

Electroweak precision data and Higgs physics

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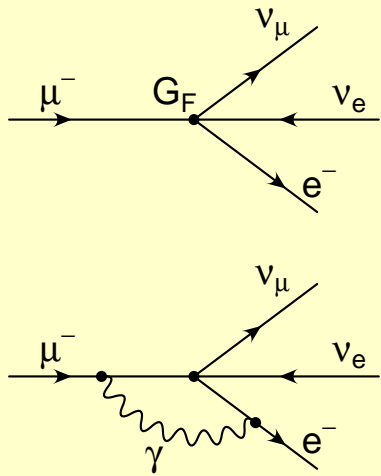
HEFT 2017

- 1. Overview of electroweak precision tests**
- 2. Effective operator description**
- 3. Connection between EWPO and HEFT**



W mass

μ decay in Fermi Model

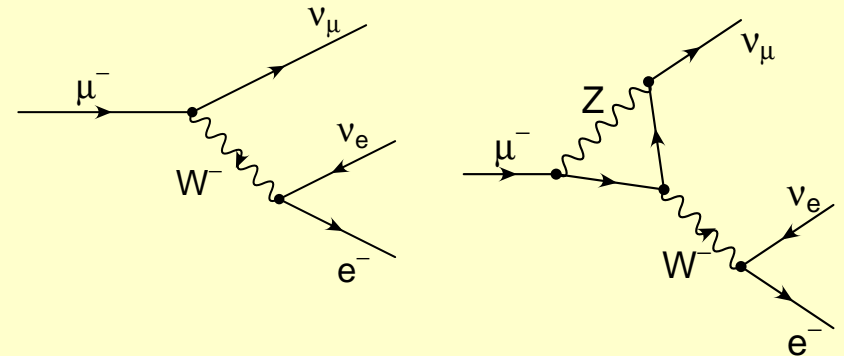


← QED corr.
(2-loop)

$$\Gamma_{\mu} = \frac{G_F^2 m_{\mu}^5}{192\pi^3} F\left(\frac{m_e^2}{m_{\mu}^2}\right) (1 + \Delta q)$$

Ritbergen, Stuart '98
Pak, Czarnecki '08

μ decay in Standard Model



$$\frac{G_F^2}{\sqrt{2}} = \frac{e^2}{8s_w^2 M_W^2} (1 + \Delta r)$$

electroweak corrections

■ Deconvolution of initial-state QED radiation:

$$\sigma[e^+e^- \rightarrow f\bar{f}] = \mathcal{R}_{ini}(s, s') \otimes \sigma_{hard}(s')$$

Kureav, Fadin '85

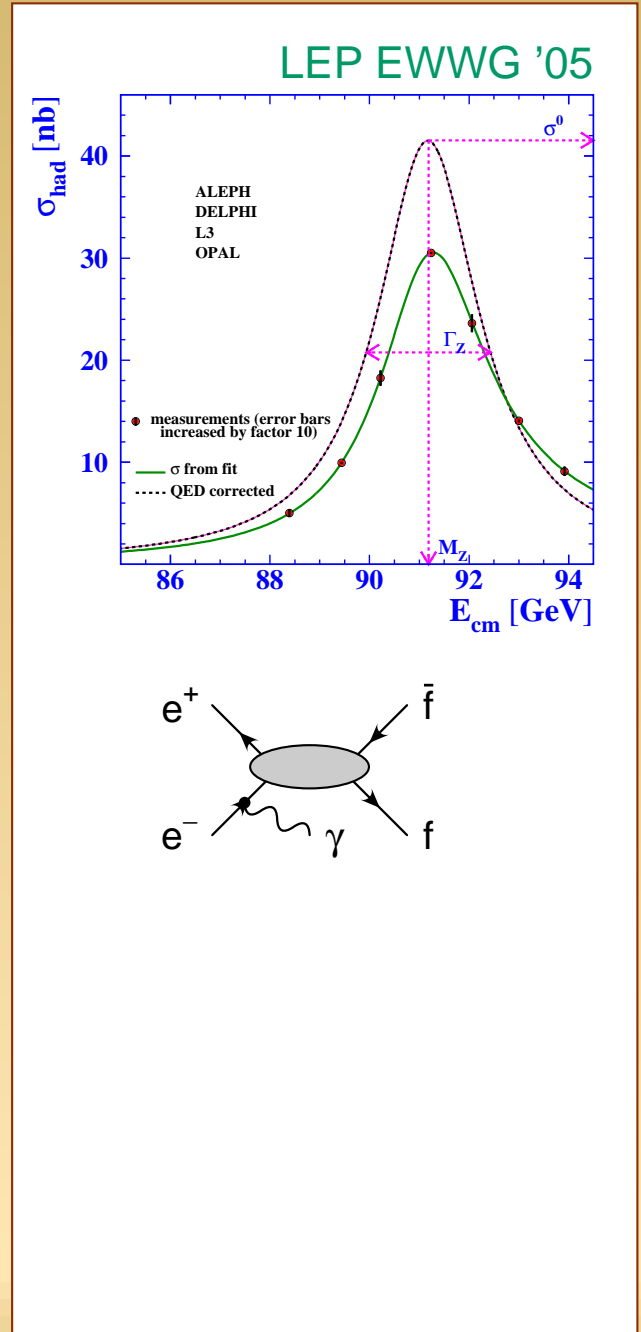
Berends, Burgers, v. Neerven '88

Kniehl, Krawczyk, Kühn, Stuart '88

Beenakker, Berends, v. Neerven '89

Skrzypek '92

Montagna, Nicosini, Piccinini '97

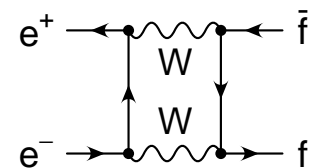
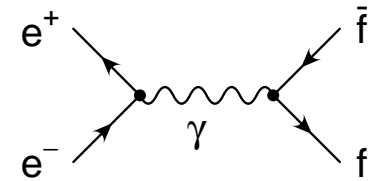
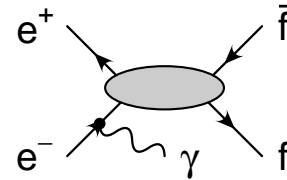
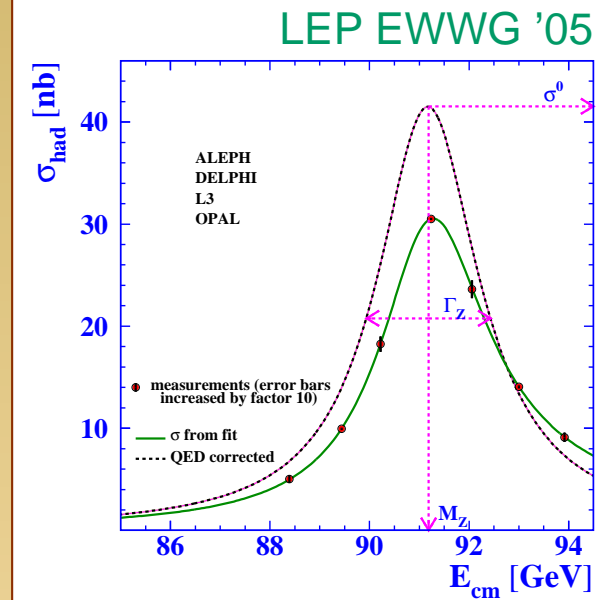


- Deconvolution of initial-state QED radiation:

$$\sigma[e^+e^- \rightarrow f\bar{f}] = \mathcal{R}_{ini}(s, s') \otimes \sigma_{\text{hard}}(s')$$

