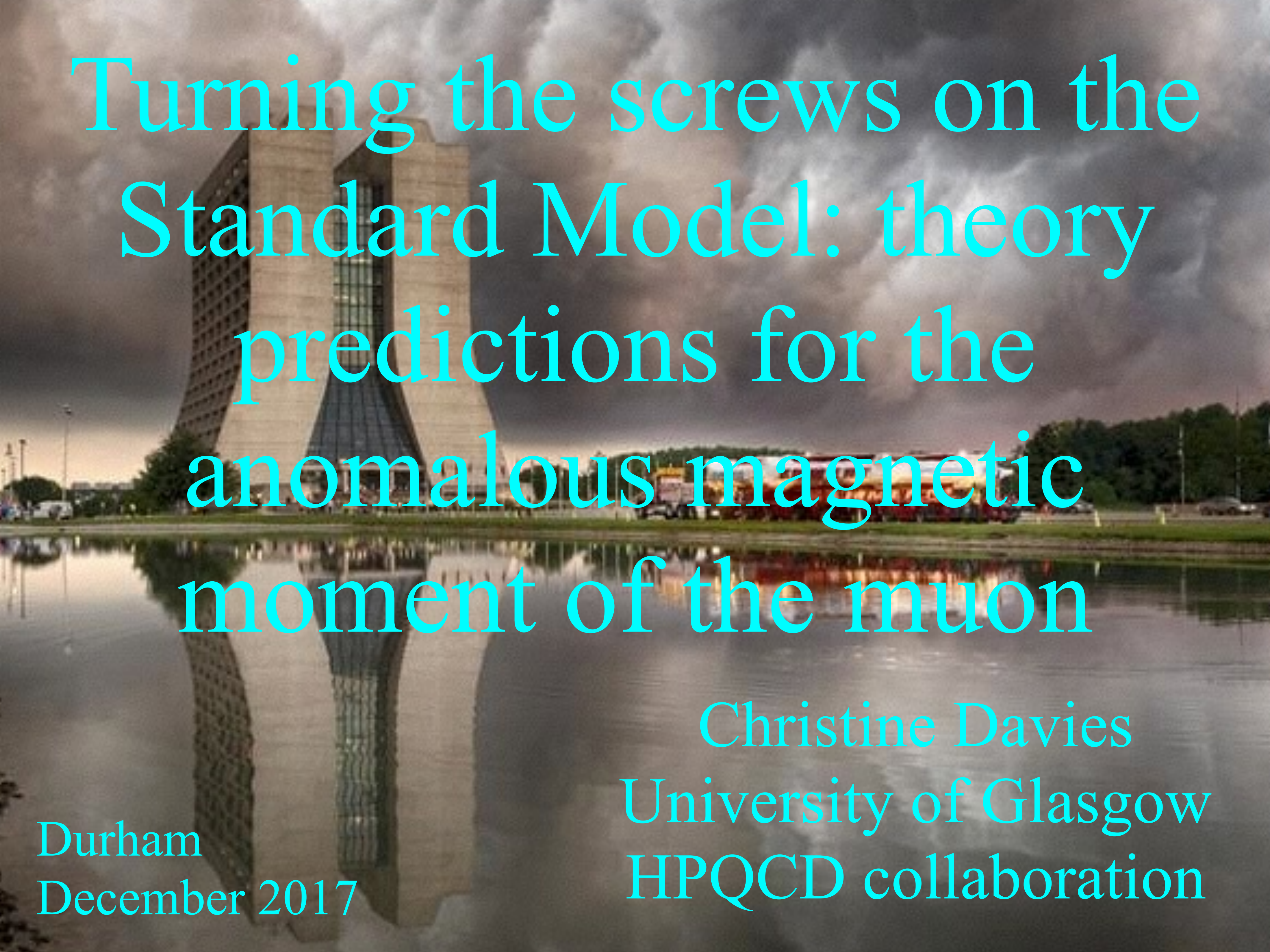


Turning the screws on the Standard Model: theory predictions for the anomalous magnetic moment of the muon

Christine Davies
University of Glasgow
HPQCD collaboration

Durham
December 2017

Outline

1) Introduction : what is the anomalous magnetic moment (a_μ) of the muon?

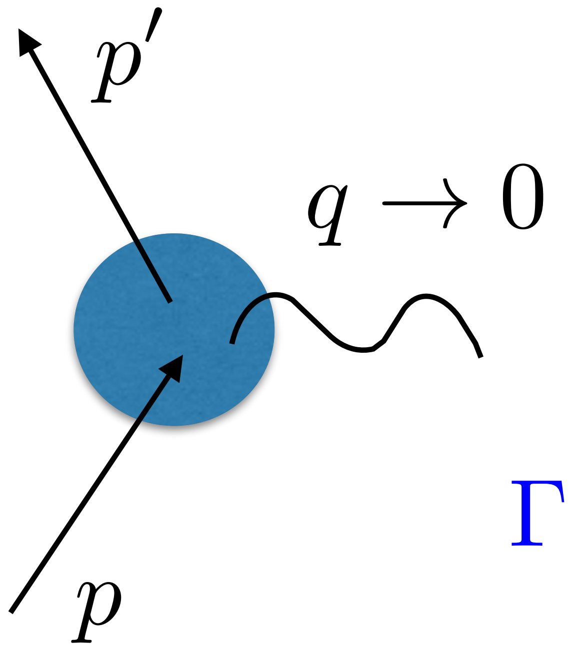
2) How is it determined (so accurately) in experiment?

3) Theory calculations in the Standard Model: QED/EW calculations

4) Pinning down QCD effects, using experimental data and using Lattice QCD calculations.

5) Conclusions and prospects

e, μ, τ have electric charge and spin



Interaction with an external em field has a magnetic component:

$$-ie\overline{u}(p')\Gamma^{\mu}(p,p')u(p)A_{\mu}(q)$$

$$\Gamma^{\mu}(p,p') = \gamma^{\mu}F_1(q^2) + \frac{i\sigma^{\mu\nu}q_{\nu}}{2m}F_2(q^2)$$

Electric field interaction (charge consvn): $F_1(0) = 1$

Magnetic field intn, equiv. to scattering from potential :

$$V(x) = -\langle\vec{\mu}\rangle \cdot \vec{B}(x)$$

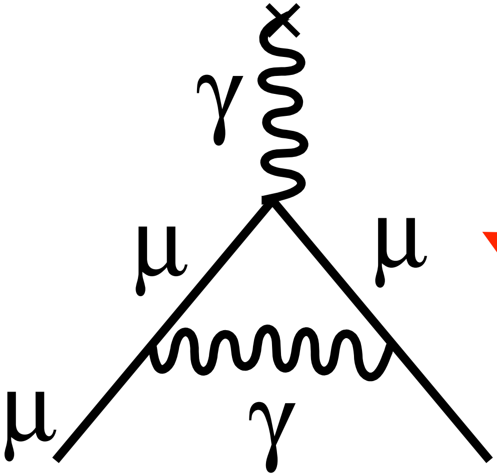
$$\vec{\mu} = \frac{e}{m}[F_1(0) + F_2(0)]\frac{\vec{\sigma}}{2} \equiv g\left(\frac{e}{2m}\right)\vec{S}$$

$$g = 2 + 2F_2(0)$$

Peskin + Schroeder

Anomalous magnetic moment

$$a_{e,\mu,\tau} = \frac{g-2}{2} = F_2(0)$$

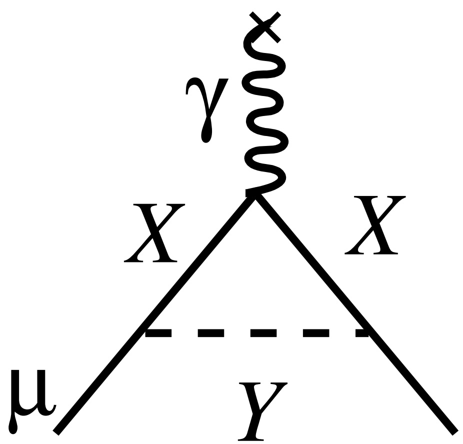


LO contribn is lepton mass independent

Schwinger 1948

$$\frac{\alpha}{2\pi} = 0.00116\ldots$$

many higher order pieces .....



New physics could appear in loops

$$\delta a_\ell^{\text{new physics}} \propto \frac{m_\ell^2}{m_X^2}$$

1 TeV?

Motivates study of μ rather than e

$\approx 10^{-8}$ $\approx 10^{-13}$

flavour,CP-conserving
chirality flipping

CURRENT STATUS

$$a_\mu^{\rm expt} = 11659209.1(6.3) \times 10^{-10}$$
$$a_\mu^{\rm SM} = 11659182.2(4.3) \times 10^{-10}$$

tantalising discrepancy! details to follow …

$$a_e^{\rm expt} = 11596521.807(3) \times 10^{-10}$$
$$a_e^{\rm SM} = 11596521.816(8) \times 10^{-10}$$

higher accuracy small-scale experiments possible (Penning trap) but discrepancies will be tiny …

τ very hard since decays in 0.3picoseconds ….

$\delta a_\tau = 5 \times 10^{-2}$ (LEP) $e^+e^- \to e^+e^-\tau^+\tau^-$

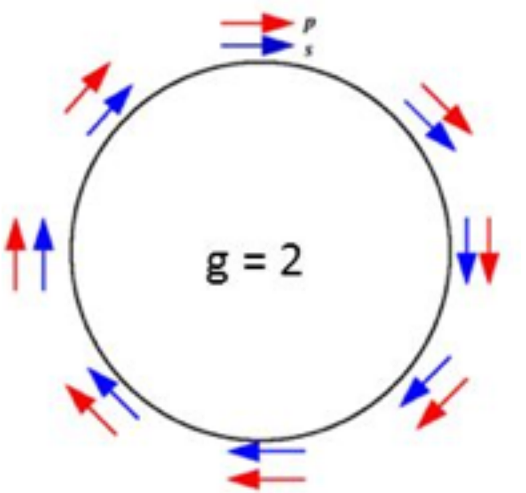
Accurate experimental results + theory calculations needed

