Durham Lumley Castle, HEFT 22nd - 24th May 2017

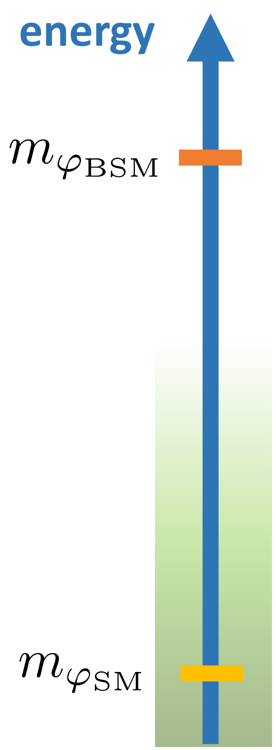
Matching with the Universal One-Loop Effective Action

Tevong You

UNIVERSITY OF CAMBRIDGE

Introduction

- See talk by Z. (Kevin) Zhang for introduction to matching:



$$\mathcal{L}_{\rm UV}[\varphi_{\rm BSM}, \varphi_{\rm SM}]$$

match *(UV ←→ EFT dictionary)*

$$\mathcal{L}_{\rm EFT}[\varphi_{\rm SM}] = \mathcal{L}_{\rm SM} + \sum_i c_i \, \mathcal{O}_i \, (\mu \sim m_{\varphi_H})$$

run *(resum large logs)*

$$\mathcal{L}_{\rm EFT}[\varphi_{\rm SM}] = \mathcal{L}_{\rm SM} + \sum_i c_i \, \mathcal{O}_i \, (\mu \sim E_{\rm exp})$$

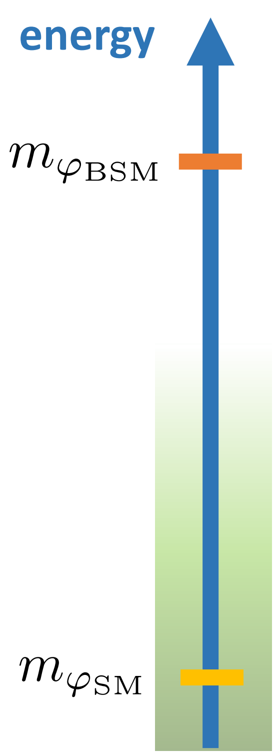
calculate *(model-independent)*

observables (cross sections, etc.)

Slide from Z. Zhang

Introduction

- Standard approach is to use Feynman diagrams



$$\mathcal{L}_{\rm UV}[\varphi_{\rm BSM}, \varphi_{\rm SM}]$$

Feynman diagrams $\langle \varphi_L \varphi_L \ldots \varphi_L \rangle$

$$\mathcal{L}_{\rm EFT}[\varphi_{\rm SM}] = \mathcal{L}_{\rm SM} + \sum_i c_i \, \mathcal{O}_i \, (\mu \sim m_{\varphi_H})$$

run *(resum large logs)*

$$\mathcal{L}_{\rm EFT}[\varphi_{\rm SM}] = \mathcal{L}_{\rm SM} + \sum_i c_i \, \mathcal{O}_i \, (\mu \sim E_{\rm exp})$$

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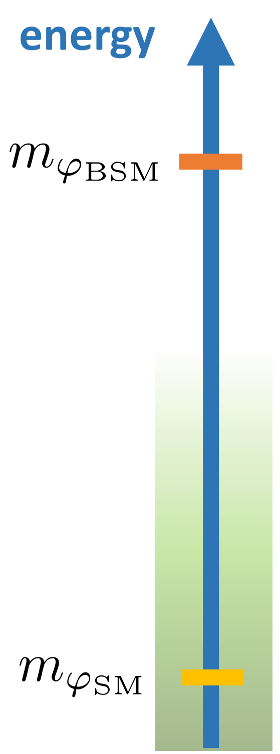
observables (cross sections, etc.)

Introduction

- Functional method more **elegant** and **direct** way of matching
- But many ways of doing this:



$$\mathcal{L}_{\rm UV}[\varphi_{\rm BSM}, \varphi_{\rm SM}]$$

Functional path integral evaluation

$$\mathcal{L}_{\rm EFT}[\varphi_{\rm SM}] = \mathcal{L}_{\rm SM} + \sum_i c_i\, \mathcal{O}_i\, (\mu \sim m_{\varphi_H})$$

run *(resum large logs)*

$$\mathcal{L}_{\rm EFT}[\varphi_{\rm SM}] = \mathcal{L}_{\rm SM} + \sum_i c_i\, \mathcal{O}_i\, (\mu \sim E_{\rm exp})$$

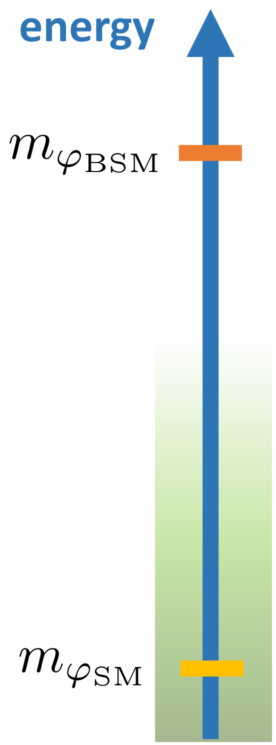
calculate *(model-independent)*

observables (cross sections, etc.)

Introduction

- Functional method more **elegant** and **direct** way of matching
- But *many* ways of doing this:

e.g. Schwinger proper time, Covariant Derivative Expansion methods, Various log expansions, heavy-light subtraction procedures, integration by regions, **covariant diagrams**, etc.



$\mathcal{L}_{\rm UV}[\varphi_{\rm BSM}$

$$\mathcal{L}_{\rm EFT}[\varphi_{\rm SM}] = \mathcal{L}_{\rm SM} + \sum_i c_i\,\mathcal{O}_i\,(\mu \sim m_{\varphi_H})$$

run *(resum large logs)*

$$\mathcal{L}_{\rm EFT}[\varphi_{\rm SM}] = \mathcal{L}_{\rm SM} + \sum_i c_i\,\mathcal{O}_i\,(\mu \sim E_{\rm exp})$$

calculate *(model-independent)*

observables (cross sections, etc.)

Outline

- Overview of recent developments in functional approach to matching
  - Original Gaillard-Cheyette CDE method (though now redundant)
  - Including heavy-light loops
- Universal One-Loop Effective Action (UOLEA)
- Examples of applying the UOLEA
  - MSSM Stops
  - Real Singlet Scalar
- Conclusion

Functional methods: Gaillard-Cheyette CDE

- **Gaillard-Cheyette** *('86, '88)* method of doing CDE reviewed/revived in **HLM** *(Henning, Lu, Murayama, 1412.1837)*

- Evaluate the path integral of the action in the usual way:
  - Expand action around minimum
  - Write Gaussian integral as determinant
  - Write determinant as trace of log in exponent

$$
\begin{aligned}
e^{iS_{\text{eff}}[\phi]} &= \int [D\Phi] e^{iS[\phi,\Phi]} \\
&= \int [D\eta] e^{i\left(S[\phi,\Phi_c] + \frac{1}{2}\frac{\delta^2 S}{\delta\Phi^2}\Big|_{\Phi=\Phi_c}\eta^2 + \mathcal{O}(\eta^3)\right)} \\
&\approx e^{iS[\phi,\Phi_c]}\left[\det\left(-\frac{\delta^2 S}{\delta\Phi^2}\Big|_{\Phi=\Phi_c}\right)\right]^{-\frac{1}{2}} \\
&\approx e^{iS[\phi,\Phi_c] - \frac{1}{2}\text{Tr}\,\ln\left(-\frac{\delta^2 S}{\delta\Phi}\Big|_{\Phi=\Phi_c}\right)},
\end{aligned}
$$

- This is common to all functional methods

- Gaillard-Cheyette also do *momentum shift* before expanding logarithm (see later slide)

- Also, different methods used for expanding log

Functional methods: Gaillard-Cheyette CDE

- For a UV Lagrangian of the form

$P_\mu \equiv iD_\mu$

$$\mathcal{L}_{\rm UV} = \mathcal{L}_{\rm SM} + (\Phi^\dagger F(x) + {\rm h.c.}) + \Phi^\dagger (P^2 - M^2 - U(x))\Phi + \mathcal{O}(\Phi^3)\,,$$

Model-dependent light fields encapsulated in F and U

$$\begin{aligned} e^{iS_{\rm eff}[\phi]} &= \int [D\Phi] e^{iS[\phi,\Phi]} \\ &= \int [D\eta] e^{i\left(S[\phi,\Phi_c] + \frac{1}{2}\frac{\delta^2 S}{\delta\Phi^2}\big|_{\Phi=\Phi_c}\eta^2 + \mathcal{O}(\eta^3)\right)} \\ &\approx e^{iS[\phi,\Phi_c]}\left[\det\left(-\frac{\delta^2 S}{\delta\Phi^2}\bigg|_{\Phi=\Phi_c}\right)\right]^{-\frac{1}{2}} \\ &\approx e^{iS[\phi,\Phi_c] - \frac{1}{2}{\rm Tr}\,\ln\left(-\frac{\delta^2 S}{\delta\Phi}\big|_{\Phi=\Phi_c}\right)}\,, \end{aligned}$$

$$\begin{aligned} S^{\rm eff}_{\text{1-loop}} &= ic_s {\rm Tr}\,\ln\left(-P^2 + M^2 + U\right) \\ &= ic_s \int d^4x \int \frac{d^4q}{(2\pi)^4} {\rm tr}\,\ln\left(-(P_\mu - q_\mu)^2 + M^2 + U\right) \end{aligned}$$

Functional methods: Gaillard-Cheyette CDE

- For a UV Lagrangian of the form

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$$\mathcal{L}_{\rm UV} = \mathcal{L}_{\rm SM} + (\Phi^\dagger F(x) + \text{h.c.}) + \Phi^\dagger (P^2 - M^2 - U(x))\Phi + \mathcal{O}(\Phi^3)\,,$$

Heavy fields can be boson or fermion

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$$\begin{aligned} S^{\rm eff}_{\text{1-loop}} &= i c_s \text{Tr}\,\ln\left(-P^2 + M^2 + U\right) \\ &= i c_s \int d^4x \int \frac{d^4q}{(2\pi)^4} \text{tr}\,\ln\left(-(P_\mu - q_\mu)^2 + M^2 + U\right) \end{aligned}$$

Functional methods: Gaillard-Cheyette CDE

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- Gaillard-Cheyette also do *momentum shift* by inserting $e^{\pm P_\mu \partial/\partial q_\mu}$

$$\begin{aligned} \mathcal{L}^{\rm eff}_{\text{1-loop}} &= ic_s \int \frac{d^4q}{(2\pi)^4} \text{tr}\ln[e^{P_\mu\partial/\partial q_\mu}(-(P_\mu - q_\mu)^2 + M^2 + U)e^{-P_\mu\partial/\partial q_\mu}] \\ &= ic_s \int \frac{d^4q}{(2\pi)^4} \text{tr}\ln[-(\tilde{G}_{\nu\mu}\partial/\partial q_\mu + q_\mu)^2 + M^2 + \tilde{U}] \end{aligned}$$

- So covariant derivatives are explicitly in commutators from beginning

$$\tilde{G}_{\nu\mu} \equiv \sum_{n=0}^{\infty} \frac{n+1}{(n+2)!}[P_{\alpha_1}, [...[P_{\alpha_n}, G'_{\nu\mu}]]]\frac{\partial^n}{\partial q_{\alpha_1}...q_{\alpha_n}}$$

$$\tilde{U} = \sum_{n=0}^{\infty} \frac{1}{n!}[P_{\alpha_1}, [...[P_{\alpha_n}, U]]]\frac{\partial^n}{\partial q_{\alpha_1}...q_{\alpha_n}}$$

Functional methods: Gaillard-Cheyette CDE

- For a UV Lagrangian of the form $P_\mu \equiv iD_\mu$

$$\mathcal{L}_{\text{UV}} = \mathcal{L}_{\text{SM}} + (\Phi^\dagger F(x) + \text{h.c.}) + \Phi^\dagger(P^2 - M^2 - U(x))\Phi + \mathcal{O}(\Phi^3)\,,$$

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- Gaillard-Cheyette also do *momentum shift* by inserting $e^{\pm P_\mu \partial/\partial q_\mu}$

$\mathcal{L}^{\text{eff}}_{\text{1-loop}} =$ … $e^{-P_\mu\partial/\partial q_\mu}]$

$=$ … $\tilde{U}]$

- So covariant de… …ors from beginning

$$\tilde{G}_{\nu\mu} \equiv \sum_{n=0}^{\infty} \frac{n+1}{(n+2)!}[P_{\alpha_1}, [\ldots[P_{\alpha_n}, G_{\nu\mu}]]] \ldots \frac{\partial^n}{\partial q_{\alpha_1}\ldots q_{\alpha_n}} \qquad \tilde{U} = \sum_{n=0}^{\infty} \frac{1}{n!}[P_{\alpha_1}, [\ldots[P_{\alpha_n}, U]]] \frac{\partial^n}{\partial q_{\alpha_1}\ldots q_{\alpha_n}}$$

But **simpler to *avoid* momentum shift!**
Instead, gather result into commutators *after* expansion evaluation.

See e.g.
Fuentes-Martin, Portoles, Ruiz-Femenia, 1607.02142;
Z. Zhang, 1610.00710.

Functional methods: Heavy-Light loops?

