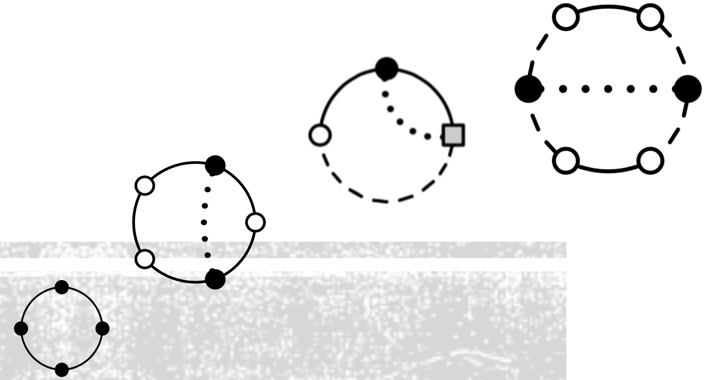


Covariant diagrams for one-loop matching

Zhengkang (Kevin) Zhang (University of Michigan)

1

Based on:

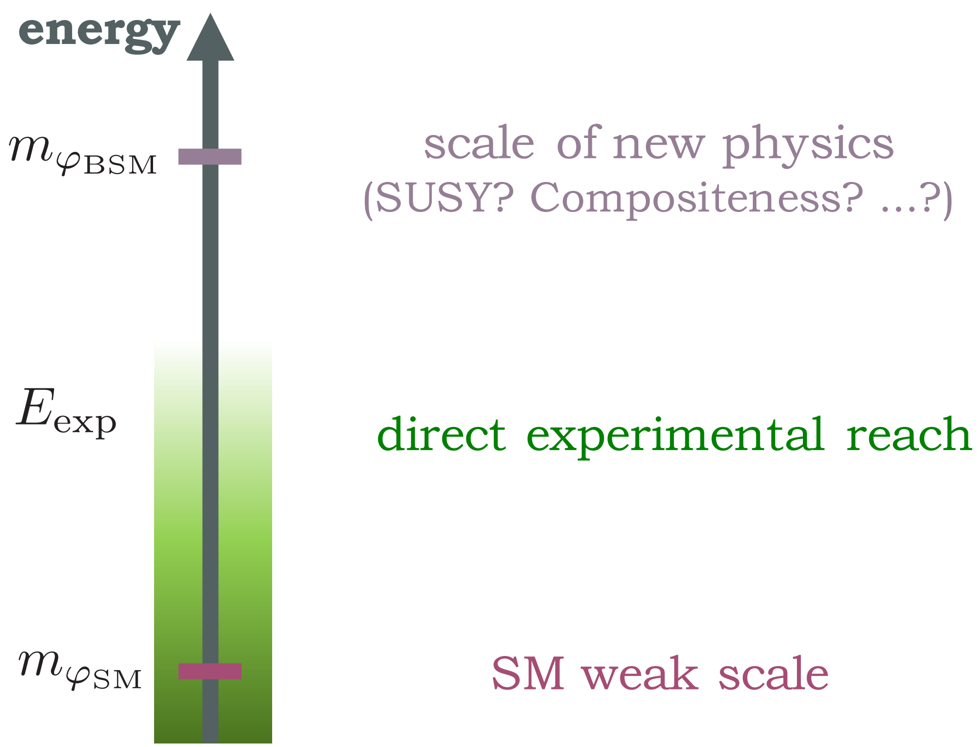
- ***ZZ [1610.00710]***

See also:

- ***Wells, ZZ [1706.xxxxx]***
- ***Ellis, Quevillon, You, ZZ [1604.02445, 1705.xxxxx]***

Intro: EFT matching

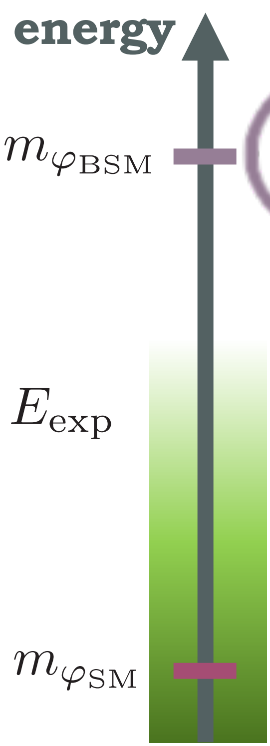
- New physics may be somewhat decoupled from weak scale.



Intro: EFT matching

- **Appropriate** and **convenient** framework: EFT.

this talk



$$\mathcal{L}_{\rm UV}[\varphi_{\rm BSM}, \varphi_{\rm SM}]$$

$\Downarrow$ ***match*** *(UV ←→ EFT dictionary)*

$$\mathcal{L}_{\rm EFT}[\varphi_{\rm SM}] = \mathcal{L}_{\rm SM} + \sum_i c_i\, \mathcal{O}_i \, (\mu \sim m_{\varphi_H})$$

$\Downarrow$ ***run*** *(resum large logs)*

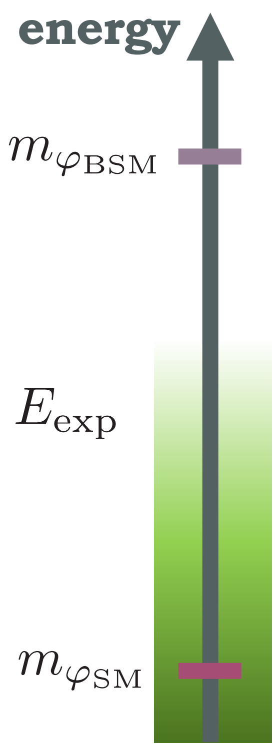
$$\mathcal{L}_{\rm EFT}[\varphi_{\rm SM}] = \mathcal{L}_{\rm SM} + \sum_i c_i\, \mathcal{O}_i \, (\mu \sim E_{\rm exp})$$

$\Downarrow$ ***calculate*** *(model-independent)*

observables (cross sections, etc.)

Intro: EFT matching

- Conventional approach to matching —



$\mathcal{L}_{\rm UV}[\varphi_{\rm BSM}, \varphi_{\rm SM}]$

Feynman diagrams

$\langle \varphi_L \varphi_L \ldots \varphi_L \rangle$

$$\mathcal{L}_{\rm EFT}[\varphi_{\rm SM}] = \mathcal{L}_{\rm SM} + \sum_i c_i\, \mathcal{O}_i\, (\mu \sim m_{\varphi_H})$$

run *(resum large logs)*

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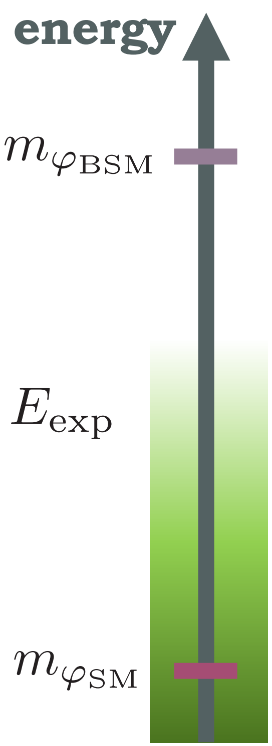
calculate *(model-independent)*

observables (cross sections, etc.)

Intro: EFT matching

- I will introduce a more **direct** and elegant approach.



$$\mathcal{L}_{\rm UV}[\varphi_{\rm BSM}, \varphi_{\rm SM}]$$

Feynman diagrams $\langle \varphi_L \varphi_L \dots \varphi_L \rangle$

covariant diagrams

$$\mathcal{L}_{\rm EFT}[\varphi_{\rm SM}] = \mathcal{L}_{\rm SM} + \sum_i c_i\, \mathcal{O}_i\, (\mu \sim m_{\varphi_H})$$

run *(resum large logs)*

$$\mathcal{L}_{\rm EFT}[\varphi_{\rm SM}] = \mathcal{L}_{\rm SM} + \sum_i c_i\, \mathcal{O}_i\, (\mu \sim E_{\rm exp})$$

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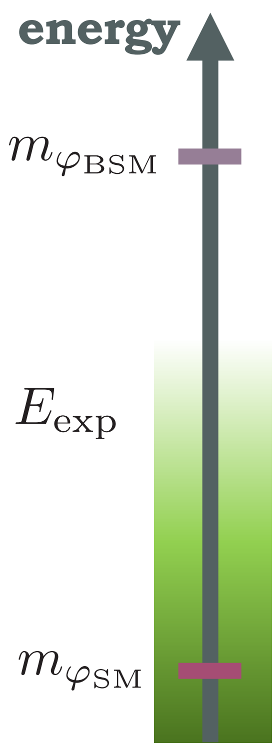
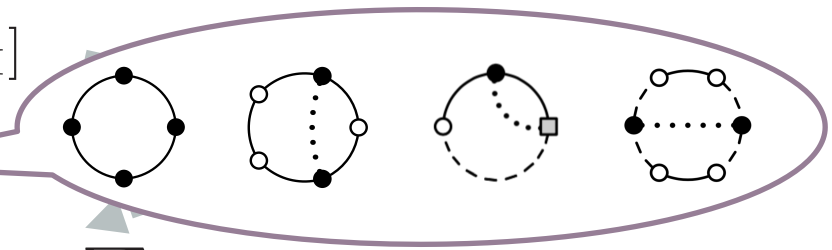


$$\mathcal{L}_{\mathrm{UV}}[\varphi_{\mathrm{BSM}}, \varphi_{\mathrm{SM}}]$$

covariant diagrams



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run *(resum large logs)*

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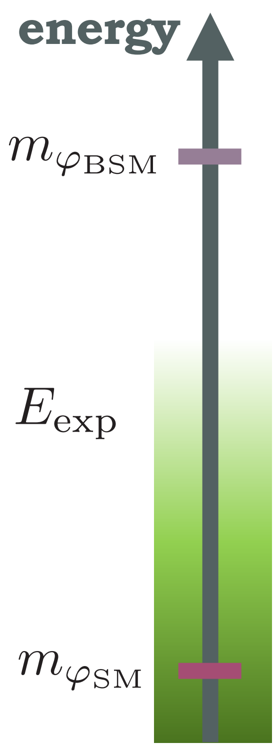
calculate *(model-independent)*

observables (cross sections, etc.)