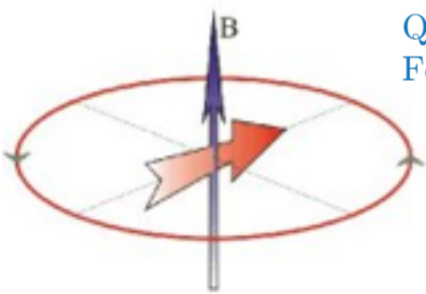
spin 0

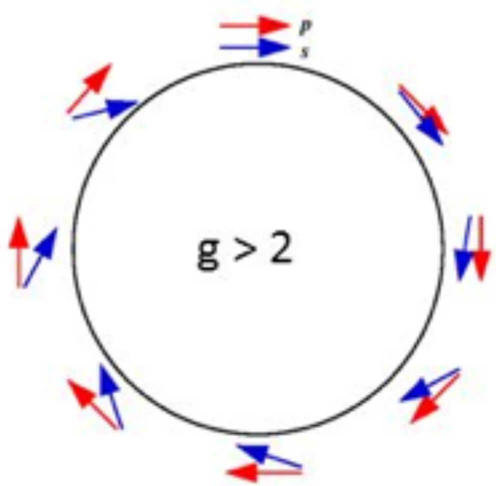
$$p \rightarrow \pi^+ \rightarrow \nu_\mu + \mu^+$$

both helicity -1 in π rest frame so get polarised μ beam pulse

B field perpendicular to ring, μ spin precesses

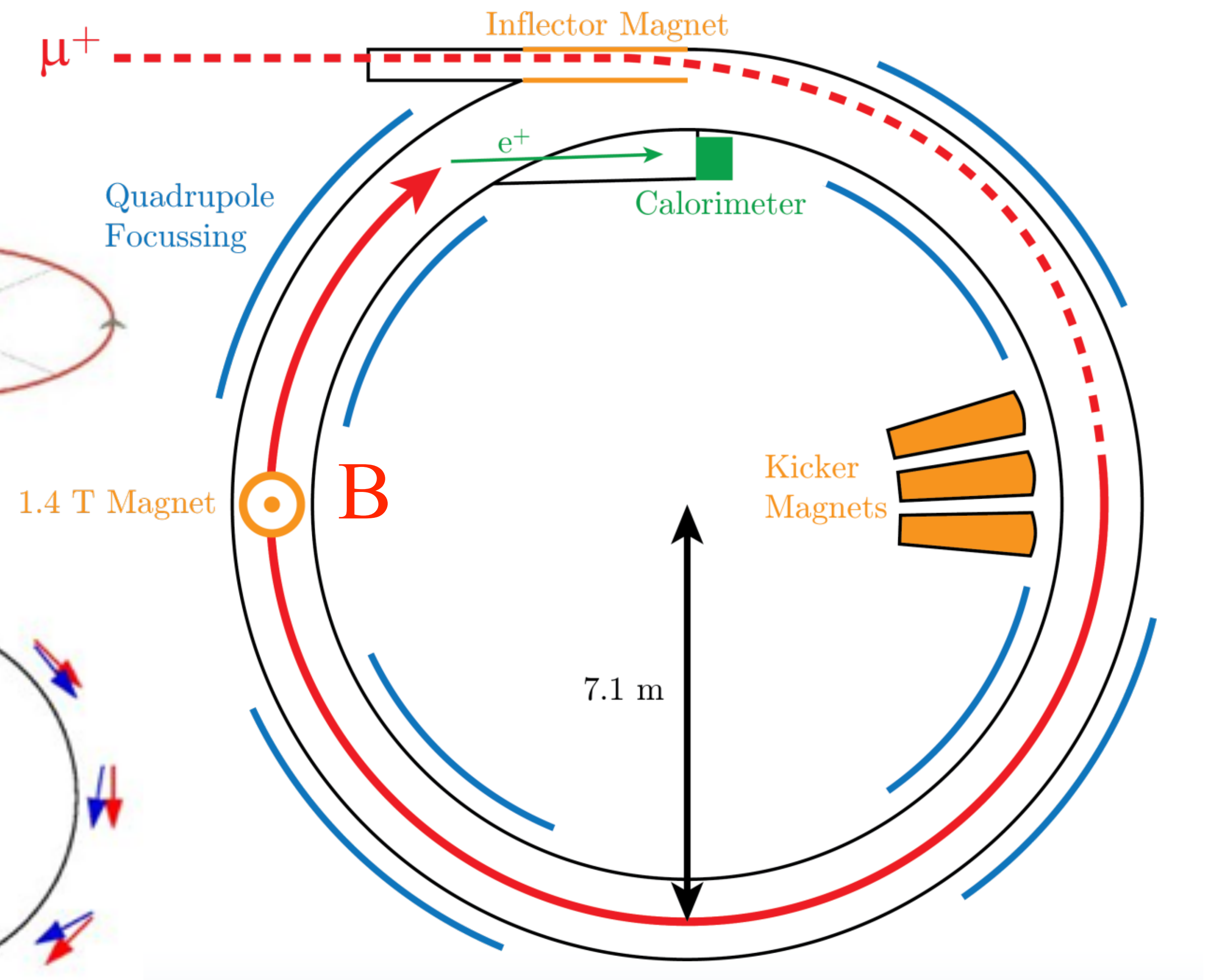






measure frequency difference

$$\omega_S - \omega_C$$



$$\vec{\omega}_a = \vec{\omega}_S - \vec{\omega}_C = -\frac{Qe}{m}\left[a_\mu \vec{B} + \left(a_\mu - \left(\frac{m}{p}\right)^2\right)\frac{\vec{\beta}\times\vec{E}}{c}\right] + ..$$

$Q = \pm 1,\ \mu^{\pm}$

need uniform stable B, measure to sub-ppm with NMR probes calibrated using g_p

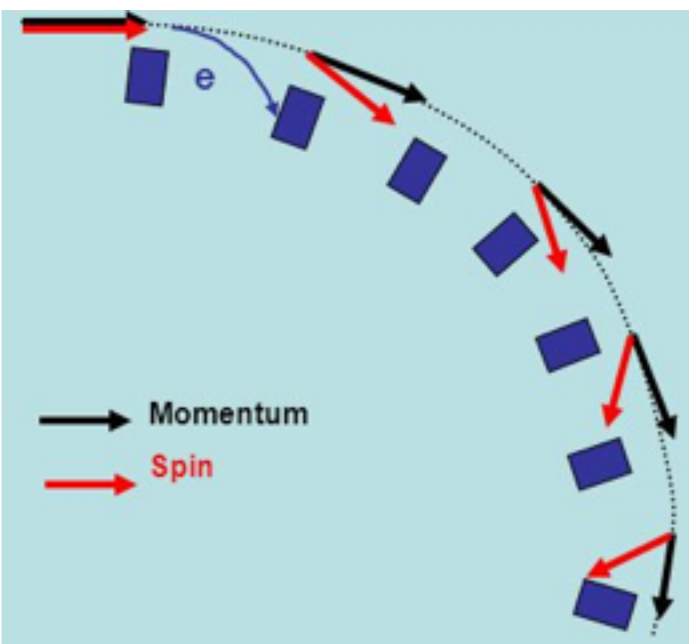
directly gives a_μ

electric field term vanishes at 'magic momentum'

$$p = 3.094\,\mathrm{GeV}/c$$

from possible EDM

$$\propto \vec{\beta} \times \vec{B}$$



measure spin direction from e produced in weak decay

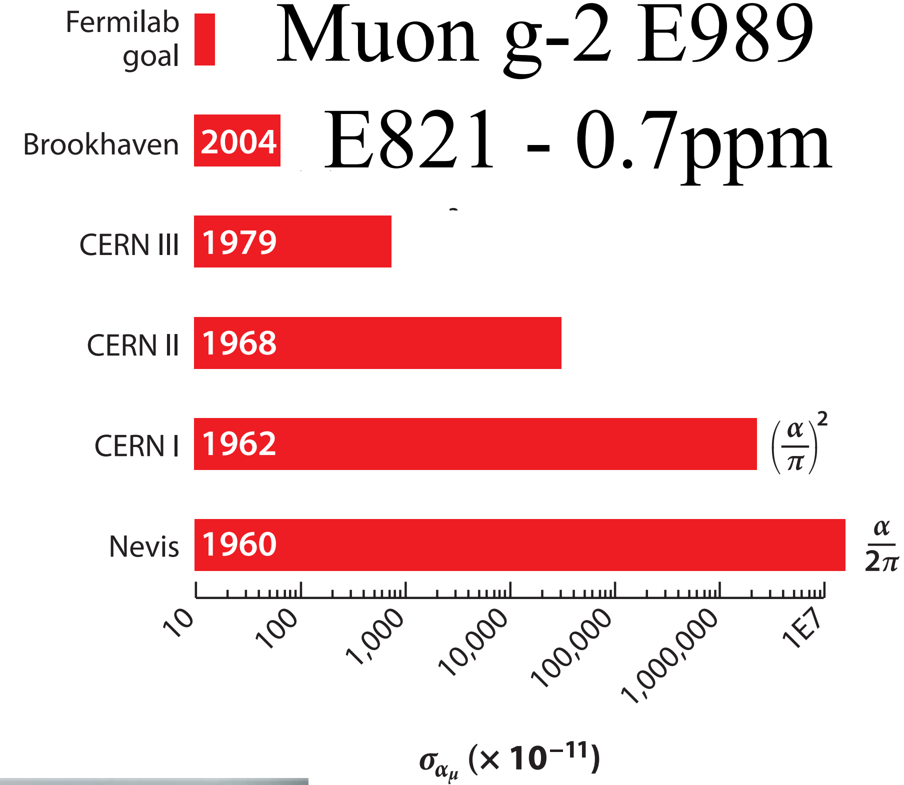
$$\mu^+ \rightarrow e^+ + \nu_e + \overline{\nu}_\mu$$

direction of highest energy e correlated with μ spin so N_e oscillates at $\omega_S - \omega_C$

