Precise LHC phenomenology: the theoretical challenge

Thomas Binoth

THE UNIVERSITY OF EDINBURGH

7. February 2007

Higgs-Maxwell workshop

Edinburgh, Scotland

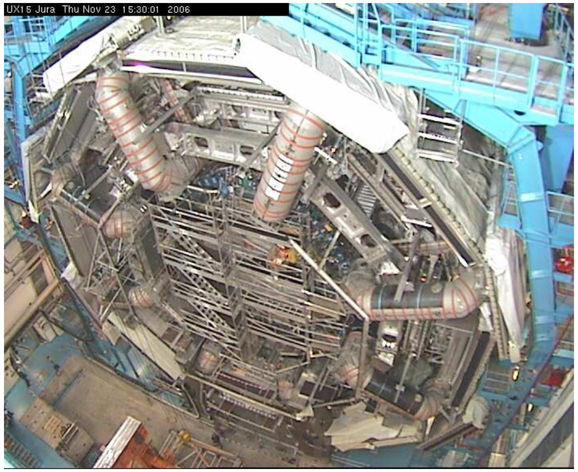
Content:

- Motivation: LHC@NLO, why going to loops?
- One loop methods
- The GOLEM project
- Going beyond one-loop
- Summary

The advent of the LHC era

LHC:

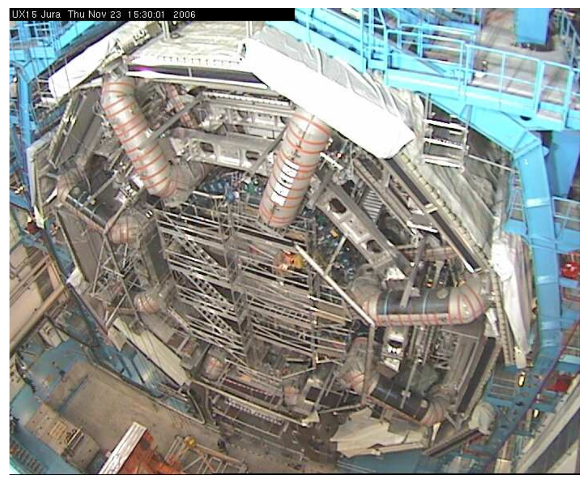
- Large Hadron Collider at CERN, $\sqrt{s} = 14$ TeV, start 2007
- Large amount of Human Capital (for experimentalists)
- Long and Hard Calculations (for theorists)



The advent of the LHC era

LHC:

- Large Hadron Collider at CERN, $\sqrt{s} = 14$ TeV, start 2007
- Large amount of Human Capital (for experimentalists)
- Long and Hard Calculations (for theorists)



What will we see?

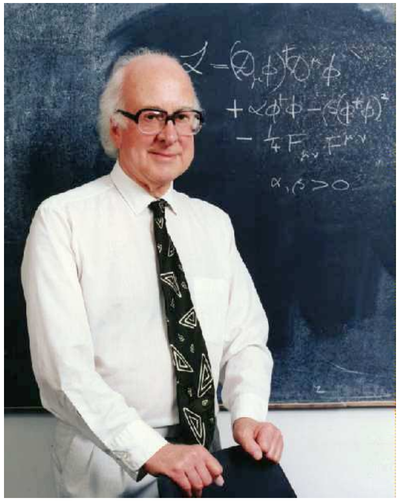
- nothing → extremely disturbing/interesting!
- Higgs boson + nothing → asks for high precision checks (ILC!)
- Higgs boson + something → investigate ”something” in SM background!

Higgs sector and beyond

- LEP: Nonabelian structure and loops important $\Rightarrow$ bounds on M_{Top}, $\log(M_H)$
- SM Higgs boson $\Rightarrow$ 114.4 GeV $< m_H <$ 200 GeV (!)
- Tevatron Run I & II: SM and nothing else!

Higgs sector and beyond

- LEP: Nonabelian structure and loops important $\Rightarrow$ bounds on $M_{\rm Top}$, $\log(M_H)$
- SM Higgs boson $\Rightarrow$ 114.4 GeV $< m_H <$ 200 GeV (!)
- Tevatron Run I & II: SM and nothing else!



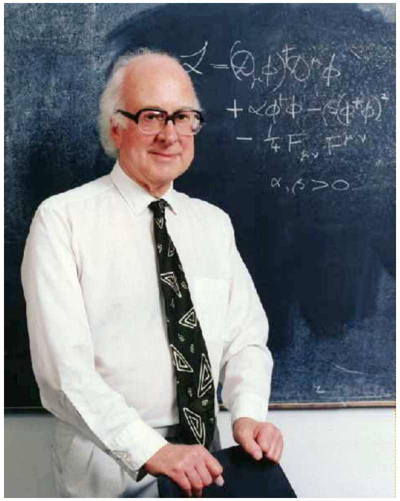
- Higgs sector not yet confirmed

$$V(H) = \frac{1}{2} M_H^2 H^2 + \lambda_3\, H^3 + \lambda_4\, H^4$$

SM: $\lambda_4 = \lambda_3/v = 3\, M_H^2/v^2$

Higgs sector and beyond

- LEP: Nonabelian structure and loops important $\Rightarrow$ bounds on M_{Top}, $\log(M_H)$
- SM Higgs boson $\Rightarrow$ 114.4 GeV $< m_H <$ 200 GeV (!)
- Tevatron Run I & II: SM and nothing else!



- Higgs sector not yet confirmed

$$V(H) = \frac{1}{2} M_H^2 H^2 + \lambda_3\, H^3 + \lambda_4\, H^4$$

SM: $\lambda_4 = \lambda_3/v = 3\, M_H^2/v^2$

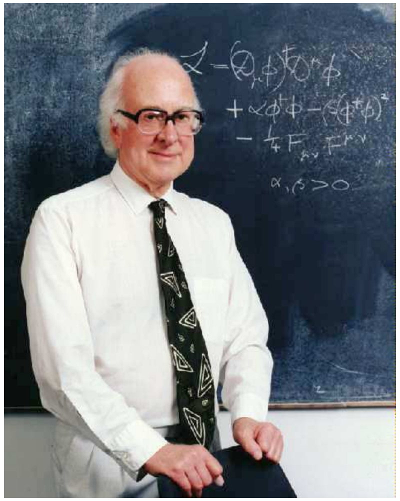
- SM $\subset$ MSSM $\subset$ SUSY GUT $\subset$ Supergravity $\subset$ Superstring $\subset$ $\mathcal{M}$-Theory
  SM $\subset$ "Extra Dimensions", "Little Higgs", "Strong interaction" Model

Higgs sector and beyond

- LEP: Nonabelian structure and loops important ⇒ bounds on $M_{\rm Top}$, $\log(M_H)$
- SM Higgs boson ⇒ 114.4 GeV $< m_H <$ 200 GeV (!)
- Tevatron Run I & II: SM and nothing else!
- Higgs sector not yet confirmed

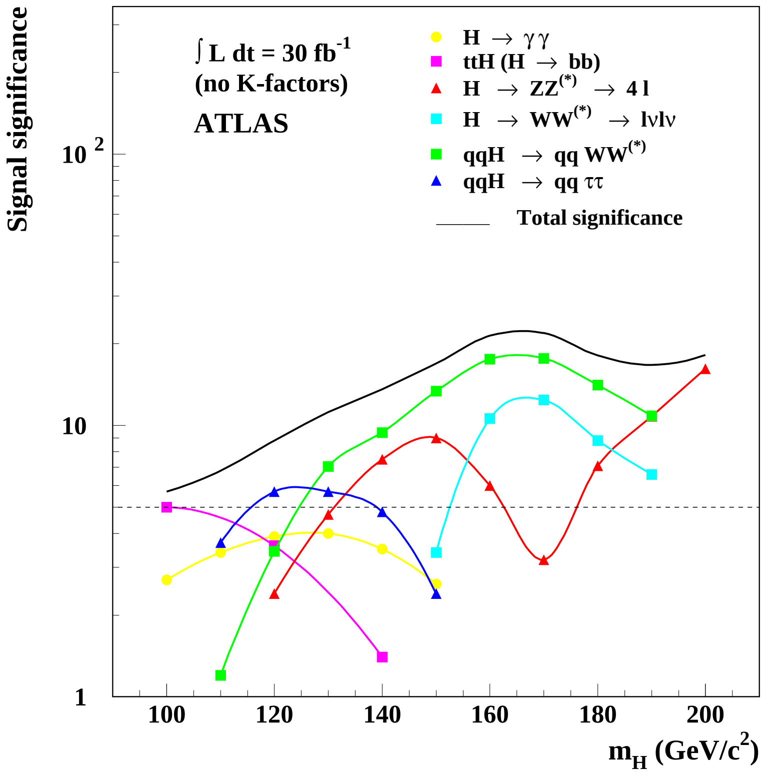
  $$V(H) = \frac{1}{2} M_H^2 H^2 + \lambda_3\, H^3 + \lambda_4\, H^4$$

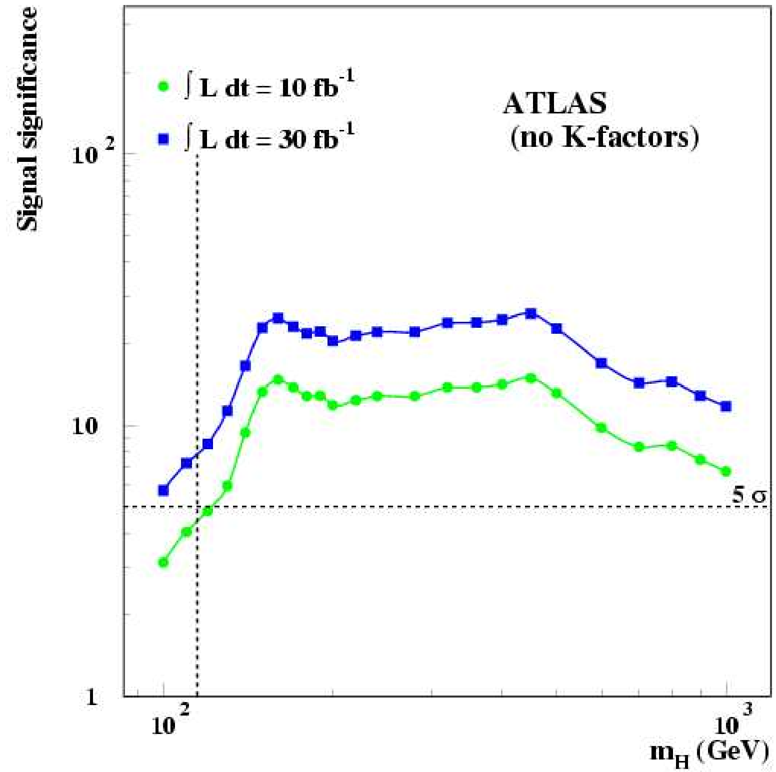
  SM: $\lambda_4 = \lambda_3/v = 3\, M_H^2/v^2$



- SM ⊂ MSSM ⊂ SUSY GUT ⊂ Supergravity ⊂ Superstring ⊂ $\mathcal{M}$-Theory
  SM ⊂ "Extra Dimensions", "Little Higgs", "Strong interaction" Model
- BSM ⇒ something around 1 TeV (?)
  Hierarchy/finetuning problem not a convincing argument !
  Renormalizability of SM ⇔ SM insensitive to cut-off scales !

Discovery potential of the Higgs boson at the LHC





- LHC designed to find the Higgs boson up to $m_H \sim 1$ TeV
- $m_H < 2\,m_Z$ most difficult
- $2\,m_Z < m_H < 1$ TeV "gold plated mode" $H \to ZZ \to \mu\mu\mu\mu$
- $m_H \sim 1$ TeV perturbative approach ceases to be valid

Nothing seen at LHC...

...due to undetectable invisible matter ?

- $H \longrightarrow$ singlet matter and missing energy signal completely washed out
- Look for excess from $PP \rightarrow H + 2\ \text{jets} \rightarrow \not{E} + 2\ \text{jets}$

Nothing seen at LHC...

...due to undetectable invisible matter ?

- $H \rightarrow$ singlet matter and missing energy signal completely washed out
- Look for excess from $PP \rightarrow H + 2\,\text{jets} \rightarrow \not{E} + 2\,\text{jets}$

...due to heavy Higgs/no Higgs scenario?

- Look for excess in $W_L W_L \rightarrow W_L W_L$ scattering

Nothing seen at LHC...

...due to undetectable invisible matter ?

- $H \rightarrow$ singlet matter and missing energy signal completely washed out
- Look for excess from $PP \rightarrow H + 2\,\text{jets} \rightarrow \not{E} + 2\,\text{jets}$

...due to heavy Higgs/no Higgs scenario?

- Look for excess in $W_L W_L \rightarrow W_L W_L$ scattering

$\Longrightarrow$ precise control of Backgrounds crucial !!!

Nothing seen at LHC...

...due to undetectable invisible matter ?

- $H \to$ singlet matter and missing energy signal completely washed out
- Look for excess from $PP \to H + 2\,\mathrm{jets} \to \not{E} + 2\,\mathrm{jets}$

...due to heavy Higgs/no Higgs scenario?

- Look for excess in $W_L W_L \to W_L W_L$ scattering

$\Longrightarrow$ precise control of Backgrounds crucial !!!

Immediate questions:

- What are the invisible decay channels?
- What fakes a light Higgs boson in the precision observables?

Nothing seen at LHC...

...due to undetectable invisible matter ?

- $H \rightarrow$ singlet matter and missing energy signal completely washed out
- Look for excess from $PP \rightarrow H + 2\,\text{jets} \rightarrow \not{E} + 2\,\text{jets}$

...due to heavy Higgs/no Higgs scenario?

- Look for excess in $W_L W_L \rightarrow W_L W_L$ scattering

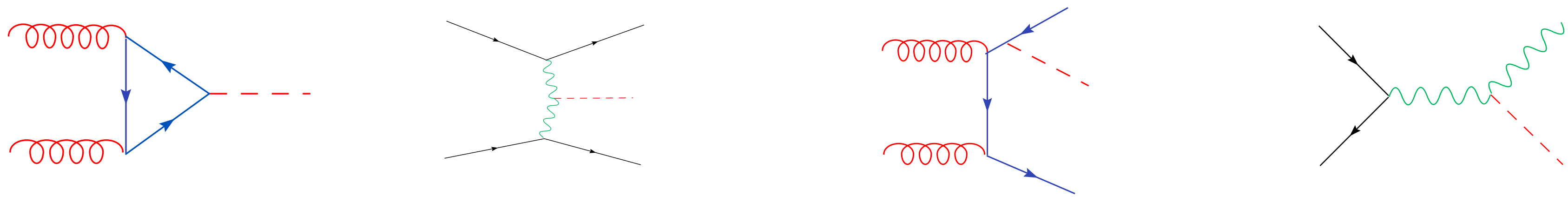
$\Longrightarrow$ precise control of Backgrounds crucial !!!

Immediate questions:

- What are the invisible decay channels?
- What fakes a light Higgs boson in the precision observables?

