SM EFT Contributions to Z Decay at One Loop

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Based on 1611.09879 with Christine Hartmann and Michael Trott

Why Loops?

- Electroweak observables have been measured with amazing precision
  - Theory calculations have to match this precision to get full value out of the data

Observable	Experimental Value	Ref.	SM Theoretical Value	Ref.
$\hat{m}_Z$[GeV]	91.1875 ± 0.0021	[38]	-	-
$\hat{m}_W$[GeV]	80.385 ± 0.015	[39]	80.365 ± 0.004	[40]
σ_h^0 [nb]	41.540 ± 0.037	[38]	41.488 ± 0.006	[41]
Γ_Z[GeV]	2.4952 ± 0.0023	[38]	2.4942 ± 0.0005	[41]
R_ℓ^0	20.767 ± 0.025	[38]	20.751 ± 0.005	[41]
R_b^0	0.21629 ± 0.00066	[38]	0.21580 ± 0.00015	[41]
R_c^0	0.1721 ± 0.0030	[38]	0.17223 ± 0.00005	[41]
A_{FB}^ℓ	0.0171 ± 0.0010	[38]	0.01616 ± 0.00008	[42]
A_{FB}^c	0.0707 ± 0.0035	[38]	0.0735 ± 0.0002	[42]
A_{FB}^b	0.0992 ± 0.0016	[38]	0.1029 ± 0.0003	[42]

Why Loops?

- What is the theory error on a tree-level prediction for EFT effects?
  - Standard loop factor is $\frac{1}{16\pi^2} \sim 1\%$
  - $\frac{v^2}{\Lambda^2} \sim 1\%$ as well
  - Numerical coefficients not known a priori
- SMEFT renormalization known, RG improvement will capture logs
  - For LHC-scale physics logs aren't so large
  - Pure-finite effects can be of comparable size

Warsaw Basis

$1: X^3$	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$
$Q_{\widetilde{G}}$	$f^{ABC} \widetilde{G}_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$
Q_W	$\epsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$
$Q_{\widetilde{W}}$	$\epsilon^{IJK} \widetilde{W}_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$

$2: H^6$	
Q_H	$(H^\dagger H)^3$

$3: H^4 D^2$	
$Q_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$
Q_{HD}	$\left(H^\dagger D_\mu H\right)^* \left(H^\dagger D_\mu H\right)$

$5: \psi^2 H^3 + \text{h.c.}$	
Q_{eH}	$(H^\dagger H)(\bar{l}_p e_r H)$
Q_{uH}	$(H^\dagger H)(\bar{q}_p u_r \widetilde{H})$
Q_{dH}	$(H^\dagger H)(\bar{q}_p d_r H)$

$4: X^2 H^2$	
Q_{HG}	$H^\dagger H\, G_{\mu\nu}^A G^{A\mu\nu}$
$Q_{H\widetilde{G}}$	$H^\dagger H\, \widetilde{G}_{\mu\nu}^A G^{A\mu\nu}$
Q_{HW}	$H^\dagger H\, W_{\mu\nu}^I W^{I\mu\nu}$
$Q_{H\widetilde{W}}$	$H^\dagger H\, \widetilde{W}_{\mu\nu}^I W^{I\mu\nu}$
Q_{HB}	$H^\dagger H\, B_{\mu\nu} B^{\mu\nu}$
$Q_{H\widetilde{B}}$	$H^\dagger H\, \widetilde{B}_{\mu\nu} B^{\mu\nu}$
Q_{HWB}	$H^\dagger \tau^I H\, W_{\mu\nu}^I B^{\mu\nu}$
$Q_{H\widetilde{W}B}$	$H^\dagger \tau^I H\, \widetilde{W}_{\mu\nu}^I B^{\mu\nu}$

$6: \psi^2 X H + \text{h.c.}$	
Q_{eW}	$(\bar{l}_p \sigma^{\mu\nu} e_r) \tau^I H W_{\mu\nu}^I$
Q_{eB}	$(\bar{l}_p \sigma^{\mu\nu} e_r) H B_{\mu\nu}$
Q_{uG}	$(\bar{q}_p \sigma^{\mu\nu} T^A u_r) \widetilde{H}\, G_{\mu\nu}^A$
Q_{uW}	$(\bar{q}_p \sigma^{\mu\nu} u_r) \tau^I \widetilde{H}\, W_{\mu\nu}^I$
Q_{uB}	$(\bar{q}_p \sigma^{\mu\nu} u_r) \widetilde{H}\, B_{\mu\nu}$
Q_{dG}	$(\bar{q}_p \sigma^{\mu\nu} T^A d_r) H\, G_{\mu\nu}^A$
Q_{dW}	$(\bar{q}_p \sigma^{\mu\nu} d_r) \tau^I H\, W_{\mu\nu}^I$
Q_{dB}	$(\bar{q}_p \sigma^{\mu\nu} d_r) H\, B_{\mu\nu}$

$7: \psi^2 H^2 D$	
$Q_{Hl}^{(1)}$	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{l}_p \gamma^\mu l_r)$
$Q_{Hl}^{(3)}$	$(H^\dagger i \overleftrightarrow{D}_\mu^I H)(\bar{l}_p \tau^I \gamma^\mu l_r)$
Q_{He}	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{e}_p \gamma^\mu e_r)$
$Q_{Hq}^{(1)}$	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{q}_p \gamma^\mu q_r)$
$Q_{Hq}^{(3)}$	$(H^\dagger i \overleftrightarrow{D}_\mu^I H)(\bar{q}_p \tau^I \gamma^\mu q_r)$
Q_{Hu}	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{u}_p \gamma^\mu u_r)$
Q_{Hd}	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{d}_p \gamma^\mu d_r)$
Q_{Hud} + h.c.	$i(\widetilde{H}^\dagger D_\mu H)(\bar{u}_p \gamma^\mu d_r)$