- Subtraction of γ -exchange, γ -Z interference, box contributions:

$$\sigma_{\text{hard}} = \sigma_Z + \sigma_\gamma + \sigma_{\gamma Z} + \sigma_{\text{box}}$$



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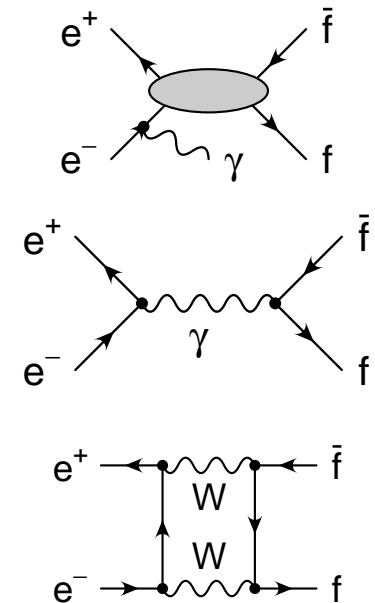
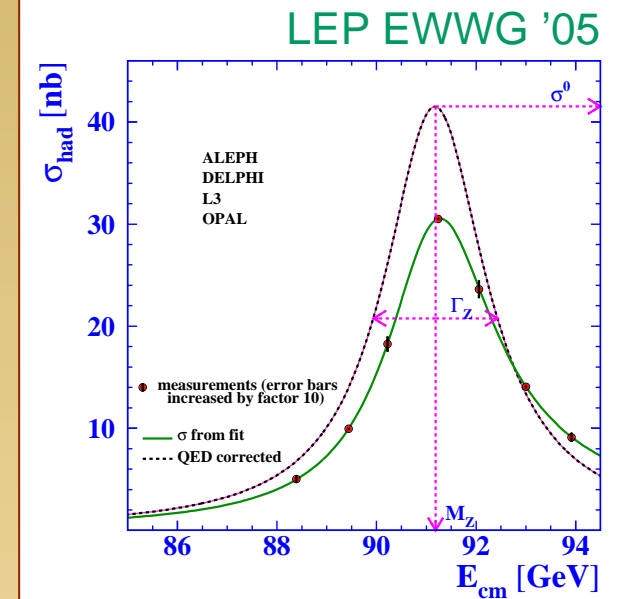
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- Z-pole contribution:

$$\sigma_Z = \frac{R}{(s - \overline{M}_Z^2)^2 + \overline{M}_Z^2 \overline{\Gamma}_Z^2} + \sigma_{\text{non-res}}$$



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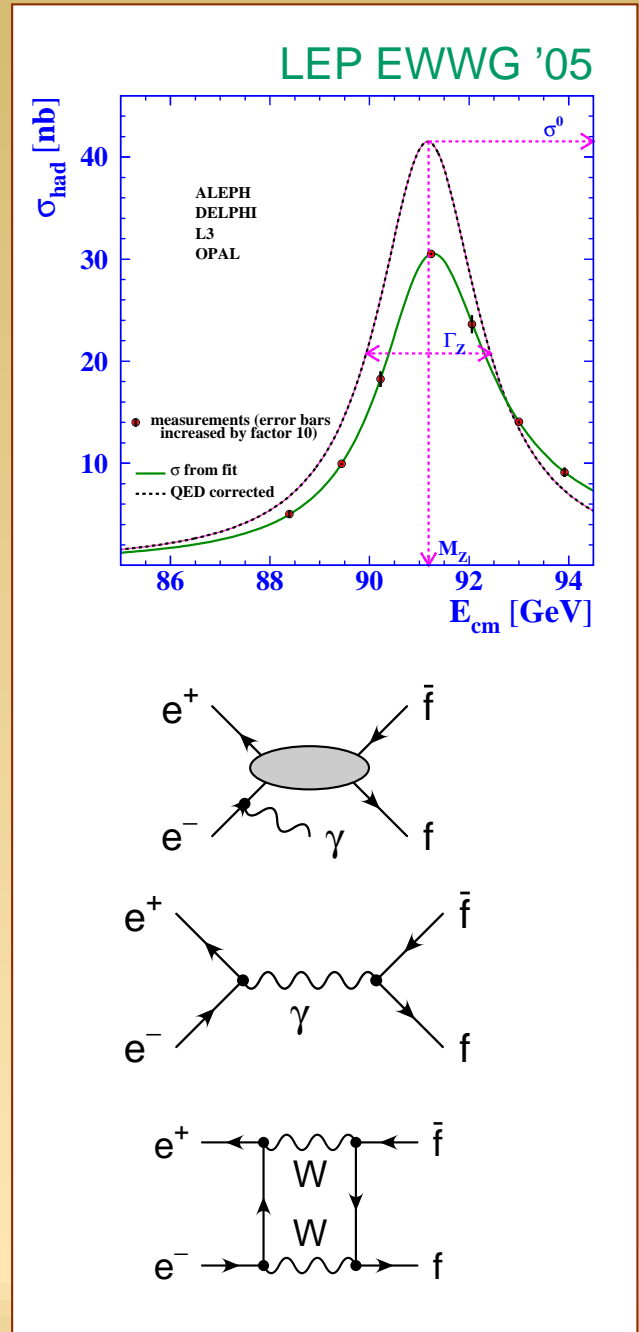
$$\sigma_Z = \frac{R}{(s - \overline{M}_Z^2)^2 + \overline{M}_Z^2 \overline{\Gamma}_Z^2} + \sigma_{\text{non-res}}$$

- In experimental analyses:

$$\sigma \sim \frac{1}{(s - M_Z^2)^2 + s^2 \Gamma_Z^2 / M_Z^2}$$

$$\overline{M}_Z = M_Z / \sqrt{1 + \Gamma_Z^2 / M_Z^2} \approx M_Z - 34 \text{ MeV}$$

$$\overline{\Gamma}_Z = \Gamma_Z / \sqrt{1 + \Gamma_Z^2 / M_Z^2} \approx \Gamma_Z - 0.9 \text{ MeV}$$

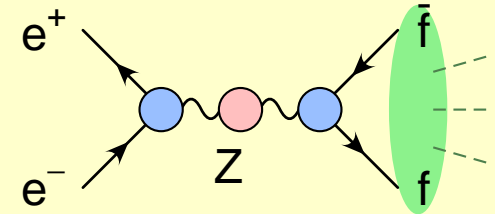


Total and partial Z widths:

$$\bar{\Gamma}_f = \Gamma[Z \rightarrow f\bar{f}]_{s=\bar{M}_Z^2} \quad \bar{\Gamma}_Z = \sum_f \bar{\Gamma}_f$$

$$\bar{\Gamma}_f \approx \frac{N_c \bar{M}_Z}{12\pi} \left[\left(\mathcal{R}_V^f |g_V^f|^2 + \mathcal{R}_A^f |g_A^f|^2 \right) \frac{1}{1 + \text{Re} \Sigma'_Z} \right]_{s=\bar{M}_Z^2}$$

$\mathcal{R}_V^f, \mathcal{R}_A^f$: Final-state QED/QCD radiation;
 g_V^f, g_A^f, Σ'_Z : Electroweak corrections



Branching ratios:

$$R_q = \bar{\Gamma}_q / \bar{\Gamma}_{\text{had}} \quad (q = b, c, \text{ probes heavy quark generations})$$

$$R_\ell = \bar{\Gamma}_{\text{had}} / \bar{\Gamma}_\ell \quad (\ell = e, \mu, \tau)$$