- Linear coupling = tree-level; quadratic coupling = *heavy-only* one-loop

$$\mathcal{L}_{\rm UV} = \mathcal{L}_{\rm SM} + (\Phi^\dagger F(x) + \text{h.c.}) + \Phi^\dagger(P^2 - M^2 - U(x))\Phi + \mathcal{O}(\Phi^3)\,,$$

- What about loops involving both heavy and light fields?
- Naively not accounted for in functional method

See e.g. Bilenky & Santamaria, hep-ph/9310302; Del Aguila, Kunszt, Santiago, 1602.00126.

- Solution: apply background field method to both **heavy** *and* **light** fields

$$\phi \to \phi_c + \phi' \quad , \quad \Phi \to \Phi_c + \Phi'$$

$$\mathcal{L}_{\rm quad} = \frac{1}{2}(\Phi', \phi')\begin{pmatrix} P^2 - M^2 - U_{\Phi\Phi} & -U_{\Phi\phi} \\ -U_{\phi\Phi} & P^2 - m^2 - U_{\phi\phi} \end{pmatrix}\begin{pmatrix} \Phi' \\ \phi' \end{pmatrix}$$

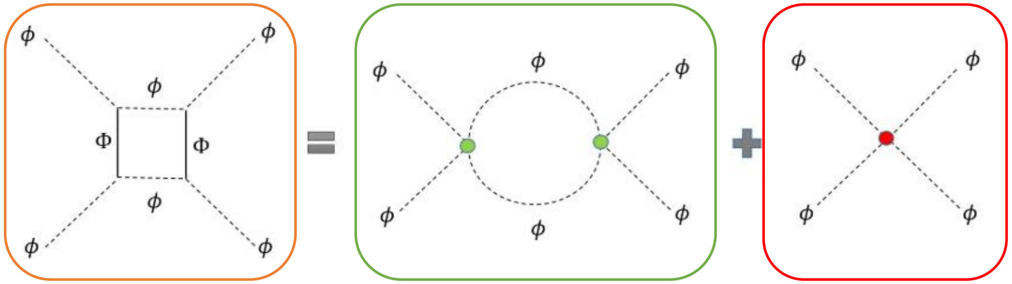
Functional methods: Heavy-Light loops?

- *Just* apply background field method to both heavy and light fields?

$$\phi \to \phi_c + \phi' \quad , \quad \Phi \to \Phi_c + \Phi'$$

- Actually, this gives the one-loop 1PI effective action and **not** $\mathcal{L}_{\rm eff}$

  - Feynman diagram intuition: Heavy-light loops in UV theory match onto both **tree-level-generated** EFT operators *inserted at one-loop*, and **one-loop-generated** EFT operators *inserted at tree-level*



(soft part) *(hard part)*

  - The **former** is not part of $\mathcal{L}_{\rm eff}$, must be subtracted to keep only the **latter**

Functional methods: Heavy-Light subtractions

- Various subtraction procedures proposed

See e.g.
Boggia, Gomez-Ambrosio, Passarino, 1603.03660;
Henning, Lu, Murayama, 1604.01019;
Ellis, Quevillon, TY, Zhang, 1604.02445;
Fuentes-Martin, Portoles, Ruiz-Femenia, 1607.02142.

Universality property also applies to heavy-light case

Integration by regions method avoids subtraction, separates **hard** and **soft** part in integral, greatly simplifies heavy-light treatment

See e.g. Beneke & Smirnov, hep-ph/9711391;
Jantzen, 1111.2589;

- Simplification of evaluating CDE from these developments lead to a **Covariant Diagram** formulation *(Z. Zhang, 1610.00710)*

- But **Universality** of CDE results means evaluation via all these different methods *gives same model-independent expression*

Henning, Lu, Murayama, 1412.1837;
Drozd, J. Ellis, Quevillon, TY, 1512.03003;
S.A.R. Ellis, Quevillon, TY, Z. Zhang; 1705.xxxxx

Universality of the One-Loop Effective Action

- *No need to reinvent the wheel*; every slide up to now can be ignored

- Universality of CDE expansion results first noticed in the **simplified** case of **degenerate mass** for heavy fields *(Henning, Lu, Murayama, 1412.1837)*

- The *general* **Universal One-Loop Effective Action** (UOLEA) subsequently derived without such assumption *(Drozd, J. Ellis, Quevillon, TY, 1512.03003)*

- Extra structures (**heavy-light terms**, "open" covariant derivatives, momentum-shifted-gamma matrices) in CDE expansion not included in initial UOLEA *(S.A.R. Ellis, Quevillon, TY, Z. Zhang, 1604.02445)*

- Universal **heavy-light** terms now done *(S.A.R. Ellis, Quevillon, TY, Z. Zhang, 1705.xxxxx)*
- A *complete* UOLEA, including all possible CDE structures, is in sight…

Universality of the One-Loop Effective Action

- Neglect these extra structures for now; derivation of universal results e.g. in Gaillard-Cheyette CDE starts from

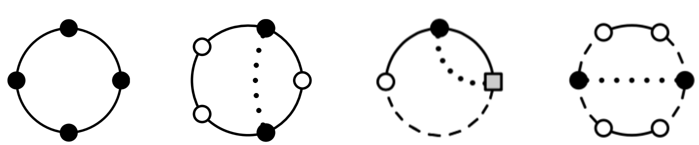
$$\mathcal{L}^{\text{eff}}_{\text{1-loop}} = ic_s \int \frac{d^4q}{(2\pi)^4} \text{tr} \ln[e^{P_\mu \partial/\partial q_\mu}(-(P_\mu - q_\mu)^2 + M^2 + U)e^{-P_\mu \partial/\partial q_\mu}]$$

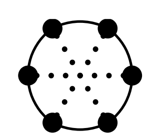
$$= ic_s \int \frac{d^4q}{(2\pi)^4} \text{tr} \ln[-(\tilde{G}_{\nu\mu} \partial/\partial q_\mu + q_\mu)^2 + M^2 + \tilde{U}]$$

$$\tilde{G}_{\nu\mu} \equiv \sum_{n=0}^{\infty} \frac{n+1}{(n+2)!}[P_{\alpha_1}, [\ldots[P_{\alpha_n}, G'_{\nu\mu}]]] \frac{\partial^n}{\partial q_{\alpha_1} \ldots q_{\alpha_n}}$$

$$\tilde{U} = \sum_{n=0}^{\infty} \frac{1}{n!}[P_{\alpha_1}, [\ldots[P_{\alpha_n}, U]]] \frac{\partial^n}{\partial q_{\alpha_1} \ldots q_{\alpha_n}}$$

- (much easier using Covariant Diagrams, see Z. Zhang talk)





$$= -i\,\frac{1}{6}\,\mathcal{I}[q^6]^6_i \cdot 2^6\, \text{tr}(P^\mu P^\nu P^\rho P_\mu P_\nu P_\rho)$$

Universality of the One-Loop Effective Action

- Whatever the method used to obtain it, the resulting UOLEA can be written as

$$
\begin{aligned}
\mathcal{L}^{\text{eff}}_{\text{1-loop}}[\phi] \supset -ic_s \Big\{ & f_1^i + f_2^i U_{ii} + f_3^i G'^2_{\mu\nu,ij} + f_4^{ij} U_{ij}^2 \\
& + f_5^{ij} (P_\mu G'_{\mu\nu,ij})^2 + f_6^{ij} (G'_{\mu\nu,ij})(G'_{\nu\sigma,jk})(G'_{\sigma\mu,ki}) + f_7^{ij} [P_\mu, U_{ij}]^2 + f_8^{ijk} (U_{ij} U_{jk} U_{ki}) \\
& + f_9^{ij} (U_{ij} G'_{\mu\nu,jk} G'_{\mu\nu,ki}) \\
& + f_{10}^{ijkl} (U_{ij} U_{jk} U_{kl} U_{li}) + f_{11}^{ijk} U_{ij} [P_\mu, U_{jk}][P_\mu, U_{ki}] \\
& + f_{12,a}^{ij} \, [P_\mu, [P_\nu, U_{ij}]] \, [P_\mu, [P_\nu, U_{ji}]] + f_{12,b}^{ij} \, [P_\mu, [P_\nu, U_{ij}]] \, [P_\nu, [P_\mu, U_{ji}]] \\
& + f_{12,c}^{ij} \, [P_\mu, [P_\mu, U_{ij}]] \, [P_\nu, [P_\nu, U_{ji}]] \\
& + f_{13}^{ijk} U_{ij} U_{jk} G'_{\mu\nu,kl} G'_{\mu\nu,li} + f_{14}^{ijk} \, [P_\mu, U_{ij}] \, [P_\nu, U_{jk}] \, G'_{\nu\mu,ki} \\
& + \left(f_{15a}^{ijk} U_{i,j} [P_\mu, U_{j,k}] - f_{15b}^{ijk} [P_\mu, U_{i,j}] U_{j,k} \right) [P_\nu, G'_{\nu\mu,ki}] \\
& + f_{16}^{ijklm} (U_{ij} U_{jk} U_{kl} U_{lm} U_{mi}) + f_{17}^{ijkl} U_{ij} U_{jk} [P_\mu, U_{kl}][P_\mu, U_{li}] + f_{18}^{ijkl} U_{ij} [P_\mu, U_{jk}] U_{kl} [P_\mu, U_{li}] \\
& + f_{19}^{ijklmn} (U_{ij} U_{jk} U_{kl} U_{lm} U_{mn} U_{ni}) \Big\} .
\end{aligned}
$$

Drozd, J. Ellis, Quevillon, TY, 1512.03003

Universality of the One-Loop Effective Action

- Whatever the method used to obtain it, the resulting UOLEA can be written as

(f3 universal term calculated by 't Hooft '73)

$$\mathcal{L}^{\text{eff}}_{\text{1-loop}}[\phi] \supset -ic_s \Big\{ f_1^i + f_2^i U_{ii} + f_3^i G'^2_{\mu\nu,ij} + f_4^{ij} U_{ij}^2$$
$$+ f_5^{ij}(P_\mu G'_{\mu\nu,ij})^2 + f_6^{ij}(G'_{\mu\nu,ij})(G'_{\nu\sigma,jk})(G'_{\sigma\mu,ki}) + f_7^{ij}[P_\mu, U_{ij}]^2 + f_8^{ijk}(U_{ij}U_{jk}U_{ki})$$
$$+ f_9^{ij}(U_{ij}G'_{\mu\nu,jk}G'_{\mu\nu,ki})$$
$$+ f_{10}^{ijkl}(U_{ij}U_{jk}U_{kl}U_{li}) + f_{11}^{ijk}U_{ij}[P_\mu, U_{jk}][P_\mu, U_{ki}]$$
$$+ f_{12,a}^{ij}\,[P_\mu,[P_\nu,U_{ij}]]\,[P_\mu,[P_\nu,U_{ji}]] + f_{12,b}^{ij}\,[P_\mu,[P_\nu,U_{ij}]]\,[P_\nu,[P_\mu,U_{ji}]]$$
$$+ f_{12,c}^{ij}\,[P_\mu,[P_\mu,U_{ij}]]\,[P_\nu,[P_\nu,U_{ji}]]$$
$$+ f_{13}^{ijk}U_{ij}U_{jk}G'_{\mu\nu,kl}G'_{\mu\nu,li} + f_{14}^{ijk}\,[P_\mu,U_{ij}]\,[P_\nu,U_{jk}]\,G'_{\nu\mu,ki}$$
$$+ \left(f_{15a}^{ijk}U_{i,j}[P_\mu,U_{j,k}] - f_{15b}^{ijk}[P_\mu,U_{i,j}]U_{j,k}\right)[P_\nu, G'_{\nu\mu,ki}]$$
$$+ f_{16}^{ijklm}(U_{ij}U_{jk}U_{kl}U_{lm}U_{mi}) + f_{17}^{ijkl}U_{ij}U_{jk}[P_\mu,U_{kl}][P_\mu,U_{li}] + f_{18}^{ijkl}U_{ij}[P_\mu,U_{jk}]U_{kl}[P_\mu,U_{li}]$$
$$+ f_{19}^{ijklmn}(U_{ij}U_{jk}U_{kl}U_{lm}U_{mn}U_{ni}) \Big\}.$$

Universal coefficients f *encapsulate dependence on combinations of momentum master integrals*

Drozd, J. Ellis, Quevillon, TY, 1512.03003

Universality of the One-Loop Effective Action

- **Universal coefficients** in terms of standard master integrals:

