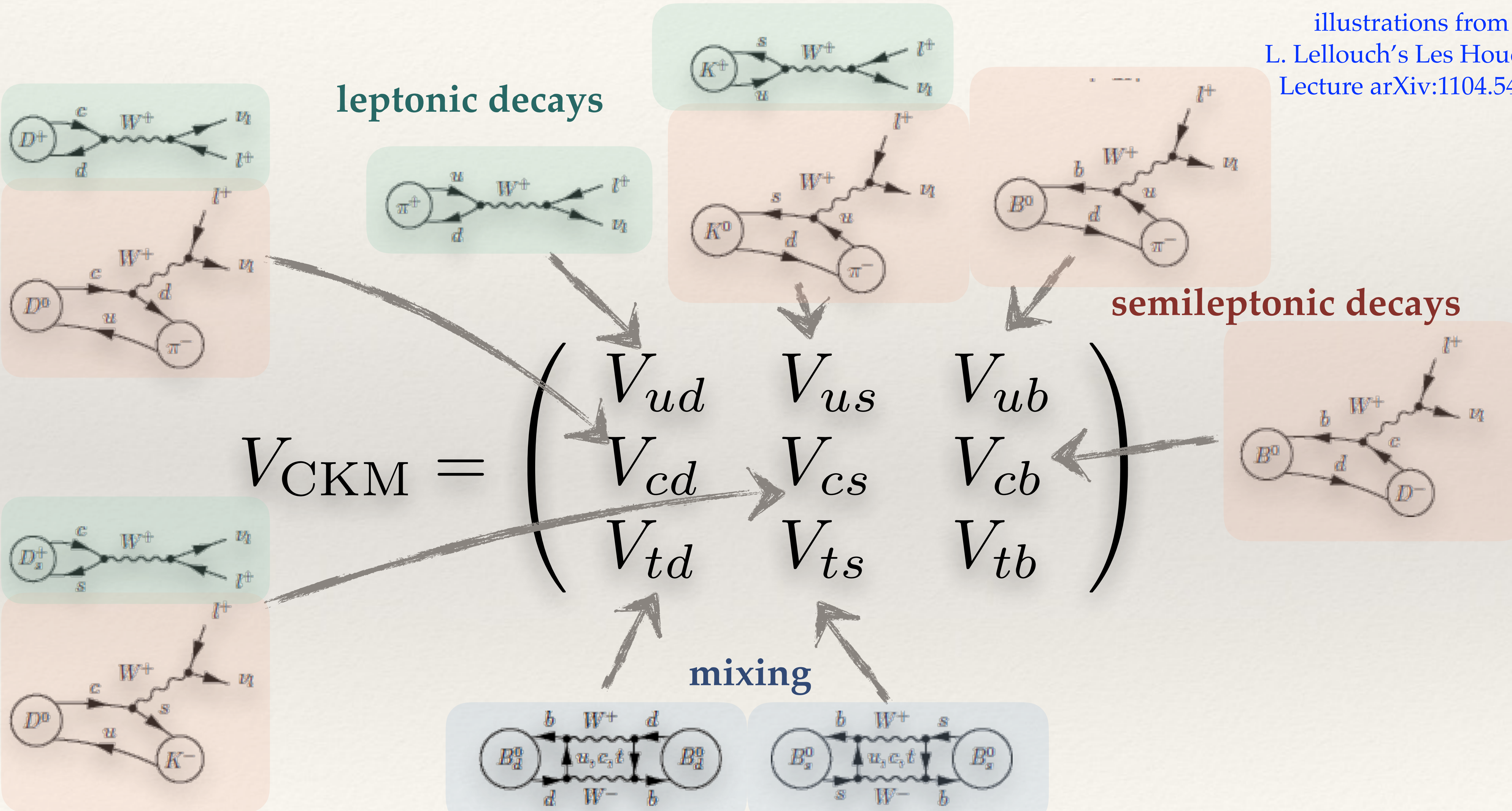
Experimental measurement + theory prediction allows for extraction of CKM MEs

Lattice flavour physics and CKM

$$V_{\mathrm{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Lattice flavour physics and CKM

illustrations from
L. Lellouch's Les Houches
Lecture arXiv:1104.5484



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“tree” kaon/pion decays

$$\Gamma(K \to \mu\bar{\nu}_\mu) = \frac{G_F^2}{8\pi} f_K^2 m_\mu^2 m_K \left(1 - \frac{m_\mu^2}{m_K^2}\right)^2 |V_{us}|^2$$

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Marciano, Phys.Rev.Lett. 2004

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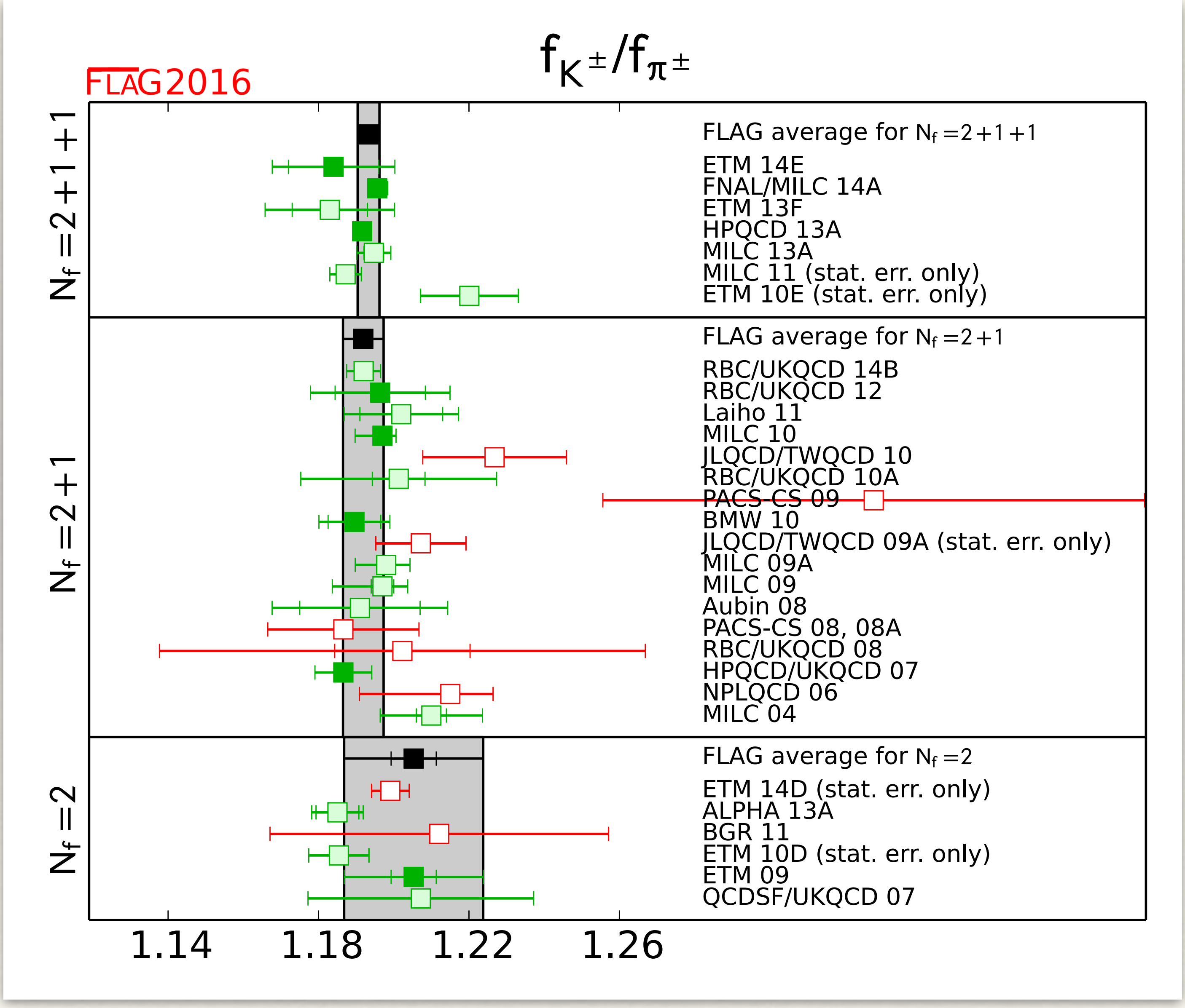

1 ‰

Standard calculations and results - FLAG

Flavour Lattice Averaging Group

"What's currently the best lattice value for a particular quantity?"

FLAG-1 (Eur. Phys. J. C71 (2011) 1695)

FLAG-2 (http://itpwiki.unibe.ch/flag/, Eur. Phys.J. C74 (2014) 2890)

FLAG-3 - about to go to press (arXiv:1607.00299)

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$$m_{u,d,s,c,b}$$

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$$f_{D_{(s)}},\, f_{B_{(s)}},\, B_{B_{(s)}},\, B_{(s)}\text{– and } D_{(s)}\text{–semileptonics}$$

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  - evaluation according to FLAG quality criteria (colour coding)
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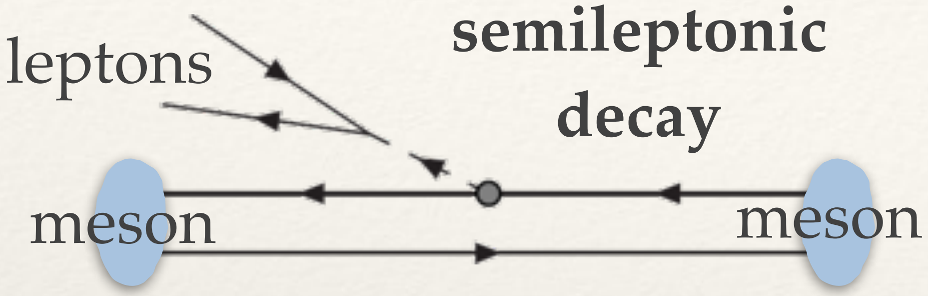
FLAG-4 kickoff meeting end of April at Higgs Centre

“tree” kaon/pion decays



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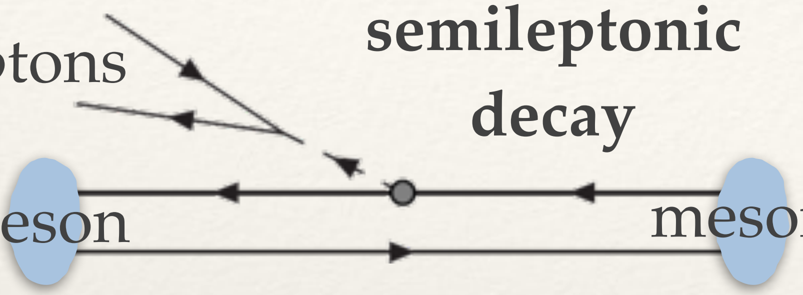

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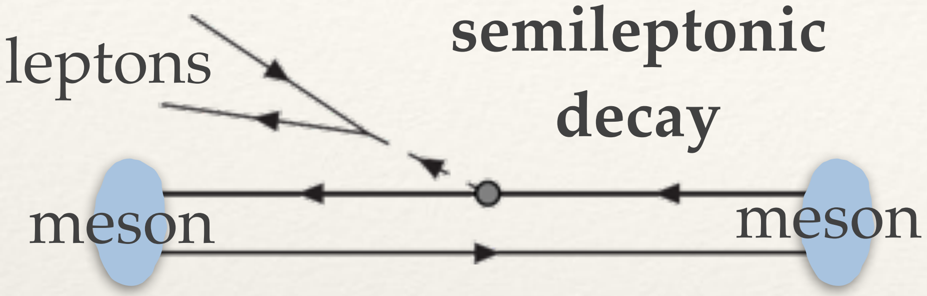

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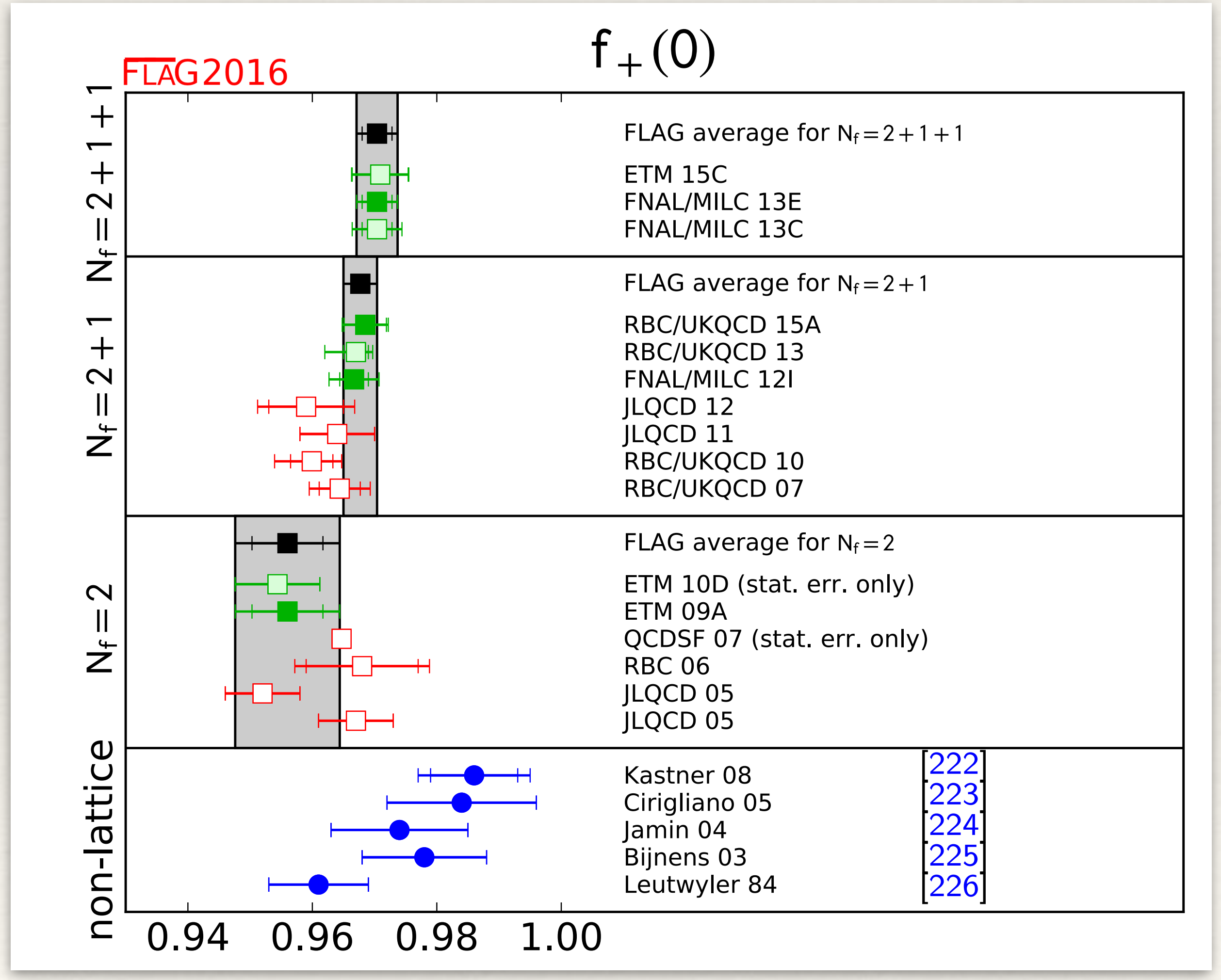

3‰!!!

