Recent Developments in Analytic Bootstrap

Annual Theory Meeting, Durham, 2017

Alexander Zhiboedov, Harvard U

How do we compute when the coupling is strong?

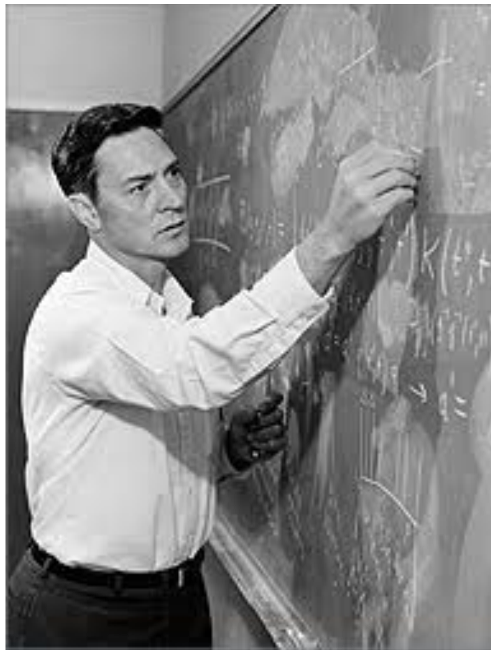
- No small coupling expansion
- No Lagrangian
- No extra symmetries/integrability

Bootstrap is an old idea of solving theories based on consistency.

Radical Bootstrap

"Nature is as it is because this is the only possible Nature consistent with itself"

G. Chew



(In other words, a consistent theory of quantum gravity compatible with all known experimental data is unique)

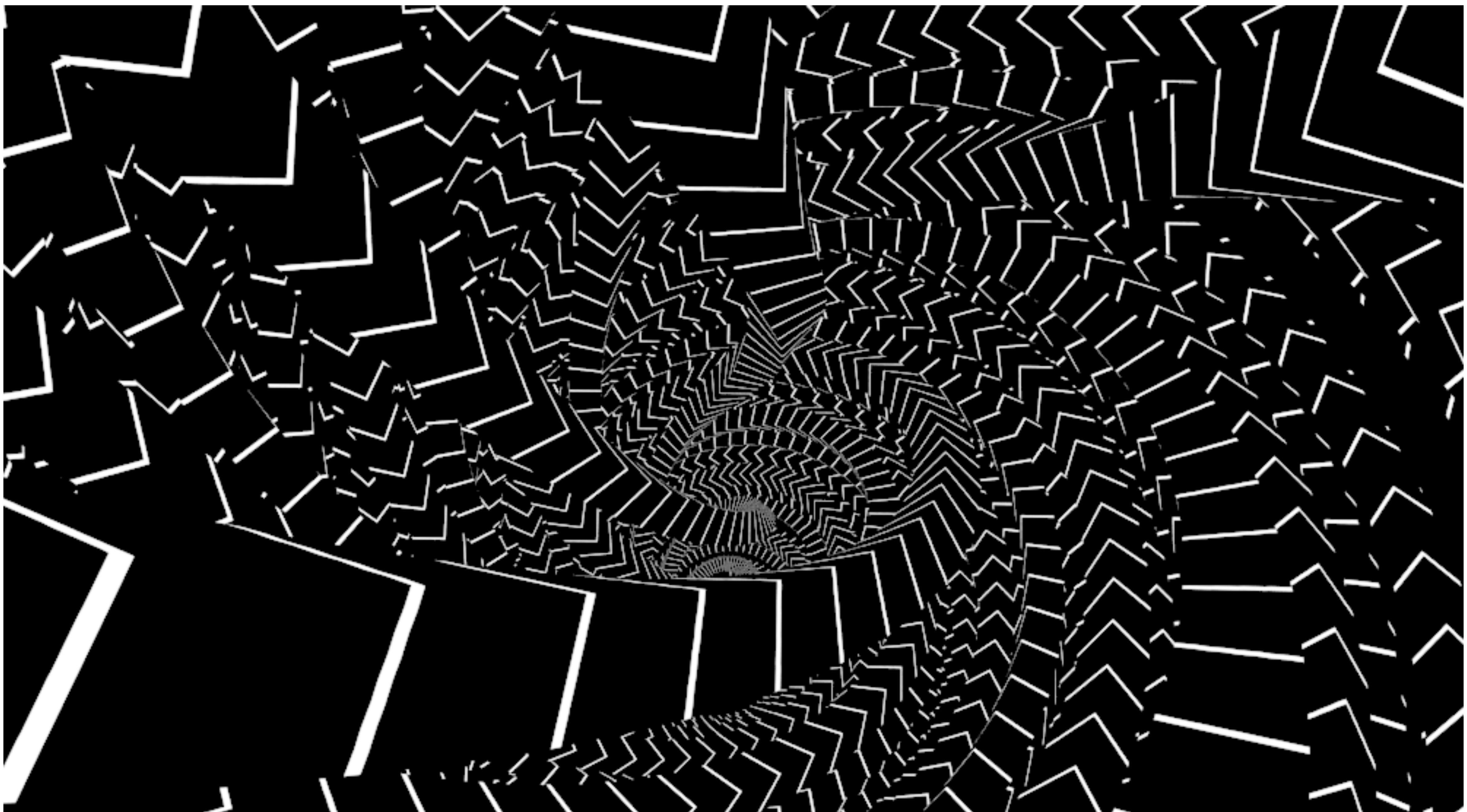
This is too ambitious! But for Conformal Field Theories (CFTs) it is almost true.

Conformal Bootstrap is a method to solve them based on consistency.

THEORETICAL PHYSICS

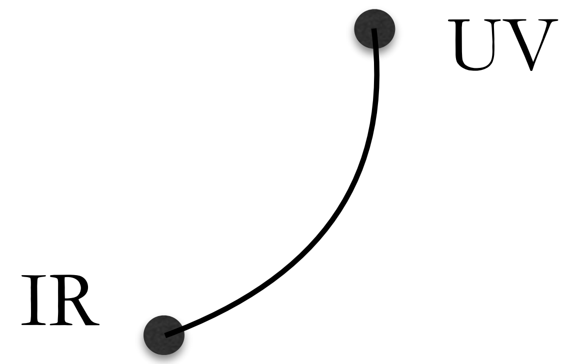
Physicists Uncover Geometric 'Theory Space'

A decades-old method called the "bootstrap" is enabling new discoveries about the geometry underlying all quantum theories.

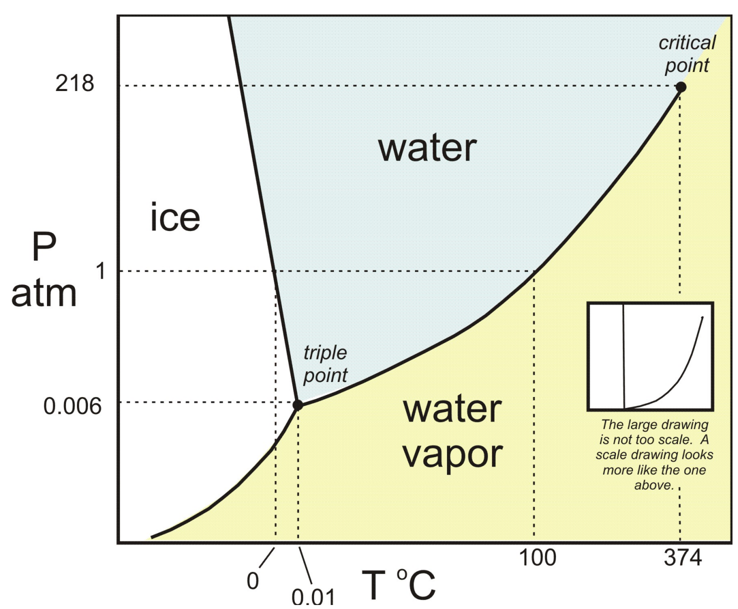


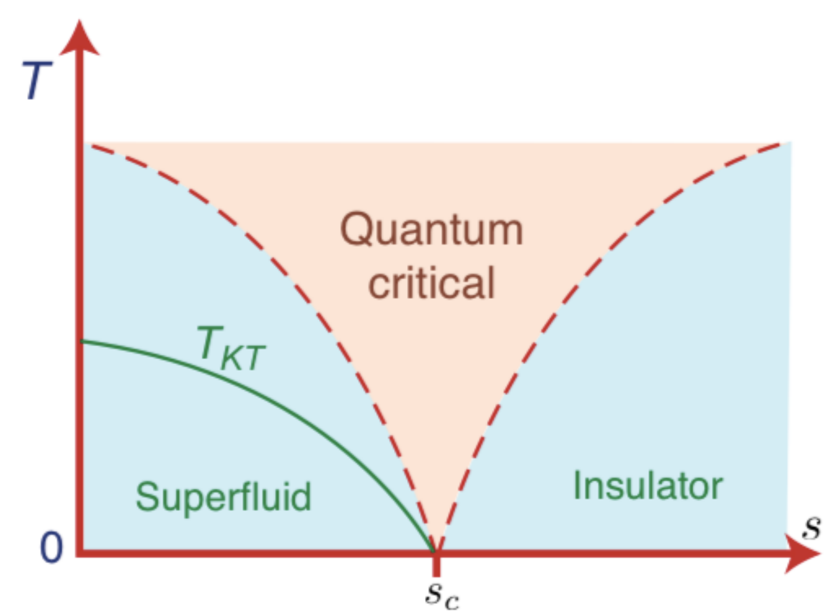
Why CFTs?