Universal coefficient	Operator
$f_2^i = \mathcal{I}_i^1$	U_{ii}
$f_3^i = 2\,\mathcal{I}[q^4]_i^4$	$G_i'^{\mu\nu} G'_{\mu\nu,i}$
$f_4^{ij} = \frac{1}{2}\,\mathcal{I}_{ij}^{11}$	$U_{ij}U_{ji}$
$f_5^i = 16\,\mathcal{I}[q^6]_i^6$	$[P^\mu, G'_{\mu\nu,i}][P_\rho, G_i'^{\rho\nu}]$
$f_6^i = \frac{32}{3}\,\mathcal{I}[q^6]_i^6$	$G'^{\mu}{}_{\nu,i} G'^{\nu}{}_{\rho,i} G'^{\rho}{}_{\mu,i}$
$f_7^{ij} = \mathcal{I}[q^2]_{ij}^{22}$	$[P^\mu, U_{ij}][P_\mu, U_{ji}]$
$f_8^{ijk} = \frac{1}{3}\,\mathcal{I}_{ijk}^{111}$	$U_{ij}U_{jk}U_{ki}$
$f_9^i = 8\,\mathcal{I}[q^4]_i^5$	$U_{ii} G_i'^{\mu\nu} G'_{\mu\nu,i}$
$f_{10}^{ijkl} = \frac{1}{4}\,\mathcal{I}_{ijkl}^{1111}$	$U_{ij}U_{jk}U_{kl}U_{li}$
$f_{11}^{ijk} = 2\big(\mathcal{I}[q^2]_{ijk}^{122} + \mathcal{I}[q^2]_{ijk}^{212}\big)$	$U_{ij}[P^\mu, U_{jk}][P_\mu, U_{ki}]$
$f_{12}^{ij} = 4\,\mathcal{I}[q^4]_{ij}^{33}$	$\big[P^\mu, [P_\mu, U_{ij}]\big]\big[P^\nu, [P_\nu, U_{ji}]\big]$
$f_{13}^{ij} = 4\big(\mathcal{I}[q^4]_{ij}^{33} + 2\,\mathcal{I}[q^4]_{ij}^{42} + 2\,\mathcal{I}[q^4]_{ij}^{51}\big)$	$U_{ij}U_{ji} G_i'^{\mu\nu} G'_{\mu\nu,i}$
$f_{14}^{ij} = -8\,\mathcal{I}[q^4]_{ij}^{33}$	$[P^\mu, U_{ij}][P^\nu, U_{ji}] G'_{\nu\mu,i}$
$f_{15}^{ij} = 4\big(\mathcal{I}[q^4]_{ij}^{33} + \mathcal{I}[q^4]_{ij}^{42}\big)$	$\big(U_{ij}[P^\mu, U_{ji}] - [P^\mu, U_{ij}]U_{ji}\big)[P^\nu, G'_{\nu\mu,i}]$
$f_{16}^{ijklm} = \frac{1}{5}\,\mathcal{I}_{ijklm}^{11111}$	$U_{ij}U_{jk}U_{kl}U_{lm}U_{mi}$
$f_{17}^{ijkl} = 2\big(\mathcal{I}[q^2]_{ijkl}^{2112} + \mathcal{I}[q^2]_{ijkl}^{1212} + \mathcal{I}[q^2]_{ijkl}^{1122}\big)$	$U_{ij}U_{jk}[P^\mu, U_{kl}][P_\mu, U_{li}]$
$f_{18}^{ijkl} = \mathcal{I}[q^2]_{ijkl}^{2121} + \mathcal{I}[q^2]_{ijkl}^{2112} + \mathcal{I}[q^2]_{ijkl}^{1221} + \mathcal{I}[q^2]_{ijkl}^{1212}$	$U_{ij}[P^\mu, U_{jk}]U_{kl}[P_\mu, U_{li}]$
$f_{19}^{ijklmn} = \frac{1}{6}\,\mathcal{I}_{ijklmn}^{111111}$	$U_{ij}U_{jk}U_{kl}U_{lm}U_{mn}U_{ni}$

$$\int \frac{d^d q}{(2\pi)^d} \frac{q^{\mu_1}\cdots q^{\mu_{2n_c}}}{(q^2-M_i^2)^{n_i}(q^2-M_j^2)^{n_j}\cdots(q^2)^{n_L}} \equiv g^{\mu_1\cdots\mu_{2n_c}}\,\mathcal{I}[q^{2n_c}]_{ij\ldots 0}^{n_i n_j \ldots n_L}$$

Drozd, J. Ellis, Quevillon, TY, 1512.03003;

Simplified form by covariant diagram computation shown here from Z. Zhang, 1610.00710.

Universality of the One-Loop Effective Action

- **Universal coefficients** in terms of standard master integrals:

Universal coefficient	Operator
$f_2^i = \mathcal{I}_i^1$	U_{ii}
$f_3^i = 2\,\mathcal{I}[q^4]_i^4$	$G_i'^{\mu\nu} G'_{\mu\nu,i}$
$f_4^{ij} = \frac{1}{2}\,\mathcal{I}_{ij}^{11}$	$U_{ij}U_{ji}$
$f_5^i = 16\,\mathcal{I}[q^6]_i^6$	$[P^\mu, G'_{\mu\nu,i}][P_\rho, G_i'^{\rho\nu}]$
$f_6^i = \frac{32}{3}\,\mathcal{I}[q^6]_i^6$	$G'^{\mu}{}_{\nu,i} G'^{\nu}{}_{\rho,i} G'^{\rho}{}_{\mu,i}$
$f_7^{ij} = \mathcal{I}[q^2]_{ij}^{22}$	$[P^\mu, U_{ij}][P_\mu, U_{ji}]$
$f_8^{ijk} = \frac{1}{3}\,\mathcal{I}_{ijk}^{111}$	$U_{ij}U_{jk}U_{ki}$
$f_9^i = 8\,\mathcal{I}[q^4]_i^5$	$U_{ii} G_i'^{\mu\nu} G'_{\mu\nu,i}$
$f_{10}^{ijkl} = \frac{1}{4}\,\mathcal{I}_{ijkl}^{1111}$	$U_{ij}U_{jk}U_{kl}U_{li}$
$f_{11}^{ijk} = 2\big(\mathcal{I}[q^2]_{ijk}^{122} + \mathcal{I}[q^2]_{ijk}^{212}\big)$	$U_{ij}[P^\mu, U_{jk}][P_\mu, U_{ki}]$
$f_{12}^{ij} = 4\,\mathcal{I}[q^4]_{ij}^{33}$	$\big[P^\mu, [P_\mu, U_{ij}]\big]\big[P^\nu, [P_\nu, U_{ji}]\big]$
$f_{13}^{ij} = 4\big(\mathcal{I}[q^4]_{ij}^{33} + 2\,\mathcal{I}[q^4]_{ij}^{42} + 2\,\mathcal{I}[q^4]_{ij}^{51}\big)$	$U_{ij}U_{ji}G_i'^{\mu\nu} G'_{\mu\nu,i}$
$f_{14}^{ij} = -8\,\mathcal{I}[q^4]_{ij}^{33}$	$[P^\mu, U_{ij}][P^\nu, U_{ji}]G'_{\nu\mu,i}$
$f_{15}^{ij} = 4\big(\mathcal{I}[q^4]_{ij}^{33} + \mathcal{I}[q^4]_{ij}^{42}\big)$	$\big(U_{ij}[P^\mu, U_{ji}] - [P^\mu, U_{ij}]U_{ji}\big)[P^\nu, G'_{\nu\mu,i}]$
$f_{16}^{ijklm} = \frac{1}{5}\,\mathcal{I}_{ijklm}^{11111}$	$U_{ij}U_{jk}U_{kl}U_{lm}U_{mi}$
$f_{17}^{ijkl} = 2\big(\mathcal{I}[q^2]_{ijkl}^{2112} + \mathcal{I}[q^2]_{ijkl}^{1212} + \mathcal{I}[q^2]_{ijkl}^{1122}\big)$	$U_{ij}U_{jk}[P^\mu, U_{kl}][P_\mu, U_{li}]$
$f_{18}^{ijkl} = \mathcal{I}[q^2]_{ijkl}^{2121} + \mathcal{I}[q^2]_{ijkl}^{2112} + \mathcal{I}[q^2]_{ijkl}^{1221} + \mathcal{I}[q^2]_{ijkl}^{1212}$	$U_{ij}[P^\mu, U_{jk}]U_{kl}[P_\mu, U_{li}]$
$f_{19}^{ijklmn} = \frac{1}{6}\,\mathcal{I}_{ijklmn}^{111111}$	$U_{ij}U_{jk}U_{kl}U_{lm}U_{mn}U_{ni}$

Degenerate limit *(Henning, Lu, Murayama, 1412.1837)*

$$f_5 = -\frac{i}{(4\pi)^2 60m^2}, \quad f_{11} = \frac{i}{(4\pi)^2 12m^4}, \quad f_{15a} = \frac{i}{(4\pi)^2 60m^4},$$

$$f_6 = -\frac{i}{(4\pi)^2 90m^2}, \quad f_{12,a} = 0, \quad f_{15b} = \frac{i}{(4\pi)^2 60m^4},$$

$$f_7 = -\frac{i}{(4\pi)^2 12m^2}, \quad f_{12,b} = 0, \quad f_{16} = -\frac{i}{(4\pi)^2 60m^6},$$

$$f_8 = -\frac{i}{(4\pi)^2 6m^2}, \quad f_{12,c} = \frac{i}{(4\pi)^2 120m^4}, \quad f_{17} = -\frac{i}{(4\pi)^2 20m^6},$$

$$f_9 = -\frac{i}{(4\pi)^2 12m^2}, \quad f_{13} = \frac{i}{(4\pi)^2 24m^4}, \quad f_{18} = -\frac{i}{(4\pi)^2 30m^6},$$

$$f_{10} = \frac{i}{(4\pi)^2 24m^4}, \quad f_{14} = \frac{-i}{(4\pi)^2 60m^4}, \quad f_{19} = \frac{i}{(4\pi)^2 120m^8}.$$

Drozd, J. Ellis, Quevillon, TY, 1512.03003;

Simplified form by covariant diagram computation shown here from Z. Zhang, 1610.00710.

Universality of the One-Loop Effective Action

- **Heavy-light** extension also done: *(S.A.R. Ellis, Quevillon, TY, Z. Zhang, 1705.xxxxx)*

$\mathcal{O}(U_H^4 P^2)$ terms	
$f_{17}^{ijkl} = 2\left(\mathcal{I}[q^2]_{ijkl}^{2112} + \mathcal{I}[q^2]_{ijkl}^{1212} + \mathcal{I}[q^2]_{ijkl}^{1122}\right)$	$U_{Hij}U_{Hjk}[P^\mu, U_{Hkl}][P_\mu, U_{Hli}]$
$f_{18}^{ijkl} = \mathcal{I}[q^2]_{ijkl}^{2121} + \mathcal{I}[q^2]_{ijkl}^{2112} + \mathcal{I}[q^2]_{ijkl}^{1221} + \mathcal{I}[q^2]_{ijkl}^{1212}$	$U_{Hij}[P^\mu, U_{Hjk}]U_{Hkl}[P_\mu, U_{Hli}]$
$\mathcal{O}(U_H^2 U_{HL}^1 U_{LH}^1 P^2)$ terms	
$f_{17A}^{ijk} = 2\left(\mathcal{I}[q^2]_{ijk0}^{1122} + \mathcal{I}[q^2]_{ijk0}^{1221} + \mathcal{I}[q^2]_{ijk0}^{2121}\right)$	$U_{Hij}U_{HLji'}[P^\mu, U_{LHi'k}][P_\mu, U_{Hki}]$ $+U_{LHi'i}U_{Hij}[P^\mu, U_{Hjk}][P_\mu, U_{HLki'}]$
$f_{17B}^{ijk} = 2\left(\mathcal{I}[q^2]_{ijk0}^{1122} + \mathcal{I}[q^2]_{ijk0}^{1212} + \mathcal{I}[q^2]_{ijk0}^{2112}\right)$	$U_{Hij}U_{Hjk}[P^\mu, U_{HLki'}][P_\mu, U_{LHi'i}]$
$f_{17C}^{ijk} = 2\left(\mathcal{I}[q^2]_{ijk0}^{1122} + \mathcal{I}[q^2]_{ijk0}^{2121} + \mathcal{I}[q^2]_{ijk0}^{1221}\right)$	$U_{HLii'}U_{LHi'j}[P^\mu, U_{Hjk}], [P_\mu, U_{Hki}]$
$f_{18A}^{ijk} = 2\left(\mathcal{I}[q^2]_{ijk0}^{1221} + \mathcal{I}[q^2]_{ijk0}^{2121} + \mathcal{I}[q^2]_{ijk0}^{1212} + \mathcal{I}[q^2]_{ijk0}^{2112}\right)$	$U_{Hij}[P^\mu, U_{HLji'}]U_{LHi'k}[P_\mu, U_{Hki}]$ $+U_{Hij}[P^\mu, U_{Hjk}]U_{HLki'}[P_\mu, U_{LHi'i}]$
$\mathcal{O}(U_H^1 U_L^1 U_{HL}^1 U_{LH}^1 P^2)$ terms	
$f_{17D}^{ij} = 2\left(2\mathcal{I}[q^2]_{ij0}^{123} + \mathcal{I}[q^2]_{ij0}^{222}\right)$	$U_{HLii'}U_{Li'j'}[P^\mu, U_{LHj'j}][P_\mu, U_{Hji}]$ $+U_{Li'j'}U_{LHj'i}[P^\mu, U_{Hij}][P_\mu, U_{HLji'}]$
$f_{17E}^{ij} = 2\left(\mathcal{I}[q^2]_{ij0}^{114} + \mathcal{I}[q^2]_{ij0}^{123} + \mathcal{I}[q^2]_{ij0}^{213}\right)$	$U_{Hij}U_{HLji'}[P^\mu, U_{Li'j'}][P_\mu, U_{LHj'i}]$ $+U_{LHi'i}U_{Hij}[P^\mu, U_{HLjj'}][P_\mu, U_{Lj'i'}]$
$f_{18B}^{ij} = 2\left(\mathcal{I}[q^2]_{ij0}^{123} + \mathcal{I}[q^2]_{ij0}^{222} + \mathcal{I}[q^2]_{ij0}^{114} + \mathcal{I}[q^2]_{ij0}^{213}\right)$	$U_{HLii'}[P^\mu, U_{Li'j'}]U_{LHj'j}[P_\mu, U_{Hji}]$
$f_{18C}^{ij} = 4\left(\mathcal{I}[q^2]_{ij0}^{123} + \mathcal{I}[q^2]_{ij0}^{213}\right)$	$U_{Hij}[P^\mu, U_{HLji'}]U_{Li'j'}[P_\mu, U_{LHj'i}]$
$\mathcal{O}(U_L^2 U_{HL}^1 U_{LH}^1 P^2)$ terms	
$f_{17F}^{i} = 2\left(2\mathcal{I}[q^2]_{i0}^{15} + \mathcal{I}[q^2]_{i0}^{24}\right)$	$U_{HLii'}U_{Li'j'}[P^\mu, U_{Lj'k'}][P_\mu, U_{LHk'i}]$ $+U_{Li'j'}U_{LHj'i}[P^\mu, U_{HLik'}][P_\mu, U_{Lk'i'}]$
$f_{17G}^{i} = 2\left(2\mathcal{I}[q^2]_{i0}^{15} + \mathcal{I}[q^2]_{i0}^{24}\right)$	$U_{LHi'i}U_{HLij'}[P^\mu, U_{Lj'k'}][P_\mu, U_{Lk'i'}]$
$f_{17H}^{i} = 6\mathcal{I}[q^2]_{i0}^{24}$	$U_{Li'j'}U_{Lj'k'}[P^\mu, U_{LHk'i}][P_\mu, U_{HLii'}]$
$f_{18D}^{i} = 4\left(\mathcal{I}[q^2]_{i0}^{15} + \mathcal{I}[q^2]_{i0}^{24}\right)$	$U_{HLii'}[P^\mu, U_{Li'j'}]U_{Lj'k'}[P_\mu, U_{LHk'i}]$ $+U_{LHi'i}[P^\mu, U_{HLij'}]U_{Lj'k'}[P_\mu, U_{Lk'i}]$
$\mathcal{O}(U_{HL}^2 U_{LH}^2 P^2)$ terms	
$f_{17I}^{ij} = 2\left(\mathcal{I}[q^2]_{ij0}^{114} + \mathcal{I}[q^2]_{ij0}^{213} + \mathcal{I}[q^2]_{ij0}^{123}\right)$	$U_{HLii'}U_{LHi'j}[P^\mu, U_{HLjj'}][P_\mu, U_{LHj'i}]$
$f_{17J}^{ij} = 2\left(\mathcal{I}[q^2]_{ij0}^{222} + 2\mathcal{I}[q^2]_{ij0}^{123}\right)$	$U_{LHi'i}U_{HLij'}[P^\mu, U_{LHj'j}][P_\mu, U_{HLji'}]$
$f_{18E}^{ij} = \mathcal{I}[q^2]_{ij0}^{114} + 2\mathcal{I}[q^2]_{ij0}^{123} + \mathcal{I}[q^2]_{ij0}^{222}$	$U_{HLii'}[P^\mu, U_{LHi'j}]U_{HLjj'}[P_\mu, U_{LHj'i}]$ $+U_{LHi'i}[P^\mu, U_{HLij'}]U_{LHj'j}[P_\mu, U_{HLji'}]$