Intro: EFT matching

- I will introduce a more direct and **elegant** approach.



$$\mathcal{L}_{\rm UV}[\varphi_{\rm BSM}, \varphi_{\rm SM}]$$

covariant diagrams

Bonus: preserve gauge covariance

$$\mathcal{L}_{\rm EFT}[\varphi_{\rm SM}] = \mathcal{L}_{\rm SM} + \sum_i c_i\, \mathcal{O}_i\, (\mu \sim m_{\varphi_H})$$

run *(resum large logs)*

$$\mathcal{L}_{\rm EFT}[\varphi_{\rm SM}] = \mathcal{L}_{\rm SM} + \sum_i c_i\, \mathcal{O}_i\, (\mu \sim E_{\rm exp})$$

calculate *(model-independent)*

observables (cross sections, etc.)

Previous literature on correlation-function-free matching that covariant diagrams build upon

- *Gaillard [Nucl.Phys.B268,669 (1986)];*
- *Chan [Phys.Rev.Lett.57,1199 (1986)];*
- *Cheyette [Nucl.Phys.B297,183 (1988)].*

early studies

- ***Henning, Lu, Murayama [1412.1837]****;* ← ***revival***
- *Chiang, Huo [1505.06334];*
- *Huo [1506.00840, 1509.05942];*

application

- ***Drozd, Ellis, Quevillon, You [1512.03003]****.* ← *generalization*
- *Del Aguila, Kunszt, Santiago [1602.00126];*
- *Boggia, Gomez-Ambrosio, Passarino [1603.03660];*

criticism

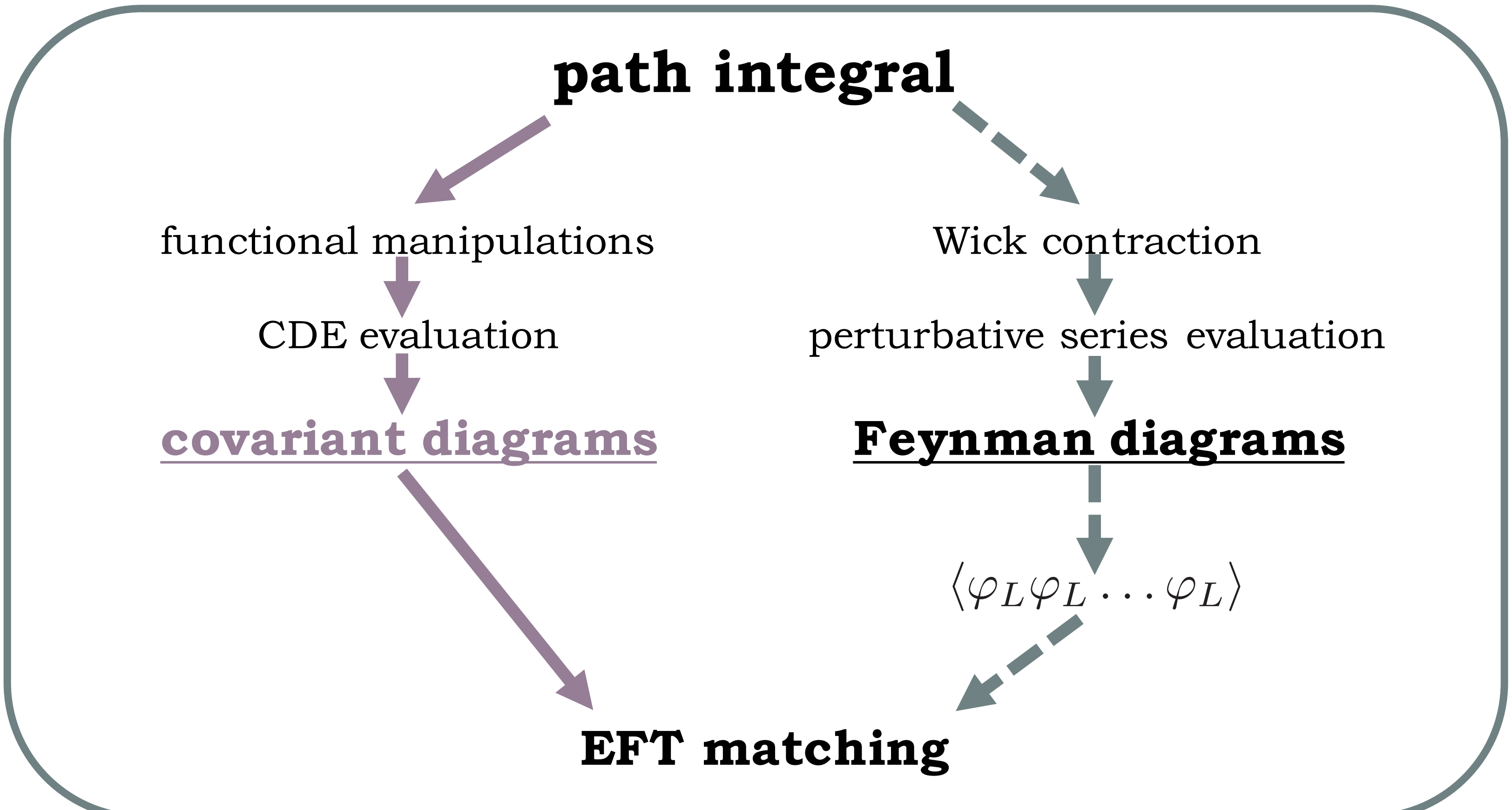
- ***Henning, Lu, Murayama [1604.01019];***
- ***Ellis, Quevillon, You, ZZ [1604.02445];***

partial resolution

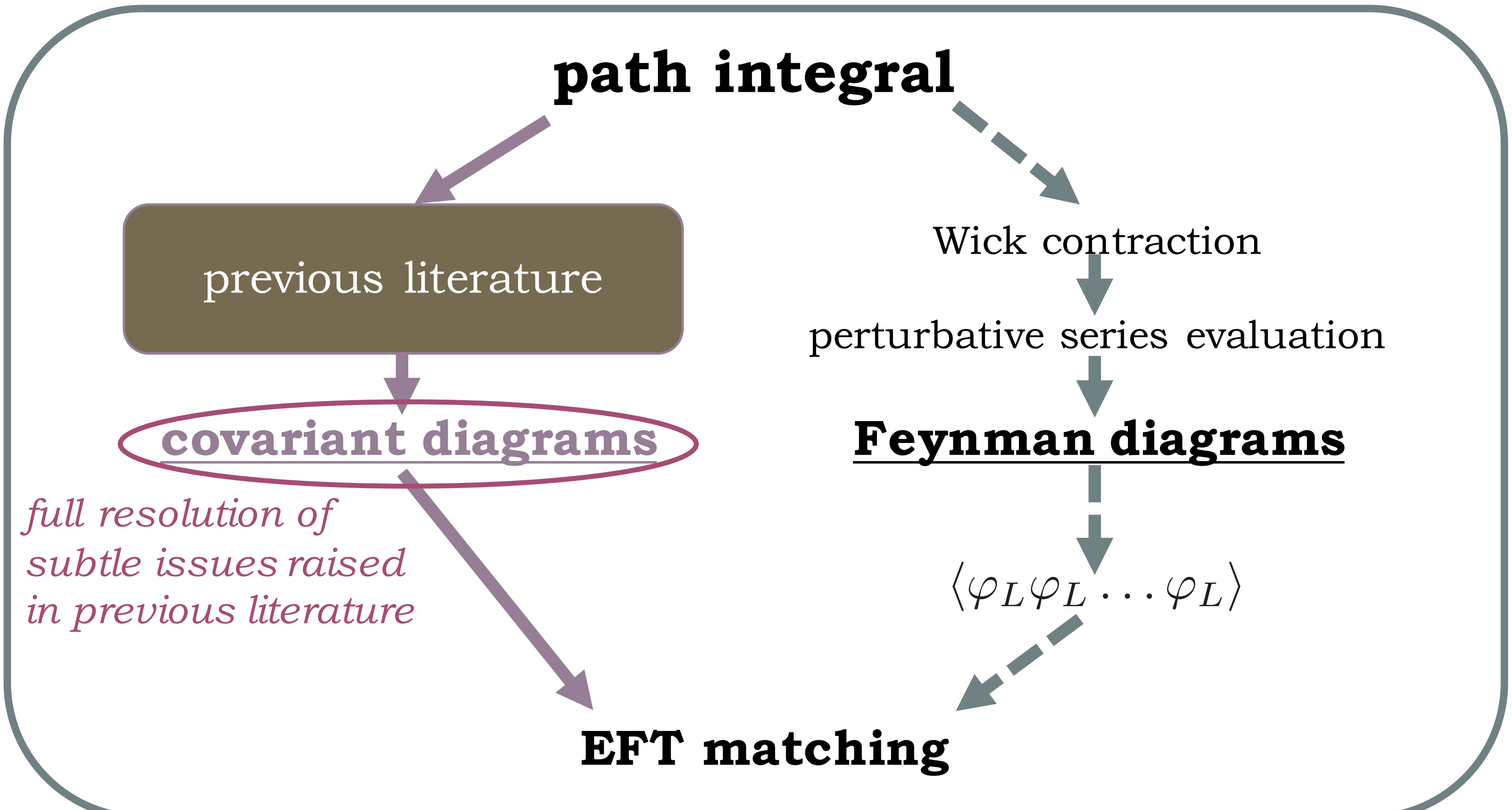
- ***Fuentes-Martin, Portoles, Ruiz-Femenia [1607.02142]****.* ← ***simplification***

See talk by Tevong You.