Peak cross section:

$$\sigma_{\text{had}}^0 = \sigma_Z(s = \overline{M_Z^2}) = \frac{12\pi}{\overline{M_Z^2}} \sum_q \frac{\overline{\Gamma}_e \overline{\Gamma}_q}{\overline{\Gamma}_Z^2} (1 + \delta X)$$

↳ NNLO correction term

Z-pole asymmetries / effective weak mixing angle:

$$A_{\text{FB}}^f \equiv \frac{\sigma(\theta < \frac{\pi}{2}) - \sigma(\theta > \frac{\pi}{2})}{\sigma(\theta < \frac{\pi}{2}) + \sigma(\theta > \frac{\pi}{2})} = \frac{3}{4} \mathcal{A}_e \mathcal{A}_f$$

$$A_{\text{LR}} \equiv \frac{\sigma(\mathcal{P}_e > 0) - \sigma(\mathcal{P}_e < 0)}{\sigma(\mathcal{P}_e > 0) + \sigma(\mathcal{P}_e < 0)} = \mathcal{A}_e$$

$$\mathcal{A}_f = 2 \frac{g_V^f / g_A^f}{1 + (g_V^f / g_A^f)^2} = \frac{1 - 4|Q_f| \sin^2 \theta_{\text{eff}}^f}{1 - 4|Q_f| \sin^2 \theta_{\text{eff}}^f + 8(|Q_f| \sin^2 \theta_{\text{eff}}^f)^2}$$

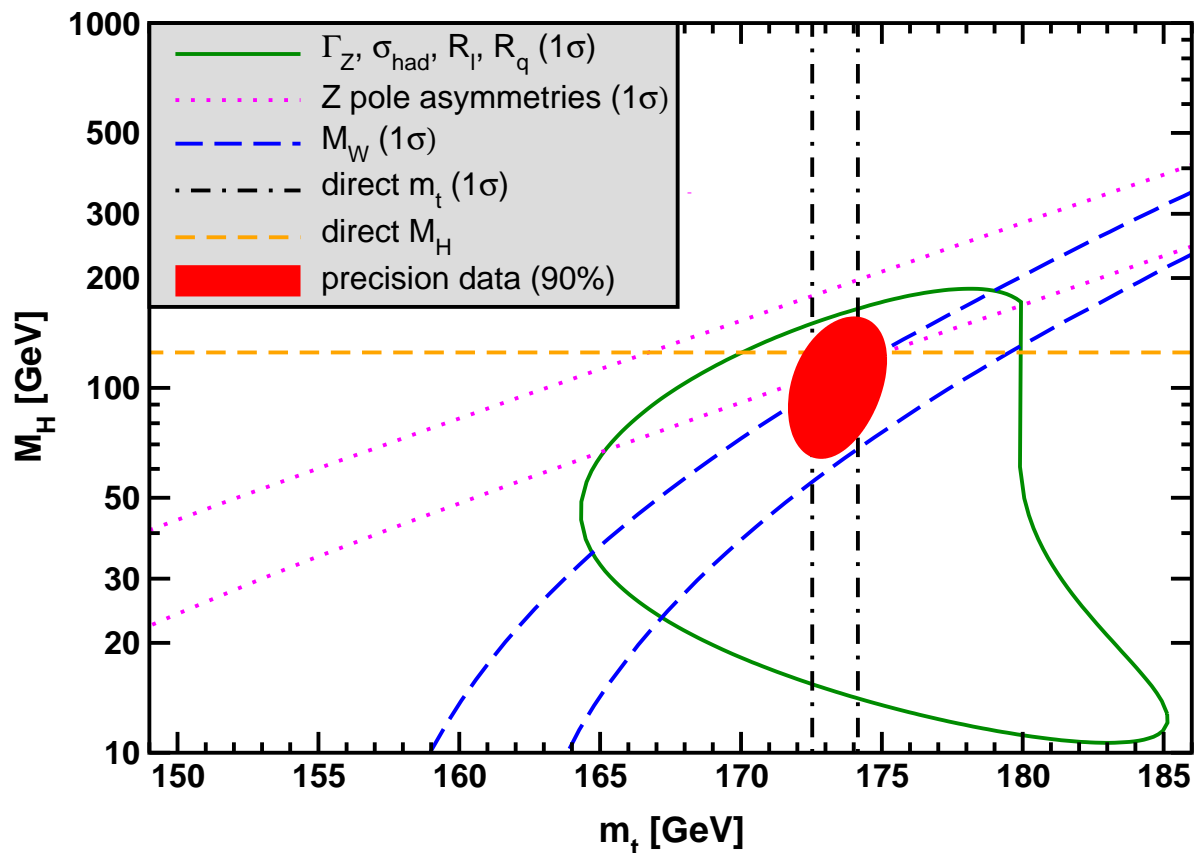
Most precisely measured for $f = \ell$ (also $f = b, c$)

	Experiment	Theory error	Main source
M_W	80385 ± 15 MeV	4 MeV	$\alpha^3, \alpha^2\alpha_s$
Γ_Z	2495.2 ± 2.3 MeV	0.5 MeV	$\alpha_{\text{bos}}^2, \alpha^3, \alpha^2\alpha_s, \alpha\alpha_s^2$
σ_{had}^0	41540 ± 37 pb	6 pb	$\alpha_{\text{bos}}^2, \alpha^3, \alpha^2\alpha_s$
R_b	0.21629 ± 0.00066	0.00015	$\alpha_{\text{bos}}^2, \alpha^3, \alpha^2\alpha_s$
$\sin^2 \theta_{\text{eff}}^l$	0.23153 ± 0.00016	4.5×10^{-5}	$\alpha^3, \alpha^2\alpha_s$

Standard Model:

- Good agreement between measured mass and indirect prediction
- Very good agreement over large number of observables

Erlar '16



Direct measurements:

$$M_H = 125.09 \pm 0.24 \text{ GeV}$$

$$m_t = 173.34 \pm 0.81 \text{ GeV}$$

Indirect prediction:

$$M_H = 126.1 \pm 1.9 \text{ GeV}$$

(with LHC BRs)

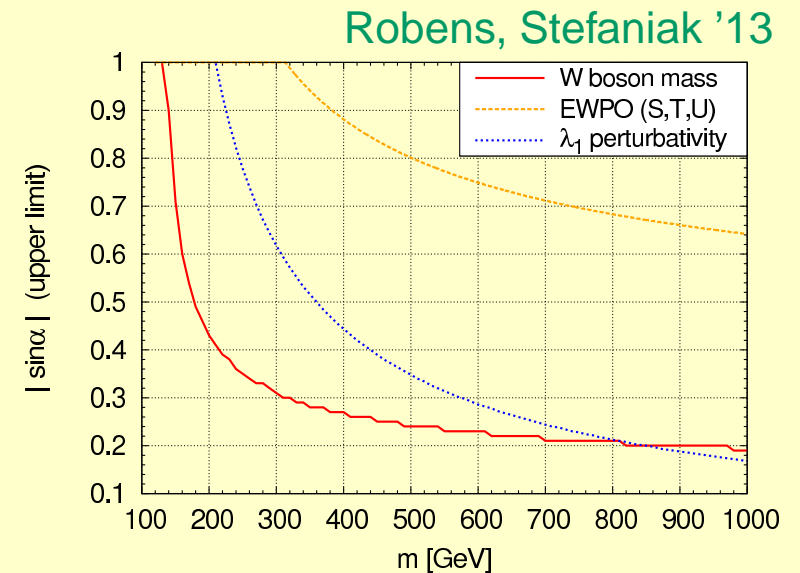
$$M_H = 96_{-19}^{+22} \text{ GeV}$$

(w/o LHC data)

$$m_t = 176.7 \pm 2.1 \text{ GeV}$$

Higgs singlet extension:

Constraints on singlet mass and mixing angle



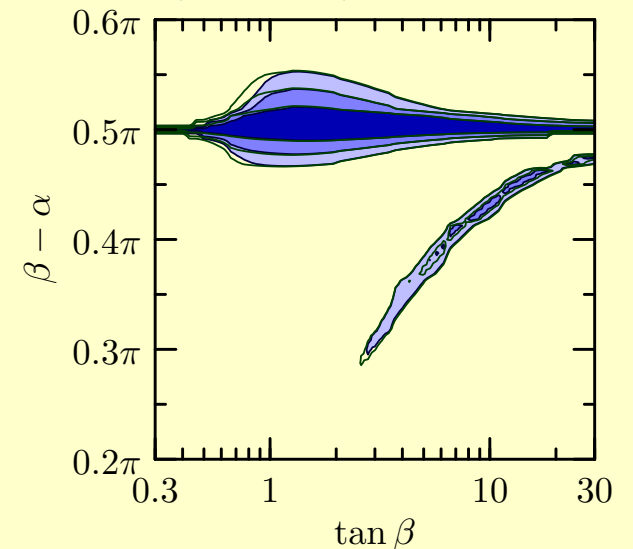
Two-Higgs-Doublet Model:

Constraints on couplings of SM-like Higgs

$$\left| \frac{g_{hVV}^{\text{THDM}}}{g_{hVV}^{\text{SM}}} \right| = \sin(\beta - \alpha),$$

$$\left| \frac{g_{hff}^{\text{THDM}}}{g_{hff}^{\text{SM}}} \right| = \frac{\cos \alpha}{\sin \alpha} \text{ or } \frac{\sin \alpha}{\cos \alpha}$$

Eberhardt, Nierste, Wiebusch '13

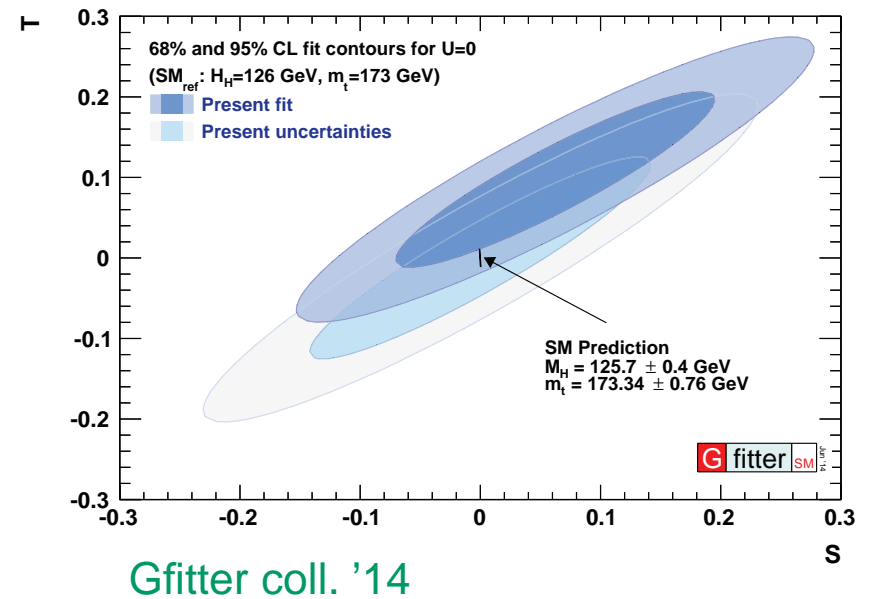


Oblique parameters:

$$\alpha T = \frac{\Sigma_{WW}(0)}{M_W} - \frac{\Sigma_{ZZ}(0)}{M_Z}$$

$$\frac{\alpha}{4s^2c^2} S = \frac{\Sigma_{ZZ}(M_Z^2) - \Sigma_{ZZ}(0)}{M_Z}$$

$$+ \frac{s^2 - c^2}{sc} \frac{\Sigma_{Z\gamma}(M_Z^2)}{M_Z} - \frac{\Sigma_{\gamma\gamma}(M_Z^2)}{M_Z}$$

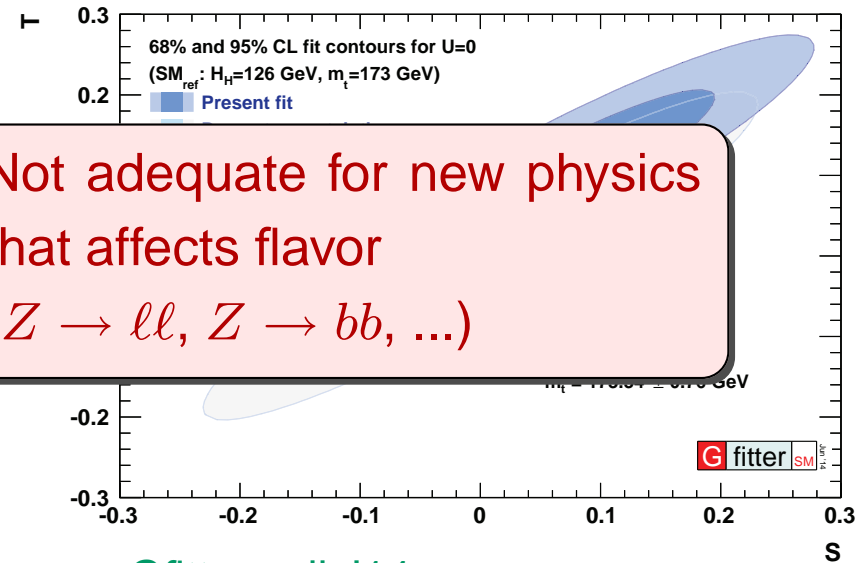


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Not adequate for new physics that affects flavor ($Z \rightarrow \ell\ell, Z \rightarrow bb, \dots$)

Gfitter coll. '14

Effective field theory: $\mathcal{L} = \sum_i \frac{c_i}{\Lambda^2} \mathcal{O}_i + \mathcal{O}(\Lambda^{-3}) \quad (\Lambda \gg M_Z)$

Contributions at tree-level:

$$\mathcal{O}_{\phi 1} = (D_\mu \Phi)^\dagger \Phi \Phi^\dagger (D^\mu \Phi)$$

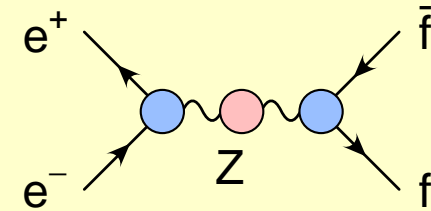
$$\mathcal{O}_{\text{BW}} = \Phi^\dagger B_{\mu\nu} W^{\mu\nu} \Phi$$

$$\mathcal{O}_{\text{LL}}^{(3)e} = (\bar{L}_L^e \sigma^a \gamma_\mu L_L^e) (\bar{L}_L^e \sigma^a \gamma^\mu L_L^e)$$

$$\mathcal{O}_R^f = i(\Phi^\dagger \overleftrightarrow{D}_\mu \Phi) (\bar{f}_R \gamma^\mu f_R)$$

$$\mathcal{O}_L^F = i(\Phi^\dagger \overleftrightarrow{D}_\mu \Phi) (\bar{F}_L \gamma^\mu F_L)$$

$$\mathcal{O}_L^{(3)F} = i(\Phi^\dagger \overleftrightarrow{D}_\mu^a \Phi) (\bar{F}_L \sigma_a \gamma^\mu F_L)$$



$$f = e, \mu, \tau, b, lq$$

$$F = \begin{pmatrix} \nu_e \\ e \end{pmatrix}, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}, \begin{pmatrix} u, c \\ d, s \end{pmatrix}, \begin{pmatrix} t \\ b \end{pmatrix}$$