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 \approx |V_{ud}|^2 + |V_{us}|^2 \stackrel{?}{=} 1$$

$|V_{us}| f_+^{K^0\pi^-}(0) = 0.2163(5)$ $\qquad \dfrac{f_{K^+}}{f_{\pi^+}} \dfrac{|V_{us}|}{|V_{ud}|} = 0.2758(5)$ FLAVIA Kaon WG EPJ C 69, 399-424 (2010) KTeV, Istra, KLOE, NA48 **arXiv:1005.2323**

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First row unitarity:

- $f_+^{K\pi}(0)$ and $|V_{ud}|$ from experiment
- f_K/f_π and $|V_{ud}|$ from experiment
- $f_+^{K\pi}(0)$ and f_K/f_π from lattice

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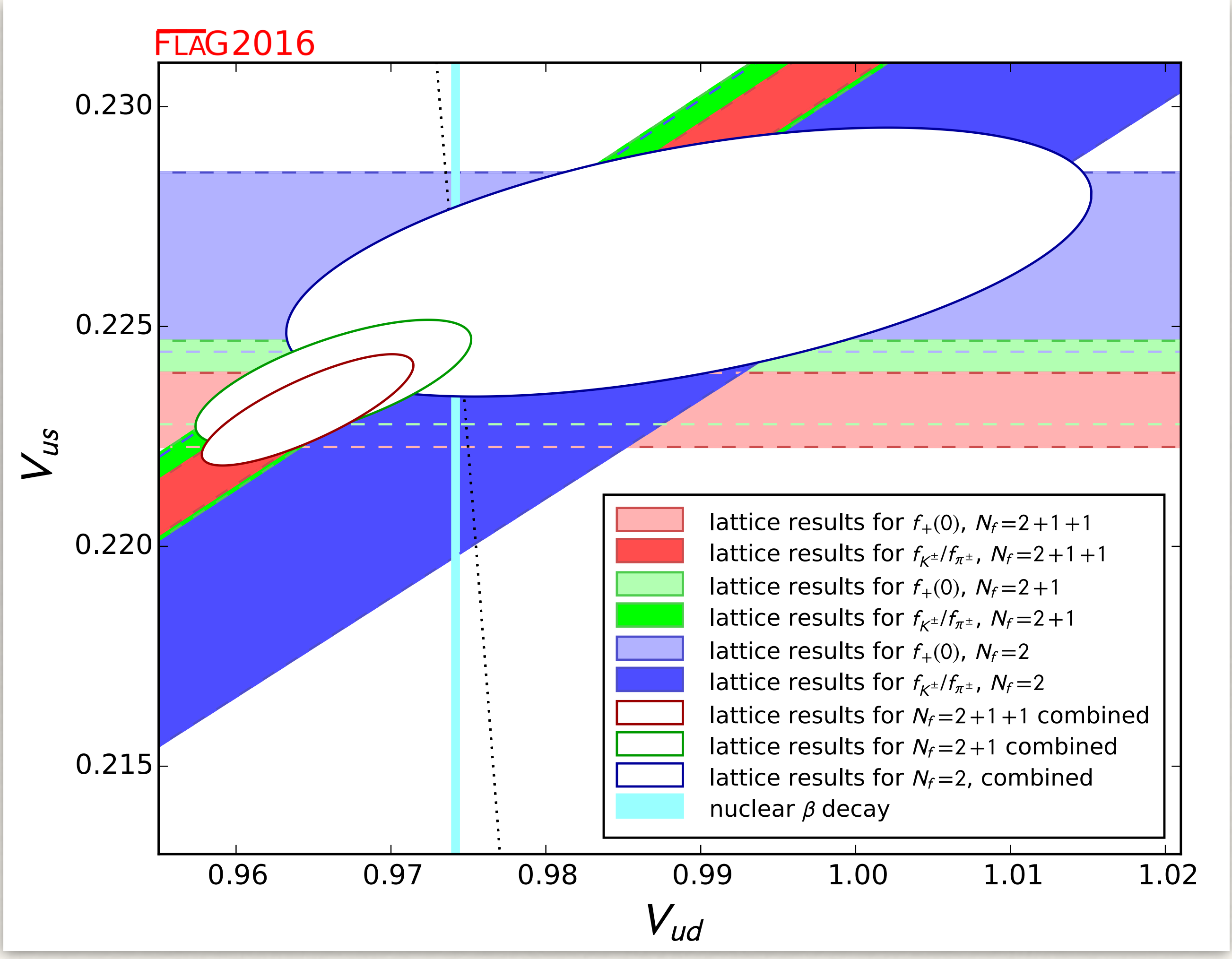

Analysis assuming CKM unitarity

Experimental results:

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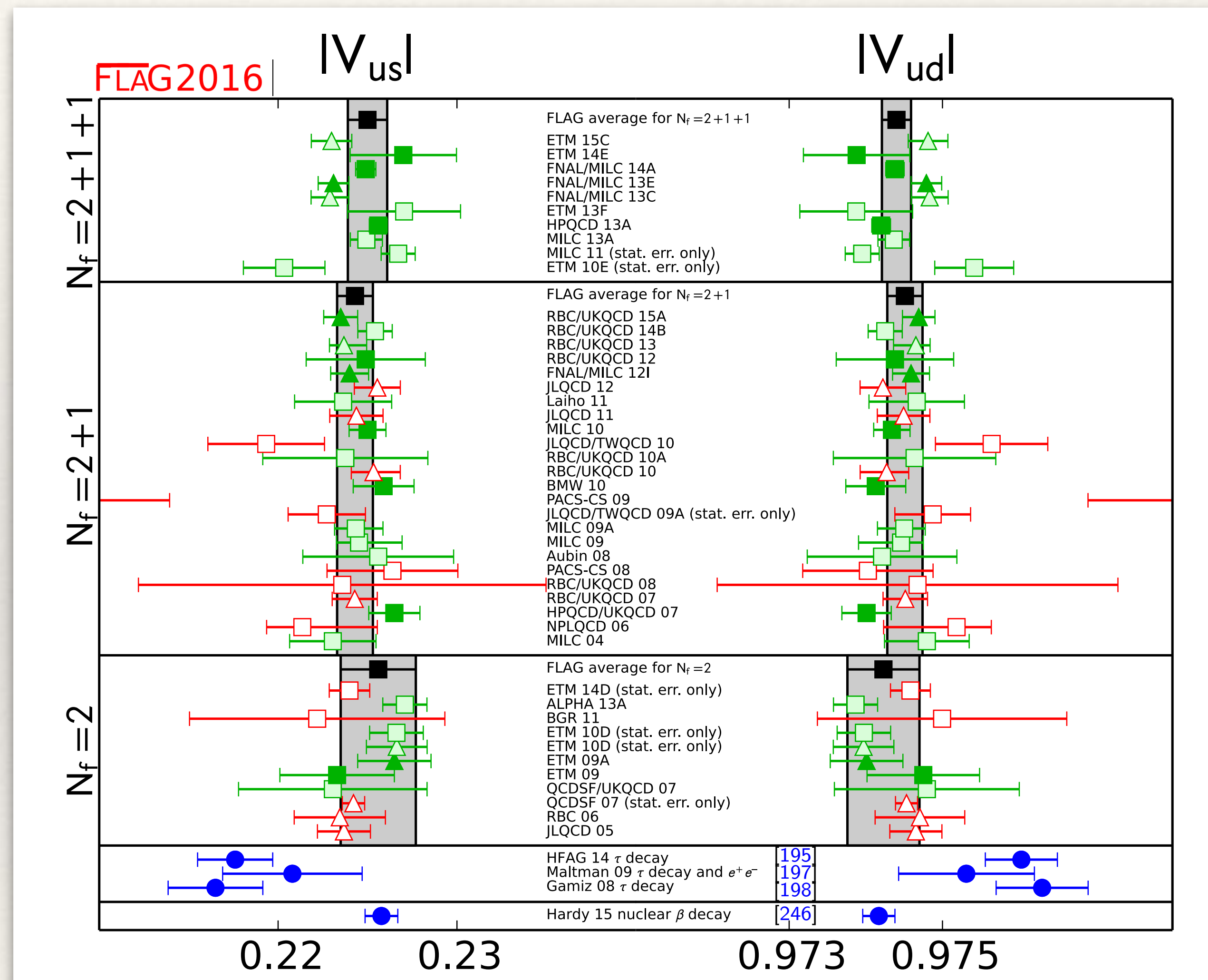
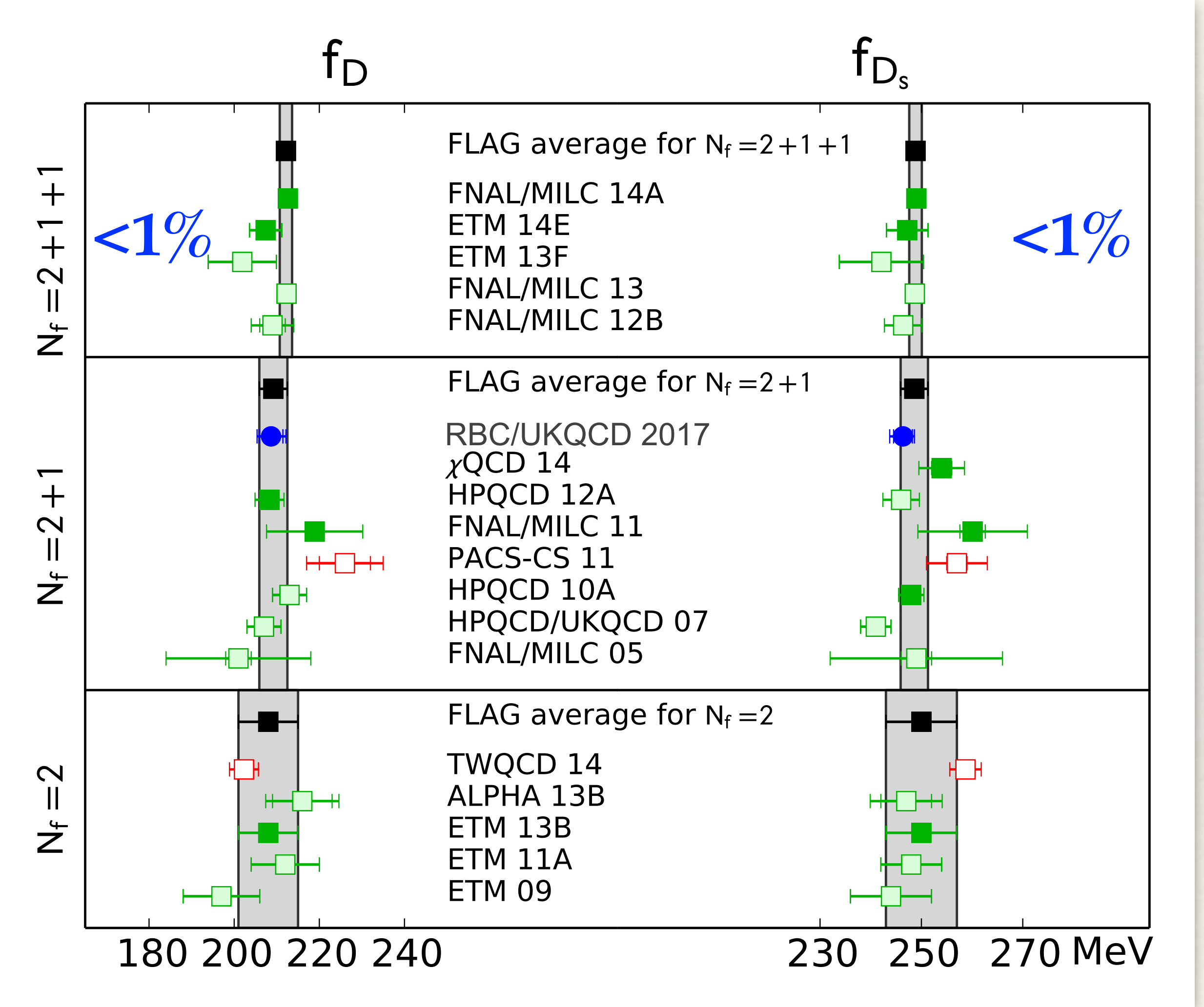
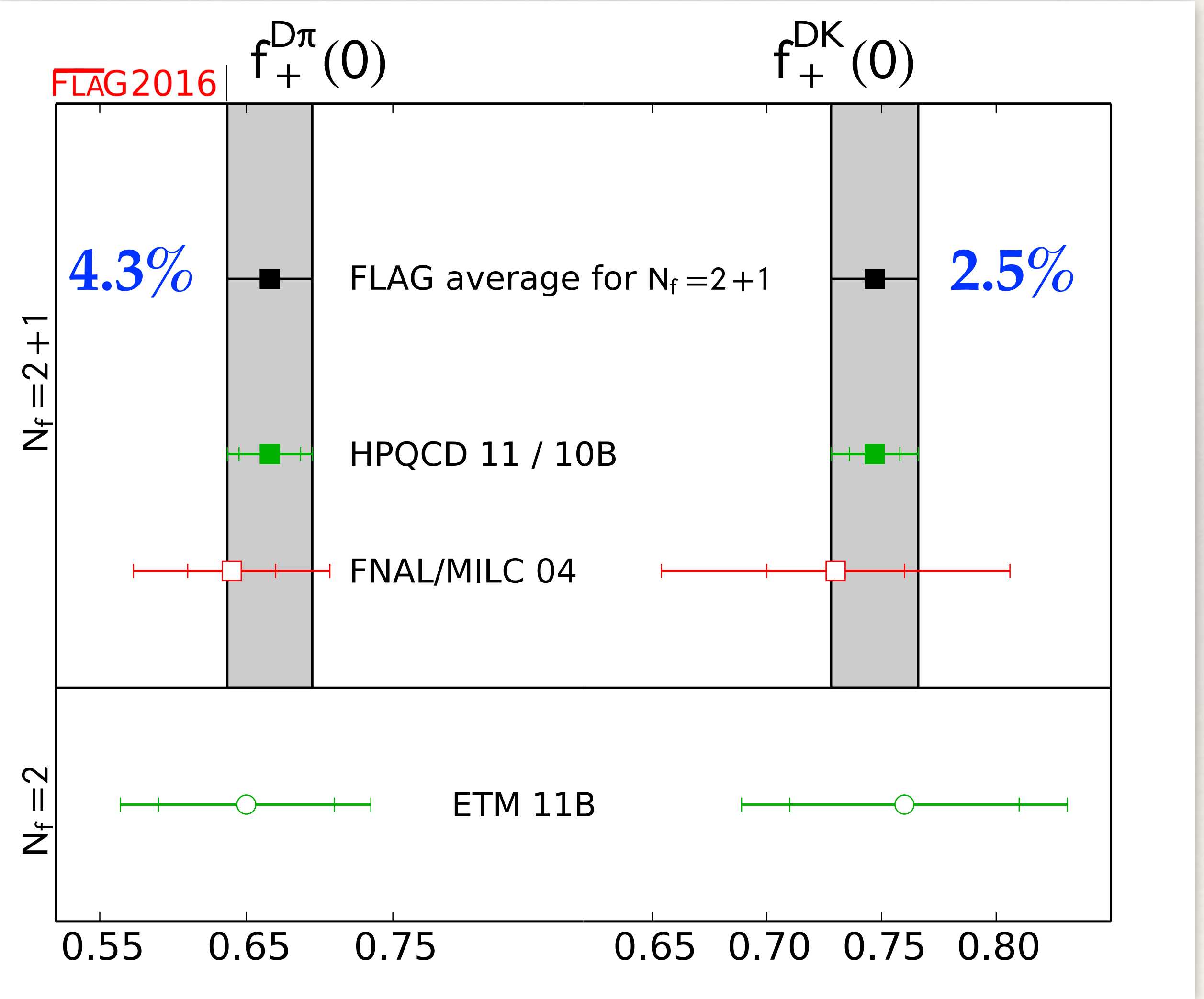