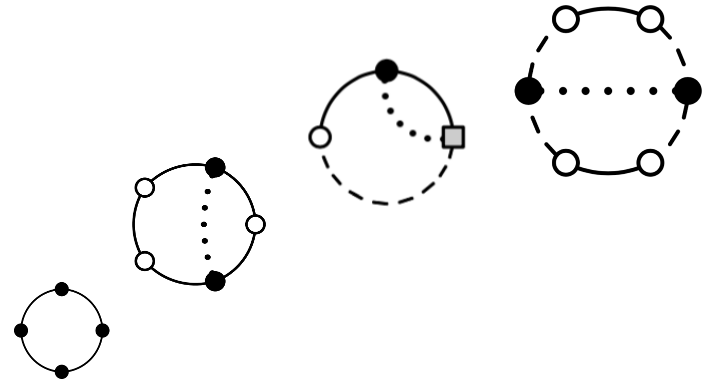
Previous literature on correlation-function-free matching that covariant diagrams build upon



Previous literature on correlation-function-free matching that covariant diagrams build upon



Outline



- **Preliminaries.**
  - **Path integral** at tree and one-loop levels.
- **Core techniques.**
  - **Expansion** by regions.
  - Covariant Derivative **Expansion** (CDE).
- **Covariant diagrams** (to systematically keep track of expansion)**.**
  - Basic rules with a simple example.
  - Application: SUSY threshold corrections in the MSSM.

Preliminary: path integral at tree level

$$\int [D\varphi_H][D\varphi_L]\, e^{i\int d^dx\, \mathcal{L}_{\mathrm{UV}}[\varphi_H,\varphi_L]} = \int [D\varphi_L]\, e^{i\int d^dx\, \mathcal{L}_{\mathrm{EFT}}[\varphi_L]}$$

- **Tree level = stationary point approximation.**
  - Solve classical equations of motion:

$$\left.\frac{\delta \mathcal{L}_{\mathrm{UV}}}{\delta \varphi_H}\right|_{\varphi_H=\varphi_{H,\mathrm{c}}} = 0$$

$$\Rightarrow \quad \mathcal{L}_{\mathrm{EFT}}^{\mathrm{tree}}[\varphi_L] = \mathcal{L}_{\mathrm{UV}}\left[\varphi_{H,\mathrm{c}}[\varphi_L],\varphi_L\right]$$

Preliminary: path integral at one-loop level

$$\int [D\varphi_H][D\varphi_L]\, e^{i\int d^dx\, \mathcal{L}_{\mathrm{UV}}[\varphi_H,\varphi_L]} = \int [D\varphi_L]\, e^{i\int d^dx\, \mathcal{L}_{\mathrm{EFT}}[\varphi_L]}$$

- **One-loop level = Gaussian approximation.**
  - Background field method:

$$\varphi_H = \varphi_{H,\mathrm{b}} + \varphi'_H\,, \quad \varphi_L = \varphi_{L,\mathrm{b}} + \varphi'_L$$

$$\begin{aligned} \Rightarrow \quad \mathcal{L}_{\mathrm{UV}}[\varphi_H,\varphi_L] + J_L\varphi_L \quad &= \quad \mathcal{L}_{\mathrm{UV}}\left[\varphi_{H,\mathrm{c}}[\varphi_{L,\mathrm{b}}],\varphi_{L,\mathrm{b}}\right] + J_L\varphi_{L,\mathrm{b}} \\ &\boxed{-\frac{1}{2}\begin{pmatrix}\varphi_H'^T & \varphi_L'^T\end{pmatrix} \mathcal{Q}_{\mathrm{UV}}\left[\varphi_{H,\mathrm{c}}[\varphi_{L,\mathrm{b}}],\varphi_{L,\mathrm{b}}\right] \begin{pmatrix}\varphi'_H \\ \varphi'_L\end{pmatrix}} + \mathcal{O}(\varphi'^3) \end{aligned}$$

*terms **quadratic** in quantum fluctuations*

  - Path integral is *Gaussian* at this order
    => functional determinant of the **quadratic operator** $\mathcal{Q}_{\mathrm{UV}}$.

1LPI effective action vs. EFT Lagrangian

- This is what we would do if we were to compute the **1LPI effective action** (Legendre transform of the path integral):

$$\Gamma_{\mathrm{L,UV}}^{\text{1-loop}}[\varphi_{L,\mathrm{b}}] = i\,c_s\, \underline{\log\det}\, \mathcal{Q}_{\mathrm{UV}}\left[\varphi_{H,\mathrm{c}}[\varphi_{L,\mathrm{b}}], \varphi_{L,\mathrm{b}}\right]$$

$$= i\,c_s\, \underline{\boxed{\mathrm{Tr}}\log}\, \mathcal{Q}_{\mathrm{UV}} = i\,c_s \boxed{\int d^d x \int \frac{d^d q}{(2\pi)^d}}\, \mathrm{tr}\, \log\, \mathcal{Q}_{\mathrm{UV}}\boxed{\Big|_{P_\mu \to P_\mu - q_\mu}}$$

  - c_s is spin factor (= +1/2 for real scalar, -1/2 for Weyl fermion).
  - Notation: $P_\mu \equiv iD_\mu$ ("kinetic momentum operator," hermitian).

- But we are interested in a **different** quantity:

$$\int d^d x\, \mathcal{L}_{\mathrm{EFT}}^{\text{1-loop}}[\varphi_L] \quad \neq \quad \Gamma_{\mathrm{L,UV}}^{\text{1-loop}}[\varphi_L]$$

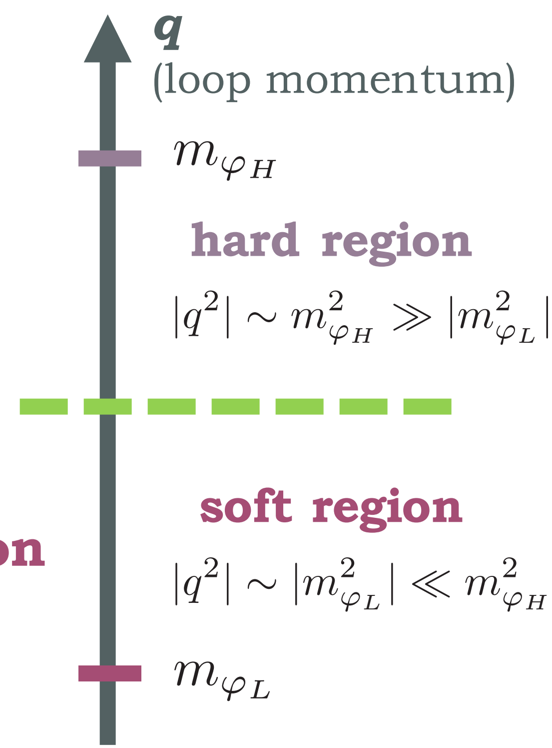
Core technique #1: expansion by regions

- After careful functional manipulations, we can show *(ZZ [1610.00710])*:

$$\boxed{\int d^d x\, \mathcal{L}_{\text{EFT}}^{\text{1-loop}}[\varphi_L] = \Gamma_{\text{L,UV}}^{\text{1-loop}}[\varphi_L]\Big|_{\text{hard}}}$$

  - Previously argued in *Fuentes-Martin, Portoles, Ruiz-Femenia [1607.02142]*.

- Expand integrand before integrating.
- **Full integral = hard region + soft region** contributions.
  - See e.g. *Beneke, Smirnov, hep-ph/9711391; Jantzen, 1111.2589*.



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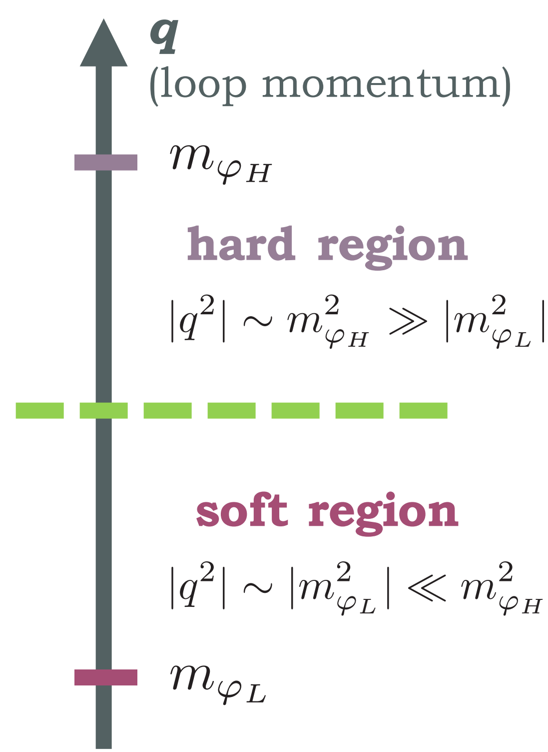
  - Previously argued in *Fuentes-Martin, Portoles, Ruiz-Femenia [1607.02142]*.