Status of experiment

Muon g-2 E989

E821 - 0.7ppm



2013: E821 ring moved to Fermilab

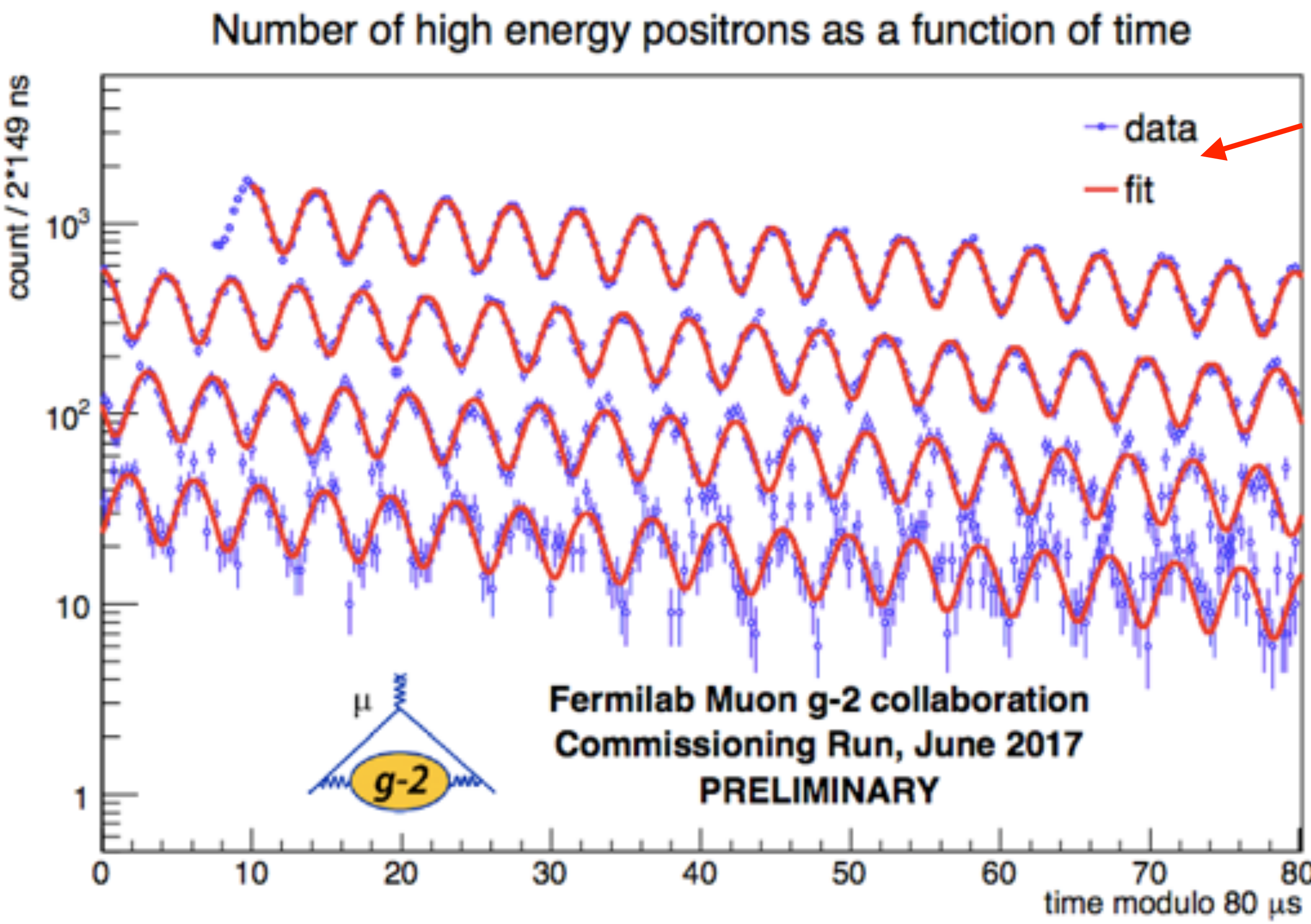
becomes E989

UK groups: Cockcroft, Lancaster, Liverpool, UCL

Aim: Much higher statistics with cleaner injection to ring, more uniform B field + temp. control : 0.15ppm i.e $\delta a_\mu = 2 \times 10^{-10}$

Muon g-2 now running at Fermilab, optimising beam

Aim: run summer 2018 for 1-3 x E821, first results 2019



commissioning run: 0.001% of final stats

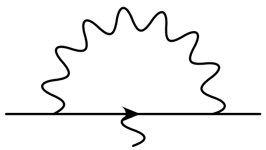
$$N_e(t) = N_0 e^{-t/\gamma t_\mu} \times [1 + A\cos(\omega_a t + \phi)]$$

$$\gamma\tau_\mu = 60 \times 10^{-6} s$$

J-PARC future plan: slow μ in 1m ring - no need for 'magic momentum'

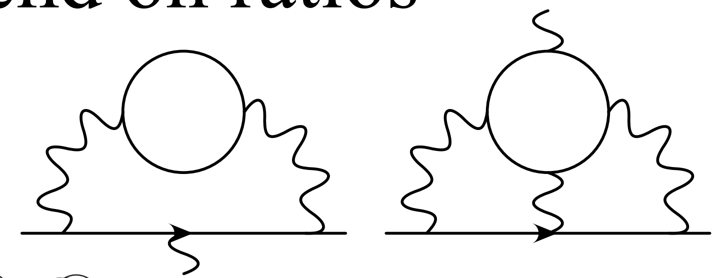
Accurate experimental results + theory calculations needed

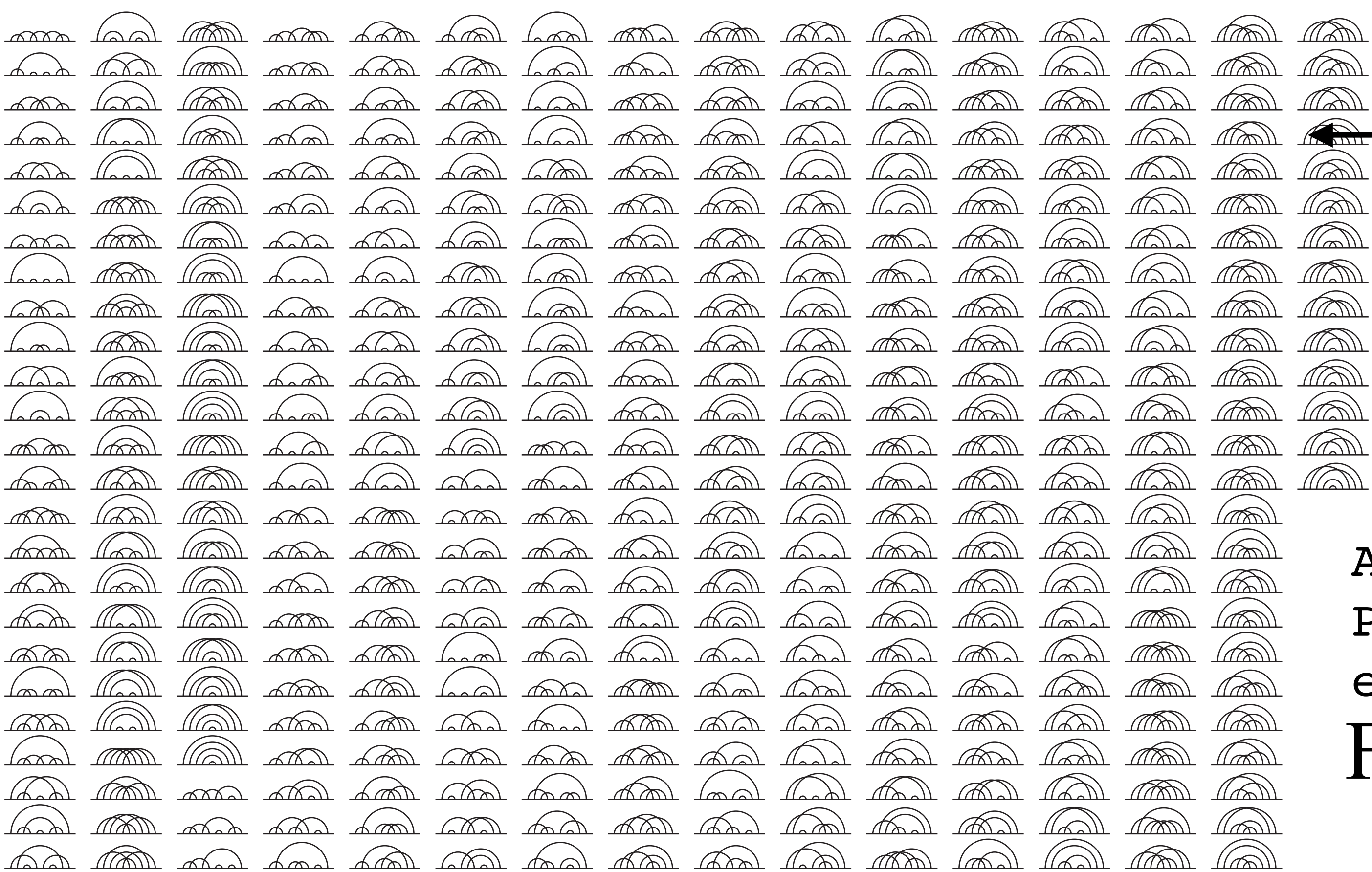
QED corrections dominate - calculate in Perturbation theory



$$0.5\frac{\alpha}{\pi}$$

higher orders depend on ratios of lepton masses:





subset of diagrams at α^5

integration challenging- use VEGAS

Aoyama, Kinoshita et al PRD91:033006(2015), err:PRD96:019901(2017)

For α use a_e or Rb

<0.5ppb

$$a_\mu^{\mathrm{QED}} = \frac{\alpha}{2\pi} + 0.765\,857\,425(17)\left(\frac{\alpha}{\pi}\right)^2 + 24.050\,509\,96(32)\left(\frac{\alpha}{\pi}\right)^3 + 130.879\,6(6\,3)\left(\frac{\alpha}{\pi}\right)^4 + 753.3(1.0)\left(\frac{\alpha}{\pi}\right)^5 + \cdots$$

Hoecker
+Marciano
RPP 2017

$$a_\mu^{\mathrm{QED}} = 0.00116 + 0.00000413 \ldots + 0.000000301 + 0.00000000381 + 0.000000000509 + \ldots$$