- RG flow fixed points

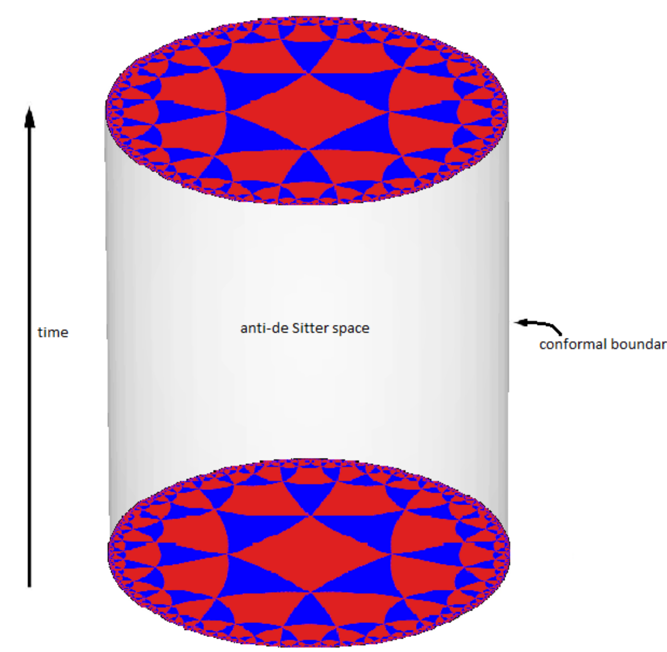


- critical points in condensed matter systems





[Sachdev et al.]



- non-perturbative quantum gravity in Anti-de Sitter (AdS/CFT) [Maldacena]

Plan

1. Basics of Conformal Bootstrap

2. Analytic Bootstrap: Spin = Expansion Parameter

3. Applications

Basics of Conformal Bootstrap

Conformal Bootstrap

Conformal Bootstrap is based on **symmetries** and **consistency conditions**:

- Conformal Symmetry
- Unitarity and the OPE

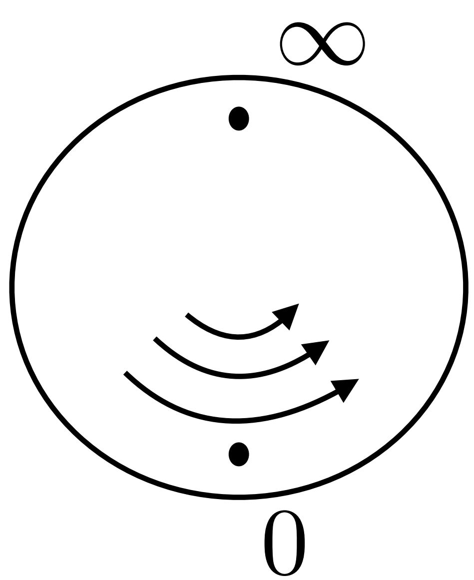
→ Crossing Equations

As such it is suitable for strongly coupled theories.

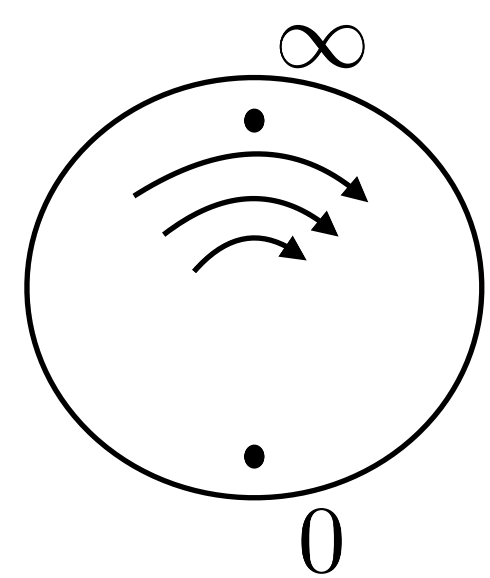
Basics of Conformal Symmetry

- Poincare symmetry: translations P_μ and rotations $M_{\mu\nu}$
- Scale or dilatation invariance D $\quad \delta x^\mu = \lambda x^\mu$
- Special conformal transformation K_μ $\quad \delta x^\mu = 2(b.x)x^\mu - b^\mu x^2$

P_μ



$$K_\mu = -I P_\mu I$$



$$[D, P_\mu] = P_\mu \ ,$$
$$[K_\nu, P_\mu] = 2\delta_{\nu\mu} D + 2M_{\mu\nu} \ ,$$
$$[D, K_\nu] = -K_\nu \ .$$

Observables

The basic observables are correlation functions of local operators

$$\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)...\mathcal{O}_n(x_n)\rangle$$

- Each operator is characterized by
  - Scaling dimension Δ
  - Representation under rotations (spin **J**)

- Primary operators $[K_\nu, \mathcal{O}_{\Delta,\mathbf{J}}(0)] = 0$

could not be written as a derivative of smth

$$\mathcal{O}(x') = \lambda^{-\Delta}\mathcal{O}(x) \qquad \lambda = \left|\frac{\partial x'}{\partial x}\right|^{\frac{1}{d}}$$

- Descendants $P_{\mu_k}...P_{\mu_1}\mathcal{O} = \partial_{\mu_k}...\partial_{\mu_1}\mathcal{O}$

Operators

Simple examples present in every CFT are

- Unit operator $\qquad \Delta = 0 \qquad J = 0$

  (lightest operator
  = dominates the OPE)

- Stress energy tensor $\qquad \partial_\mu T^{\mu\nu} = 0 \qquad T^\mu_{\ \mu} = 0$

  (gravity dual) $\qquad \Delta = d \qquad J = 2$

- Unitarity bounds $\qquad \Delta \geq d - 2 + J$

Two- and Three-point Functions

Correlation functions are invariant under symmetries.

Conformal symmetry fixes 1-, 2-, and 3-point functions.

critical exponents
(measured in experiments)

$$\langle \mathcal{O}_i(x) \rangle = 0$$

$$\langle \mathcal{O}_i(x_1) \mathcal{O}_j(x_2) \rangle = \frac{\delta_{ij}}{x_{ij}^{2\Delta}}$$

$$\langle \mathcal{O}_{\Delta_i}(x_1) \mathcal{O}_{\Delta_j}(x_2) \mathcal{O}_{\Delta_k}(x_3) \rangle = \frac{\lambda_{ijk}}{(x_{12}^2)^{\frac{\Delta_i+\Delta_j-\Delta_k}{2}} (x_{13}^2)^{\frac{\Delta_i+\Delta_k-\Delta_j}{2}} (x_{23}^2)^{\frac{\Delta_j+\Delta_k-\Delta_i}{2}}}$$

[Polyakov 70’]

CFT data: $(\Delta, \mathbf{J})$ λ_{ijk}

Goal: Find it!