Warsaw Basis: 4-fermion

$8 : (\bar{L}L)(\bar{L}L)$	
Q_{ll}	$(\bar{l}_p \gamma_\mu l_r)(\bar{l}_s \gamma^\mu l_t)$
$Q_{qq}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r)(\bar{q}_s \gamma^\mu q_t)$
$Q_{qq}^{(3)}$	$(\bar{q}_p \gamma_\mu \tau^I q_r)(\bar{q}_s \gamma^\mu \tau^I q_t)$
$Q_{lq}^{(1)}$	$(\bar{l}_p \gamma_\mu l_r)(\bar{q}_s \gamma^\mu q_t)$
$Q_{lq}^{(3)}$	$(\bar{l}_p \gamma_\mu \tau^I l_r)(\bar{q}_s \gamma^\mu \tau^I q_t)$

$8 : (\bar{R}R)(\bar{R}R)$	
Q_{ee}	$(\bar{e}_p \gamma_\mu e_r)(\bar{e}_s \gamma^\mu e_t)$
Q_{uu}	$(\bar{u}_p \gamma_\mu u_r)(\bar{u}_s \gamma^\mu u_t)$
Q_{dd}	$(\bar{d}_p \gamma_\mu d_r)(\bar{d}_s \gamma^\mu d_t)$
Q_{eu}	$(\bar{e}_p \gamma_\mu e_r)(\bar{u}_s \gamma^\mu u_t)$
Q_{ed}	$(\bar{e}_p \gamma_\mu e_r)(\bar{d}_s \gamma^\mu d_t)$
$Q_{ud}^{(1)}$	$(\bar{u}_p \gamma_\mu u_r)(\bar{d}_s \gamma^\mu d_t)$
$Q_{ud}^{(8)}$	$(\bar{u}_p \gamma_\mu T^A u_r)(\bar{d}_s \gamma^\mu T^A d_t)$

$8 : (\bar{L}L)(\bar{R}R)$	
Q_{le}	$(\bar{l}_p \gamma_\mu l_r)(\bar{e}_s \gamma^\mu e_t)$
Q_{lu}	$(\bar{l}_p \gamma_\mu l_r)(\bar{u}_s \gamma^\mu u_t)$
Q_{ld}	$(\bar{l}_p \gamma_\mu l_r)(\bar{d}_s \gamma^\mu d_t)$
Q_{qe}	$(\bar{q}_p \gamma_\mu q_r)(\bar{e}_s \gamma^\mu e_t)$
$Q_{qu}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r)(\bar{u}_s \gamma^\mu u_t)$
$Q_{qu}^{(8)}$	$(\bar{q}_p \gamma_\mu T^A q_r)(\bar{u}_s \gamma^\mu T^A u_t)$
$Q_{qd}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r)(\bar{d}_s \gamma^\mu d_t)$
$Q_{qd}^{(8)}$	$(\bar{q}_p \gamma_\mu T^A q_r)(\bar{d}_s \gamma^\mu T^A d_t)$

$8 : (\bar{L}R)(\bar{R}L) + \mathrm{h.c.}$	
Q_{ledq}	$(\bar{l}_p^j e_r)(\bar{d}_s q_{tj})$

$8 : (\bar{L}R)(\bar{L}R) + \mathrm{h.c.}$	
$Q_{quqd}^{(1)}$	$(\bar{q}_p^j u_r)\epsilon_{jk}(\bar{q}_s^k d_t)$
$Q_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r)\epsilon_{jk}(\bar{q}_s^k T^A d_t)$
$Q_{lequ}^{(1)}$	$(\bar{l}_p^j e_r)\epsilon_{jk}(\bar{q}_s^k u_t)$
$Q_{lequ}^{(3)}$	$(\bar{l}_p^j \sigma_{\mu\nu} e_r)\epsilon_{jk}(\bar{q}_s^k \sigma^{\mu\nu} u_t)$

Operator Normalization

- We're interested in physics at the weak scale
  - Flavor, CPV, and other probes provide strong limits
- We therefore impose MFV, CP relations in the EFT
  - $U(3)^5$, with dominant breaking by SM Yukawas
- Operators which violate flavor symmetry must be normalized by appropriate coupling
- Convenient to normalize operators with field strengths by gauge coupling as well
  - Retains familiar gauge relations from SM
  - Consistent with tree-vs-loop distinction in UV a la Artz, Einhorn, Wudka hep-ph/9405214

Large y_t, λ limit

- These two couplings are known to be sizeable
  - Only QCD coupling compares
- Calculations are simpler in vanishing gauge coupling limit
  - Gauge fixing in the presence of D=6 operators leads to additional subtleties
  - Gauge independence assured here
- A good first step toward a full NLO treatment of the problem

Tree-level Effects

- At tree level SM parameters are modified, e.g.

$$\delta M_Z^2 \equiv \frac{1}{2\sqrt{2}}\frac{\hat{m}_Z^2}{\hat{G}_F}C_{HD} + \frac{2^{1/4}\sqrt{\pi}\,\sqrt{\hat{\alpha}}\,\hat{m}_Z}{\hat{G}_F^{3/2}}C_{HWB},$$

$$\delta M_W^2 = -\hat{m}_W^2\left(\frac{\delta s_{\hat{\theta}}^2}{s_{\hat{\theta}}^2} + \frac{c_{\hat{\theta}}}{s_{\hat{\theta}}\sqrt{2}\hat{G}_F}C_{HWB} + \sqrt{2}\delta G_F\right),$$