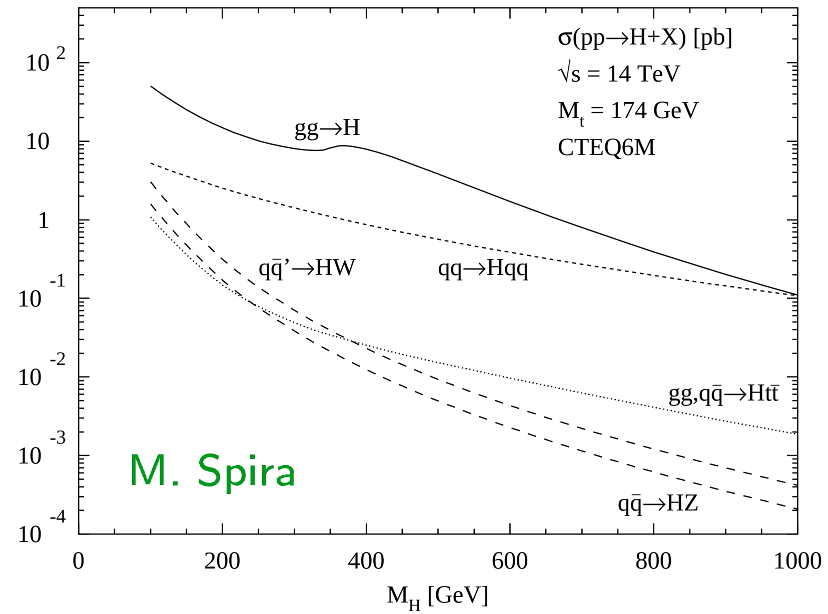
Nothing@LHC $\Rightarrow$ Manifestation of New physics !

Higgs boson + nothing...

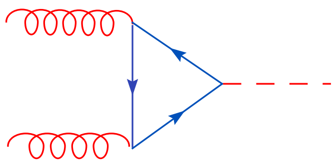


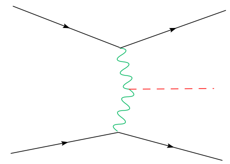
Signal:

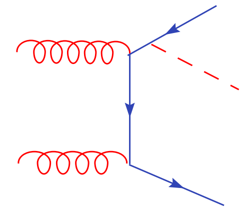
- Decays: $H \rightarrow \gamma\gamma$, $H \rightarrow WW^{(*)}$, $H \rightarrow ZZ^{(*)}$, $H \rightarrow \tau^+\tau^-$
- $PP \rightarrow H + 0, 1, 2$ jets Gluon Fusion
- $PP \rightarrow Hjj$ Weak Boson Fusion
- $PP \rightarrow H + t\bar{t}$
- $PP \rightarrow H + W, Z$

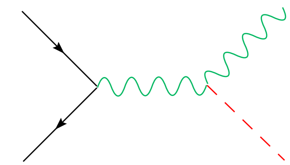


Higgs boson + nothing...



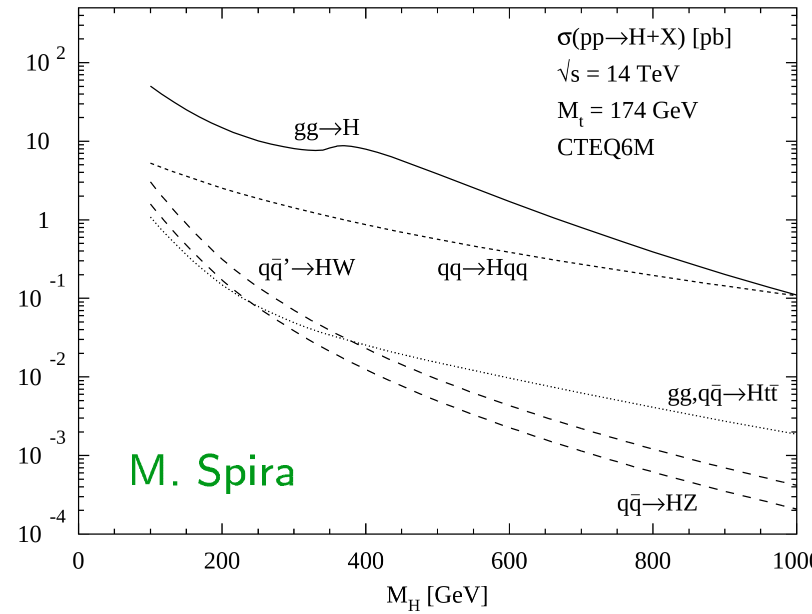






Signal:

- Decays: $H \to \gamma\gamma$, $H \to WW^{(*)}$, $H \to ZZ^{(*)}$, $H \to \tau^+\tau^-$
- $PP \to H + 0, 1, 2$ jets Gluon Fusion
- $PP \to Hjj$ Weak Boson Fusion
- $PP \to H + t\bar{t}$
- $PP \to H + W, Z$



Backgrounds:

- $PP \to \gamma\gamma + 0, 1, 2$ jets
- $PP \to WW^*, ZZ^* + 0, 1, 2$ jets
- $PP \to t\bar{t} + 0, 1, 2$ jets
- $PP \to V +$ up to 3 jets $\quad (V = \gamma, W, Z)$
- $PP \to VVV + 0, 1$ jet

Higgs boson + nothing...

After discovery of a Higgs like boson:

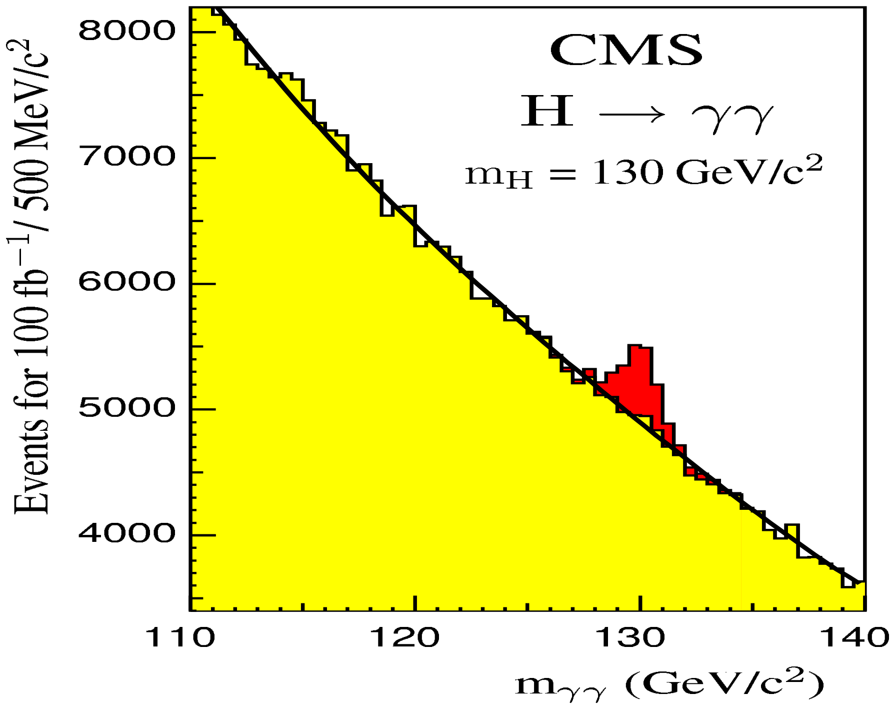
- measure Standard Model properties
- quantitative analysis of Higgs/Matter couplings
- Crucial: reliable background control
- not all backgrounds can be measured: theoretical input necessary!

Higgs boson + nothing...

After discovery of a Higgs like boson:

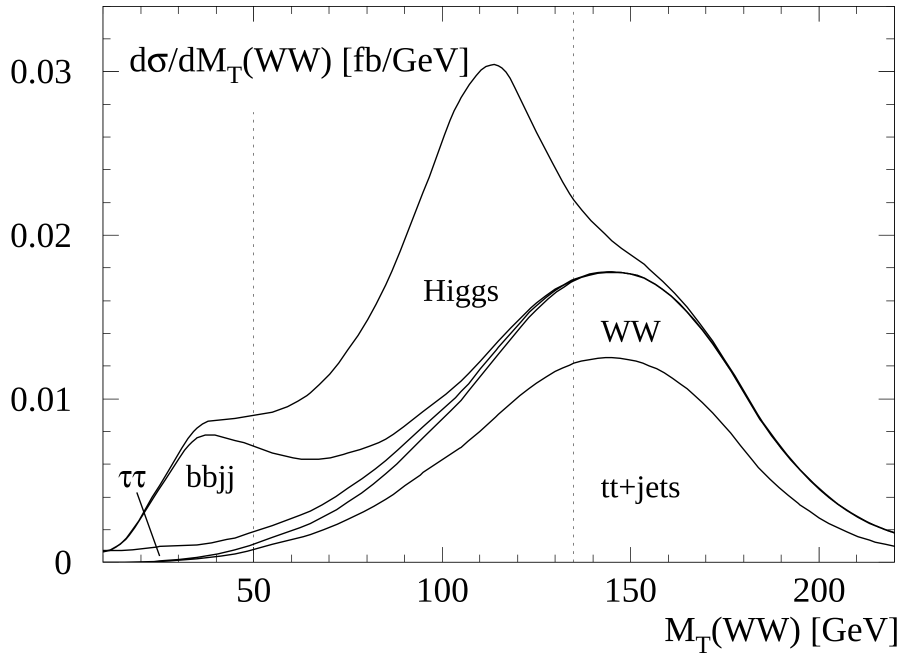
- measure Standard Model properties
- quantitative analysis of Higgs/Matter couplings
- Crucial: reliable background control
- not all backgrounds can be measured: theoretical input necessary!

$PP \rightarrow H + X \rightarrow \gamma\gamma + X$



$H \rightarrow WW \rightarrow l^+ l^- + \not{p}_T$

Kauer, Plehn, Rainwater, Zeppenfeld (2001)

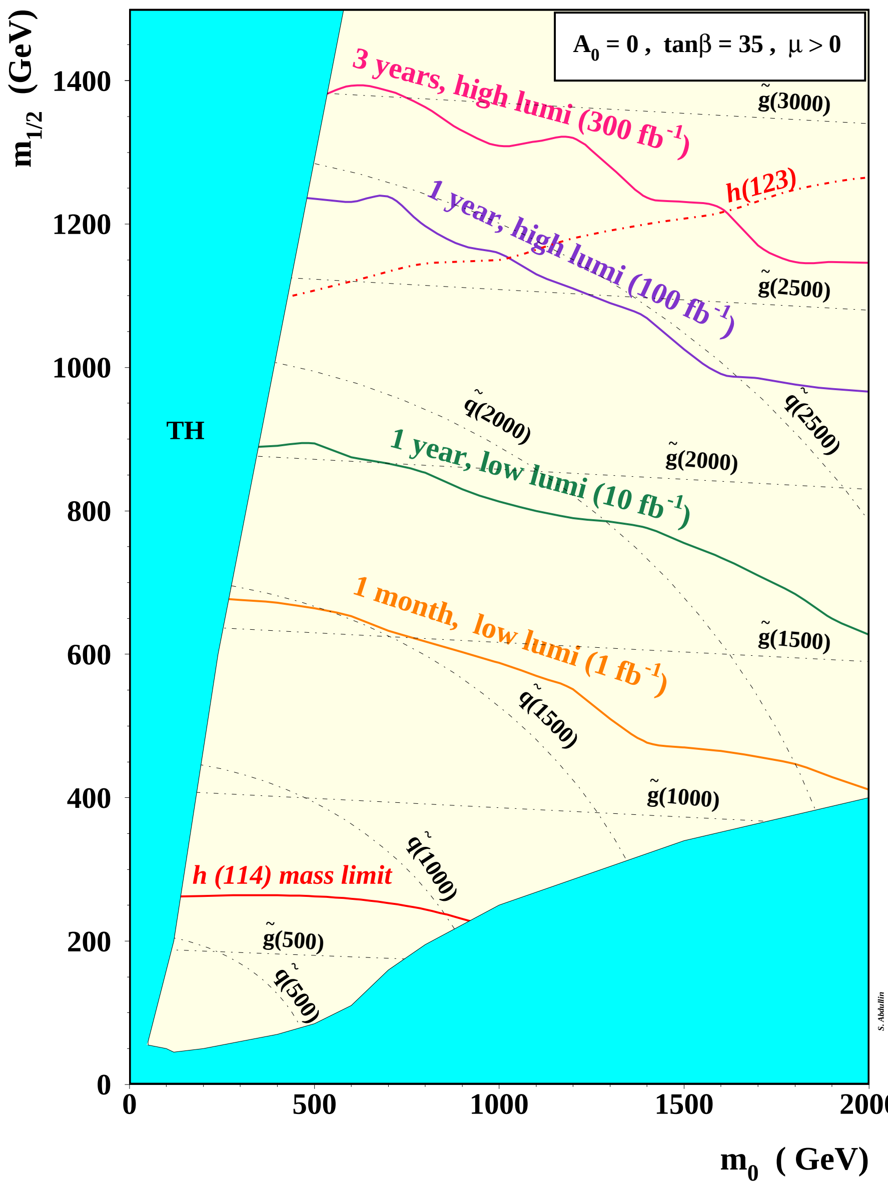


Higgs boson + something...

All Standard Model processes are background to new physics!