Operator Product Expansion

Operators form an algebra (OPE)

$$\mathcal{O}_i(x)\mathcal{O}_j(0) = \sum_k \lambda_{ijk} |x|^{\Delta_i+\Delta_j-\Delta_k} \left(\mathcal{O}_k(0) + \underline{x^\mu \partial_\mu \mathcal{O}_k(0) + \dots}\right)$$

expansion in powers of distance

fixed

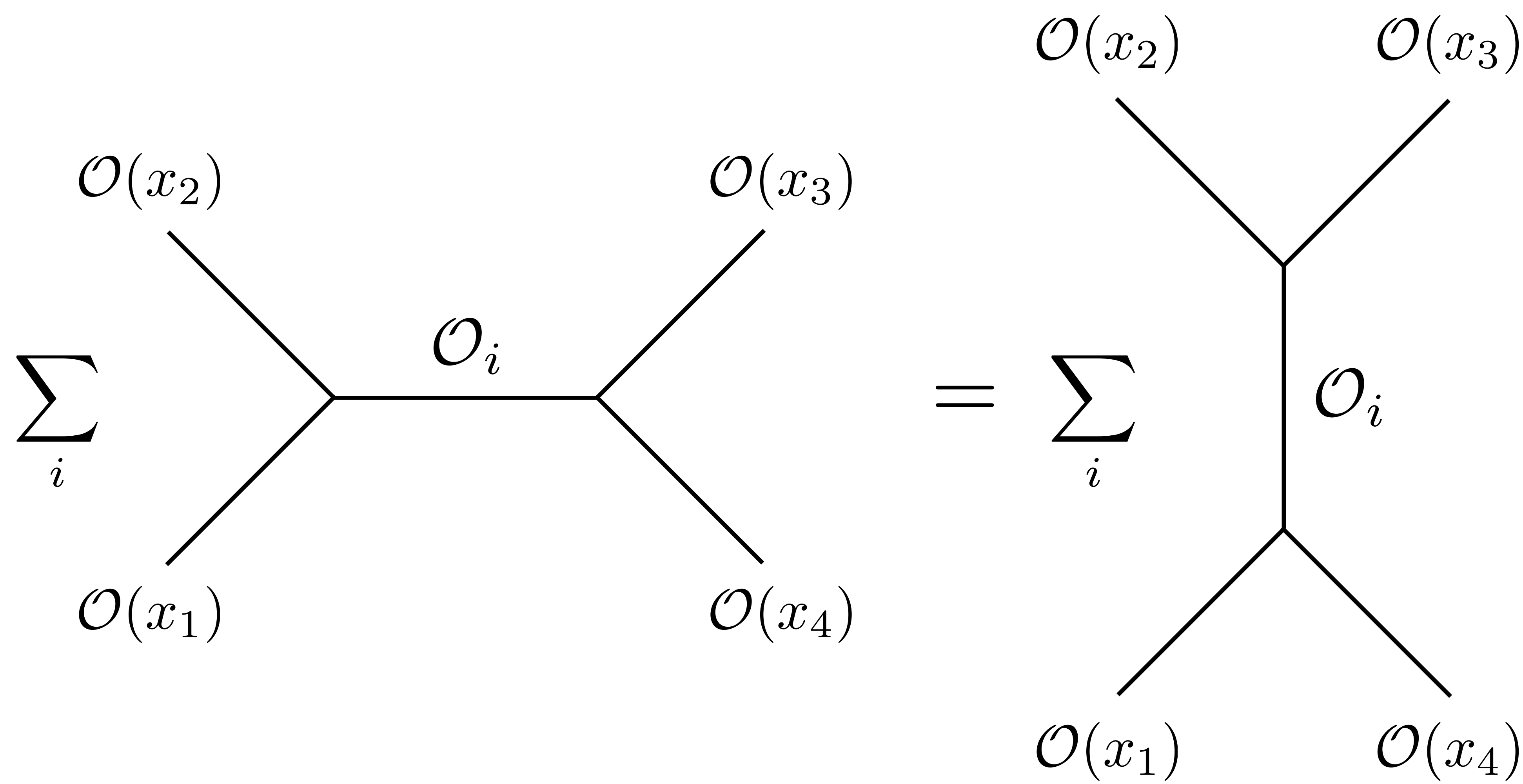
Consider now the four-point function of identical operators:

$$\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_3)\mathcal{O}(x_4)\rangle = \frac{\mathcal{G}(u,v)}{(x_{12}^2 x_{34}^2)^\Delta}$$

$$u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}\,, \qquad v = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}$$

Crossing Equations

We can apply the OPE inside the correlation function



Nonperturbative!

Conformal Bootstrap

- Original Idea

  [Ferrara, Gatto, Grillo 73′] [Polyakov 74′]

- Realization in 2d

  [Belavin, Polyakov, Zamolodchikov 83′]

- Realization in 4d (based on results of [Dolan, Osborn 00′])

  [Rattazzi, Rychkov, Tonni, Vichi 08′]

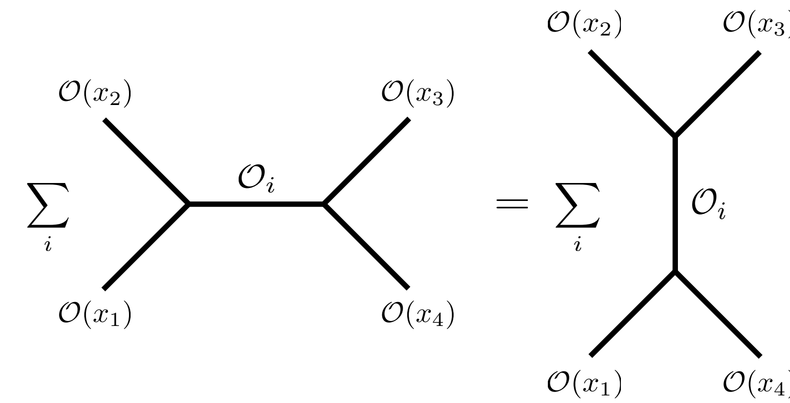
- Numerical Solution of the Critical 3d Ising Model

  [El-Showk, Paulos, Poland, Rychkov, Simmons-Duffin 12′-14′]

- Analytic Bootstrap

  [Fitzpatrick, Kaplan, Poland, Simmons-Duffin 12′]
  [Komargodski, AZ 12′]

Crossing Equations



$$v^{\Delta} \sum_{\Delta,\mathbf{J}} \lambda^2_{\Delta,\mathbf{J}} g_{\Delta,\mathbf{J}}(u,v) = u^{\Delta} \sum_{\Delta,\mathbf{J}} \lambda^2_{\Delta,\mathbf{J}} g_{\Delta,\mathbf{J}}(v,u)$$

conformal block
(known functions)

Conformal block = contribution of the primary and its descendants

$$(\mathcal{O}, \partial\mathcal{O}, \partial^2\mathcal{O}, \ldots)$$

Conformal bootstrap = solve these equations

Functional constraints on CFT data. Must be satisfied for all values of the cross ratios.

Conformal Blocks (Technical Details)

Let us list few basic properties of conformal blocks:

