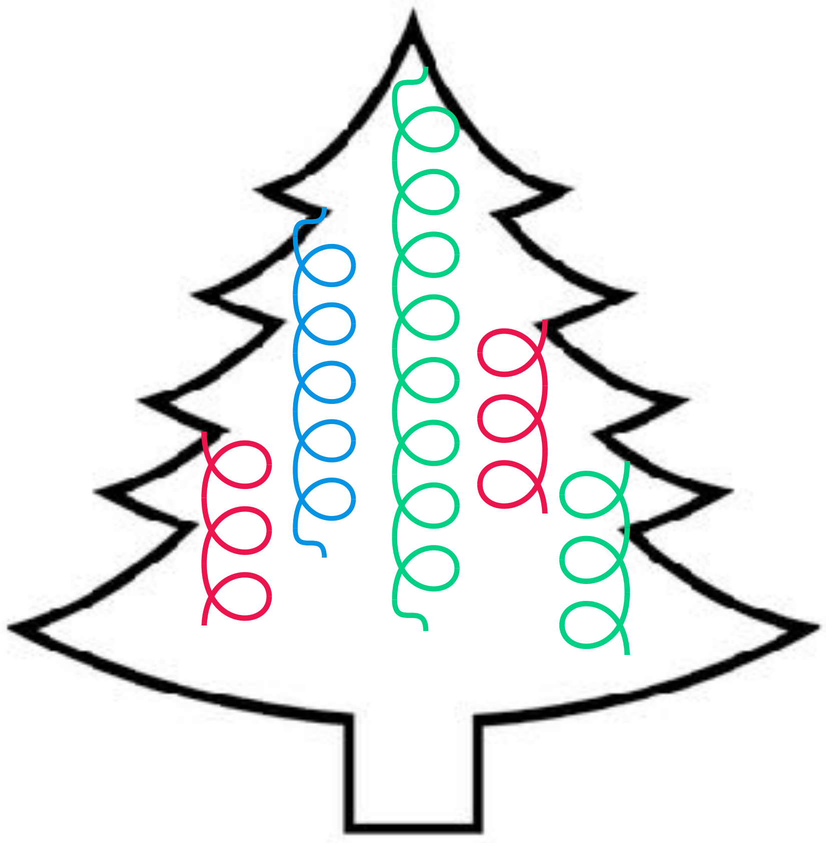

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FOR FUNDAMENTAL PHYSICS

Effective field theory for collider processes

Thomas Becher
University of Bern

Annual Theory Christmas Meeting, Durham, December 19

Tools for QFT computations

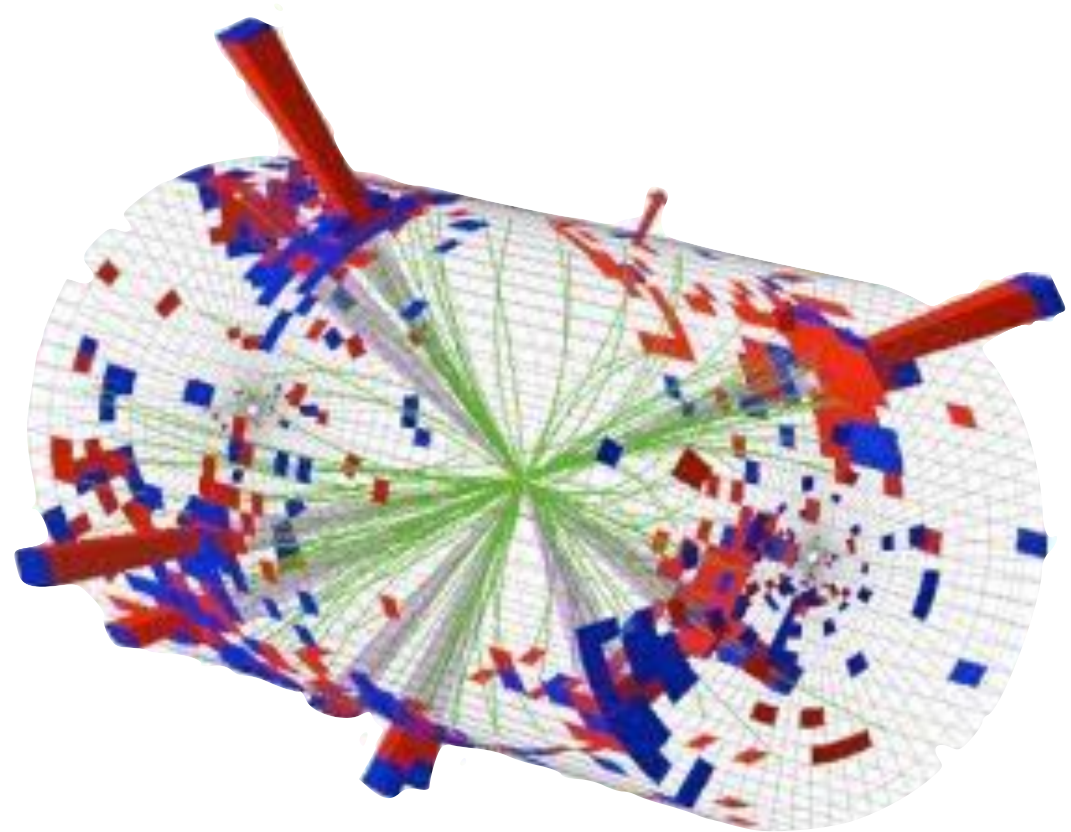
Expansion
in the interaction
strength:
perturbation
theory

Expansion in
scale ratios:
effective field
theories

Numerical
methods:
lattice simulations

Toy models:
solvable models
SUSY theories
AdS/CFT

Jet physics at the LHC



Many scale hierarchies!

$$\sqrt{s} \gg p_{\mathrm{Jet}}^{T} \gg M_{\mathrm{Jet}} \gg E_{\mathrm{out}} \gg m_{\mathrm{proton}} \sim \Lambda_{\mathrm{QCD}}$$

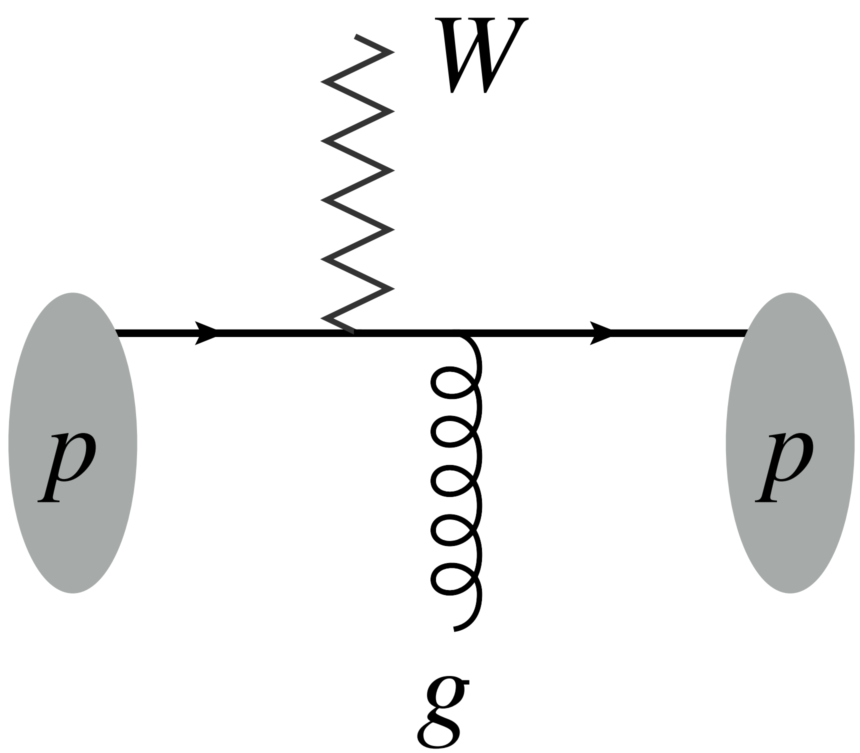
→ Soft-Collinear Effective Theory (SCET)

Bauer, Pirjol, Stewart et al. 2001, 2002; Beneke, Diehl et al. 2002; …
Introduction: TB, Broggio and Ferroglia 2015

Outline

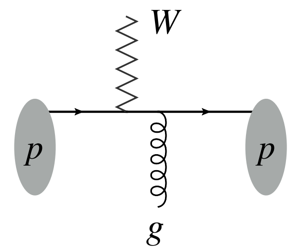
- Introduction
  - Collider physics…
  - … and effective field theory
- Physics of soft gluons
- Recent developments in SCET
  - Resummation for jet processes and non-global logarithms
  - Hadron colliders: Glauber gluons and super-leading logs

Sketch of a hadron collider process



Hard scattering at short distances: perturbation theory

Sketch of a hadron collider process



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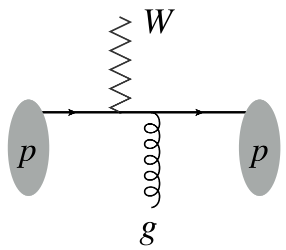

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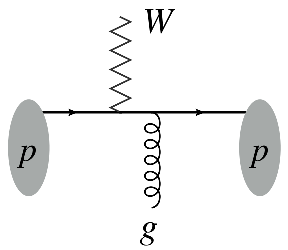

Hard scattering at short distances: perturbation theory

Soft and collinear emissions: parton shower, resummation, SCET

Sketch of a hadron collider process

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Hadronisation, hadron decays: modelled by MCs

Sketch of a hadron collider process

Hard scattering at short distances:
perturbation theory

Soft and collinear emissions:
parton shower, resummation, SCET

Hadronisation, hadron decays:
modelled by MCs

This picture can be misleading: it depends on the observable to which aspect of QCD one is sensitive!

For inclusive observables, sensitive only to a single high-energy scale Q, we have

$$\sigma = \sum_{a,b} \int_0^1 dx_1 dx_2\, \hat{\sigma}_{ab}(Q, x_1, x_2, \mu_f)\, f_a(x_1, \mu_f)\, f_b(x_2, \mu_f) + \mathcal{O}(\Lambda_{\mathrm{QCD}}/Q)$$

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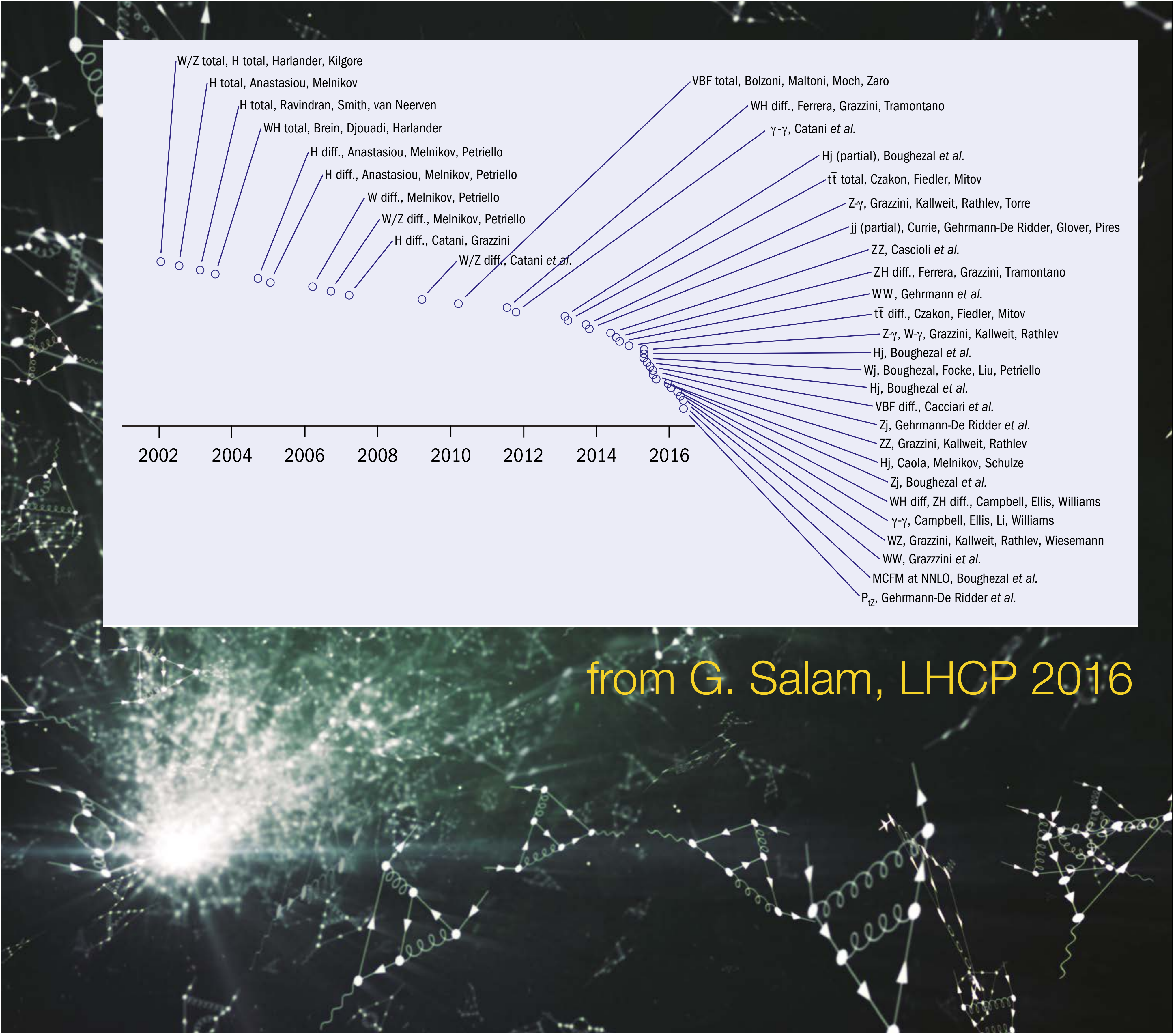
partonic cross sections: perturbation theory

parton distribution functions (PDFs): nonperturbative



Theory

The two-loop explosion

During the past two years there has been a burst of activity in next-to-next-to-leading order calculations to ensure that theory keeps up with the increasing precision of LHC measurements.