- Intuition:
  - **1PI effective actions** encode quantum fluctuations at **all scales**.
  - Extract **short-distance** fluctuations => **local** operators in **EFT Lagrangian**.



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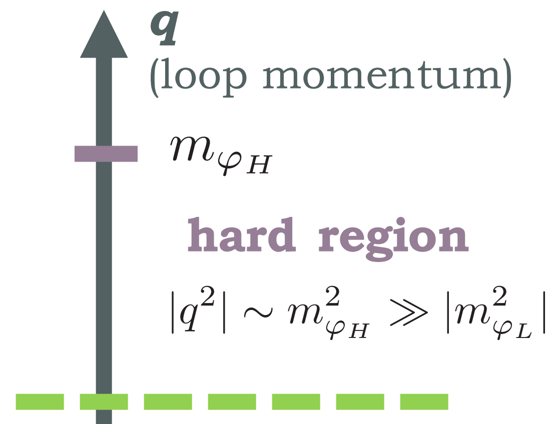

$$\mathcal{L}_{\text{EFT}}^{\text{1-loop}}[\varphi_L] = i\, c_s \,\text{tr} \int \frac{d^dq}{(2\pi)^d} \left[\log \mathcal{Q}_{\text{UV}}|_{P_\mu \to P_\mu - q_\mu}\right]_{\text{expand for } |q^2| \sim m_{\varphi_H}^2 \gg |m_{\varphi_L}^2|}$$

➢ Covariant diagrams keep track of this series expansion.

Core technique #2: Covariant Derivative Expansion

- CDE = expansion where derivatives are **covariant**.
  - We never separate D_μ into ∂_μ and $-igA_\mu$.

$$\mathcal{L}_{\text{EFT}}^{\text{1-loop}}[\varphi_L] = i\, c_s \,\text{tr} \int \frac{d^d q}{(2\pi)^d} \left[\log \boxed{\mathcal{Q}_{\text{UV}}} \big|_{P_\mu \to P_\mu - q_\mu} \right]_{\text{expand for } |q^2| \sim m^2_{\varphi_H} \gg |m^2_{\varphi_L}|}$$

- General form of quadratic operator Q_{UV}:

$$\boxed{\mathcal{Q}_{\text{UV}}[\varphi\,,\, P_\mu = iD_\mu\,;\, m_{\varphi_H}\,,\, m_{\varphi_L}]}$$

$$= \left\{ \begin{matrix} -P^2 + \mathbf{M}^2 & \text{(boson)} \\ -\not{P} + \mathbf{M} & \text{(fermion)} \end{matrix} \right\} + \mathbf{U}[\varphi] + P_\mu \,\mathbf{Z}^\mu[\varphi] + \mathbf{Z}^{\dagger\mu}[\varphi]\, P_\mu + \dots$$

- Recall: $\varphi_H = \varphi_{H,\text{b}} + \varphi'_H\,, \quad \varphi_L = \varphi_{L,\text{b}} + \varphi'_L$

$$\Rightarrow \quad \mathcal{L}_{\text{UV}}^{\text{quadratic}} = -\frac{1}{2} \begin{pmatrix} \varphi'^T_H & \varphi'^T_L \end{pmatrix} \boxed{\mathcal{Q}_{\text{UV}}\left[\varphi_{H,\text{c}}[\varphi_{L,\text{b}}], \varphi_{L,\text{b}}\right]} \begin{pmatrix} \varphi'_H \\ \varphi'_L \end{pmatrix}$$

Core technique #2: Covariant Derivative Expansion

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  - We never separate D_μ into

$$\mathcal{L}_{\text{EFT}}^{\text{1-loop}}[\varphi_L] = i\, c_s \,\text{tr} \int \frac{d^dq}{(2\pi)^d} \Big[\log \quad \Big|_{H} \gg |m^2_{\varphi_L}|$$

> Example: real singlet scalar S coupling to SM Higgs H
>
> $$\mathbf{U} = \begin{pmatrix} \boxed{U_{SS}} & U_{SH} & \dots \\ U_{HS} & U_{HH} & \dots \\ \dots & \dots & \dots \end{pmatrix}$$

- General form of quadrati

$$\mathcal{Q}_{\text{UV}}[\varphi\,,\, P_\mu = iD_\mu\,;\, m_{\varphi_H}\,,\, m_{\varphi_L}]$$

$$= \begin{Bmatrix} -P^2 + \mathbf{M}^2 & \text{(boson)} \\ -\not{P} + \mathbf{M} & \text{(fermion)} \end{Bmatrix} + \mathbf{U}[\varphi] + P_\mu\, \mathbf{Z}^\mu[\varphi] + \mathbf{Z}^{\dagger\mu}[\varphi]\, P_\mu + \dots$$

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Core technique #2:
Covariant Derivative Expansion

- CDE = expansion where derivatives are **covariant**.
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$$\mathcal{L}_{\text{EFT}}^{\text{1-loop}}[\varphi_L] = i\, c_s \,\text{tr} \int \frac{d^d q}{(2\pi)^d} \Big[\log \quad \ldots \quad {}_H \gg |m^2_{\varphi_L}|$$

Example: real singlet scalar S coupling to SM Higgs H

$$\mathcal{L}_{\text{UV}} \supset -\frac{1}{2}\lambda_{HS}|H|^2 S^2$$

$$\Rightarrow \boxed{U_{SS} \supset \lambda_{HS}|H|^2}$$

- General form of quadrati

$$\mathcal{Q}_{\text{UV}}[\varphi\,,\, P_\mu = iD_\mu\,;\, m_{\varphi_H}\,,\, m_{\varphi_L}]$$

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- General form of quadratic operator Q_{UV}:

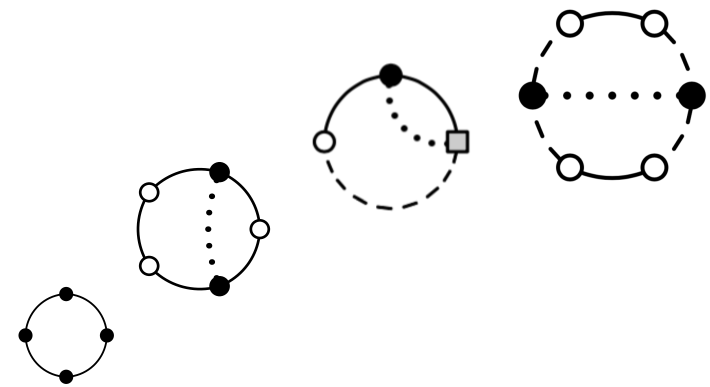
$$\mathcal{Q}_{\text{UV}}[\varphi, P_\mu = iD_\mu; m_{\varphi_H}, m_{\varphi_L}]$$
$$= \begin{Bmatrix} -P^2 + \mathbf{M}^2 & \text{(boson)} \\ -\not{P} + \mathbf{M} & \text{(fermion)} \end{Bmatrix} + \mathbf{U}[\varphi] + P_\mu \mathbf{Z}^\mu[\varphi] + \mathbf{Z}^{\dagger\mu}[\varphi] P_\mu + \dots$$

- Result of expansion: **operators** made of fields φ and **covariant** derivatives $P_\mu \equiv iD_\mu$ (rather than corr. functions)

  => automatically **gauge-invariant**!

Covariant diagrams

(to keep track of CDE)



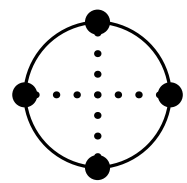
$$\mathcal{L}^{\text{1-loop}}_{\text{EFT}}[\varphi_L] = i\,c_s\,\text{tr}\int\frac{d^dq}{(2\pi)^d}\left[\log \mathcal{Q}_{\text{UV}}\big|_{P_\mu\to P_\mu - q_\mu}\right]_{\text{expand for } |q^2|\sim m^2_{\varphi_H}\gg|m^2_{\varphi_L}|}$$

$$\mathcal{Q}_{\text{UV}} = \begin{Bmatrix} -P^2 + \mathbf{M}^2 & \text{(boson)} \\ -\not{P}+\mathbf{M} & \text{(fermion)}\end{Bmatrix} + \mathbf{U}[\varphi] + P_\mu \mathbf{Z}^\mu[\varphi] + \mathbf{Z}^{\dagger\mu}[\varphi]P_\mu + \dots$$

Building block	Bosonic	Fermionic
Propagator	$= 1$	$= M$; $= -\gamma^\mu$
P insertion	$= 2P_\mu$	$= -\not{P}$
U insertion	$i \quad j \quad = U_{ij}$	
Z insertion	$i \quad j \quad = P_\mu Z^\mu_{ij}$	$i \quad j \quad = -Z^\mu_{ij}$
Contraction	$\mu \cdots\cdots \nu = g^{\mu\nu}$	