New physics signatures:

- Z' easy
- n jets + $\not{E}_T$
- multiparticle cascades



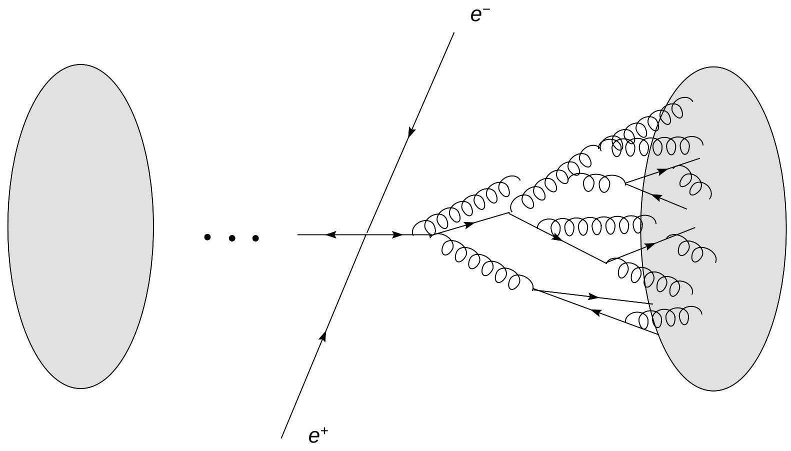
Tools for experimental analysis

Pythia

Herwig

Sherpa

- LO Matrixelements + parton shower + hadronization model



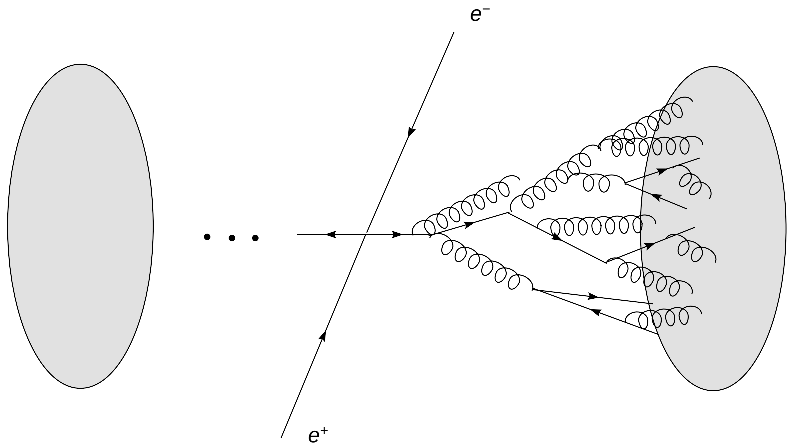
Tools for experimental analysis

Pythia

Herwig

Sherpa

- LO Matrixelements + parton shower + hadronization model

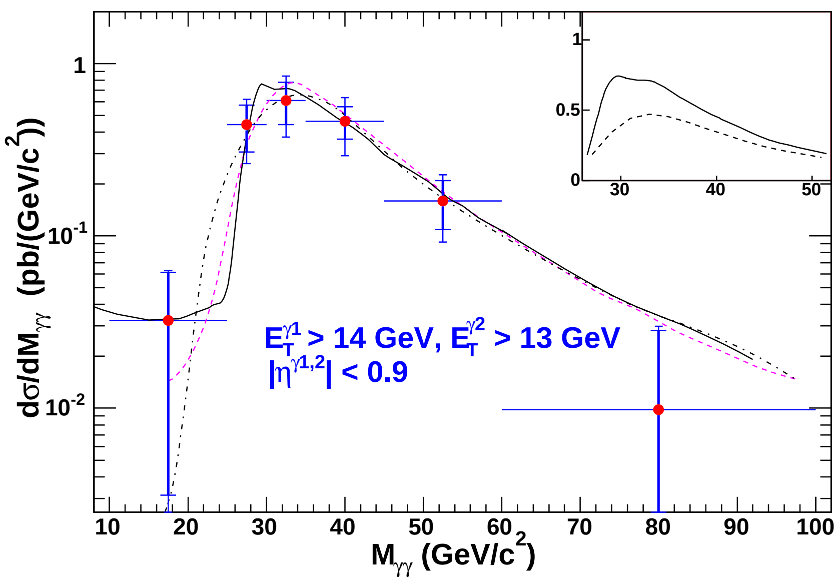


- $2 \to N$ Matrixelements: shapes, jet structure, well described after tuning
- LO absolute rates intrinsically unreliable!

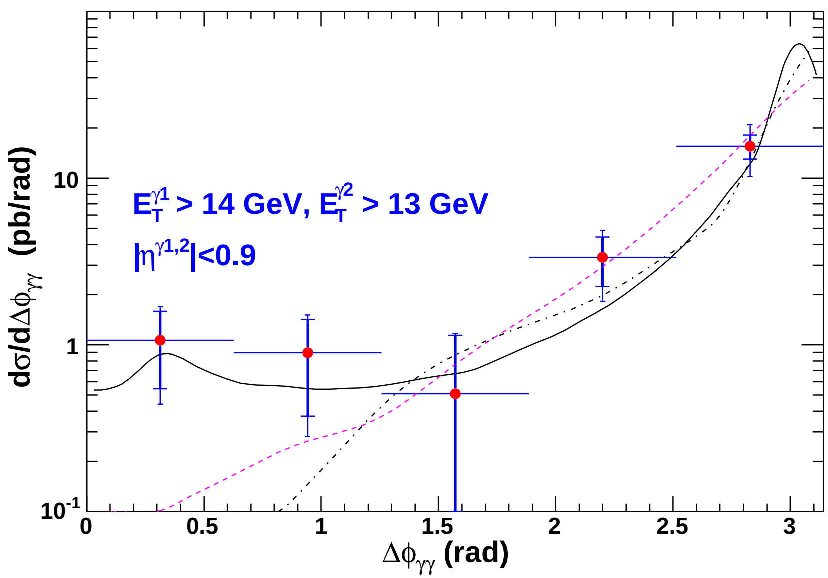
Example: $\gamma\gamma$ rate at Tevatron Run II [hep-ex/0412050]

- DIPHOX: NLO code for $\gamma\gamma$, $\gamma\pi^0$, $\pi^0\pi^0$ production (including fragmentation)
- http://lappweb.in2p3.fr/lapth/PHOX_FAMILY/diphox.html
  [T.B., J.P. Guillet, E. Pilon, M. Werlen]

$M_{\gamma\gamma}$ distribution

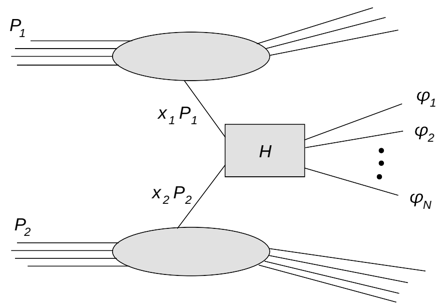


$\Delta\phi_{\gamma\gamma}$ distribution



DIPHOX (solid), RESBOS (dashed), PYTHIA$\times 2$!!! (dot-dashed)

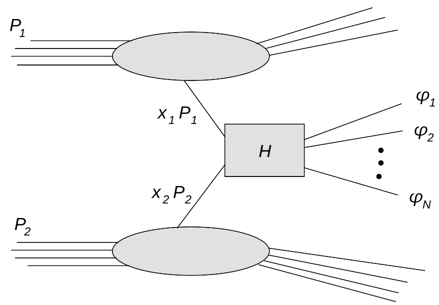
Parton model and scale uncertainties



$$d\sigma(H_1 H_2 \to \phi_1 + \ldots \phi_N + X) = \sum_{j,l} \int dx_1 dx_2 f_{j/H_1}(x_1, \mu_F) f_{l/H_2}(x_2, \mu_F)$$

$$\times\, d\hat{\sigma}(\mathrm{parton}_j(x_1 P_1) + \mathrm{parton}_l(x_2 P_2) \to \varphi_1 + \ldots \varphi_N, \alpha_s(\mu), \mu_F)$$

Parton model and scale uncertainties

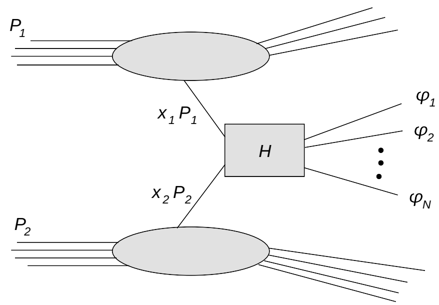


$$d\sigma(H_1H_2 \to \phi_1 + \ldots \phi_N + X) = \sum_{j,l} \int dx_1 dx_2 f_{j/H_1}(x_1, \mu_F) f_{l/H_2}(x_2, \mu_F)$$

$$\times\, d\hat{\sigma}(\mathrm{parton}_j(x_1P_1) + \mathrm{parton}_l(x_2P_2) \to \varphi_1 + \ldots \varphi_N, \alpha_s(\mu), \mu_F)$$

Scale dependence remnant of UV/IR divergencies: $\frac{Q^\epsilon}{\epsilon} - \frac{\mu^\epsilon}{\epsilon} = \log(Q/\mu)$

Parton model and scale uncertainties



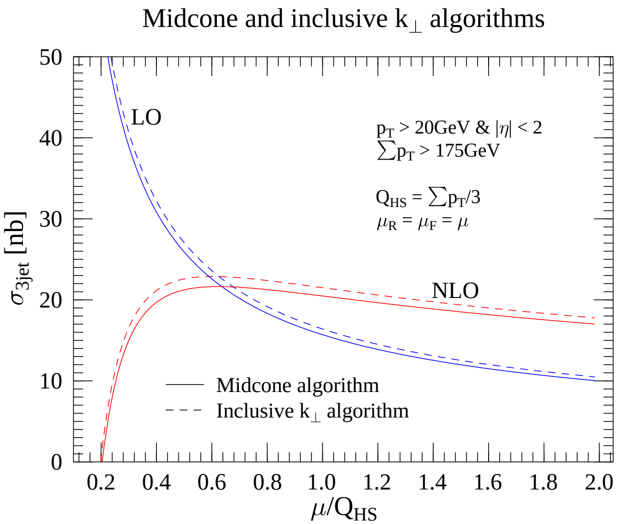
$$d\sigma(H_1 H_2 \to \phi_1 + \ldots \phi_N + X) = \sum_{j,l} \int dx_1 dx_2 f_{j/H_1}(x_1, \mu_F) f_{l/H_2}(x_2, \mu_F)$$

$$\times\, d\hat{\sigma}(\text{parton}_j(x_1 P_1) + \text{parton}_l(x_2 P_2) \to \varphi_1 + \ldots \varphi_N, \alpha_s(\mu), \mu_F)$$

Scale dependence remnant of UV/IR divergencies: $\frac{Q^\epsilon}{\epsilon} - \frac{\mu^\epsilon}{\epsilon} = \log(Q/\mu)$

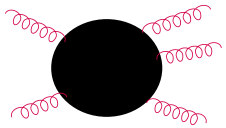
Example: 3 jet cross section at NLO

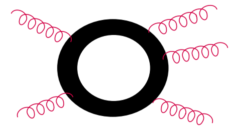
[Z. Nagy, Phys.Rev. D68 (2003)]

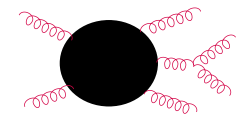


Higher order QCD calculations are mandatory to soften scale dependence !!!

Framework for NLO calculations

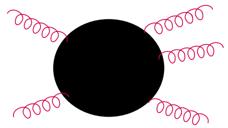
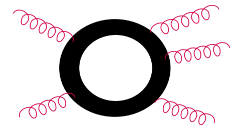
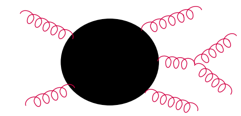
$\mathcal{M}_{\mathrm{LO}}$: 

$\mathcal{M}_{\mathrm{NLO,virtual}}$: 

$\mathcal{M}_{\mathrm{NLO,real}}$: 

$$
\begin{aligned}
\sigma &= \sigma_{LO} + \sigma_{NLO} \\
\sigma_{LO} &= \frac{1}{2s} \int dPS_N \;\; \mathcal{O}_N(\{p_j\}) \;\; |\mathcal{M}_{\mathrm{LO}}|^2 \\
\sigma_{NLO} &= \frac{1}{2s} \int dPS_N \; \alpha_s \; \Big(\mathcal{O}_N(\{p_j\}) \left[\mathcal{M}_{\mathrm{LO}} \mathcal{M}^*_{\mathrm{NLO,V}} + \mathcal{M}^*_{\mathrm{LO}} \mathcal{M}_{\mathrm{NLO,V}} \right] \\
&\qquad + \int dPS_1 \, \mathcal{O}_{N+1}(\{p_j\}) \;\; |\mathcal{M}_{\mathrm{NLO,R}}|^2 \Big)
\end{aligned}
$$

Framework for NLO calculations

$\mathcal{M}_{\mathrm{LO}}$:  $\mathcal{M}_{\mathrm{NLO,virtual}}$:  $\mathcal{M}_{\mathrm{NLO,real}}$: 

$$
\begin{aligned}
\sigma &= \sigma_{LO} + \sigma_{NLO} \\
\sigma_{LO} &= \frac{1}{2s} \int dPS_N \;\; \mathcal{O}_N(\{p_j\}) \;\; |\mathcal{M}_{\mathrm{LO}}|^2 \\
\sigma_{NLO} &= \frac{1}{2s} \int dPS_N \; \alpha_s \; \Big(\mathcal{O}_N(\{p_j\}) \left[\mathcal{M}_{\mathrm{LO}} \mathcal{M}^*_{\mathrm{NLO,V}} + \mathcal{M}^*_{\mathrm{LO}} \mathcal{M}_{\mathrm{NLO,V}} \right] \\
&\qquad + \int dPS_1 \, \mathcal{O}_{N+1}(\{p_j\}) \;\; |\mathcal{M}_{\mathrm{NLO,R}}|^2 \Big)
\end{aligned}
$$

- IR divergences cancel between real and virtual corrections
- $\mathcal{O}_N$, $\mathcal{O}_{N+1}$ define observable, e.g. N, $N+1$ jets

Cancellation of IR divergences

$$
\begin{aligned}
F_{NLO} &= F^{V}_{NLO} + \int dPS_1 F^{R}_{NLO} \\
F^{V}_{NLO} &= \mathcal{O}_N(\{p_j\}) \left[\frac{A(0)}{\epsilon} + \text{finite terms}\right] \\
\int dPS_1\, F^{R}_{NLO} &= \int \frac{dQ^2}{(Q^2)^{1+\epsilon}} \mathcal{O}_{N+1}(\{p_j\})\, A(Q^2) \\
&= \int dQ^2 \mathcal{O}_{N+1}(\{p_j\}) \left[\frac{A(0)}{(Q^2)^{1+\epsilon}} + \frac{A(Q^2) - A(0)}{Q^2} + \ldots\right]
\end{aligned}
$$

Cancellation of IR divergences

$$
\begin{aligned}
F_{NLO} &= F^V_{NLO} + \int dPS_1 F^R_{NLO} \\
F^V_{NLO} &= \mathcal{O}_N(\{p_j\}) \left[\frac{A(0)}{\epsilon} + \text{finite terms}\right] \\
\int dPS_1\ F^R_{NLO} &= \int \frac{dQ^2}{(Q^2)^{1+\epsilon}} \mathcal{O}_{N+1}(\{p_j\})\, A(Q^2) \\
&= \int dQ^2 \mathcal{O}_{N+1}(\{p_j\}) \left[\frac{A(0)}{(Q^2)^{1+\epsilon}} + \frac{A(Q^2) - A(0)}{Q^2} + \dots\right]
\end{aligned}
$$

If and only if (!) $\mathcal{O}_{N+1}(\{p_j\}) \to \mathcal{O}_N(\{p_j\})$ for $Q^2 \to 0$ the poles cancel:

$$
\int dPS_1\ F^R_{NLO} = -\mathcal{O}_N(\{p_j\})\ \frac{A(0)}{\epsilon} + \int dQ^2 \text{ finite terms}
$$

Cancellation of IR divergences

$$F_{NLO} = F^V_{NLO} + \int dPS_1 F^R_{NLO}$$

$$F^V_{NLO} = \mathcal{O}_N(\{p_j\}) \left[\frac{A(0)}{\epsilon} + \text{finite terms}\right]$$

$$\int dPS_1\, F^R_{NLO} = \int \frac{dQ^2}{(Q^2)^{1+\epsilon}} \mathcal{O}_{N+1}(\{p_j\})\, A(Q^2)$$

$$= \int dQ^2 \mathcal{O}_{N+1}(\{p_j\}) \left[\frac{A(0)}{(Q^2)^{1+\epsilon}} + \frac{A(Q^2) - A(0)}{Q^2} + \ldots\right]$$

If and only if (!) $\mathcal{O}_{N+1}(\{p_j\}) \to \mathcal{O}_N(\{p_j\})$ for $Q^2 \to 0$ the poles cancel:

$$\int dPS_1\, F^R_{NLO} = -\mathcal{O}_N(\{p_j\})\, \frac{A(0)}{\epsilon} + \int dQ^2 \text{ finite terms}$$

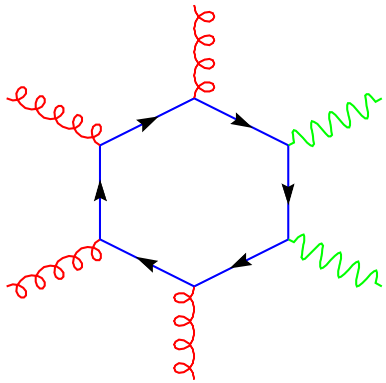
- IR safe measurement functions do not resolve soft or collinear partons
  IR safety is essential for applying perturbation theory
- combination of real/virtual corrections well understood at 1-loop
- Bottleneck for NLO computations: virtual corrections

Status QCD@NLO for LHC:

$2 \to 2:$ everything you want

$2 \to 3:$ $PP \to 3\,j,\ Vjj,\ \gamma\gamma j,\ Vb\bar{b},\ t\bar{t}H,\ b\bar{b}H,\ jjH,\ HHH, (t\bar{t}j)$

$2 \to 4:$ everything remains to be done !