Studying matter at the highest energies possible has transformed our understanding of the microscopic world. CERN's Large Hadron Collider (LHC), which generates proton collisions at the highest energy ever produced in a laboratory (13 TeV), provides a controlled environment in which to search for new phenomena and to address fundamental questions about the nature of the interactions between elementary particles. Specifically, the LHC's main detectors – ATLAS, CMS, LHCb and ALICE – allow us to measure the cross-sections of elementary processes with remarkable precision. A great challenge for theorists is to match the experimental precision with accurate theoretical predictions. This is necessary to establish the Higgs sector of the Standard Model of particle physics and to look for deviations that could signal the existence of new particles or forces. Pushing our current capabilities further is key to the success of the LHC physics programme.

Underpinning the prediction of LHC observables at the highest levels of precision are perturbative computations of cross-sections. Perturbative calculations have been carried out since the early days of quantum electrodynamics (QED) in the 1940s. Here, the smallness of the QED coupling constant is exploited to allow the expressions for physical quantities to be expanded in terms of the coupling constant – giving rise to a series of terms with decreasing magnitude. The first example of such a calculation was the one-loop QED correction to the magnetic moment of the electron, which was carried out by Schwinger in 1948. It demonstrated for the first time that QED was in agreement with the experimental discovery of the anomalous magnetic moment of the electron, g_e-2 (the latter quantity was dubbed "anomalous" precisely because, prior to Schwinger's calculation, it did not agree with predictions from Dirac's theory). In 1957, Sommerfeld and Petermann computed the two-loop correction, and it ▷

Next-to-next-to-leading order (NNLO) Feynman diagrams relevant to the LHC physics programme. (Image credit: Daniel Dominguez, CERN.)

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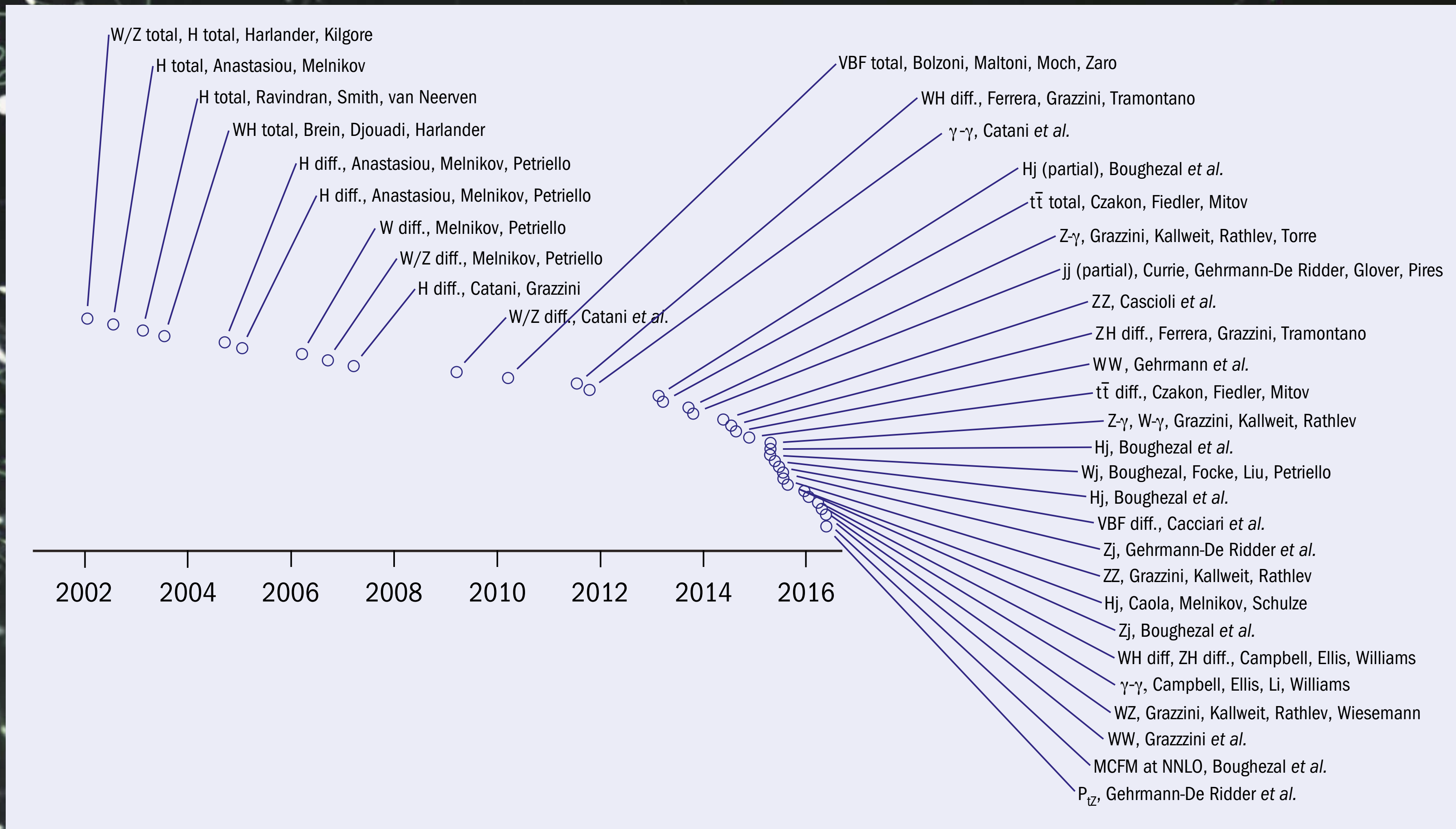
by G. Zanderighi, 2017



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... n diagrams ... credit: Daniel

Note: Many computations based on effective field theory (q_T and N-jettiness subtractions):

$$\text{NNLO (QCD)} \approx \text{NNLO (SCET)} + \text{NLO (QCD)}$$

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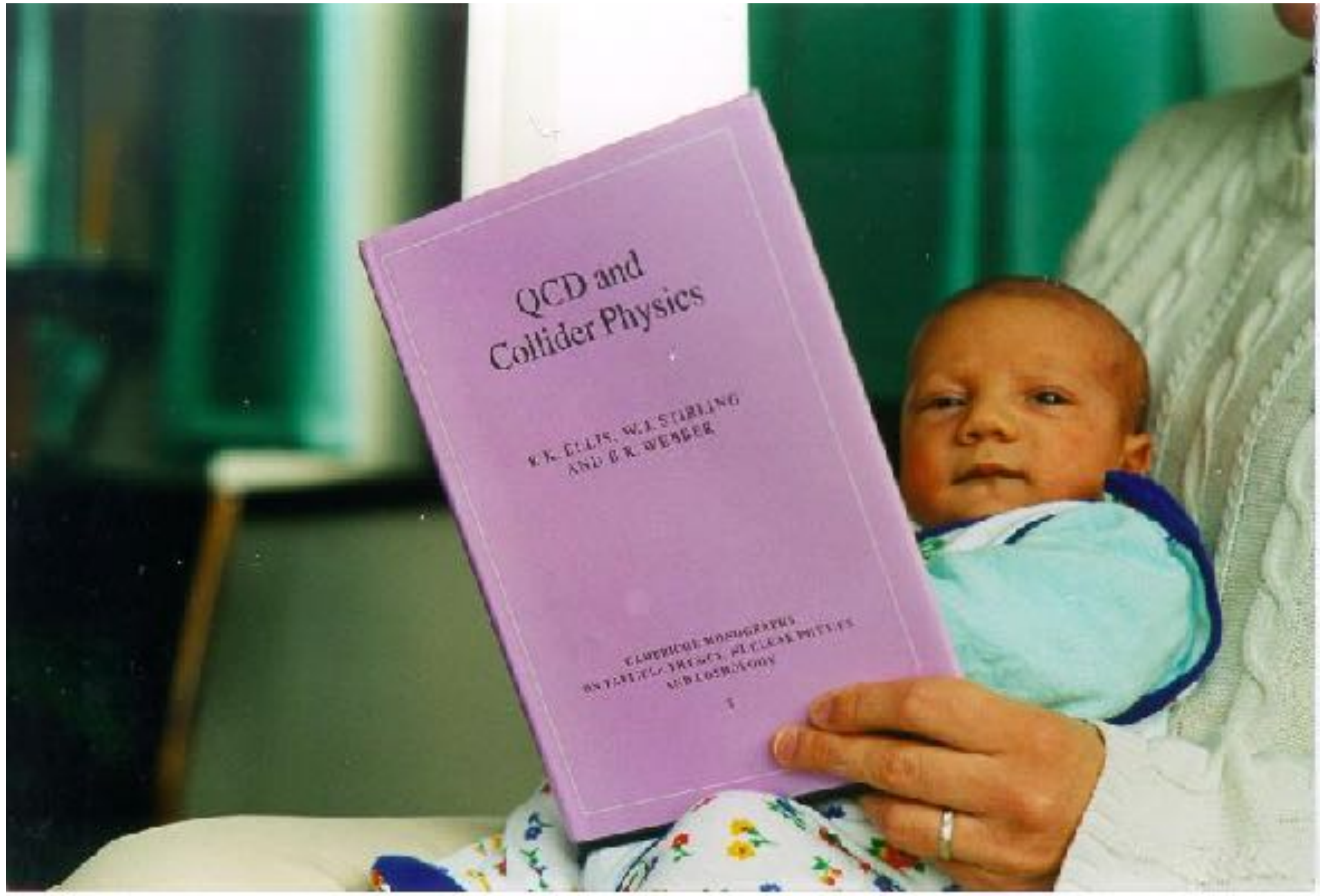

$$\sigma = \sum_{a,b} \int_0^1 dx_1 dx_2 \, \hat{\sigma}_{ab}(Q, x_1, x_2, \mu_f) \, f_a(x_1, \mu_f) \, f_b(x_2, \mu_f) + \mathcal{O}(\Lambda_{\mathrm{QCD}}/Q)$$

The right way to look at this formula is (soft-collinear) effective field theory

$$\sigma = \sum_{a,b} \int_0^1 dx_1 dx_2\, C_{ab}(Q, x_1, x_2, \mu) \langle P(p_1)|O_a(x_1)|P(p_1)\rangle\, \langle P(p_2)|O_b(x_2)|P(p_2)\rangle + \mathcal{O}(\Lambda_{\mathrm{QCD}}/Q)$$

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Wilson coefficient:
matching at $\mu \approx Q$
perturbation theory

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RG-evolution

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low-energy proton
matrix elements
nonperturbative

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Wilson coefficient:
matching at $\mu \approx Q$
perturbation theory

RG-evolution

low-energy proton matrix elements
nonperturbative

power suppressed operators

The matching coefficient C_{ab} is independent of external states and insensitive to physics below the matching scale μ.

Can use quark and gluon states to perform the matching.

- Trivial matrix elements

$$\langle q_{a'}(x'p)|O_a(x)|q_{a'}(x'\,p)\rangle = \delta_{aa'}\,\delta(x'-x)$$

- Wilson coefficients are partonic cross section

$$C_{ab}(Q,x_1,x_2) = \hat{\sigma}_{ab}(Q,x_1,x_2)$$

- Bare Wilson coefficients have divergencies. Renormalization induces dependence on μ.

If disparate hard scales are present, one encounters large logarithms in the matching coefficient.

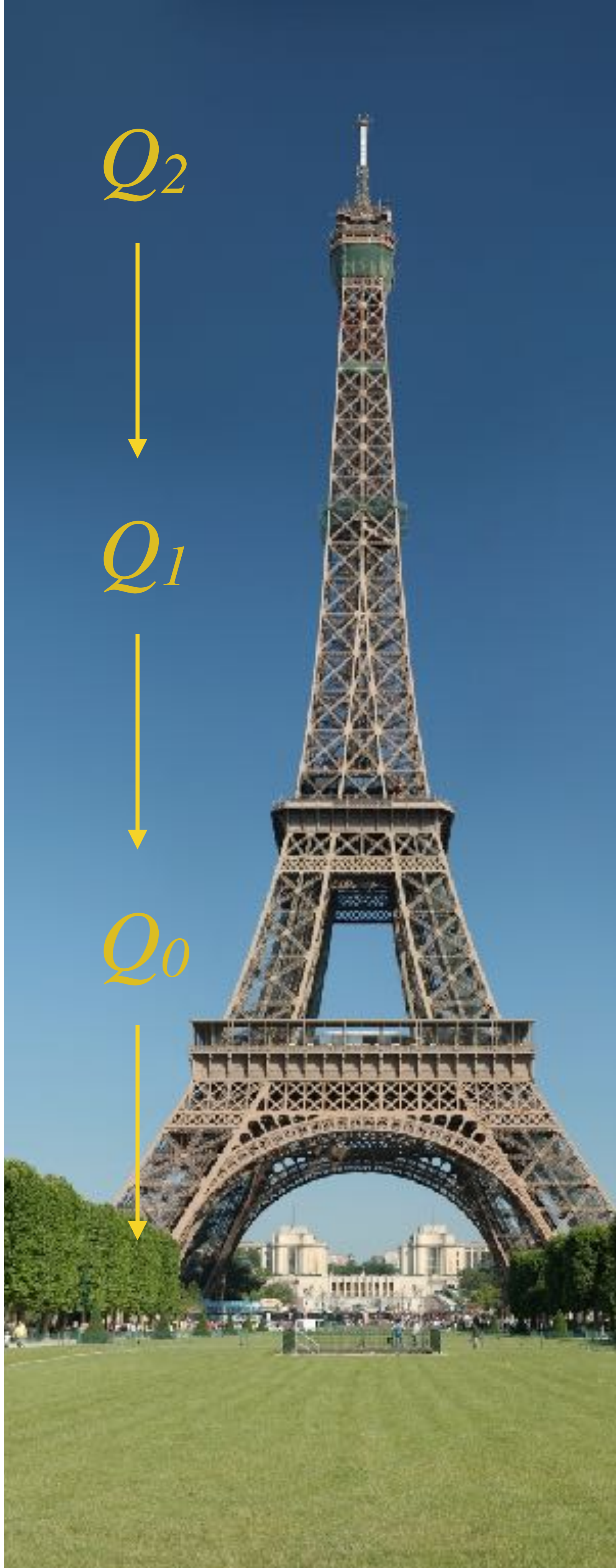
- Can spoil convergence of perturbation theory

Solution: use a tower of effective theories. Integrate out the contributions at the different scales, one after another.