- Eigenfunctions of the Casimir operator

$$\hat{C} g_{\Delta,J}(u,v) = (\Delta + J)(\Delta + J - 1) g_{\Delta,J}(u,v)$$

- Small u<<1 limit

twist

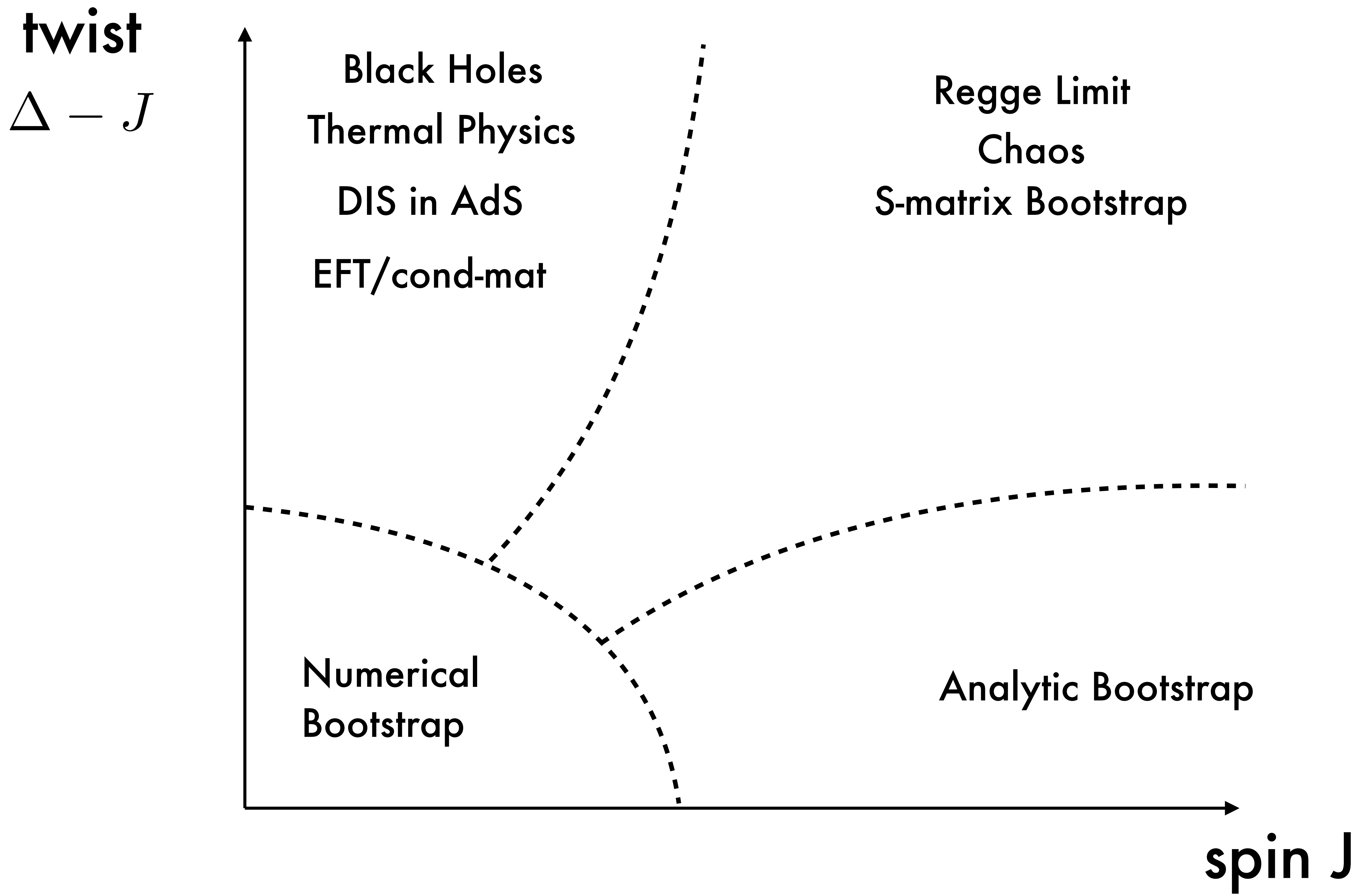
$$g_{\Delta,J}(u,v) \sim u^{\frac{\tau}{2}} f_{\tau,J}(v), \quad \tau = \Delta - J$$

expansion in powers
of cross ratios

- Small v<<1 limit

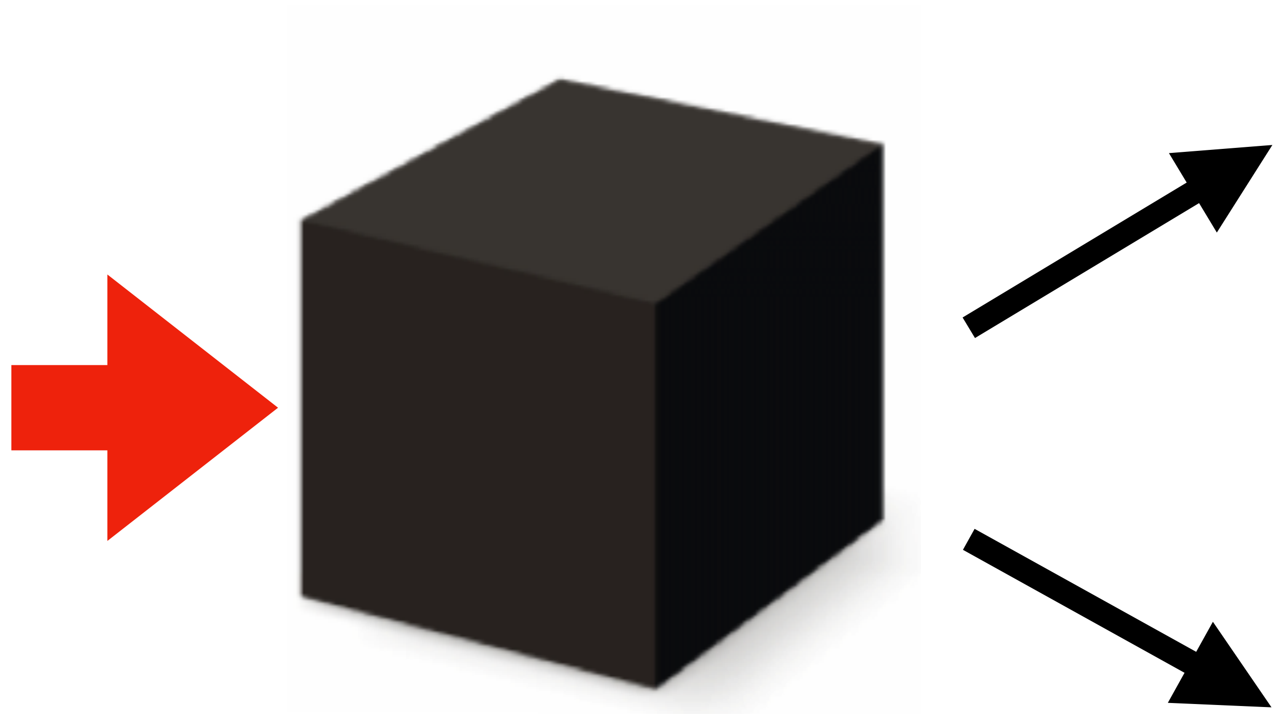
$$g_{\Delta,J}(u,v) \sim \log v$$

Conformal Map of the World



Numerical Bootstrap (Conformal Oracle)

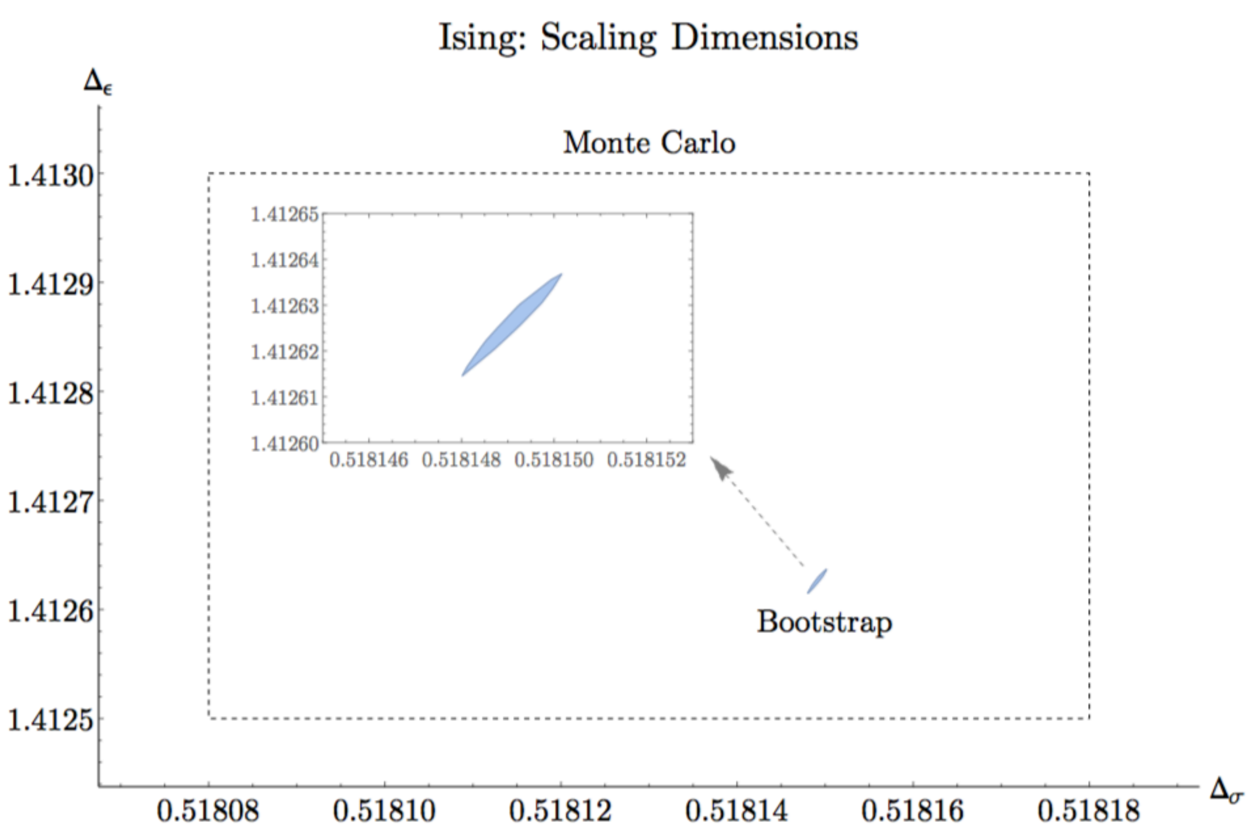
Tentative
CFT data



NO

MAYBE

[Talk by Slava Rychkov '14]



[Kos, Poland, Simmons-Duffin, Vichi '16]

Input:

a) Z_2 symmetry
b) 1 even relevant scalar
c) 1 odd relevant scalar

Analytic Bootstrap

Analytic Methods

- Protected Observables

  [Dolan, Osborn; Beem, Lemos, Liendo, Peelaers, Rastelli, van Rees; Chesler, Lee, Pufu, Yacoby, ...]