Leptonic $D_{(s)}$ meson decays

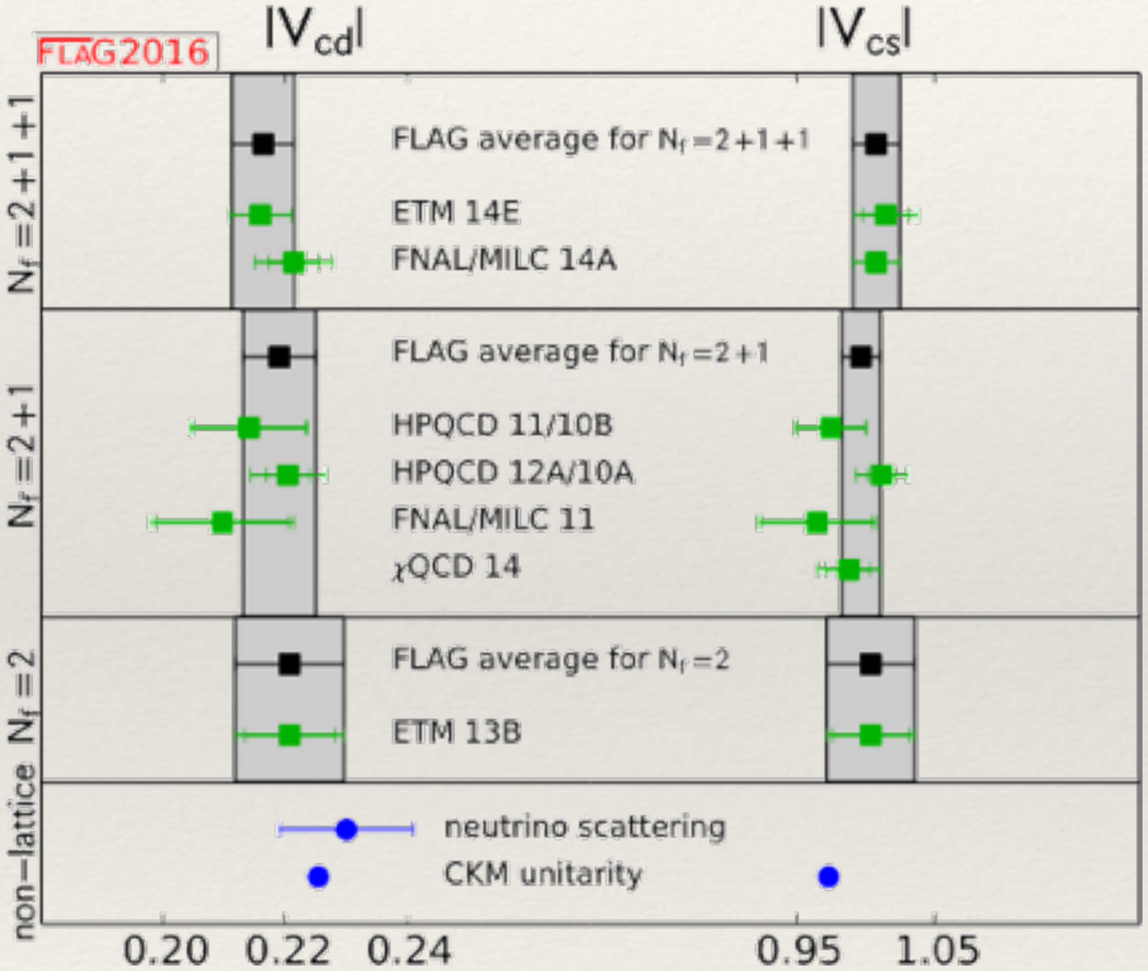
Once the technicalities behind controlling charm and bottom on the lattice the form factor computation is very similar, in particular for D-mesons:





Results for $|V_{cd}|$ and $|V_{cs}|$

$f_D|V_{cd}| = 45.91(1.05)$ MeV , $f_{D_s}|V_{cs}| = 250.9(4.0)$ MeV PDG



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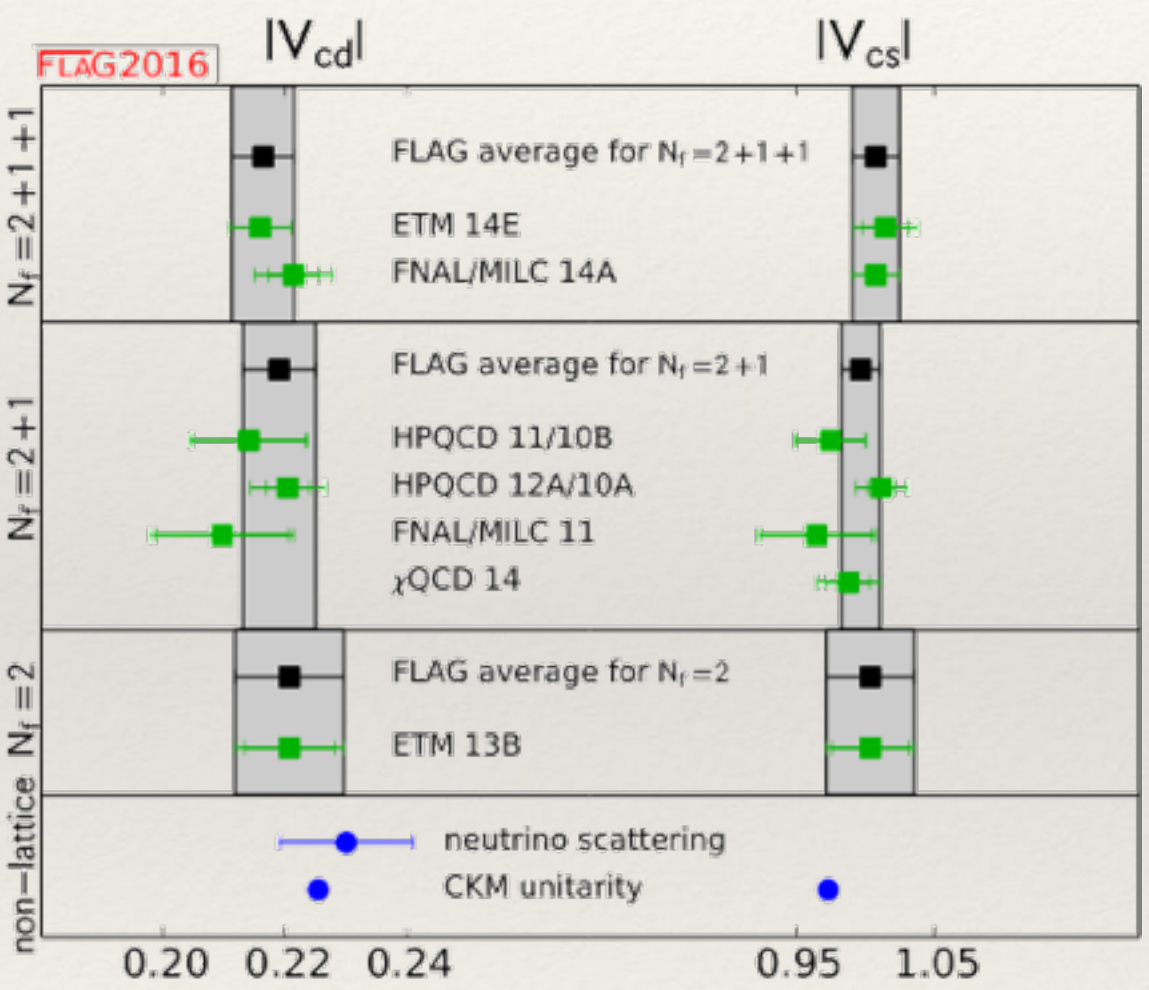
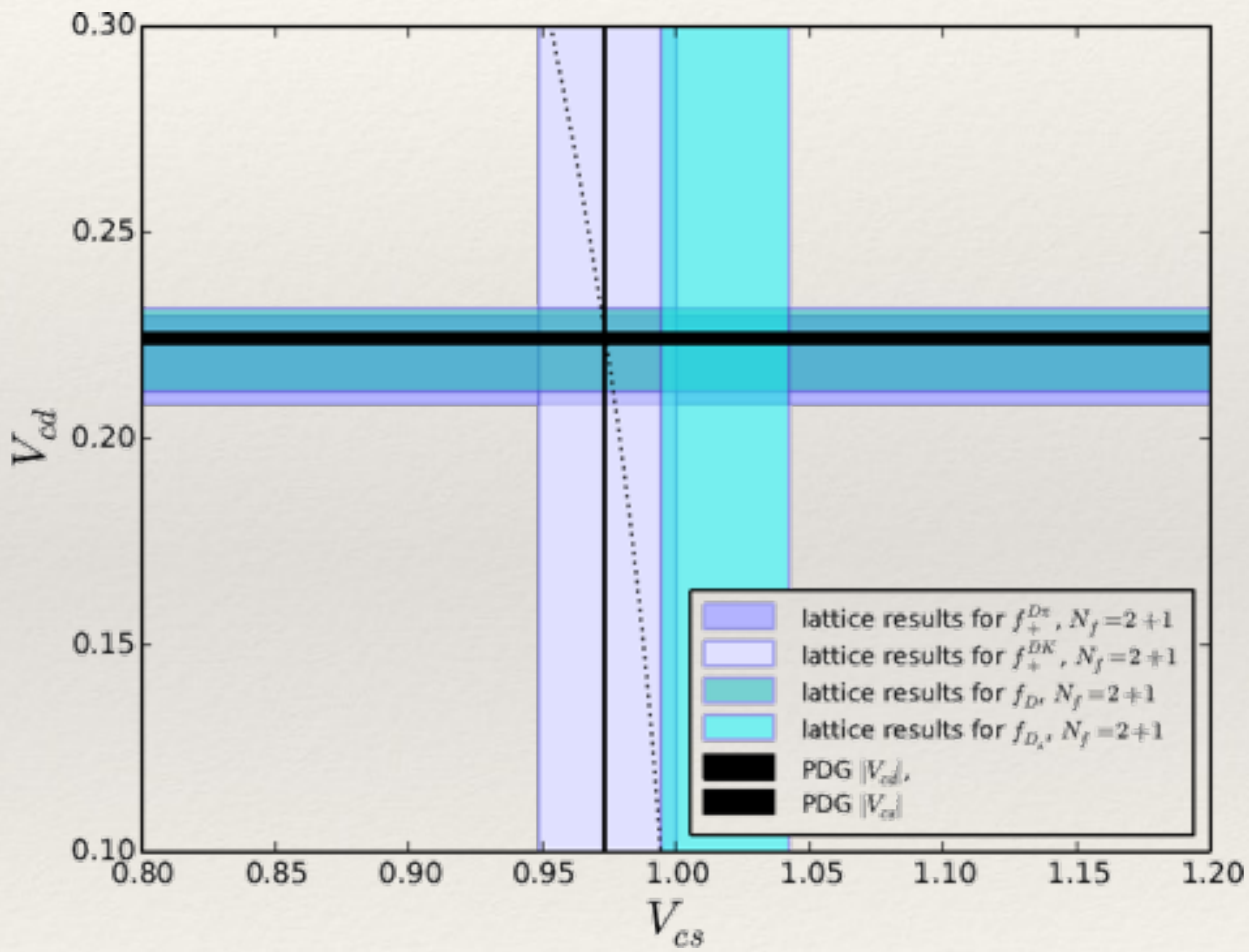
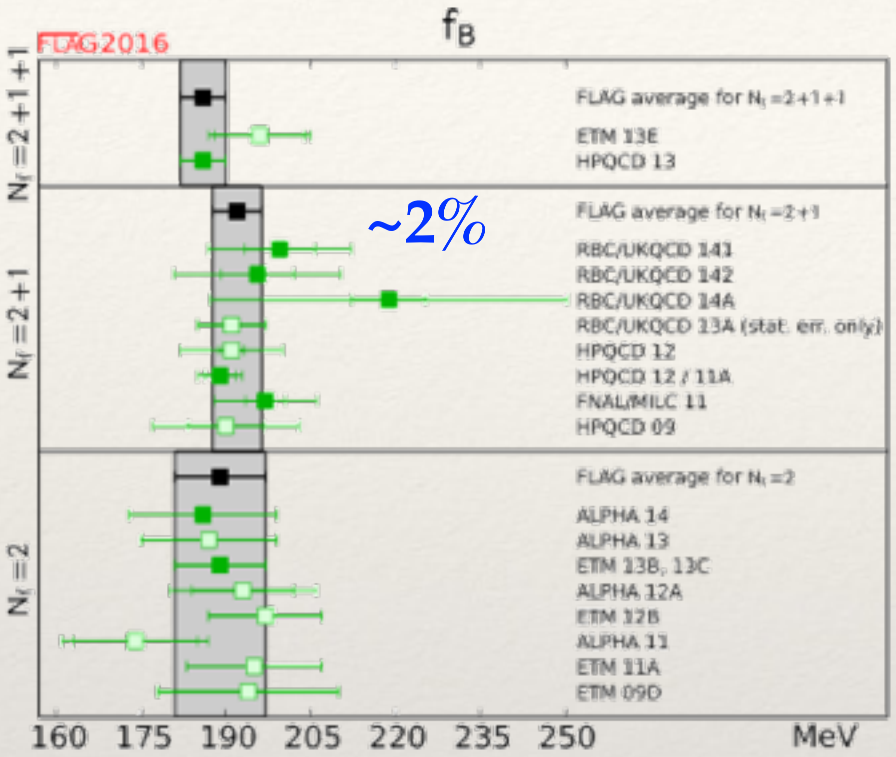
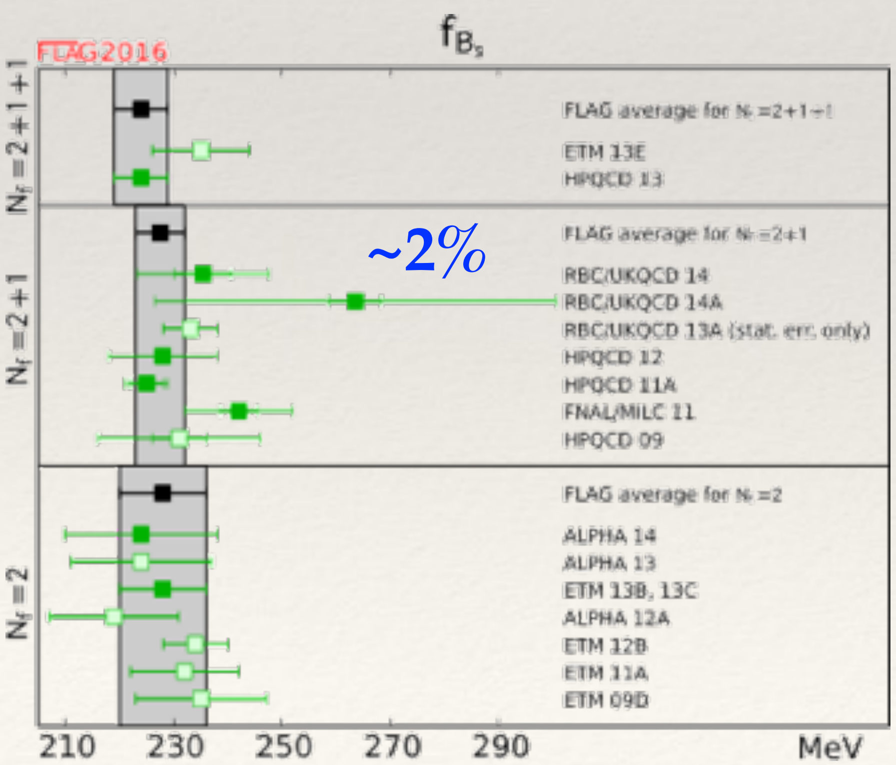