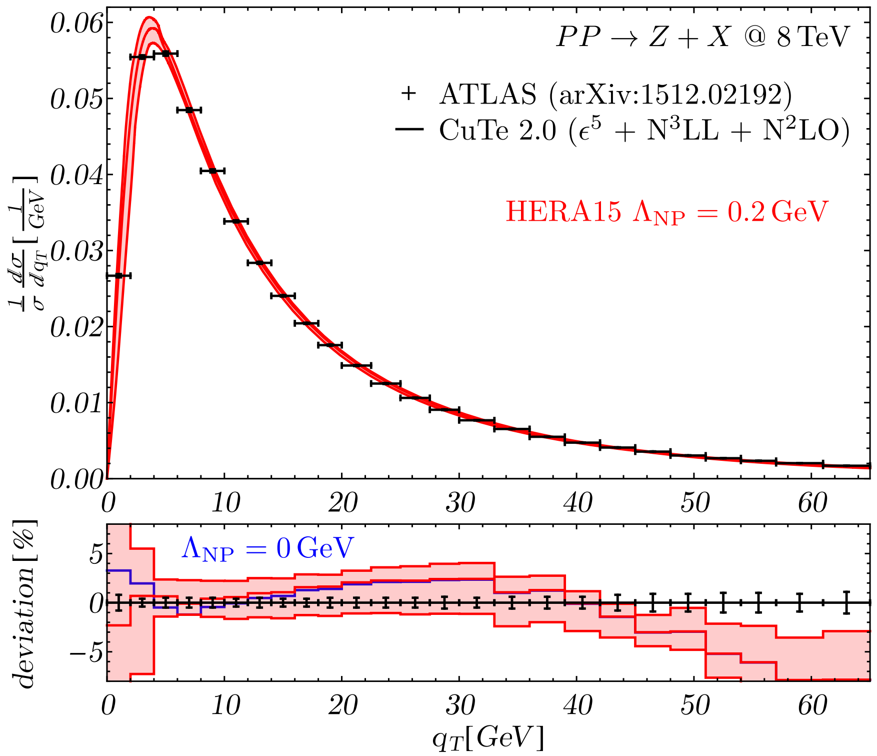
- Resummation by RG evolution

Challenges

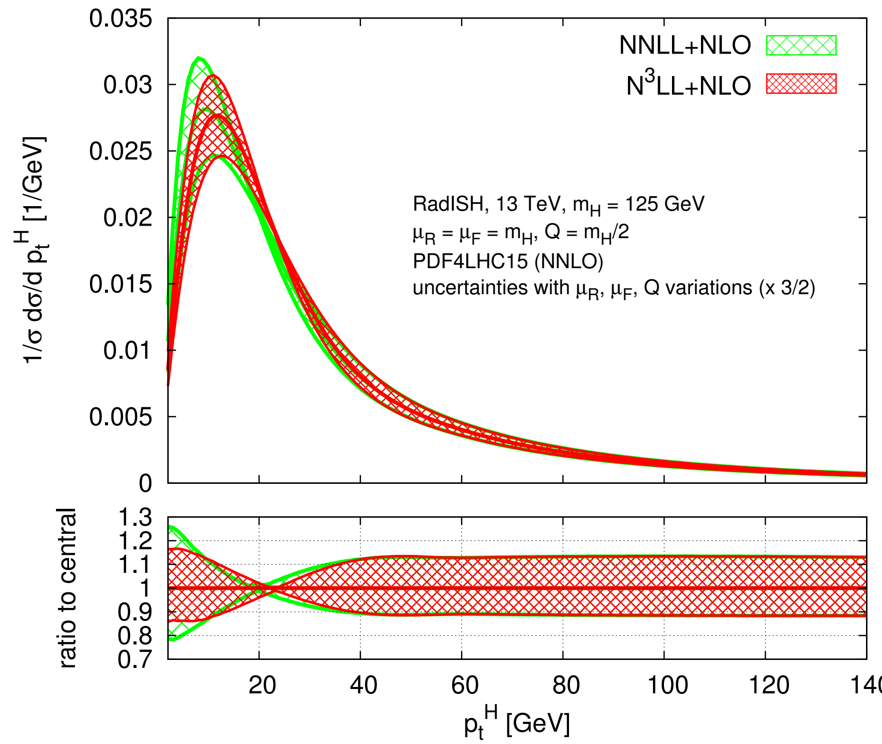
- Need the EFT relevant for the given kinematics. By now, we know how to handle many kinematic situations.
- Need to compute and match the results in different hierarchies, e.g. for $Q_1 \to Q_2$



Example: Z and H production $q_T \ll M_{Z,H}$



CuTe 2.0 TB, Lübbert, Neubert, Wilhelm



Bizon, Monni, Re, Rottoli and Torrielli, 1705.09127

- *Cannot* use fixed-order computation in peak region, but need to match onto fixed order at larger q_T.
- NNNLL resummation using three-loop anomalous dimensions Li, Zhu '16; Vladimirov, '16 obtained from three-loop computations of soft-gluon matrix element.
- Experimental precision for Z is fantastic! Non-perturbative effects?

Relation of EFT to MC parton showers?

A crucial tool for detailed simulation / modeling of collider processes

- based on soft and collinear limit of amplitudes
- evolution parameter, similar to RG scale
- resum *some* large logarithms
- exclusive events, by adding successive emissions (branchings)
- so far restricted to large N_c limit. No interference, no phases, …

Improved parton showers?

- A lot of progress in matching showers to fixed order: automated matching at NLO, some results at NNLO, some based on SCET (GENEVA by Alioli et al.)
- Recent papers incorporate *some* higher-order effects into the shower Nagy, Soper ’16; Hoeche, Prestel, + Krauss '17

But theoretical underpinnings of shower resummation are murky

- QFT derivation? Work in this direction Bauer, Schwartz ’07; Nagy, Soper ’07-‘17
- For which classes of observables does one obtain *full* LL or NLL? Resummation of subleading logs? Study using CAESAR: Hoeche, Reichelt, Siegert, ‘17

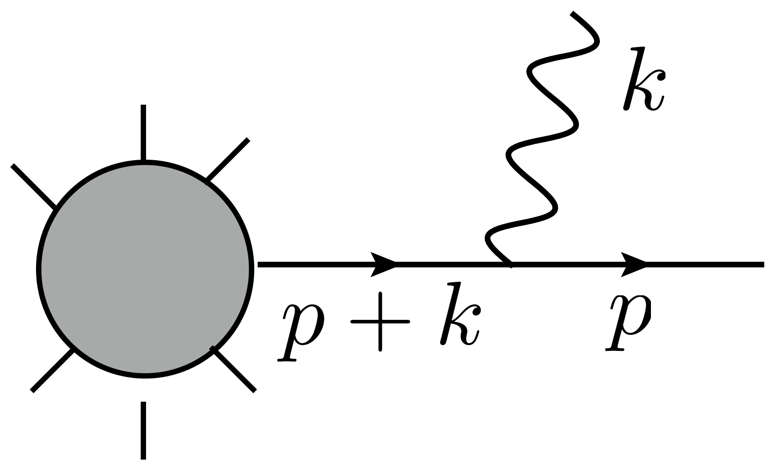
I will show that for a certain class of observables we can derive parton shower equation from RG evolution in EFT.

- Operator framework: clear what is needed for NLL accuracy.

The physics of soft gluons

Soft limit

When particles with small energy and momentum are emitted, the amplitudes greatly simplify:



$$\bar{u}(p)\not{\varepsilon}(k,\lambda)\frac{\not{p}+\not{k}+m}{(p+k)^2-m^2}\cdots$$

$$\approx \boxed{\frac{p\cdot\varepsilon(k,\lambda)}{p\cdot k}}\bar{u}(p)\ldots$$

Soft emission

- factors from the rest of the amplitude.
- only depends on the direction p^{μ}/E
- sees charge, but not spin of emitting particle

Wilson lines

Multiple emissions can be obtained from

$$S_i = \mathbf{P} \exp\left[ig \int_0^\infty ds\, n_i \cdot A_s^a(sn_i) \boldsymbol{T}_i^a\right]$$

$n_i^\mu = p_i^\mu / E$ is a vector in the direction of the energetic particle, and $\boldsymbol{T}_i^a$ is its color charge. $\mathbf{P}$ indicates that the color matrices are path ordered.

Wilson line can be obtained by considering a pointlike classical source moving along the line $x^\mu = sn^\mu$

$0 \qquad\qquad x^\mu = sn^\mu \qquad\qquad \infty$

Soft emissions in process with m energetic particles are obtained from the matrix elements of the operator

$$\boldsymbol{S}_1(n_1)\,\boldsymbol{S}_2(n_2)\,\ldots\,\boldsymbol{S}_m(n_m)|\mathcal{M}_m(\{\underline{p}\})\rangle$$

soft Wilson lines along the directions of the energetic particles / jets (color matrices)

hard scattering amplitude with *m* particles (vector in color space)

To get the amplitudes with additional soft partons, one takes the matrix element of the multi-Wilson-line operators:

$$\langle X_s|\,\boldsymbol{S}_1(n_1)\,\ldots\,\boldsymbol{S}_m(n_m)\,|0\rangle$$

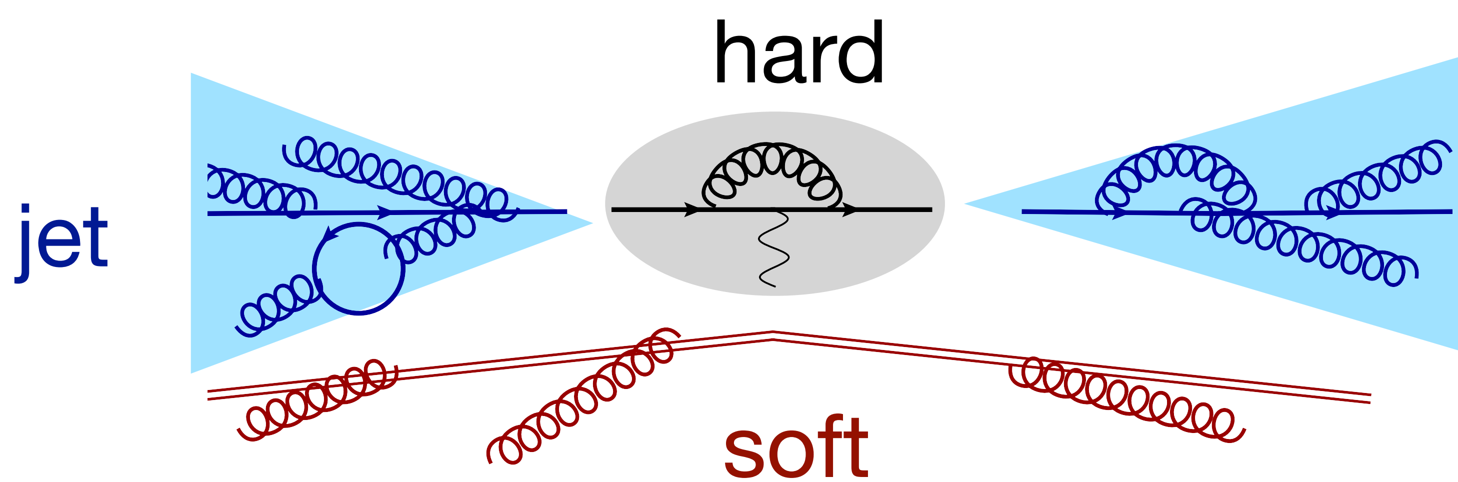
Soft-Collinear Effective Theory (SCET)

Implements interplay between soft and collinear partons on the operator level into an effective field theory

Hard } **high-energy**

Collinear *fields*

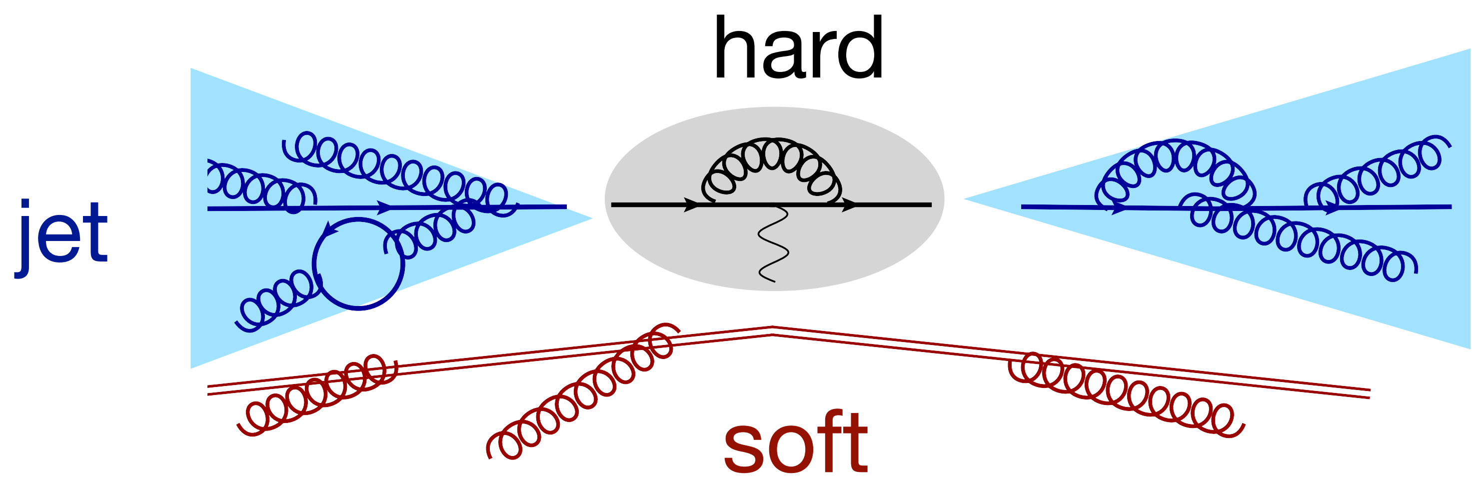
Soft *fields* } **low-energy part**



Factorization of cross sections and perform resummations of large Sudakov logarithms using effective field theory.