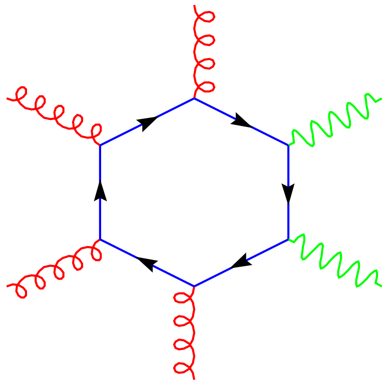


Status QCD@NLO for LHC:

$2 \to 2:$ everything you want

$2 \to 3:$ $PP \to 3\,j,\ Vjj,\ \gamma\gamma j,\ Vb\bar{b},\ t\bar{t}H,\ b\bar{b}H,\ jjH,\ HHH, (t\bar{t}j)$

$2 \to 4:$ everything remains to be done !



- mostly standalone computations
  Exception: MCFM Campbell, Ellis
- LHC induces a lot of very recent activity !

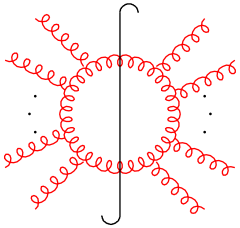
Milestones:

- 4 partons @ NLO Ellis/Sexton, 1985
- 5 g @ NLO Bern/Dixon/Kosower, 1993
- Unitarity based and twistor space inspired methods
- "Modern" Algebraic/Seminumerical techniques
- 6 g @ NLO 2006

Unitarity based/Twistor space inspired approach:

- get loop amplitudes by sewing tree amplitudes using unitarity

$$\mathcal{A}_{1-\mathrm{loop}} \sim \sum_C \int d\mathrm{PS}_C$$

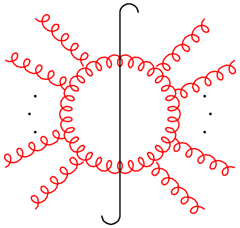


- tree amplitudes gauge invariant

Unitarity based/Twistor space inspired approach:

- get loop amplitudes by sewing tree amplitudes using unitarity

$$\mathcal{A}_{1-\text{loop}} \sim \sum_C \int d\text{PS}_C$$

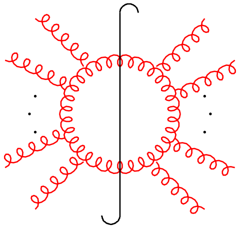


- tree amplitudes gauge invariant
- Bern,Dixon,Dunbar,Kosower-Theorem on cut-constructability:
  Sufficient condition for cut.-con. is that tensor integrals
  $\int d^D k k^R/(k^2 - M^2)^N$ obey $R \leq N - 2$

Unitarity based/Twistor space inspired approach:

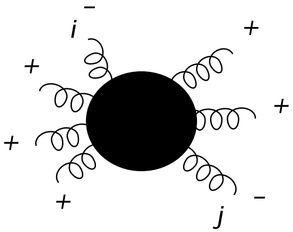
- get loop amplitudes by sewing tree amplitudes using unitarity

$$\mathcal{A}_{1-\text{loop}} \sim \sum_C \int d\text{PS}_C$$



- tree amplitudes gauge invariant
- Bern,Dixon,Dunbar,Kosower-Theorem on cut-constructability:
  Sufficient condition for cut.-con. is that tensor integrals
  $\int d^D k k^R/(k^2 - M^2)^N$ obey $R \leq N - 2$
- Revived by "Twistor space approach" [Cachazo, Svrcek, Witten (2004)]
- maximally helicity violating QCD tree amplitudes are lines in "Twistor space".

$$\mathcal{A}_{\text{MHV}} \sim ig^{N-2} \frac{\langle ij \rangle}{\langle 12 \rangle \langle 23 \rangle ... \langle n{-}1,n \rangle \langle n1 \rangle} \sim$$



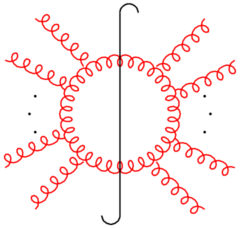
- novel perturbative expansion: MHV-vertices + scalar propagators $\sim 1/P^2$

$\langle ij \rangle := \langle i^- | j^+ \rangle$, $[ij] := \langle i^+ | j^- \rangle$, $|j^+\rangle$ defined by $\not{p}_j |j^+\rangle = 0$, $|j^-\rangle = |j^+\rangle^C$

Unitarity based/Twistor space inspired approach:

- BDDK: $d = 4$ cuts do not fully determine one-loop amplitude

$$\mathcal{A}_{1-\mathrm{loop}} = \sum_C \int d\mathrm{PS}_C$$



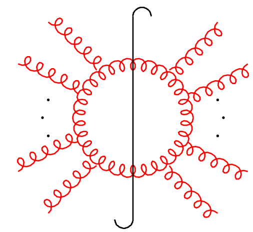
$$+ \ \mathcal{R}$$

- cut constructible part of all 1-loop 6-gluon helicity amplitudes known [Feng, Britto, Mastrolia (2006)]

Unitarity based/Twistor space inspired approach:

- BDDK: $d = 4$ cuts do not fully determine one-loop amplitude

$$\mathcal{A}_{1-\mathrm{loop}} = \sum_C \int d\mathrm{PS}_C$$



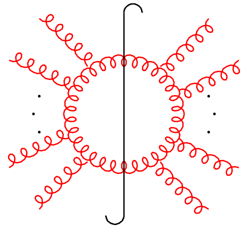
$$+ \ \mathcal{R}$$

- cut constructible part of all 1-loop 6-gluon helicity amplitudes known [Feng, Britto, Mastrolia (2006)]
- "bootstrap" approach exists to evaluate $\mathcal{R}$ by collinear limits and "auxiliary relations", done for $\mathcal{A}_{6-\mathrm{gluon}}^{+++---}$ [Bern, Dixon, Kosower (2006)]
- d-dimensional cut techniques under investigation
- Feynman diagrammatic approach by Chinese group [Xiao, Yang, Zhu (2006)] $\mathcal{R}[\mathcal{A}_{6-\mathrm{gluon}}^{\pm\cdots}]$ from tensor form factors.
- virtual part of 6g@NLO done!!!

Unitarity based/Twistor space inspired approach:

- BDDK: $d=4$ cuts do not fully determine one-loop amplitude

$$\mathcal{A}_{1-\mathrm{loop}} = \sum_C \int d\mathrm{PS}_C$$



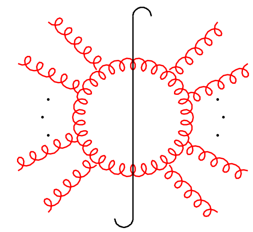
$$+ \ \mathcal{R}$$

- cut constructible part of all 1-loop 6-gluon helicity amplitudes known [Feng, Britto, Mastrolia (2006)]
- "bootstrap" approach exists to evaluate $\mathcal{R}$ by collinear limits and "auxiliary relations", done for $\mathcal{A}^{+++---}_{6-\mathrm{gluon}}$ [Bern, Dixon, Kosower (2006)]
- d-dimensional cut techniques under investigation
- Feynman diagrammatic approach by Chinese group [Xiao, Yang, Zhu (2006)] $\mathcal{R}[\mathcal{A}^{\pm\cdots}_{6-\mathrm{gluon}}]$ from tensor form factors.
- virtual part of 6g@NLO done!!!
- formalisms mainly for multi-gluon amplitudes
- external quarks, massive internal particles (?), big issue: automation

Unitarity based/Twistor space inspired approach:

- BDDK: $d = 4$ cuts do not fully determine one-loop amplitude

$$\mathcal{A}_{1-\text{loop}} = \sum_C \int d\text{PS}_C$$



$$+ \ \mathcal{R}$$

- cut constructible part of all 1-loop 6-gluon helicity amplitudes known [Feng, Britto, Mastrolia (2006)]
- "bootstrap" approach exists to evaluate $\mathcal{R}$ by collinear limits and "auxiliary relations", done for $\mathcal{A}^{+++---}_{6-\text{gluon}}$ [Bern, Dixon, Kosower (2006)]
- d-dimensional cut techniques under investigation
- Feynman diagrammatic approach by Chinese group [Xiao, Yang, Zhu (2006)] $\mathcal{R}[\mathcal{A}^{\pm\cdots}_{6-\text{gluon}}]$ from tensor form factors.
- virtual part of 6g@NLO done!!!
- formalisms mainly for multi-gluon amplitudes
- external quarks, massive internal particles (?), big issue: automation

Unitarity based/Twistor space inspired methods have good potential
further research necessary to establish a general method!!!

Feynman diagrammatic approach:

$$\Gamma^{c,\lambda}(p_j,m_j) = \sum_{\{c_i\},\alpha} f^{\{c_i\}}\mathcal{G}_\alpha^{\{\lambda\}}$$

$$\mathcal{G}_\alpha^{\{\lambda\}} = \int \frac{d^n k}{i\pi^{n/2}} \frac{\mathcal{N}^{\{\lambda\}}}{(q_1^2-m_1^2)\dots(q_N^2-m_N^2)} = \sum_R \mathcal{N}^{\{\lambda\}}_{\mu_1,\dots,\mu_R}\, I_N^{\mu_1\cdots\mu_R}(p_j,m_j)$$

$$I_N^{\mu_1\cdots\mu_R}(p_j,m_j) = \int \frac{d^n k}{i\pi^{n/2}} \frac{k^{\mu_1}\dots k^{\mu_R}}{(q_1^2-m_1^2)\dots(q_N^2-m_N^2)} \quad , \quad q_j = k - r_j = k - p_1 \cdots - p_j$$

- Passarino-Veltman: momentum space reduction $\rightarrow 1/\det(G)^R, G_{ij} = 2\,r_i\cdot r_j$
- Lorentz Tensor Integrals $\rightarrow$ Formfactor representation à la Davydychev:

$$I_N^{\mu_1\cdots\mu_R} = \sum \tau^{\mu_1\cdots\mu_R}(r_{j_1},\dots,r_{j_r},g^m)\, I_N^{n+2m}(j_1,\dots,j_r)$$

$$I_N^D(j_1,\dots,j_r) = (-1)^N\Gamma(N-\frac{D}{2})\int_0^\infty d^N z\,\delta(1-\sum_{l=1}^N z_l)\,\frac{z_{j_1}\dots z_{j_r}}{(-\frac{1}{2}z\cdot\mathcal{S}\cdot z)^{N-D/2}}$$

$$\mathcal{S}_{ij} = (r_i-r_j)^2 - m_i^2 - m_j^2\ ,\ r_j = p_1+\cdots+p_j$$

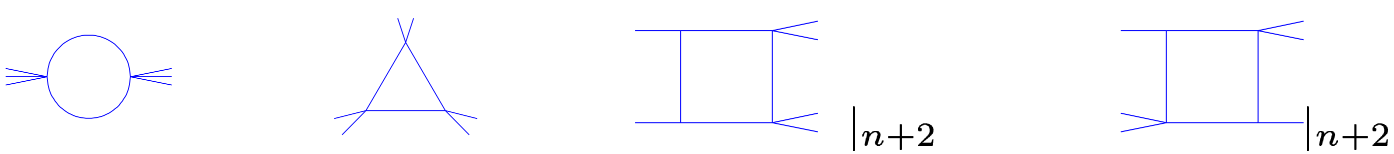
Reduction of Feynman parameter integrals

Bern, Dixon, Kosower (1993);
T.B., Guillet, Heinrich, (2000)

Each N–point integral with a non–trivial numerator can be represented by scalar integrals with shifted dimensions.

- $I_{N=5,6}^{n+2m}$ drop out.
- $I_N^{n+2m} \longrightarrow (I_N^{n+2m-2}, I_{N-1}^{n+2m-2})$

Each N–point integral with non–trivial numerator can be represented by scalar integrals $I_1^n, I_2^n, I_3^n, I_4^{n+2}$. But $1/\det(G)$ unavoidable!



The GOLEM project

- Evaluation of 1-loop amplitudes bottleneck for LHC@NLO
- Automated evaluation of one-loop amplitudes
- Combinatorial complexity $\leftrightarrow$ Numerical instabilities
  $\Rightarrow$ flexibility to switch between algebraic/numeric representations

The GOLEM project

- Evaluation of 1-loop amplitudes bottleneck for LHC@NLO
- Automated evaluation of one-loop amplitudes
- Combinatorial complexity $\leftrightarrow$ Numerical instabilities
  $\Rightarrow$ flexibility to switch between algebraic/numeric representations

G eneral

0 ne

L oop

E valuator for

M atrix elements

The GOLEM project

- Evaluation of 1-loop amplitudes bottleneck for LHC@NLO
- Automated evaluation of one-loop amplitudes
- Combinatorial complexity ↔ Numerical instabilities
  ⇒ flexibility to switch between algebraic/numeric representations

G eneral

0 ne

L oop

E valuator for

M atrix elements

- The GOLEM team: T.B., A. Guffanti, J.Ph. Guillet, G. Heinrich, S. Karg, N. Kauer, F. Mahmoudi, E. Pilon, T. Reiter, C. Schubert, G. Burton

The GOLEM algorithm

Step 1: Amplitude organization

- Split amplitude into gauge invariant subamplitudes
  $\longrightarrow$ No compensations between subamplitudes

$$\mathcal{A}(|p_j\rangle, \epsilon_j^\lambda, \ldots) = \sum_I \mathcal{A}_I(|p_j\rangle, \epsilon_j^\lambda, \ldots)$$

The GOLEM algorithm

Step 1: Amplitude organization

- Split amplitude into gauge invariant subamplitudes
  $\longrightarrow$ No compensations between subamplitudes

$$\mathcal{A}(|p_j\rangle, \epsilon_j^\lambda, \ldots) = \sum_I \mathcal{A}_I(|p_j\rangle, \epsilon_j^\lambda, \ldots)$$

Step 2: Graph generation

- generate Feynman diagrams
- project onto gauge invariant structures defined in step 1

$$\begin{aligned} \mathcal{A}(|p_j\rangle, \epsilon_j^\lambda, \ldots) &= \sum_G \mathcal{G}_G(|p_j\rangle, \epsilon_j^\lambda, \ldots) \\ &= \sum_I \sum_G \mathcal{C}_{IG}(s_{jk})\, \mathcal{T}_I(|p_j\rangle, \epsilon_j^\lambda, \ldots) \end{aligned}$$