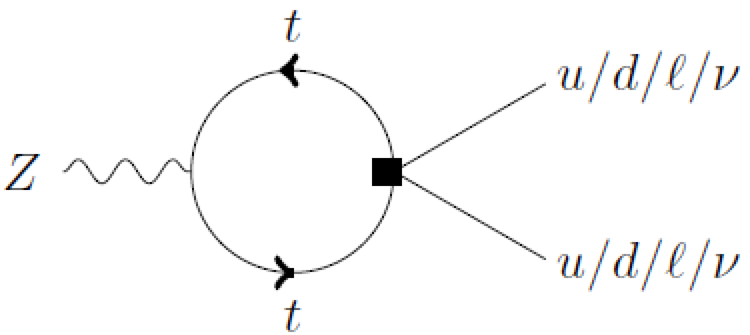
$$\delta G_F = \frac{1}{\sqrt{2}\,\hat{G}_F}\left(\sqrt{2}\,C_{Hl}^{(3)} - \frac{C_{ll}}{\sqrt{2}}\right),$$

$$\delta s_{\theta}^2 = -\frac{s_{\hat{\theta}}\,c_{\hat{\theta}}}{2\sqrt{2}\,\hat{G}_F(1-2s_{\hat{\theta}}^2)}\left[s_{\hat{\theta}}\,c_{\hat{\theta}}\,(C_{HD} + 4\,C_{Hl}^{(3)} - 2\,C_{ll}) + 2\,C_{HWB}\right],$$

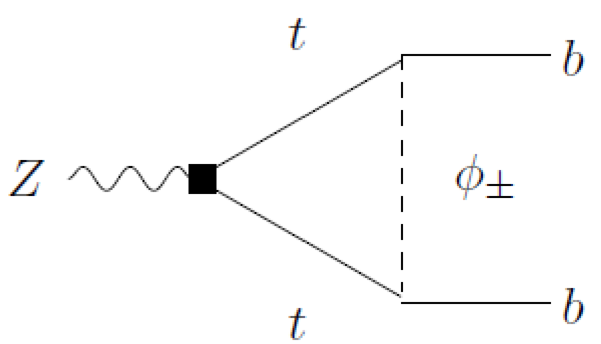
- Direct EFT effects also appear in processes at higher energies or with higher multiplicity

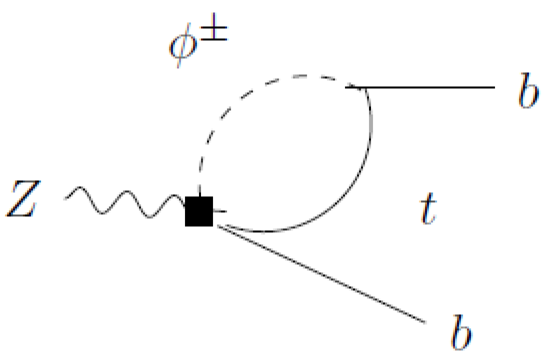
Contributing Operators

- 4-fermion operators:



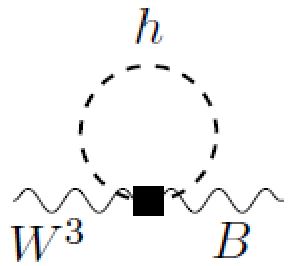
- Scalar-fermionic current operators:



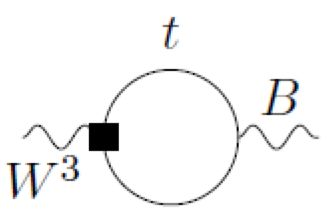


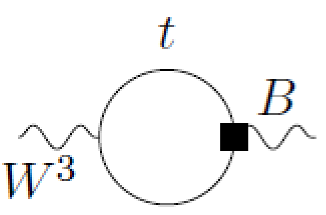
Contributing Operators

- Gauge-Higgs operators:



- Dipole operators:





Amplitude: Divergences

- Many 4-fermion operators contribute
  - Fairly simple form throughout:

$$
\begin{aligned}
i\,\Delta\mathcal{A}_{\epsilon}^{C8} = & -C_\epsilon\, N_C\, \bar{U}_L^p\, \not{\epsilon}_Z\, U_L^p \left[-C^{(1)}_{\substack{qu\\pp33}} + C^{(1)}_{\substack{qq\\33pp}} + C^{(1)}_{\substack{qq\\pp33}} + C^{(3)}_{\substack{qq\\33pp}} + C^{(3)}_{\substack{qq\\pp33}} \right], \\
& - C_\epsilon\, N_C\, \bar{U}_R^p\, \not{\epsilon}_Z\, U_R^p \left[C^{(1)}_{\substack{qu\\33pp}} - C_{\substack{uu\\33pp}} - C_{\substack{uu\\pp33}} \right], \\
& + C_\epsilon\, \bar{D}_L^r\, \not{\epsilon}_Z\, D_L^r \left[N_C \left(C^{(1)}_{\substack{qu\\rr33}} - C^{(1)}_{\substack{qq\\33rr}} - C^{(1)}_{\substack{qq\\rr33}} + C^{(3)}_{\substack{qq\\33rr}} + C^{(3)}_{\substack{qq\\rr33}} \right) - 4\, C^{(3)}_{\substack{qq\\rr33}}\, \delta_{rb} \right], \\
& - C_\epsilon\, N_C\, \bar{D}_R^r\, \not{\epsilon}_Z\, D_R^r \left[-C^{(1)}_{\substack{qd\\33rr}} + C^{(1)}_{\substack{ud\\33rr}} \right] - C_\epsilon\, N_C\, \bar{\nu}_L^t\, \not{\epsilon}_Z\, \nu_L^t \left[C^{(1)}_{\substack{lq\\tt33}} + C^{(3)}_{\substack{lq\\tt33}} - C_{\substack{lu\\tt33}} \right], \\
& - C_\epsilon\, N_C\, \bar{\ell}_L^s\, \not{\epsilon}_Z\, \ell_L^s \left[C^{(1)}_{\substack{lq\\ss33}} - C_{\substack{lu\\rr33}} - C^{(3)}_{\substack{lq\\ss33}} \right] - C_\epsilon\, N_C\, \bar{\ell}_R^s\, \not{\epsilon}_Z\, \ell_R^s \left[C_{\substack{qe\\33ss}} - C_{\substack{eu\\ss33}} \right].
\end{aligned}
$$

$$
C_\epsilon = \frac{y_t^2\, C_Z}{16\,\pi^2\,\epsilon}. \qquad\qquad C_Z = i\,\sqrt{\bar{g}_1^2 + \bar{g}_2^2}\,\bar{v}_T^2/2
$$