Resummation for jet processes: theory of non-global logarithms

The standard factorization

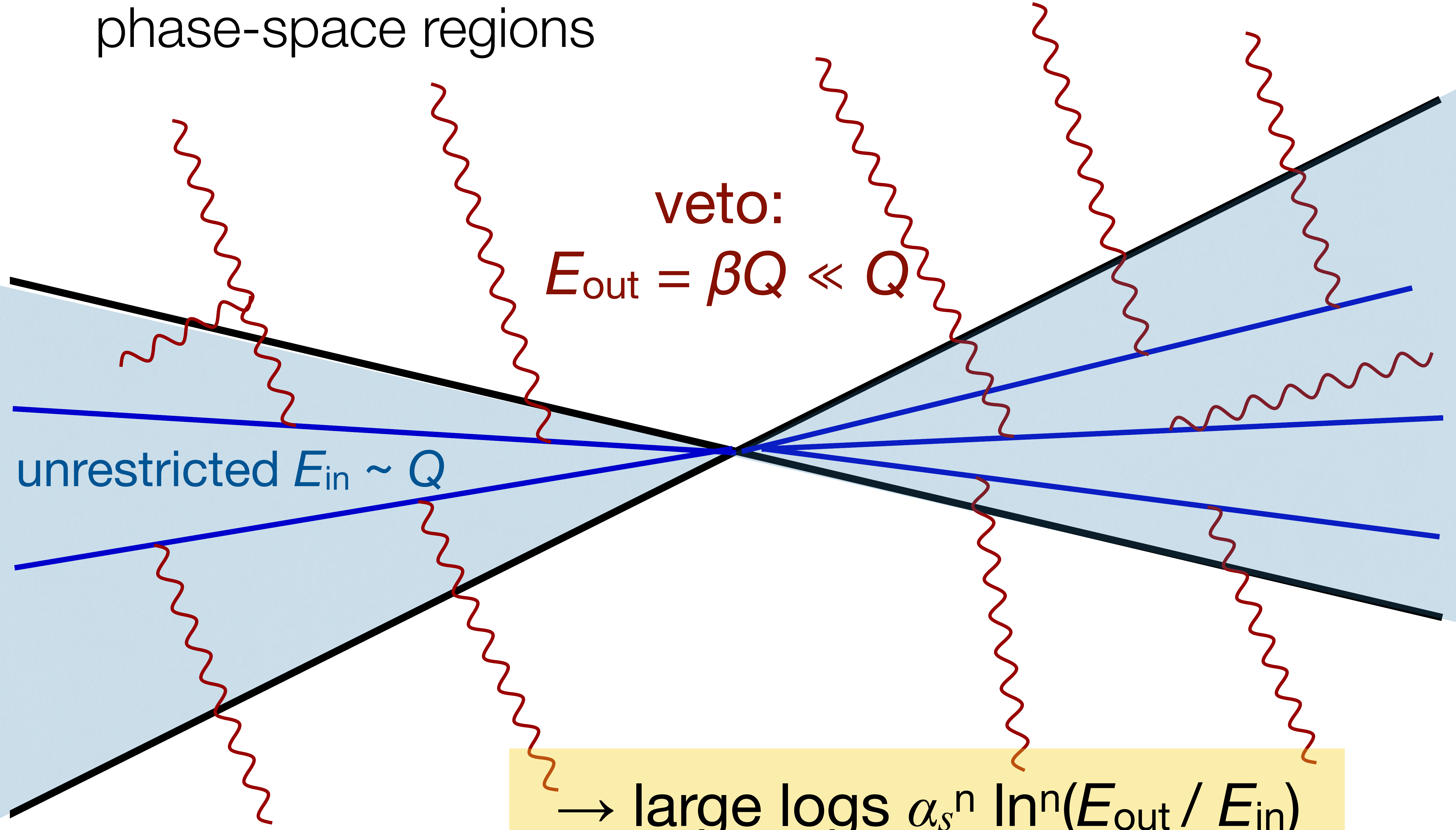


involving a soft function with two Wilson lines applies to many event shape variables

- thrust, total broadening, C-parameter, heavy-jet mass, *N*-jettiness, …

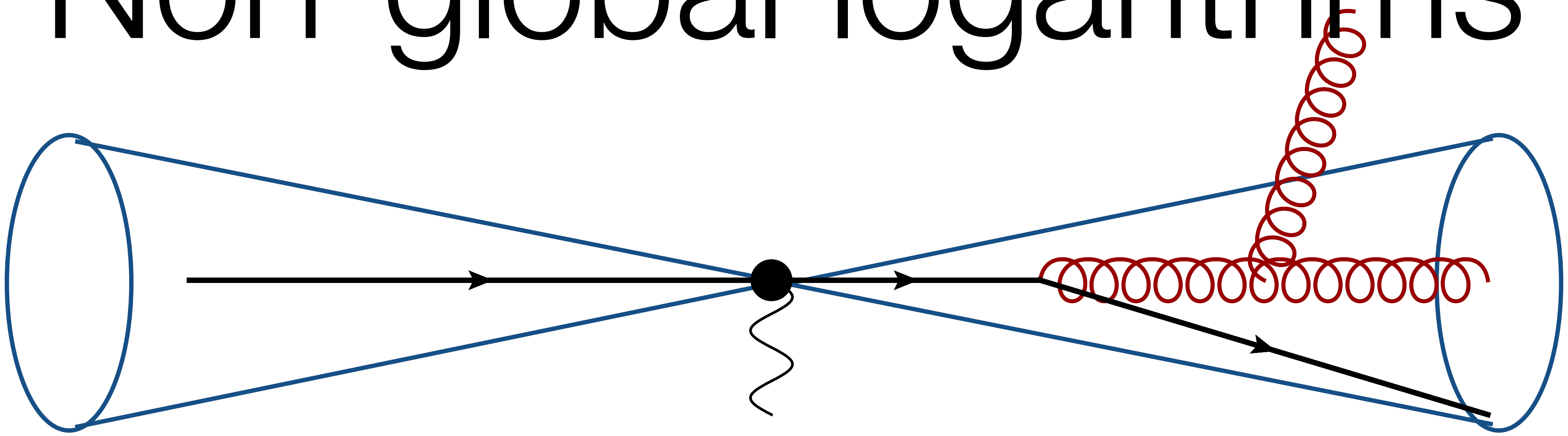
but fails for many others, in particular for all jet observables and other “nonglobal” observables

Characteristic feature of these “non-global” observables is unrestricted radiation in certain phase-space regions



$\longrightarrow$ large logs $\alpha_s^n \ln^n(E_{out} / E_{in})$

Non-global logarithms

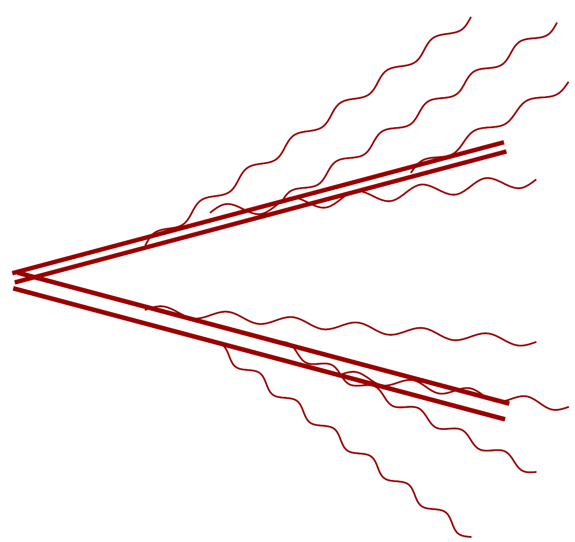


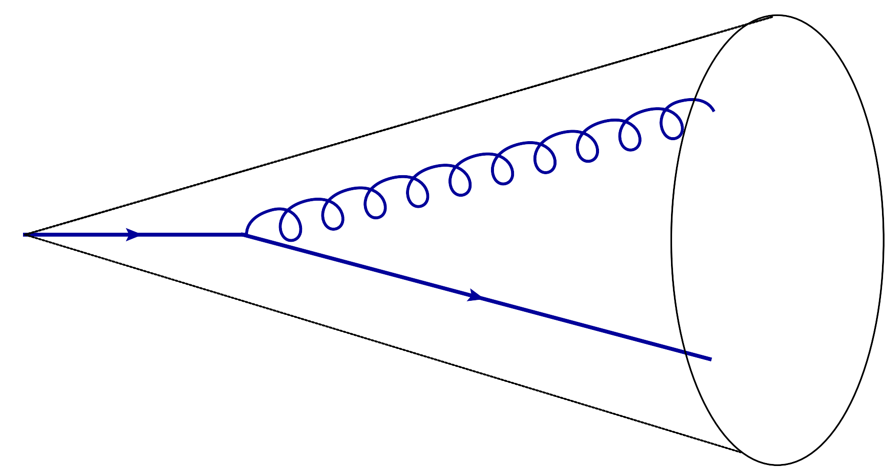
Dasgupta and Salam ’02: soft gluons from emissions inside the jets source lead to complicated pattern of logs $\alpha_s^n \ln^m(\beta)$

- Even leading logarithms do not simply exponentiate!
- At large N_c logs can be obtained with parton shower (Dasgupta and Salam ‘02) or by solving a non-linear integral equation Banfi, Marchesini, Smye ’02.
- Also some finite N_c results Hatta and Ueda ’13 + Hagiwara, ‘15 based on Weigert ‘03

Soft emissions in process with m energetic particles are obtained from the matrix elements of the operator

$$\boldsymbol{S}_1(n_1)\,\boldsymbol{S}_2(n_2)\;\ldots\;\boldsymbol{S}_m(n_m)|\mathcal{M}_m(\{\underline{p}\})\rangle$$





soft Wilson lines along the directions of the energetic particles / jets (color matrices)

soft particles can be inside or outside

hard scattering amplitude with *m* particles (vector in color space)

energetic partons must be inside

For a jet of several (nearly) collinear energetic particles, one can combine

$$\boldsymbol{S}_1(n)\,\boldsymbol{S}_2(n) = \mathbf{P}\exp\left(ig_s\int_0^\infty ds\, n\cdot A_s^a(sn)\,(\boldsymbol{T}_1^a+\boldsymbol{T}_2^a)\right)$$

into a single Wilson line with the total color charge.

For non-global observables one cannot combine the soft Wilson lines → complicated structure of logs!

- For a wide-angle jet, the energetic particles are not collinear.

- For a narrow-angle jets, we find that small-angle soft radiation plays an important role. Resolves directions of individual energetic partons!

Factorization theorem

TB, Neubert, Rothen, Shao ’15 ’16, see also Caron-Huot ‘15

Hard function.
m hard partons along
fixed directions $\{n_1, \ldots, n_m\}$

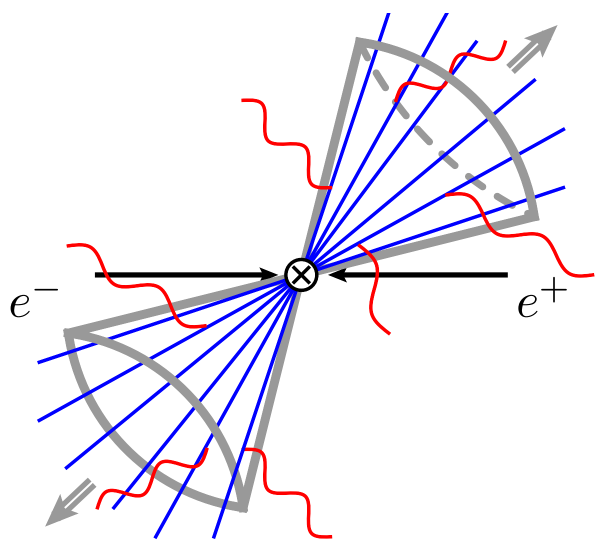
Soft function
with *m* Wilson lines

$$\sigma(\beta) = \sum_{m=2}^{\infty} \big\langle \boldsymbol{\mathcal{H}}_m(\{\underline{n}\}, Q, \mu) \otimes \boldsymbol{\mathcal{S}}_m(\{\underline{n}\}, Q\beta, \mu) \big\rangle \, ;$$

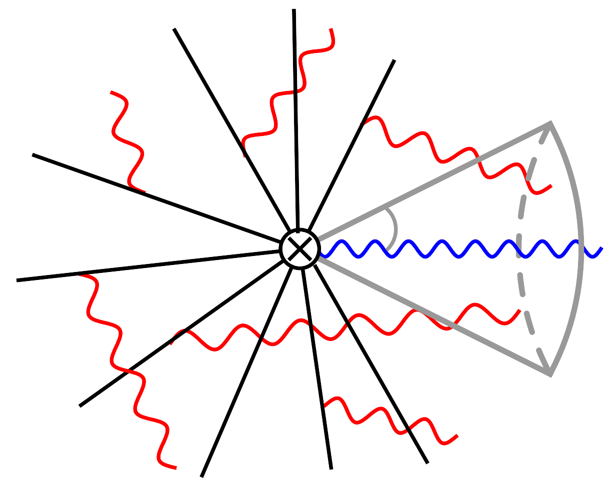
color trace

integration over the *m* directions

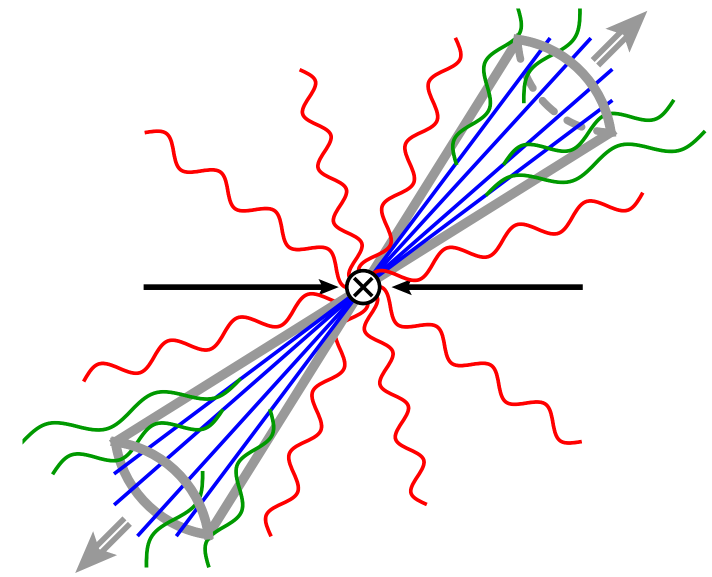
First all-order factorization theorem for non-global observable. Achieves full scale separation!



