UNIVERSITY OF CAMBRIDGE

The anatomy of double heavy quark initiated processes

M. A. Lim F. Maltoni G. Ridolfi M. Ubiali

IPPP, University of Durham, 8th September 2017

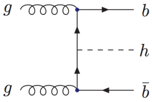
Outline

1. Heavy quark schemes
2. 4F schemes and collinear logs
3. 5F schemes and resummation effects
4. Scale choices in $b\bar{b} \to H$

Heavy quark schemes

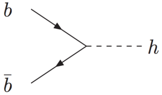
4F ("massive") scheme

- Heavy quarks only created in high Q^2 interactions
- Generated only as massive final states
- b/t-PDFs set to zero



5F ("massless") schemes

- Hard scale Q of process $\gg m_q$
- Heavy quark contributes to proton wave function, running of α_s
- Logarithms $\log \frac{Q^2}{m_q^2}$ are resummed



Heavy quark schemes

4F ("massive") scheme

- Kinematics completely accounted for
- Implementation in a parton shower easier
- Calculations more involved
- Potentially large logs threaten convergence

5F ("massless") schemes

- Simplified calculation - NNLO available
- Possible to include mass effects at higher orders
- Logs from collinear splittings are resummed into PDFs

Heavy quark schemes

- To all orders in P.T., schemes are identical - at fixed order, however, they vary
- Difference in schemes at LO can be significant (see later...)

Questions

- What is the effect of resummation of logs in a b PDF as compared to keeping explicit leading and subleading logs at fixed order?
- What are the typical sizes of the logs in a 4F scheme for processes of interest at a hadron collider?

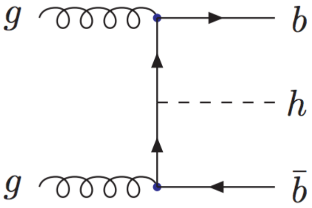
Answers

- For processes with a single b, unless a very heavy particle is produced in the final state the initial state collinear logs are modest and convergence of the P.T. is not spoiled in a 4F calculation (*F. Maltoni, G. Ridolfi, M. Ubiali*).
- Resummation effects are relevant primarily at large Bjorken x and keeping only explicit logs at NLO is a good approximation.
- Scale Q appearing in logs is suppressed by phase space factors, reducing their size

Some more questions

- What is the analogous effect in boson production processes involving two heavy quarks?
  - Lowest order 4F contribution appears in NNLO 5F real corrections
  - Latter now readily available in public codes
- What choice of the factorisation/renormalisation scales should be made and why?

Bottom fusion initiated Higgs production in the 4F scheme



- Final state bs treated as massive
- $M_H \gg m_b \implies$ cross section dominated by arrangements with bs collinear to gluons

4F Cross section and origins of logs

The partonic cross-section is given by

$$\hat{\sigma}^{\mathrm{4F,coll}}(\hat{\tau}) = \hat{\tau}\frac{\alpha_s^2}{4\pi^2}\frac{G_F\pi}{3\sqrt{2}}\frac{m_b^2}{M_H^2}2\int_0^1 dz_1\int_0^1 dz_2\, P_{qg}(z_1)P_{qg}(z_2)L(z_1,\hat{\tau})L(z_2,\hat{\tau})\delta\left(z_1 z_2 - \hat{\tau}\right), \tag{1}$$

where

$$\hat{\tau} = \frac{M_H^2}{\hat{s}}, \tag{2}$$

$P_{qg}(z)$ is the leading-order quark-gluon Altarelli-Parisi splitting function

$$P_{qg}(z) = \frac{1}{2}[z^2 + (1-z)^2], \tag{3}$$

and

$$L(z,\hat{\tau}) = \log\left[\frac{M_H^2}{m_b^2}\frac{(1-z)^2}{\hat{\tau}}\right]. \tag{4}$$