Amplitude: Divergences

- These poles are all cancelled by the counterterms induced for Higgs-fermionic ops

$$i\, Z_{C7}\, \delta\mathcal{A}^{C7} \to i\, \delta\mathcal{A}^{C7} - i\, \Delta\mathcal{A}^{C8}_{\epsilon}.$$

$$\begin{aligned} i\,\delta\mathcal{A}^{C7} = & \; C_Z\, \bar{U}^p_L\, \not{\epsilon}_Z\, U^p_L \left[C^{(1)}_{\substack{Hq\\pp}} - C^{(3)}_{\substack{Hq\\pp}} \right] + C_Z\, \bar{U}^p_R\, \not{\epsilon}_Z\, U^p_R \left[C_{\substack{Hu\\pp}} \right], \\ & + C_Z\, \bar{D}^r_L\, \not{\epsilon}_Z\, D^r_L \left[C^{(1)}_{\substack{Hq\\rr}} + C^{(3)}_{\substack{Hq\\rr}} \right] + C_Z\, \bar{D}^r_R\, \not{\epsilon}_Z\, D^r_R \left[C_{\substack{Hd\\rr}} \right], \\ & + C_Z\, \bar{\ell}^s_L\, \not{\epsilon}_Z\, \ell^s_L \left[C^{(1)}_{\substack{H\ell\\ss}} + C^{(3)}_{\substack{H\ell\\ss}} \right] + C_Z\, \bar{\ell}^s_R\, \not{\epsilon}_Z\, \ell^s_R \left[C_{\substack{He\\ss}} \right] + C_Z\, \bar{\nu}^t_L\, \not{\epsilon}_Z\, \nu^t_L \left[C^{(1)}_{\substack{H\ell\\tt}} - C^{(3)}_{\substack{H\ell\\tt}} \right] \end{aligned} \quad ($$

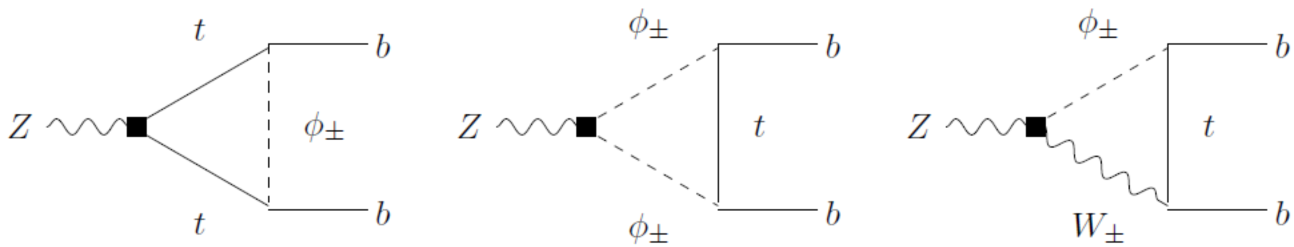
Higgs-Fermionic Divergences

- These only contribute to b-quark final states in our limit
  - Straightforward diagram contributions
  - Contributions from SM vev renormalization
  - B-quark wave function renormalization
- All poles resulting from this cancelled by self-renormalization of these operators

$$\langle \epsilon_Z^\alpha | \bar{b}_L \gamma_\alpha b_L | \bar{b}_L b_L \rangle Z_b (\sqrt{Z_v} + \frac{\Delta v}{\bar{v}_T})^2_{div} = -\frac{C_\epsilon}{C_Z} i \, \delta \mathcal{A}^{C7} + \frac{C_\epsilon}{2} \bar{b}_L \not{\epsilon}_Z b_L \left[7 \, C^{(3)}_{\substack{Hq \\ bb}} - C^{(1)}_{\substack{Hq \\ bb}} + C_{\substack{Hu \\ bb}} \right]$$

Gauge Operator Divergences

- Direct divergences from correction to $\sin\theta_W$:



$$i\,\Delta\mathcal{A}_{\epsilon}^{HWB} = C_{\epsilon}\,\bar{b}_L\,\not{\epsilon}_Z\,b_L\,\left(\mathcal{Q}_t - 1\right)\left[\frac{\bar{g}_1^2\,\bar{g}_2^2(\bar{g}_2^2 - \bar{g}_1^2)}{(\bar{g}_1^2 + \bar{g}_2^2)^2}\right] C_{HWB}$$

- Cancelled by b-quark wavefunction shift introduced to tree-level EFT effect

Input Parameters

- Any calculation depends on the inputs used to set the theory parameters
- We use a canonical set of inputs for the SM
  - $\alpha_{EM}, G_F, M_Z, M_t, M_h$
- EFT gives corrections to the extraction of each
- We treat the Wilson coefficients in $\overline{MS}$ at the NP scale as EFT input parameters to be measured and/or constrained

α_{EM} Corrections

- Matching contributions at low scales where α is measured are proportional to lepton masses
- Running cannot be neglected for α
- Two EFT effects here
  - Shift in Weinberg angle from HWB operator
  - Different running from HB, HW operators