$(s_{jk} = (p_j + p_k)^2)$

The GOLEM algorithm

Step 3: Reduction to integral basis

- Choose integral basis $\{I_B\}$ (see below)
- apply algebraic or semi-numerical reduction methods to map onto $\{I_B\}$
- semi-numerical reduction done with Fortran/C code

$$\mathcal{A}(|p_j\rangle, \epsilon_j^{\lambda}, \ldots) = \sum_B \sum_I \sum_G \mathcal{C}_{BIG}(s_{jk}, \ldots)\, I_B\, \mathcal{T}_I(|p_j\rangle, \epsilon_j^{\lambda}, \ldots)$$

The GOLEM algorithm

Step 3: Reduction to integral basis

- Choose integral basis $\{I_B\}$ (see below)
- apply algebraic or semi-numerical reduction methods to map onto $\{I_B\}$
- semi-numerical reduction done with Fortran/C code

$$\mathcal{A}(|p_j\rangle, \epsilon_j^\lambda, \ldots) = \sum_B \sum_I \sum_G \mathcal{C}_{BIG}(s_{jk}, \ldots)\, I_B\, \mathcal{T}_I(|p_j\rangle, \epsilon_j^\lambda, \ldots)$$

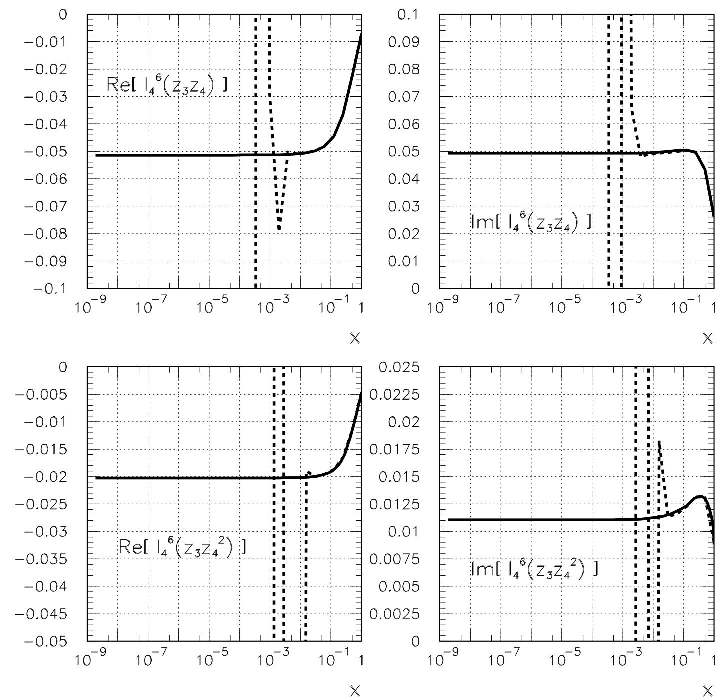
Step 4: Export/manipulate coefficients C_{BIG}'s (optional)

- Denominator structure and size of C_{BIG}'s critical for numerical evaluation
- Export C_{BIG} to MAPLE/MATHEMATICA $\rightarrow$ simplification/factorization
- Export C_{BIG} to Fortran/C code $\rightarrow$ produce optimized output

$$\mathcal{A}(|p_j\rangle, \epsilon_j^\lambda, \ldots) = \sum_B \sum_I \sum_G \text{simplify}[\mathcal{C}_{BIG}(s_{jk}, \ldots)]\, I_B\, \mathcal{T}_I(|p_j\rangle, \epsilon_j^\lambda, \ldots)$$

GOLEM Basis integrals

$$I^{n,n+2}_{N=3,4}(j_1,\ldots,j_r) \quad \sim \quad \int_0^1 \prod_{i=1}^{4} dz_i\,\delta(1-\sum_{l=1}^{4} z_l)\,\frac{z_{j_1}\cdots z_{j_r}}{(-\frac{1}{2}\,z\cdot\mathcal{S}\cdot z - i\delta)^{3-n/2}}$$



Three alternatives for evaluation:

1. algebraic reduction to “standard” basis I_2^n ,I_3^n, I_4^{n+2} (“Master integrals”)
2. semi-numerical reduction to scalar integrals
   $[1.\&2. \to$ Gram determinants $\sim 1/\det(G)^r]$
3. direct numerical evaluation

Computations with GOLEM:

All algorithms coded in FORM and FORTRAN 90:

- $\phi\phi \rightarrow \phi\phi\phi\phi$, $\gamma\gamma \rightarrow \phi\phi\phi\phi$
- $gg \rightarrow HH, HHH$
- $gg \rightarrow W^*W^* \rightarrow l\nu l'\nu'$ (30% effect with Higgs search cuts)
- $qq \rightarrow qqqq$ under construction, goal: $PP \rightarrow bbbb$, $PP \rightarrow 4$ jets
- GOLEM can be used to evaluate rational terms of amplitudes!
  $\rightarrow$ complementary to unitarity based methods

Other semi-numerical approach by Ellis, Giele, Zanderighi:

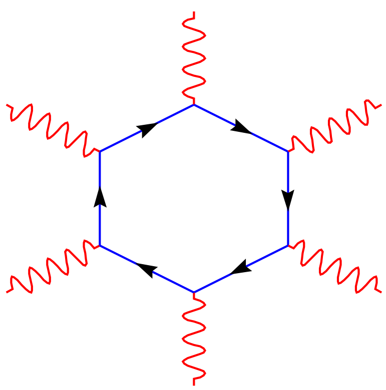
- 6-gluon amplitude (for some phase space points)
- $gg \rightarrow Hgg$ ($m_{Top} \rightarrow \infty$)

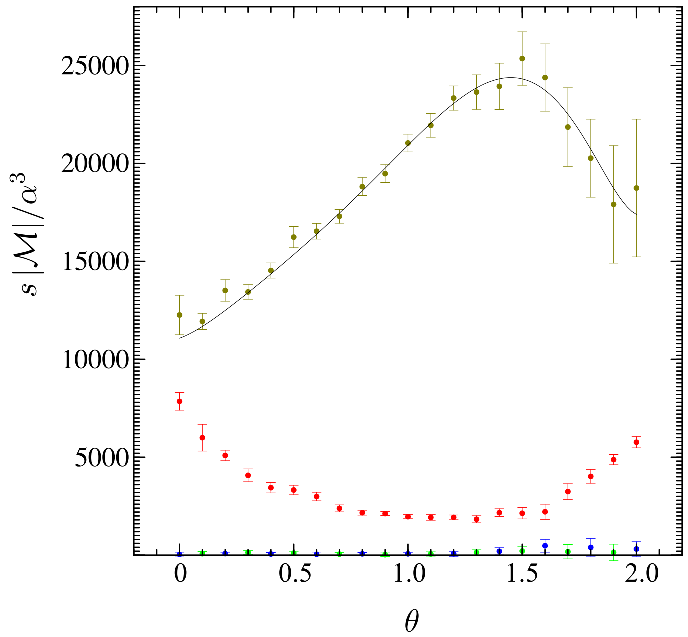
Numerical evaluation of amplitudes

Efficient numerical evaluation of unreduced integrals would avoid proliferation of terms:

$$I_N^D(j_1,\ldots,j_r) \quad \sim \quad \int_0^\infty d^N z\, \delta(1-\sum_{l=1}^{N} z_l)\, \frac{z_{j_1}\ldots z_{j_r}}{(-\frac{1}{2}z\cdot\mathcal{S}\cdot z)^{N-D/2}}$$

6-photon amplitude using "multi-dimensional contour deformation" Soper, Nagy 2006





T.B., Heinrich, Kauer (2002), T.B., Guillet, Heinrich, Pilon, Schubert (2005)
Soper (2000); Ferroglia, Passera, Passarino, Uccirati (2002);
Y. Kurihara, T. Kaneko, (2005); Anastasiou, Daleo (2005); Soper, Nagy (2006).

Going beyond NLO

Exclusive observables induce logarithmic sensitivity in fixed order computations!

$$\sigma_{NLO} \sim \alpha_s \log^2\left(\frac{p_T^2}{Q^2}\right) \cdots \rightarrow \infty \;\; \text{for} \;\; p_T \rightarrow 0$$

If $\alpha_s \log^2\left(\frac{p_T^2}{Q^2}\right) \sim 1 \Rightarrow$ perturbation theory breaks down !

Going beyond NLO

Exclusive observables induce logarithmic sensitivity in fixed order computations!

$$\sigma_{NLO} \sim \alpha_s \log^2\left(\frac{p_T^2}{Q^2}\right)\cdots \to \infty \ \text{ for } \ p_T \to 0$$

If $\alpha_s \log^2\left(\frac{p_T^2}{Q^2}\right) \sim 1 \Rightarrow$ perturbation theory breaks down !

Include, i.e. “resum”, higher order contributions:

$$\sigma = 1 + \alpha_s \log^2\Big(\frac{p_T^2}{Q^2}\Big) + \frac{\alpha_s^2}{2}\log^4\Big(\frac{p_T^2}{Q^2}\Big) + \cdots = \exp(\alpha_s \log^2\Big(\frac{p_T^2}{Q^2}\Big)) \ \to 0 \ \text{ for } \ p_T \to 0$$

Going beyond NLO

Exclusive observables induce logarithmic sensitivity in fixed order computations!

$$\sigma_{NLO} \sim \alpha_s \log^2\left(\frac{p_T^2}{Q^2}\right) \cdots \to \infty \quad \text{for} \quad p_T \to 0$$

If $\alpha_s \log^2\left(\frac{p_T^2}{Q^2}\right) \sim 1 \Rightarrow$ perturbation theory breaks down !

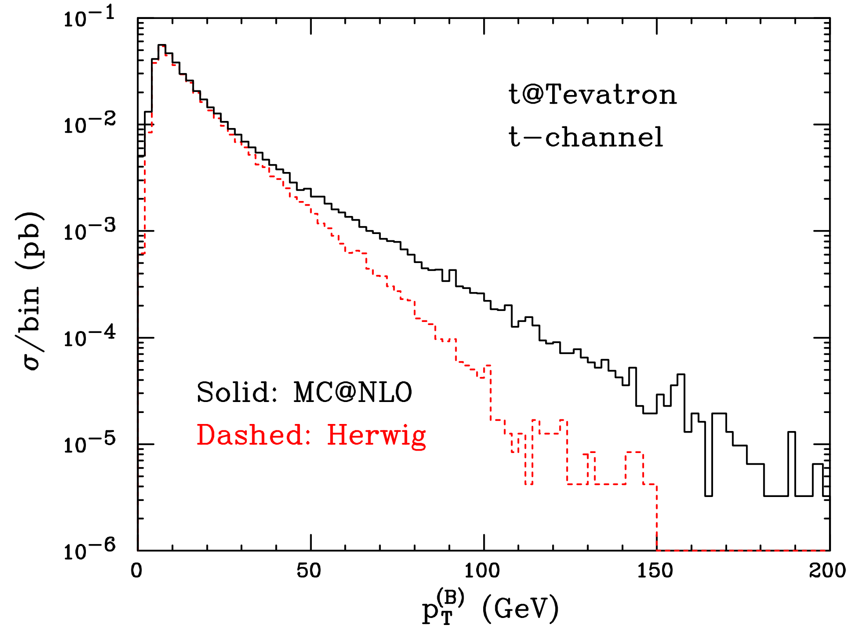
Include, i.e. "resum", higher order contributions:

$$\sigma = 1 + \alpha_s \log^2\left(\frac{p_T^2}{Q^2}\right) + \frac{\alpha_s^2}{2}\log^4\left(\frac{p_T^2}{Q^2}\right) + \cdots = \exp(\alpha_s \log^2\left(\frac{p_T^2}{Q^2}\right)) \to 0 \quad \text{for} \quad p_T \to 0$$

- Parton shower in LO Monte-Carlo programs contain all order information in collinear direction
- Hard radiation better desribed by including higher order Matrix elements
- Control over scale variation needs virtual higher order contributions
- Final goal: Monte Carlo simulations at NLO !

Combining NLO with parton showers

- Contributions by: Collins, Zu (2002); Frixione, Nason, Webber (2002); GRACE-collab. (2003); Krämer, Soper (2004)
- public MC@NLO code [Frixione, Webber] contains processes: W, Z, γ^*, H, $b\bar{b}$, $t\bar{t}$, HW, HZ, WW, WZ, ZZ, $t+X$



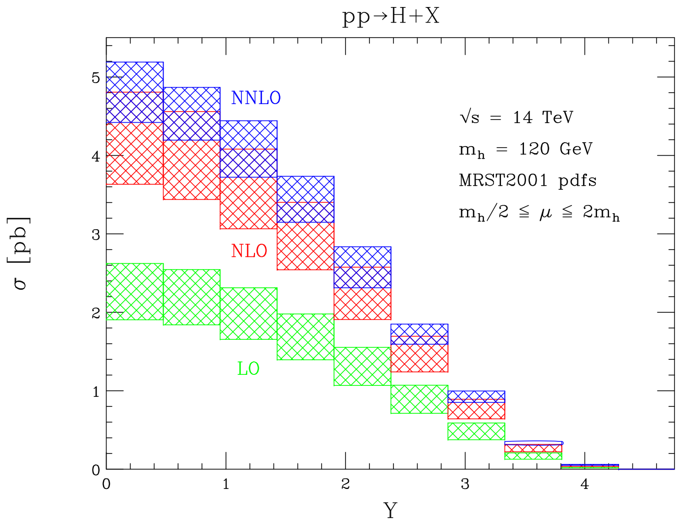
Frixione, Laenen, Motylinski, Webber (2005)

Going to NNLO

- NLO leads to $\mathcal{O}(10\%)$ precision
- for certain paradigm processes we need $\mathcal{O}(1\%)$, i.e. NNLO !
- $PP \to H, W, Z$ done (mostly inclusive!)

Going to NNLO

- NLO leads to $\mathcal{O}(10\%)$ precision
- for certain paradigm processes we need $\mathcal{O}(1\%)$, i.e. NNLO !
- $PP \to H, W, Z$ done (mostly inclusive!)
- Subtraction method for NNLO processes not yet established
- Method to isolate IR poles algorithmically from loop and phase space integrals does exist ["sector decomposition", T.B., G.Heinrich (2000)]
- Applied to $PP \to H \to \gamma\gamma$, by Anastasiou, Melnikov, Petriello (2004) differential NNLO result !
- has potential for automation.