4F Cross section and origins of logs

We can rewrite this as

$$\hat{\sigma}^{4\mathrm{F,coll}}(\hat{\tau}) = 2\int_{\hat{\tau}}^{1} dz_1 \int_{\frac{\hat{\tau}}{z_1}}^{1} dz_2 \left[\frac{\alpha_s}{2\pi} P_{qg}(z_1) L(z_1,\hat{\tau})\right] \left[\frac{\alpha_s}{2\pi} P_{qg}(z_2) L(z_2,\hat{\tau})\right] \hat{\sigma}^{5\mathrm{F}}\left(\frac{\hat{\tau}}{z_1 z_2}\right). \tag{5}$$

This is logarithmically divergent as $m_b \to 0$.
The subleading terms $(1-z_i)^2/\hat{\tau}$ arise from the kinematics and so are 'universal'. For completeness, the hadronic cross section is given by

$$\sigma^{4\mathrm{F,coll}}(\tau) = 2\int_{\tau}^{1} dx_1 \int_{\frac{\tau}{x_1}}^{1} dx_2\, \hat{\sigma}^{5\mathrm{F}}\left(\frac{\tau}{x_1 x_2}\right) \int_{x_1}^{1} \frac{dz_1}{z_1} \left[\frac{\alpha_s}{2\pi} P_{qg}(z_1) L\left(z_1, z_1 z_2\right)\right] g\left(\frac{x_1}{z_1}, \mu_F^2\right) \int_{x_2}^{1} \frac{dz_2}{z_2} \left[\frac{\alpha_s}{2\pi} P_{qg}(z_2) L\left(z_2, z_1 z_2\right)\right] g\left(\frac{x_2}{z_2}, \mu_F^2\right). \tag{6}$$

UNIVERSITY OF CAMBRIDGE

The collinear approximation

How good is the approximation we have made?

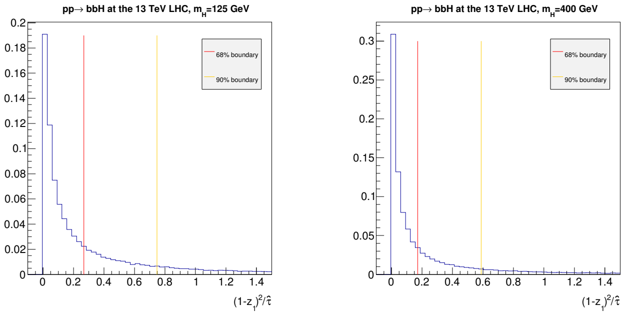
M_H	exact	collinear ME	collinear ME and PS
125 GeV	$4.71 \cdot 10^{-1}$ pb	$5.15 \cdot 10^{-1}$ pb	$5.82 \cdot 10^{-1}$ pb
400 GeV	$5.42 \cdot 10^{-3}$ pb	$5.58 \cdot 10^{-3}$ pb	$5.91 \cdot 10^{-3}$ pb

Total cross sections for Higgs boson production at the LHC 13 TeV in the 4F scheme.

We see an order 20% effect on the total cross section.

Kinematic suppression of the logarithms

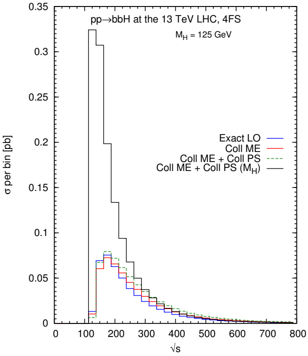
We noted that the arguments of the logs are suppressed by subleading terms of kinematic origin. What is the numerical effect?

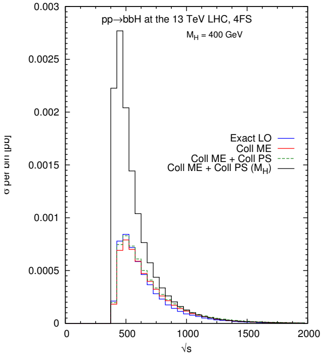


Kinematic suppression of the logarithms

- We note that the distributions are peaked around values smaller than 1.
- Though formally subleading with respect to $\log \frac{M_H^2}{m_b^2}$, these give sizeable contributions to the total cross section.