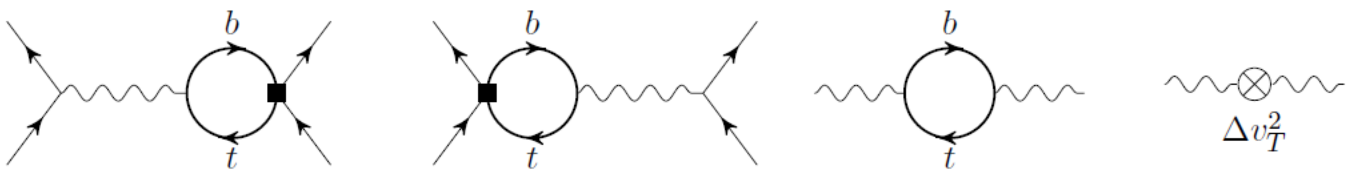
$$\delta\alpha = -\sqrt{2}\,\frac{4\pi\,\tilde{\alpha}}{\hat{G}_F} C^{(r)}_{HWB},$$

$$\Delta\alpha = -\sqrt{2}\,\frac{4\pi\,\tilde{\alpha}}{\hat{G}_F} C^{(r)}_{HWB}\left(\Delta V^2 + \frac{\Delta G_F}{\hat{G}_F}\right) + \frac{\tilde{\alpha}}{\pi}\,\hat{m}_h^2\,\left(C^{(r)}_{HB} + C^{(r)}_{HW}\right)\log\left[\frac{\hat{m}_Z^2}{p^2}\right],$$

$$\simeq -\sqrt{2}\,\frac{4\pi\,\tilde{\alpha}}{\hat{G}_F} C^{(r)}_{HWB}\left(\Delta V^2 + \frac{\Delta G_F}{\hat{G}_F}\right) + 0.03\,\hat{m}_h^2\,\left(C^{(r)}_{HB} + C^{(r)}_{HW}\right).$$

G_F Corrections

- G_F is extracted from the muon lifetime
  - Dominated by $\mu \to e\nu\nu$
- Corrected by 4-fermion ops and W mass shifts



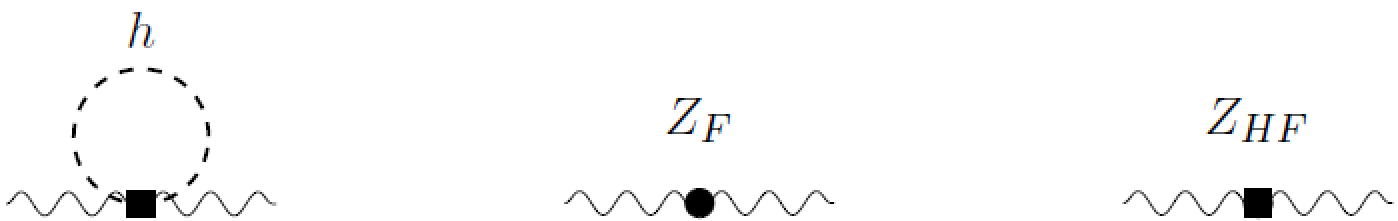
- Lagrangian parameter extracted as

$$-\frac{4\mathcal{G}_F}{\sqrt{2}} = -\frac{2}{\bar{v}_T^2}\left(1 - \frac{\Delta V^2}{\bar{v}_T^2} - \frac{\Delta m_W^2}{\bar{m}_W^2}\right) - 4\,\hat{G}_F\,\delta G_F - \Delta\psi^4$$

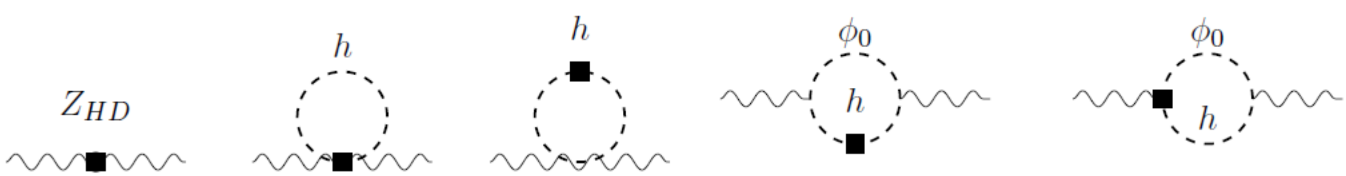
$$\Delta G_F = -\hat{G}_F\,\Delta V^2\,(1 - 2\sqrt{2}\,\delta G_F) - \frac{\Delta m_W^2}{\sqrt{2}\,\hat{m}_W^2} + \frac{\Delta\psi^4}{4\,\hat{G}_F} - \frac{\Delta m_W^2}{\sqrt{2}\,\hat{m}_W^2}\,\frac{\delta m_W^2}{\hat{m}_W^2}$$

M_Z Corrections

- Gauge-Higgs operators correct the Z mass through the graphs



- The Higgs-derivative operator contributes too



M_Z Corrections

- VEV and Weinberg angle get involved as well
- In terms of Lagrangian parameters

$$\Delta M_Z^2 = \left[\Delta V^2 + \frac{\Delta \bar{v}_T^4}{2} C_{HD}^{(r)}\right] \frac{(\bar{g}_1^2 + \bar{g}_2^2)}{4} + \frac{\bar{g}_1^2\,\bar{g}_2^2}{2}\,\bar{v}_T^2 \left[(\Delta \bar{v}_Z^2 + \Delta V^2)\, C_{HWB}^{(r)} + \Delta_{HWB}^{yt}\right]$$