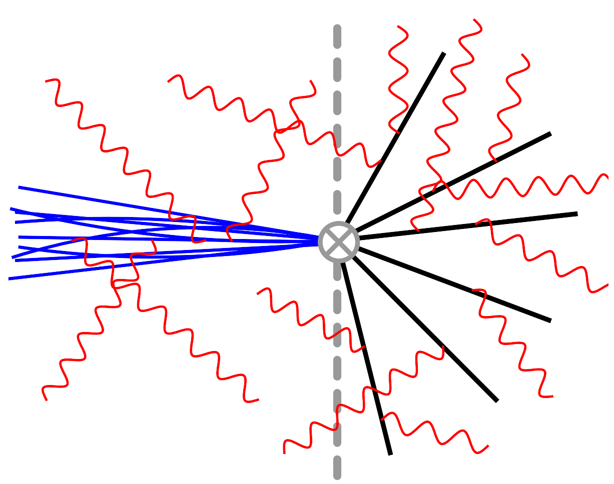
1.) 2.) cone jets,
gap between jets



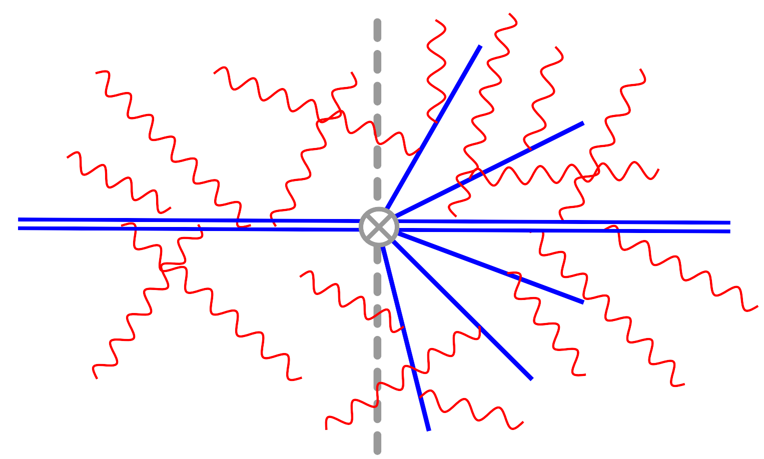
5.) isolation cones



1.) narrow cone jets



3.) light-jet mass
4.) narrow broadening



3.) hemisphere
soft function

1.) 2.) TB, Neubert, Rothen, Shao ’15 ’16

3.) TB, Pecjak, Shao ’16

4.) TB, Rahn, Shao ’17

5.) Balsiger, TB, Shao, in prep

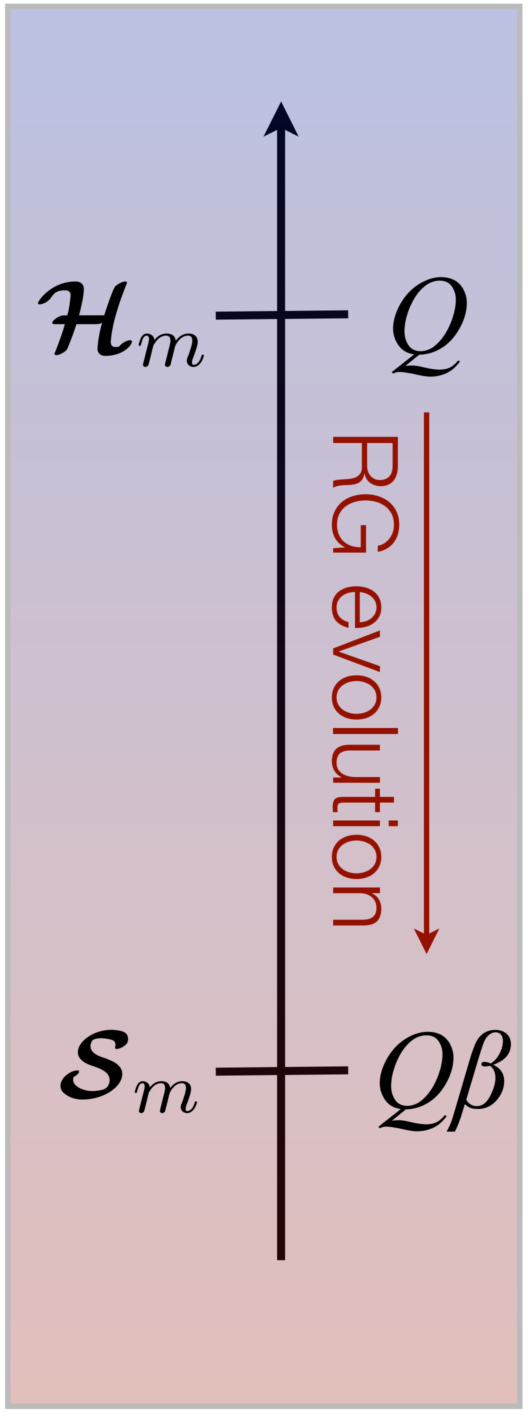
Resummation by RG evolution

Wilson coefficients fulfill renormalization group (RG) equations

$$\frac{d}{d\ln\mu}\boldsymbol{\mathcal{H}}_m(Q,\mu) = -\sum_{l=2}^{m}\boldsymbol{\mathcal{H}}_l(Q,\mu)\,\boldsymbol{\Gamma}^H_{lm}(Q,\mu)$$

1. Compute $\mathcal{H}_\mathrm{m}$ at a characteristic high scale $\mu_h \sim Q$
2. Evolve $\mathcal{H}_\mathrm{m}$ to the scale of low energy physics $\mu_l \sim Q\beta$

Avoids large logarithms $\alpha_s{}^n \ln^n(\beta)$ of scale ratios which can spoil convergence of perturbation theory.



RG = Parton Shower

- Ingredients for LL

$$\boldsymbol{\mathcal{H}}_2(\mu = Q) = \sigma_0$$
$$\boldsymbol{\mathcal{H}}_m(\mu = Q) = 0 \text{ for } m > 2$$
$$\boldsymbol{\mathcal{S}}_m(\mu = \beta Q) = 1$$

$$\boldsymbol{\Gamma}^{(1)} = \begin{pmatrix} \boldsymbol{V}_2 & \boldsymbol{R}_2 & 0 & 0 & \dots \\ 0 & \boldsymbol{V}_3 & \boldsymbol{R}_3 & 0 & \dots \\ 0 & 0 & \boldsymbol{V}_4 & \boldsymbol{R}_4 & \dots \\ 0 & 0 & 0 & \boldsymbol{V}_5 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

- RG

$$\frac{d}{dt}\boldsymbol{\mathcal{H}}_m(t) = \boldsymbol{\mathcal{H}}_m(t)\boldsymbol{V}_m + \boldsymbol{\mathcal{H}}_{m-1}(t)\boldsymbol{R}_{m-1}\,. \qquad t = \int_{\alpha(\mu)}^{\alpha(Q)} \frac{d\alpha}{\beta(\alpha)}\frac{\alpha}{4\pi}$$

- equivalent to parton shower equation

$$\boldsymbol{\mathcal{H}}_m(t) = \boldsymbol{\mathcal{H}}_m(t_1)e^{(t-t_1)V_n} + \int_{t_1}^{t} dt'\boldsymbol{\mathcal{H}}_{m-1}(t')\boldsymbol{R}_{m-1}e^{(t-t')\boldsymbol{V}_n}$$

1-loop anomalous dimension

$$\boldsymbol{V}_m = 2\sum_{(ij)}\int \frac{d\Omega(n_k)}{4\pi}\left(\boldsymbol{T}_{i,L}\cdot\boldsymbol{T}_{j,L}+\boldsymbol{T}_{i,R}\cdot\boldsymbol{T}_{j,R}\right)W_{ij}^{k}$$

$$-\,2\,i\pi\sum_{(ij)}\left(\boldsymbol{T}_{i,L}\cdot\boldsymbol{T}_{j,L}-\boldsymbol{T}_{i,R}\cdot\boldsymbol{T}_{j,R}\right)\Pi_{ij}$$

$$\boldsymbol{R}_m = -\,4\sum_{(ij)}\boldsymbol{T}_{i,L}\cdot\boldsymbol{T}_{j,R}\,W_{ij}^{m+1}\,\Theta_{\rm in}(n_{m+1})$$

$$\boldsymbol{\mathcal{H}}_m \propto |\mathcal{M}_m\rangle\langle\mathcal{M}_m|$$

$\boldsymbol{T}_{i,L}$: acts on $|\mathcal{M}_m\rangle$

$\boldsymbol{T}_{i,R}$: acts on $\langle\mathcal{M}_m|$

Glauber phase, see later!

- Dipoles $\rightarrow$ dipole shower

$$W_{ij}^{k} = \frac{n_i\cdot n_j}{n_i\cdot n_k\, n_j\cdot n_k}$$

product of two eikonal factors

- Trivial color structure at large N_c :

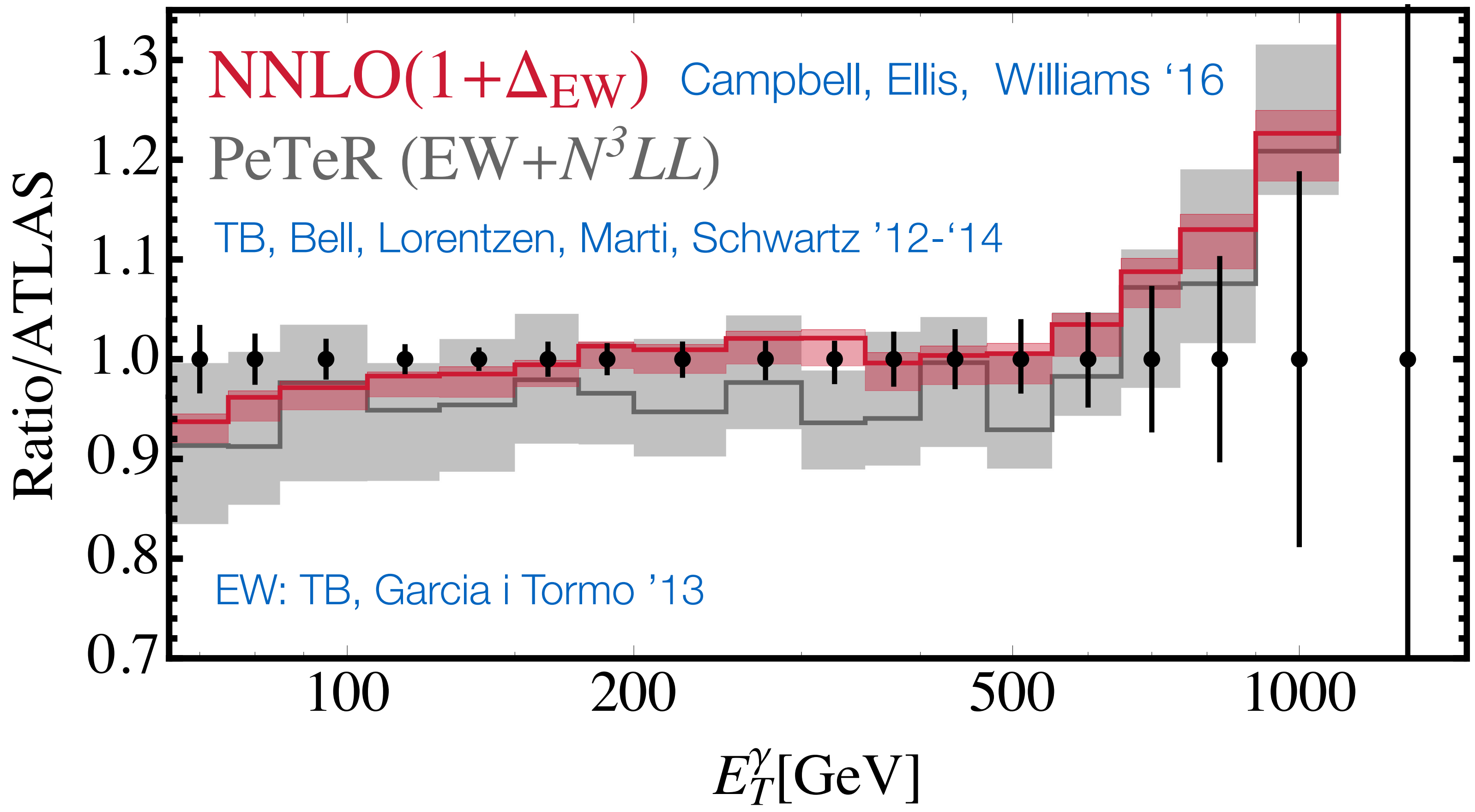
$$\boldsymbol{T}_i\cdot\boldsymbol{T}_j \rightarrow -\frac{N_c}{2}\,\delta_{j,i\pm 1}$$

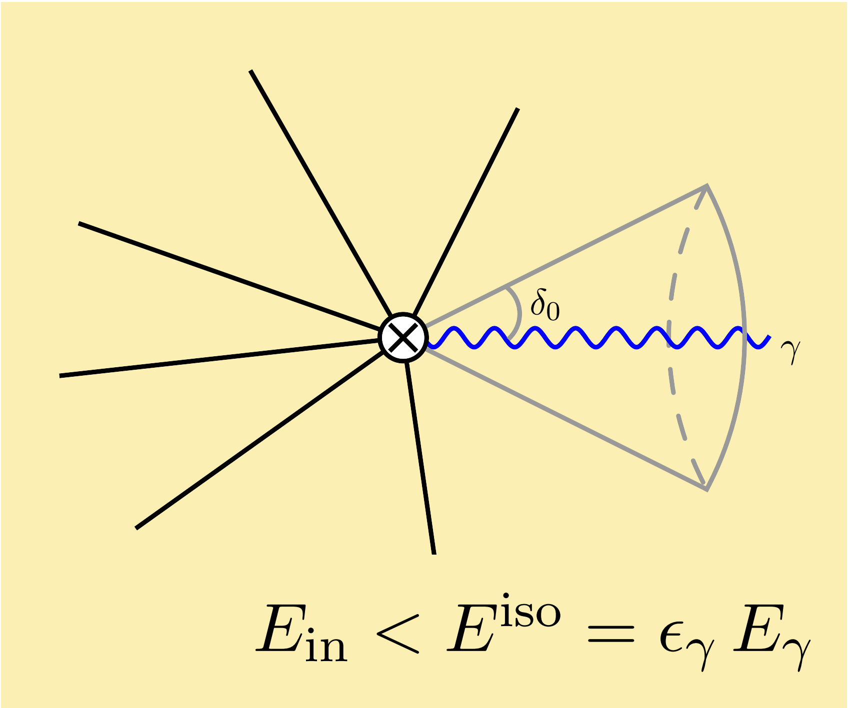
Implementation of leading-log resummation