Kinematic suppression of the logarithms





Clear that collinear phase space result differs substantially!

UNIVERSITY OF CAMBRIDGE

5F cross section

We have a physical cross section given by

$$\sigma^{5\mathrm{F}}(\tau) = 2\int_{\tau}^{1} dx_1\, b(x_1,\mu_F^2) \int_{\frac{\tau}{x_1}}^{1} dx_2\, b(x_2,\mu_F^2) \hat{\sigma}^{5\mathrm{F}}\left(\frac{\tau}{x_1 x_2}\right). \quad (7)$$

We can expand the b PDF in α_s:

$$\begin{aligned} b(x,\mu_F^2) &= \frac{\alpha_s}{2\pi} L_b \int_x^1 \frac{dy}{y} P_{qg}(y) g\left(\frac{x}{y},\mu_F^2\right) + \mathcal{O}(\alpha_s^2) \\ &= \tilde{b}^{(1)}(x,\mu_F^2) + \mathcal{O}(\alpha_s^2), \end{aligned} \quad (8)$$

where

$$L_b = \log\frac{\mu_F^2}{m_b^2}. \quad (9)$$

5F cross section

Defining a truncated 5F cross section with only one power of log m_b^2 for each b, we obtain

$$\sigma^{5\mathrm{F},(1)}(\tau) = 2\int_{\tau}^{1} dx_1 \int_{\frac{\tau}{x_1}}^{1} dx_2\, \hat{\sigma}^{5F}\left(\frac{\tau}{x_1 x_2}\right) \int_{x_1}^{1} \frac{dy}{y} \left[\frac{\alpha_s}{2\pi} P_{qg}(y) L_b\right] g\left(\frac{x_1}{y}, \mu_F^2\right) \int_{x_2}^{1} \frac{dz}{z} \left[\frac{\alpha_s}{2\pi} P_{qg}(z) L_b\right] g\left(\frac{x_2}{z}, \mu_F^2\right) \tag{10}$$

which has the same form as the collinear approximation to the 4F result but with constant arguments in the logs.

UNIVERSITY OF CAMBRIDGE

Renormalisation scale choices

We therefore suggest a scale choice for the 5F calculation such that the two schemes give the same results:

$$\sigma^{5\mathrm{F},(1)}(\tau) = \sigma^{4\mathrm{F,coll}}(\tau). \tag{11}$$

For $\sqrt{s} = 13$ GeV, and $m_b = 4.75$ GeV, we find the following values for $\tilde{\mu}_F$:

$$\begin{aligned} b\bar{b}H, M_H = 125\,\mathrm{GeV}: &\qquad \tilde{\mu}_F \approx 0.36\, M_H \\ b\bar{b}Z', M_{Z'} = 91.2\,\mathrm{GeV}: &\qquad \tilde{\mu}_F \approx 0.38\, M_{Z'} \\ b\bar{b}Z', M_{Z'} = 400\,\mathrm{GeV}: &\qquad \tilde{\mu}_F \approx 0.29\, M_{Z'} \end{aligned} \tag{12}$$

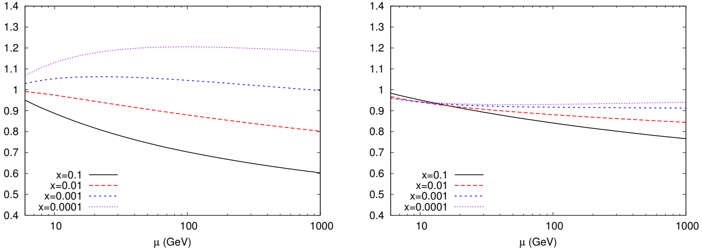
Renormalisation scale choices

- We note that $\tilde{\mu}_F$ is considerably smaller than the mass of the produced heavy particle.
- For the Higgs case, the appropriate scale choice is $\tilde{\mu}_F \approx M_H/3$.
- Further differences can occur from
  - power-like mass terms in the 4F scheme (shown to be small in previous works)
  - collinear resummation in the 5F scheme.