- With as many input parameters as possible

$$\frac{\Delta m_Z^2}{\hat{m}_Z^2} = \sqrt{2}\,\Delta \bar{v}_Z^2\,\hat{G}_F \left[1 - \sqrt{2}\,\delta G_F - \frac{\delta m_Z^2}{\hat{m}_Z^2}\right] + C_{HD}^{(r)} \left(\frac{4}{2^{1/4}}\,\frac{\Delta v}{\sqrt{\hat{G}_F}} + \frac{\hat{m}_h^2}{32\,\pi^2} - 3\,\hat{m}_h^2\,\Delta_1\right)$$
$$+ 8\,\pi\tilde{\alpha}\left[(\Delta \bar{v}_Z^2 + \Delta V^2)\, C_{HWB}^{(r)} + \Delta_{HWB}^{yt}\right].$$

$$\Delta V^2 = \frac{2^{3/4}\,\Delta v}{\sqrt{\hat{G}_F}}\,(1 + \delta G_F) - \hat{m}_h^2\,\Delta_1, \qquad \bar{g}_2^2 = \frac{4\,\pi\,\hat{\alpha}}{s_{\hat{\theta}}^2}\left[1 + \frac{\delta s_\theta}{s_{\hat{\theta}}} + 4\,\hat{m}_W^2\,C_{HWB}\right]$$

M_t, M_h Corrections

- These don't enter the process at tree-level
  - Only tree-level EFT effects needed
- Equivalent to correcting couplings y_t, λ
  - $\bar{\lambda} = \lambda + 15v^2 C_H$
  - $\bar{y_t} = y_t + 3v^2 C_{tH}$
- Retains the SM relations for masses as functions of couplings at this order
  - As these masses are input parameters, these corrections ultimately cancel

Aside: Chirality Schemes

- These calculations include closed fermion loops with projectors and 4 gamma matrices
  - Long known to depend on treatment of γ^5
- In renormalizable theories these scheme dependences are understood and benign
- We investigated the contributions of these effects in the EFT context and found similar results

Aside: Chirality Schemes

- Scheme-dependent traces appear for 4-fermion operator contributions
  - Difference only in finite terms
- In NDA scheme, these graphs give no non-logarithmic finite terms
- In t'Hooft-Veltmann scheme one finds

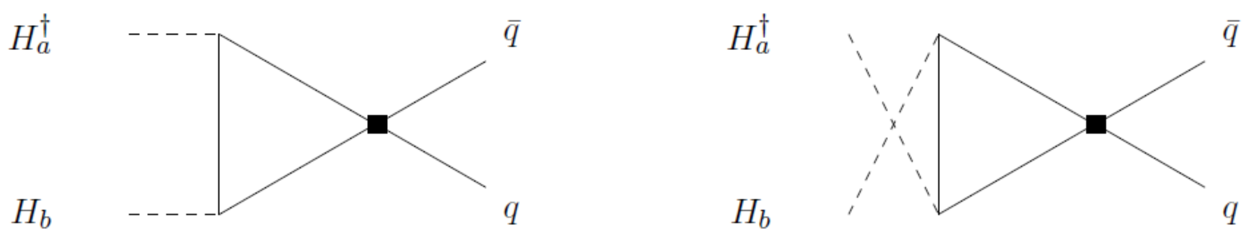
$$i\mathcal{A}^{HV-NDR} = -\frac{i\,\delta_{pr}}{16\,\pi^2}\,(C^{(1)}_{\substack{qq\\33pr}} + C^{(1)}_{\substack{qq\\pr33}})\,m_z\,v\,y_t^2\,\bar{u}_p\,\tilde{\gamma}_\alpha\,P_L\,u_r$$

Aside: Chirality Schemes

- Scheme dependences like this can only appear in an internal calculation
  - Physical input to physical observable must be unaffected, or we have a problem
- Here we need to think more carefully about what our input is for the Wilson Coefficient
- Most sensible definition would be based on measuring the scattering predicted at tree-level

Aside: Chirality Schemes

- If we adopt a definition of that form, we have to include finite contributions to the scattering



- These graphs generate a Higgs-fermionic current interaction from the 4-fermion interaction we started with

Aside: Chirality Schemes

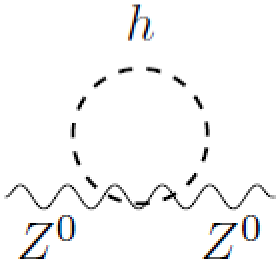
- Difference between the matching terms in the two schemes:

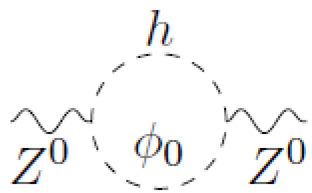
$$C^{(1)}_{\substack{Hq\\pr}} = C^{(1)}_{\substack{Hq\\pr}} + \frac{1}{48\pi^2}\left(C^{(1)}_{\substack{qq\\prst}} + C^{(1)}_{\substack{qq\\stpr}}\right)(2[Y_u^\dagger Y_u]^{st} + [Y_u Y_u^\dagger]^{st})$$

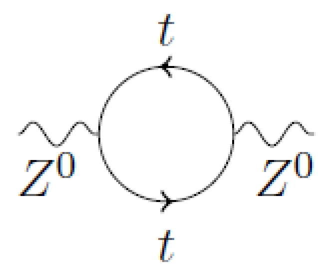
- Inserting this matching term to tree-level contribution of operator exactly cancels loop-level scheme dependence
- Particularly interesting: SU(2) violating graph is necessary to properly cancel the scheme dependence

Finite Field Normalizations

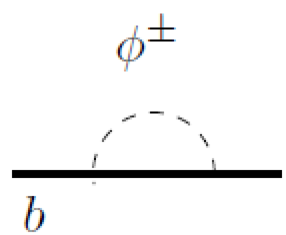
- Z boson R-factor arises from the graphs:

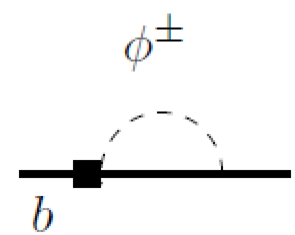






- B-quark R-factor:





$$= \frac{\hat{m}_t^2}{16\,\pi^2}\left(\sqrt{2}\hat{G}_F(1-2\delta G_F) + C^{\star}_{\substack{uH\\33}} - 2\,C^{(3)}_{Hq}\right)\left[-\frac{3}{4}-\frac{1}{2}\log\left[\frac{\Lambda^2}{\hat{m}_t^2}\right]\right]$$