$\mathcal{O}(U_H^3 P^2)$ terms	
$f_{11}^{ijk} = 2\left(\mathcal{I}[q^2]_{ijk}^{122} + \mathcal{I}[q^2]_{ijk}^{212}\right)$	$U_{Hij}[P^\mu, U_{Hjk}][P_\mu, U_{Hki}]$
$\mathcal{O}(U_H^1 U_{HL}^1 U_{LH}^1 P^2)$ terms	
$f_{11A}^{ij} = 2\left(\mathcal{I}[q^2]_{ij0}^{122} + \mathcal{I}[q^2]_{ij0}^{212}\right)$ $f_{11B}^{ij} = 2\left(\mathcal{I}[q^2]_{ij0}^{221} + \mathcal{I}[q^2]_{ij0}^{122}\right)$	$U_{Hij}[P^\mu, U_{HLji'}][P_\mu, U_{LHi'i}$ $U_{LHi'i}[P^\mu, U_{Hij}][P_\mu, U_{HLji'}] + U_{HLii'}[P^\mu, U_{LHi'j}][P_\mu, U_{Hji}]$
$\mathcal{O}(U_L^1 U_{HL}^1 U_{LH}^1 P^2)$ terms	
$f_{11C}^{ij} = 4\mathcal{I}[q^2]_{i0}^{23}$ $f_{11D}^{ij} = 2\left(\mathcal{I}[q^2]_{i0}^{14} + \mathcal{I}[q^2]_{i0}^{23}\right)$	$U_{Li'j'}[P^\mu, U_{LHj'i}][P_\mu, U_{HLii'}]$ $U_{HLii'}[P^\mu, U_{Li'j'}][P_\mu, U_{LHj'i}] + U_{LHi'i}[P^\mu, U_{HLij'}][P_\mu, U_{Lj'i'}]$

P-only terms	
$f_3^i = 2\mathcal{I}[q^4]_i^4$	$G_i'^{\mu\nu} G'_{\mu\nu_i}$
$f_5^i = 16\mathcal{I}[q^6]_i^6$	$[P^\mu, G'_{\mu\nu_i}][P_\rho, G_i'^{\rho\nu}]$
$f_6^i = (32/3)\mathcal{I}[q^6]_i^6$	$G_{\nu i}'^{\mu} G_{\rho i}'^{\nu} G_{\mu i}'^{\rho}$

$\mathcal{O}(U_H^2 P^4)$ terms	
$f_{12}^{ij} = 4\mathcal{I}[q^4]_{ij}^{33}$	$[P^\mu, [P_\mu, U_{Hij}]][P^\nu, [P_\nu, U_{Hji}]]$
$f_{13}^{ij} = 4\left(\mathcal{I}[q^4]_{ij}^{33} + 2\mathcal{I}[q^4]_{ij}^{33} + 2\mathcal{I}[q^4]_{ij}^{51}\right)$	$U_{Hij}U_{Hji}G_i'^{\mu\nu} G'_{\mu\nu_i}$
$f_{14}^{ij} = -8\mathcal{I}[q^4]_{ij}^{33}$	$[P^\mu, U_{Hij}][P^\nu, U_{Hji}]G'_{\nu\mu_i}$
$f_{15}^{ij} = 4\left(\mathcal{I}[q^4]_{ij}^{33} + \mathcal{I}[q^4]_{ij}^{42}\right)$	$\left(U_{Hij}[P^\mu, U_{Hji}] - [P^\mu, U_{Hij}]U_{Hji}\right)[P^\nu, G'_{\nu\mu_i}]$
$\mathcal{O}(U_{HL}^1 U_{LH}^1 P^4)$ terms	
$f_{12A}^{i} = 8\mathcal{I}[q^4]_{i0}^{33}$	$[P^\mu, [P_\mu, U_{HLii'}]][P^\nu, [P_\nu, U_{LHi'i}]]$
$f_{13A}^{i} = 2\left(\mathcal{I}[q^4]_{i0}^{24} + 2\mathcal{I}[q^4]_{i0}^{33} + 3\mathcal{I}[q^4]_{i0}^{42} + 4\mathcal{I}[q^4]_{i0}^{51}\right)$ $f_{13B}^{i} = 2\left(4\mathcal{I}[q^4]_{i0}^{15} + 3\mathcal{I}[q^4]_{i0}^{24} + 2\mathcal{I}[q^4]_{i0}^{33} + \mathcal{I}[q^4]_{i0}^{42}\right)$	$U_{HLii'}U_{LHi'i}G_i'^{\mu\nu} G'_{\mu\nu_i}$ $U_{LHi'i}U_{HLii'}G_{i'}'^{\mu\nu} G'_{\mu\nu_{i'}}$
$f_{14A}^{i} = 4\left(-\mathcal{I}[q^4]_{i0}^{24} - 2\mathcal{I}[q^4]_{i0}^{33} + \mathcal{I}[q^4]_{i0}^{42}\right)$ $f_{14B}^{i} = 4\left(\mathcal{I}[q^4]_{i0}^{24} - 2\mathcal{I}[q^4]_{i0}^{33} - \mathcal{I}[q^4]_{i0}^{42}\right)$	$[P^\mu, U_{HLii'}][P^\nu, U_{LHi'i}]G'_{\nu\mu_i}$ $[P^\mu, U_{LHi'i}][P^\nu, U_{HLii'}]G'_{\nu\mu_{i'}}$
$f_{15A}^{i} = 2\left(\mathcal{I}[q^4]_{i0}^{24} + 2\mathcal{I}[q^4]_{i0}^{33} + \mathcal{I}[q^4]_{i0}^{42}\right)$	$\left(U_{HLii'}[P^\mu, U_{LHi'i}] - [P^\mu, U_{HLii'}]U_{LHi'i}\right)[P^\nu, G'_{\nu\mu_i}]$ $+\left(U_{LHi'i}[P^\mu, U_{HLii'}] - [P^\mu, U_{LHi'i}]U_{HLii'}\right)[P^\nu, G'_{\nu\mu_{i'}}]$

$\mathcal{O}(U_H^2 P^2)$ terms	
$f_7^{ij} = \mathcal{I}[q^2]_{ij}^{22}$	$[P^\mu, U_{Hij}][P_\mu, U_{Hji}]$
$\mathcal{O}(U_{HL}^1 U_{LH}^1 P^2)$ terms	
$f_{7A}^{ij} = 2\mathcal{I}[q^2]_{i0}^{22}$	$[P^\mu, U_{HLii'}][P_\mu, U_{LHi'i}]$

$\mathcal{O}(U_H^1 P^4)$ terms	
$f_9^i = 8\mathcal{I}[q^4]_i^5$	$U_{Hij}G_i'^{\mu\nu} G'_{\mu\nu_i}$

$\mathcal{O}(U)$ term		$\mathcal{O}(U^3)$ terms	
$f_2^i = \mathcal{I}_i^1$	U_{Hii}	$f_8^{ijk} = \frac{1}{3}\mathcal{I}_{ijk}^{111}$	$U_{Hij}U_{Hjk}U_{Hki}$
$\mathcal{O}(U^2)$ terms		$f_{8A}^{ij} = \mathcal{I}_{ij0}^{111}$	$U_{Hij}U_{HLji'}U_{LHi'i}$
$f_4^{ij} = \frac{1}{2}\mathcal{I}_{ij}^{11}$	$U_{Hij}U_{Hji}$	$f_{8B}^{i} = \mathcal{I}_{i0}^{12}$	$U_{HLii'}U_{Li'j'}U_{LHj'i}$
$f_{4A}^{ij} = \mathcal{I}_{i0}^{11}$	$U_{HLii'}U_{LHi'i}$	$\mathcal{O}(U^6)$ terms	
$\mathcal{O}(U^4)$ terms		$f_{19}^{ijklmn} = \frac{1}{6}\mathcal{I}_{ijklmn}^{111111}$	$U_{Hij}U_{Hjk}U_{Hkl}U_{Hlm}U_{Hmn}U_{Hni}$
$f_{10}^{ijkl} = \frac{1}{4}\mathcal{I}_{ijkl}^{1111}$	$U_{Hij}U_{Hjk}U_{Hkl}U_{Hli}$	$f_{19A}^{ijklm} = \mathcal{I}_{ijklm0}^{111111}$	$U_{Hij}U_{Hjk}U_{Hkl}U_{Hlm}U_{HLmi'}U_{LHi'i}$
$f_{10A}^{ijk} = \mathcal{I}_{ijk0}^{1111}$	$U_{Hij}U_{Hjk}U_{HLki'}U_{LHi'i}$	$f_{19B}^{ijkl} = \mathcal{I}_{ijkl0}^{11112}$	$U_{Hij}U_{Hjk}U_{Hkl}U_{HLli'}U_{Li'j'}U_{LHj'i}$
$f_{10B}^{ij} = \mathcal{I}_{ij0}^{112}$	$U_{Hij}U_{HLji'}U_{Li'j'}U_{LHj'i}$	$f_{19C}^{ijkl} = \mathcal{I}_{ijkl0}^{11112}$	$U_{Hij}U_{Hjk}U_{HLki'}U_{LHi'l}U_{HLlj'}U_{LHj'i}$
$f_{10C}^{ij} = \frac{1}{2}\mathcal{I}_{ij0}^{112}$	$U_{HLii'}U_{LHi'j}U_{HLjj'}U_{LHj'i}$	$f_{19D}^{ijk} = \mathcal{I}_{ijk0}^{1113}$	$U_{Hij}U_{Hjk}U_{HLki'}U_{Li'j'}U_{Lj'k'}U_{LHk'i}$
$f_{10D}^{i} = \mathcal{I}_{i0}^{13}$	$U_{HLii'}U_{Li'j'}U_{Lj'k'}U_{LHk'i}$	$f_{19E}^{ijkl} = \frac{1}{2}\mathcal{I}_{ijkl0}^{11112}$	$U_{Hij}U_{HLji'}U_{LHi'k}U_{Hkl}U_{HLlj'}U_{LHj'i}$
$\mathcal{O}(U^5)$ terms		$f_{19F}^{ijk} = \mathcal{I}_{ijk0}^{1113}$	$U_{Hij}U_{HLji'}U_{LHi'k}U_{HLkj'}U_{Lj'k'}U_{LHk'i}$
$f_{16}^{ijklm} = \frac{1}{5}\mathcal{I}_{ijklm}^{11111}$	$U_{Hij}U_{Hjk}U_{Hkl}U_{Hlm}U_{Hmi}$	$f_{19G}^{ijk} = \mathcal{I}_{ijk0}^{1113}$	$U_{Hij}U_{HLji'}U_{Li'j'}U_{LHj'k}U_{HLkk'}U_{LHk'i}$
$f_{16A}^{ijkl} = \mathcal{I}_{ijkl0}^{11111}$	$U_{Hij}U_{Hjk}U_{Hkl}U_{HLli'}U_{LHi'i}$	$f_{19H}^{ij} = \mathcal{I}_{ij0}^{114}$	$U_{Hij}U_{HLji'}U_{Li'j'}U_{Lj'k'}U_{Lk'l'}U_{LHl'i}$
$f_{16B}^{ijk} = \mathcal{I}_{ijk0}^{1112}$	$U_{Hij}U_{Hjk}U_{HLki'}U_{Li'j'}U_{LHj'i}$	$f_{19I}^{ijk} = \frac{1}{3}\mathcal{I}_{ijk0}^{1113}$	$U_{HLii'}U_{LHi'j}U_{HLjj'}U_{LHj'k}U_{HLkk'}U_{LHk'i}$
$f_{16C}^{ijk} = \mathcal{I}_{ijk0}^{1112}$	$U_{Hij}U_{HLji'}U_{LHi'k}U_{HLkj'}U_{LHj'i}$	$f_{19J}^{ij} = \mathcal{I}_{ij0}^{114}$	$U_{HLii'}U_{LHi'j}U_{HLjj'}U_{Lj'k'}U_{Lk'l'}U_{LHl'i}$
$f_{16D}^{ij} = \mathcal{I}_{ij0}^{113}$	$U_{Hij}U_{HLji'}U_{Li'j'}U_{Lj'k'}U_{LHk'i}$	$f_{19K}^{ij} = \frac{1}{2}\mathcal{I}_{ij0}^{114}$	$U_{HLii'}U_{Li'j'}U_{LHj'j}U_{HLjk'}U_{Lk'l'}U_{LHl'i}$
$f_{16E}^{ij} = \mathcal{I}_{ij0}^{113}$	$U_{HLii'}U_{LHi'j}U_{HLjj'}U_{Lj'k'}U_{LHk'i}$	$f_{19L}^{i} = \mathcal{I}_{i0}^{15}$	$U_{HLii'}U_{Li'j'}U_{Lj'k'}U_{Lk'l'}U_{Ll'm'}U_{LHm'i}$
$f_{16F}^{i} = \mathcal{I}_{i0}^{14}$	$U_{HLii'}U_{Li'j'}U_{Lj'k'}U_{Lk'l'}U_{LHl'i}$		