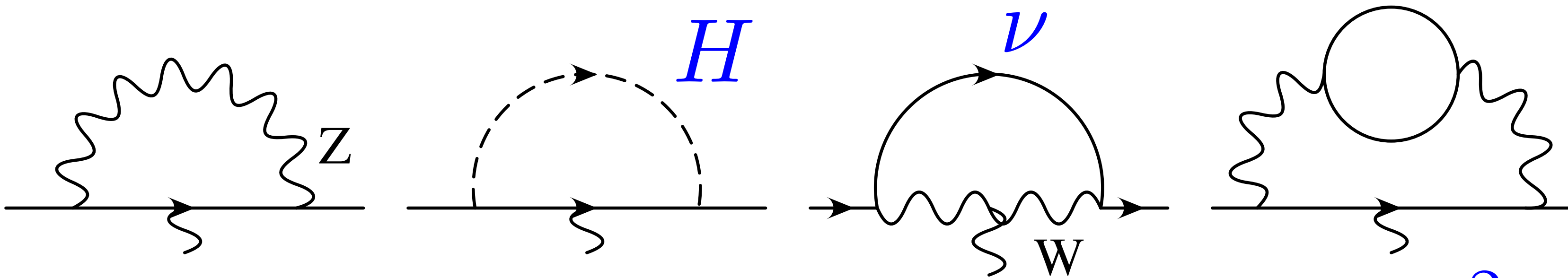
using Rb α

$$= 11{,}658{,}471.895(8) \times 10^{-10}$$

uncertainty from error in α but missing α^6 (light-by-light) also this size

Electroweak contributions from Z, W, H

Gnendiger et al, 1306.5546



a_μ^{EW} is small - suppressed by powers of $\dfrac{m_\mu^2}{m_W^2}$

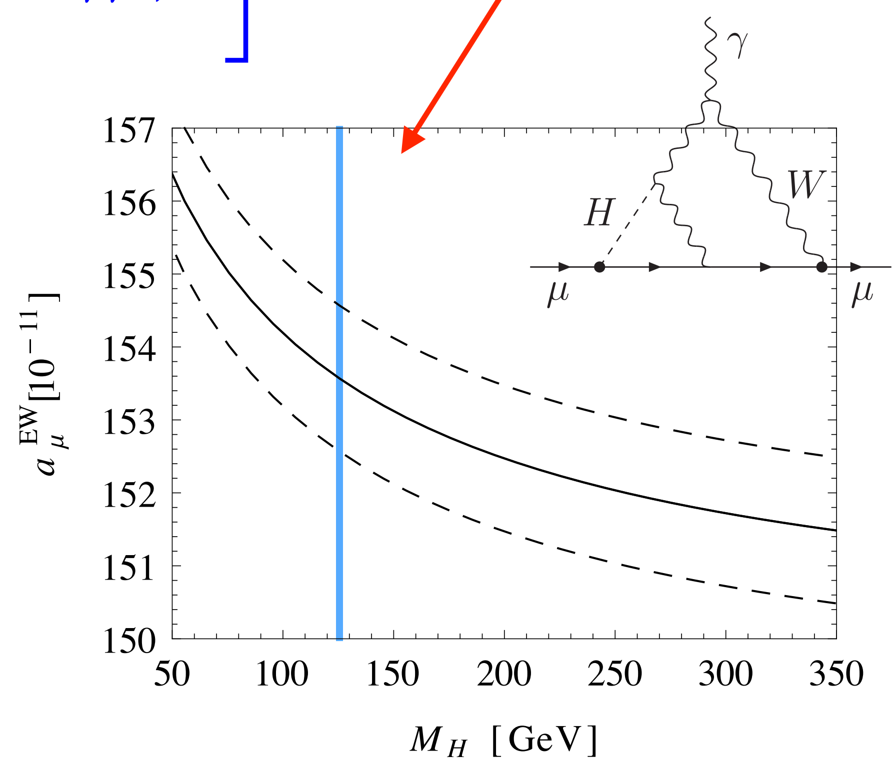
$$a_\mu^{\mathrm{EW}(1)} = \frac{G_F m_\mu^2}{\sqrt{2}\, 8\pi^2}\left[\frac{5}{3} + \frac{1}{3}(1-4s_W^2)^2\right]$$

$$= 19.480(1) \times 10^{-10}$$

$$a_\mu^{\mathrm{EW}(2)} = -4.12(10) \times 10^{-10}$$

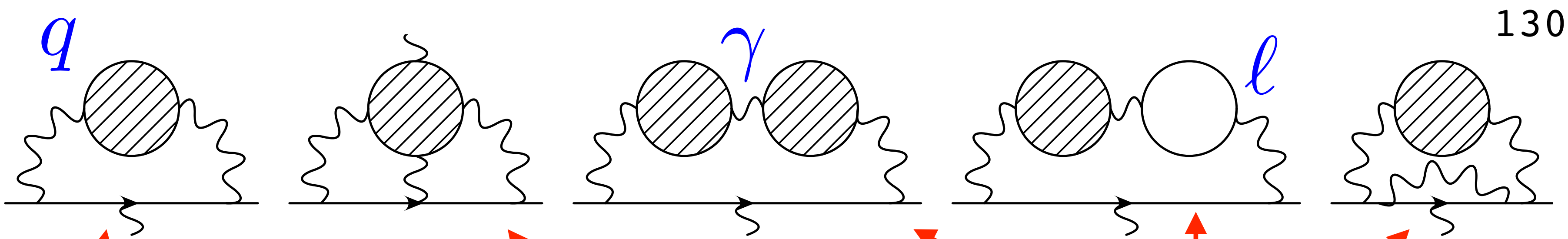
$$a_\mu^{\mathrm{EW}} = 15.36(10) \times 10^{-10}$$

H piece tiny at 1-loop; 2-loops



QCD contributions to a_μ start at α^2, nonpert. in QCD

Blum et al,
1301.2607



LO Hadronic vacuum polarisation (HVP) dominates uncertainty in SM result

Hadronic light-by-light, not well known but small

Higher order Hadronic vacuum polarisation (HOHVP)

Since QED, EW known accurately, subtract from expt and compare QCD calculations to remainder

$$a_\mu^{E821} = 11659209.1(6.3) \times 10^{-10}$$

$$a_\mu^{\rm QED} = 11658471.895(8) \times 10^{-10} \quad a_\mu^{\rm EW} = 15.36(10) \times 10^{-10}$$

Hadronic (and other) contributions = EXPT - QED - EW

$$a_\mu^{E821} - a_\mu^{\rm QED} - a_\mu^{\rm EW} = 721.9(6.3) \times 10^{-10}$$

$$= a_\mu^{HVP} + a_\mu^{HOHVP} + a_\mu^{HLBL} + a_\mu^{new\,physics}$$

Focus on lowest order hadronic vacuum polarisation (HVP), so take:

$$a_\mu^{HLbL} = 10.5(2.6) \times 10^{-10}$$ will return to this

$$a_\mu^{HOHVP} = -8.85(9) \times 10^{-10}$$ NLO+NNLO

Kurz et al, 1403.6400

$$a_\mu^{\rm HVP, \mathit{no\,new\,physics}} = 720.2(6.8) \times 10^{-10}$$

Note: much larger than $a_\mu^{\rm EW}$

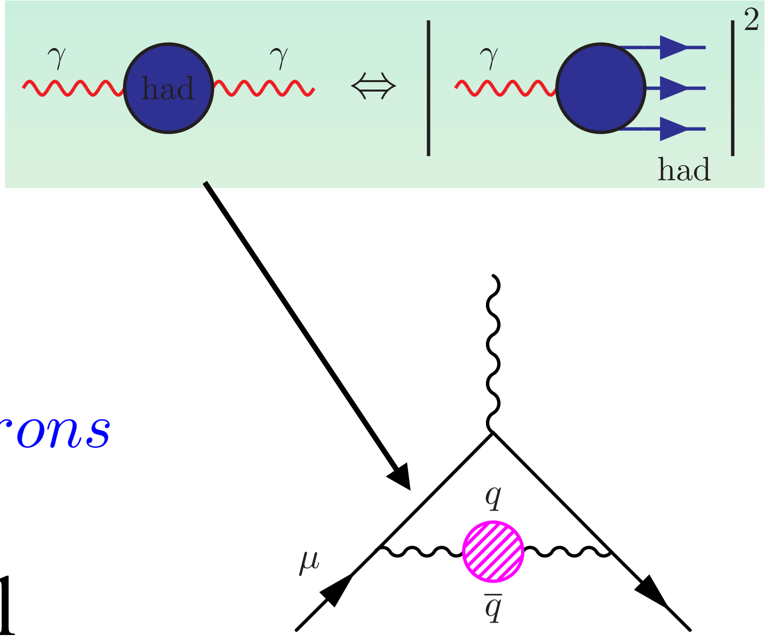
How to calculate $a_\mu^{\rm HVP}$ - Two approaches:

1) $\sigma(e^+e^- \rightarrow \text{hadrons})$ + dispersion relations.

2) lattice QCD

1) $\sigma(e^+e^- \rightarrow \text{ hadrons})$

Analyticity+optical theorem



$$a_\mu^{HVP} = \frac{1}{4\pi^3}\int_{m_\pi^2}^{\infty} ds\sigma^0_{had}(s)K(s)$$

$\pi^0\gamma$ threshold

$e^+e^- \rightarrow \gamma^* \rightarrow hadrons$

K(s) kernel emphasises low s - integral dominated by $\rho, \pi^+\pi^-$. Use pert. QCD at high s.

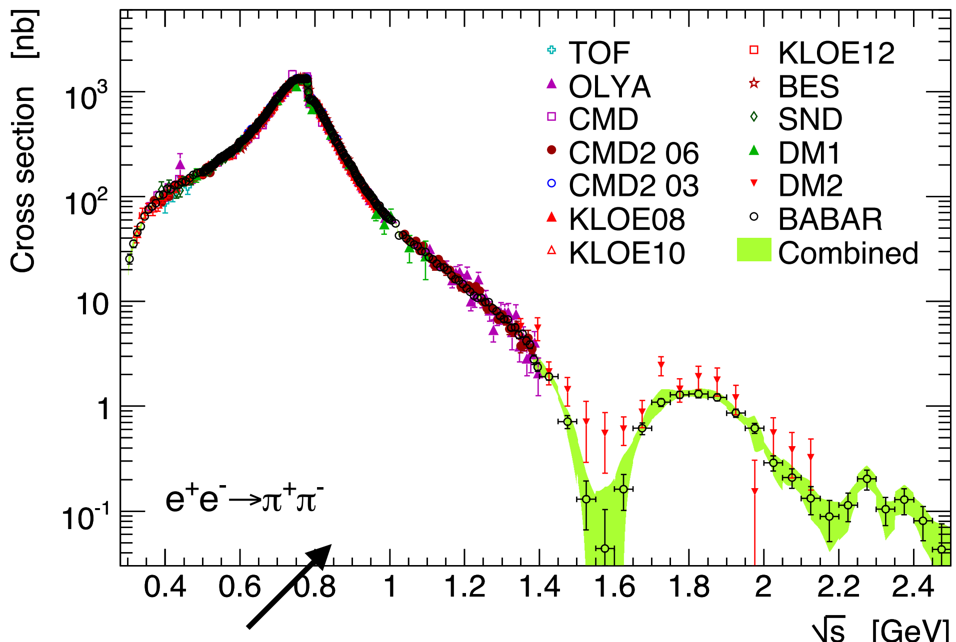
$$R_{e^+e^-} = \frac{\sigma}{\sigma_{pt}}$$

σ^0 is 'bare', with running α effects removed .