Sample Results

$$
\begin{aligned}
\Delta(g_L^d)_{rr} =\ & \Delta\bar{g}_Z (g_L^d)^{SM}_{rr} + \frac{N_c \hat{m}_t^2}{16\pi^2}\log\left[\frac{\Lambda^2}{\hat{m}_t^2}\right]\left[C^{(1)}_{\substack{qq\\33rr}} + C^{(1)}_{\substack{qq\\rr33}} - C^{(3)}_{\substack{qq\\33rr}} - C^{(3)}_{\substack{qq\\rr33}} - C^{(1)}_{\substack{qu\\rr33}}\right], \\
& - \frac{1}{2}\left(\frac{\Delta G_F}{\hat{G}_F} + \Delta V^2\right)\left(C^{(1)}_{\substack{Hq\\rr}} + C^{(3)}_{\substack{Hq\\rr}}\right) + \delta_{br}\frac{\hat{m}_t^2}{4\,\pi^2}\left[C^{(3)}_{\substack{qq\\3333}}\left(-1 + \log\left[\frac{\Lambda^2}{\hat{m}_t^2}\right]\right)\right] - \frac{1}{3}\Delta s_{\theta}^2, \\
& - \delta_{br}\,\frac{\hat{m}_t^2}{16\,\pi^2}\left[\left(\frac{1}{4} - \frac{1}{2}\log\left[\frac{\Lambda^2}{\hat{m}_t^2}\right]\right)C_{Hu} + C^{(1)}_{Hq}\right] - \delta_{br}\,\Delta R_b^L\,\left((g_L^d)^{SM}_{rr} + \delta(g_L^d)_{rr}\right), \\
& - \delta_{br}\,\frac{\hat{m}_t^2}{16\,\pi^2}\,C^{(3)}_{Hq}\left[\frac{1}{2} - \mathcal{Q}_b\, s^2_{\hat{\theta}} + (3 - 2\,\mathcal{Q}_b\, s^2_{\hat{\theta}})\log\left[\frac{\Lambda^2}{\hat{m}_t^2}\right]\right], \\
& - \delta_{br}\,\frac{\hat{m}_t^2}{4\,\pi}\,\tilde{\alpha}\,(c^2_{\hat{\theta}} - s^2_{\hat{\theta}})\,C_{HWB}\,(\mathcal{Q}_u - 1)\left[\frac{3}{2} + \log\left[\frac{\Lambda^2}{\hat{m}_t^2}\right]\right],
\end{aligned}
$$

$$
\begin{aligned}
\Delta\Gamma_{Z\to Had} =\ & 2\,\Delta\Gamma_{Z\bar{u}u} + 2\,\Delta\Gamma_{Z\bar{d}d} + \Delta\Gamma_{Z\bar{b}b}, \\
=\ & \frac{\sqrt{2}\,\hat{G}_F\hat{m}_Z^3}{6\,\pi}\left[4\,(g_R^u + \delta g_R^u)\,\Delta g_R^u + 4\,(g_L^u + \delta g_L^u)\,\Delta g_L^u + 4\,(g_R^d + \delta g_R^d)\,\Delta g_R^d\right] \\
& + \frac{\sqrt{2}\,\hat{G}_F\hat{m}_Z^3}{6\,\pi}\left[4\,(g_L^d + \delta g_L^d)\,\Delta g_L^d + 2\,(g_R^b + \delta g_R^b)\,\Delta g_R^b + 2\,(g_L^b + \delta g_L^b)\,\Delta g_L^b\right]
\end{aligned}
$$

Phenomenology

- Counting is all that's needed for the most important point
- Tree amplitude depends on:
  - 1 Higgs-gauge WC
  - 1 Higgs-derivative WC
  - 7 Higgs-fermion WCs
  - 1 four-fermion WC
- NLO corrections have introduced additional dependence on:
  - 2 Higgs-gauge WCs
  - 1 Yukawa-correcting WC
  - 2 Dipole WCs
  - 11 four-fermion WCs
- At this level of precision, we can measure only 5 Z pole observables (A_{FB} goes beyond NWA)

Phenomenology

- Recall that at tree level there were flat directions in Z pole observables
  - Lifted by TGC measurements
- With this increase in relevant parameters, all of EWPD not enough to constrain the EFT
- The lesson: loop corrections cannot be constrained by EWPD alone, thus EWPD bounds (at tree level) can never be more precise than a loop factor on WCs

Numerics

The δ correction to $\bar{\Gamma}_{Z\to\bar{d}\,d}$ (where $d = \{d, s, b\}$) is given by

$$\frac{\delta\bar{\Gamma}_{Z\to\bar{d}\,d}}{10^{-2}} = -0.939\,C_{Hd} - 1.58\,C_{HD} - 6.31\,C^{(3)}_{H\ell} + 5.10\left(C^{(1)}_{Hq} + C^{(3)}_{Hq}\right) - 0.510\,C_{HWB} + 3.15\,C_{\ell\ell}. \tag{7.21}$$