Application of the UOLEA: MSSM Stops

- Write UV Lagrangian for heavy multiplet in appropriate form to extract U matrix, mass matrix, and covariant derivative:

$$\boxed{\mathcal{L}_{\rm UV} = \mathcal{L}_{\rm SM} + \cancel{(\Phi^\dagger F(x) + \text{h.c.})} + \Phi^\dagger (P^2 - M^2 - U(x))\Phi + \mathcal{O}(\Phi^3)}$$

(R-parity)

$$\Phi = (\tilde{Q}\,, \tilde{t}_R^*), \qquad M^2 = \begin{pmatrix} m_{\tilde{Q}}^2 & 0 \\ 0 & m_{\tilde{t}_R}^2 \end{pmatrix} \qquad G'_{\mu\nu} = \begin{pmatrix} W'^a_{\mu\nu}\tau^a + Y_{\tilde{Q}} B'_{\mu\nu}\mathbb{1} & 0 \\ 0 & -Y_{\tilde{t}_R} B'_{\mu\nu} \end{pmatrix}$$

$$U = \begin{pmatrix} (h_t^2 + \frac{1}{2}g_2^2 c_\beta^2)\tilde{H}\tilde{H}^\dagger + \frac{1}{2}g_2^2 s_\beta^2 H H^\dagger - \frac{1}{2}(g_1^2 Y_{\tilde{Q}} c_{2\beta} + \frac{1}{2}g_2^2)|H|^2 & h_t X_t \tilde{H} \\ h_t X_t \tilde{H}^\dagger & (h_t^2 - \frac{1}{2}g_1^2 Y_{\tilde{t}_R} c_{2\beta})|H|^2 \end{pmatrix}$$

Application of the UOLEA: MSSM Stops

- Write UV Lagrangian for heavy multiplet in appropriate form to extract U matrix, mass matrix, and covariant derivative:

$$\mathcal{L}_{\rm UV} = \mathcal{L}_{\rm SM} + (\Phi^\dagger F(x) + \text{h.c.}) + \Phi^\dagger (P^2 - M^2 - U(x))\Phi + \mathcal{O}(\Phi^3),$$

(R-parity)

$$\Phi = (\tilde{Q}\,, \tilde{t}_R^*),$$

$$M^2 = \begin{pmatrix} m_{\tilde{Q}}^2 & 0 \\ 0 & m_{\tilde{t}_R}^2 \end{pmatrix}$$

$$G'_{\mu\nu} = \begin{pmatrix} W'^a_{\mu\nu}\tau^a + Y_{\tilde{Q}} B'_{\mu\nu}\mathbb{1} & 0 \\ 0 & -Y_{\tilde{t}_R} B'_{\mu\nu} \end{pmatrix}$$

$$U = \begin{pmatrix} (h_t^2 + \frac{1}{2}g_2^2 c_\beta^2)\tilde{H}\tilde{H}^\dagger + \frac{1}{2}g_2^2 s_\beta^2 HH^\dagger - \frac{1}{2}(g_1^2 Y_{\tilde{Q}} c_{2\beta} + \frac{1}{2}g_2^2)|H|^2 & h_t X_t \tilde{H} \\ h_t X_t \tilde{H}^\dagger & (h_t^2 - \frac{1}{2}g_1^2 Y_{\tilde{t}_R} c_{2\beta})|H|^2 \end{pmatrix}$$

Application of the UOLEA: MSSM Stops

- Pick the relevant operators by counting operator dimensions

Universal coefficient	Operator
$f_2^i = \mathcal{I}_i^1$	U_{ii}
$f_3^i = 2\,\mathcal{I}[q^4]_i^4$	$G_i'^{\mu\nu} G'_{\mu\nu,i}$
$f_4^{ij} = \frac{1}{2}\,\mathcal{I}_{ij}^{11}$	$U_{ij}U_{ji}$
$f_5^i = 16\,\mathcal{I}[q^6]_i^6$	$[P^\mu, G'_{\mu\nu,i}][P_\rho, G_i'^{\rho\nu}]$
$f_6^i = \frac{32}{3}\,\mathcal{I}[q^6]_i^6$	$G'^{\mu}{}_{\nu,i} G'^{\nu}{}_{\rho,i} G'^{\rho}{}_{\mu,i}$
$f_7^{ij} = \mathcal{I}[q^2]_{ij}^{22}$	$[P^\mu, U_{ij}][P_\mu, U_{ji}]$
$f_8^{ijk} = \frac{1}{3}\,\mathcal{I}_{ijk}^{111}$	$U_{ij}U_{jk}U_{ki}$
$f_9^i = 8\,\mathcal{I}[q^4]_i^5$	$U_{ii} G_i'^{\mu\nu} G'_{\mu\nu,i}$
$f_{10}^{ijkl} = \frac{1}{4}\,\mathcal{I}_{ijkl}^{1111}$	$U_{ij}U_{jk}U_{kl}U_{li}$
$f_{11}^{ijk} = 2\big(\mathcal{I}[q^2]_{ijk}^{122} + \mathcal{I}[q^2]_{ijk}^{212}\big)$	$U_{ij}[P^\mu, U_{jk}][P_\mu, U_{ki}]$
$f_{12}^{ij} = 4\,\mathcal{I}[q^4]_{ij}^{33}$	$\big[P^\mu, [P_\mu, U_{ij}]\big]\big[P^\nu, [P_\nu, U_{ji}]\big]$
$f_{13}^{ij} = 4\big(\mathcal{I}[q^4]_{ij}^{33}$ $+2\,\mathcal{I}[q^4]_{ij}^{42} + 2\,\mathcal{I}[q^4]_{ij}^{51}\big)$	$U_{ij}U_{ji} G_i'^{\mu\nu} G'_{\mu\nu,i}$
$f_{14}^{ij} = -8\,\mathcal{I}[q^4]_{ij}^{33}$	$[P^\mu, U_{ij}][P^\nu, U_{ji}] G'_{\nu\mu,i}$
$f_{15}^{ij} = 4\big(\mathcal{I}[q^4]_{ij}^{33} + \mathcal{I}[q^4]_{ij}^{42}\big)$	$\big(U_{ij}[P^\mu, U_{ji}] - [P^\mu, U_{ij}]U_{ji}\big)[P^\nu, G'_{\nu\mu,i}]$
$f_{16}^{ijklm} = \frac{1}{5}\,\mathcal{I}_{ijklm}^{11111}$	$U_{ij}U_{jk}U_{kl}U_{lm}U_{mi}$
$f_{17}^{ijkl} = 2\big(\mathcal{I}[q^2]_{ijkl}^{2112}$ $+\mathcal{I}[q^2]_{ijkl}^{1212} + \mathcal{I}[q^2]_{ijkl}^{1122}\big)$	$U_{ij}U_{jk}[P^\mu, U_{kl}][P_\mu, U_{li}]$
$f_{18}^{ijkl} = \mathcal{I}[q^2]_{ijkl}^{2121} + \mathcal{I}[q^2]_{ijkl}^{2112}$ $+\mathcal{I}[q^2]_{ijkl}^{1221} + \mathcal{I}[q^2]_{ijkl}^{1212}$	$U_{ij}[P^\mu, U_{jk}]U_{kl}[P_\mu, U_{li}]$
$f_{19}^{ijklmn} = \frac{1}{6}\,\mathcal{I}_{ijklmn}^{111111}$	$U_{ij}U_{jk}U_{kl}U_{lm}U_{mn}U_{ni}$

$$U = \begin{pmatrix} (h_t^2 + \frac{1}{2}g_2^2 c_\beta^2)\tilde{H}\tilde{H}^\dagger + \frac{1}{2}g_2^2 s_\beta^2 H H^\dagger - \frac{1}{2}(g_1^2 Y_{\tilde{Q}} c_{2\beta} + \frac{1}{2}g_2^2)|H|^2 & h_t X_t \tilde{H} \\ h_t X_t \tilde{H}^\dagger & (h_t^2 - \frac{1}{2}g_1^2 Y_{\tilde{t}_R} c_{2\beta})|H|^2 \end{pmatrix}$$

	X_t^0	X_t^2	X_t^4	X_t^6
c_6	f_8	f_{10}	f_{16}	f_{19}
c_H	f_7	f_{11}	f_{17}, f_{18}	-
c_T	f_7	f_{11}	f_{17}, f_{18}	-
c_R	f_7	f_{11}	f_{17}	-
c_{GG}	f_9	f_{13}	-	-
c_{WW}	f_9	f_{13}, f_{14}	-	-
c_{BB}	f_9	f_{13}, f_{14}	-	-
c_{WB}	f_9	f_{13}, f_{14}	-	-
c_W	-	f_{15a}, f_{15b}	-	-
c_B	-	f_{15a}, f_{15b}	-	-
c_D	-	f_{12c}	-	-

Application of the UOLEA: MSSM Stops

- Example: $\mathcal{O}_{GG} \;=\; g_s^2\, |H|^2\, G^a_{\mu\nu} G^{a,\mu\nu}$

Universal coefficient	Operator
$f_2^i = \mathcal{I}_i^1$	U_{ii}
$f_3^i = 2\,\mathcal{I}[q^4]_i^4$	$G_i^{\prime\mu\nu} G'_{\mu\nu,i}$
$f_4^{ij} = \frac{1}{2}\,\mathcal{I}_{ij}^{11}$	$U_{ij}U_{ji}$
$f_5^i = 16\,\mathcal{I}[q^6]_i^6$	$[P^\mu, G'_{\mu\nu,i}][P_\rho, G_i^{\prime\rho\nu}]$
$f_6^i = \frac{32}{3}\,\mathcal{I}[q^6]_i^6$	$G'^{\mu}{}_{\nu,i} G'^{\nu}{}_{\rho,i} G'^{\rho}{}_{\mu,i}$
$f_7^{ij} = \mathcal{I}[q^2]_{ij}^{22}$	$[P^\mu, U_{ij}][P_\mu, U_{ji}]$
$f_8^{ijk} = \frac{1}{3}\,\mathcal{I}_{ijk}^{111}$	$U_{ij}U_{jk}U_{ki}$
$f_9^i = 8\,\mathcal{I}[q^4]_i^5$	$U_{ii} G_i^{\prime\mu\nu} G'_{\mu\nu,i}$
$f_{10}^{ijkl} = \frac{1}{4}\,\mathcal{I}_{ijkl}^{1111}$	$U_{ij}U_{jk}U_{kl}U_{li}$
$f_{11}^{ijk} = 2\big(\mathcal{I}[q^2]_{ijk}^{122} + \mathcal{I}[q^2]_{ijk}^{212}\big)$	$U_{ij}[P^\mu, U_{jk}][P_\mu, U_{ki}]$
$f_{12}^{ij} = 4\,\mathcal{I}[q^4]_{ij}^{33}$	$\big[P^\mu,[P_\mu,U_{ij}]\big]\big[P^\nu,[P_\nu,U_{ji}]\big]$
$f_{13}^{ij} = 4\big(\mathcal{I}[q^4]_{ij}^{33} + 2\,\mathcal{I}[q^4]_{ij}^{42} + 2\,\mathcal{I}[q^4]_{ij}^{51}\big)$	$U_{ij}U_{ji} G_i^{\prime\mu\nu} G'_{\mu\nu,i}$
$f_{14}^{ij} = -8\,\mathcal{I}[q^4]_{ij}^{33}$	$[P^\mu, U_{ij}][P^\nu, U_{ji}] G'_{\nu\mu,i}$
$f_{15}^{ij} = 4\big(\mathcal{I}[q^4]_{ij}^{33} + \mathcal{I}[q^4]_{ij}^{42}\big)$	$\big(U_{ij}[P^\mu, U_{ji}] - [P^\mu, U_{ij}]U_{ji}\big)[P^\nu, G'_{\nu\mu,i}]$
$f_{16}^{ijklm} = \frac{1}{5}\,\mathcal{I}_{ijklm}^{11111}$	$U_{ij}U_{jk}U_{kl}U_{lm}U_{mi}$
$f_{17}^{ijkl} = 2\big(\mathcal{I}[q^2]_{ijkl}^{2112} + \mathcal{I}[q^2]_{ijkl}^{1212} + \mathcal{I}[q^2]_{ijkl}^{1122}\big)$	$U_{ij}U_{jk}[P^\mu, U_{kl}][P_\mu, U_{li}]$
$f_{18}^{ijkl} = \mathcal{I}[q^2]_{ijkl}^{2121} + \mathcal{I}[q^2]_{ijkl}^{2112} + \mathcal{I}[q^2]_{ijkl}^{1221} + \mathcal{I}[q^2]_{ijkl}^{1212}$	$U_{ij}[P^\mu, U_{jk}]U_{kl}[P_\mu, U_{li}]$
$f_{19}^{ijklmn} = \frac{1}{6}\,\mathcal{I}_{ijklmn}^{111111}$	$U_{ij}U_{jk}U_{kl}U_{lm}U_{mn}U_{ni}$