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Contributions at tree-level:

$$\mathcal{O}_{\phi 1} = (D_\mu \Phi)^\dagger \Phi \Phi^\dagger (D^\mu \Phi)$$

$$\alpha \Delta T = -\frac{v^2}{2} \frac{c_{\phi 1}}{\Lambda^2}$$

$$\mathcal{O}_{\text{BW}} = \Phi^\dagger B_{\mu\nu} W^{\mu\nu} \Phi$$

$$\alpha \Delta S = -e^2 v^2 \frac{c_{\text{BW}}}{\Lambda^2}$$

$$\mathcal{O}_{\text{LL}}^{(3)e} = (\bar{L}_L^e \sigma^a \gamma_\mu L_L^e) (\bar{L}_L^e \sigma^a \gamma^\mu L_L^e)$$

$$\Delta G_F = -\sqrt{2} \frac{c_{\text{LL}}^{(3)e}}{\Lambda^2}$$

$$\mathcal{O}_R^f = i(\Phi^\dagger \overleftrightarrow{D}_\mu \Phi) (\bar{f}_R \gamma^\mu f_R)$$

$$\mathcal{O}_L^F = i(\Phi^\dagger \overleftrightarrow{D}_\mu \Phi) (\bar{F}_L \gamma^\mu F_L)$$

$$\mathcal{O}_L^{(3)F} = i(\Phi^\dagger \overleftrightarrow{D}_\mu^a \Phi) (\bar{F}_L \sigma_a \gamma^\mu F_L)$$

} effect on $Z \rightarrow f \bar{f}$

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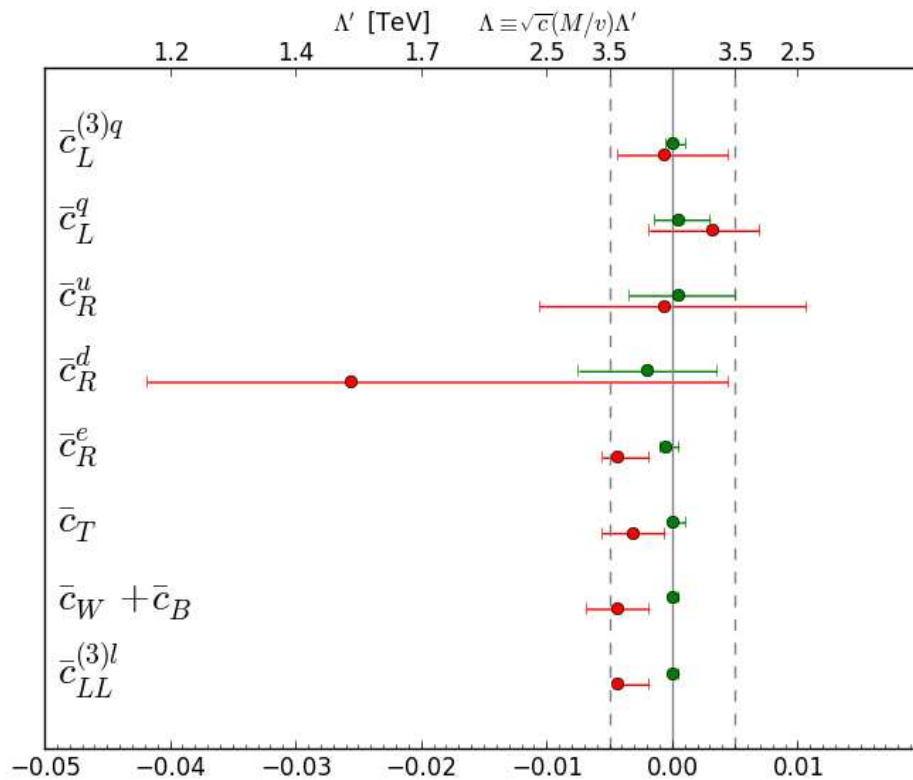
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$$\mathcal{O}_L^{(3)F} = i(\Phi^\dagger \overleftrightarrow{D}_\mu^a \Phi) (\bar{F}_L \sigma_a \gamma^\mu F_L)$$

} relevant for Higgs physics,
but strongly bounded from EWPO

} irrelevant for Higgs physics

Assuming flavor universality:



Significant correlation/
degeneracy between
different operators

Pomaral, Riva '13
Ellis, Sanz, You '14

Contributions at 1-loop-level:

$$\mathcal{O}_H = \frac{1}{2} \partial_\mu (\Phi^\dagger \Phi) \partial^\mu (\Phi^\dagger \Phi)$$

$$\mathcal{O}_B = i (D_\mu \Phi)^\dagger B^{\mu\nu} (D_\nu \Phi)$$

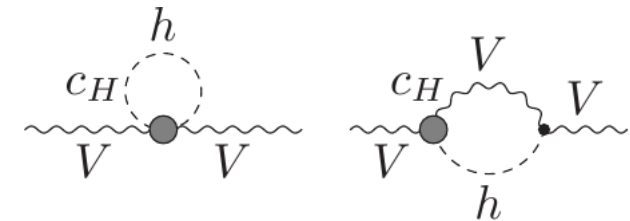
$$\mathcal{O}_W = i (D_\mu \Phi)^\dagger W^{\mu\nu} (D_\nu \Phi)$$