- Integrability

  [Escobedo, Gromov, Sever, Vieira; Basso, Coronado, Komatsu, Tat Lam, Vieira, Zhong; Bargheer, Caetano, Fleury, Komatsu, ...]

- Large Central Charge

  [Heemskerk, Penedones, Polchinski, Sully; Fitzpatrick, Kaplan; Alday, Bissi, Lukowski; Rastelli, Zhou, ...]

- Bootstrap in 2d

  [Cardy; Hellerman; Hartman, Keller, Stoica; Fitzpatrick, Kaplan, Walters; Lin, Shao, Simmons-Duffin, Wang, Yin, ...]

- Crossing in Mellin space

  [Mack; Penedones; Gopakumar, Kaviraj, Sen, Sinha; Dey; Rastelli, Zhou; Alday, Bissi, Lukowski, ...]

- Large Global Charge

  [Hellerman, Orlando, Reffert, Watanabe; Alvarez-Gaume, Loukas; Monin, Pirtskhalava, Rattazzi, Seibold; Jafferis, Mukhametzhanov, AZ ...]

- S-matrix bootstrap

  [Caron-Huot, Komargodski, Sever, AZ; Paulos, Penedones, Toledo, van Rees, Vieira]

Analytic Bootstrap

Study of the crossing equations in the Lorentzian regime.

- Analytic/Light-Cone Bootstrap
- Analyticity in Spin
- Regge limit, ANEC, chaos, gravity, etc

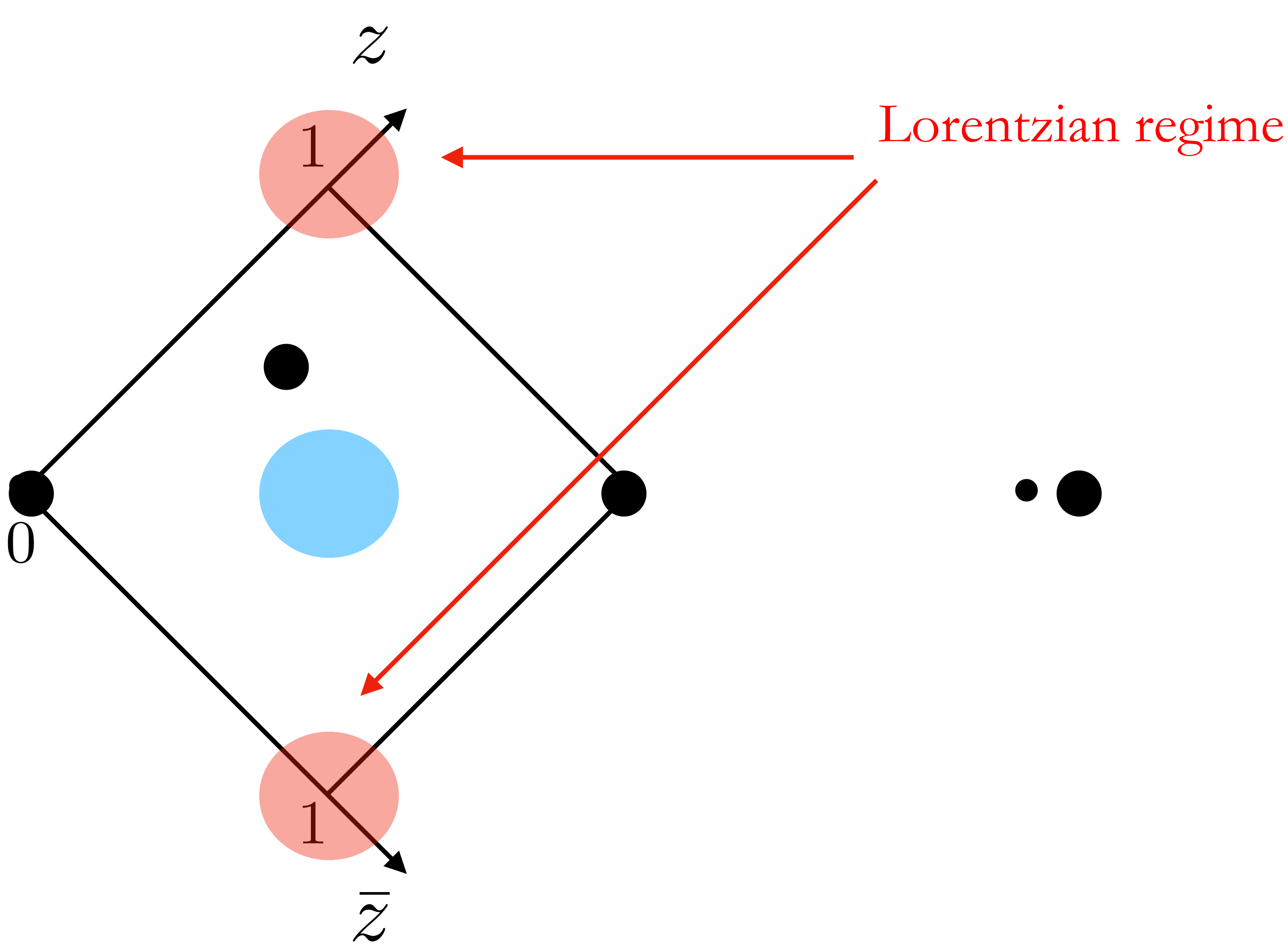
Analytic Bootstrap 101:

Minimal Solution to Crossing

Analytic Bootstrap

Consider the crossing equation in the light-cone limit

$$v \ll u \ll 1$$



$$u = z\bar{z} = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}$$

$$v = (1-z)(1-\bar{z}) = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}$$

Analytic Bootstrap

We can use the OPE in one channel

$$\mathcal{G}(v,u) = 1 + O(v^{\frac{\Delta - J}{2}}) \qquad v \ll u \ll 1$$

Crossing equation $v^{\Delta}\mathcal{G}(u,v) = u^{\Delta}\mathcal{G}(v,u)$ becomes

$$\mathcal{G}(u,v) = 1 + \sum_{\Delta,\mathbf{J}} \lambda^2_{\Delta,\mathbf{J}} g_{\Delta,\mathbf{J}}(u,v) = \frac{u^{\Delta}}{v^{\Delta}}(1+\ldots)$$

diverges!

Puzzle 1: $\lim_{v \ll 1} g_{\Delta,\mathbf{J}}(u,v) \sim \log v$

Generalized Free Field (GFF)

Example: Generalized Free Field

$$\langle \mathcal{O}\mathcal{O}\mathcal{O}\mathcal{O} \rangle = \langle \mathcal{O}\mathcal{O} \rangle \langle \mathcal{O}\mathcal{O} \rangle + \text{permutations}$$

$$\mathcal{G}^{(0)}(u,v) = 1 + u^{\Delta} + \left(\frac{u}{v}\right)^{\Delta}$$

- Spectrum contains operators $\mathcal{O}\Box^n \partial_{\mu_1}...\partial_{\mu_J}\mathcal{O}$

(double-twist operators)

$$\Delta_{n,J} = 2\Delta_{\mathcal{O}} + 2n + J$$

- Sum over spins produces the divergence $\dfrac{u^{\Delta}}{v^{\Delta}}$

Analytic Bootstrap

Resolution:

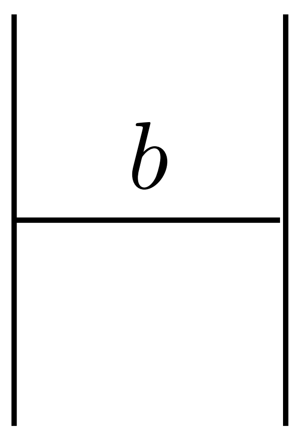
Every solution to crossing equations has an infinite number of operators of every spin.