$$U = \begin{pmatrix} (h_t^2 + \frac{1}{2}g_2^2 c_\beta^2)\tilde{H}\tilde{H}^\dagger + \frac{1}{2}g_2^2 s_\beta^2 H H^\dagger - \frac{1}{2}(g_1^2 Y_{\tilde{Q}} c_{2\beta} + \frac{1}{2}g_2^2)|H|^2 & h_t X_t \tilde{H} \\ h_t X_t \tilde{H}^\dagger & (h_t^2 - \frac{1}{2}g_1^2 Y_{\tilde{t}_R} c_{2\beta})|H|^2 \end{pmatrix}$$

	X_t^0	X_t^2	X_t^4	X_t^6
c_6	f_8	f_{10}	f_{16}	f_{19}
c_H	f_7	f_{11}	f_{17}, f_{18}	-
c_T	f_7	f_{11}	f_{17}, f_{18}	-
c_R	f_7	f_{11}	f_{17}	-
c_{GG}	f_9	f_{13}	-	-
c_{WW}	f_9	f_{13}, f_{14}	-	-
c_{BB}	f_9	f_{13}, f_{14}	-	-
c_{WB}	f_9	f_{13}, f_{14}	-	-
c_W	-	f_{15a}, f_{15b}	-	-
c_B	-	f_{15a}, f_{15b}	-	-
c_D	-	f_{12c}	-	-

Application of the UOLEA: MSSM Stops

- Example: $\mathcal{O}_{GG} = g_s^2 \, |H|^2 \, G^a_{\mu\nu} G^{a,\mu\nu}$

Universal coefficient	Operator
$f_2^i = \mathcal{I}_i^1$	U_{ii}
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$f_5^i = 16\,\mathcal{I}[q^6]_i^6$	$[P^\mu, G'_{\mu\nu,i}][P_\rho, G_i'^{\rho\nu}]$
$f_6^i = \frac{32}{3}\,\mathcal{I}[q^6]_i^6$	$G'^{\mu}{}_{\nu,i} G'^{\nu}{}_{\rho,i} G'^{\rho}{}_{\mu,i}$
$f_7^{ij} = \mathcal{I}[q^2]_{ij}^{22}$	$[P^\mu, U_{ij}][P_\mu, U_{ji}]$
$f_8^{ijk} = \frac{1}{3}\,\mathcal{I}_{ijk}^{111}$	$U_{ij}U_{jk}U_{ki}$
$f_9^i = 8\,\mathcal{I}[q^4]_i^5$	$U_{ii} G_i'^{\mu\nu} G'_{\mu\nu,i}$
$f_{10}^{ijkl} = \frac{1}{4}\,\mathcal{I}_{ijkl}^{1111}$	$U_{ij}U_{jk}U_{kl}U_{li}$
$f_{11}^{ijk} = 2\big(\mathcal{I}[q^2]_{ijk}^{122} + \mathcal{I}[q^2]_{ijk}^{212}\big)$	$U_{ij}[P^\mu, U_{jk}][P_\mu, U_{ki}]$
$f_{12}^{ij} = 4\,\mathcal{I}[q^4]_{ij}^{33}$	$[P^\mu, [P_\mu, U_{ij}]]\,[P^\nu, [P_\nu, U_{ji}]]$
$f_{13}^{ij} = 4\big(\mathcal{I}[q^4]_{ij}^{33} + 2\,\mathcal{I}[q^4]_{ij}^{42} + 2\,\mathcal{I}[q^4]_{ij}^{51}\big)$	$U_{ij}U_{ji} G_i'^{\mu\nu} G'_{\mu\nu,i}$
$f_{14}^{ij} = -8\,\mathcal{I}[q^4]_{ij}^{33}$	$[P^\mu, U_{ij}][P^\nu, U_{ji}] G'_{\nu\mu,i}$
$f_{15}^{ij} = 4(\mathcal{I}[q^4]_{ij}^{33} + \mathcal{I}[q^4]_{ij}^{42})$	$(U_{ij}[P^\mu, U_{ji}] - [P^\mu, U_{ij}]U_{ji})[P^\nu, G'_{\nu\mu,i}]$
f_{16}^{ijklm} = [illegible]	[illegible]
f_{17}^{ijkl} = [illegible]	[illegible]
f_{18}^{ijkl} = [illegible]	[illegible]
f_{19}^{ijklmn} [illegible]	[illegible]

$$U = \begin{pmatrix} (h_t^2 + \frac{1}{2}g_2^2 c_\beta^2)\tilde{H}\tilde{H}^\dagger + \frac{1}{2}g_2^2 s_\beta^2 H H^\dagger - \frac{1}{2}(g_1^2 Y_{\tilde{Q}} c_{2\beta} + \frac{1}{2}g_2^2)|H|^2 & h_t X_t \tilde{H} \\ h_t X_t \tilde{H}^\dagger & (h_t^2 - \frac{1}{2}g_1^2 Y_{\tilde{t}_R} c_{2\beta})|H|^2 \end{pmatrix}$$

	X_t^0	X_t^2	X_t^4	X_t^6
c_6	f_8	f_{10}	f_{16}	f_{19}
c_H	f_7	f_{11}	f_{17}, f_{18}	-
c_T	f_7	f_{11}	f_{17}, f_{18}	-
c_R	f_7	f_{11}	f_{17}	-
c_{GG}	f_9	f_{13}	-	-
c_{WW}	f_9	f_{13}, f_{14}	-	-
c_{BB}	f_9	f_{13}, f_{14}	-	-
[illegible]	[illegible]	[illegible]	[illegible]	-
[illegible]	[illegible]	[illegible]	[illegible]	-
[illegible]	[illegible]	[illegible]	[illegible]	-
[illegible]	[illegible]	[illegible]	[illegible]	-

$$c_{GG} = \frac{1}{24}\left(\frac{h_t^2 - \frac{1}{6}g_1^2 c_{2\beta}}{m_{\tilde{Q}}^2} + \frac{h_t^2 + \frac{1}{3}g_1^2 c_{2\beta}}{m_{\tilde{t}_R}^2} - \frac{\bar{X}_t^2}{m_{\tilde{Q}}^2 m_{\tilde{t}_R}^2}\right)$$

Application of the UOLEA: MSSM Stops

- Example: $\mathcal{O}_{GG} = g_s^2 \, |H|^2 \, G^a_{\mu\nu} G^{a,\mu\nu}$

For full results see Drozd, J. Ellis, Quevillon, TY, 1512.03003.

(note: error in previous results by R. Huo, 1509.05942)

Universal coefficient	Operator
$f_2^i = \mathcal{I}_i^1$	U_{ii}
$f_3^i = 2\,\mathcal{I}[q^4]_i^4$	$G_i'^{\mu\nu} G'_{\mu\nu,i}$
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$f_6^i = \frac{32}{3}\,\mathcal{I}[q^6]_i^6$	$G'^{\mu}{}_{\nu,i} G'^{\nu}{}_{\rho,i} G'^{\rho}{}_{\mu,i}$
$f_7^{ij} = \mathcal{I}[q^2]_{ij}^{22}$	$[P^\mu, U_{ij}][P_\mu, U_{ji}]$
$f_8^{ijk} = \frac{1}{3}\,\mathcal{I}_{ijk}^{111}$	$U_{ij}U_{jk}U_{ki}$
$f_9^i = 8\,\mathcal{I}[q^4]_i^5$	$U_{ii} G_i'^{\mu\nu} G'_{\mu\nu,i}$
$f_{10}^{ijkl} = \frac{1}{4}\,\mathcal{I}_{ijkl}^{1111}$	$U_{ij}U_{jk}U_{kl}U_{li}$
$f_{11}^{ijk} = 2\big(\mathcal{I}[q^2]_{ijk}^{122} + \mathcal{I}[q^2]_{ijk}^{212}\big)$	$U_{ij}[P^\mu, U_{jk}][P_\mu, U_{ki}]$
$f_{12}^{ij} = 4\,\mathcal{I}[q^4]_{ij}^{33}$	$[P^\mu, [P_\mu, U_{ij}]]\,[P^\nu, [P_\nu, U_{ji}]]$
$f_{13}^{ij} = 4\big(\mathcal{I}[q^4]_{ij}^{33} + 2\,\mathcal{I}[q^4]_{ij}^{42} + 2\,\mathcal{I}[q^4]_{ij}^{51}\big)$	$U_{ij}U_{ji} G_i'^{\mu\nu} G'_{\mu\nu,i}$
$f_{14}^{ij} = -8\,\mathcal{I}[q^4]_{ij}^{33}$	$[P^\mu, U_{ij}][P^\nu, U_{ji}] G'_{\nu\mu,i}$
$f_{15}^{ij} = 4(\mathcal{I}[q^4]_{ij}^{33} + \mathcal{I}[q^4]_{ij}^{42})$	$(U_{ij}[P^\mu, U_{ji}] - [P^\mu, U_{ij}]U_{ji})[P^\nu, G' \ldots]$
f_{16}^{ijklm}	
f_{17}^{ijkl}	
f_{18}^{ijkl}	
f_{19}^{ijklmn}	

$$U = \begin{pmatrix} (h_t^2 + \frac{1}{2}g_2^2 c_\beta^2)\tilde{H}\tilde{H}^\dagger + \frac{1}{2}g_2^2 s_\beta^2 H H^\dagger - \frac{1}{2}(g_1^2 Y_{\tilde{Q}} c_{2\beta} + \frac{1}{2}g_2^2)|H|^2 & h_t X_t \tilde{H} \\ h_t X_t \tilde{H}^\dagger & (h_t^2 - \frac{1}{2}g_1^2 Y_{\tilde{t}_R} c_{2\beta})|H|^2 \end{pmatrix}$$

	X_t^0	X_t^2	X_t^4	X_t^6
c_6	f_8	f_{10}	f_{16}	f_{19}
c_H	f_7	f_{11}	f_{17}, f_{18}	-
c_T	f_7	f_{11}	f_{17}, f_{18}	-
c_R	f_7	f_{11}	f_{17}	-
c_{GG}	f_9	f_{13}	-	-
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c_{BB}	f_9	f_{13}, f_{14}	-	-

$$c_{GG} = \frac{1}{24}\left(\frac{h_t^2 - \frac{1}{6}g_1^2 c_{2\beta}}{m^2_{\tilde{Q}}} + \frac{h_t^2 + \frac{1}{3}g_1^2 c_{2\beta}}{m^2_{\tilde{t}_R}} - \frac{\bar{X}_t^2}{m^2_{\tilde{Q}} m^2_{\tilde{t}_R}}\right)$$

Application of the UOLEA: Real Singlet Scalar

$$\Delta\mathcal{L} = \frac{1}{2}\left(\partial_\mu \Phi\right)^2 - \frac{1}{2}m^2\Phi^2 - \underline{A\left|H\right|^2\Phi} - \frac{1}{2}k\left|H\right|^2\Phi^2 - \frac{1}{3!}\mu\Phi^3 - \frac{1}{4!}\lambda_\Phi\Phi^4$$

- U matrix includes heavy-light contributions due to linear coupling

$$\mathcal{U} = \begin{pmatrix} \mathcal{U}_\phi & \left(\mathcal{U}^\dagger_{H\phi}\right)_{1\times 2} \\ \left(\mathcal{U}_{H\phi}\right)_{2\times 1} & \left(\mathcal{U}_H\right)_{2\times 2} \end{pmatrix}$$