$$\mathcal{O}_{BB} = -\Phi^\dagger B^{\mu\nu} B_{\mu\nu} \Phi$$

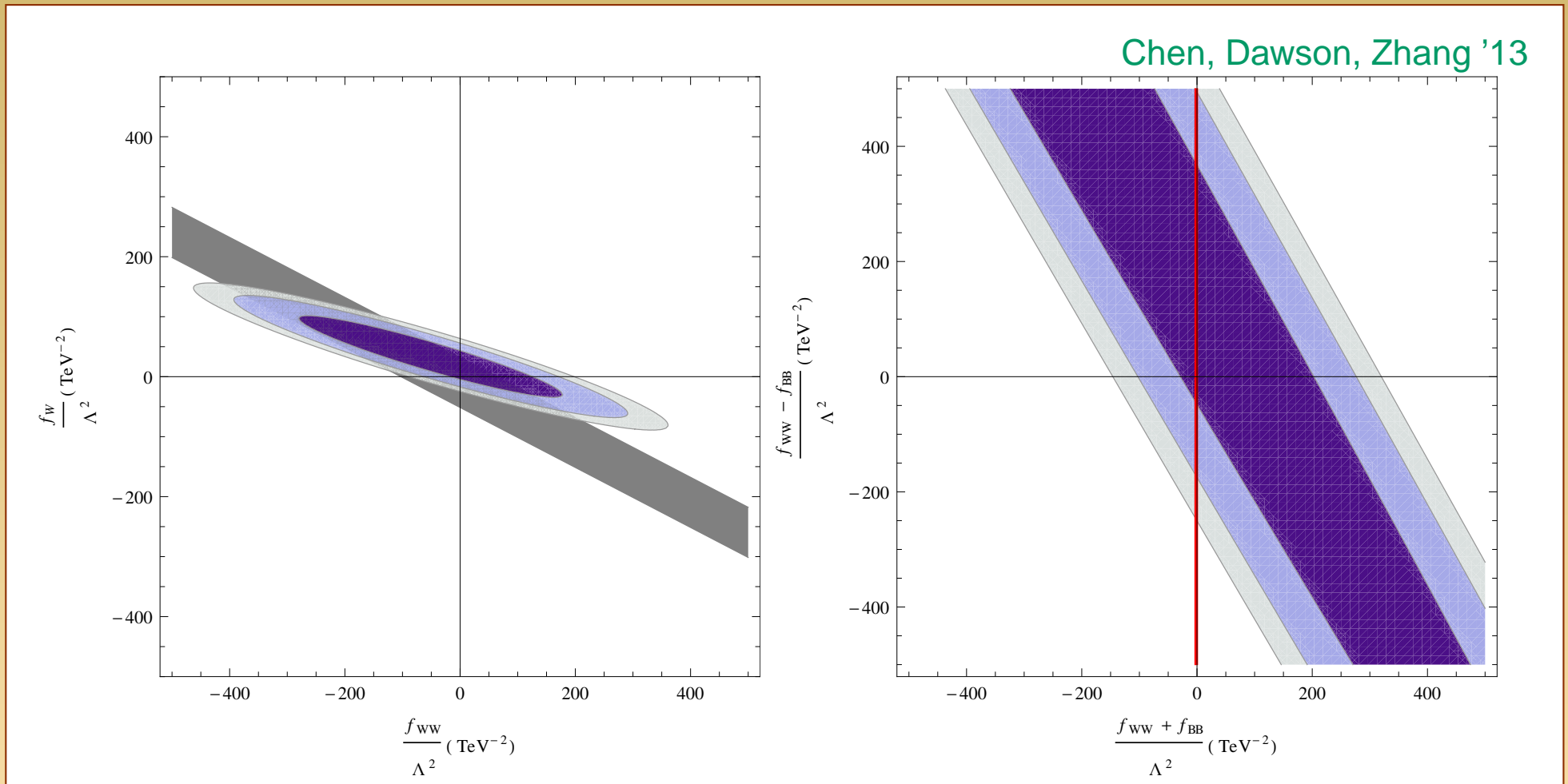
$$\mathcal{O}_{WW} = -\Phi^\dagger W^{\mu\nu} W_{\mu\nu} \Phi$$

+ few more

→ Direct correlation with Higgs production and decay rates



Chen, Dawson, Zhang '13
Hartmann, Shephard, Trott '16



Assumptions:

- Tree-level EWPO operators are zero
- Loop contributions are UV-divergent → must choose renormalization scheme

- Competitive and complementary sensitivity to HEFT operators from Higgs physics and EWPO @ NLO
- SMEFT @ NLO requires (model-dependent) assumptions

More operators than EWPOs

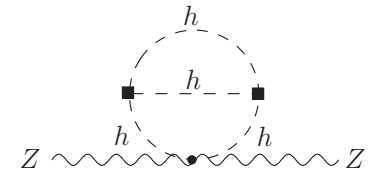
- a) 1-loop contributions from $\mathcal{O}_H, \mathcal{O}_B, \mathcal{O}_W, \mathcal{O}_{BB}, \mathcal{O}_{WW}$ can be absorbed into tree-level contributions from $\mathcal{O}_{\phi 1}, \mathcal{O}_{BW}, \dots$
 - No sensitivity on $\mathcal{O}_H, \mathcal{O}_B, \dots$ since $\mathcal{O}_{\phi 1}, \mathcal{O}_{BW}, \dots$ dominate
- b) Assume some operators ($\mathcal{O}_{\phi 1}, \mathcal{O}_{BW}, \dots$) vanish
 - Depends on renormalization scheme and scale
 - Compatible with some UV-completions, but not all

Renormalization

- Loop diagrams with HEFT operator insertions are in general UV-divergent

[Exceptions e.g. $\mathcal{O}_6 = (\Phi^\dagger \Phi)^3$ in 2-loop diagrams

Kribs, Maier, Rzehak, Spannowsky, Waite '17]



- Divergencies absorbed in operators entering at tree-level ($\mathcal{O}_{\phi 1}$, \mathcal{O}_{BW} , ...)

- Ambiguity due to choice of renormalization

(\overline{MS} , \overline{MS} with BFM, ...)

Mebane, Greiner, Zhang, Willenbrock '13

Hartmann, Trott '15

- Ambiguity due to choice of scale:

$\mathcal{O}_{\phi 1}$, \mathcal{O}_{BW} , ... can only be zero at one scale μ_0

RG running and mixing

- Operators in loops and renormalization can be treated through RG evolution (as well as resummation of $\log \frac{\Lambda}{\mu}$ terms)

Shiftman, Vainshtein, Zakharov '77; Floratos, Ross, Sachrajda '77; Gilman, Wise '79
Morozov '84; Hagiwara, Ishihara, Szalapski, Zeppenfeld '93; Han, Skiba '04
Grojean, Jenkins, Manohar, Trott '13; Jenkins, Manohar, Trott '13
Alonso, Jenkins, Manohar, Trott '13; Elias-Miro, Espinosa, Masso, Pomarol '13

...

- Currently only leading-log (LL) approximation known;
for Higgs physics today $\Lambda/\mu < 10$

Englert et al. '14

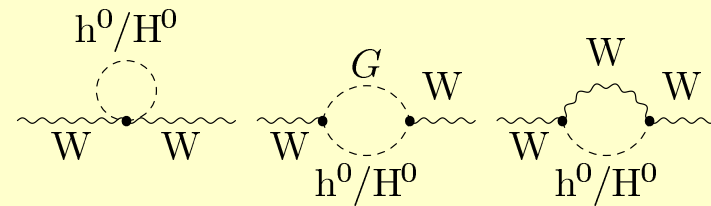
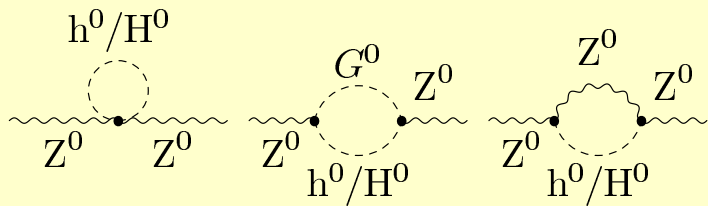
SM + singlet scalar S

$$V(\Phi, S) = \mu_1^2 \Phi^\dagger \Phi + \lambda_1 (\Phi^\dagger \Phi)^2 + \mu_2^2 S^2 + \lambda_2 S^4 + \lambda_3 (\Phi^\dagger \Phi) S^2$$

$$\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & v \end{pmatrix}^\top \quad \langle S \rangle = v_s / \sqrt{2}$$

For $v_s \gg v$: $m_H^2 \approx \Lambda^2 \equiv 2\lambda_2 v_s^2$ $\sin^2 \alpha \approx \frac{\lambda_3^2}{2\lambda_2} \frac{v^2}{\Lambda^2}$