Summary

LHC phenomenology needs and deserves (!) at least NLO precision

Summary

LHC phenomenology needs and deserves (!) at least NLO precision

The NLO multi-leg challenge:

- Lots of activity: algebraic, numeric, string inspired $\longrightarrow$ 6-gluon amplitude done...
- ... but still no complete $2 \rightarrow 4$ process
- GOLEM approach for 1-loop multi-leg processes
  - $gg \rightarrow W^*W^* \rightarrow l\nu l'\nu'$ $\longrightarrow$ sizable Higgs background
  - $gg \rightarrow HH, HHH$ $\longrightarrow$ Multi-Higgs physics
  - $q\bar{q} \rightarrow q\bar{q}q\bar{q}$ at NLO in progress
  - complementary to unitarity based methods for rational terms

Summary

LHC phenomenology needs and deserves (!) at least NLO precision

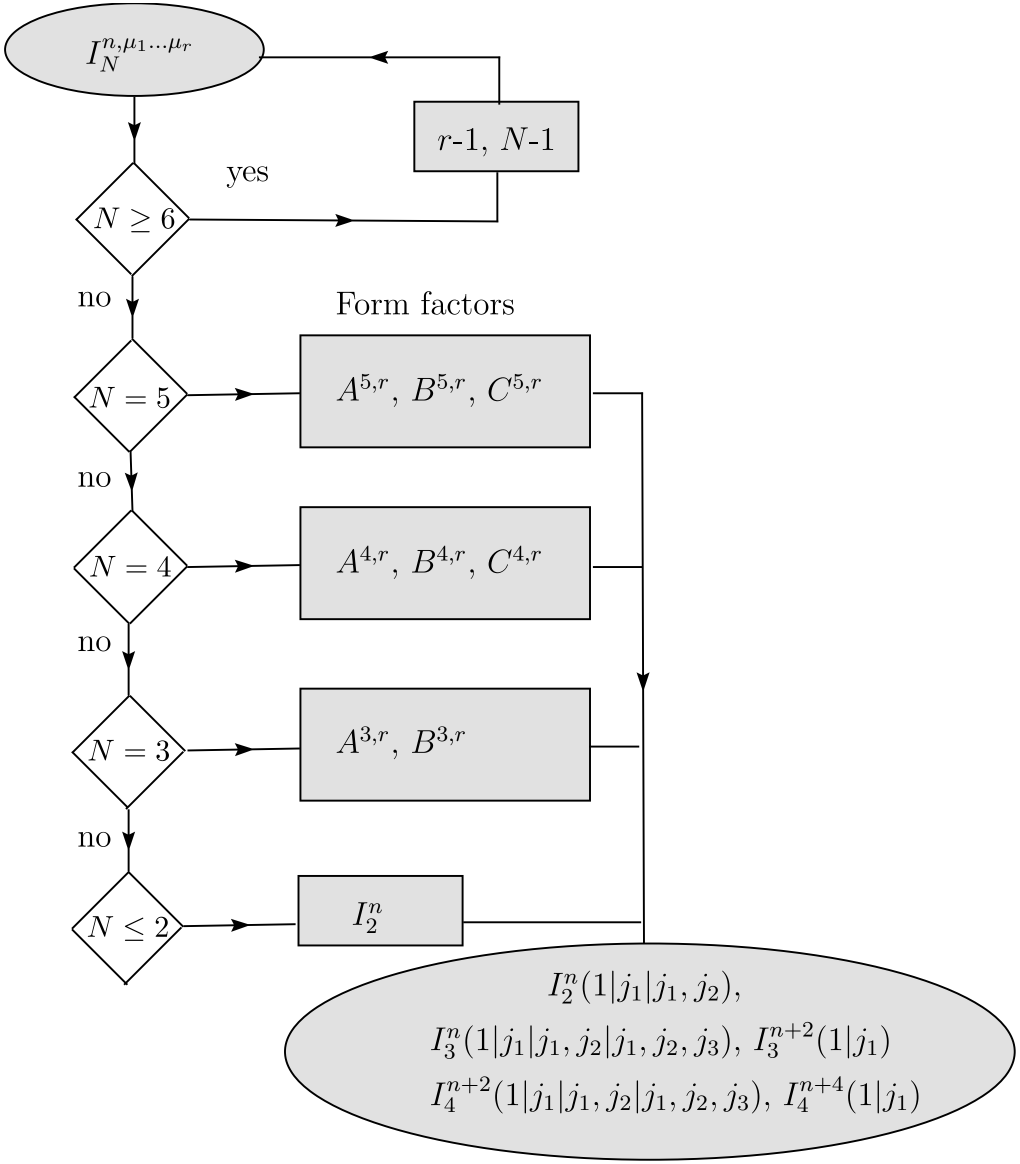
The NLO multi-leg challenge:

- Lots of activity: algebraic, numeric, string inspired $\rightarrow$ 6-gluon amplitude done...
- ... but still no complete $2 \rightarrow 4$ process
- GOLEM approach for 1-loop multi-leg processes
  - $gg \rightarrow W^*W^* \rightarrow l\nu l'\nu'$ $\rightarrow$ sizable Higgs background
  - $gg \rightarrow HH, HHH$ $\rightarrow$ Multi-Higgs physics
  - $q\bar{q} \rightarrow q\bar{q}q\bar{q}$ at NLO in progress
  - complementary to unitarity based methods for rational terms

Beyond NLO challenges:

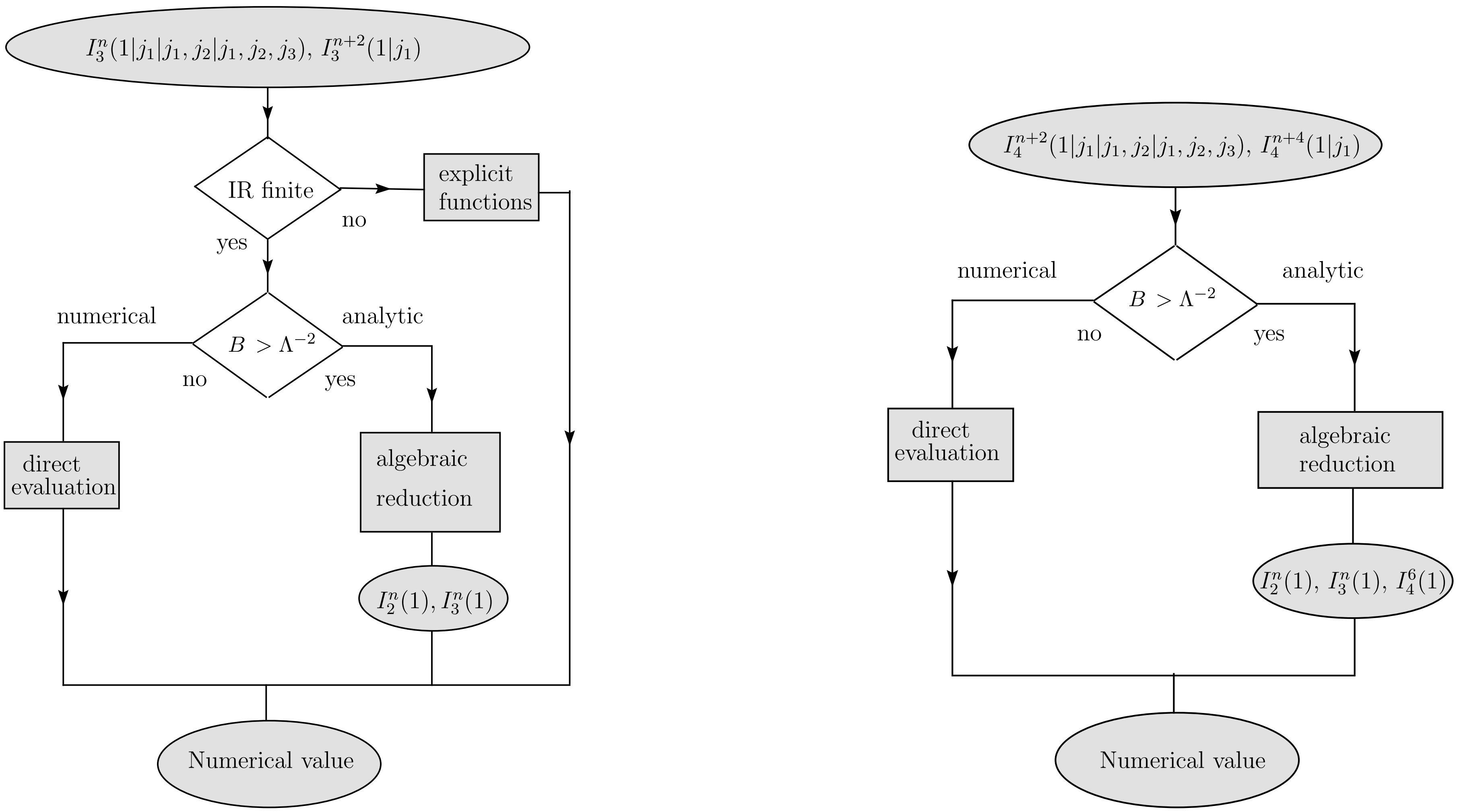
- combining NLO with parton shower
- implementation of known 2-loop matrix elements
- efficient NNLO IR subtraction formalism $\rightarrow$ differential distributions

Schematic overview of N-point tensor integral reduction



Treatment of basis integrals:

$B = |\det(G)/\det(\mathcal{S})|$

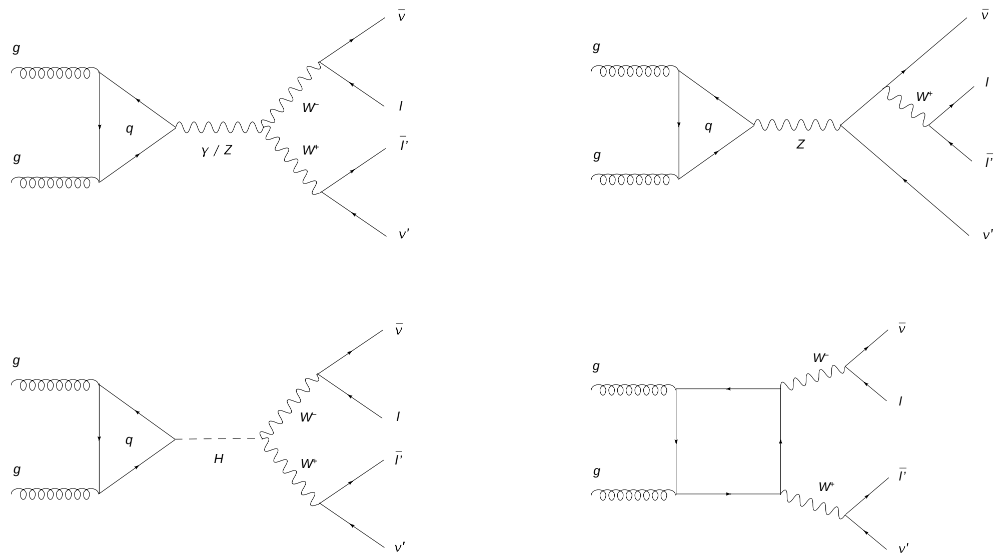


The $gg \to W^*W^* \to l\nu l'\nu'$ amplitude

- missing background for $gg \to H \to W^*W^*$
  [T.B., Ciccolini, Kauer, Krämer, 2005/2006. $m_q \neq 0$, W's offshell.]
  [Dührssen, Jacobs, Marquard, van der Bij, 2005. $m_q \neq 0$, W's onshell.]
- On-shell amplitude known since a long time
  [N. Glover, J.J. van der Bij (1989) $m_q = 0$; C. Kao, D. A. Dicus (1991) $m_q \neq 0$]

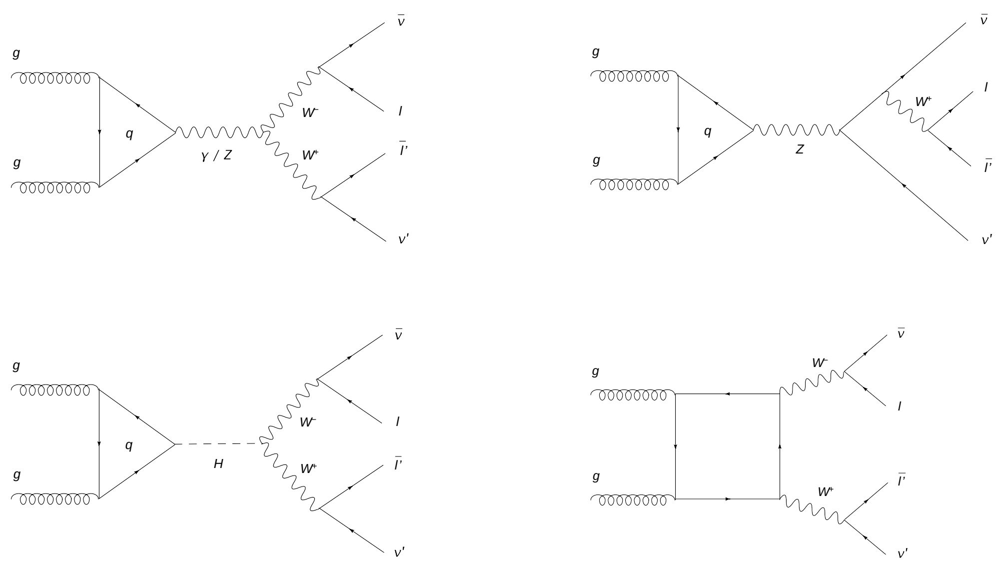
The $gg \to W^*W^* \to l\nu l'\nu'$ amplitude

- missing background for $gg \to H \to W^*W^*$
  [T.B., Ciccolini, Kauer, Krämer, 2005/2006. $m_q \neq 0$, W's offshell.]
  [Dührssen, Jacobs, Marquard, van der Bij, 2005. $m_q \neq 0$, W's onshell.]
- On-shell amplitude known since a long time
  [N. Glover, J.J. van der Bij (1989) $m_q = 0$; C. Kao, D. A. Dicus (1991) $m_q \neq 0$]



The $gg \to W^*W^* \to l\nu l'\nu'$ amplitude

- missing background for $gg \to H \to W^*W^*$
  [T.B., Ciccolini, Kauer, Krämer, 2005/2006. $m_q \neq 0$, W's offshell.]
  [Dührssen, Jacobs, Marquard, van der Bij, 2005. $m_q \neq 0$, W's onshell.]
- On-shell amplitude known since a long time
  [N. Glover, J.J. van der Bij (1989) $m_q = 0$; C. Kao, D. A. Dicus (1991) $m_q \neq 0$]

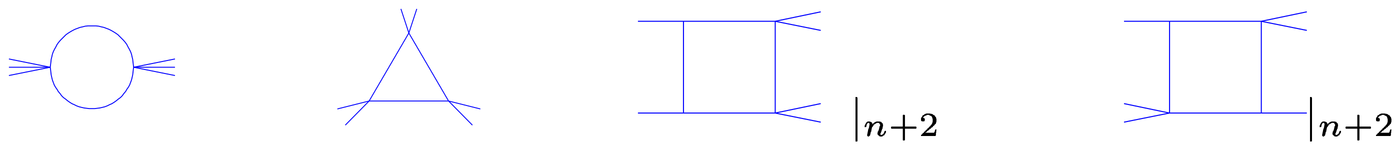


- single resonant graphs add to zero
- interference between Higgs signal and background also below WW threshold

The $gg \to W^*W^* \to l\nu l'\nu'$ amplitude

Helicity amplitudes Γ^{++}, Γ^{+-}, off-shell W's, $m_q \neq 0$, S/B interference