Quantifying the effects of resummation

- We wish to assess the accuracy of the $\mathcal{O}(\alpha_s^1)$ approximation to the b PDF as compared to the full expression.
- The truncated expression $\tilde{b}^{(p)}(x, \mu^2)$ does not feature the full resummation of logs but contains powers of the log with $1 \leq n \leq p$.
- We can examine the effects of truncation by looking at the ratio $\frac{\tilde{b}^{(p)}(x,\mu^2)}{b(x,\mu^2)}$ for $p = 1, 2...$

Quantifying the effects of resummation

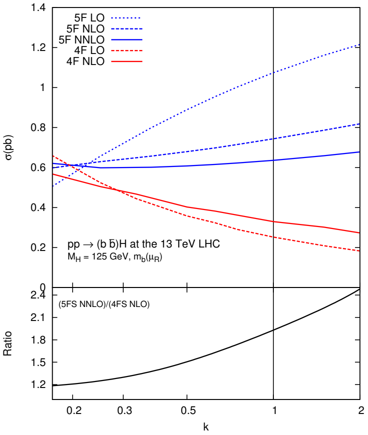


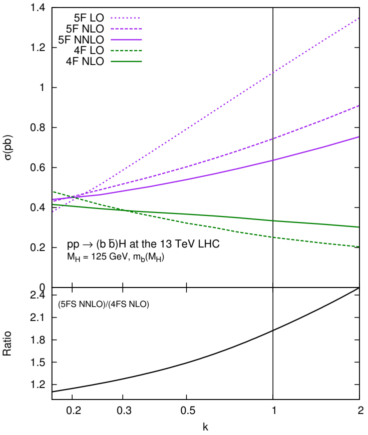
At LO higher order logs are important - $\tilde{b}^{(1)}(x, \mu^2)$ is a poor approximation of the fully resummed result. Things are better at NLO, particularly at smaller values of x.

More on $b\bar{b} \to H$

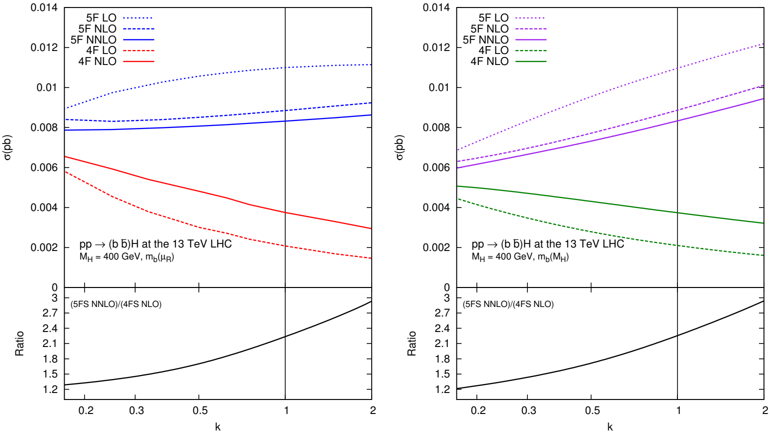
- Many calculations of b-initiated Higgs production are available, at NNLO in the 5F scheme and NLO in the 4F.
- Fully differential calculations are available in MadGraph5_aMC@NLO - studies conclude that 4F results are generally more accurate than 5F for exclusive observables.
- Scheme differences for inclusive observables are mild if judicious choices for the scale are made.