Covariant diagrams
(to keep track of CDE)

- For example, to integrate out a complex scalar, we need to compute diagrams like —



$\propto \mathrm{tr}(2P^{\mu} \cdot 2P^{\nu} \cdot 2P_{\mu} \cdot 2P_{\nu})$

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Covariant diagrams
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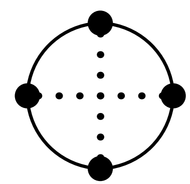

$\propto \mathrm{tr}(2P^\mu \cdot 2P^\nu \cdot 2P_\mu \cdot 2P_\nu)$

- **Prefactor rule:** $\boxed{-ic_s \cdot \frac{1}{S} \cdot \mathcal{I}[q^{2n_c}]^{n_i n_j \dots}_{ij\dots}}$

Spin factor *c_s*	1/2 for each real scalar/vector, -1/2 for each Weyl fermion
Symmetry factor *1/S*	if diagram has Z_S symmetry
Master integral $\mathcal{I}[q^{2n_c}]^{n_i n_j \dots}_{ij\dots}$	n_i propagators with mass M_i n_c Lorentz contractions

$c_s = 1$

$S = 4$

$n_i = 4$
$n_c = 2$

Covariant diagrams
(to keep track of CDE)

- For exa
  compu

Defined by $\displaystyle\int \frac{d^dq}{(2\pi)^d} \frac{q^{\mu_1}\cdots q^{\mu_{2n_c}}}{(q^2-M_i^2)^{n_i}(q^2-M_j^2)^{n_j}\cdots} \equiv g^{\mu_1\cdots\mu_{2n_c}}\,\mathcal{I}[q^{2n_c}]^{n_i n_j\cdots}_{ij\cdots}$

$\tilde{\mathcal{I}}[q^{2n_c}]^{n_i}_i$	$n_c=0$	$n_c=1$	$n_c=2$	$n_c=3$
$n_i=1$	$M_i^2\left(1-\log\frac{M_i^2}{\mu^2}\right)$	$\frac{M_i^4}{4}\left(\frac{3}{2}-\log\frac{M_i^2}{\mu^2}\right)$	$\frac{M_i^6}{24}\left(\frac{11}{6}-\log\frac{M_i^2}{\mu^2}\right)$	$\frac{M_i^8}{192}\left(\frac{25}{12}-\log\frac{M_i^2}{\mu^2}\right)$
$n_i=2$	$-\log\frac{M_i^2}{\mu^2}$	$\frac{M_i^2}{2}\left(1-\log\frac{M_i^2}{\mu^2}\right)$	$\frac{M_i^4}{8}\left(\frac{3}{2}-\log\frac{M_i^2}{\mu^2}\right)$	$\frac{M_i^6}{48}\left(\frac{11}{6}-\log\frac{M_i^2}{\mu^2}\right)$
$n_i=3$	$-\frac{1}{2M_i^2}$	$-\frac{1}{4}\log\frac{M_i^2}{\mu^2}$	$\frac{M_i^2}{8}\left(1-\log\frac{M_i^2}{\mu^2}\right)$	$\frac{M_i^4}{32}\left(\frac{3}{2}-\log\frac{M_i^2}{\mu^2}\right)$
$n_i=4$	$\frac{1}{6M_i^4}$	$-\frac{1}{12M_i^2}$	$-\frac{1}{24}\log\frac{M_i^2}{\mu^2}$	$\frac{M_i^2}{48}\left(1-\log\frac{M_i^2}{\mu^2}\right)$
$n_i=5$	$-\frac{1}{12M_i^6}$	$\frac{1}{48M_i^4}$	$-\frac{1}{96M_i^2}$	$-\frac{1}{192}\log\frac{M_i^2}{\mu^2}$
$n_i=6$	$\frac{1}{20M_i^8}$	$-\frac{1}{120M_i^6}$	$\frac{1}{480M_i^4}$	$-\frac{1}{960M_i^2}$

Table 7. Commonly-used degenerate master integrals $\tilde{\mathcal{I}}[q^{2n_c}]^{n_i}_i \equiv \mathcal{I}[q^{2n_c}]^{n_i}_i/\frac{i}{16\pi^2}$, with $\frac{2}{\bar\epsilon}=\frac{2}{\epsilon}-\gamma+\log 4\pi$ dropped. All nondegenerate (including mixed heavy-light) master integrals can be reduced to degenerate master integrals by Eq. (A.2).

- **Prefacto**

Spin fact	
Symmetry	
Master integral $\mathcal{I}[q^{2n_c}]^{n_i n_j\cdots}_{ij\cdots}$	n_i propagators with mass M_i n_c Lorentz contractions

$n_i = 4$
$n_c = 2$

Covariant diagrams
(to keep track of CDE)

- For exa…
  compu…

- **Prefacto…**

Defined by $\int \frac{d^dq}{(2\pi)^d} \frac{q^{\mu_1}\cdots q^{\mu_{2n_c}}}{(q^2-M_i^2)^{n_i}(q^2-M_j^2)^{n_j}\cdots} \equiv g^{\mu_1\cdots\mu_{2n_c}}\,\mathcal{I}[q^{2n_c}]^{n_in_j\cdots}_{ij\cdots}$

$\tilde{\mathcal{I}}[q^{2n_c}]^{n_i}_i$	$n_c=0$	$n_c=1$	$n_c=2$	$n_c=3$
$n_i=1$	$M_i^2\left(1-\log\frac{M_i^2}{\mu^2}\right)$	$\frac{M_i^4}{4}\left(\frac{3}{2}-\log\frac{M_i^2}{\mu^2}\right)$	$\frac{M_i^6}{24}\left(\frac{11}{6}-\log\frac{M_i^2}{\mu^2}\right)$	$\frac{M_i^8}{192}\left(\frac{25}{12}-\log\frac{M_i^2}{\mu^2}\right)$
$n_i=2$	$-\log\frac{M_i^2}{\mu^2}$	$\frac{M_i^2}{2}\left(1-\log\frac{M_i^2}{\mu^2}\right)$	$\frac{M_i^4}{8}\left(\frac{3}{2}-\log\frac{M_i^2}{\mu^2}\right)$	$\frac{M_i^6}{48}\left(\frac{11}{6}-\log\frac{M_i^2}{\mu^2}\right)$
$n_i=3$	$-\frac{1}{2M_i^2}$	$-\frac{1}{4}\log\frac{M_i^2}{\mu^2}$	$\frac{M_i^2}{8}\left(1-\log\frac{M_i^2}{\mu^2}\right)$	$\frac{M_i^4}{32}\left(\frac{3}{2}-\log\frac{M_i^2}{\mu^2}\right)$
$n_i=4$	$\frac{1}{6M_i^4}$	$\frac{1}{12M_i^2}$	$-\frac{1}{24}\log\frac{M_i^2}{\mu^2}$	$\frac{M_i^2}{48}\left(1-\log\frac{M_i^2}{\mu^2}\right)$
$n_i=5$	$-\frac{1}{12M_i^6}$	$\frac{1}{48M_i^4}$	$-\frac{1}{96M_i^2}$	$-\frac{1}{192}\log\frac{M_i^2}{\mu^2}$
$n_i=6$	$\frac{1}{20M_i^8}$	$-\frac{1}{120M_i^6}$	$\frac{1}{480M_i^4}$	$-\frac{1}{960M_i^2}$

Table 7. Commonly-used degenerate master integrals $\tilde{\mathcal{I}}[q^{2n_c}]^{n_i}_i \equiv \mathcal{I}[q^{2n_c}]^{n_i}_i/\frac{i}{16\pi^2}$, with $\frac{2}{\bar\epsilon}=\frac{2}{\epsilon}-\gamma+\log 4\pi$ dropped. All nondegenerate (including mixed heavy-light) master integrals can be reduced to degenerate master integrals by Eq. (A.2).

Spin fact…	
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$n_i = 4$
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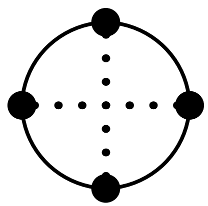
Covariant diagrams
(to keep track of CDE)

- Now let's put the pieces together —



$$= -i \cdot \frac{1}{4} \cdot \mathcal{I}[q^4]_i^4 \cdot \operatorname{tr}(2P^\mu \cdot 2P^\nu \cdot 2P_\mu \cdot 2P_\nu)$$

$$= -\frac{1}{96\pi^2} \log \frac{M_i^2}{\mu^2} \boxed{\operatorname{tr}(P^\mu P^\nu P_\mu P_\nu)}$$

- This is part of

$$\begin{aligned} g^2 \operatorname{tr}(G^{\mu\nu} G_{\mu\nu}) &= -\operatorname{tr}([D^\mu, D^\nu][D_\mu, D_\nu]) = -\operatorname{tr}([P^\mu, P^\nu][P_\mu, P_\nu]) \\ &= 2 \operatorname{tr}(P^2 P^2) - 2 \boxed{\operatorname{tr}(P^\mu P^\nu P_\mu P_\nu)} \end{aligned}$$

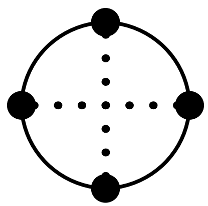
- In fact, this is the only possible contribution to the 2nd term.