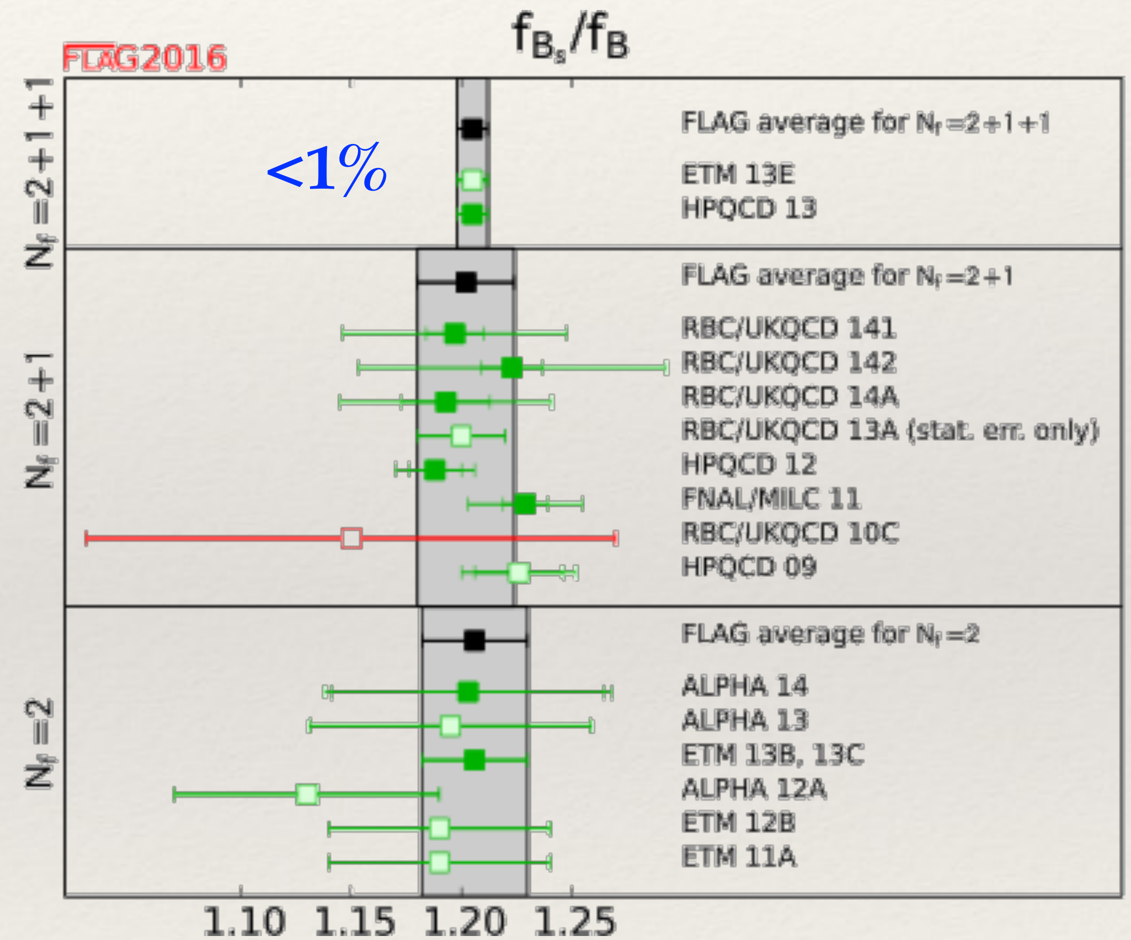
N_f=2+1 unitarity analysis



Leptonic beauty decays







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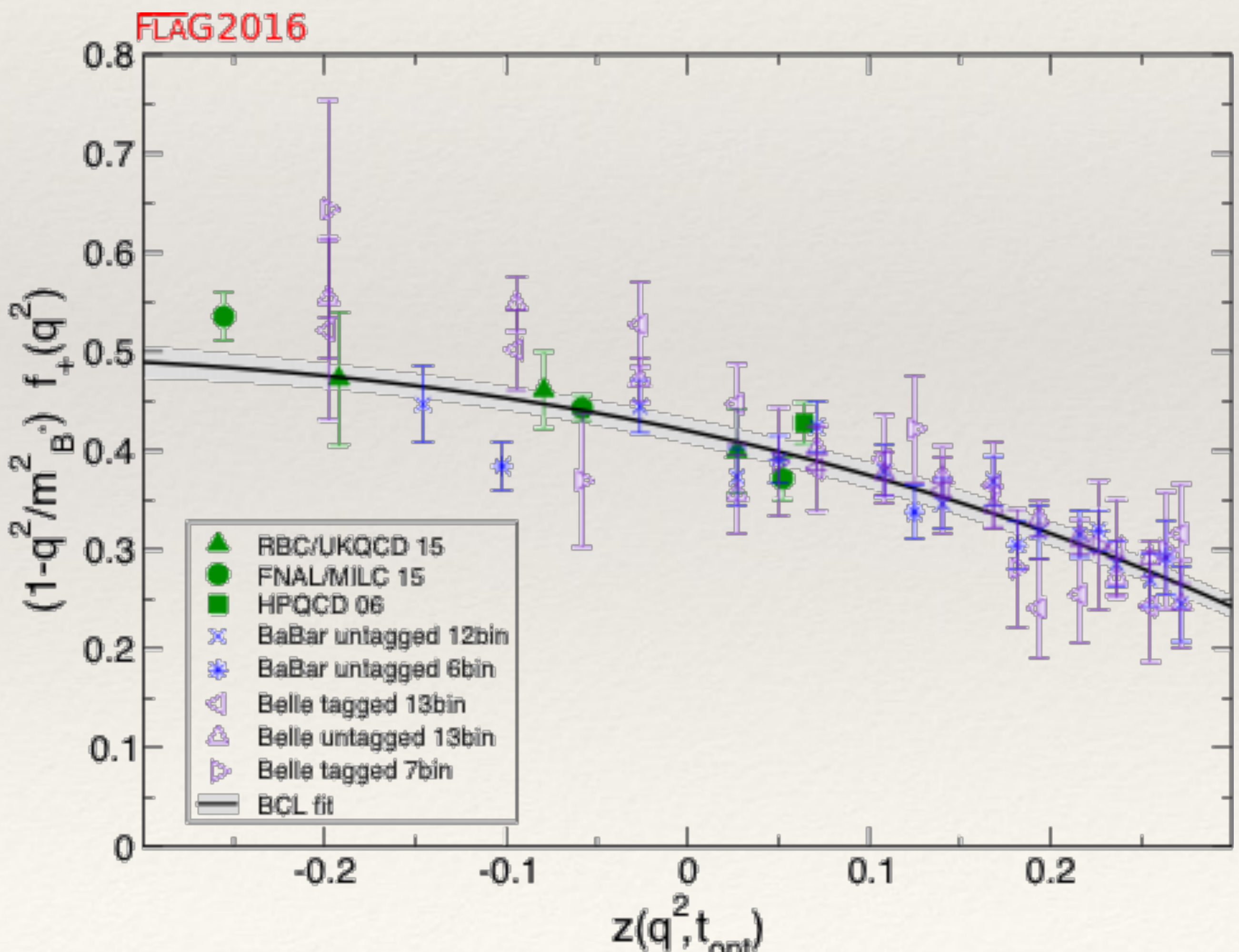

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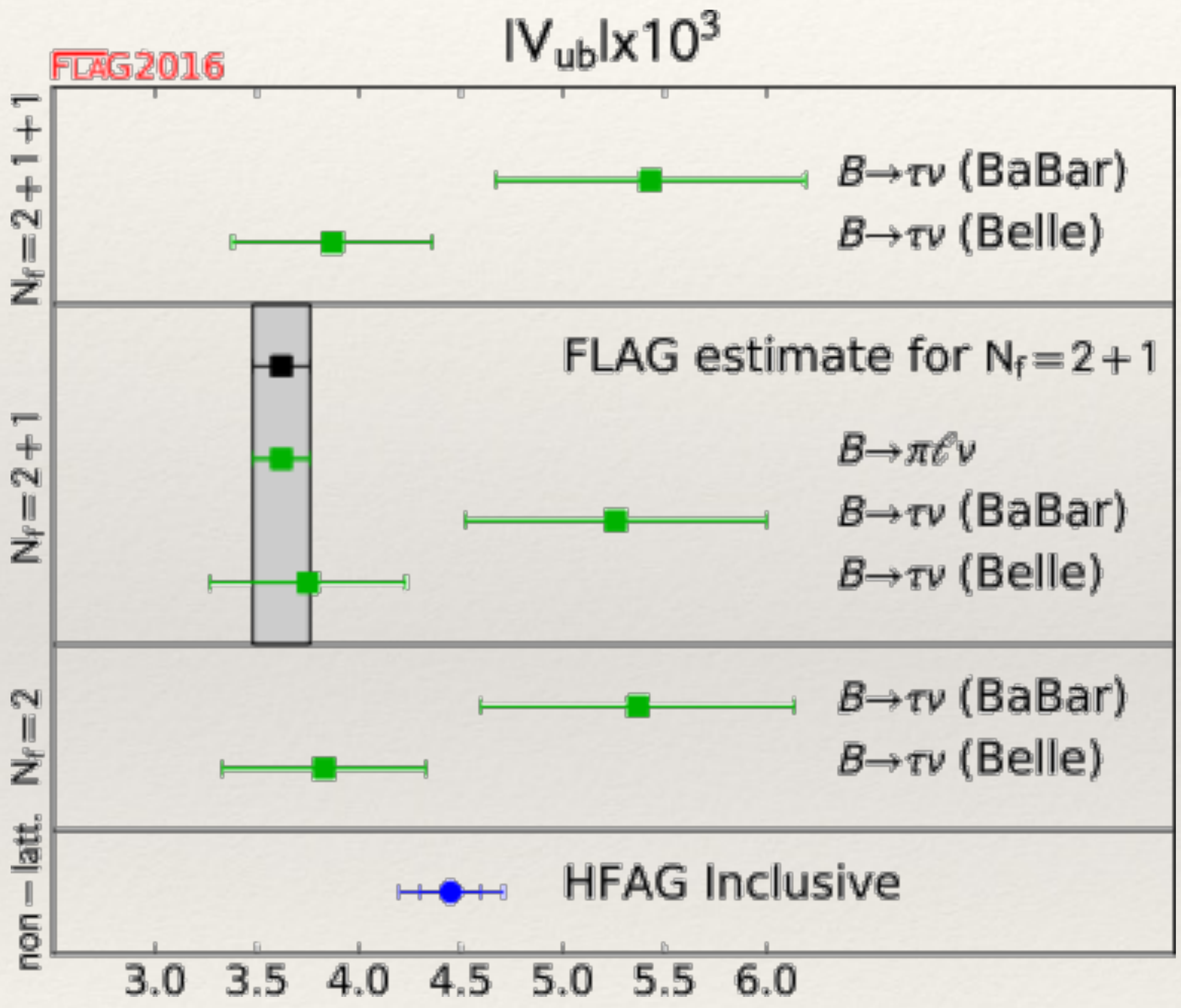

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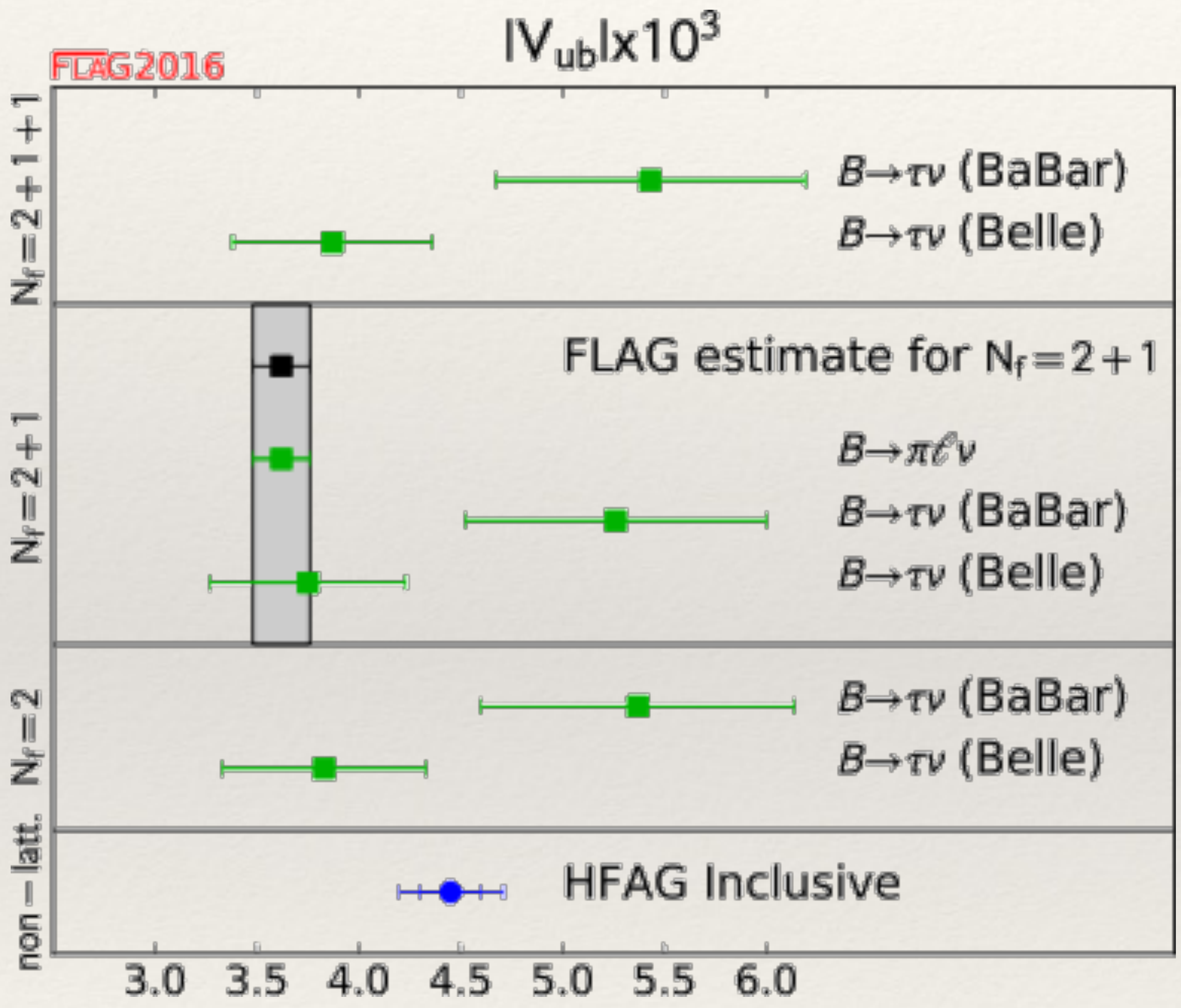

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- slight tension in exp data Belle/BaBar
- looking forward to Belle II

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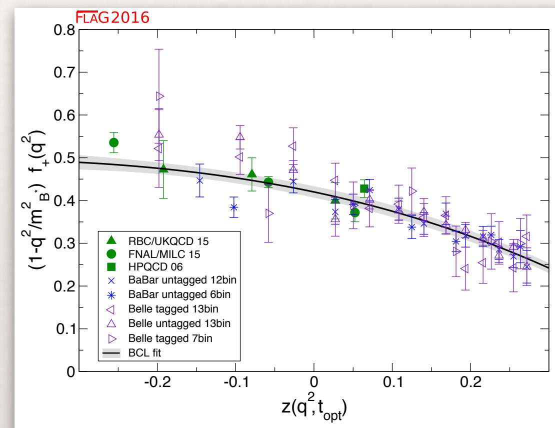