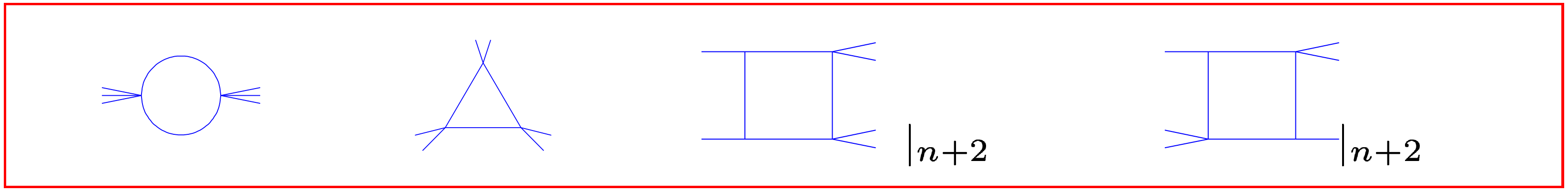
Fully algebraic reduction:



The $gg \to W^*W^* \to l\nu l'\nu'$ amplitude

Helicity amplitudes Γ^{++}, Γ^{+-}, off-shell W's, $m_q \neq 0$, S/B interference

Fully algebraic reduction:



- box/triangle topologies $\to$ 27 Basis functions: $I_4^{d=6}$, $I_3^{d=4}$, $I_2^{d=n}$, 1.
- Decomposition of amplitude by gauge invariant structures (9 independent)
- Coefficients at most $1/\det(G)$, 6 scales $(s, t, s_3, s_4, M_b^2, M_t^2)$
- Instability region: $p_T^2(W) = \det(G)/s^2 < 0.01$ GeV^2, $|s_{3,4} - M_W^2| \gg M_W\Gamma_W$.
- Code available: http://hepsource.sf.net/GG2WW for $m_q = 0$, $m_q \neq 0$
- Shortly: All $gg \to VV$ $(V = \gamma, Z, W)$ box processes

Results: 2 Massless Generations, 3 Generations

LHC (pp, $\sqrt{s} = 14$ TeV)

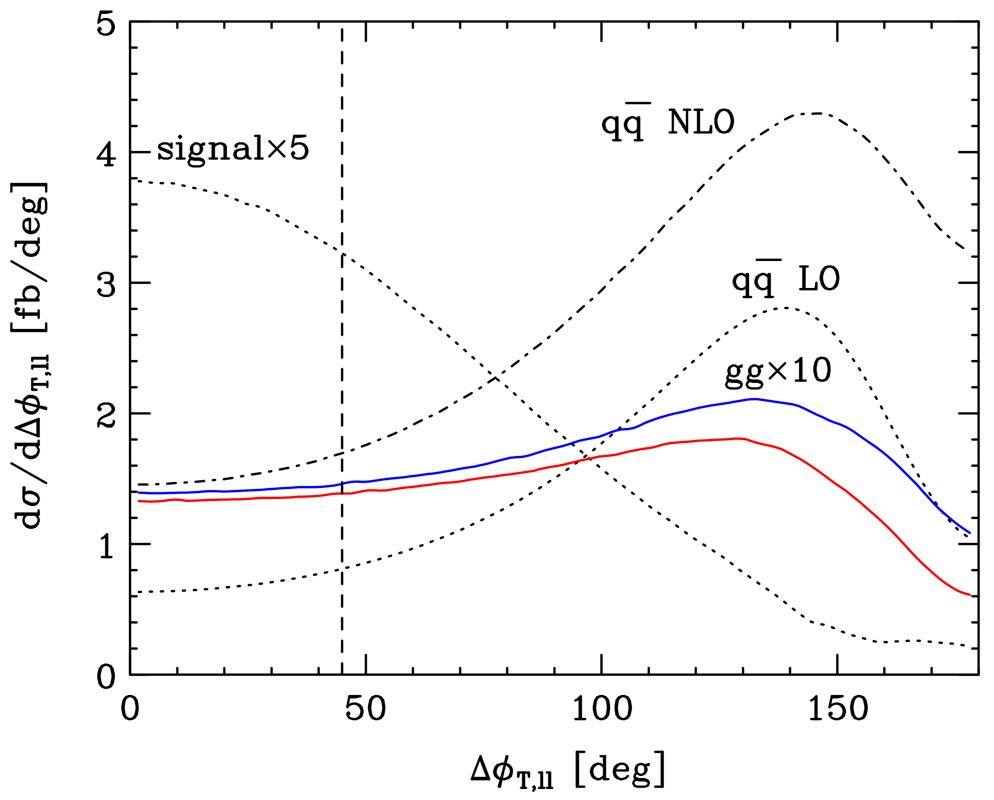
	$\sigma(pp \to W^*W^* \to \ell\bar{\nu}\bar{\ell}'\nu')$ [fb]					
	gg	$\frac{\sigma_{gg,3gen}}{\sigma_{gg,2gen}}$	$q\bar{q}$ LO	$q\bar{q}$ NLO	$\frac{\sigma_{\mathrm{NLO}}}{\sigma_{\mathrm{LO}}}$	$\frac{\sigma_{\mathrm{NLO}+gg}}{\sigma_{\mathrm{NLO}}}$
σ_{tot}	$\frac{60.12(7)}{53.61(2)^{+14.0}_{-10.8}}$	1.12	$875.8(1)^{+54.9}_{-67.5}$	$1373(1)^{+71}_{-79}$	1.57	$\frac{1.04}{1.04}$
σ_{std}	$\frac{29.79(2)}{25.89(1)^{+6.85}_{-5.29}}$	1.15	$270.5(1)^{+20.0}_{-23.8}$	$491.8(1)^{+27.5}_{-32.7}$	1.82	$\frac{1.06}{1.05}$
σ_{bkg}	$\frac{1.416(3)}{1.385(1)^{+0.40}_{-0.31}}$	1.02	$4.583(2)^{+0.42}_{-0.48}$	$4.79(3)^{+0.01}_{-0.13}$	1.05	$\frac{1.30}{1.29}$

$M_W/2 \le \mu_{\mathrm{ren,fac}} \le 2M_W$ ($q\bar{q} \to WW$ from MCFM by J. Campbell, R.K. Ellis)

standard cuts: $p_{T,\ell} > 20$ GeV, $|\eta_\ell| < 2.5$, $\not{p}_T > 25$ GeV

search cuts: $\Delta\phi_{T,\ell\ell} < 45°$, $M_{\ell\ell} < 35$ GeV, 25 GeV $< p_{T,\min}$, 35 GeV $< p_{T,\max} < 50$ GeV
jet veto removes jets with: $p_{T,\mathrm{jet}} > 20$ GeV, $|\eta_{\mathrm{jet}}| < 3$

The $gg \to W^*W^* \to l\nu l'\nu'$ cross section



- $\Rightarrow$ severe Higgs search cuts amplify $ggWW$ contribution $\sim 30\%$!

The $\gamma\gamma \to ggg$ amplitude

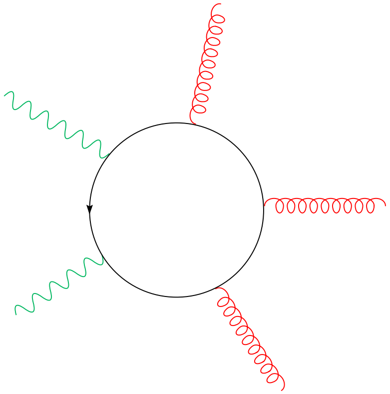
[T.B., J.-Ph. Guillet, F. Mahmoudi, (2004)]

- Relevant for $\gamma\gamma +$ jet background for Higgs+jet production
  [D. de Florian, Z. Kunszt, (1999)]
- Amplitude indirectly known from $gg \to ggg$
  [Z. Bern, L. Dixon, D. Kosower, (1993)]

The $\gamma\gamma \to ggg$ amplitude

[T.B., J.-Ph. Guillet, F. Mahmoudi, (2004)]

- Relevant for $\gamma\gamma +$ jet background for Higgs+jet production
  [D. de Florian, Z. Kunszt, (1999)]
- Amplitude indirectly known from $gg \to ggg$
  [Z. Bern, L. Dixon, D. Kosower, (1993)]

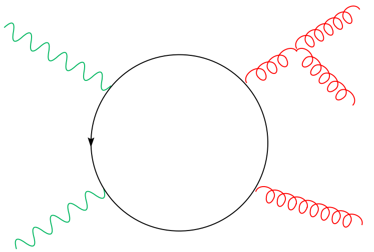


Independent helicity structures:

$\Gamma^{+++++}, \Gamma^{++++-}, \Gamma^{+++--}, \Gamma^{+-+++}, \Gamma^{+-++-}, \Gamma^{--+++}$

All helicity amplitudes calculated by algebraic reduction

- Box, pentagon topologies, 5 scales
- One colour structure: $\sim f^{abc}$
- Sorted by scalar integrals and gauge independent structures



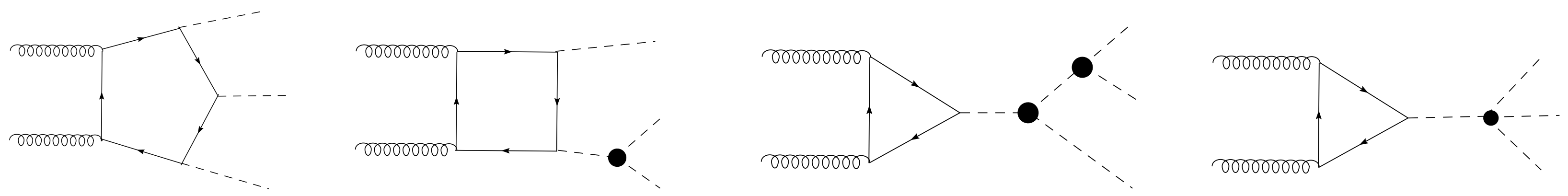
The $gg \to HH, HHH$ amplitude

- Cross sections for multi-Higgs production by gluon fusion
  [T.B., S. Karg, N. Kauer]
- $gg \to HH$ and effective amplitudes $M_T \to \infty$ known since a long time
  [N. Glover, J.J. van der Bij (1987/1988)]
- $gg \to HHH$ recently calculated by T. Plehn, M. Rauch.

The $gg \to HH, HHH$ amplitude

- Cross sections for multi-Higgs production by gluon fusion
  [T.B., S. Karg, N. Kauer]
- $gg \to HH$ and effective amplitudes $M_T \to \infty$ known since a long time
  [N. Glover, J.J. van der Bij (1987/1988)]
- $gg \to HHH$ recently calculated by T. Plehn, M. Rauch.

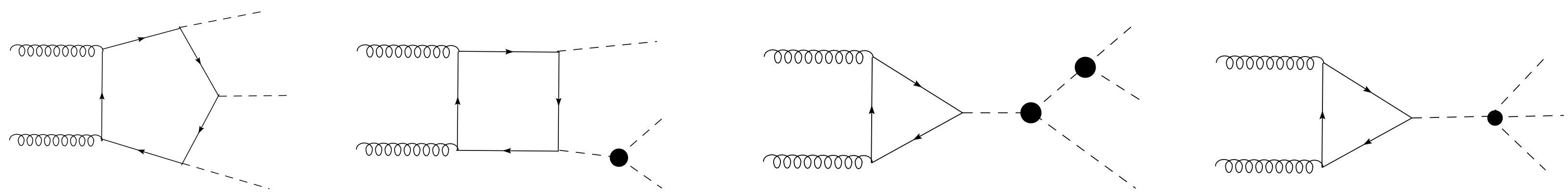
Helicity amplitudes Γ^{++}, Γ^{+-}, algebraic reduction:



The $gg \to HH, HHH$ amplitude

- Cross sections for multi-Higgs production by gluon fusion [T.B., S. Karg, N. Kauer]
- $gg \to HH$ and effective amplitudes $M_T \to \infty$ known since a long time [N. Glover, J.J. van der Bij (1987/1988)]
- $gg \to HHH$ recently calculated by T. Plehn, M. Rauch.

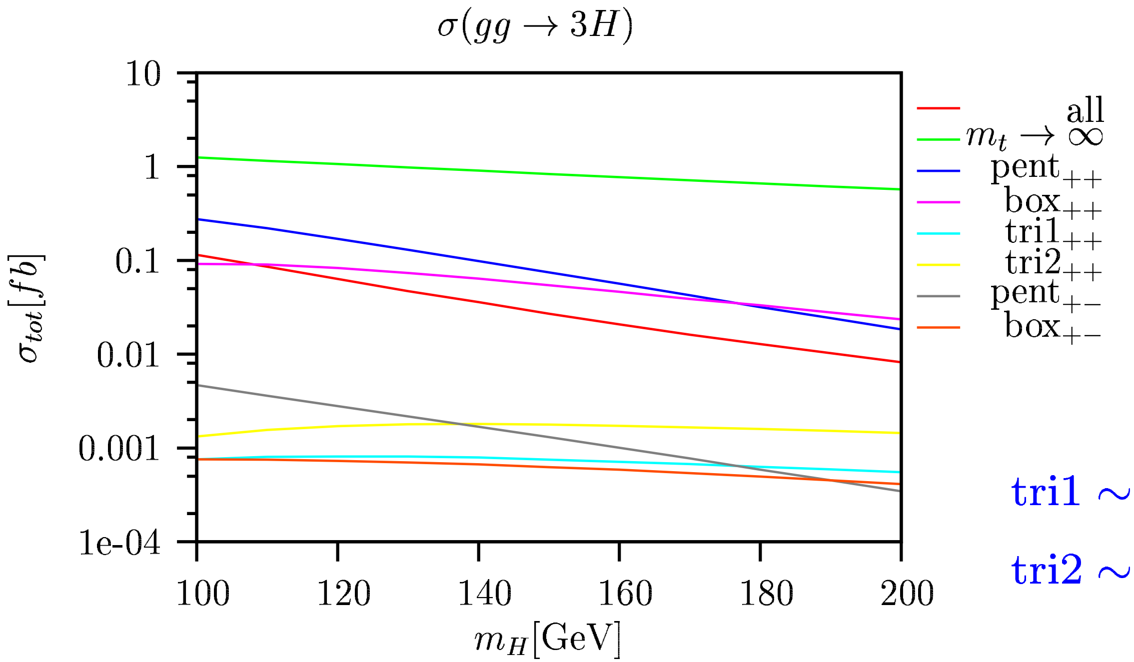
Helicity amplitudes Γ^{++}, Γ^{+-}, algebraic reduction:



- box/triangle/pentagon topologies, 7 scales $(s_{12}, s_{23}, s_{34}, s_{45}, s_{51}, M_H^2, M_t^2)$
- Gauge invariant structures: tr$(\mathcal{F}_1\mathcal{F}_2)$, $p_2.\mathcal{F}_1.p_i\ p_1.\mathcal{F}_2.p_j$, $\mathcal{F}_j^{\mu\nu} = p_j^\mu \varepsilon_j^\nu - p_j^\nu \varepsilon_j^\mu$
- Basis functions: $I_4^{d=6}$, $I_3^{d=4}$, $I_2^{d=n}$, 1. Coefficients at most $1/\det(G)$

The $gg \to HHH$ cross section

- perfect agreement with Plehn/Rauch
- Numerically stable result
- CPU time: 1 h for inclusive cross section on pentium 4 PC (2.8 GHz)