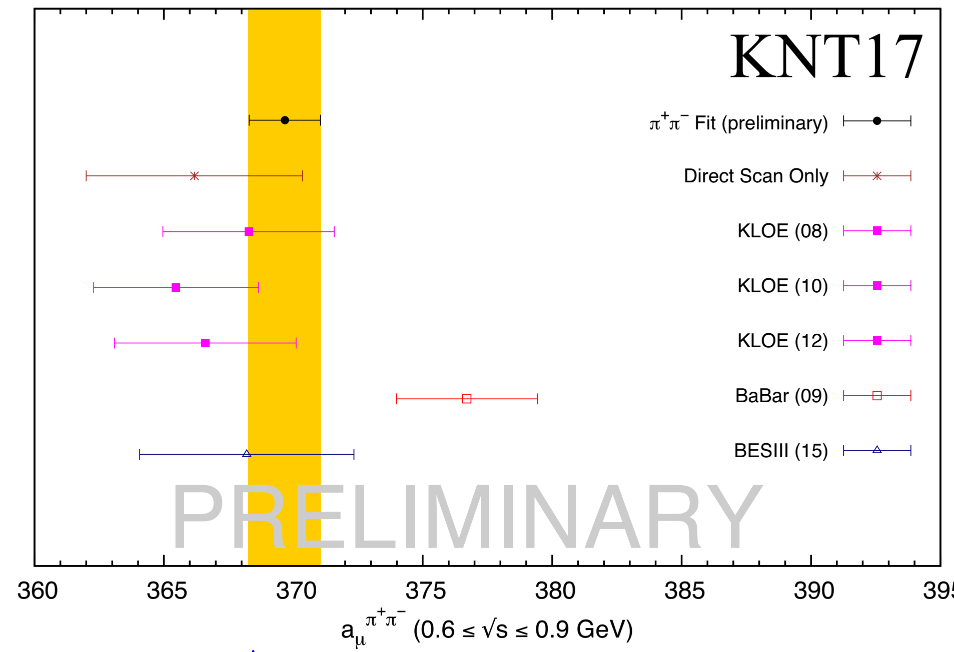
Final state em radiation IS included - γ inside hadron bubble

Need to combine multiple sets of experimental data from many hadronic channels (+ inclusive) inc. correlations

New data sets from KLOE, BESIII, SND(Novosibirsk) ..

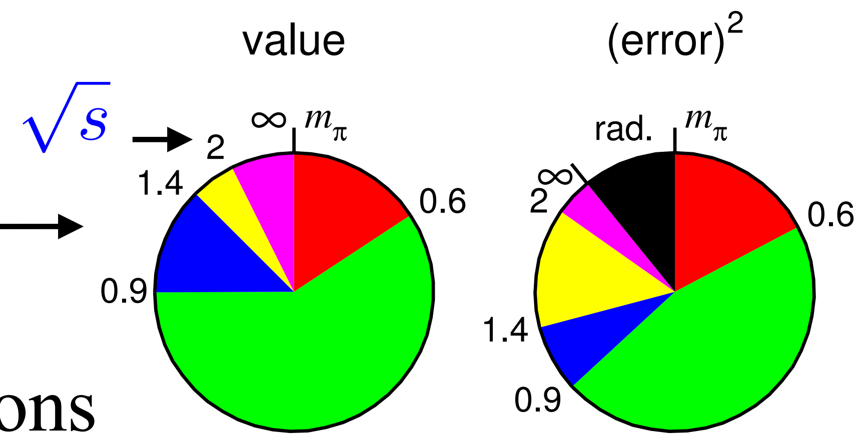


Davier et al
1706.09436



$\pi^+\pi^-,\ 0.6 < \sqrt{s} < 0.9\text{GeV}$

New results



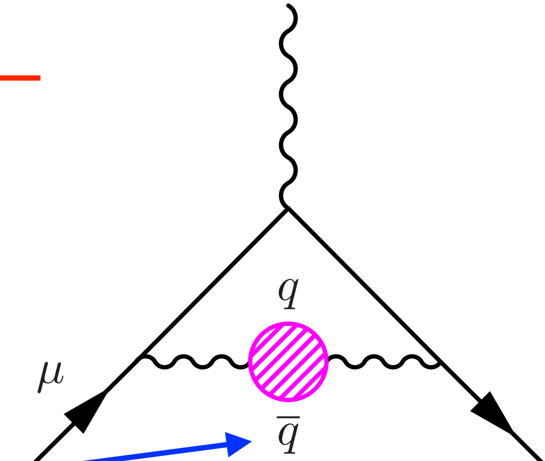
Keshavarzi, Nomura, Teubner
Liverpool-Kyoto ’17 :
70% redn in uncty since 2011.
New data, more channels, correlations

KNT17 $a_\mu^{\rm HVP} = 692.2(2.5) \times 10^{-10}$

Davier et al, 2017 $a_\mu^{\rm HVP} = 693.1(3.4) \times 10^{-10}$

Jegerlehner 1705.00263 $a_\mu^{\rm HVP} = 688.8(3.4) \times 10^{-10}$

agree well - 0.4% uncty 3.5σ from no new physics.



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2) Lattice QCD

Blum, hep-lat/0212018

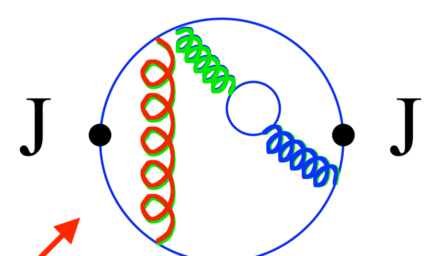
$$a_\mu^{HVP,i} = \frac{\alpha}{\pi}\int_0^\infty dq^2 f(q^2)(4\pi\alpha e_i^2)\hat{\Pi}_i(q^2)$$

'connected' contribution for flavour i

Integrate over Euclidean q^2 – $f(q^2)$ diverges at small q^2 with scale set by m_μ so $q^2 \approx 0$ dominates

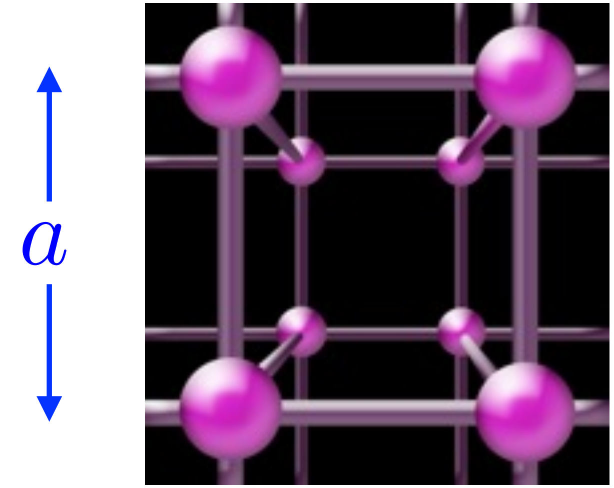
Renormalised vacuum polarisation function

$\hat{\Pi}(q^2) = \Pi(q^2) - \Pi(0)$ vanishes at q^2=0



This is (fourier transform of) vector meson correlators

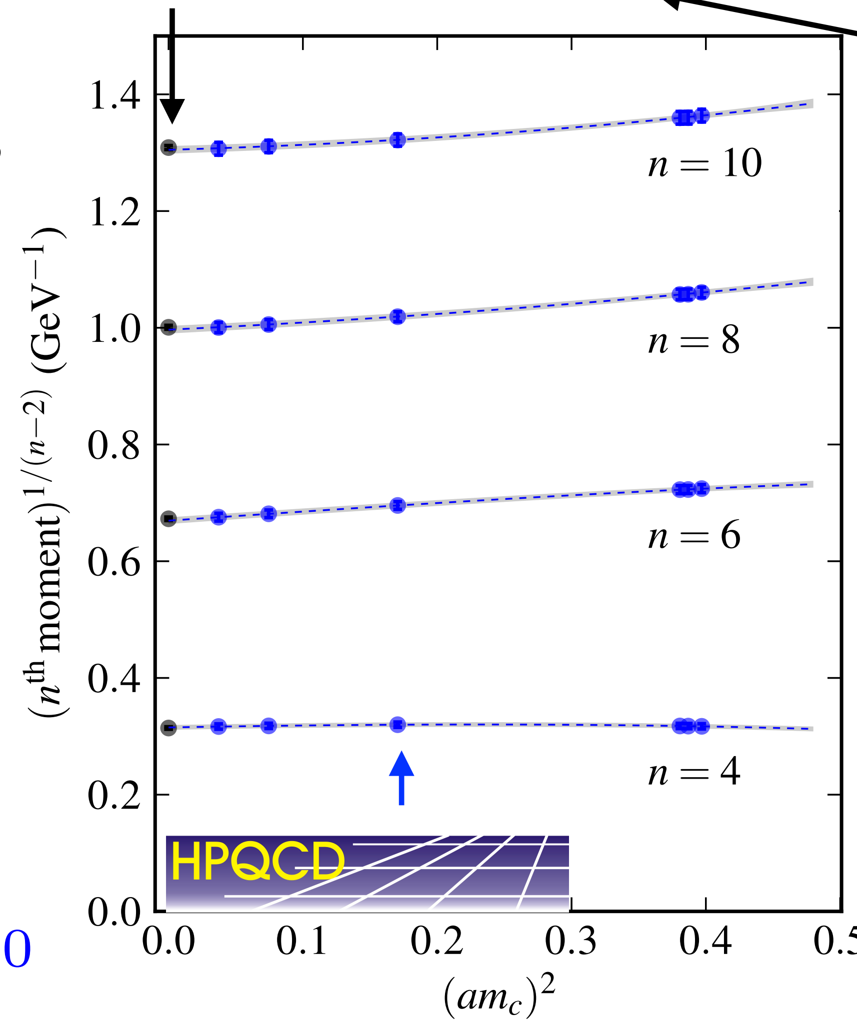
Lattice QCD - perform QCD Feynman PI by averaging correlators on lattice gluon fields that include the effect of sea quarks. NOW: realistic sea quarks, multiple values of lattice spacing

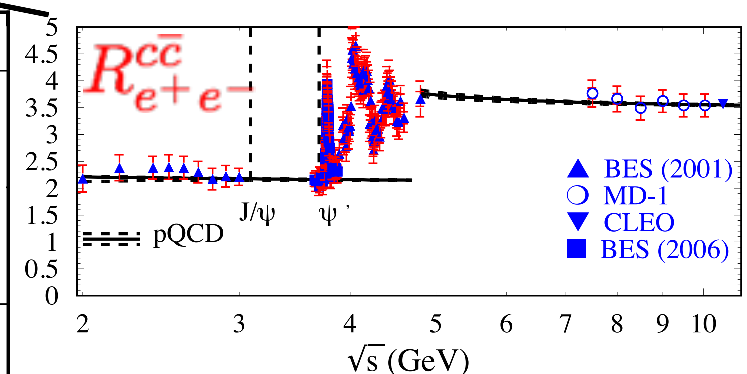


Test $c\overline{c}$ correlator time-moments vs. expt

- agree to 1.5%

$\sigma(e^+e^- \to \text{hadrons via } c\overline{c})$





DiRAC

Lattice QCD:

$a_\mu^{\rm HVP,c} = 14.4(4) \times 10^{-10}$

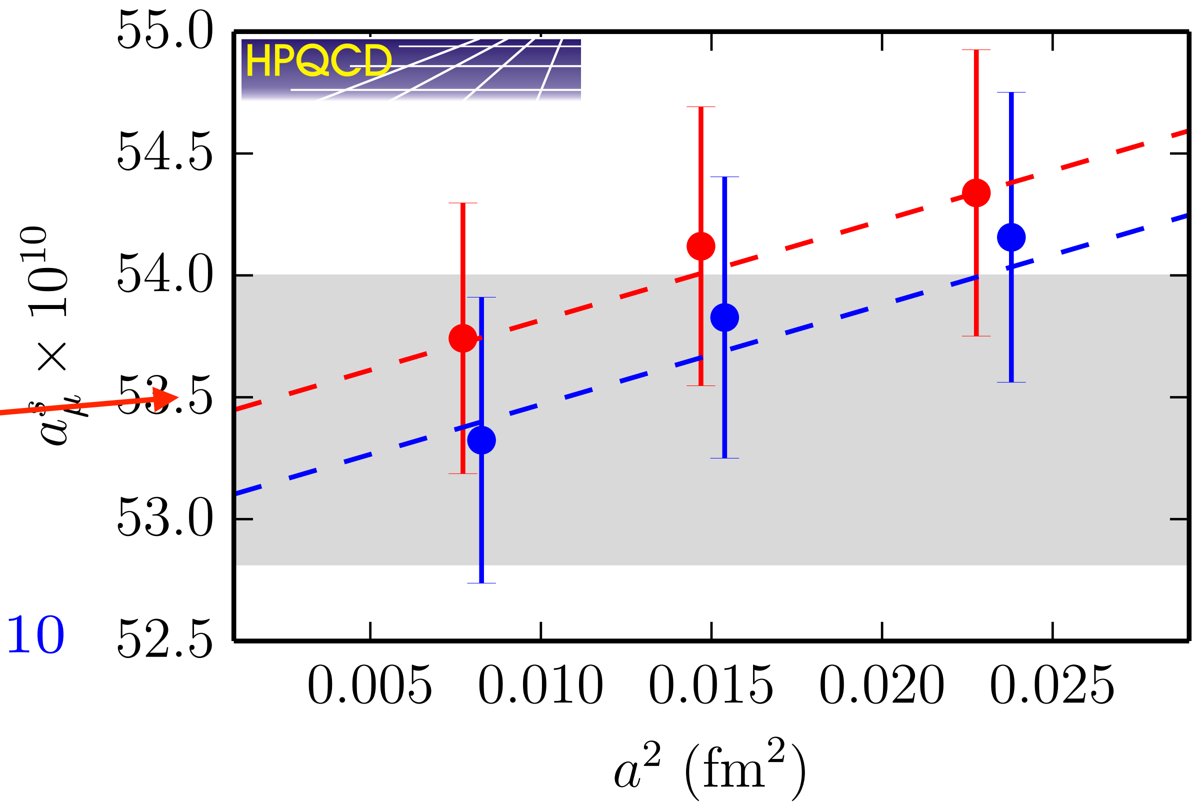
$a_\mu^{\rm HVP,b} = 0.27(4) \times 10^{-10}$

HPQCD,1208.2855,1403.1778

‘connected’ s quark contribution to a_μ

Chakraborty et al, HPQCD 1403.1778

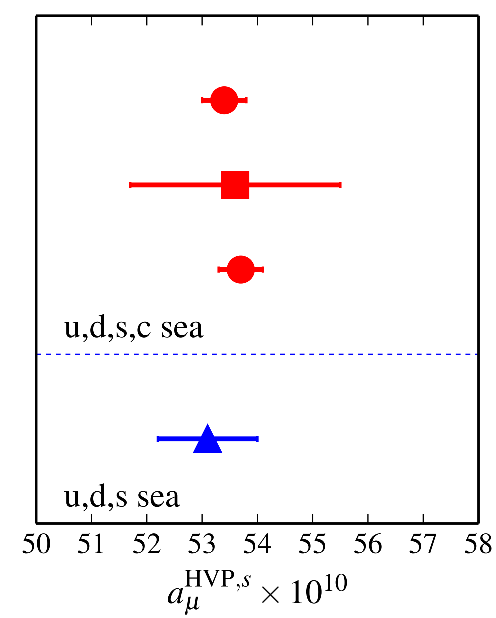
HISQ quarks on configs with u, d, s and c sea.
Local J_v - nonpert. Z_v.
multiple a (fixed by w_0),
m_l (inc. phys.), volumes.
Tune s from η_s



$$a_\mu^{HVP,s} = 53.4(4) \times 10^{-10}$$

allowing for missing QED

	a_μ^s
Uncertainty in lattice spacing (w_0, r_1):	0.4%
Uncertainty in Z_V:	0.4%
Monte Carlo statistics:	0.1%
$a^2 \to 0$ extrapolation:	0.1%
QED corrections:	0.1%
Quark mass tuning:	0.4%
Finite lattice volume:	$< 0.1\%$
Padé approximants:	$< 0.1\%$
Total:	0.7%



HPQCD 1403.1778

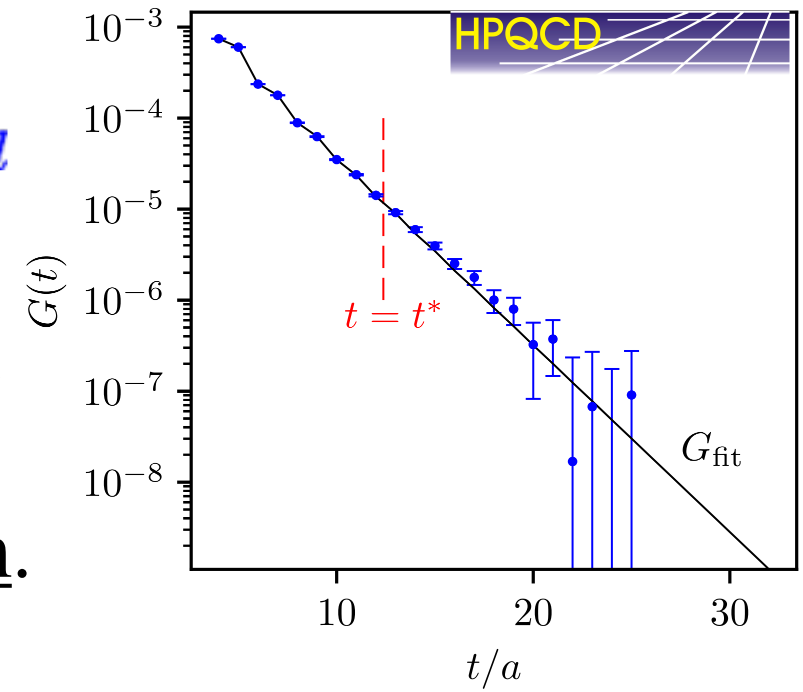
ETMC 1411.0705

BMW 1711.04980

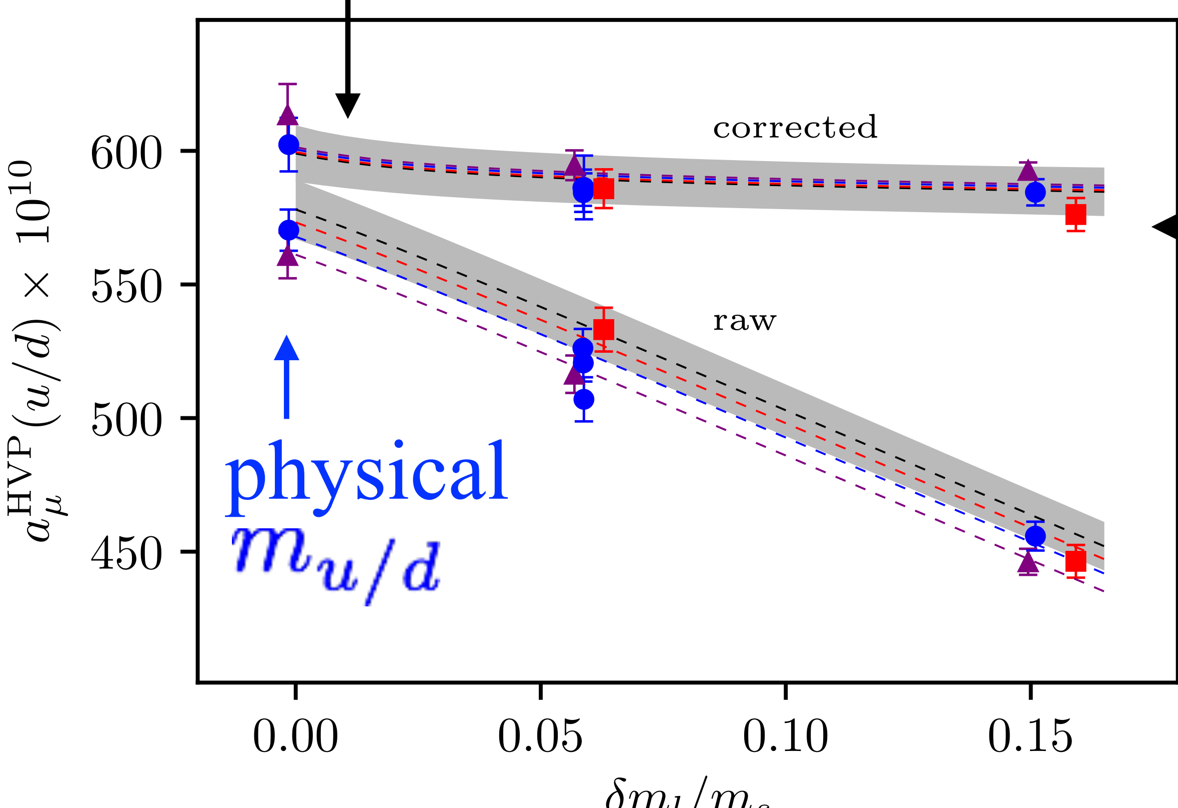
RBC/UKQCD 1606.01767

$R_{e^+e^-} <\approx 55 \times 10^{-10}$

UP/DOWN contribution, largest and most difficult - signal/noise worse and results sensitive to u/d mass $m_u = m_d = m_l$