- Classify possible contributions by counting operator dimensions

$$\mathcal{U}_\phi \equiv -\left.\frac{\delta^2\mathcal{L}}{\delta\phi^2}\right|_{\phi_c} = \kappa|H|^2 + \mu\phi_c + \frac{1}{2}\lambda_\phi\phi_c^2 \qquad \supset \mathcal{O}(H^2, \partial^2H^2, H^4, \partial^2H^4, H^6)$$

$$\mathcal{U}_{H\phi} \equiv -\left.\frac{\delta^2\mathcal{L}}{\delta H^\dagger\delta\phi}\right|_{\phi_c} = AH + \kappa H\phi_c \qquad \supset \mathcal{O}(H, H^3, H^5, H\partial^2H^2)$$

$$\mathcal{U}_{\tilde{H}\phi} \equiv -\left.\frac{\delta^2\mathcal{L}}{\delta\tilde{H}^\dagger\delta\phi}\right|_{\phi_c} = A\tilde{H} + \kappa\tilde{H}\phi_c \qquad \supset \mathcal{O}(H, H^3, H^5, H\partial^2H^2)$$

etc…

See S.A.R. Ellis, Quevillon, TY, Z. Zhang, 1705.xxxxx

Application of the UOLEA: Real Singlet Scalar

$$\mathcal{O}_6 = |H|^6$$

$$\mathcal{O}_H = \tfrac{1}{2}\left(\partial_\mu |H|^2\right)^2 \qquad \mathcal{O}_T = \tfrac{1}{2}\left(H^\dagger \overleftrightarrow{D}_\mu H\right)^2 \qquad \mathcal{O}_R = |H|^2 |D_\mu H|^2$$

- Classify possible contributions by counting operator dimensions

$\mathcal{O}(U)$ term		$\mathcal{O}(U^3)$ terms	
✓ $f_2^i = \mathcal{I}_i^1$ ✓	U_{Hii}	$f_8^{ijk} = \frac{1}{3}\mathcal{I}_{ijk}^{111}$ ✓	$U_{Hij}U_{Hjk}U_{Hki}$
$\mathcal{O}(U^2)$ terms		✓ $f_{8A}^{ij} = \mathcal{I}_{ij0}^{111}$ ✓	$U_{Hij}U_{HLji'}U_{LHi'i}$
✓ $f_4^{ij} = \frac{1}{2}\mathcal{I}_{ij}^{11}$ ✓	$U_{Hij}U_{Hji}$	✓ $f_{8B}^{i} = \mathcal{I}_{i0}^{12}$ ✓	$U_{HLii'}U_{Li'j'}U_{LHj'i}$
✓ $f_{4A}^{ij} = \mathcal{I}_{i0}^{11}$ ✓	$U_{HLii'}U_{LHi'i}$	$\mathcal{O}(U^6)$ terms	
$\mathcal{O}(U^4)$ terms		$f_{19}^{ijklmn} = \frac{1}{6}\mathcal{I}_{ijklmn}^{111111}$	$U_{Hij}U_{Hjk}U_{Hkl}U_{Hlm}U_{Hmn}U_{Hni}$
$f_{10}^{ijkl} = \frac{1}{4}\mathcal{I}_{ijkl}^{1111}$	$U_{Hij}U_{Hjk}U_{Hkl}U_{Hli}$	$f_{19A}^{ijklm} = \mathcal{I}_{ijklm0}^{111111}$	$U_{Hij}U_{Hjk}U_{Hkl}U_{Hlm}U_{HLmi'}U_{LHi'i}$
$f_{10A}^{ijk} = \mathcal{I}_{ijk0}^{1111}$ ✓	$U_{Hij}U_{Hjk}U_{HLki'}U_{LHi'i}$	$f_{19B}^{ijkl} = \mathcal{I}_{ijkl0}^{11112}$	$U_{Hij}U_{Hjk}U_{Hkl}U_{HLli'}U_{Li'j'}U_{LHj'i}$
$f_{10B}^{ij} = \mathcal{I}_{ij0}^{112}$ ✓	$U_{Hij}U_{HLji'}U_{Li'j'}U_{LHj'i}$	$f_{19C}^{ijkl} = \mathcal{I}_{ijkl0}^{11112}$	$U_{Hij}U_{Hjk}U_{HLki'}U_{LHi'l}U_{HLlj'}U_{LHj'i}$
$f_{10C}^{ij} = \frac{1}{2}\mathcal{I}_{ij0}^{112}$ ✓	$U_{HLii'}U_{LHi'j}U_{HLjj'}U_{LHj'i}$	$f_{19D}^{ijk} = \mathcal{I}_{ijk0}^{1113}$	$U_{Hij}U_{Hjk}U_{HLki'}U_{Li'j'}U_{Lj'k'}U_{LHk'i}$
$f_{10D}^{i} = \mathcal{I}_{i0}^{13}$ ✓	$U_{HLii'}U_{Li'j'}U_{Lj'k'}U_{LHk'i}$	$f_{19E}^{ijkl} = \frac{1}{2}\mathcal{I}_{ijkl0}^{11112}$	$U_{Hij}U_{HLji'}U_{LHi'k}U_{Hkl}U_{HLlj'}U_{LHj'i}$
$\mathcal{O}(U^5)$ terms		$f_{19F}^{ijk} = \mathcal{I}_{ijk0}^{1113}$	$U_{Hij}U_{HLji'}U_{LHi'k}U_{HLkj'}U_{Lj'k'}U_{LHk'i}$
$f_{16}^{ijklm} = \frac{1}{5}\mathcal{I}_{ijklm}^{11111}$	$U_{Hij}U_{Hjk}U_{Hkl}U_{Hlm}U_{Hmi}$	$f_{19G}^{ijk} = \mathcal{I}_{ijk0}^{1113}$	$U_{Hij}U_{HLji'}U_{Li'j'}U_{LHj'k}U_{HLkk'}U_{LHk'i}$
$f_{16A}^{ijkl} = \mathcal{I}_{ijkl0}^{11111}$	$U_{Hij}U_{Hjk}U_{Hkl}U_{HLli'}U_{LHi'i}$	$f_{19H}^{ij} = \mathcal{I}_{ij0}^{114}$	$U_{Hij}U_{HLji'}U_{Li'j'}U_{Lj'k'}U_{Lk'l'}U_{LHl'i}$
$f_{16B}^{ijk} = \mathcal{I}_{ijk0}^{1112}$	$U_{Hij}U_{Hjk}U_{HLki'}U_{Li'j'}U_{LHj'i}$	$f_{19I}^{ijk} = \frac{1}{3}\mathcal{I}_{ijk0}^{1113}$ ✓	$U_{HLii'}U_{LHi'j}U_{HLjj'}U_{LHj'k}U_{HLkk'}U_{LHk'i}$
$f_{16C}^{ijk} = \mathcal{I}_{ijk0}^{1112}$ ✓	$U_{Hij}U_{HLji'}U_{LHi'k}U_{HLkj'}U_{LHj'i}$	$f_{19J}^{ij} = \mathcal{I}_{ij0}^{114}$	$U_{HLii'}U_{LHi'j}U_{HLjj'}U_{Lj'k'}U_{Lk'l'}U_{LHl'i}$
$f_{16D}^{ij} = \mathcal{I}_{ij0}^{113}$	$U_{Hij}U_{HLji'}U_{Li'j'}U_{Lj'k'}U_{LHk'i}$	$f_{19K}^{ij} = \frac{1}{2}\mathcal{I}_{ij0}^{114}$	$U_{HLii'}U_{Li'j'}U_{LHj'j}U_{HLjk'}U_{Lk'l'}U_{LHl'i}$
$f_{16E}^{ij} = \mathcal{I}_{ij0}^{113}$ ✓	$U_{HLii'}U_{LHi'j}U_{HLjj'}U_{Lj'k'}U_{LHk'i}$	$f_{19L}^{i} = \mathcal{I}_{i0}^{15}$	$U_{HLii'}U_{Li'j'}U_{Lj'k'}U_{Lk'l'}U_{Ll'm'}U_{LHm'i}$
$f_{16F}^{i} = \mathcal{I}_{i0}^{14}$	$U_{HLii'}U_{Li'j'}U_{Lj'k'}U_{Lk'l'}U_{LHl'i}$		

$\mathcal{O}(U_{HL}^2 U_{LH}^2 P^2)$ terms	
✓ $f_{17I}^{ij} = 2\left(\mathcal{I}[q^2]_{ij0}^{114} + \mathcal{I}[q^2]_{ij0}^{213} + \mathcal{I}[q^2]_{ij0}^{123}\right)$	$U_{HLii'}U_{LHi'j}[P^\mu, U_{HLjj'}][P_\mu, U_{LHj'i}]$
✓ $f_{17J}^{ij} = 2\left(\mathcal{I}[q^2]_{ij0}^{222} + 2\mathcal{I}[q^2]_{ij0}^{123}\right)$	$U_{LHi'i}U_{HLij'}[P^\mu, U_{LHj'j}][P_\mu, U_{HLji'}]$
✓ $f_{18E}^{ij} = \mathcal{I}[q^2]_{ij0}^{114} + 2\mathcal{I}[q^2]_{ij0}^{123} + \mathcal{I}[q^2]_{ij0}^{222}$	$U_{HLii'}[P^\mu, U_{LHi'j}]U_{HLjj'}[P_\mu, U_{LHj'i}]$ $+U_{LHi'i}[P^\mu, U_{HLij'}]U_{LHj'j}[P_\mu, U_{HLji'}]$

$\mathcal{O}(U_H^1 U_{HL}^1 U_{LH}^1 P^2)$ terms	
✓ $f_{11A}^{ij} = 2\left(\mathcal{I}[q^2]_{ij0}^{122} + \mathcal{I}[q^2]_{ij0}^{212}\right)$	$U_{Hij}[P^\mu, U_{HLji'}][P_\mu, U_{LHi'i}$
✓ $f_{11B}^{ij} = 2\left(\mathcal{I}[q^2]_{ij0}^{221} + \mathcal{I}[q^2]_{ij0}^{122}\right)$	$U_{LHi'i}[P^\mu, U_{Hij}][P_\mu, U_{HLji'}] + U_{HLii'}[P^\mu, U_{LHi'j}][P_\mu, U_{Hji}]$

$\mathcal{O}(U_L^1 U_{HL}^1 U_{LH}^1 P^2)$ terms	
✓ $f_{11C}^{ij} = 4\mathcal{I}[q^2]_{i0}^{23}$	$U_{Li'j'}[P^\mu, U_{LHj'i}][P_\mu, U_{HLii'}]$
✓ $f_{11D}^{ij} = 2\left(\mathcal{I}[q^2]_{i0}^{14} + \mathcal{I}[q^2]_{i0}^{23}\right)$	$U_{HLii'}[P^\mu, U_{Li'j'}][P_\mu, U_{LHj'i}] + U_{LHi'i}[P^\mu, U_{HLij'}][P_\mu, U_{Lj'i'}]$

$\mathcal{O}(U_H^2 P^2)$ terms	
✓ $f_7^{ij} = \mathcal{I}[q^2]_{ij}^{22}$	$[P^\mu, U_{Hij}][P_\mu, U_{Hji}]$

$\mathcal{O}(U_{HL}^1 U_{LH}^1 P^2)$ terms	
✓ $f_{7A}^{ij} = 2\mathcal{I}[q^2]_{i0}^{22}$	$[P^\mu, U_{HLii'}][P_\mu, U_{LHi'i}]$

Application of the UOLEA: Real Singlet Scalar

$$\mathcal{O}_6 = |H|^6$$

$$\mathcal{O}_H = \tfrac{1}{2}(\partial_\mu |H|^2)^2 \quad \mathcal{O}_T = \tfrac{1}{2}(H^\dagger \overleftrightarrow{D}_\mu H)^2 \quad \mathcal{O}_R = |H|^2 |D_\mu H|^2$$

- Evaluate sum over each term to get full result for e.g. O6:

$$\begin{aligned}\mathrm{O6}\Bigg(&\left(2A^2\kappa^2 - \frac{4A^3\kappa\mu}{M^2} + \frac{2A^4\mu^2}{M^4}\right)\mathrm{f10A}[1,1,1] - \frac{16A^4\kappa\,\mathrm{f10C}[1,1]}{M^2} + \left(4A^4\kappa - \frac{4A^5\mu}{M^2}\right)\mathrm{f16C}[1,1,1] + 8A^6\,\mathrm{f19I}[1,1,1] + \frac{6A^2\kappa^2\,\mathrm{f4A}[1,1]}{M^4} + \\ &\left(\kappa^3 - \frac{3A\kappa^2\mu}{M^2} + \frac{3A^2\kappa\mu^2}{M^4} - \frac{A^3\mu^3}{M^6}\right)\mathrm{f8}[1,1,1] + \mathrm{f16E}[1,1]\left(-\frac{4A^6}{M^2} + 12A^4\lambda_h\right) + \mathrm{f8B}[1]\left(\frac{7A^4\kappa}{M^4} - \frac{12A^2\kappa\lambda_h}{M^2}\right) + \mathrm{f10B}[1,1]\left(-\frac{2A^4\kappa}{M^2} + \frac{2A^5\mu}{M^4} + 6A^2\kappa\lambda_h - \frac{6A^3\mu\lambda_h}{M^2}\right) + \\ &\mathrm{f10D}[1]\left(\frac{2A^6}{M^4} - \frac{12A^4\lambda_h}{M^2} + 18A^2\lambda_h^2\right) - \frac{A^2\kappa\,\mathrm{f2}[1]\,\lambda_s}{M^6} + \mathrm{f8A}[1,1]\left(-\frac{4A^2\kappa^2}{M^2} + \frac{6A^3\kappa\mu}{M^4} + \frac{A^4\lambda_s}{M^4}\right) + \mathrm{f4}[1,1]\left(\frac{2A\kappa^2\mu}{M^4} - \frac{2A^2\kappa\mu^2}{M^6} + \frac{A^2\kappa\lambda_s}{M^4} - \frac{A^3\mu\lambda_s}{M^6}\right)\Bigg)\end{aligned}$$