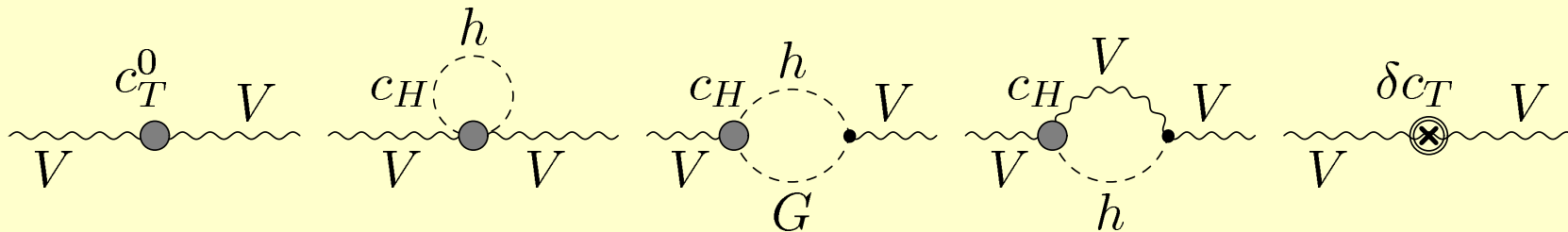
Oblique parameters:



$$S \approx \frac{\lambda_3^2}{24\pi\lambda_2} \frac{v^2}{m_H^2} \ln \frac{m_H^2}{m_h^2} + \dots$$

$$T \approx -\frac{3\lambda_3^2}{32\pi c_W^2 \lambda_2} \frac{v^2}{m_H^2} \ln \frac{m_H^2}{m_h^2} + \dots$$

EFT description:



LO:

$$\alpha T = -c_{\phi 1} \frac{v^2}{\Lambda^2}$$

$$\mathcal{O}_{\phi 1} = (D_\mu \Phi)^\dagger \Phi \Phi^\dagger (D^\mu \Phi)$$

NLO:

$$\alpha T = \frac{3\alpha c_H}{16\pi s_w^2 M_W^2} \frac{v^2}{\Lambda^2} \left[\left\{ \frac{M_Z^2 m_h^2}{m_h^2 - M_Z^2} \ln \frac{m_h^2}{M_Z^2} \right\} - \{M_Z \leftrightarrow M_W\} \right]$$

$$\mathcal{O}_H = \frac{1}{2} \partial_\mu (\Phi^\dagger \Phi) \partial^\mu (\Phi^\dagger \Phi)$$

Note: NLO contribution is finite, so cannot be obtained through RG mixing

Matching: $(\mu = M_W)$

Freitas, Lopez-Val, Plehn '16

LL-L:
$$c_{\phi 1} = \frac{3\alpha_s^2 \lambda_3^2}{32\pi c_W^2 \lambda_2} \ln \frac{\Lambda^2}{\mu^2}$$

$$c_H = 0$$

LL-TL:
$$c_{\phi 1} = \frac{3\alpha_s^2 \lambda_3^2}{32\pi c_W^2 \lambda_2} \ln \frac{\Lambda^2}{\mu^2}$$

$$c_H = \lambda_3^2 / (2\lambda_2)$$

BP-TL:
$$c_{\phi 1} = \frac{\alpha_s^2 \lambda_3^2}{32\pi c_W^2 \lambda_2} \left(3 \ln \frac{\Lambda^2}{\mu^2} - \frac{5}{2} \right)$$

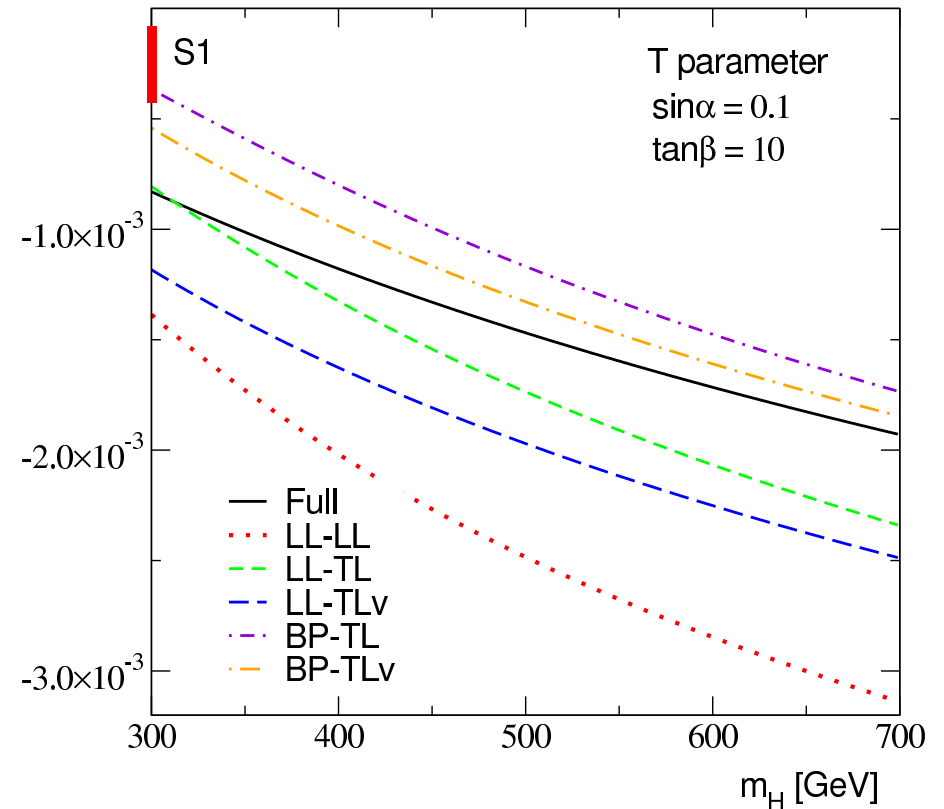
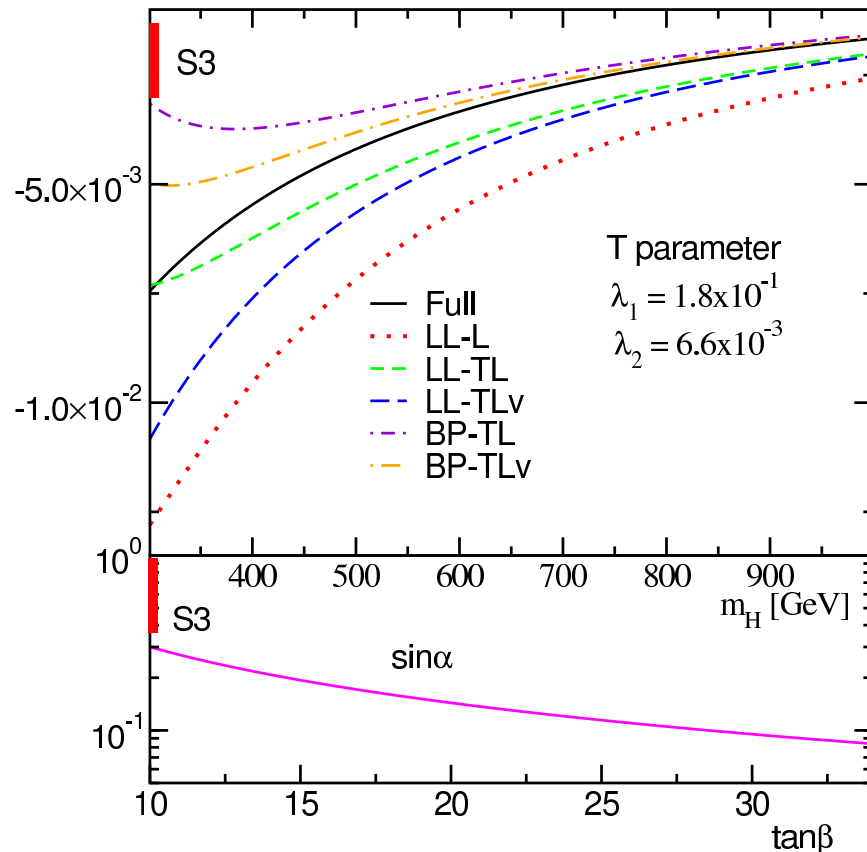
$$c_H = \lambda_3^2 / (2\lambda_2)$$

v -improv.: $\Lambda \rightarrow m_H, \frac{\lambda_3^2}{2\lambda_2} \rightarrow \sin^2 \alpha \frac{\Lambda^2}{v^2}$