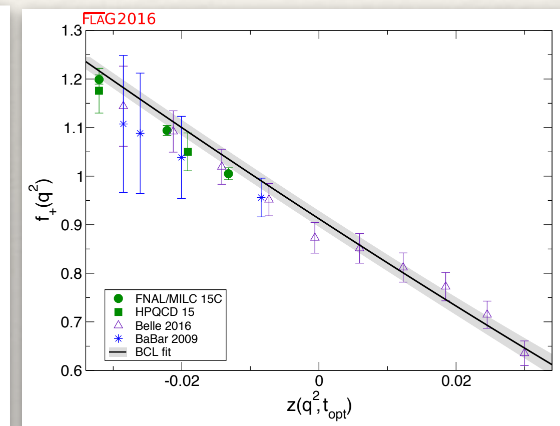

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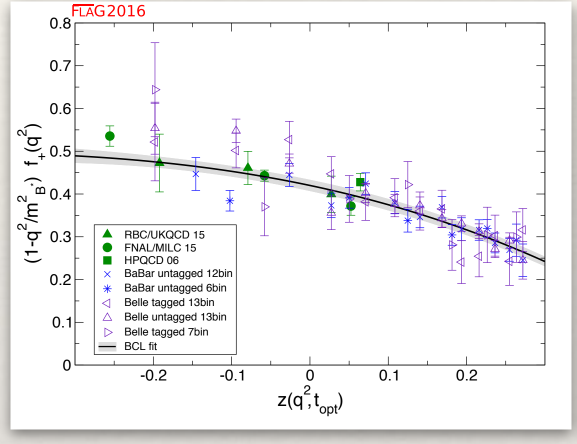

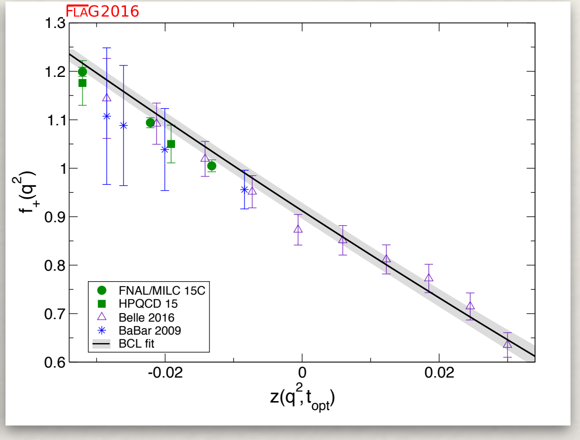

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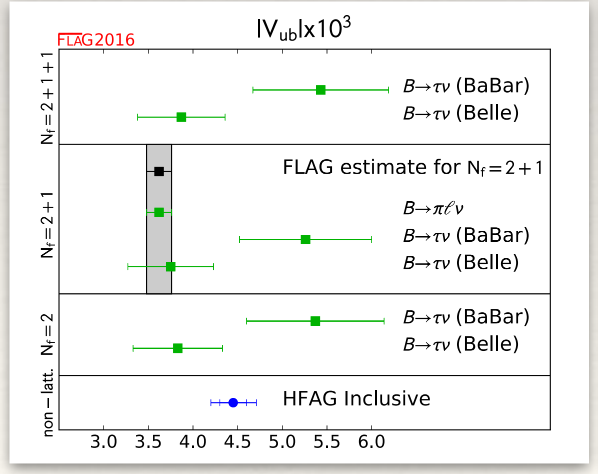

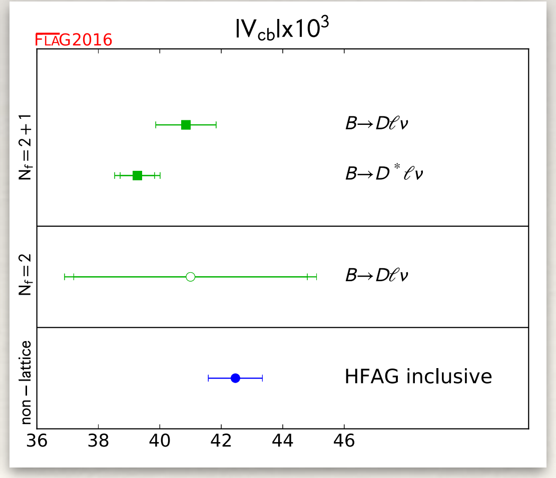

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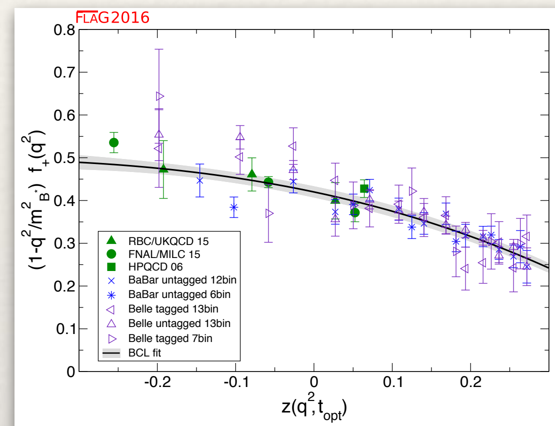

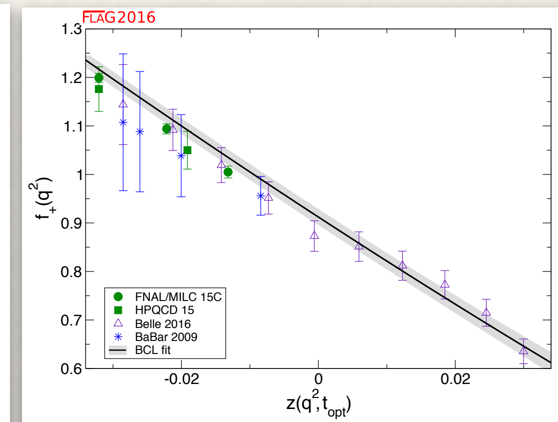

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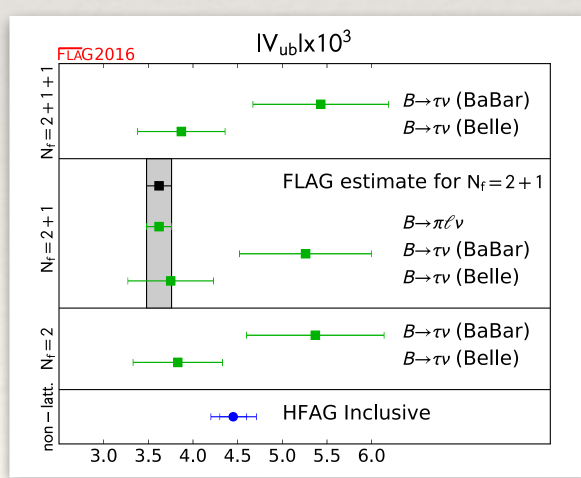

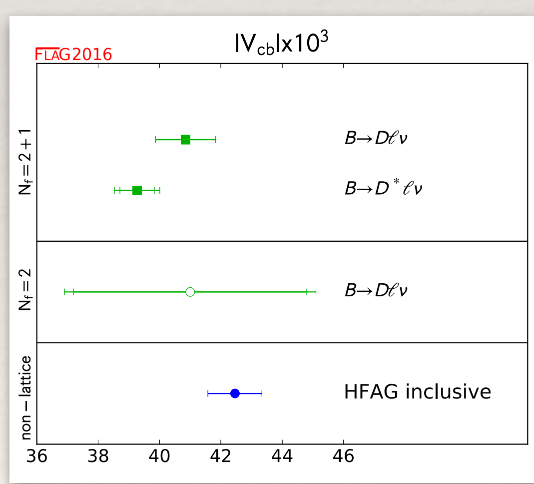

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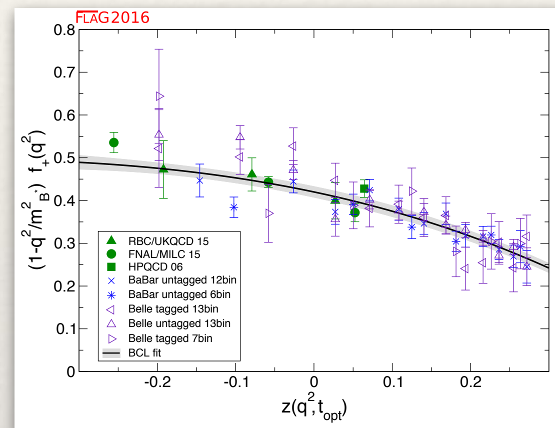

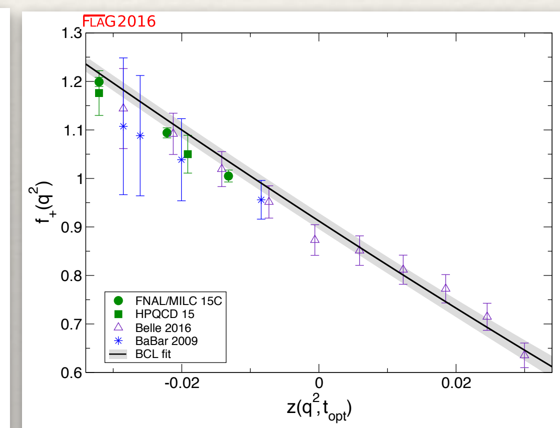

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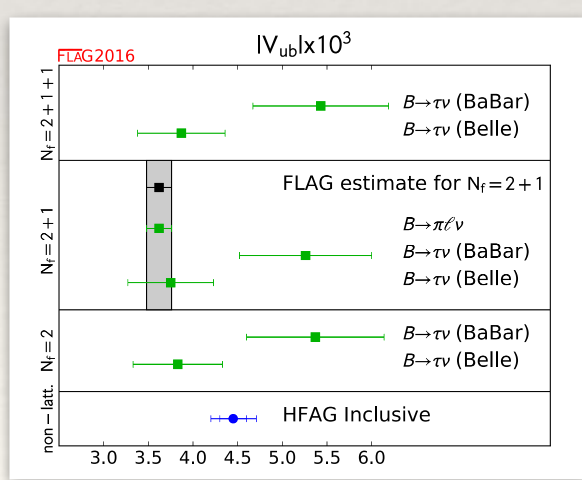

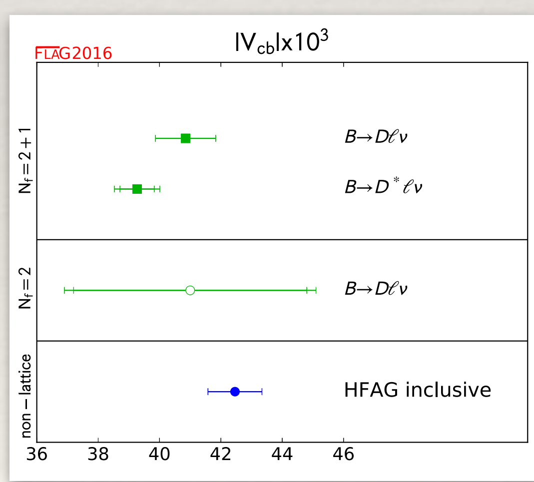

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Plenty more going on, e.g. spectroscopy …

Summary I

- (non-rare) Lattice Flavour Physics is a mature research field
- several independent groups competing
- (sub-)percent precision for *standard* quantities feasible
- FLAG summarises particularly mature quantities
  for use in SM and BSM phenomenology

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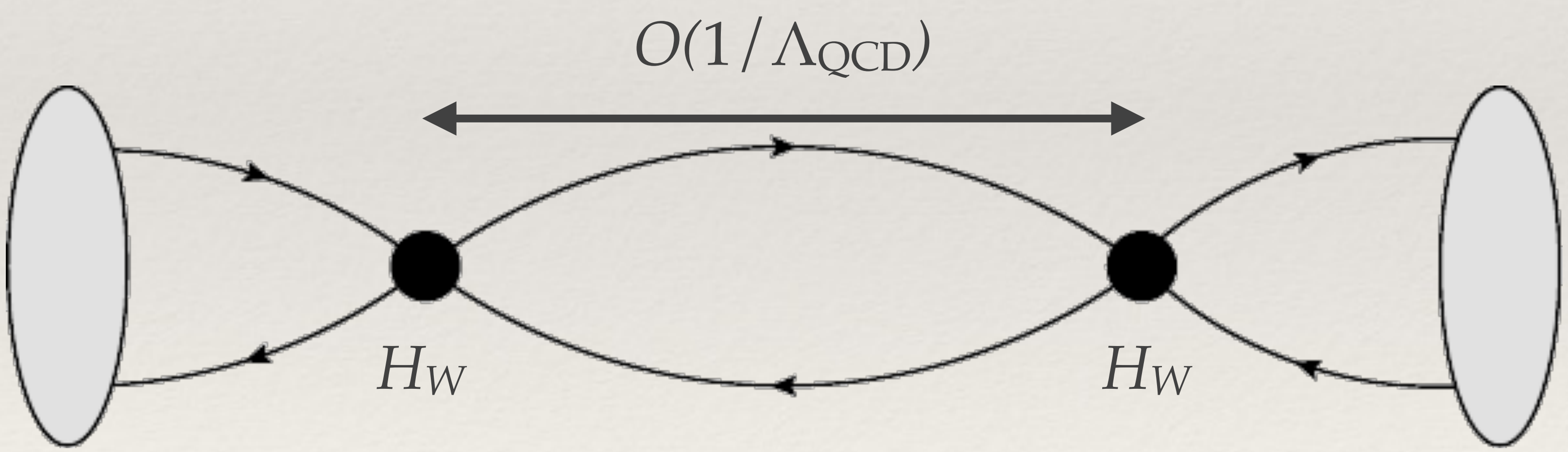
Go beyond short distance physics



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- we are starting go beyond EFT treatment
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- already many results in spectroscopy but not for matrix elements
- finite size effects with photons pose a substantial problem

Isospin corrections are important

- e.g. $K \to \pi l \nu$: $\Gamma_{K\to\pi l\nu} = C_K^2 \frac{G_F^2 m_K^5}{192\pi^2} S_{\mathrm{EW}} (1 + \Delta_{SU(2)} + \Delta_{\mathrm{EM}})^2 \, I \, |f_+^{K\pi}(0)|^2 |V_{us}|^2$

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Kastner & Neufeld
Eur. Phys. J. C 57 (2008) 541

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- precision now such that corrections need to be improved:

			Approx contrib to % err			
Mode	$V_{us} f_+(0)$	% err	BR	τ	Δ	I
$K_{L e3}$	0.2163(6)	0.26	0.09	0.20	0.11	0.05
$K_{L \mu3}$	0.2166(6)	0.28	0.15	0.18	0.11	0.06
$K_{S e3}$	0.2155(13)	0.61	0.60	0.02	0.11	0.05
$K_{e3}^{\pm}$	0.2172(8)	0.36	0.27	0.06	0.23	0.05
$K_{\mu3}^{\pm}$	0.2170(11)	0.51	0.45	0.06	0.23	0.06

Moulson@CKM 2014
arXiv:1411.5252

QCD+QED

Action:

$$S[U, A, \bar{\psi}, \psi] = S_g[U; g] + S_\gamma[A] + \sum_f \bar{\psi}_f D[U, A; e, q_f, m_f]\psi_f$$

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- sufficiently large volumes currently not feasible, so use effective field theory to subtract finite volume effects