$$\Rightarrow \boxed{\mathcal{L}_{\text{eff}} \supset -\frac{g^2}{48\pi^2} \log \frac{M_i^2}{\mu^2} \left[-\frac{1}{4} \operatorname{tr}(G^{\mu\nu} G_{\mu\nu}) \right]} \quad \Rightarrow \quad \frac{g_{\text{eff}}^2(\mu)}{g^2(\mu)} = 1 + \frac{g^2}{48\pi^2} T(R_i) \log \frac{M_i^2}{\mu^2}$$

Dynkin index

Covariant diagrams
(to keep track of CDE)

- Now let's put the pieces together —



$$= -i \cdot \frac{1}{4} \cdot \mathcal{I}[q^4]_i^4 \cdot \mathrm{tr}(2P^\mu \cdot 2P^\nu \cdot 2P_\mu \cdot 2P_\nu)$$

$$= -\frac{1}{96\pi^2} \log \frac{M_i^2}{\mu^2} \boxed{\mathrm{tr}(P^\mu P^\nu P_\mu P_\nu)}$$

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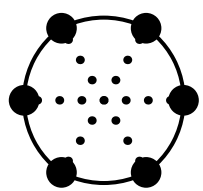
$$\begin{aligned} g^2\,\mathrm{tr}(G^{\mu\nu}G_{\mu\nu}) &= -\mathrm{tr}([D^\mu, D^\nu][D_\mu, D_\nu]) = -\mathrm{tr}([P^\mu, P^\nu][P_\mu, P_\nu]) \\ &= 2\,\mathrm{tr}(P^2 P^2) - 2\boxed{\mathrm{tr}(P^\mu P^\nu P_\mu P_\nu)} \end{aligned}$$

Aside: 1^{st} term comes from other terms in the CDE.

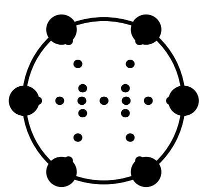
➢ **Additional rule:** we only need to compute covariant diagrams where no Lorentz contraction is between adjacent P's — those are sufficient to fix all independent EFT operator coefficients.

Covariant diagrams
(to keep track of CDE)

- Similarly, we can compute **dim-6 pure-gauge operators**.
  - Need covariant diagrams with **6 *P* insertions**.
  - 2 ways of Lorentz contraction => **2 independent operators**.



$$= -i\,\frac{1}{6}\,\mathcal{I}[q^6]_i^6 \cdot 2^6\,\mathrm{tr}(P^\mu P^\nu P^\rho P_\mu P_\nu P_\rho)$$



$$= -i\,\frac{1}{2}\,\mathcal{I}[q^6]_i^6 \cdot 2^6\,\mathrm{tr}(P^\mu P^\nu P^\rho P_\nu P_\mu P_\rho)$$

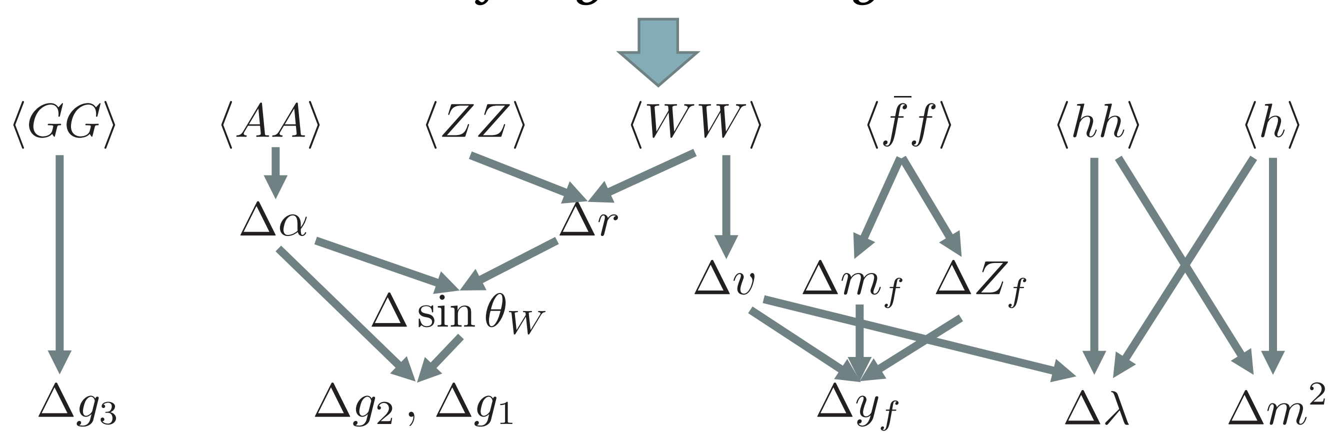
$$\Rightarrow \boxed{\mathcal{L}_{\text{eff}} \supset \frac{g^2}{480\pi^2}\frac{T(R_i)}{M_i^2}\left[-\frac{1}{2}\left(D^\mu G^a_{\mu\nu}\right)^2 + \frac{g}{3!} f^{abc} G^{a\nu}_{\mu} G^{b\rho}_{\nu} G^{c\mu}_{\rho}\right]}$$

Application: reformulating 1-loop SUSY threshold corrections calculation

(Wells, ZZ [1706.xxxxx])

- Old way of doing this calculation:
  - See classic paper *Bagger, Matchev, Pierce, Zhang [hep-ph/ 9606211]*.

<lots of Feynman diagrams>



- In the decoupling limit, we can use covariant diagrams to easily reproduce all their results.

Application: reformulating 1-loop SUSY threshold corrections calculation

(Wells, ZZ [1706.xxxxx])

- This is how we formulate the calculation —

$$\int [D\varphi_{\mathrm{BSM}}][D\varphi_{\mathrm{SM}}]\, e^{i\int d^dx\, \mathcal{L}\,[\varphi_{\mathrm{BSM}},\varphi_{\mathrm{SM}}]} = \int [D\varphi_{\mathrm{SM}}]\, e^{i\int d^dx\, \boxed{\mathcal{L}_{\mathrm{SMEFT}}[\varphi_{\mathrm{SM}}]}}$$

$$\boxed{\begin{aligned}\mathcal{L}_{\mathrm{SMEFT}} \;=\; & \mathcal{L}_{\mathrm{SM}} + \boxed{\delta Z_\phi}|D_\mu\phi|^2 + \sum_{f=q,u,d,l,e} \bar{f}\,\boxed{\boldsymbol{\delta Z_f}}\, i\not{D} f - \frac{1}{4}\boxed{\delta Z_G} G^A_{\mu\nu}G^{A\mu\nu} - \frac{1}{4}\boxed{\delta Z_W} W^I_{\mu\nu}W^{I\mu\nu} - \frac{1}{4}\boxed{\delta Z_B} B_{\mu\nu}B^{\mu\nu} \\ & + \boxed{\delta m^2}|\phi|^2 + \boxed{\delta\lambda}|\phi|^4 + \left(\bar{u}\,\boxed{\boldsymbol{\delta y_u}}\, q\cdot\epsilon\cdot\phi + \bar{d}\,\boxed{\boldsymbol{\delta y_d}}\, q\cdot\phi^* + \bar{e}\,\boxed{\boldsymbol{\delta y_e}}\, l\cdot\phi^* + \text{h.c.}\right) + \text{dimension 6} \ldots\end{aligned}}$$

$$g_3 - g_3^{\mathrm{eff}} = \frac{1}{2}\, g_3 \boxed{\delta Z_G}, \quad g_2 - g_2^{\mathrm{eff}} = \frac{1}{2}\, g_2 \boxed{\delta Z_W}, \quad g_1 - g_1^{\mathrm{eff}} = \frac{1}{2}\, g_1 \boxed{\delta Z_B},$$

$$m^2 - m^2_{\mathrm{eff}} = \boxed{\delta m^2} + m^2 \boxed{\delta Z_\phi}, \quad \lambda - \lambda_{\mathrm{eff}} = \boxed{\delta\lambda} + 2\lambda \boxed{\delta Z_\phi},$$

$$\boldsymbol{y_u} - \boldsymbol{y_u}^{\mathrm{eff}} = \boxed{\boldsymbol{\delta y_u}} + \frac{1}{2}\left(\boldsymbol{y_u}\boxed{\boldsymbol{\delta Z_q}} + \boxed{\boldsymbol{\delta Z_u}}\boldsymbol{y_u} + \boldsymbol{y_u}\boxed{\delta Z_\phi}\right),$$

$$\boldsymbol{y_d} - \boldsymbol{y_d}^{\mathrm{eff}} = \boxed{\boldsymbol{\delta y_d}} + \frac{1}{2}\left(\boldsymbol{y_d}\boxed{\boldsymbol{\delta Z_q}} + \boxed{\boldsymbol{\delta Z_d}}\boldsymbol{y_d} + \boldsymbol{y_d}\boxed{\delta Z_\phi}\right),$$

$$\boldsymbol{y_e} - \boldsymbol{y_e}^{\mathrm{eff}} = \boxed{\boldsymbol{\delta y_e}} + \frac{1}{2}\left(\boldsymbol{y_e}\boxed{\boldsymbol{\delta Z_l}} + \boxed{\boldsymbol{\delta Z_e}}\boldsymbol{y_e} + \boldsymbol{y_e}\boxed{\delta Z_\phi}\right).$$

Application: reformulating 1-loop SUSY threshold corrections calculation

(Wells, ZZ [1706.xxxxx])