Brehmer, Freitas, Lopez-Val, Plehn '15

Sample benchmarks:

Freitas, Lopez-Val, Plehn '16



- LL approximation not adequate
- Full 1-loop matching (with v -induced terms) necessary to relate EWPO–HEFT
- “Phenomenological” operators to be defined at weak scale

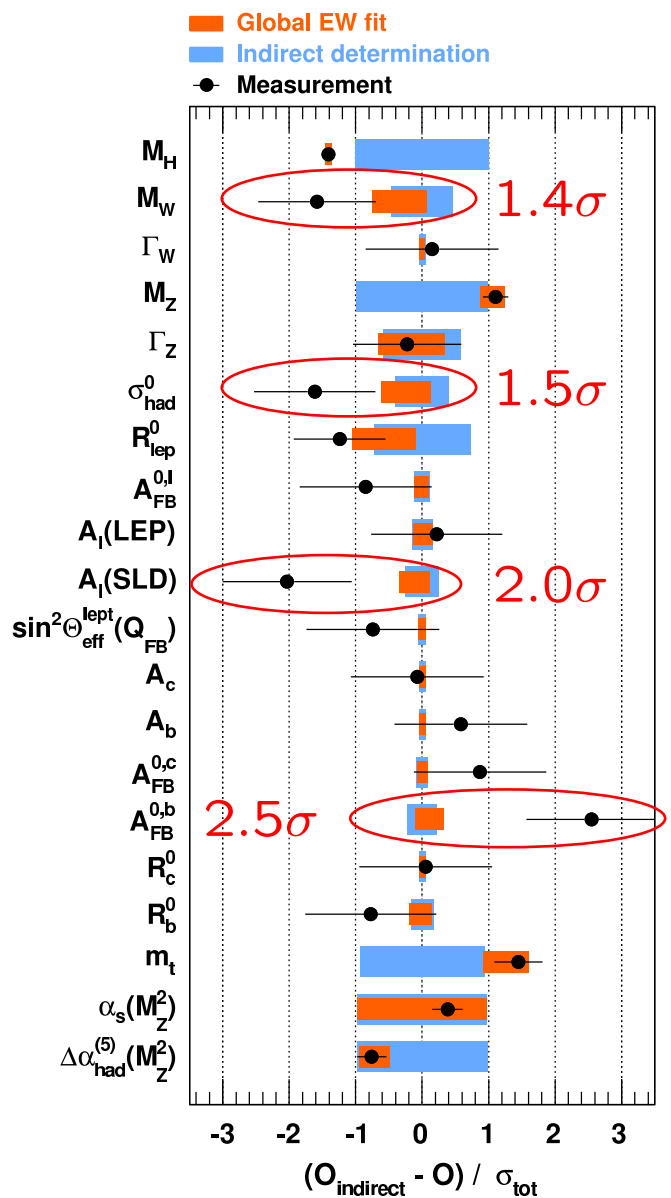
- **Electroweak precision tests** can probe Higgs physics beyond the Standard Model
- Model independent description of new physics through **dim-6 operators**
- Sensitivity of EWPO to HEFT operators mostly through **loop contributions**
 - Analysis depends on (model-dependent) assumptions
- Full **1-loop matching** important for connecting HEFT to UV-complete models

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Backup slides

Current status of electroweak precision tests



Surprisingly good agreement:

$$\chi^2/d.o.f. = 18.1/14 \quad (p = 20\%)$$

Most quantities measured with
1%–0.1% precision

A few interesting deviations:

$$M_W \quad (\sim 1.4\sigma)$$

$$\sigma_{had}^0 \quad (\sim 1.5\sigma)$$

$$A_l(SLD) \quad (\sim 2\sigma)$$

$$A_{FB}^b \quad (\sim 2.5\sigma)$$

$$(g_\mu - 2) \quad (\sim 3\sigma)$$

GFitter coll. '14

Future projections

ILC: High-energy e^+e^- linear collider, running at $\sqrt{s} \approx M_Z$ with 30 fb^{-1}

CEPC: Circular e^+e^- collider, running at $\sqrt{s} \approx M_Z$ with $2 \times 150 \text{ fb}^{-1}$

FCC-ee: Circular e^+e^- collider, running at $\sqrt{s} \approx M_Z$ with $4 \times 3000 \text{ fb}^{-1}$

	Current exp.	ILC	CEPC	FCC-ee	Current perturb.
M_W [MeV]	15	3–4	3	1	4
Γ_Z [MeV]	2.3	0.8	0.5	0.1	0.5
R_b [10^{-5}]	66	14	17	6	15
$\sin^2 \theta_{\text{eff}}^l$ [10^{-5}]	16	1	2.3	0.6	4.5

→ Existing theoretical calculations adequate for LEP/SLC/LHC,
but not ILC/CEPC/FCC-ee!

Theory and parametric uncertainties

	ILC	CEPC	perturb. error with 3-loop [†]	Param. error ILC*	Param. error CEPC**
M_W [MeV]	3–4	3	1	2.6	2.1
Γ_Z [MeV]	0.8	0.5	$\lesssim 0.2$	0.5	0.15
R_b [10^{-5}]	14	17	5–10	< 1	< 1
$\sin^2 \theta_{\text{eff}}^l$ [10^{-5}]	1	2.3	1.5	2	2

[†] **Theory scenario:** $\mathcal{O}(\alpha\alpha_s^2)$, $\mathcal{O}(N_f\alpha^2\alpha_s)$, $\mathcal{O}(N_f^2\alpha^2\alpha_s)$
 (N_f^n = at least n closed fermion loops)

Parametric inputs:

* **ILC:** $\delta m_t = 100$ MeV, $\delta\alpha_s = 0.001$, $\delta M_Z = 2.1$ MeV

****CEPC:** $\delta m_t = 600$ MeV, $\delta\alpha_s = 0.0002$, $\delta M_Z = 0.5$ MeV

also: $\delta(\Delta\alpha) = 5 \times 10^{-5}$

Theory uncertainties in extraction of pseudo-observables

■ Subtraction of QED radiation contributions

→ Known to $\mathcal{O}(\alpha^2)$, $\mathcal{O}(\alpha^3 L^3)$ for **ISR**,
 $\mathcal{O}(\alpha^2)$ for **FSR** and $\mathcal{O}(\alpha^2 L^2)$ for **A_{FB}**

$$(L = \log \frac{s}{m_e^2})$$

Berends, Burgers, v.Neerven '88

Kniehl, Krawczyk, Kühn, Stuart '88

Beenakker, Berends, v.Neerven '89

Skrzypek '92; Montagna, Nicrosini, Piccinini '97

→ $\mathcal{O}(0.1\%)$ uncertainty on σ_Z , A_{FB}

→ Improvement needed for ILC/CEPC/FCC-ee

■ Subtraction of non-resonant γ -exchange, γ -Z interf., box contributions, Bhabha scattering

see, e.g., Bardin, Grünewald, Passarino '99

→ $\mathcal{O}(0.01\%)$ uncertainty within SM

(improvements may be needed)

→ Sensitivity to some NP beyond EWPO