Balsiger, TB, Shao, in prep

- Use Madgraph5 tree-level generator
  - event file with directions and large-N_c color connections of hard partons
  - provides lowest multiplicity hard function for given process
- Run our shower on each event to generate additional partons
  - obtain reduction of cross section in the presence of of veto on radiation, write result back into event file
- Analyze events, according to cuts on hard partons, obtain resummed cross section for non-global observable as a function of hard cuts and veto scale

Example: isolation cone in γ production

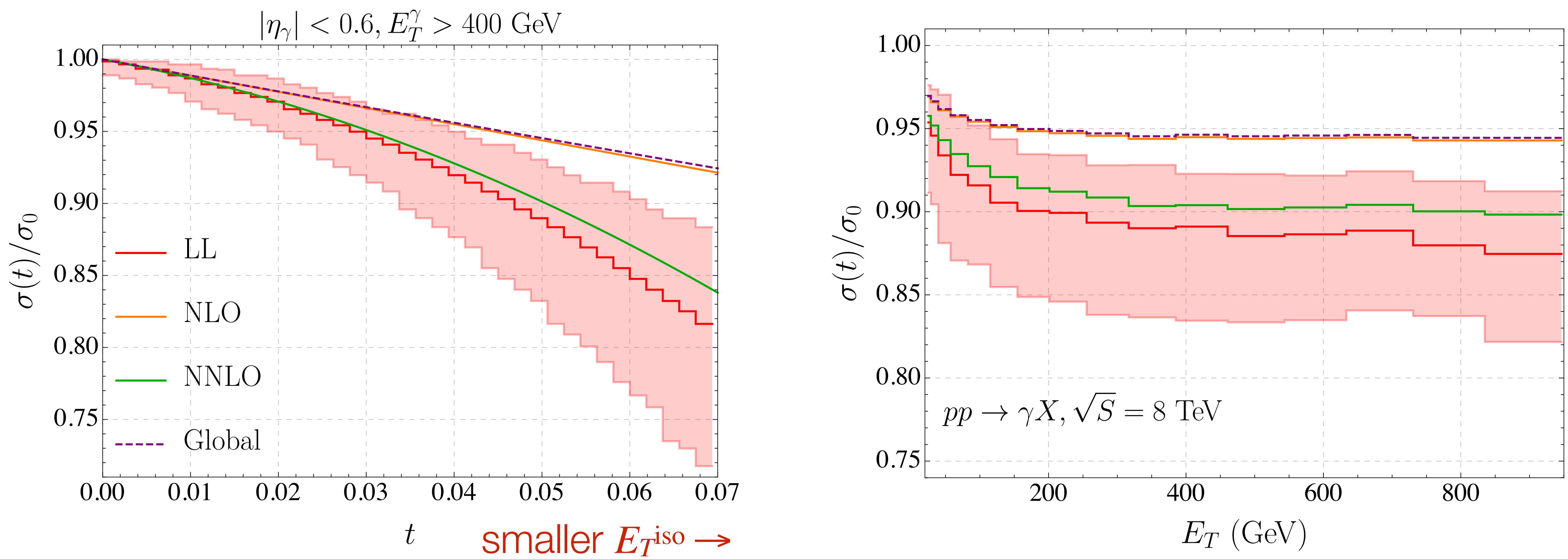




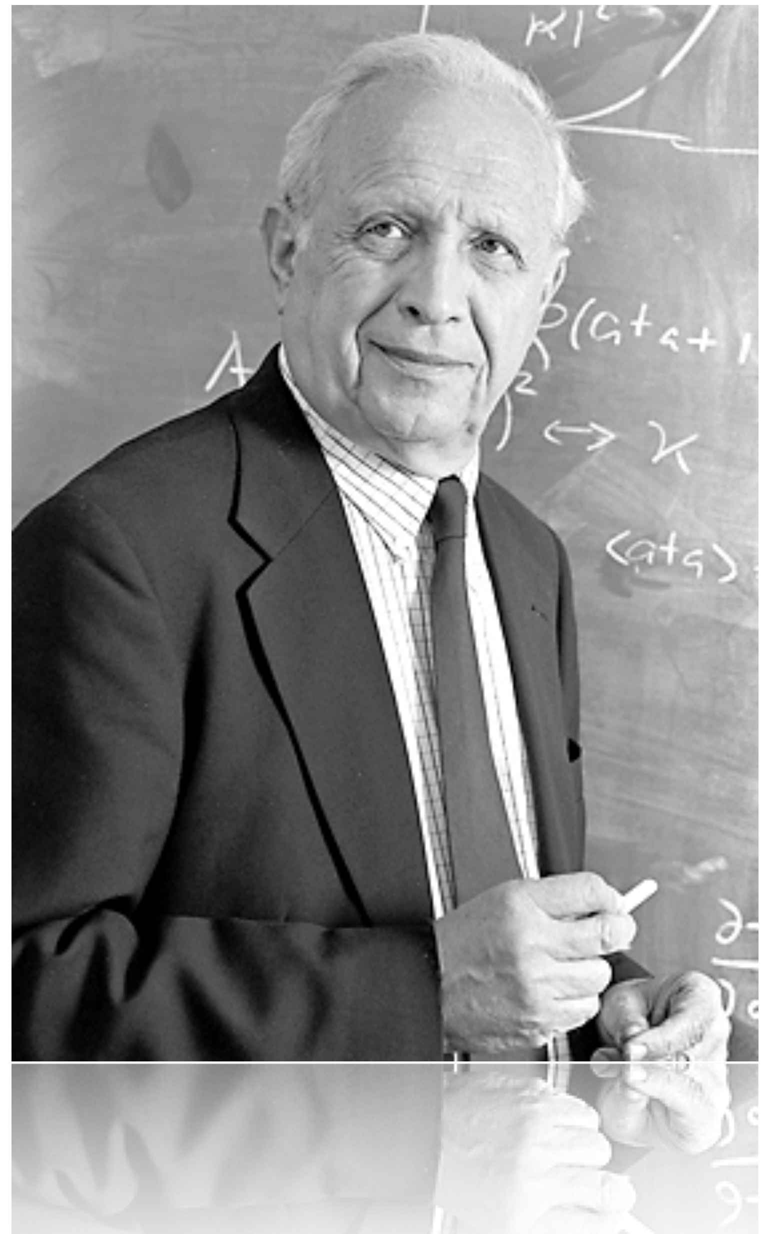
- Experiments use isolation cone to separate photon from hard scattering from photons due to hadron decays.
- ATLAS imposes $E^T_{\rm iso} \approx 5\,\mathrm{GeV}$ on hadronic energy in cone.
- Large logs of $\epsilon_\gamma = E^T_\gamma / E^T_{\rm iso}$ but suppressed by the angular size $R = 0.4$ of the isolation cone.
- Scaling: $R \times \alpha_s^n \, \ln^n \epsilon_\gamma \, \ln^{n-1} R$ see Hatta et al. 1710.06722

Effect of γ isolation at LHC

Balsiger, TB, Shao, in prep.

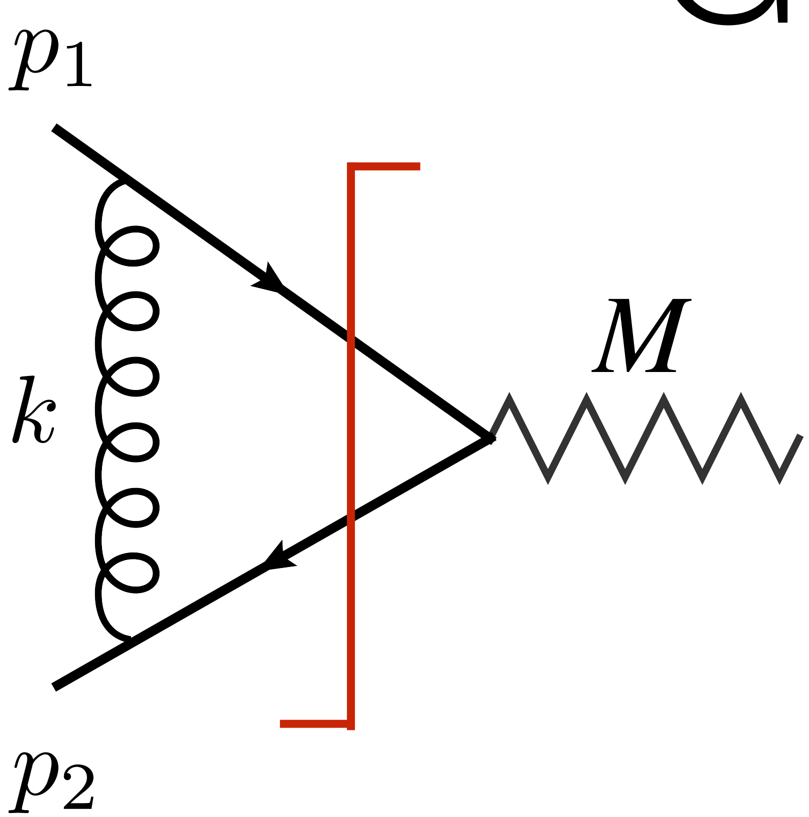


- Value of evolution parameter for ATLAS isolation t ≈ 0.05
- NLO: ~5% reduction, NNLO ~10%, **resummed** ~ 12%
- NGL dominates over global contribution: naive exponentiation (dashed) not appropriate!



Glauber Gluons & Superleading Logs

Glauber phase



s+c divs.

$$
= \frac{\alpha_s(\mu)}{4\pi} C_F \left(-\frac{2}{\varepsilon^2} - \frac{3}{\varepsilon} + \ldots \right) \left(\frac{-M^2}{\mu^2} \right)^{-\varepsilon}
$$

$$
= \frac{\alpha_s(\mu)}{4\pi} C_F \left(-\frac{2}{\varepsilon^2} - \frac{2}{\varepsilon} \left[\ln \frac{M^2}{\mu^2} + i\pi \right] - \frac{3}{\varepsilon} + \ldots \right)
$$

Imaginary part, from a region where gluon is soft and

$$
\begin{aligned}
0 &= (p_1 + k)^2 \approx 2 p_1 \cdot k \\
0 &= (p_2 - k)^2 \approx -2 p_2 \cdot k
\end{aligned}
$$

Glauber region $k^\mu \approx k_\perp^\mu$

Glauber Phase in V_m

$$\mathrm{Im}\left[\boldsymbol{V}_m\right] = -\,2\pi \sum_{(ij)} \left(\boldsymbol{T}_{i,L} \cdot \boldsymbol{T}_{j,L} - \boldsymbol{T}_{i,R} \cdot \boldsymbol{T}_{j,R}\right) \Pi_{ij}$$

$\Pi_{ij}=1$ if (ij) both incoming or outgoing

$\Pi_{ij}=0$ otherwise

Amplitudes conserve color charge $\sum_i \boldsymbol{T}_i = 0$

- If all particles outgoing $\Pi_{ij}=1$ and the sum vanishes. No Glauber phases in e^+e^- !
- But sum is non-zero for $T_1 + T_2 \to T_3 + \ldots + T_m$

Super-leading logarithms

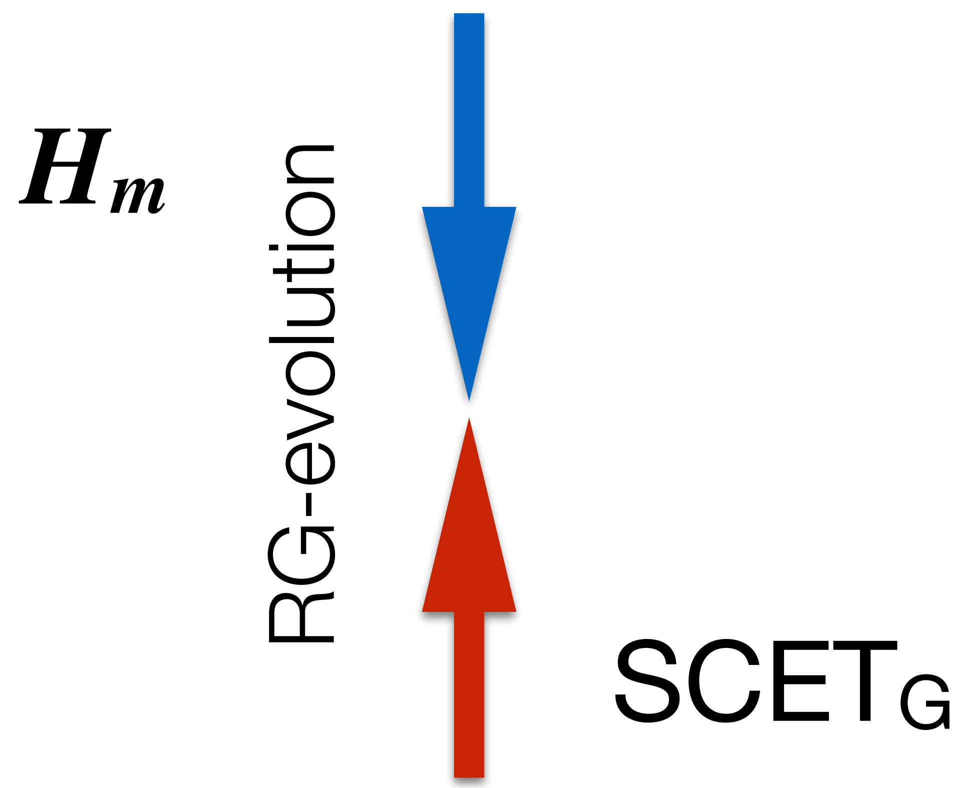
Forshaw, Kyrieleis, Seymour ’06 have analyzed the effect of Glauber phases in non-global observables directly in QCD

- Non-zero contributions starting at 3 loops
- Collinear logarithms starting at 4 loops (in observables, which are single-log in e^+e^-)

We have verified the gap-between-jets results of Keates and Seymour ’09 up to 5 loops by iterating our anomalous dimension and evaluating the color structures order-by-order using ColorMath (Sjodahl ’12)