$f_{10B}^{[illegible]} = \mathcal{I}^{[illegible]}_{ij0}$ ✓	$U_{Hij}U_{HLji'}U_{Li'j'}U_{LHj'i}$	$f_{19C}^{[illegible]} = \mathcal{I}^{[illegible]}_{ijkl0}$	$U_{Hij}U_{Hjk}U_{HLki'}U_{LHi'l}U_{HLlj'}U_{LHj'i}$
$f_{10C}^{ij} = \frac{1}{2}\mathcal{I}^{112}_{ij0}$ ✓	$U_{HLii'}U_{LHi'j}U_{HLjj'}U_{LHj'i}$	$f_{19D}^{ijk} = \mathcal{I}^{1113}_{ijk0}$	$U_{Hij}U_{Hjk}U_{HLki'}U_{Li'j'}U_{Lj'k'}U_{LHk'i}$
$f_{10D}^{i} = \mathcal{I}^{13}_{i0}$ ✓	$U_{HLii'}U_{Li'j'}U_{Lj'k'}U_{LHk'i}$	$f_{19E}^{ijkl} = \frac{1}{2}\mathcal{I}^{11112}_{ijkl0}$	$U_{Hij}U_{HLji'}U_{LHi'k}U_{Hkl}U_{HLlj'}U_{LHj'i}$
$\mathcal{O}(U^5)$ terms		$f_{19F}^{ijk} = \mathcal{I}^{1113}_{ijk0}$	$U_{Hij}U_{HLji'}U_{LHi'k}U_{HLkj'}U_{Lj'k'}U_{LHk'i}$
$f_{16}^{ijklm} = \frac{1}{5}\mathcal{I}^{11111}_{ijklm}$	$U_{Hij}U_{Hjk}U_{Hkl}U_{Hlm}U_{Hmi}$	$f_{19G}^{ijk} = \mathcal{I}^{1113}_{ijk0}$	$U_{Hij}U_{HLji'}U_{Li'j'}U_{LHj'k}U_{HLkk'}U_{LHk'i}$
$f_{16A}^{ijkl} = \mathcal{I}^{11111}_{ijkl0}$	$U_{Hij}U_{Hjk}U_{Hkl}U_{HLli'}U_{LHi'i}$	$f_{19H}^{ij} = \mathcal{I}^{114}_{ij0}$	$U_{Hij}U_{HLji'}U_{Li'j'}U_{Lj'k'}U_{Lk'l'}U_{LHl'i}$
$f_{16B}^{ijk} = \mathcal{I}^{1112}_{ijk0}$	$U_{Hij}U_{Hjk}U_{HLki'}U_{Li'j'}U_{LHj'i}$	$f_{19I}^{ijk} = \frac{1}{3}\mathcal{I}^{1113}_{ijk0}$ ✓	$U_{HLii'}U_{LHi'j}U_{HLjj'}U_{LHj'k}U_{HLkk'}U_{LHk'i}$
$f_{16C}^{ijk} = \mathcal{I}^{1112}_{ijk0}$ ✓	$U_{Hij}U_{HLji'}U_{LHi'k}U_{HLkj'}U_{LHj'i}$	$f_{19J}^{ij} = \mathcal{I}^{114}_{ij0}$	$U_{HLii'}U_{LHi'j}U_{HLjj'}U_{Lj'k'}U_{Lk'l'}U_{LHl'i}$
$f_{16D}^{ij} = \mathcal{I}^{113}_{ij0}$	$U_{Hij}U_{HLji'}U_{Li'j'}U_{Lj'k'}U_{LHk'i}$	$f_{19K}^{ij} = \frac{1}{2}\mathcal{I}^{114}_{ij0}$	$U_{HLii'}U_{Li'j'}U_{LHj'j}U_{HLjk'}U_{Lk'l'}U_{LHl'i}$
$f_{16E}^{ij} = \mathcal{I}^{113}_{ij0}$ ✓	$U_{HLii'}U_{LHi'j}U_{HLjj'}U_{Lj'k'}U_{LHk'i}$	$f_{19L}^{i} = \mathcal{I}^{15}_{i0}$	$U_{HLii'}U_{Li'j'}U_{Lj'k'}U_{Lk'l'}U_{Ll'm'}U_{LHm'i}$
$f_{16F}^{i} = \mathcal{I}^{14}_{i0}$	$U_{HLii'}U_{Li'j'}U_{Lj'k'}U_{Lk'l'}U_{LHl'i}$		

✓ $f_{11B}^{ij} = 2\left(\mathcal{I}[q^2]^{[illegible]}_{ij0} + \mathcal{I}[q^2]^{[illegible]}_{ij0}\right)$	$U_{LHi'i}[P^\mu, U_{Hij}][P_\mu, U_{HLji'}] + U_{HLii'}[P^\mu, U_{LHi'j}][P_\mu, U_{Hji}]$
$\mathcal{O}(U_L^1 U_{HL}^1 U_{LH}^1 P^2)$ terms	
✓ $f_{11C}^{ij} = 4\mathcal{I}[q^2]^{23}_{i0}$	$U_{Li'j'}[P^\mu, U_{LHj'i}][P_\mu, U_{HLii'}]$
✓ $f_{11D}^{ij} = 2\left(\mathcal{I}[q^2]^{14}_{i0} + \mathcal{I}[q^2]^{23}_{i0}\right)$	$U_{HLii'}[P^\mu, U_{Li'j'}][P_\mu, U_{LHj'i}] + U_{LHi'i}[P^\mu, U_{HLij'}][P_\mu, U_{Lj'i'}]$

$\mathcal{O}(U_H^2 P^2)$ terms	
✓ $f_7^{ij} = \mathcal{I}[q^2]^{22}_{ij}$	$[P^\mu, U_{Hij}][P_\mu, U_{Hji}]$
$\mathcal{O}(U_{HL}^1 U_{LH}^1 P^2)$ terms	
✓ $f_{7A}^{ij} = 2\mathcal{I}[q^2]^{22}_{i0}$	$[P^\mu, U_{HLii'}][P_\mu, U_{LHi'i}]$

Application of the UOLEA: Real Singlet Scalar

- Can (partially) **automate** evaluation of each term, e.g.

f_{10A}

```
sumf10A = NCExpand[Sum[f10A[1, 1, 1] * Uϕ1x1 ** Uϕ1x1 ** UϕH1x2[[1]][[ip]] ** UHϕ2x1[[ip]][[1]], {ip, 1, 2}] /.
   subϕcforO6]

sumf10Adimcount = sumf10A /. subDimCounting

sumf10Adim6only = sumf10Adimcount /. autoremovenondim6step1 /. autoremovenondim6step2 /.
  autoremovenondim6step3
```

$$A^2 \kappa^2 \,\mathtt{f10A}[1,1,1]\, \mathtt{HdagH ** HdagH ** Hdag ** H} - \frac{2 A^3 \kappa \mu \,\mathtt{f10A}[1,1,1]\, \mathtt{HdagH ** HdagH ** Hdag ** H}}{M^2} + \frac{A^4 \mu^2 \,\mathtt{f10A}[1,1,1]\, \mathtt{HdagH ** HdagH ** Hdag ** H}}{M^4} + A^2 \kappa^2 \,\mathtt{f10A}[1,1,1]\, \mathtt{HdagH ** HdagH ** Htdag ** Ht} - \frac{2 A^3 \kappa \mu \,\mathtt{f10A}[1,1,1]\, \mathtt{HdagH ** HdagH ** Htdag ** Ht}}{M^2} + \frac{A^4 \mu^2 \,\mathtt{f10A}[1,1,1]\, \mathtt{HdagH ** HdagH ** Htdag ** Ht}}{M^4}$$

```
sumf10Adim6op = sumf10Adim6only /. subO6op

Collect[sumf10Adim6op, {f10A[1, 1, 1], O6}]
```

$$\mathtt{O6}\left(2 A^2 \kappa^2 - \frac{4 A^3 \kappa \mu}{M^2} + \frac{2 A^4 \mu^2}{M^4}\right) \mathtt{f10A}[1,1,1]$$

- Substitute operator structure relations for desired basis, worked out by hand
  - This example trivial but in general most of the work involved is in this step
  - Possible automation: dictionary of operator relations, or work out algorithm

Conclusion

- When calculating Feynman diagrams we don't Wick contract and calculate symmetry factors by hand every time
- Similar redundancy in evaluating CDE from the beginning every time we use functional methods for one-loop matching

$$\mathcal{L}_{\rm UV}[\varphi_{\rm BSM}, \varphi_{\rm SM}]$$

Setup path integral

Evaluate CDE expansion

Covariant diagrams

Matrix trace evaluation

Setup correlation functions

Wick contractions

Perturbation series evaluation

Feynman diagrams

$$\langle \varphi_L \varphi_L \ldots \varphi_L \rangle$$

UV-EFT matching

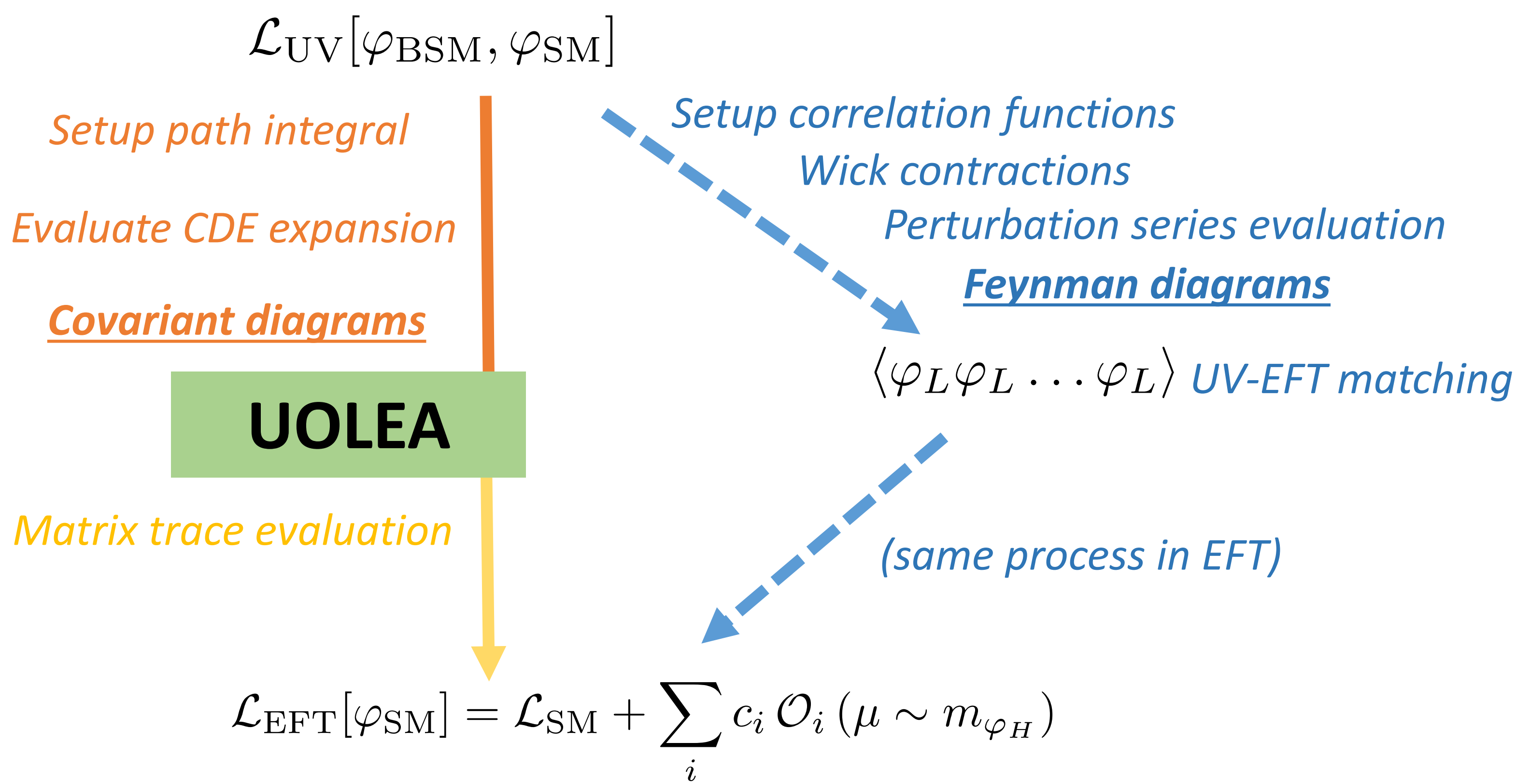
(same process in EFT)

$$\mathcal{L}_{\rm EFT}[\varphi_{\rm SM}] = \mathcal{L}_{\rm SM} + \sum_i c_i\, \mathcal{O}_i\, (\mu \sim m_{\varphi_H})$$

- Standardise functional one-loop matching procedure…

Conclusion

- When calculating Feynman diagrams we don't Wick contract and calculate symmetry factors by hand every time
- Similar redundancy in evaluating CDE from the beginning every time we use functional methods for one-loop matching



- **Start directly from UOLEA!**