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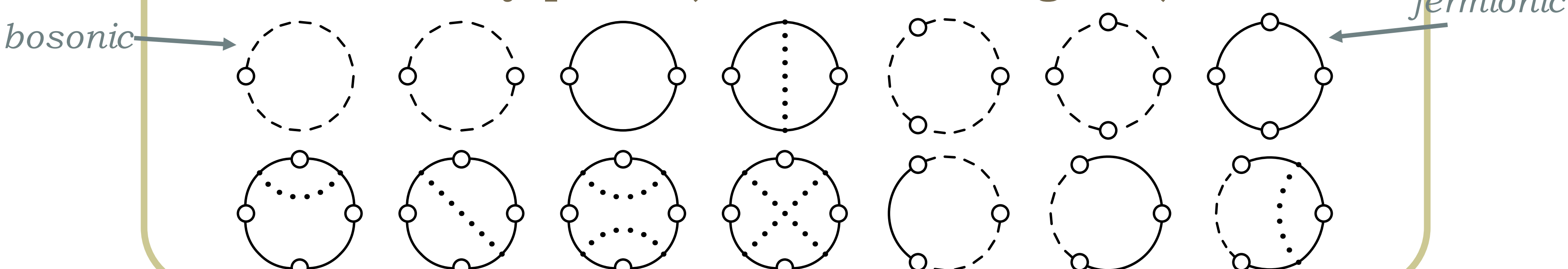
$$\int [D\varphi_{\rm BSM}][D\varphi_{\rm SM}]\, e^{i\int d^dx\, \mathcal{L}[\varphi_{\rm BSM},\varphi_{\rm SM}]} = \int [D\varphi_{\rm SM}]\, e^{i\int d^dx\, \boxed{\mathcal{L}_{\rm SMEFT}[\varphi_{\rm SM}]}}$$

$$\boxed{\begin{aligned}\mathcal{L}_{\rm SMEFT} \;=\;& \mathcal{L}_{\rm SM} + \delta Z_\phi |D_\mu\phi|^2 + \sum_{f=q,u,d,l,e} \bar{f}\,\boldsymbol{\delta Z_f}\, i\not{D} f - \frac{1}{4}\delta Z_G G^A_{\mu\nu}G^{A\mu\nu} - \frac{1}{4}\delta Z_W W^I_{\mu\nu}W^{I\mu\nu} - \frac{1}{4}\delta Z_B B_{\mu\nu}B^{\mu\nu} \\ & + \delta m^2 |\phi|^2 + \delta\lambda |\phi|^4 + \left(\bar{u}\,\boldsymbol{\delta y_u}\, q\cdot\epsilon\cdot\phi + \bar{d}\,\boldsymbol{\delta y_d}\, q\cdot\phi^* + \bar{e}\,\boldsymbol{\delta y_e}\, l\cdot\phi^* + \text{h.c.}\right) + \text{dimension 6} \ldots\end{aligned}}$$

U-only pieces (14 covariant diagrams)



Application: reformulating 1-loop SUSY threshold corrections calculation

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- This is how we formulate the calculation —

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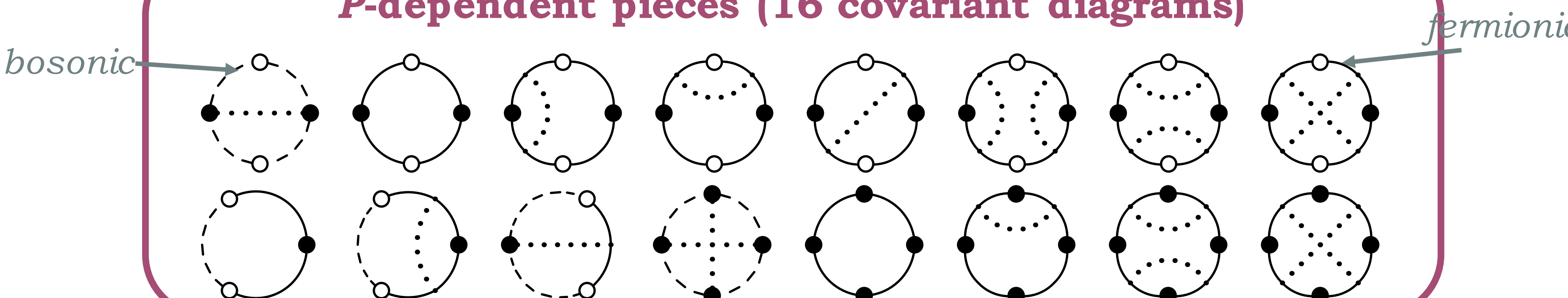
P-dependent pieces (16 covariant diagrams)

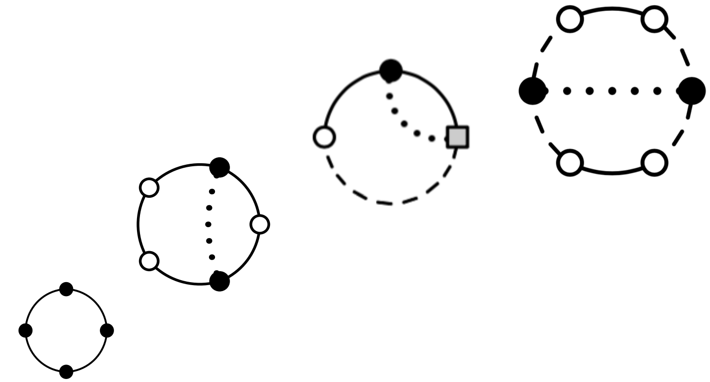


Application: reformulating 1-loop SUSY threshold corrections calculation

(Wells, ZZ [1706.xxxxx])

- **30** covariant diagrams => full 1-loop SUSY threshold corrections in the MSSM.
- **Generally, the number of covariant diagrams is bounded by dimensional analysis.**
  - Building blocks of covariant diagrams represent P, U, Z.
  - $\dim[P_\mu] = 1$, $\dim[U] \geq 1$, $\dim[Z] \geq 1$.
  - If we have a more complicated UV theory => more terms in U, Z matrices, but ***no more covariant diagrams*** *to compute*.
  - In contrast, the number of Feynman diagram is unbounded.
- In a sense, CDE and covariant diagrams are **universal**.
  - *See talk by Tevong You.*

Summary



- Covariant diagrams: a new systematic approach to one-loop matching, which
  - avoids the detour of computing correlation functions;
  - preserves gauge covariance;
  - can make EFT matching calculations easier.

$$\mathcal{L}_{\mathrm{UV}}[\varphi_{\mathrm{BSM}}, \varphi_{\mathrm{SM}}]$$

Feynman diagrams

path integral

expansion by regions

CDE

covariant diagrams

$$\langle \varphi_L \varphi_L \cdots \varphi_L \rangle$$

$$\mathcal{L}_{\mathrm{EFT}}[\varphi_{\mathrm{SM}}] = \mathcal{L}_{\mathrm{SM}} + \sum_i c_i \, \mathcal{O}_i \, (\mu \sim m_{\varphi_H})$$

Backup: matching the MSSM onto the SMEFT

(Wells, ZZ [1706.xxxxx])

- The MSSM **U** matrix (schematic, assuming R-parity):

		Φ	$\tilde{q}$	$\tilde{u}$	$\tilde{d}$	$\tilde{l}$	$\tilde{e}$	$\tilde{\chi}$	$\tilde{g}$	$\tilde{W}$	$\tilde{B}$	ϕ	q	u	d	l	e	G	W	B
heavy fields	Φ	φ^2										v^2,φ^2	u,d	q	q	e	l		$D\Phi$	$D\Phi$
	$\tilde{q}$		φ^2	φ	φ			u,d	q	q	q									
	$\tilde{u}$		φ	φ^2	$\Phi\phi$			q	u		u									
	$\tilde{d}$		φ	$\Phi\phi$	φ^2			q	d		d									
	$\tilde{l}$					φ^2	φ	e		l	l									
	$\tilde{e}$					φ	φ^2	l			e									
	$\tilde{\chi}$		u,d	q	q	e	l			φ	φ									
	$\tilde{g}$		q	u	d															
	$\tilde{W}$		q			l		φ												
	$\tilde{B}$		q	u	d	l	e	φ												
light fields	ϕ	v^2,φ^2										φ^2	u,d	q	q	e	l		$D\phi$	$D\phi$
	q	u,d										u,d		φ	φ			q	q	q
	u	q										q	φ					u		u
	d	q										q	φ					d		d
	l	e										e					φ		l	l
	e	l										l				φ				e
	G												q	u	d			$G_{\mu\nu}$		
	W	$D\Phi$										$D\phi$	q			l			$W_{\mu\nu},\phi^2,\Phi^2$	
	B	$D\Phi$										$D\phi$	q	u	d	l	e			$B_{\mu\nu},\phi^2,\Phi^2$