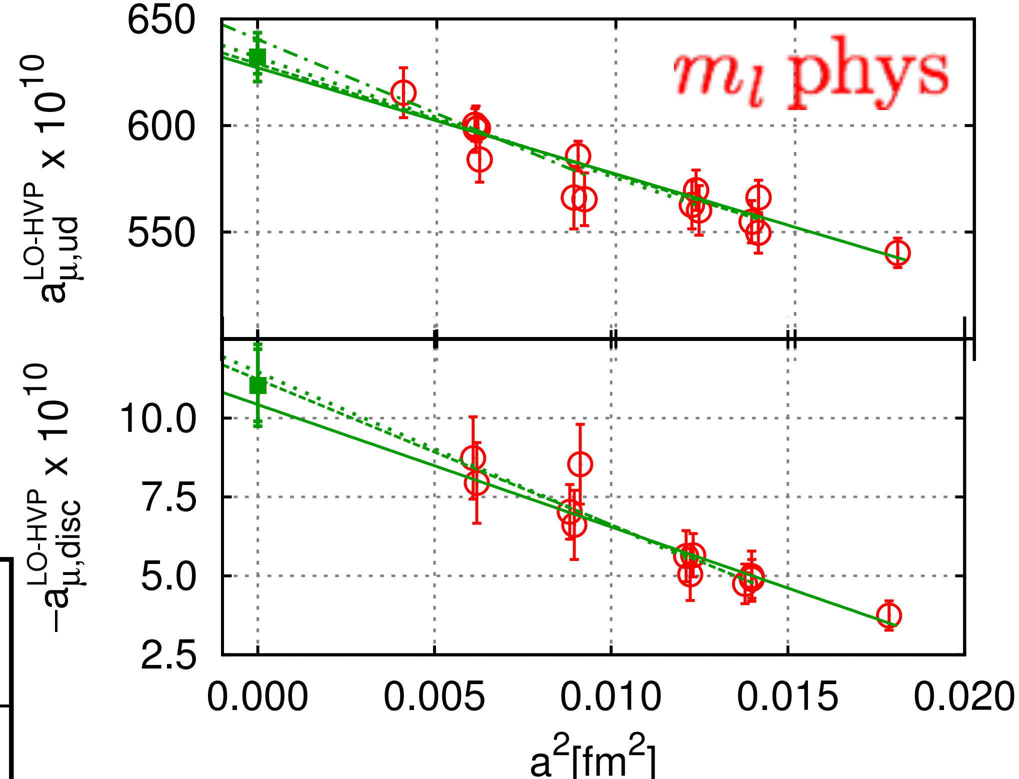
HPQCD (1601.03071): 64,000 correlators per point, use fit to data at large t. Correct for lattice effect on $\pi^+\pi^-$ /finite vol using chiral p.th.

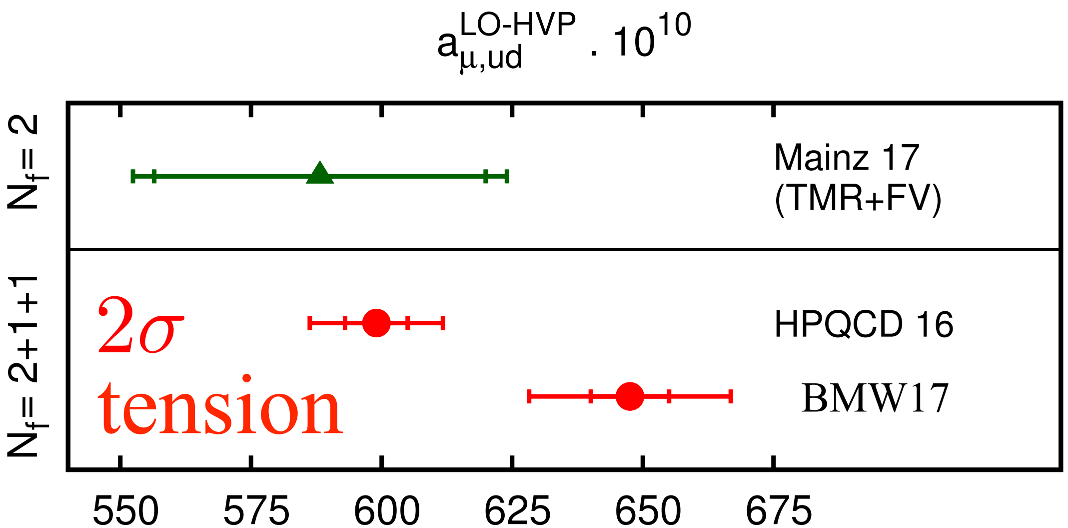


Rescale Π_j by $(m_\rho^{latt}/m_\rho^{expt})^{2j}$ very little dependence on m_l, a^2, volume - simple to fit.

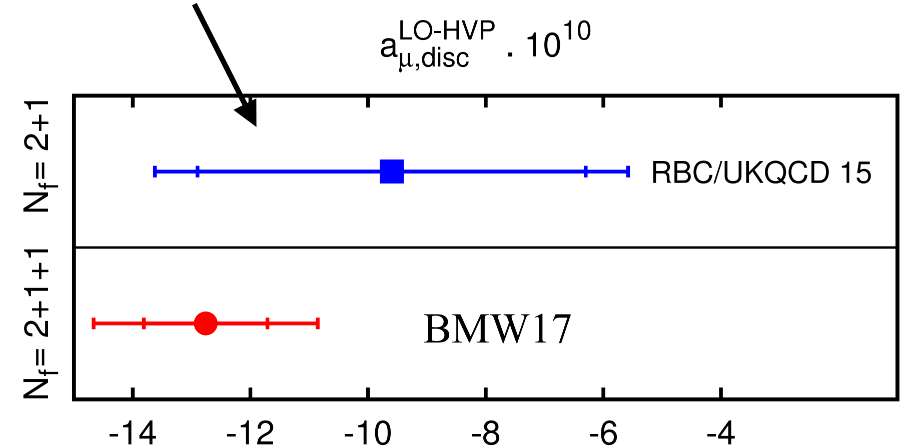
Errors from missing QED, $m_u \neq m_d$

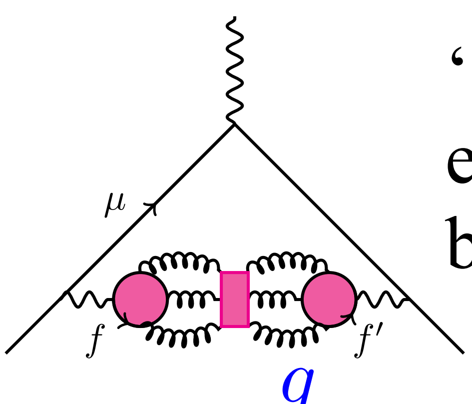
BMW(1711.04980): ~1million correlators per point, constrain $\pi^+\pi^-$from data. Large a-dependence (handled by extrapolation).





Also calculate small -ve ‘disconnected contribn’





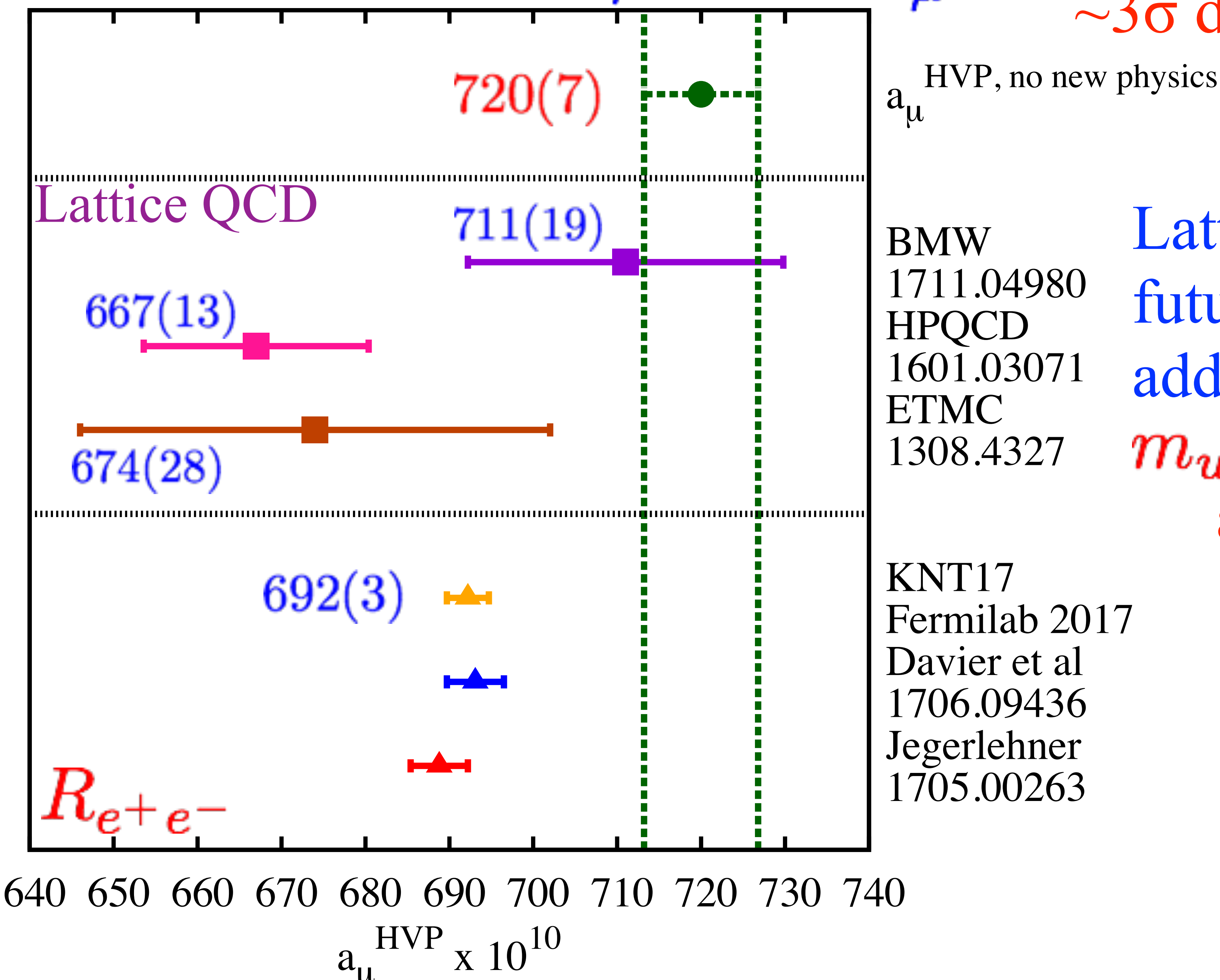
‘disc’ has u, d, s on each side, suppressed by q masses since

$$\sum_{u,d,s} Q_f = 0$$

Total LO HVP contribution - compare lattice QCD and e^+e^-

equivalent to testing a_μ^{expt} vs a_μ^{SM}

Most results show ~3σ discrepancy.