Effective theory of Glauber gluons

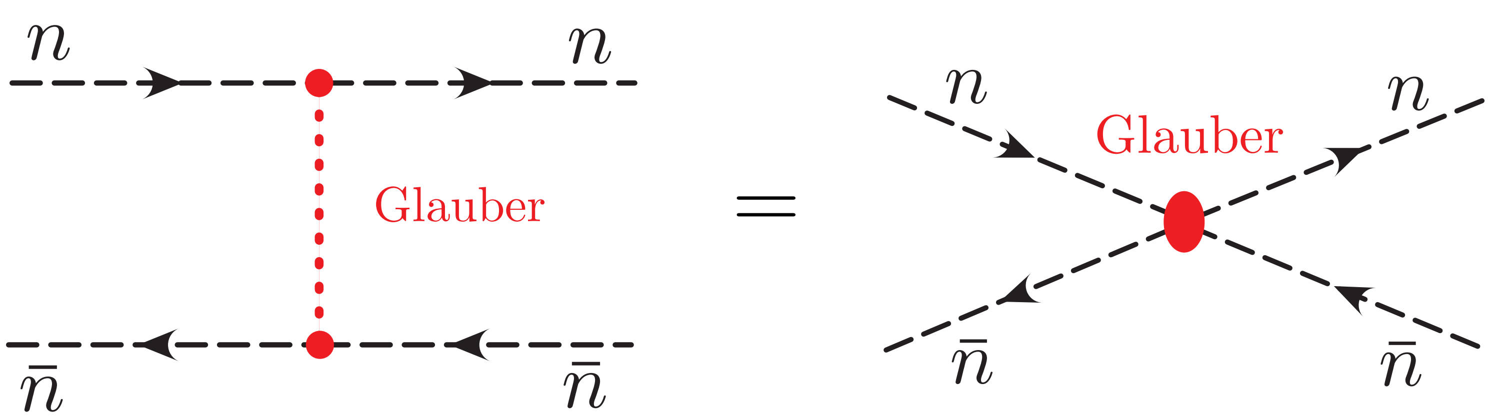
The Glauber effects discussed so far are part of the hard anomalous dimension



RG evolution must match up with low-energy theory: SCET + Glauber gluons

- formulating the EFT proved difficult…

Technical challenges



- Glauber gluons are offshell $k^\mu \approx k_\perp^\mu$
  - $k_T \gg E$, like Coulomb gluons, must be included as potential, not dynamical field in $\mathcal{L}_{\text{eff}}$
- Glauber region is not well defined without additional rapidity regulator (on top of dim.reg.)
  - separation among soft, collinear and Glauber gluons scheme dependent

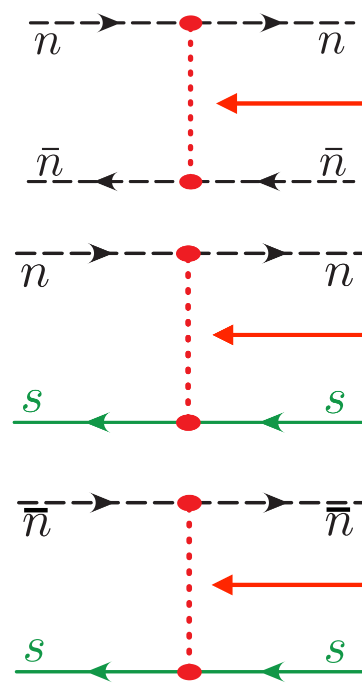
Glauber exchanges

$(n \cdot k, \bar{n} \cdot k, k_\perp)$

n-$\bar{n}$
fwd. scattering

n-s
fwd. scattering

$\bar{n}$-s
fwd. scattering



$p^\mu \sim Q(\lambda^2, \lambda^2, \lambda)$

$p^\mu \sim Q(\lambda^2, \lambda, \lambda)$

$p^\mu \sim Q(\lambda, \lambda^2, \lambda)$

- Exploratory studies by several groups (Liu et al., Idilbi et al, Bauer et al., Donoghue et al., Fleming, …).
- Last year Rothstein and Stewart published an EFT framework for Glauber exchanges [JHEP 1608 (2016) 025 (204pp!)]

Applications

Rothstein and Stewart ’16 mainly focussed on the construction of $\mathscr{L}_{\text{eff}}$, but the framework has many possible application

- Forward scattering, Reggeization (for quarks: Moult, Solon, Stewart, Vita ‘17), BFKL, …
- Collinear factorization violation Schwartz, Yan, Zhu ’17
- PDF factorization of hadron collider cross sections?
    - Collins Soper and Sterman ‘85 have proven this for Drell-Yan; a proof for the general case is still missing.
- Non-global logs at hadron collider; super-leading logs

Conclusions

- A lot of progress to extend Soft-Collinear Effective Theory to more observables. Better understanding of soft physics was key
  - multi-Wilson line operators for non-global observables
  - Glauber gluons for hadronic collisions, forward scattering, …
  - (Finite N_c) + Glauber + nonglobal = superleading logs
- For non-global observables, we obtain a parton shower from effective field theory
  - first-principles derivation of shower, based on RG evolution
  - not restricted to leading logarithms or large N_c
  - not a general purpose shower, but helpful to understand how to extend showers to higher accuracy
  - flexible implementation of LL shower using LHE event files
    - used to study photon isolation, gaps between jets

Diagrammatic Factorization

The simple structure of soft and collinear emissions forms the basis of the classic factorization proofs, which were obtained by analyzing Feynman diagrams.

Collins, Soper, Sterman 80's ...

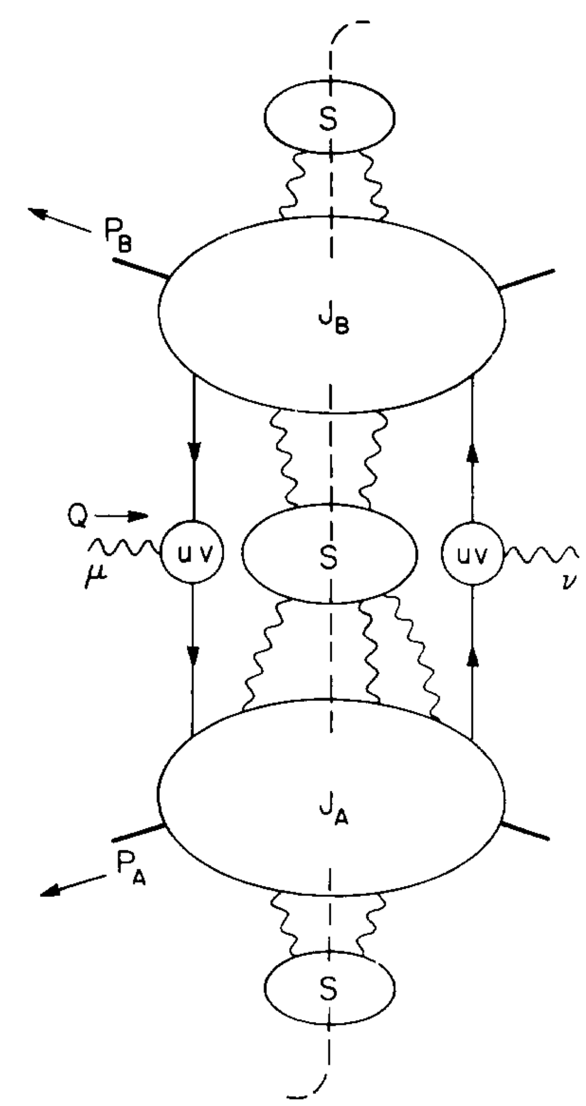
Advantages of the the SCET approach:

Simpler to exploit gauge invariance on the Lagrangian level

Operator definitions for the soft and collinear contributions

Resummation with renormalization group

Can include power corrections



Collins and Soper '81

Im[$\boldsymbol{V}_\mathrm{m}$] for $T_1 + T_2 \to T_3 + \ldots + T_\mathrm{m}$

$$\sum_{(ij)} \boldsymbol{T}_i \cdot \boldsymbol{T}_j \, \Pi_{ij} = 2\,\boldsymbol{T}_1 \cdot \boldsymbol{T}_2 + \sum_{i=3}^{m} \boldsymbol{T}_i \cdot (-\boldsymbol{T}_1 - \boldsymbol{T}_2 - \boldsymbol{T}_i)$$

$$= 2\,\boldsymbol{T}_1 \cdot \boldsymbol{T}_2 + (\boldsymbol{T}_1 + \boldsymbol{T}_2) \cdot (\boldsymbol{T}_1 + \boldsymbol{T}_2) - \sum_{i=3}^{m} C_i^2$$

$$= 4\,\boldsymbol{T}_1 \cdot \boldsymbol{T}_2 + C_1^2 + C_2^2 - \sum_{i=3}^{m} C_i^2$$

constants cancel in $\boldsymbol{T}_{i,L} \cdot \boldsymbol{T}_{j,L} - \boldsymbol{T}_{i,R} \cdot \boldsymbol{T}_{j,R}$

