# A determination of the strong coupling constant from a global PDF analysis

Final HiggsTools meeting, University of Durham

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# Why $\alpha_S$?

- Rule of thumb: If an observable starts at $\alpha_S^N$, the relative uncertainty is $\sim n\left(\frac{\Delta\alpha_S}{\alpha_S}\right)$. Has not decreased for a while.
- Example: $ggH \sim \alpha_S^2$ :

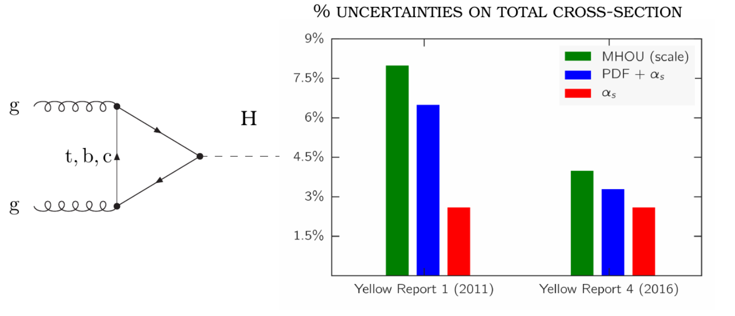



**Figure 1:** (S.Forte, Lattice 2017)

(and backgrounds like $t\bar{t}jj \sim \alpha_S^4$ ).

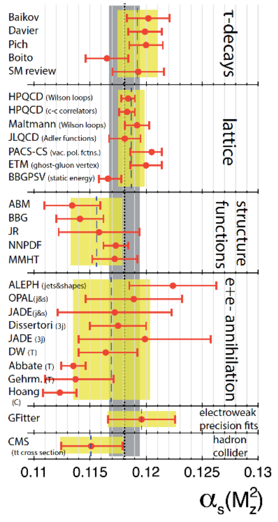
# Current value and error on $\alpha_S$?

The usually accepted value comes from the PDG Average (PDG review 2016, Bethke, Dissertori, Salam):

$$\alpha_S(M_Z^2) = 0.1181 \pm 0.0013$$

- “Combined” from determinations from several physical processes.

# PDG combination




- Only NNLO or better determinations considered.
- “Pre averaging”: Take the unweighted mean and the mean error from each process.
- Final number obtained as a weighted “$\chi^2$ average” over the processes.

# Ways to combine quantity determinations

Assume we have two determinations of the quantity $x$ characterized by uncorrelated probability densities $P_1(x)$ and $P_2(x)$.

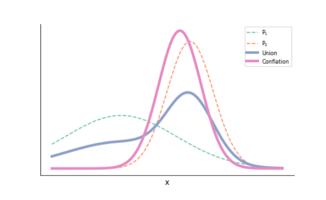
**Uncertainty as prediction output** "Union" (e.g. PDF combination)

$$P(x) = \frac{P_1(x) + P_2(x)}{2}$$

**Combined uncertainties** "Intersection" or "conflation" (Hill, 2008)

$$P(x) = \frac{P_1(x)P_2(x)}{\int P_1(x)P_2(x)dx}$$

($\chi^2$ averaging for Gaussians)



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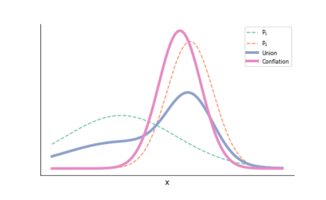


**Determination vs test to the methodology**

# $\alpha_S$ combination as a sociological construct

*"In my opinion one should select few theoretically simplest processes for measuring $\alpha_S$ and consider all other ways as tests of the theory."*

*G.Altarelli, 2013*

- Rule of thumb: Larger uncertainties easier to trust.
- Example: EW precision fits.
    - Small theoretical uncertainties, but must assume Standard Model.
    - PDG value (from Gfitter group, 2014): $\alpha_S(M_Z^2) = 0.1196 \pm 0.0030$
- Example: $\tau$ decays
    - Leptonic initial state (no PDF dependence) but at very small scale.
    - PDG value: $\alpha_S(M_Z^2) = 0.1192 \pm 0.0018$
- Combined $\alpha_S(M_Z^2) = 0.1193 \pm 0.0015$