The $\delta\,\Delta$ correction to $\bar{\Gamma}_{Z\to\bar{d}\,d}$ (where $d = \{d, s\}$) has the contributions

$$\begin{aligned}\frac{\delta\Delta\bar{\Gamma}_{Z\to\bar{d}\,d}}{10^{-3}} = \Big[&(0.071\,\Delta\bar{v}_T + 0.201)\,C_{Hd} - (0.115\,\Delta\bar{v}_T + 0.144)\,C_{HD}, - (1.45\,\Delta\bar{v}_T + 1.08)\,C^{(3)}_{H\ell}\\ &+ (0.316\,\Delta\bar{v}_T - 0.206)\left(C^{(1)}_{Hq} + C^{(3)}_{Hq}\right) - (0.024\,\Delta\bar{v}_T + 0.064)\,C_{HWB} + 4.23\,\Delta\bar{v}_T,\\ &+ (0.727\,\Delta\bar{v}_T + 0.541)\,C_{\ell\ell} + 0.593\,C^{(3)}_{\ell q} + 0.072\,(C_{HB} + C_{HW})\Big],\end{aligned} \tag{7.22}$$

and the $\delta\,\Delta$ corrections to $\bar{\Gamma}_{Z\to\bar{d}\,d}$ (where $d = \{d, s\}$) also has the logarithmic terms

$$\begin{aligned}\frac{\delta\Delta\bar{\Gamma}_{Z\to\bar{d}\,d}}{10^{-3}} = \Big[&0.342\,C_{Hd} - 0.266\,C_{HD} - 0.995\,C^{(3)}_{H\ell} - 0.225\left(C^{(1)}_{Hq} + C^{(3)}_{Hq}\right) - 0.110\,C_{HWB},\\ &+1.09\,C_{\ell\ell} - 1.19\,C^{(3)}_{\ell q} + 0.176\left(C^{(1)}_{qd} - C^{(1)}_{ud}\right) + 1.92\left(C^{(3)}_{qq} - C^{(1)}_{qq}\right) + 0.958\,C^{(1)}_{qu},\\ &-0.091\,C_{uW} - 0.055\,C_{uB}\Big]\log\left[\frac{\Lambda^2}{\hat{m}_t^2}\right] + \Big[(2.43\times 10^{-5}\,C_{HD} + 0.015\,C_{Hd},\\ &+0.103\,C^{(3)}_{Hl} - 0.083\left(C^{(1)}_{hq} + C^{(3)}_{hq}\right) - 0.005\,C_{HWB} - 0.052\,C_{\ell\ell}\Big]\log\left[\frac{\Lambda^2}{\hat{m}_h^2}\right]\end{aligned} \tag{7.23}$$

Numerics

The δ correction to $\bar{R}^b_\ell$ is given by

$$\frac{\delta R_b^0}{10^{-2}} = -0.192\, C_{Hd} + 0.039\, C_{HD} + 0.158\, C_{H\ell}^{(3)} + 2.13\, C_{Hq}^{(1)} - 0.055\, C_{Hq}^{(3)}, \\ -0.494\, C_{Hu} + 0.043\, C_{HWB} - 0.079\, C_{\ell\ell}. \tag{7.35}$$

Similarly, the $\delta\,\Delta$ correction to $\bar{R}^0_b$ has the contributions

$$\frac{\delta\Delta R_b^0}{10^{-3}} = \Big[(0.036\,\Delta\bar{v}_T + 0.083)\, C_{Hd} + (0.011\,\Delta\bar{v}_T + 0.013)\, C_{HD} + (0.084\,\Delta\bar{v}_T - 0.014)\, C_{H\ell}^{(3)}, \\ -(0.085\,\Delta\bar{v}_T + 0.152)\, C_{Hq}^{(1)} - (0.016\,\Delta\bar{v}_T + 0.019)\, C_{Hq}^{(3)} + (0.099\,\Delta\bar{v}_T + 0.208)\, C_{Hu}, \\ -(0.042\,\Delta\bar{v}_T - 0.007)\, C_{\ell\ell} + (0.013\,\Delta\bar{v}_T + 0.009)\, C_{HWB} - 0.015\, C_{\ell q}^{(3)}, \\ +0.597\, C_{qq}^{(3)} + 0.047\, C_{uH} - 0.006\,(C_{HB} + C_{HW}) - 0.106\,\Delta v\Big], \tag{7.36}$$

and the $\delta\,\Delta$ correction to R^0_b also has the logarithmic terms

$$\frac{\delta\Delta R_b^0}{10^{-3}} = \Big[0.129\, C_{Hd} + 0.025\, C_{HD} + 0.067\, C_{H\ell}^{(3)} - 0.559\, C_{Hq}^{(1)} + 0.383\, C_{Hq}^{(3)} + 0.240\, C_{Hu}, \\ +0.023\, C_{HWB} - 0.049\, C_{\ell\ell} + 0.030\, C_{\ell q}^{(3)} + 0.036\left(C_{qd}^{(1)} - C_{ud}^{(1)}\right) - 0.618\, C_{qq}^{(3)}, \\ -0.803\, C_{qq}^{(1)} + 0.494\, C_{qu}^{(1)} - 0.002\, C_{uB} + 0.032\, C_{uH} - 0.004\, C_{uW} - 0.186\, C_{uu}\Big] \log\left[\frac{\Lambda^2}{\hat{m}_t^2}\right] \\ + \Big[-8.94 \times 10^{-7}\, C_{HD} + \Big(0.313\, C_{Hd} - 3.49\, C_{Hq}^{(1)} + 0.090\, C_{Hq}^{(3)} - 0.258\, C_{H\ell}^{(3)}, \\ +0.808\, C_{Hu} + 0.129\, C_{\ell\ell} - 0.020\, C_{HWB}\Big) 10^{-2}\Big] \log\left[\frac{\Lambda^2}{\hat{m}_h^2}\right]. \tag{7.37}$$

Conclusions

- We have excellent data available, and must have enough respect for that to understand our new physics predictions at comparable precision
- In the case of LEP data, especially at the Z pole, this requires NLO accuracy
- In the most model-independent formulation of heavy new physics, the NLO predictions are under-constrained by low energy data
  - Setting shifts in EW observables to zero for the purposes of further searches does not give model-independent results
- A truly global analysis will be needed to properly constrain the EFT without UV assumptions
- Thank goodness we have the LHC with its forthcoming unprecedented data set to constrain new physics at higher energies!

Thank You!