Analytic Bootstrap 201:

Large Spin Universality

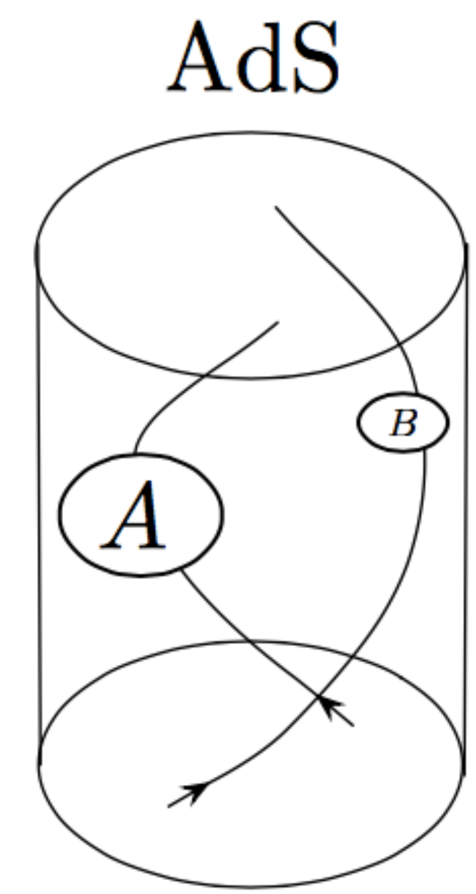
Analytic Bootstrap

Impact parameter **b** is dual to spin **J**.



Flat Space: $b \sim J$

AdS (CFT): $b \sim \log J$



[Cornalba, Costa, Penedones, Schiappa]
[Alday, Maldacena]

scattering phase shift ⟷ CFT energy levels

$$\delta(s,b) \sim e^{-mb}$$

$$\delta\Delta(J) \sim \frac{1}{J^m}$$

Large Spin Universality

This mechanism of reproducing operators on one side by summing large spin operators on the other side is completely universal.

(inner workings of crossing equations)

[Fitzpatrick, Kaplan, Poland, Simmons-Duffin 12′]

[Komargodski, AZ 12′]

Every CFT is GFF at large spin

Every CFT admits an infinite family of operators with the properties

$$\Delta_{n,J} = \Delta_{\mathcal{O}_1} + \Delta_{\mathcal{O}_2} + 2n + J + O(\frac{1}{J})$$

$\mathcal{O}\Box^n \partial_{\mu_1} ... \partial_{\mu_J} \mathcal{O}$

$$\lambda_{n,J} = \lambda_{n,J}^{GFF} \left(1 + O(\frac{1}{J})\right)$$

Analytic Bootstrap

Let us add a first nontrivial correction to the previous exercise

$$\sum_{\Delta,J} \lambda^2_{\Delta,J} g_{\Delta,J}(u,v) = \frac{u^\Delta}{v^\Delta}\left(1 + \frac{d^2}{(d-1)^2}\frac{\Delta^2}{c_T} v^{\frac{d-2}{2}} \log u + \dots\right)$$

GFF result

leading 'correction due to stress tensor

$$\sum_{\Delta,J} \lambda^2_{\Delta,J} g_{\Delta,J}(u,v) \simeq \sum_J u^\Delta \left(1 + \frac{\gamma_J}{2}\log u\right)\lambda_J^{GFF} f_J(v) + \dots$$

anomalous dimension

known collinear conformal block

By matching the two we get

$$\gamma_J = -\frac{d^2}{2(d-1)^2}\frac{\Delta^2}{c_T}\frac{\Gamma(\Delta)^2\Gamma(d+2)}{\left(\Gamma(\frac{d+2}{2})\Gamma(\Delta - \frac{d-2}{2})\right)^2}\frac{1}{J^{d-2}}$$

Analytic Bootstrap

The method works not only for singular terms, but also for Casimir-singular terms (act on the Casimir equation on the crossing equations).

[Alday, Bissi, Lukowski]

These correspond to terms that become singular upon acting on them with the Casimir operator

$$v^a$$

Casimir-regular terms are

$$v^n, \; v^n \log v$$

Equivalently, these are terms with non-zero double discontinuity

$$\mathrm{dDisc}[f(v)] = f(v) - \frac{1}{2}\left(f(ve^{2\pi i}) - f(ve^{-2\pi i})\right)$$

Perturbative Analytic Bootstrap/Large Spin Perturbation Theory

[Alday et al.]

Feynman Rules ⟷ Unperturbed Spectrum

Feynman Diagrams ⟷ Large Spin Expansion + Crossing

- Critical O(N) models

[Alday, Bissi, Lukowski; Gopakumar, Kaviraj, Sen, Sinha; Dey, Kaviraj; Alday, AZ, Giombi, Kirilin, Skvortsov, ...]

$d = 4 - \epsilon$

$$\frac{C_T}{C_{\text{free}}} = 1 - \frac{5}{324}\epsilon^2 - \frac{233}{8748}\epsilon^3 - \left(\frac{100651}{3779136} - \frac{55}{2916}\zeta_3\right)\epsilon^4 + \cdots$$

$\langle T_{\mu\nu} T_{\rho\sigma} \rangle \sim C_T$

[Alday, Henriksson, van Loon]

- Gauge Theories

$\Delta(J) \sim \log J$

[Alday, Bissi; Lukowski; Li, Meltzer, Poland; Korchemsky; Alday, AZ; Henriksson, Lukowski, ...]

- Loops in AdS

[Aharony, Alday, Bissi, Perlmutter; Alday, Bissi; Aprile, Drummond, Heslop, Paul; Ye Yuan; Alday, Caron-Huot,...]

Analytic Bootstrap 301:

Analyticity in Spin

Few Questions

- Spin is discrete, not continuous

- How large is “‘``large spin’``’’’’’’’’’’’?

- What are the errors?

All these problems are solved due to **analyticity in spin**.

[Caron-Huot 17’]

Complex J-plane and Regge Limit

Already in the 50's it was understood that it is natural to think about the complex angular momentum. [Regge]

Consider an ``amplitude''''`` f(E) that is: [Caron-Huot 17']

basic clash

- Admits the low-energy Taylor expansion

$$f(E) = \sum_{J=0}^{\infty} f_J E^J$$

- Analytic away from two branch cuts at $|E| > 1$

- Bounded at infinity $\lim_{E \to \infty} \left| \frac{f(E)}{E} \right| \leq c$

Complex J-plane and Regge Limit

We can write a simple dispersion integral which is manifestly **analytic in spin**

$$f_J = \frac{1}{2\pi}\int_1^\infty \frac{dE}{E} E^{-J}\left(\mathrm{Disc} f(E) + (-1)^J \mathrm{Disc} f(-E)\right)$$

The same idea applies to scattering amplitudes and CFTs!

Taylor expansion ⟷ Partial wave expansion

Analyticity ⟷ Unitarity

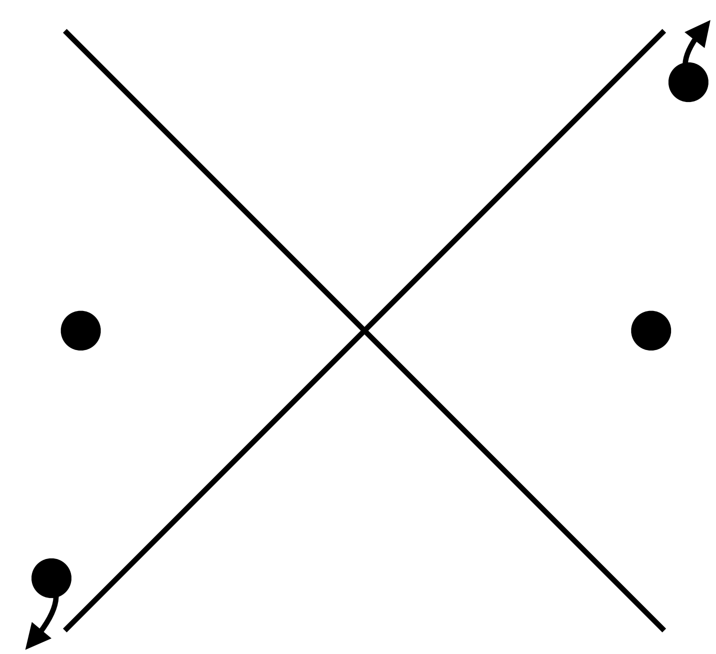
Bound at infinity ⟷ Regge limit/Causality