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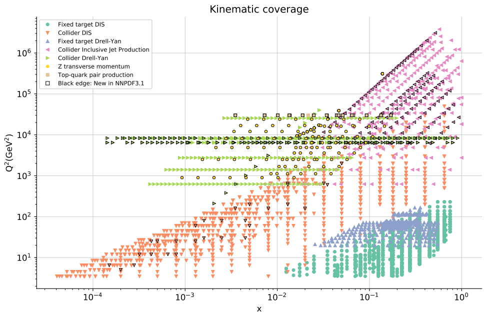
# $\alpha_S$ from PDFs

- Physical mechanism: Scale violations

$$\mu \frac{d}{d\mu} f_\alpha(x,\mu) = \frac{\alpha_S(\mu)}{2\pi} \int_x^1 \frac{d\xi}{\xi} \sum_b P_a^b(\xi, \alpha_s(\mu)) f_b(\frac{x}{\xi}, \mu)$$

- Correct value of $\alpha_S$ required to describe data at different scales $\rightarrow$ can obtain $\alpha_S$ as best fit to the data (simultaneously with the PDFs).




- Different physical processes included in the fit.

# Challenges extracting $\alpha_S$ from PDFs

- PDF parametrization may bias the $\alpha_S$ value
- Correct treatment of experimental systematics (particularly normalization uncertainties).
- Hidden uncertainties in theoretical description of PDFs (e.g. heavy quark treatment).
- Inclusion of PDF uncertainty in the $\alpha_S$ determination.

- Accurate estimation of missing higher order uncertainties

# Challenges extracting $\alpha_S$ from PDFs

- PDF parametrization may bias the $\alpha_S$ value[NNPDF]
- Correct treatment of experimental systematics (particularly normalization uncertainties). [NNPDF]
- Hidden uncertainties in theoretical description of PDFs (e.g. heavy quark treatment). [NNPDF 3.1]
- Inclusion of PDF uncertainty in the $\alpha_S$ determination. [NNPDF 3.1-$\alpha_S$]
- Accurate estimation of missing higher order uncertainties [NNPDF 3.1-$\alpha_S$]

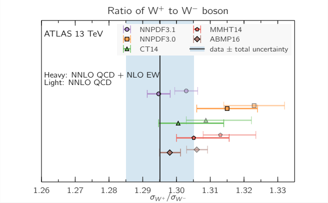
Made lots of progress since latest NNPDF determination in 2011.

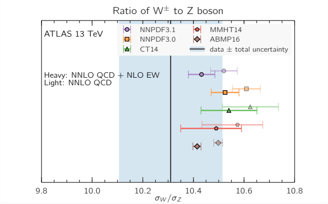
# NNPDF 3.1

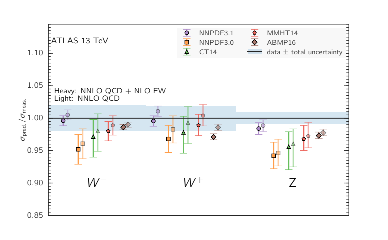
NNPDF 3.1 2017 is an interesting baseline for an $\alpha_S$ determination.

- PDFs generally constrained at percent level in the data region.
- Wealth of new data. Particularly sensitive to $\alpha_s$ are inclusive jets, $t\bar{t}$ distributions, $Z\,p_T$ distribution.
    - Used only NNLO jets for $\alpha_S$ fits where available (upgraded from 3.1).
- Charm PDF explicitly fitted.
- Improved numeric stability.

# Impact on the W and Z cross sections



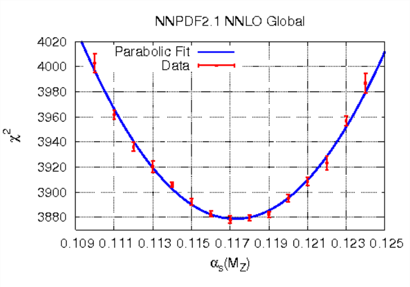




- Much improved accuracy and precision for standard candles.