QCD+QED

BMW Collaboration
Science 347 (2015) 1452-1455
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Example: FV correction to mass of a spin-1/2 particle in QED

analytically compute the difference of the *finite volume* and *infinite volume* self energies Σ:

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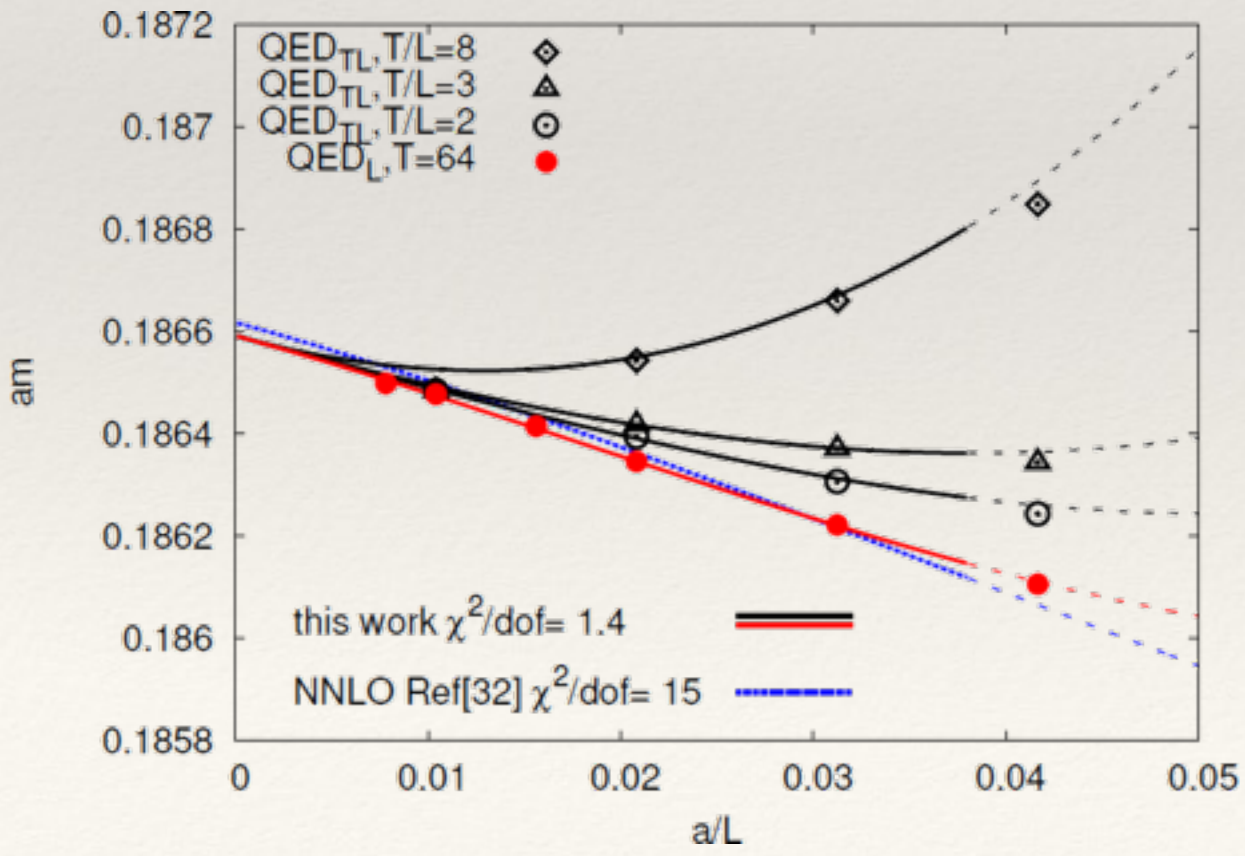

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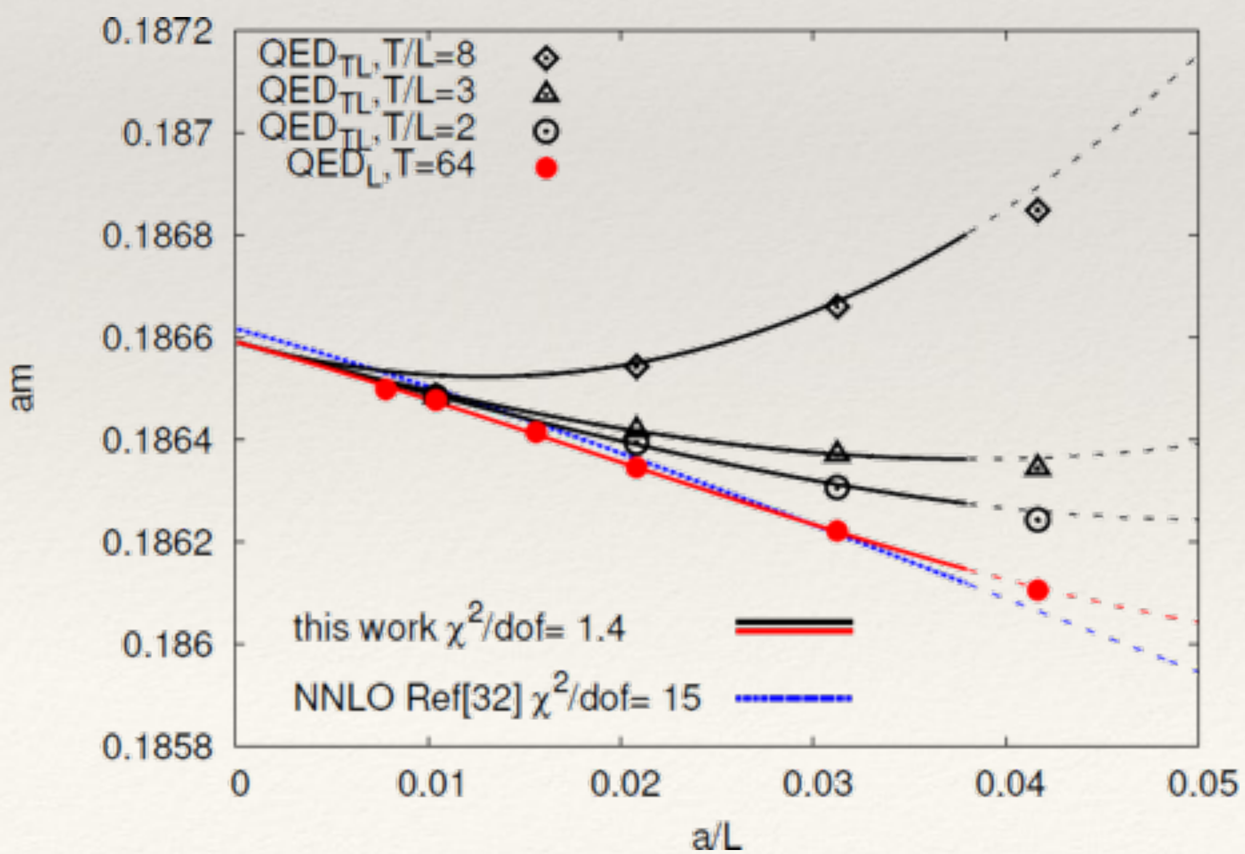
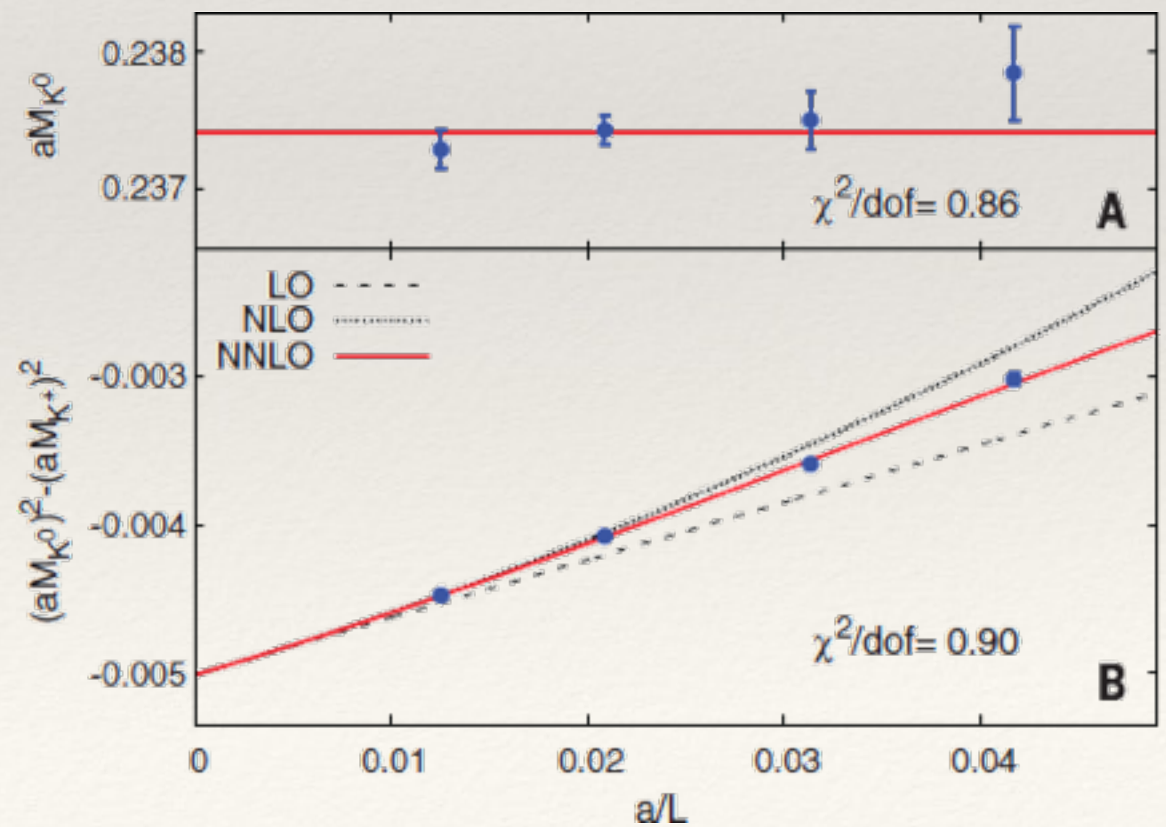
Example: FV correction to mass of a spin-1/2 particle in QED

analytically compute the difference of the *finite volume* and *infinite volume* self energies Σ:

$$m^2(T,L) \overset{T,L\to\infty}{\propto} m^2\left\{1 - q^2\alpha\left[\frac{\kappa}{2mL}\left(1+\frac{2}{mL}\right) - \frac{3\pi}{(mL)^3}\right]\right\}$$

leading behaviour universal in κ (structure- and spin-independent)





QCD+QED: baryon mass splitting

- relative neutron-proton mass difference found in nature 0.14%
- the value has significant implication for nature
  - smaller value → inverse β-decay of H
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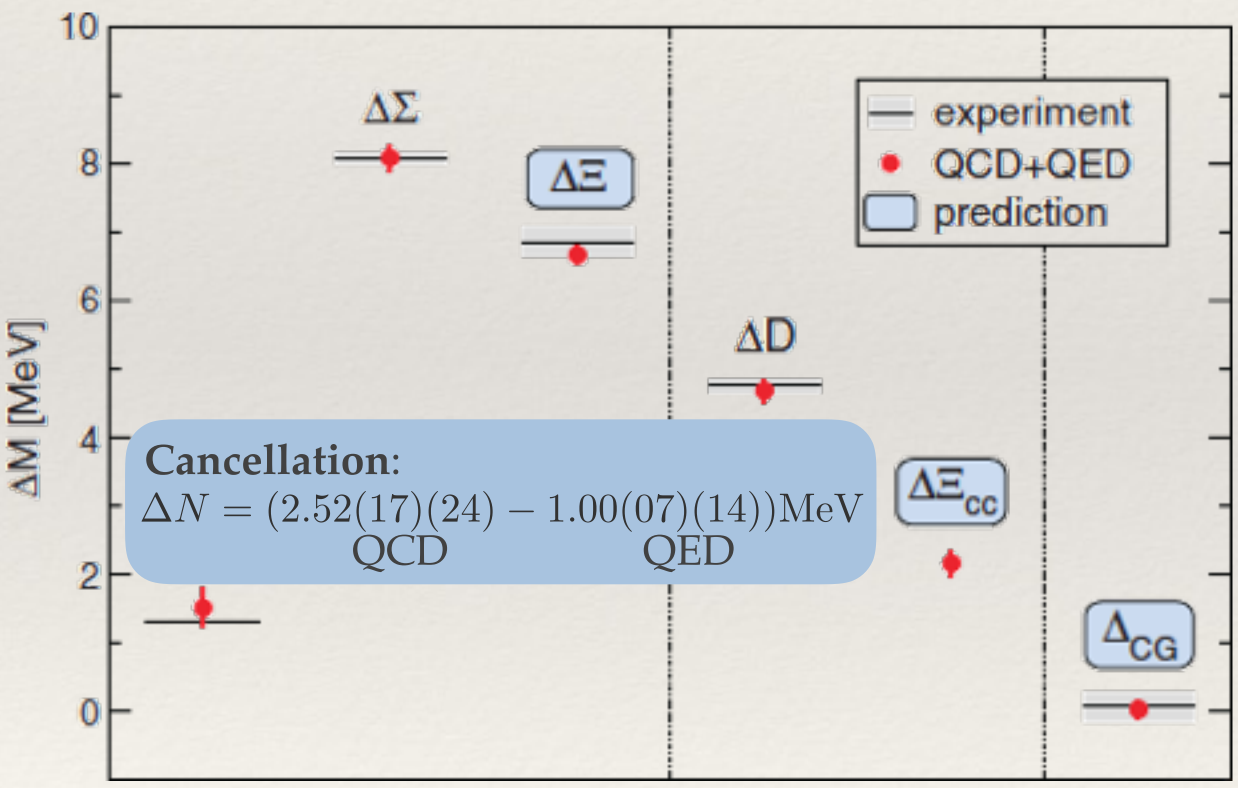

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Including QED in meson decay MEs

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- leptonic decay at $O(\alpha^0)$:

$$\Gamma(\pi^+ \to l^+ \nu_l) = \frac{G_F^2 |V_{ud}|^2 f_\pi^2}{8\pi} m_\pi m_l^2 \left(1 - \frac{m_l^2}{m_\pi^2}\right)^2$$

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$$\equiv \quad \Gamma_0 \quad + \quad \Gamma_1$$

IR div. cancel between terms on r.h.s.
between virtual and real photons
(Bloch Nordsieck)

Including QED in meson decay MEs

Carrasco et al. PRD 91 074506 (2015) **arXiv:1502.00257**

- cut on small photon momentum $< \Delta E$ → γ *sees* point-like π
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$\Gamma(\pi^+ \rightarrow l^+ \nu_l)$

lattice and analytical finite V

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analytically in $V \rightarrow \infty$

both terms separately IR finite, gauge invariant on its own

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first simulations are under way!