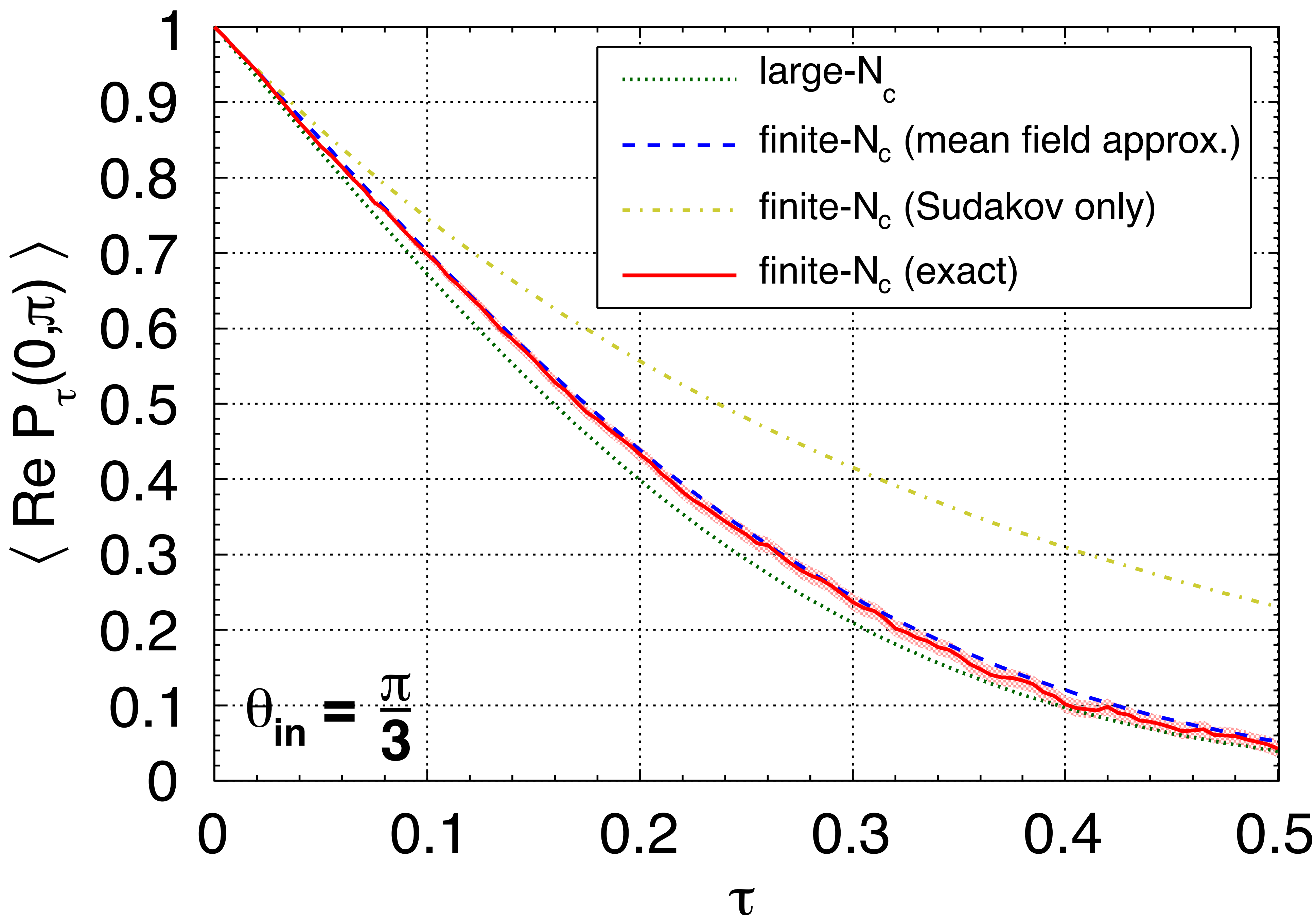
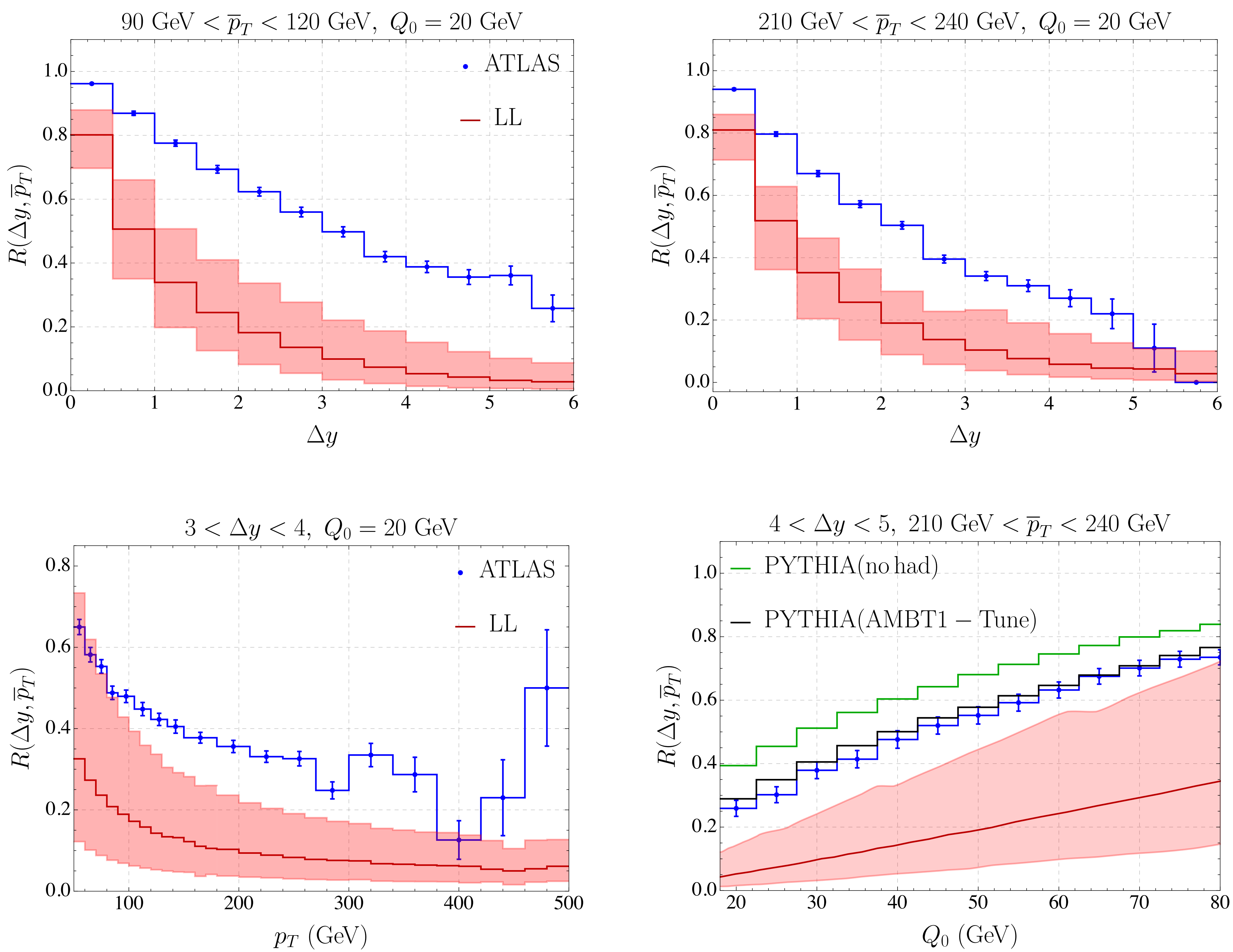
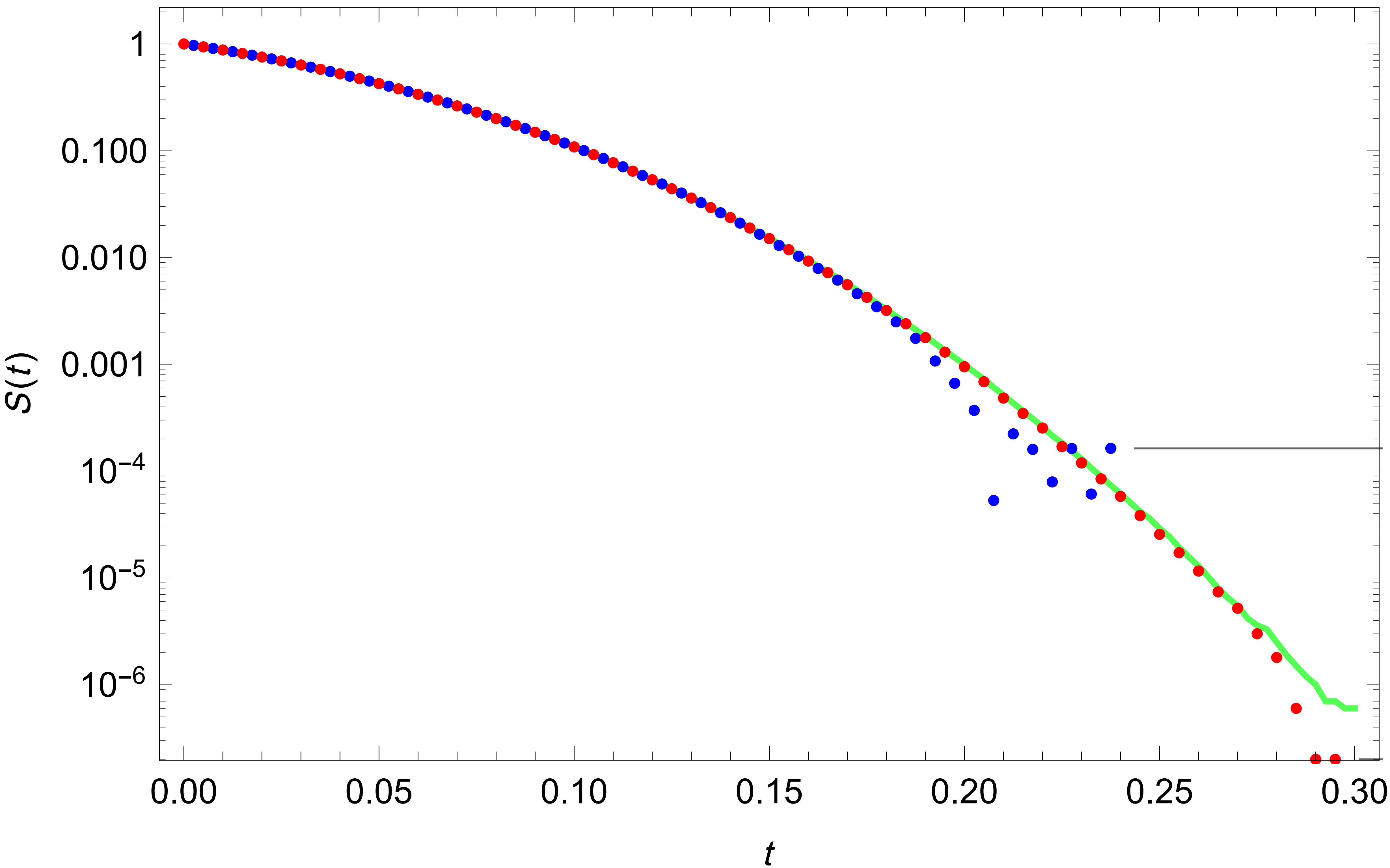


Fig. 2. Solid line (red): exact $N_c = 3$ solution to (21). The band indicates the standard error. Dashed line (blue): $N_c = 3$, mean–field solution to (58). Dotted line (green): solution to the BMS equation (16) from [22]. Dash–dotted line (yellow): result with only the Sudakov term.

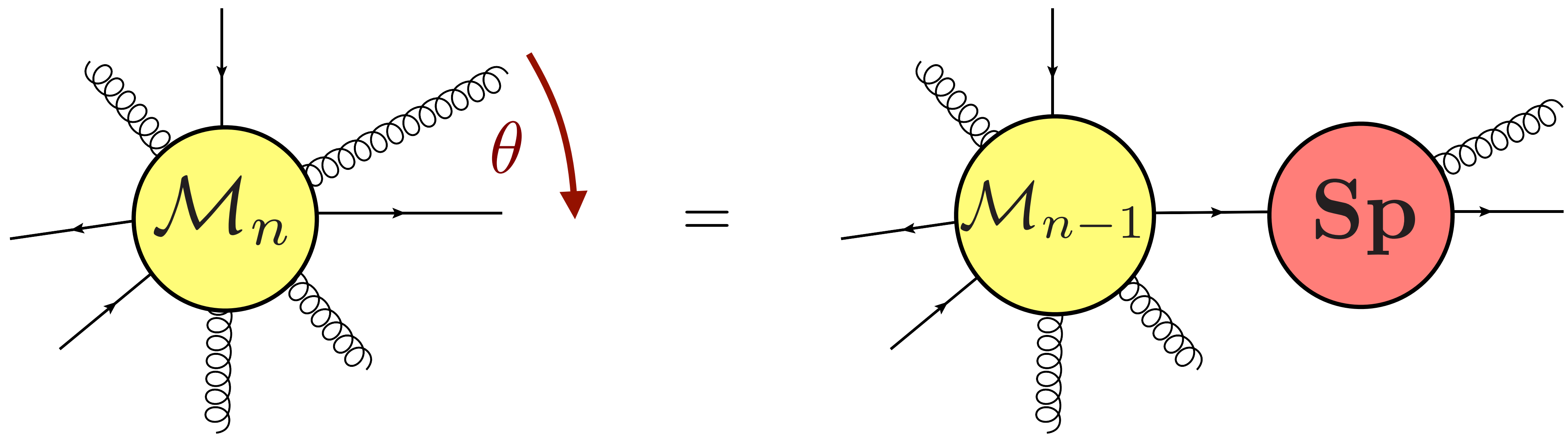
Hatta & Ueda, 1304.6930

Gaps between jets, comparison to ATLAS ‘14




1
0.100
0.010
0.001
10^{-4}
10^{-5}
10^{-6}
$S(t)$
0.00
0.05
0.10
0.15
0.20
0.25
0.30
t

Collinear factorization



When partons become collinear, the amplitude factorizes into a lower-point amplitude times a splitting amplitude $\mathbf{Sp}$.

- Leading contribution to the squared amplitude does not involve interference with the other particles!
- Can be violated by Glauber phases for process with collinear in- and outgoing particles. Catani, de Florian, Rodrigo '11; Forshaw, Seymour, Siodmok '12; Schwartz, Yan, Zhu '17

Have derived similar factorization formulas for many classic non-global observables

- cone-jets, also for small cone angle δ (collinear logs); isolation-cone cross sections for photon production
- event shapes: light-jet mass, hemisphere soft function, narrow broadening (soft recoil, rapidity logs)

Crucial element is always multi-Wilson-line operators sourced by hard partons in certain phase-space regions.

- have tested that we reproduce the full logarithmic structure at NNLO by computing ingredients up to α_s^2
- compare against full NNLO (using Event2 by Seymour) as well as analytical results (hemisphere soft function by Kelley, Schwartz, Schabinger and Zhu '11; Hornig, Lee, Stewart, Walsh and Zuberi '11)

$$\sigma(\beta) = \sum_{m=2}^{\infty} \langle \boldsymbol{\mathcal{H}}_m(\{\underline{n}\}, Q, \mu) \otimes \boldsymbol{\mathcal{S}}_m(\{\underline{n}\}, Q\beta, \mu) \rangle \, ,$$

High-*E* physics
Wilson coefficients

Low-*E* physics
EFT Operator

- Renormalization of hard Wilson coefficients

$$\boldsymbol{\mathcal{H}}_m(\{\underline{n}\}, Q, \delta, \epsilon) = \sum_{l=2}^{m} \boldsymbol{\mathcal{H}}_l(\{\underline{n}\}, Q, \delta, \mu) \, \boldsymbol{Z}^H_{lm}(\{\underline{n}\}, Q, \delta, \epsilon, \mu)$$

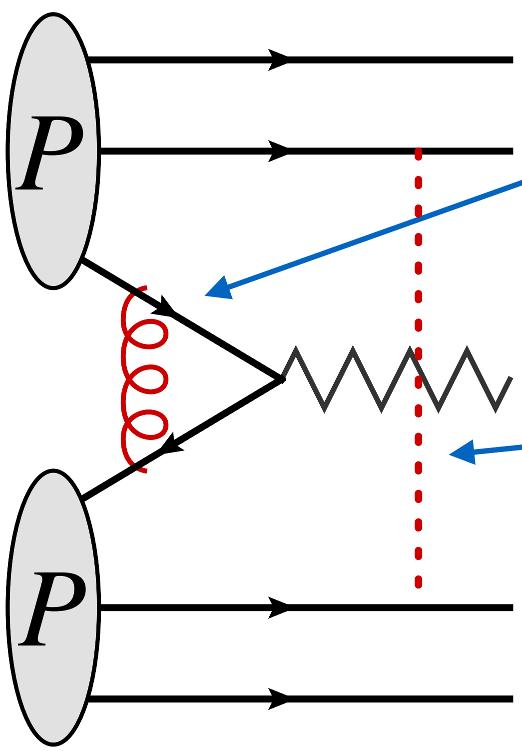
- Same *Z*-factor must render $\mathcal{S}_\mathrm{m}$ finite!
- Associated anomalous dimension $\boldsymbol{\Gamma}^H$

$$\frac{d}{d\ln\mu} \boldsymbol{Z}^H_{km}(\{\underline{n}\}, Q, \delta, \epsilon, \mu) = \sum_{l=k}^{m} \boldsymbol{Z}^H_{kl}(\{\underline{n}\}, Q, \delta, \epsilon, \mu) \, \hat{\otimes} \, \boldsymbol{\Gamma}^H_{lm}(\{\underline{n}\}, Q, \delta, \mu)$$

Quite nontrivial that the low-energy matrix element factorizes into a product

$$\langle P(p_1)|O_a(x_1)|P(p_1)\rangle\,\langle P(p_2)|O_b(x_2)|P(p_2)\rangle$$

One should be worried about long-distance interactions mediated by soft gluons



standard soft gluon

Glauber (aka Coulomb) gluon

$p^\mu \approx p_\perp^\mu$

Factorization proof?

- It is relatively easy to show that standard soft gluon contributions cancel. Collins, Soper, Sterman ’85. SCET: Bauer, Fleming, Pirjol, Rothstein and Stewart ’02
- Glauber contribution is more delicate
  - CSS showed that it is absent for inclusive Drell-Yan process.
  - Examples where Glauber gluons *do* contribute in perturbation theory: super-leading logs, collinear factorization breaking, forward scattering…
  - SCET formulation with Glauber gluons available since last year Rothstein, Stewart ’16, 204pp (!)

Wilson line and eikonal interaction

Consider one-gluon matrix element of Wilson line

$$\langle k,\lambda,b|\,\boldsymbol{S}_i\,|0\rangle = ig_s\,\boldsymbol{T}^a\int_0^\infty ds\,\langle k,\lambda,b|n_i\cdot A^a(sn_i)|0\rangle + \mathcal{O}(g_s^2)$$

$$= ig_s\,\boldsymbol{T}^a\int_0^\infty ds\,e^{isn_i\cdot k}\langle k,\lambda,b|n_i\cdot A^a_\mu(0)|0\rangle$$

$$= ig_s\,\boldsymbol{T}^b n_i\cdot\varepsilon(k,\lambda)\,\frac{e^{isn_i\cdot k}}{in_i\cdot k}\bigg|_0^\infty$$

need small imaginary part n · k ≡ n · k + iε

$$= -g_s\boldsymbol{T}^b\,\frac{n_i\cdot\varepsilon(k,\lambda)}{n_i\cdot k} = \boxed{-g_s\boldsymbol{T}^b\,\frac{p_i\cdot\varepsilon(k,\lambda)}{p_i\cdot k}}$$

eikonal interaction