# Fitting $\alpha_S$: General strategy

- Quality of PDF fit is characterized by error function $\chi^2$.
- Produce best fit-PDFs for a range of values in $\alpha_S(M_Z^2)$.
- Determine $\alpha_s$ as the minimum of $\chi^2(\alpha_S)$.

# $\alpha_S$ uncertainty: the old way



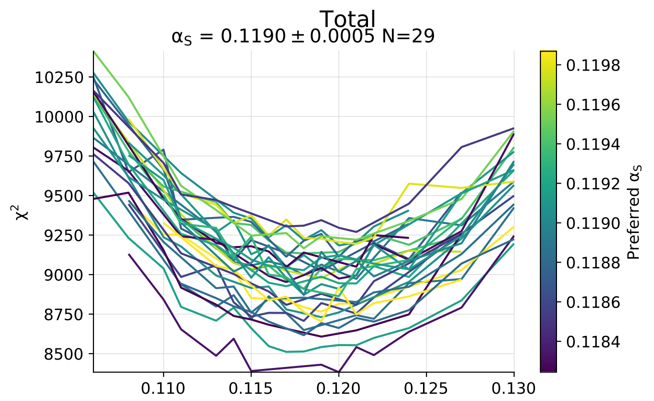

- Read off $\Delta\chi^2 = 1$ from the parabolic fit to the central $\chi^2(\alpha_S)$.
- Unclear relation with the usual PDF uncertainty.
- Ideally we would determine $\alpha_S$ simultaneously with the PDFs , instead of as an one parameter fit.

# Propagating uncertainties: General strategy

- Our data has uncertainties.
- We view the data as random variables from the distribution $\mathcal{N}(d_i, \Sigma_{i,j})$
  - $d_i$ is the experimentally measured central value for the point $i$.
  - $\Sigma_{ij}$ *a* covariance between the points $i, j$.
- We sample $N_{\text{rep}}$ datasets from the distribution, and train a neural network “replica” to each dataset to minimize an error function $\chi^2$.
- PDF dependent quantities are calculated from statistics over the ensemble of replicas. E.g. “*PDF uncertainty*” is usually the standard deviation over the replicas.

# PDF error on $\alpha_s$

- Produce $N_{\text{rep}}$ datasets.
- Fit each one for a range of values in $\alpha_S$ (we repeat this two times and take the best for each point and apply some selection criteria).
- Fit resulting $\chi^2(\alpha_S)$ to a parabola.
- Compute the error over the ensemble of best fits.
- $\alpha_S$ determined on the same footing as the PDF.



# Preliminary results

- Note, still preliminary, *will* change.

NNLO

$$\alpha_S(M_Z^2) = 0.11903 \pm 0.00053(\text{pdf}) \pm 9 \times 10^{-5}(\text{stat})$$

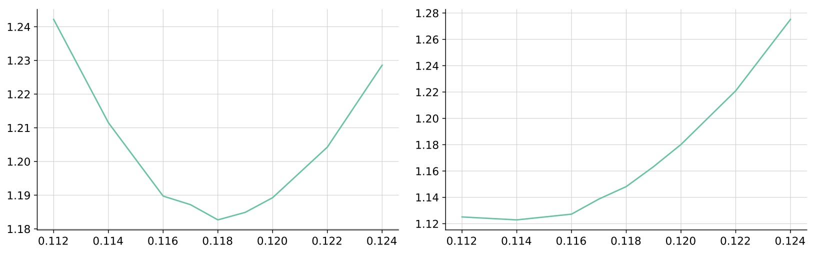
NLO

$$\alpha_S(M_Z^2) = 0.1214 \pm 0.0007(\text{pdf})$$

- Difference between NLO and NNLO sizable within uncertainties.
- Proton only fits in preparation.

# Minimization function

- If an experiment has *normalization* uncertainties:
  - E.g. $\Sigma_{i,j} = \Sigma_{i,j}^{(unnorm)} + t\Sigma_{i,j}^{(norm)}$, with $t$ the prediction for some normalization.
  - Usual$\chi^2$ minimization leads to smaller cross sections for affected datasets (d'Agostini, 1994).
- $t0$ procedure (NNPDF, 2009) is an effective solution. Essentially, fix the normalization with the result of a previous fit and iterate.
- Large effect when fitting $\alpha_S$.