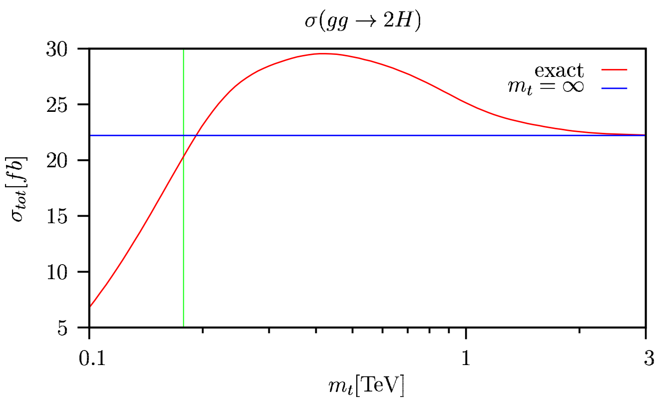
tri1 $\sim \lambda_{4H}$

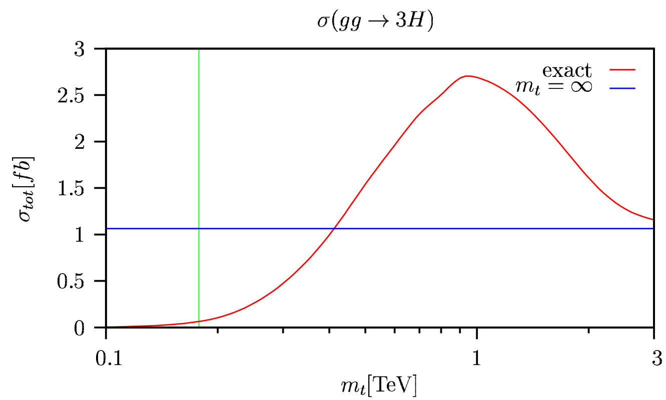
tri2 $\sim \lambda_{3H}^2$

- $\Rightarrow$ quartic Higgs coupling can not be tested at the LHC

The $gg \to HH, HHH$ cross section

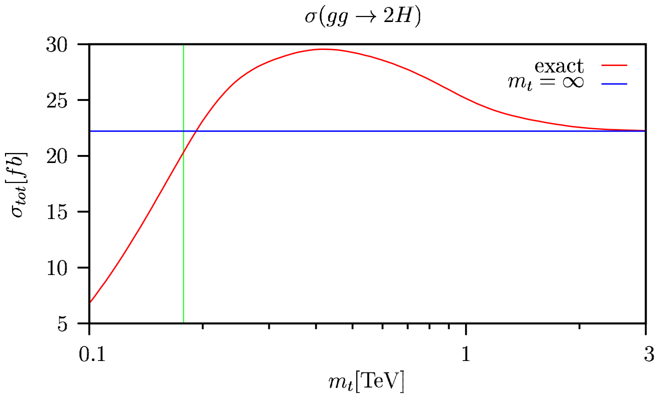
- $L_{M_T \to \infty} = \frac{\alpha_s}{12\pi} \mathcal{F}^a_{\mu\nu} \mathcal{F}^{\mu\nu\ a} \ \log(1 + H/v) \Rightarrow gg + nH$ effective vertices
- effective vertices not a good description at LHC for $n \geq 2$

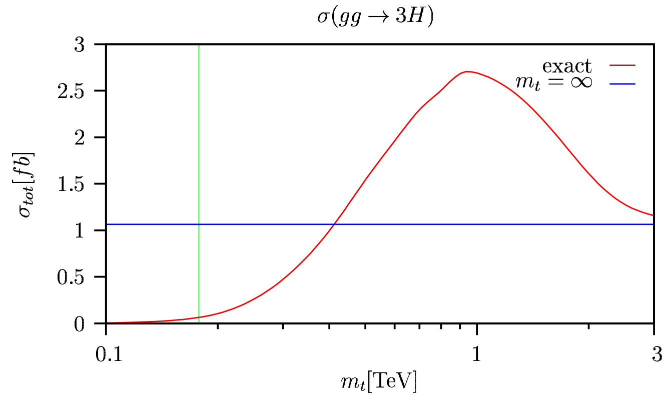




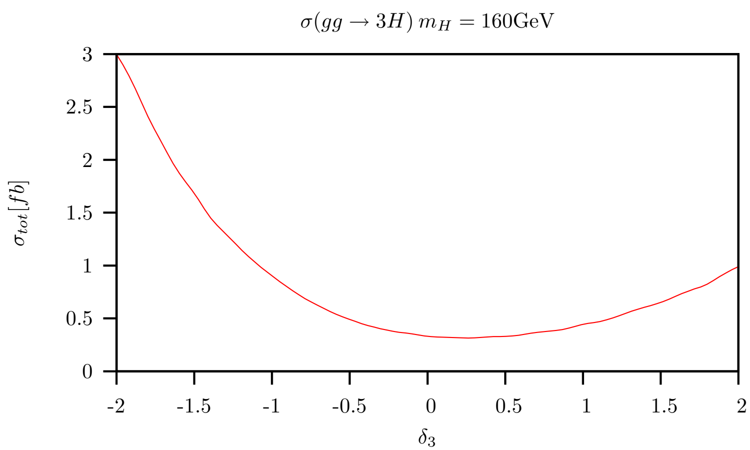
The $gg \to HH, HHH$ cross section

- $L_{M_T \to \infty} = \frac{\alpha_s}{12\pi} \mathcal{F}^a_{\mu\nu} \mathcal{F}^{\mu\nu\ a}\ \log(1 + H/v) \Rightarrow gg + nH$ effective vertices
- effective vertices not a good description at LHC for $n \geq 2$



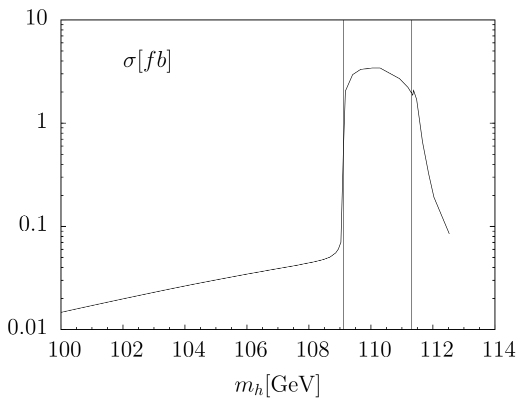


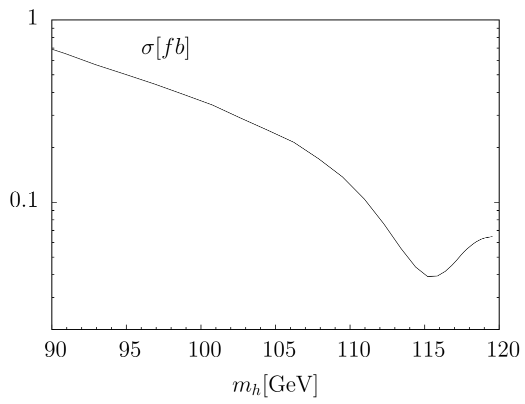
- cross section enhanced by BSM physics, $\delta_3 = (\lambda_{3H,BSM} - \lambda_{3H,SM})/\lambda_{3H,SM}$
- trilinear Higgs coupling not uniquely fixed at LHC (if at all)



The $gg \to HH, HHH$ cross section

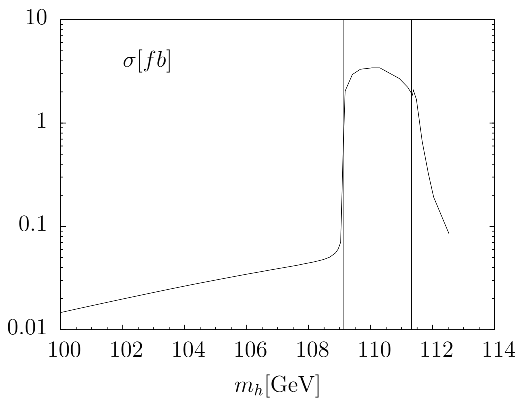
- amplification possible in two Higgs doublet models
- resonant amplification does, $\tan\beta$ amplification does not help

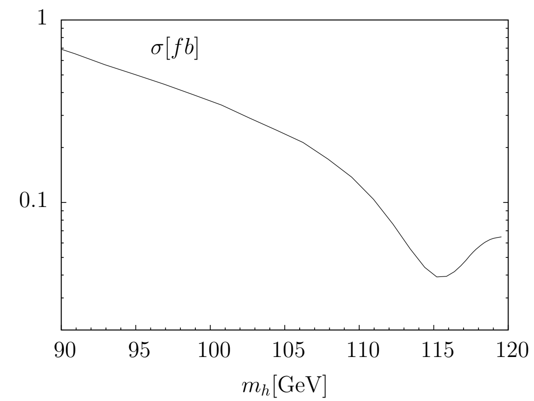




The $gg \to HH, HHH$ cross section

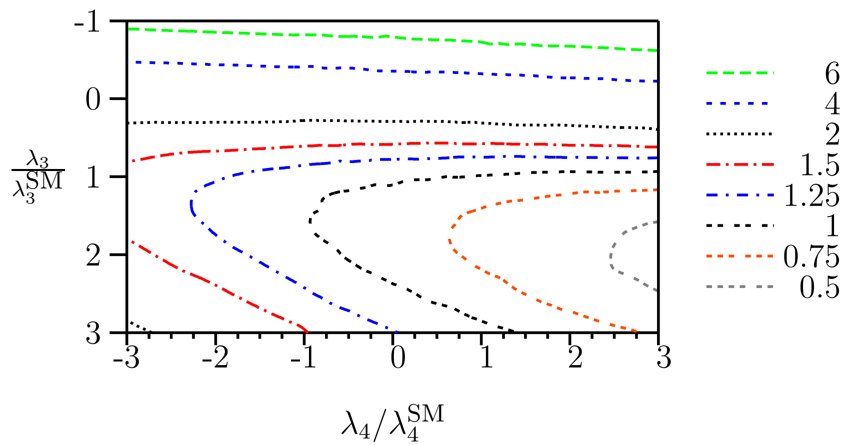
- amplification possible in two Higgs doublet models
- resonant amplification does, $\tan\beta$ amplification does not help





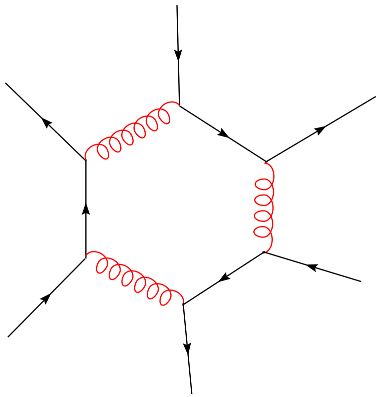
- Higher dimensional operators $\Rightarrow \lambda_{3H}, \lambda_{4H}$ free parameters

$$V = \sum_{k=1}^{\infty} \frac{g_k}{\Lambda^{2k}} \left(\Phi^\dagger \Phi - \frac{v^2}{2} \right)^{2+k}$$



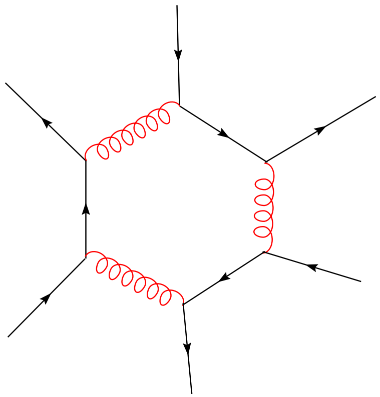
The $q\bar{q} \to q\bar{q}q\bar{q}$ amplitude (in progress)

- Contribution of $PP \to 4$ jets, $bbbb$ at NLO [$\sigma \sim \mathcal{O}$(nb) at LHC!]
- Two helicity amplitudes needed: $\mathcal{A}^{++++++}$, $\mathcal{A}^{++++--}$
- Other partonic contributions: $gg \to gggg$, $gg \to q\bar{q}gg$, $gg \to q\bar{q}q\bar{q}$ plus crossings
  $\to$ accessible with twistor space inspired/unitarity based methods (?!)



The $q\bar{q} \to q\bar{q}q\bar{q}$ amplitude (in progress)

- Contribution of $PP \to 4$ jets, $bbbb$ at NLO [$\sigma \sim \mathcal{O}$(nb) at LHC!]
- Two helicity amplitudes needed: $\mathcal{A}^{++++++}$, $\mathcal{A}^{++++--}$
- Other partonic contributions: $gg \to gggg$, $gg \to q\bar{q}gg$, $gg \to q\bar{q}q\bar{q}$ plus crossings
  $\to$ accessible with twistor space inspired/unitarity based methods (?!)



- algebraic reduction done $\to$ Masterintegrals
- semi-numerical reduction $\to$ Golem basis with Fortran 90 code "golem90 v0.2"
- Amplitude evaluation $\mathcal{O}(s)$, rank 3 6-point form factor ~ 40 ms
  (Pentium4, 1.6 GHz)
- Evaluation time of virtual corrections small compared to real emission corrections

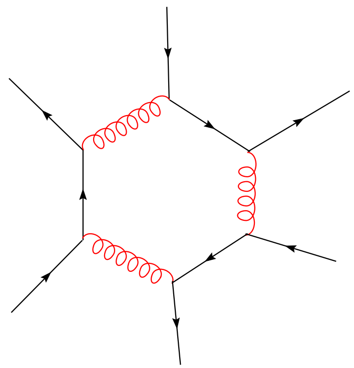
The $q\bar{q} \to q\bar{q}q\bar{q}$ amplitude

Numerical results of hexagon diagram of helicity Amplitude A^{++++++}:

$$A^{++++++}(k_1,\ldots,k_6) = \frac{g_s^6}{(4\pi)^{n/2}} \frac{1}{s} [\frac{A}{\epsilon^2} + \frac{B}{\epsilon} + C + \mathcal{O}(\epsilon)]$$

Spinor lines closed by multiplying $1 = \frac{\langle 1^+|4|2^+\rangle}{\sqrt{s_{14}s_{24}}} \frac{\langle 4^+|1|3^+\rangle}{\sqrt{s_{14}s_{13}}} \frac{\langle 6^+|1|5^+\rangle}{\sqrt{s_{15}s_{16}}} e^{i\Phi}$

Kinemtical point:



k	$= ($	$k^0,$	$k^1,$	$k^2,$	$k^4)$
k_1	$= ($	0.5,	0.,	0.,	0.5)
k_2	$= ($	0.5,	0.,	0.,	-0.5)
k_3	$= ($	0.1917819,	0.1274118,	0.08262477,	0.1171311)
k_4	$= ($	0.3366271,	-0.06648281,	-0.3189379,	-0.08471424)
k_5	$= ($	0.2160481,	-0.2036314,	0.04415762,	0.05710657)
k_6	$= ($	0.2555428,	0.1427024,	0.1921555,	-0.08952338)

Up to phase/color factor:

$\mathrm{Re}(A)$	$\mathrm{Im}(A)$	$\mathrm{Re}(B)$	$\mathrm{Im}(B)$	$\mathrm{Re}(C)$	$\mathrm{Im}(C)$
-5.313592	-1.245007	-23.74344	-23.54086	-14.37056	-96.23081