More on $b\bar{b} \to H$





More on $b\bar{b} \to H$



More on $b\bar{b} \to H$

At LO, the 4F and 5F schemes show different behaviour:

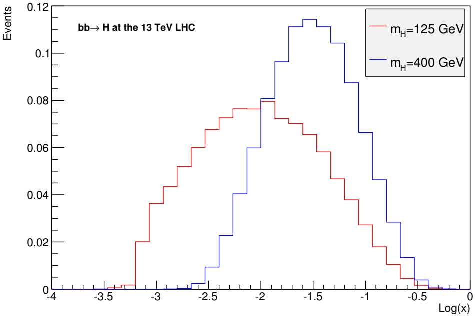
- The 4F scale dependence is driven by the running of α_s and decreases with the scale;
- The 5F is driven by the scale dependence of the b PDF.

The scale dependence is significantly reduced at higher orders, but we still see an order 80% difference between scheme predictions at the central value of the scale.

More on $b\bar{b} \to H$

- Comparing calculations at the value of the scale $\tilde{\mu}_F$ derived earlier, we see a difference of about 20%. This can be accounted for by considering the effects of resummation of the 5F calculation.
- We can quantify these effects by examining the range of x that contributes to the production channel.

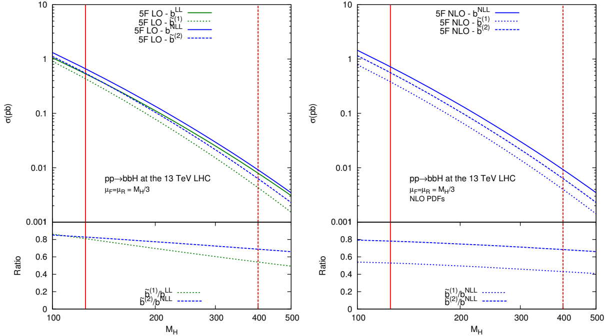
More on $b\bar{b} \to H$



The range of x probed is centred around $x \approx 10^{-2}$ for the SM Higgs, where resummation effects are sizeable.

UNIVERSITY OF CAMBRIDGE

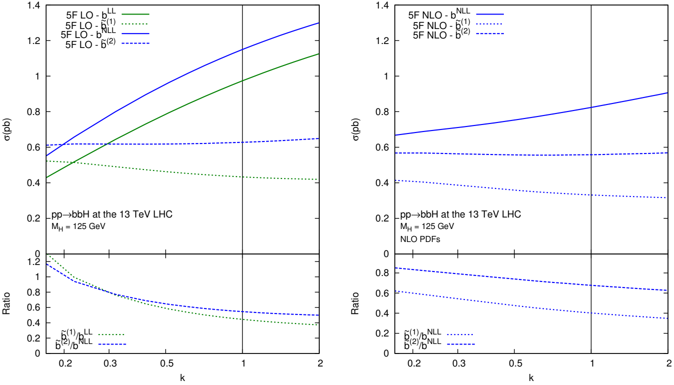
Higgs mass dependence of the cross section



We note the effect of omitting higher order logs is smaller that 20% for the SM Higgs at LO and similar at NLO. Choosing a higher scale would have made the resummation effects much more significant.

UNIVERSITY OF CAMBRIDGE

Scale dependence of the 5F resummed cross section



Effect of resummation of higher order logs is highly scale dependent, but relatively small at the suggested value of $\tilde{\mu}_F$.

Conclusions

- We have studied the behaviour of processes initiated by two heavy quarks, including the production of a Higgs from two b quarks.
- By comparing the 4F and 5F cross sections without resummation, one can identify the size of the collinear logs and hence propose a scale choice at which results in the two schemes may reasonably be compared.
- The effects of the resummation have been evaluated by comparing results for the total cross section computed with the fully evolved b PDF with those computed with a truncated expansion.

Conclusions

- We conclude that in such processes the resummation causes an increase in the cross section by order 20% at the LHC and in general leads to a better precision.
- We also note that the 4F predictions at NLO also display a consistent perturbative behaviour when evaluated at suitable scales. They remain, therefore, appropriate in cases with b quarks in the final state and where mass effects are non-negligible.
- We observe similar behaviour in Z production in association with bs at the LHC and in Z' production in association with ts at a potential 100 TeV future collider.