Regge Limit

The relevant limit is the so-called Regge limit

$$\lim_{s\to\infty,\ t-\text{fixed}} A(s,t)$$ (high energy, small angle)

$$\lim_{z\to 1,\frac{1-z}{1-\bar{z}}-\text{fixed}} G(ze^{-2\pi i},\bar{z})$$



bounded using OPE

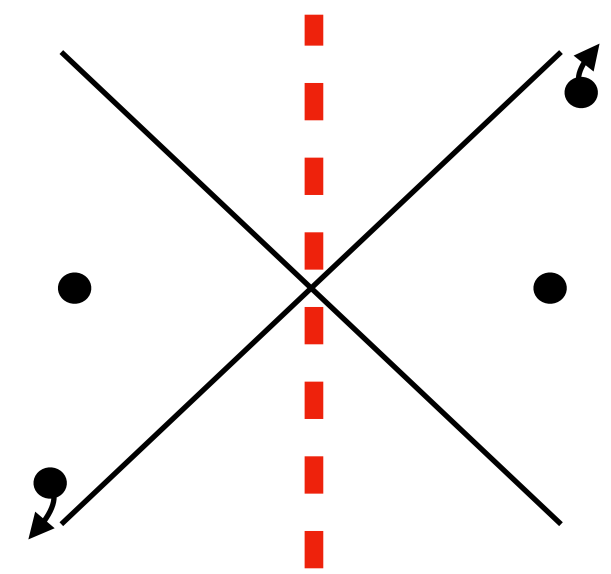
Bound on the Regge Limit in CFTs

Using the OPE it is trivial to bound the Regge limit

$$G_{Regge}(z,\bar{z}) = G(ze^{-2\pi i},\bar{z})$$

$$G(z,\bar{z}) = (z\bar{z})^{-\Delta_{\mathcal{O}}} \sum_{\Delta,J} a_{\Delta,J} z^{\frac{\Delta \pm J}{2}} \bar{z}^{\frac{\Delta \mp J}{2}}$$

$$a_{\Delta,J} \geq 0$$



$$G_{Regge}(ze^{-2\pi i},\bar{z}) = e^{2\pi i \Delta}(z\bar{z})^{-\Delta_{\mathcal{O}}} \sum_{\Delta,J} a_{\Delta,J} e^{-i\pi(\Delta \pm J)} z^{\frac{\Delta \pm J}{2}} \bar{z}^{\frac{\Delta \mp J}{2}}$$

Adding Time = Adding Phases $\quad e^{-iHt}$

$$|G_{Regge}(z,\bar{z})| \leq G_{Eucl}(|z|,|\bar{z}|)$$

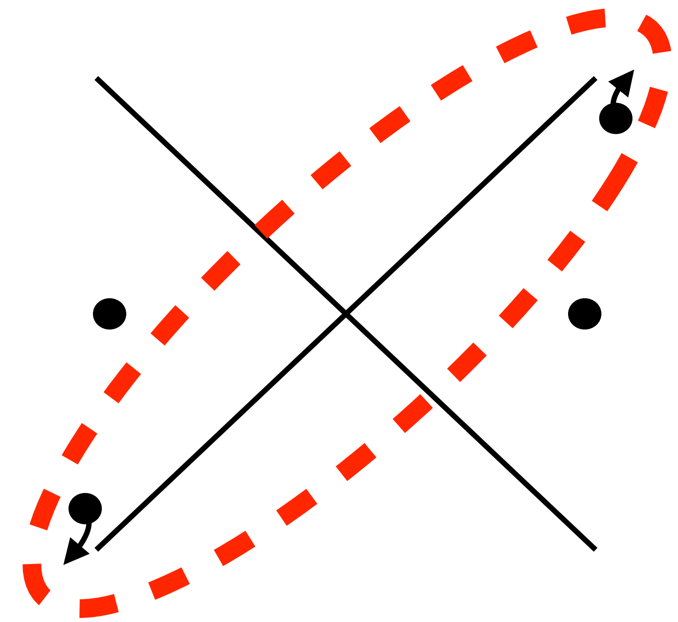
Bound on the Regge Limit in CFTs

The effect is dramatic in the other channel

$$G_{Regge}(z,\bar{z}) = G(ze^{-2\pi i},\bar{z})$$

$$G(1-z,1-\bar{z}) \sim (1-z)^{\frac{\Delta\pm J}{2}}(1-\bar{z})^{\frac{\Delta\mp J}{2}}$$

$$G(1-ze^{-2\pi i},1-\bar{z}) \sim \frac{1}{(1-z)^J}$$



Taming these divergences requires conspiracy in spin.

Bound on Regge Limit in CFTs

$$|G_{Regge}(z,\bar{z})| \le G_{Eucl}(|z|,|\bar{z}|)$$

- ANEC in QFT

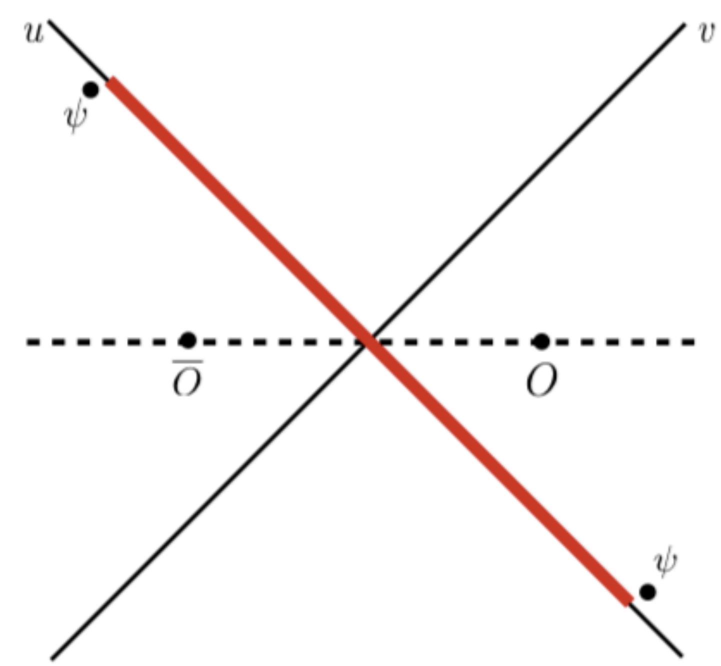
[Hofman, Li, Meltzer, Poland, Rejon-Barrera; Komargodski, Kulaxizi, Parnachev, AZ; Faulkner Leigh, Parrikar, Wang; Hartman, Kundu, Tajdini]

$$\int_{-\infty}^{\infty} d\lambda \ \langle \Psi | T_{\mu\nu} | \Psi \rangle u^{\mu} u^{\nu} \ge 0$$

(the argument uses Rindler positivity)

[Hartman, Kundu, Tajdini]

$$\int_{-\infty}^{\infty} d\lambda \ \langle \Psi | X_{\mu_1 \mu_2 \ldots \mu_s} | \Psi \rangle u^{\mu_1} u^{\mu_2} \ldots u^{\mu_s} \ge 0$$



- Bound on chaos

[Maldacena, Shenker, Stanford]

$$\langle [V(t), W(0)]^2 \rangle \sim e^{\lambda_L t} \qquad \lambda_L \le \frac{2\pi}{\beta}$$

Lorentzian OPE Inversion Formula

Similarly, one can write partial wave expansion for CFTs

conformal Fourier transform

$$G(z,\bar{z}) = 1 + \sum_{J=0}^{\infty} \int_{\frac{d}{2}-i\infty}^{\frac{d}{2}+i\infty} \frac{d\Delta}{2\pi i} c(\Delta, J) F_{J,\Delta}(z,\bar{z})$$

conformal block plus its shadow"

- Closing the contour leads to the OPE

$$c^{FG}(\Delta, J) = c^t(\Delta, J) + (-1)^J c^u(\Delta, J)$$

$$c^t(\Delta, J) = \int_0^1 dz d\bar{z} \mu(z,\bar{z}) G_{J+d-1,\Delta+1-d}(z,\bar{z}) \mathrm{dDisc}[G(z,\bar{z})]$$