(a) t0 Error function

(b) Experimental $\chi^2$

**Figure 2:** Normalized $\chi^2(\alpha_S)$

# Finite-size uncertainties

We estimate the uncertainties due to fitting a finite number of replicas by bootstrapping.

1. Take the set of $N$ minima.
2. Sample with replacement from it $M$ sets of $N$ values, with $M$ large.
3. Compute the $M$ means of each of the $M$ sets.
4. Compute the standard deviation of the $M$ means.

$$\Delta_{\text{stat}} = 9.5 \times 10^{-5}$$

Effect negligible compared to the PDF uncertainty.

# Why parabolas?

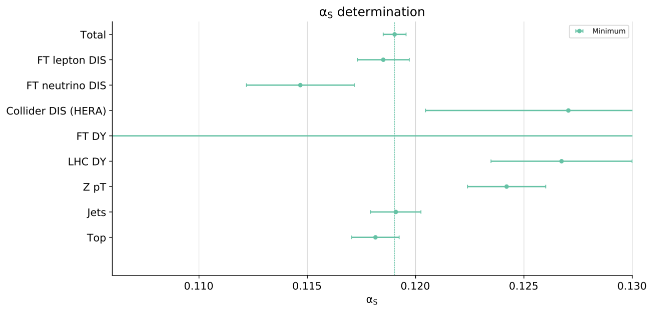
- Expect $\chi^2(\alpha_s)$ to Taylor-expand like a parabola around the minimum.
- Not obvious that the expansion is good in the whole range of $\alpha_s$ values (from 1.06 to 0.130).
- Computed "Akaike information criterion"

| | AIC score |
|---|---|
| Quadratic polynomial | $153 \pm 14$ |
| Cubic polynomial | $155 \pm 15$ |

- No evidence that a more complicated functional form is advantageous.
- Also tried fitting $\chi^2(\exp(\alpha_s))$, $\chi^2(\log(\alpha_S))$. Differences much smaller than uncertainties.

# $\alpha_S$ process by process

- Decompose the error function as $\chi^2(\alpha_S) = \sum_p \chi_p$, $\{p\}$ is a set of physical processes
- Define the *preferred* $\alpha_S$ *value for the process:* $\alpha_S^{(p)} = \min \chi_p^2(\alpha_s)$

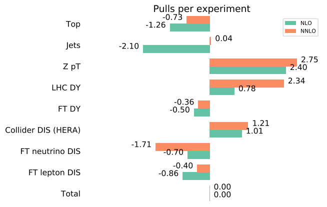



- Note, this is not equivalent to a refit including only that process.
- The values do depend on everything else (how hard is to fit).

# “Pulls”

Define

$$\text{pull} = \frac{\alpha_S^{(p)} - \alpha_S^{\text{min}}}{\sqrt{\Delta^2_{\alpha_S^{(p)}} + \Delta^2_{\alpha_S^{\text{min}}}}}$$




- LHC experiments prefer larger values.
- FT DIS prefer lower values, but not as much as expected from other determinations (this isn’t necessarily inconsistent).
- Outliers (Neutrino DIS, $Z\,p_T$) still have ~small pull.

# Theoretical uncertainties

- Still discussing these.
- Clearly, will be of the same order as the PDF uncertainties.
- Consider:
    - Difference between NLO and NNLO (e.g. Cacciari-Houdeau method).
    - Dispersion among preferred values.

# Revised combination

From $\tau$ decays and the global electroweak fit, we had:

$$\alpha_S(M_Z^2)^{(\tau+\mathrm{EW})} = 0.1193 \pm 0.0015$$

From the NNPDF fit, we have (assuming $\Delta_{\mathrm{th}}$=0.0005):

$$\alpha_S(M_Z^2)^{(\mathrm{NNPDF})} = 0.11903 \pm 0.0007$$

Combining them we have:

$$0.11908 \pm 0.0006$$

# THANK YOU!

Thank you!