QCD+QED: first applications for ME

Southampton-Edinburgh particularly interested in studying IB effects on:

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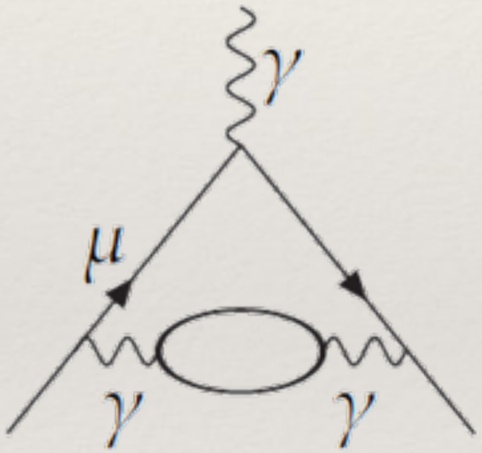

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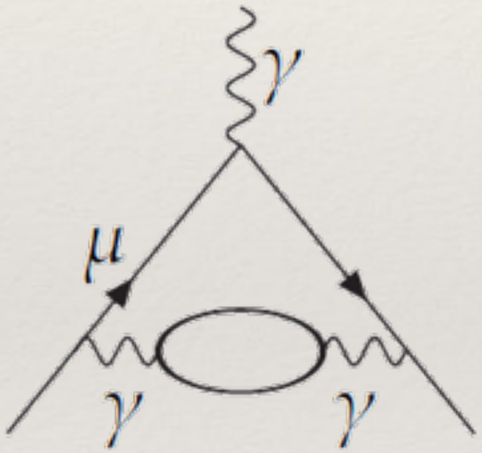

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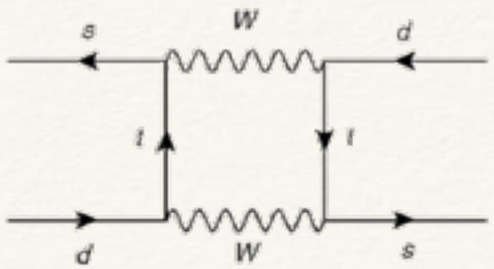
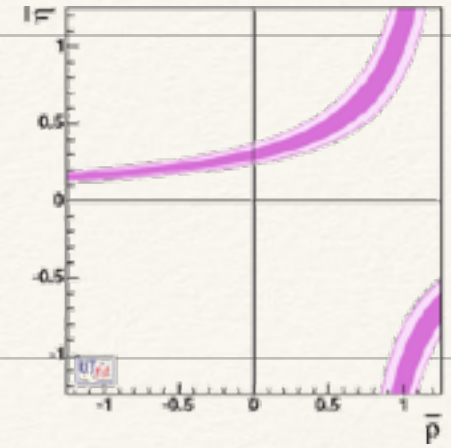
inclusion of QED effects will be one of the big challenges in Lattice phenomenology over the next years

Long distance effects in kaon physics - mixing





$$\epsilon_K = \frac{A(K_L \to (\pi\pi)_{I=0})}{A(K_S \to (\pi\pi)_{I=0})} = e^{i\Phi_\epsilon} \sin\phi_\epsilon \left(\frac{\mathrm{Im}\langle \bar{K}^0 | H_W^{\Delta S=2} | K^0 \rangle}{\Delta M_K} + \text{L.D. effects} \right)$$

L.D. effects
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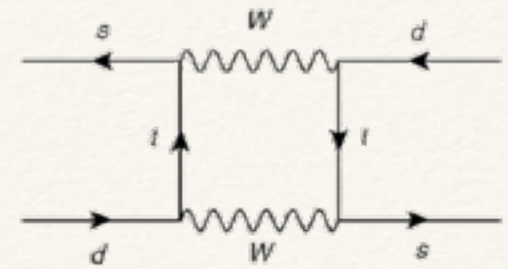

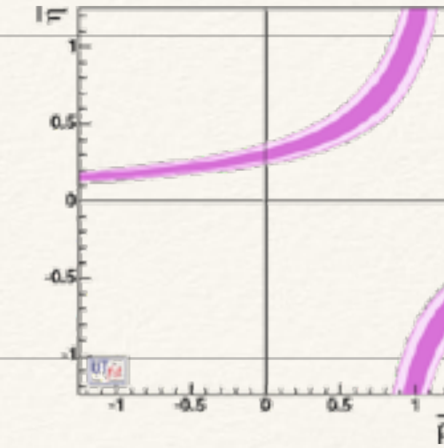

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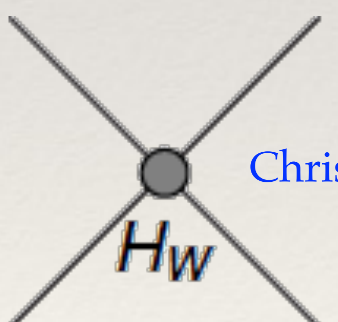
Long Distance effects amount to O(5%), so certainly worth considering on the lattice

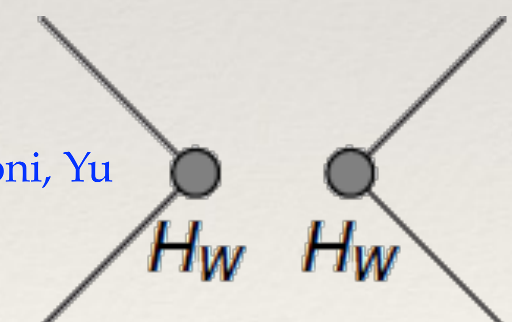
1st order Weak



single 4-quark OP, length scale 10^{-18}m

Christ, Izubuchi, Sachrajda, Soni, Yu
arXiv:1212.5931

2nd order Weak

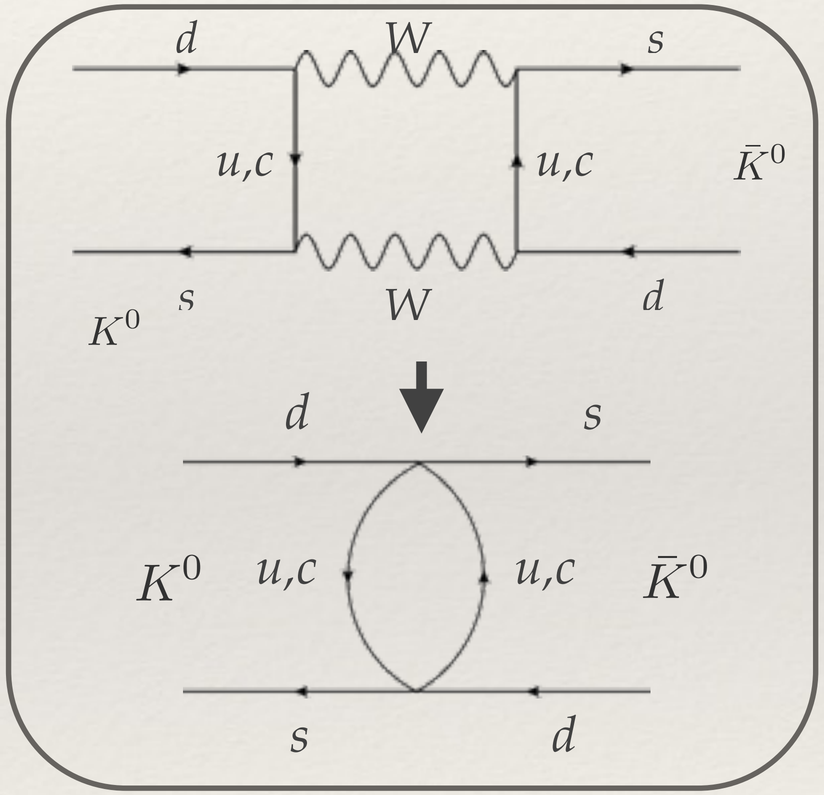


two 4-quark OP
length scale $1 / \Lambda_{QCD}$

Beyond short distance: e.g. ΔM_K

$$\boxed{\Delta M_K = m_{K_S} - m_{K_L} = \boxed{2\mathrm{Re}M_{\bar{0}0}}} \qquad M_{\bar{0}0} = \mathcal{P}\sum_{\lambda} \frac{\langle \overline{K^0}|H_W|\lambda\rangle\langle\lambda|H_W|K^0\rangle}{m_K - E_\lambda}$$

- experimentally ΔM_K=3.483(6)×10^{-12}MeV (PDG)
- suppressed by 14 orders of magnitude with respect to QCD → poses strong BSM constraints (e.g. $(1/\Lambda)^2$ $\bar{s}d\bar{s}d$ BSM contribution) knowing ΔM_K at 10%-level → **$\Lambda \geq 10^4$TeV**
- SD about 70% of experimental value - rest LD?
- PT large contributions at $\mu \sim m_c$ where PT turns out to converge badly (NLO->NNLO constitutes 36% correction) Brod, Gorbahn PRL 108 121801 (2012) arXiv:1108.2036



long distance effects

N. Christ et al. PRD 88 (2013) 014508 **arXiv:1212.5931**
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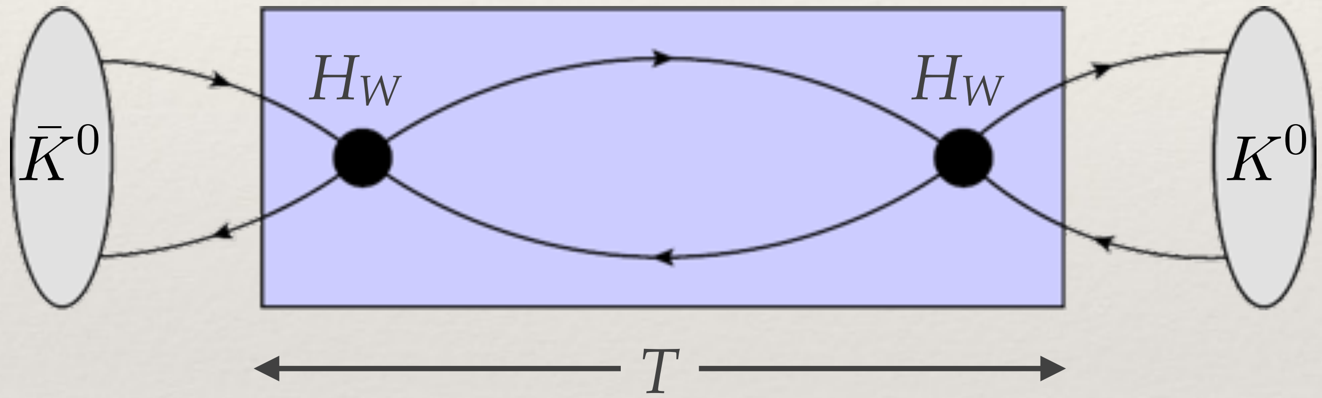
Integrate operators (here H_W) over time interval where initial and final kaon dominate



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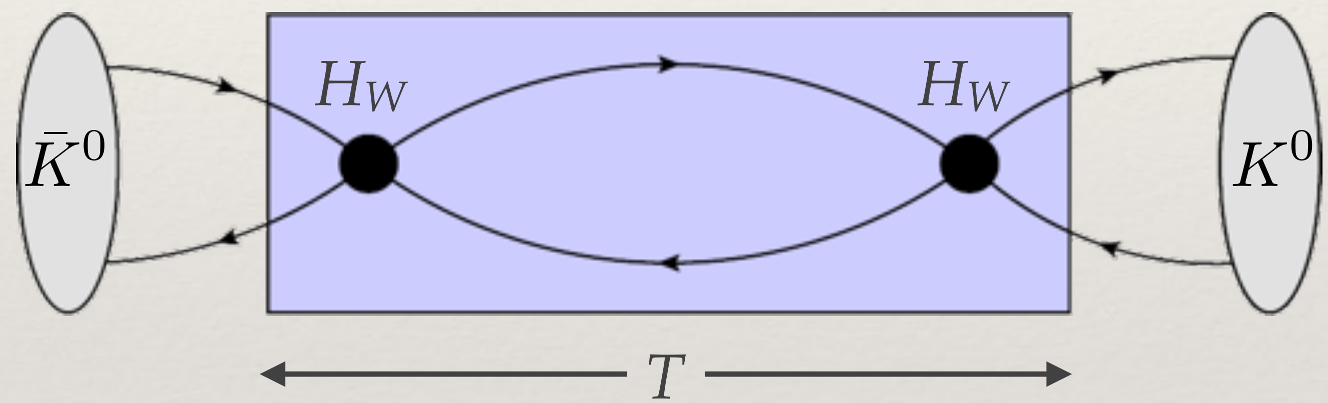

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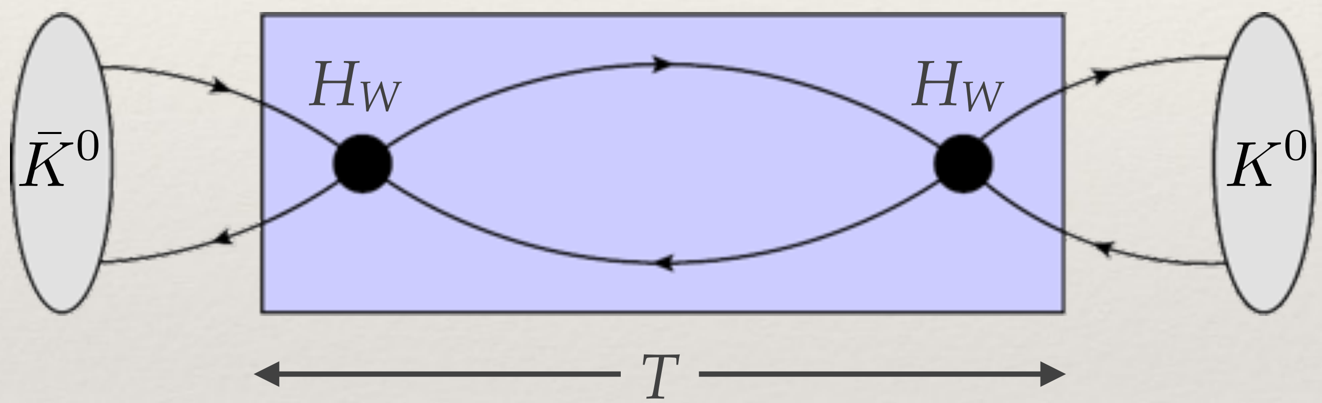

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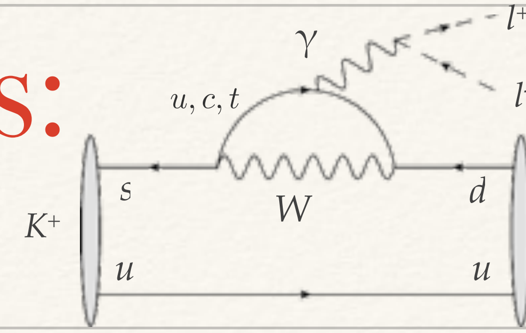
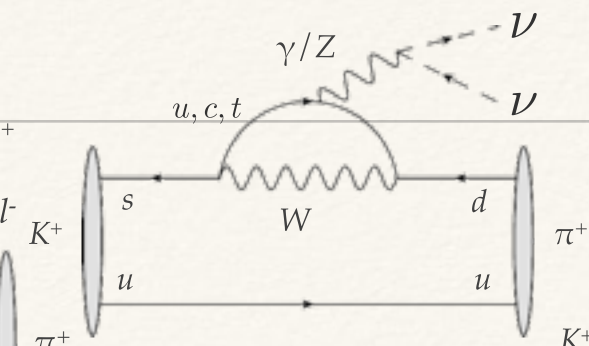
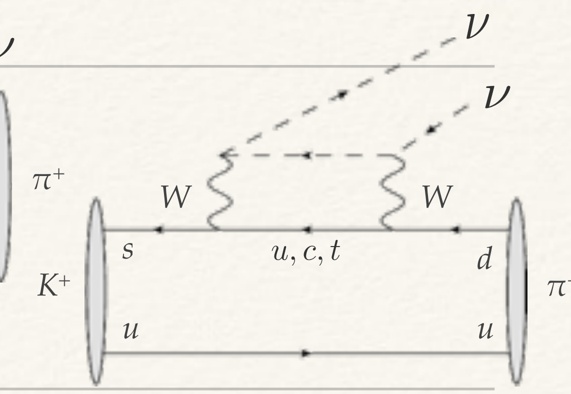
Exciting new work on ε_K and M_K in progress …

long distance effects: Rare kaon decays







Two new experiments dedicated to rare kaon decays NA62 (CERN) and KOTO (J-PARC) are running