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heavy fields

light fields

	Φ	$\tilde q$	$\tilde u$	$\tilde d$	$\tilde l$	$\tilde e$	$\tilde\chi$	$\tilde g$	$\tilde W$	$\tilde B$	ϕ	q	u	d	l	e	G	W	B
Φ	φ^2										v^2,φ^2	u,d	q	q	e	l		$D\Phi$	$D\Phi$
$\tilde q$		φ^2	φ	φ			u,d	q	q	q									
$\tilde u$		φ	φ^2	$\Phi\phi$			q	u	u										
$\tilde d$		φ	$\Phi\phi$	φ^2			q	d											
$\tilde l$					φ^2	φ	e		l										
$\tilde e$					φ	φ^2	l												
$\tilde\chi$		u,d	q	q	e	l			φ										
$\tilde g$		q	u	d															
$\tilde W$		q			l		φ												
$\tilde B$		q	u	d	l	e	φ												
ϕ	v^2,φ^2										φ^2	u,d	q	q	e	l		$D\phi$	$D\phi$
q	u,d										u,d		φ	φ			q	q	q
u	q										q	φ					u		u
d	q										q	φ					d		d
l	e										e					φ		l	l
e	l										l				φ				e
G												q	u	d			$G_{\mu\nu}$		
W	$D\Phi$										$D\phi$	q			l			$W_{\mu\nu},\phi^2,\Phi^2$	
B	$D\Phi$										$D\phi$	q	u	d	l	e			$B_{\mu\nu},\phi^2,\Phi^2$

squark-gluino interaction with background quark field

$$U_{\tilde q\tilde g} = \sqrt{2}\, g_3 \begin{pmatrix} (T^B \bar\psi_q^c)_{i\alpha} \\ (\bar\psi_q T^B)^{i\alpha} \end{pmatrix}$$

Backup: matching the MSSM onto the SMEFT

(Wells, ZZ [1706.xxxxx])

- The MSSM **U** matrix (schematic, assuming *R*-parity):

heavy fields

light fields

	Φ	$\tilde{q}$	$\tilde{u}$	$\tilde{d}$	$\tilde{l}$	$\tilde{e}$	$\tilde{\chi}$	$\tilde{g}$	$\tilde{W}$	$\tilde{B}$	ϕ	q	u	d	l	e	G	W	B
Φ	φ^2										v^2, φ^2	u, d	q	q	e	l		$D\Phi$	$D\Phi$
$\tilde{q}$		φ^2	φ	φ			u, d	q	q	q									
$\tilde{u}$																			
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$\tilde{l}$																			
$\tilde{e}$																			
$\tilde{\chi}$																			
$\tilde{g}$																			
$\tilde{W}$																			
$\tilde{B}$																			
ϕ	v^2, φ^2										φ^2	u, d	q	q	e	l		$D\phi$	$D\phi$
q	u, d										u, d		φ	φ			q	q	q
u	q										q	φ					u		u
d	q										q	φ					d		d
l	e										e					φ		l	l
e	l										l				φ				e
G												q	u	d			$G_{\mu\nu}$		
W	$D\Phi$										$D\phi$	q			l			$W_{\mu\nu}, \phi^2, \Phi^2$	
B	$D\Phi$										$D\phi$	q	u	d	l	e			$B_{\mu\nu}, \phi^2, \Phi^2$

heavy Higgs-light quark interaction with background light quark fields

$$U_{\Phi q} = \begin{pmatrix} -\delta_\alpha^\beta \, \bar{\psi}_d^j \, \boldsymbol{y_d} \tan\beta & \epsilon_{\beta\alpha} \, \bar{\psi}_{uj}^c \, \boldsymbol{y_u^*} \cot\beta \\ \epsilon^{\beta\alpha} \, \bar{\psi}_u^j \, \boldsymbol{y_u} \cot\beta & -\delta_\beta^\alpha \, \bar{\psi}_{dj}^c \, \boldsymbol{y_d^*} \tan\beta \end{pmatrix}$$

Backup: matching the MSSM onto the SMEFT

(Wells, ZZ [1706.xxxxx])

- Two example operators:

$$\bar{d}\,\boldsymbol{\delta y_d}\,q\cdot\phi^* + \text{h.c.} \quad \text{and} \quad |\phi|^2\bar{d}\,\boldsymbol{C_{d\phi}}\,q\cdot\phi^* + \text{h.c.}$$

dim-4 => Yukawa coupling threshold correction
[right size for (t-)b-tau Yukawa unification?]

dim-6 => h→bb modification
[observable at future Higgs factories?]

Backup: matching the MSSM onto the SMEFT

(Wells, ZZ [1706.xxxxx])

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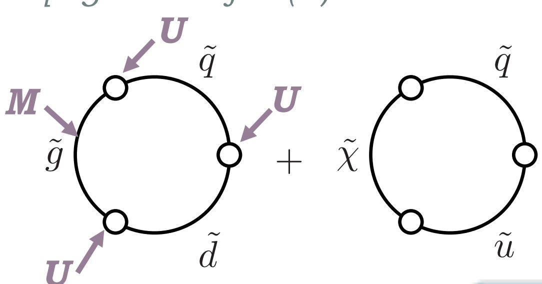


$$+\,\text{h.c.} = -\frac{i}{2} M_3\, \mathcal{I}^{111}_{\tilde q\tilde d\tilde g}\, \mathrm{tr}\big(U_{\tilde d\tilde g} U_{\tilde g\tilde q} U_{\tilde q\tilde d}\big) - \frac{i}{2}\mu\, \mathcal{I}^{111}_{\tilde q\tilde u\tilde\chi}\, \mathrm{tr}\big(U_{\tilde u\tilde\chi} U_{\tilde\chi\tilde q} U_{\tilde q\tilde u}\big) + \text{h.c.}$$

$$\Rightarrow \quad \boxed{\boldsymbol{\delta y_d} \simeq \boldsymbol{y_d}\cdot\frac{\tan\beta}{16\pi^2}\left[\frac{8}{3}\, g_3^2\, \mu\, M_3\, \tilde{\mathcal{I}}^{111}_{\tilde q\tilde d\tilde g} + \boldsymbol{y_u^\dagger y_u}\, s_\beta^{-2}\, \mu\, A_u\, \tilde{\mathcal{I}}^{111}_{\tilde q\tilde u\tilde\chi}\right]}$$

where $\tilde{\mathcal{I}}^{111}_{ijk} \equiv \mathcal{I}^{111}_{ijk} \Big/ \frac{i}{16\pi^2} = \frac{M_i^2\log M_i^2}{(M_k^2-M_i^2)(M_i^2-M_j^2)} + \frac{M_j^2\log M_j^2}{(M_i^2-M_j^2)(M_j^2-M_k^2)} + \frac{M_k^2\log M_k^2}{(M_j^2-M_k^2)(M_k^2-M_i^2)}$

Backup: matching the MSSM onto the SMEFT

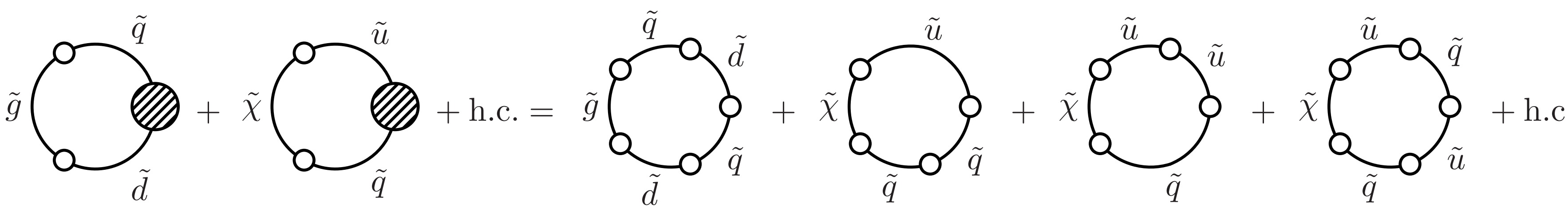
(Wells, ZZ [1706.xxxxx])

- Two example operators:

$$\bar{d}\,\boldsymbol{\delta y_d}\,q\cdot\phi^* + \text{h.c.} \quad \text{and} \quad \boxed{|\phi|^2\bar{d}\,\boldsymbol{C_{d\phi}}\,q\cdot\phi^* + \text{h.c.}}$$

dim-4 => Yukawa coupling threshold correction
[right size for (t-)b-tau Yukawa unification?]

dim-6 => h→bb modification
[observable at future Higgs factories?]



- In terms of bottom Yukawa threshold correction $\dfrac{y_b - y_b^{\text{eff}}}{y_b} \simeq \boxed{\delta_b^{\tilde{q}\tilde{d}\tilde{g}}} + \boxed{\delta_b^{\tilde{q}\tilde{u}\tilde{\chi}}}$

$$\boxed{\Delta\kappa_b = -\frac{v^2}{y_b}C_{b\phi} = -\frac{2\,m_b^2\tan^2\beta}{\underbrace{\mathcal{I}^{111}_{\tilde{q}\tilde{d}\tilde{g}}/\left(\mu^2\,\mathcal{I}^{221}_{\tilde{q}\tilde{d}\tilde{g}}\right)}_{6M^2}}\,\boxed{\delta_b^{\tilde{q}\tilde{d}\tilde{g}}} - \frac{2\,m_t^2}{\underbrace{\mathcal{I}^{111}_{\tilde{q}\tilde{u}\tilde{\chi}}/\left(\mathcal{I}^{211}_{\tilde{q}\tilde{u}\tilde{\chi}} + \mathcal{I}^{121}_{\tilde{q}\tilde{u}\tilde{\chi}} + A_u^2\,\mathcal{I}^{221}_{\tilde{q}\tilde{u}\tilde{\chi}}\right)}_{-2M^2}}\,\boxed{\delta_b^{\tilde{q}\tilde{u}\tilde{\chi}}}}$$

degenerate limit: $6M^2$ $\quad$ $-2M^2$