[Caron-Huot 17']

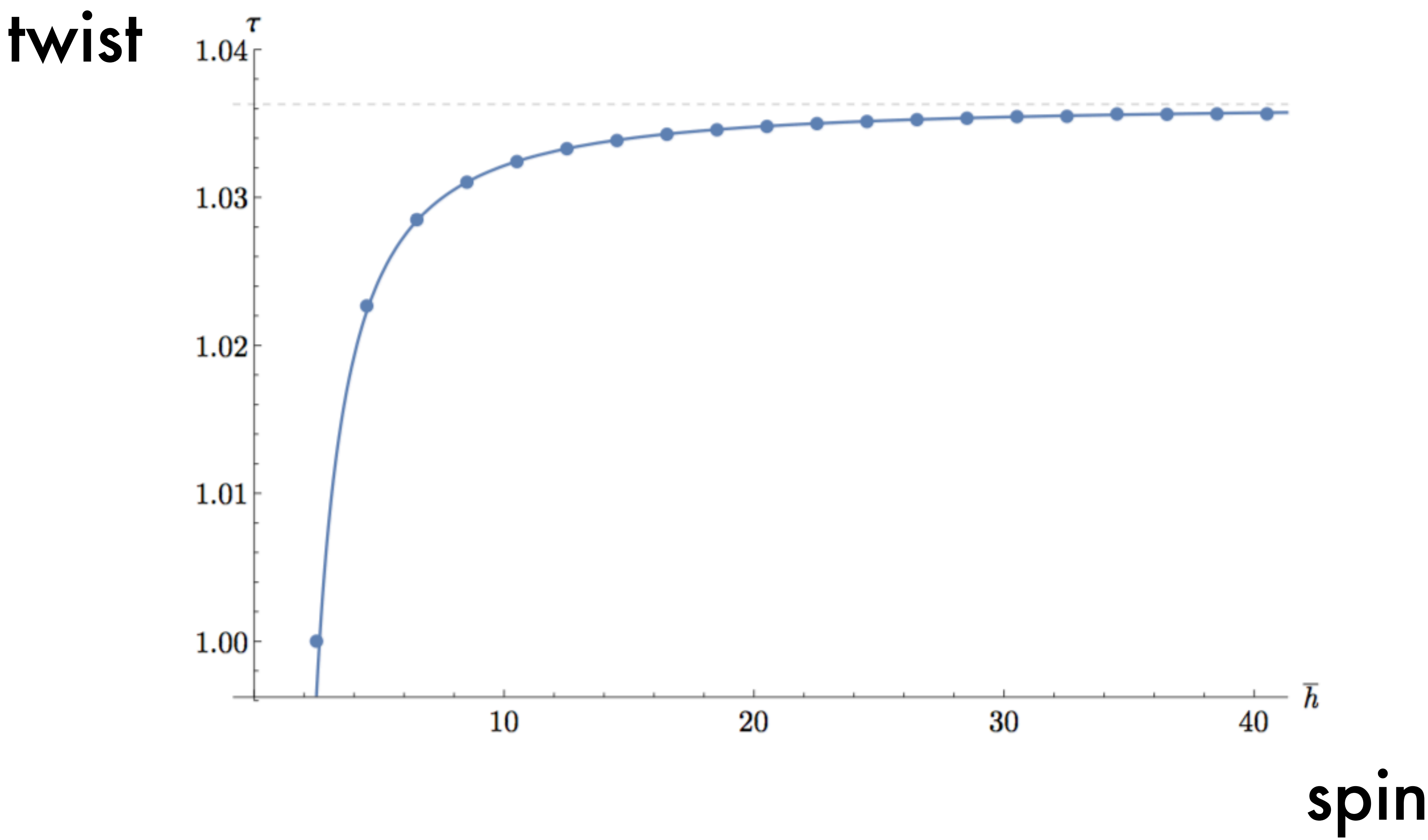
Analytic in spin!

(see also [Alday, Caron-Huot 17']
[Simmons-Duffin, Stanford, Witten 17'])

Result

In 3d Ising J=2 is already large (1% precision)!

[Simmons-Duffin; Alday, AZ]



Corollaries of the Inversion Formula

- CFT data is analytic in spin for J>1
- Analytic bootstrap with errors
- Large N theories made simple
- Step towards deriving the dual Einstein gravity

Conclusions

- Numerical+Analytic Bootstrap (powerful and rigorous!)

  [Simons Collaboration on the Nonperturbative Bootstrap]

- Time is very useful (Lorentzian constraints)
- Spin matters (Unitarity/Causality)
- Large Spin Expansion/Light-Cone Crossing
- Regge limit/Analyticity in spin
- ANEC, bound on chaos, Einstein gravity, etc.

Thank you!

Back up: **Lorentzian OPE Inversion Formula**

$$c^t(\Delta, J) = \int_0^1 dz d\bar{z} \mu(z,\bar{z}) G_{J+d-1,\Delta+1-d}(z,\bar{z}) \mathrm{dDisc}[G(z,\bar{z})]$$

- Valid for J>1 (in the planar limit J>2)

- Equal to the square of a commutator

$$\mathrm{dDisc}[G(z,\bar{z})] = -\frac{1}{2}\langle [\mathcal{O}_2(-1), \mathcal{O}_3(-\rho)][\mathcal{O}_1(1), \mathcal{O}_4(\rho)]\rangle \geq 0$$

$$\mathrm{dDisc}[G(z,\bar{z})] \sim \sum_{\mathcal{O}',J'} \sin^2\left(\frac{\pi(\Delta' - 2\Delta - J')}{2}\right) \lambda^2_{\mathcal{O}',J'} \left|\frac{1-\sqrt{\rho}}{1+\sqrt{\rho}}\right|^{\Delta'+J'} \left|\frac{1-\sqrt{\bar{\rho}}}{1+\sqrt{\bar{\rho}}}\right|^{\Delta'-J'}$$

- Only single trace operators contribute in the planar limit

$$\Delta' - J' - 2\Delta = 2 \text{ integer} + \gamma_{d.tr.}$$

Back up: Froissart-Gribov Formula

For scattering amplitudes this is result is well-known

$$A(s,t) = \sum_{J=0}^{\infty} a_J(s) P_J(\cos\theta) \qquad \cos\theta = 1 + \frac{2t}{s}$$

$$a_J^{FG}(s) = a_J^t(s) + (-1)^J a_J^u(s)$$

$$a_J^t(s) = \int_1^\infty d(\cosh\eta)(\sinh\eta)^{d-4} Q_J(\cosh\eta) \mathrm{Disc}_t A(s,t(\eta))$$

Partial waves are analytic in spin.

Back up: Einstein Gravity Dual

HPPS Conjecture: [Heemskerk, Penedones, Polchinski, Sully 09’]

Every CFT with large N and large gap in the spectrum of higher spin (J>2) operators is dual to Einstein gravity.

$$-\frac{1}{l_P^{d-1}}\int d^{d+1}x\sqrt{g}\left(R+\frac{d(d-1)}{L_{AdS}^{d-1}}+\alpha' R^2+\dots\right)$$

Recently there a was a lot of progress towards proving that.
[Camanho, Edelstein, Maldacena, A.Z.; Afkhami-Jeddi, Hartman, Kundu, Tajdini; Kulaxizi, Parnachev, A.Z.; Li, Meltzer, Poland; Meltzer, Perlmutter]

What is the deep reason for that universality?

In d=2 there is Virasoro symmetry.

Back up: Large N QCD Bootstrap

[Caron-Huot, Komargoski, Sever, A.Z. 16']

[Sever, A.Z. 17']

At large energies and imaginary scattering angles the scattering amplitude is universal

limit of the Veneziano amplitude

$$\lim_{\substack{s,t\to\infty \\ s/t \text{ fixed}}} \log A(s,t) = \alpha' \left[(s+t)\log(s+t) - s\log(s) - t\log(t)\right] \sim E^2 \log E$$

corrections are O(log E)

$$\sim E^{1/2} \log E$$

$$-\frac{16\sqrt{\pi}}{3}\alpha' m^{3/2} \left(\frac{s\,t}{s+t}\right)^{\frac{1}{4}} \left[\mathrm{K}\left(\frac{s}{s+t}\right) + \mathrm{K}\left(\frac{t}{s+t}\right)\right] + \ldots$$

correction due to the slowdown of the string (massive endpoints)/spectrum non-degeneracy

elliptic integral of the first kind EllipticK[x]