Lattice QCD future: better $\pi^+\pi^-$ add QED, $m_u \neq m_d$

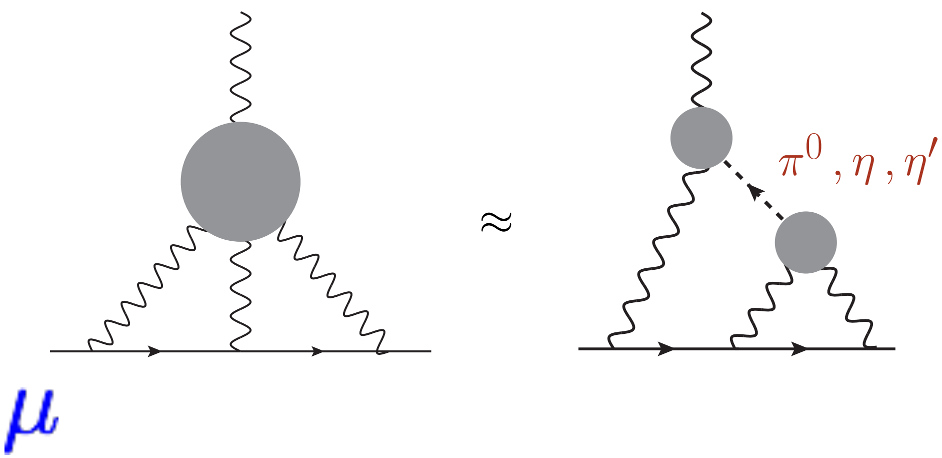
a small effect

RBC/UKQCD 1706.05293, HPQCD/FNAL/MILC 1710.11212

Elephant in the room? hadronic light-by-light contribution

Not simply related to experiment, values obtained use large N_c, chiral pert. th. etc.

'Glasgow Consensus' 2009: $a_\mu^{HLbL} = 10.5(2.6) \times 10^{-10}$



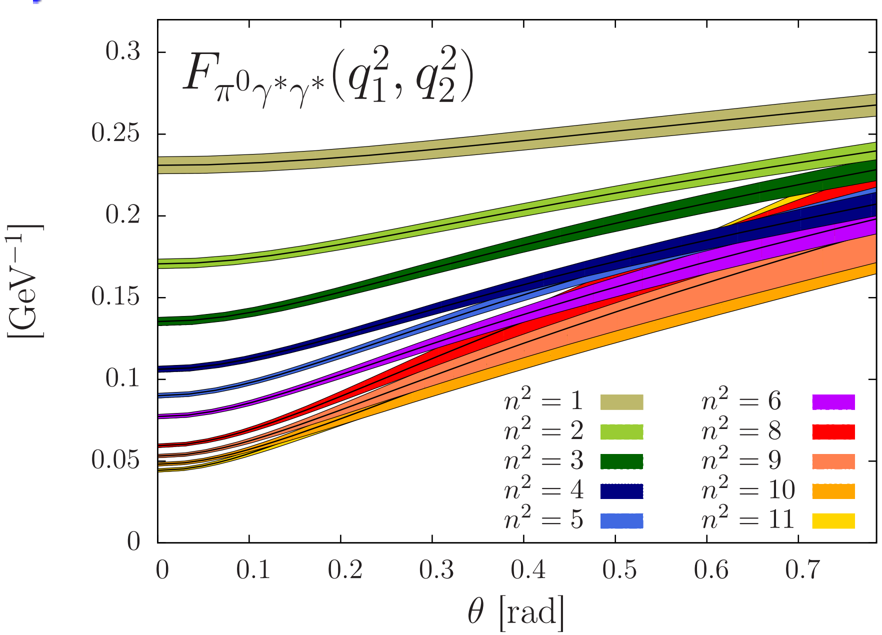
dominated by π^0 exchange :

there also OPE constraints

10% possible? with improved dispersive approaches (with imp. expt for e.g. $\pi^0 \to \gamma^*\gamma^*$

Nyffeler, 1602.03398

Colangelo et al, 1702.07347

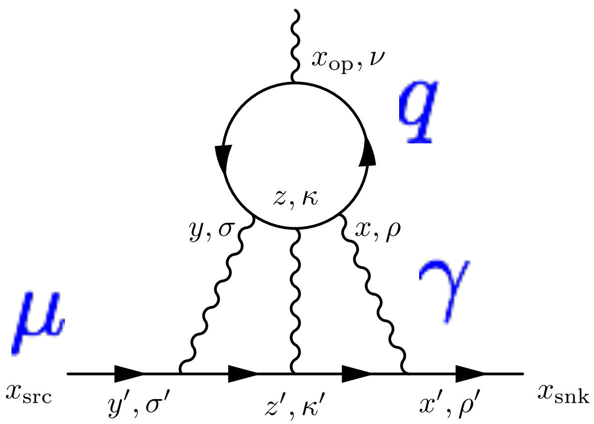


Lattice QCD calcs of $\mathcal{F}_{\pi^0\gamma^*\gamma^*}$ can test these approaches

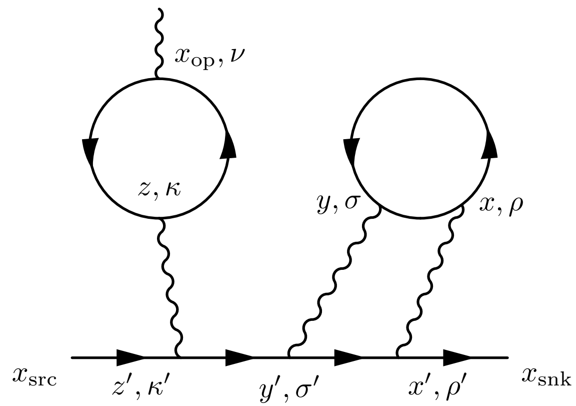
Mainz, 1607.08174,1712.00421

Direct computation of a_μ^{HLbL} in lattice QCD

RBC 1610.04603



‘connected’



leading ‘disconnected’

Note: gluons NOT shown

Calculate 4 quark propagators and combine with factors from muon and photon propagators, sum over points. Massless photon means that finite volume is an issue.

First result:
1 lattice spacing
physical m_l

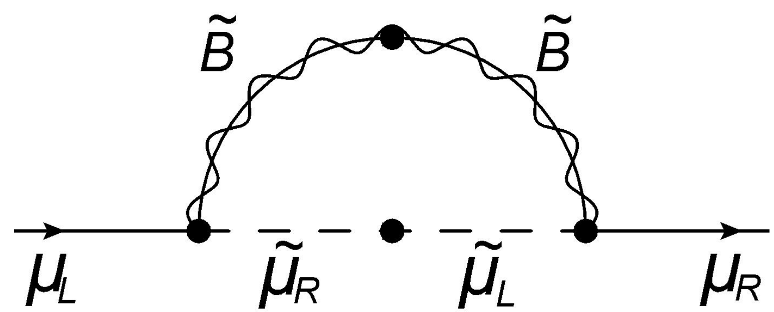
$$a_\mu^{HLbL} = 5.4(1.4) \times 10^{-10}$$ ← stat. errors only

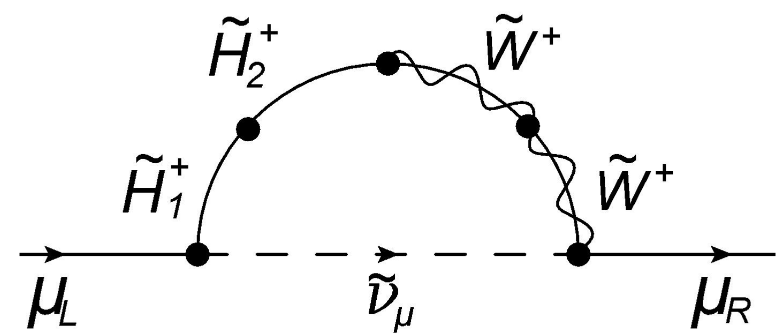
connected: 11.6 ; disc. : -6.3

improving finite-volume systematics:
Mainz, 1711.02466; RBC 1705.01067

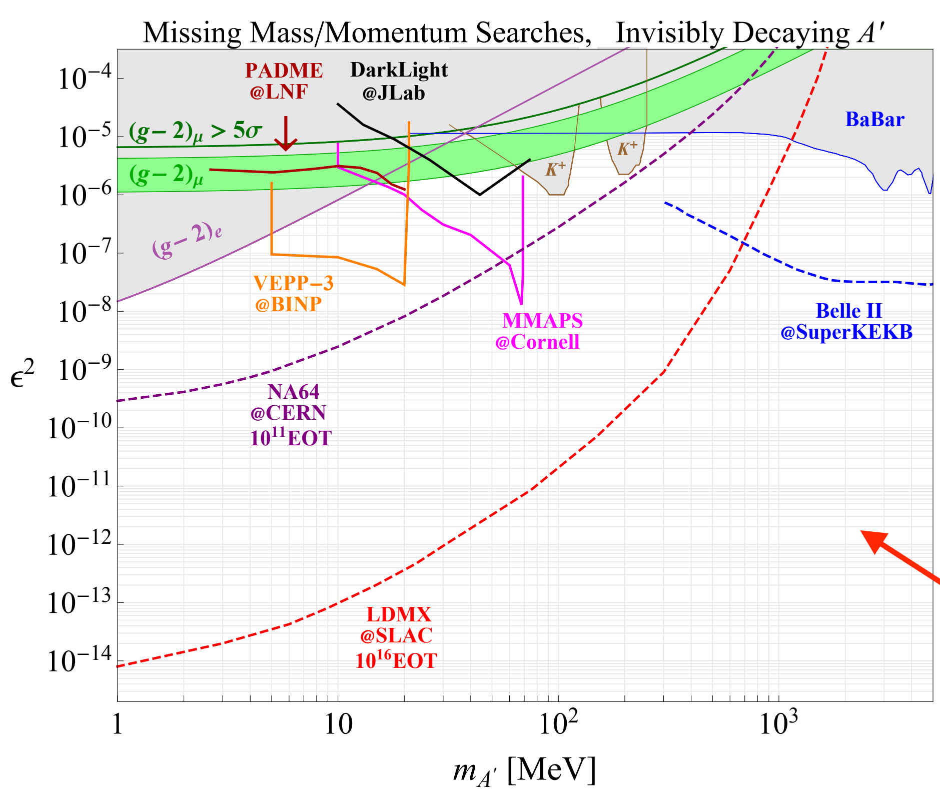
Beyond the Standard Model explanations for the discrepancy in a_μ ?

SUSY still a viable explanation
- more constrained now by LHC searches since need relatively light smuon and more fine-tuning.





e.g. Belyaev et al, MSSM with Pati-Salam at GUT scale 1605.02072



simple 'dark photon' mixing with ordinary photon now disfavoured

dark-sectors WG - 1608.08632

Conclusion

- $a_\mu^{E821} = 11659209.1(6.3) \times 10^{-10}$
  $a_\mu^{\rm SM} = 11659182.2(4.3) \times 10^{-10}$

disagreement $a_\mu^{\rm expt} - a_\mu^{\rm SM} = 27(8) \times 10^{-10}$

- SM uncertainty dominated by HVP.
 Methods using $R_{e^+e^-}$ have improved to 0.4%; lattice QCD results now at 2-3% - aim is <1% with QED and isospin-breaking included.
- HLbL determination will also improve - first direct lattice QCD results now available. It seems clearly small.
- Muon g-2 @FNAL will report its first new exptl result in 2019 - final aim is to reduce uncty by factor of 4.
If central value remains, this will be 5σ evidence for BSM