- FCNC (W-W or γ/Z-exchange diagrams)
- deep probe into flavour mixing and SM/BSM due to suppression in the SM
- can determine V_{td}, V_{ts} and test SM

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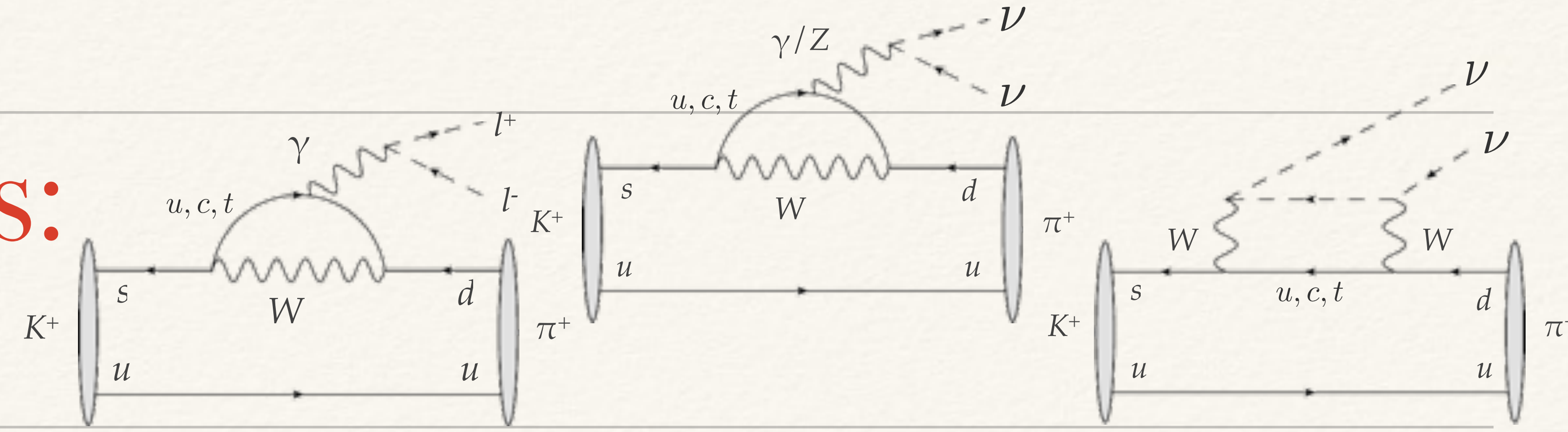

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- exp. BR $\leq 2.6 \times 10^{-8}$
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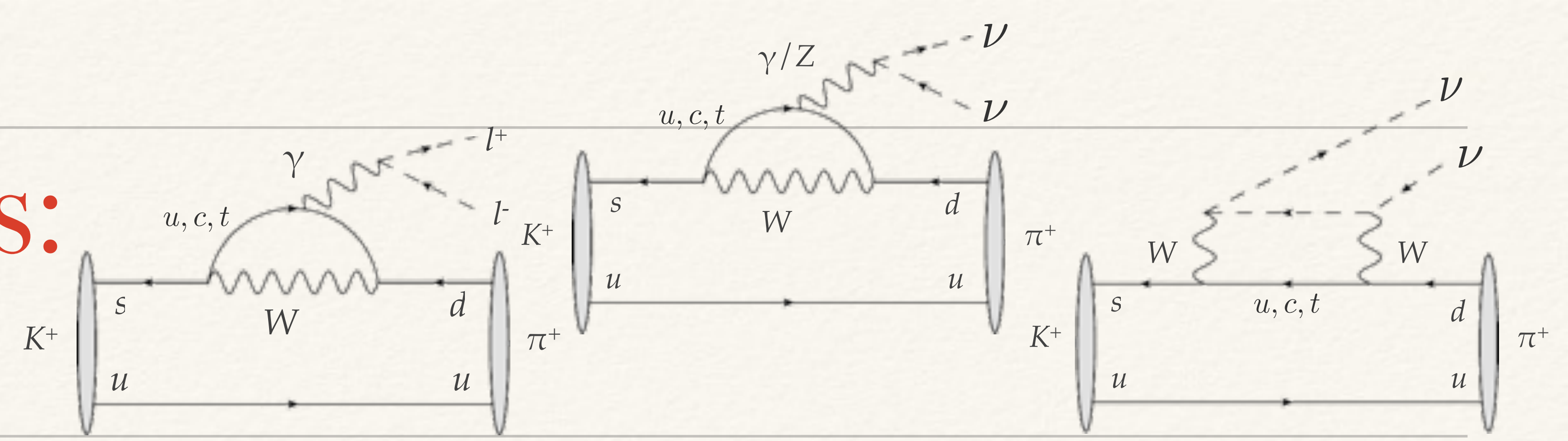

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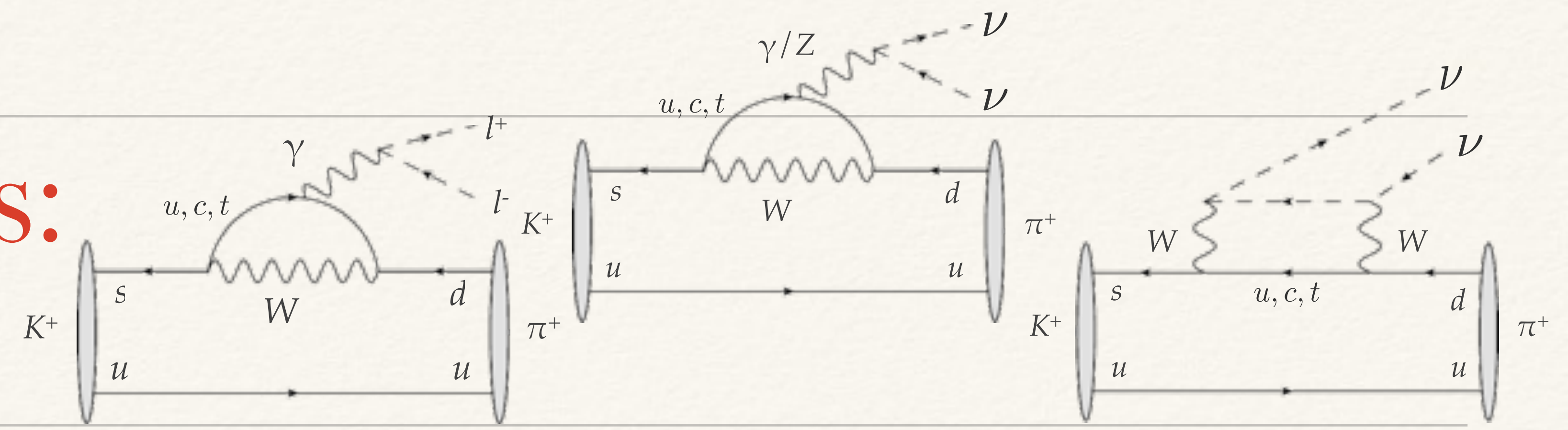

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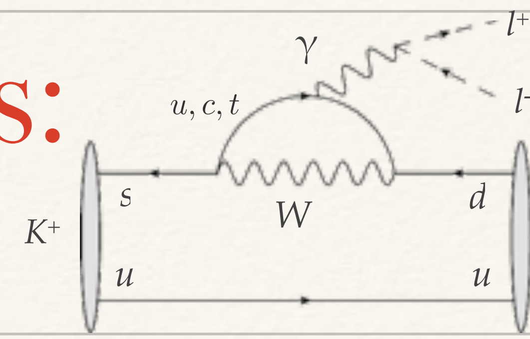

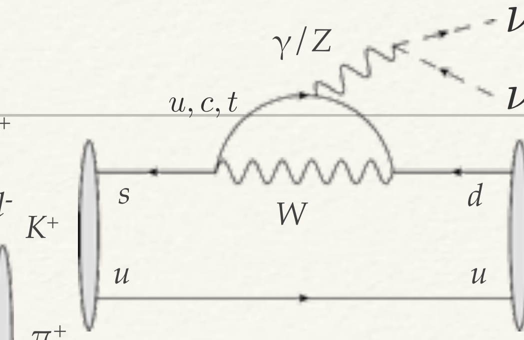

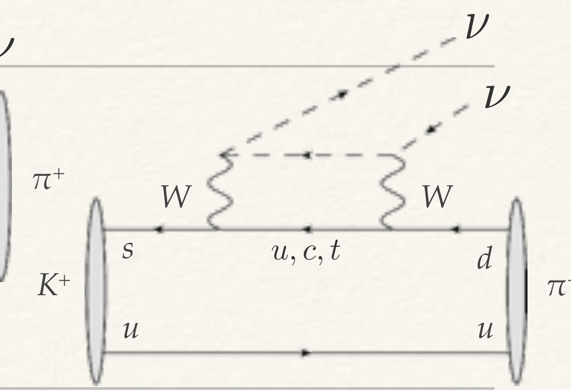

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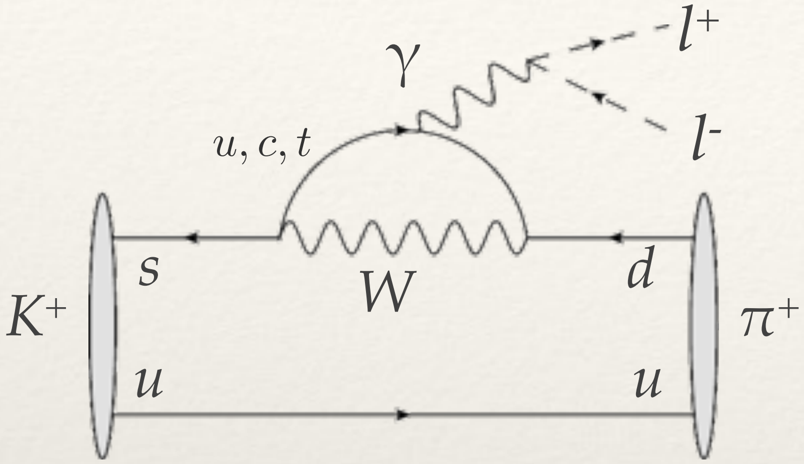

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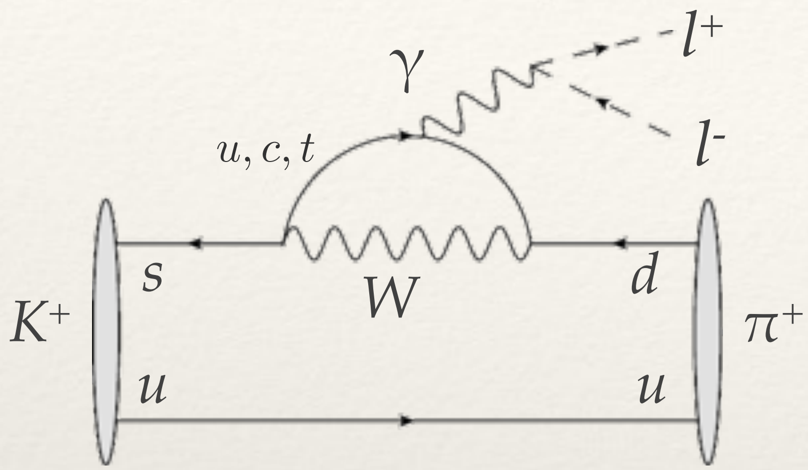

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Decay amplitude in terms of elm. transition form factor

$$A_i = -\frac{G_F \alpha}{4\pi} V_i(z)(k+p)^\mu \bar{u}_l(p_-)\gamma_\mu \nu_l(p_+) \qquad (i = +, S)$$

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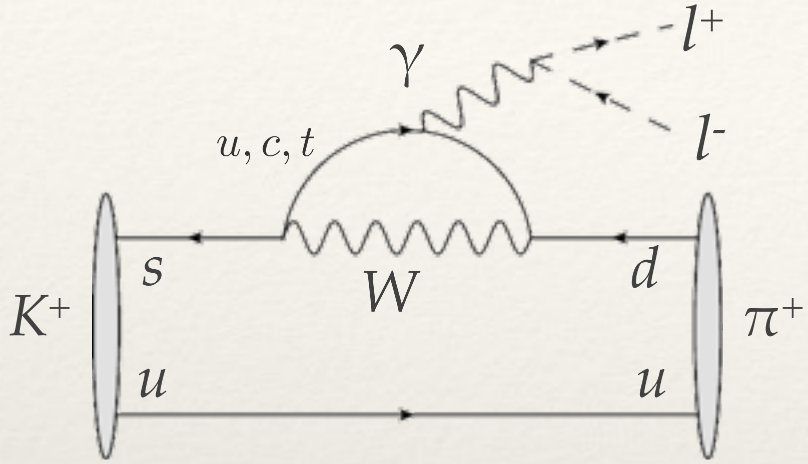

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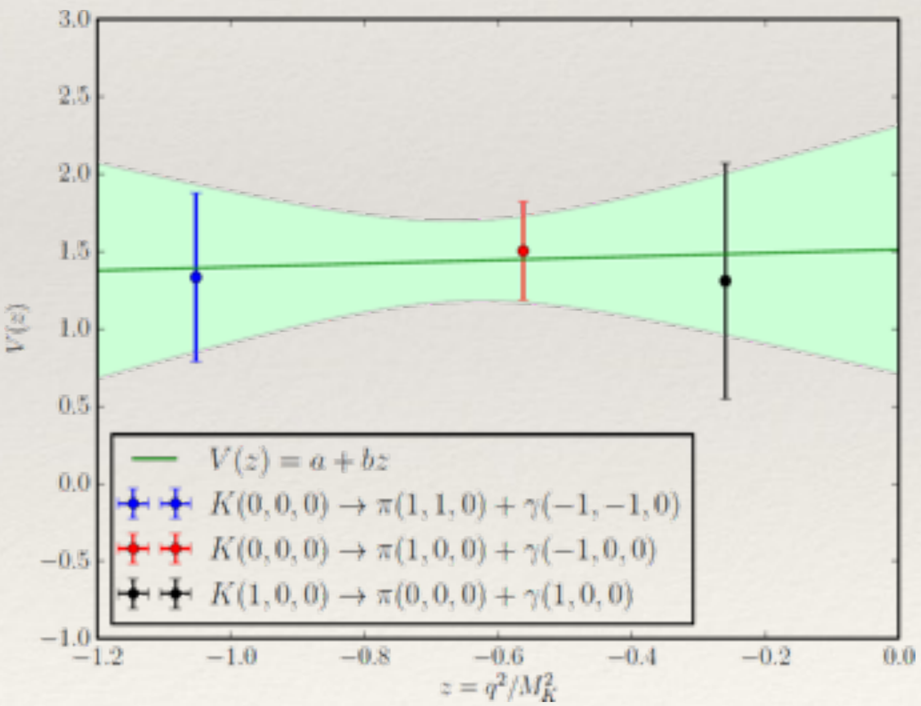
- the a_S and a_+ can be extracted from experiment or lattice
- a_S parameterises also the CP-violating contribution to the K_L decay



Summary II

- considerable set of SM parameters, spectra and matrix elements now reliably and precisely predicted in full lattice QCD — "bread and butter"

- results with good control over systematics summarised by Flavour Lattice Averaging Group (FLAG) (3rd edition is out)

- New challenges in Flavour physics:
  - precision on "bread and butter" such that isospin breaking in matrix elements and spectra needs to be taken into account
  - long distance effects (neutral main mixing, rare kaon decays, …)

loads of new questions and theoretical problems and potential impact on SM and BSM phenomenology